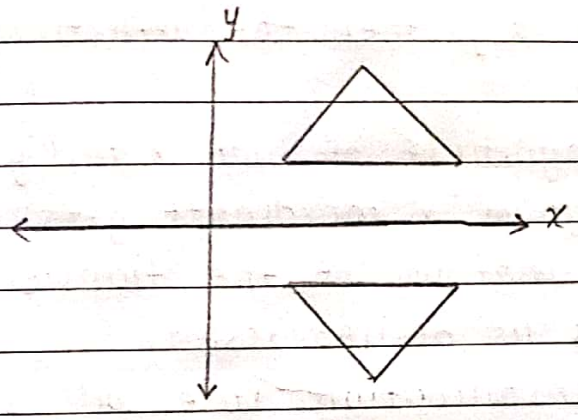
Assignment no - B4

Problem Statement - Write C++/Java program to implement reflection of 2-D object about x-axis, y-axis and about x=y axis. Also rotate object about arbitary point given by user

Objective - To study the reflection of 2D object about x axis, y axis and about x=y along with its rotation about arbitary point

Theory -

① Reflection about x-axis

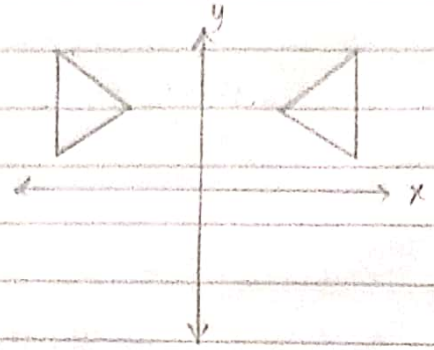


Multiplication matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection at x-axis is similar to placing a mirror at y-axis and taking the mirror image of an object so if the point is (x,y) then its reflection at x-axis will become as (x,-y). Here, after reflection the value of x coordinate remains same, but the y coordinate gets changed by sign only.

② Reflection about Y-axis



Reflection matrix

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = T$$

Reflection at y-axis will be very similar to reflection at x-axis. Here we have to change the rolls of x and y only. It means that we are assuming that we are keeping mirror at y-axis. So if the point is (x,y) then the reflection at y-axis will become (-x,y)

Here the magnitude of both x and y remains same only the sign of x coordinate gets changed. Now let us derive what will be the transformation matrix which gives us desired result.

Our basic transformation rule will be

$P_1 * T = P_2$

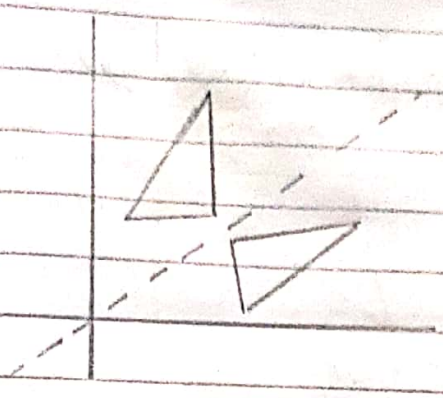
Now, $P_1 = [x,y]$ and we need P_2 as $[-x,y]$

$\therefore (x,y) * T = (-x,y)$

For Non-homogenous system

$$T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

③ Reflection about $y = x$



Reflection Matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here, we are not going to take reflection at any standard axis or in the origin, but it is at line $y = x$ we are drawing a line whose x and y values are same i.e the line is exactly at 45° and passes through the origin.

The dashed line shows a line $y = x$. Now if we want to take reflection of a point (x, y) with respect to this line, then the point (x, y) will become (y, x) It means as if we are placing mirror at line $y = x$. In this case the values of x and y get interchanged

$$P_1 * T = P_2$$

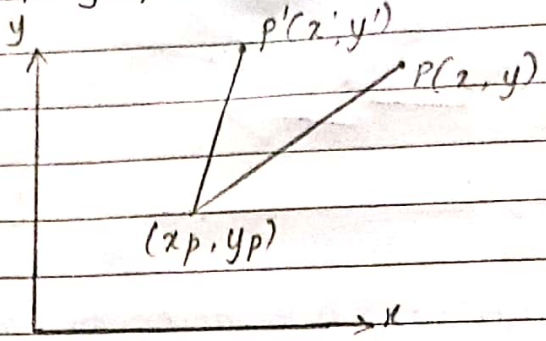
$$\therefore (x, y) * T = (y, x)$$

∴ For non-homogenous coordinate system

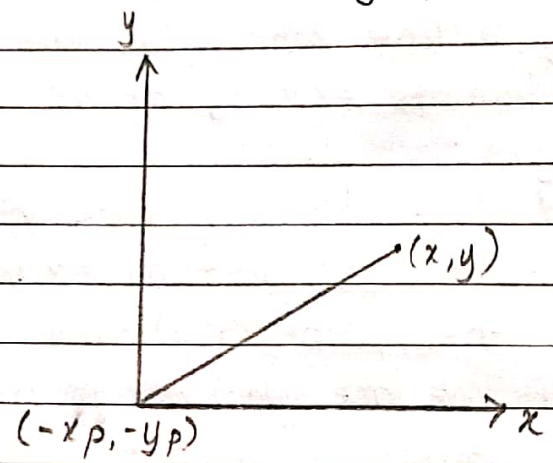
$$T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

④ Rotation about arbitary point

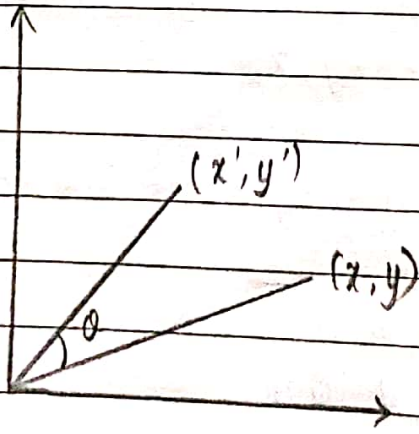
Consider Arbitary point is (x_p, y_p)



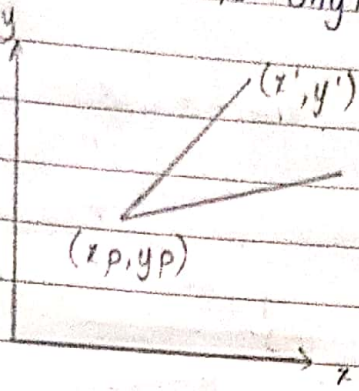
Step 1 - Translate arbitary point to origin



Step 2 :- Rotate at origin



Step 3- Translate back to original position



$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_p & -y_p & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{bmatrix}$$

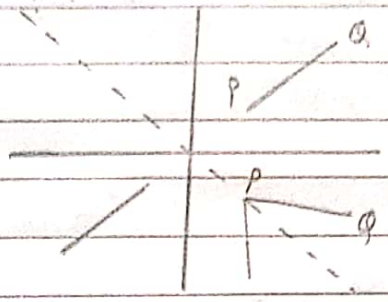
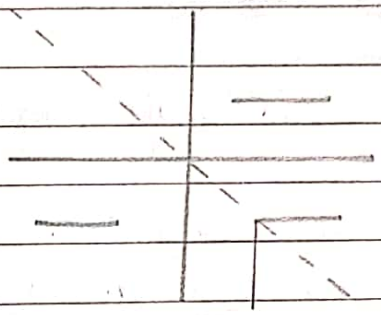
$$T_1.R.T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_p & -y_p & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -x_p\cos\theta - y_p\sin\theta & -x_p\sin\theta - y_p\cos\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{bmatrix}$$

$\therefore$ composite transformation =

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -x_p\cos\theta - y_p\cos\theta + x_p & -x_p\sin\theta - y_p\cos\theta + y_p & 1 \end{bmatrix}$$

Testcases

Testcase	Expected o/p	Actual o/p	result
① Reflection about x, y x=y axis and rotation about arbitary		same as expected	pass
② Reflection about x, y x=y axis and rotation about arbitary		same as expected	pass