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# 1. Implement functions for encoding and decoding an image using the following methods:

A. Transform Coding (using DCT for forward transform)

B. Huffman Encoding

C. LZW Encoding

D. Run-Length Encoding

E. Arithmetic Coding

## 1.Transform Coding (Using DCT)

The Discrete Cosine Transform (DCT) is commonly used in image compression to convert the image data from the spatial domain to the frequency domain, making it easier to reduce redundancies.

```
In [2]: 1 pip install numpy scipy Pillow
```

```
Requirement already satisfied: numpy in c:\users\barot nandni\anaconda3\lib\site-packages (1.26.4)  
Requirement already satisfied: scipy in c:\users\barot nandni\anaconda3\lib\site-packages (1.13.0)  
Requirement already satisfied: Pillow in c:\users\barot nandni\anaconda3\lib\site-packages (8.4.0)  
Note: you may need to restart the kernel to use updated packages.
```

```
[notice] A new release of pip is available: 24.2 -> 24.3.1  
[notice] To update, run: python.exe -m pip install --upgrade pip
```

```
In [7]: 1 import numpy as np  
2 from scipy.fftpack import dct, idct  
3 from PIL import Image  
4 import matplotlib.pyplot as plt
```

```
In [8]: 1 # Function to perform DCT encoding on an image  
2 def dct_encode(image_array):  
3     # Apply 2D DCT to the entire image  
4     dct_image = dct(dct(image_array.T, norm='ortho').T, norm='ortho')  
5     return dct_image  
6  
7 # Function to perform inverse DCT decoding to reconstruct the image  
8 def dct_decode(dct_image):  
9     # Apply Inverse 2D DCT to reconstruct the image  
10    return idct(idct(dct_image.T, norm='ortho').T, norm='ortho')
```

```
In [9]: 1 def calculate_compression_ratio(original, compressed):
2       original_size = original.size * 8 # size in bits (assuming 8 bits per pixel)
3       compressed_size = np.count_nonzero(compressed) * 8 # size in bits
4       return original_size / compressed_size
5
6 def calculate_rmse(original, reconstructed):
7     return np.sqrt(np.mean((original - reconstructed) ** 2))
8
```

```
In [10]: 1 # Load the image and convert to a grayscale numpy array
2 image = Image.open('Image_compres1.jpg').convert('L') # 'L' converts the image to grayscale
3 image_array = np.array(image)
4
5 # Perform DCT encoding and decoding
6 dct_encoded = dct_encode(image_array)
7 dct_reconstructed = dct_decode(dct_encoded)
8
9 # Clip values to be in the 0-255 range and convert to uint8
10 reconstructed_image_array = np.uint8(np.clip(dct_reconstructed, 0, 255))
```

```
In [11]: 1 # Calculate Compression Ratio
2 compression_ratio = calculate_compression_ratio(image_array, dct_encoded)
3 print(f"Compression Ratio: {compression_ratio:.2f}")
4
5 # Calculate RMSE
6 rmse = calculate_rmse(image_array, reconstructed_image_array)
7 print(f"Root Mean Square Error (RMSE): {rmse:.2f}")
8
```

Compression Ratio: 1.00  
Root Mean Square Error (RMSE): 0.54

```
In [12]: 1 plt.figure(figsize=(10, 5))
2
3 # Display the original image
4 plt.subplot(1, 2, 1)
5 plt.imshow(image_array, cmap='gray')
6 plt.title("Original Image")
7 plt.axis('off')
8
9 # Display the reconstructed image
10 plt.subplot(1, 2, 2)
11 plt.imshow(reconstructed_image_array, cmap='gray')
12 plt.title("Reconstructed Image")
13 plt.axis('off')
14
15 plt.show()
16
```

Original Image



Reconstructed Image



## 2. Huffman Encoding for Image Compression

Huffman Coding is a lossless compression technique that assigns variable-length codes to input characters, with shorter codes assigned to more frequently occurring symbols. This method efficiently reduces the file size by minimizing the average code length compared to fixed-length encoding schemes.

```
In [13]: 1 import numpy as np
2 from PIL import Image
3 import matplotlib.pyplot as plt
4 import heapq
5 from collections import defaultdict
```

```
In [14]: 1 class HuffmanNode:
2     def __init__(self, symbol, freq):
3         self.symbol = symbol
4         self.freq = freq
5         self.left = None
6         self.right = None
7
8     def __lt__(self, other):
9         return self.freq < other.freq
```

```
In [15]: 1 def build_huffman_tree(frequencies):
2         heap = [HuffmanNode(symbol, freq) for symbol, freq in frequencies.items()]
3         heapq.heapify(heap)
4         while len(heap) > 1:
5             node1 = heapq.heappop(heap)
6             node2 = heapq.heappop(heap)
7             merged = HuffmanNode(None, node1.freq + node2.freq)
8             merged.left = node1
9             merged.right = node2
10            heapq.heappush(heap, merged)
11        return heap[0]
12
```

```
In [16]: 1 def generate_codes(node, code="", codebook=None):
2         if codebook is None:
3             codebook = {}
4         if node is not None:
5             if node.symbol is not None:
6                 codebook[node.symbol] = code
7             generate_codes(node.left, code + "0", codebook)
8             generate_codes(node.right, code + "1", codebook)
9         return codebook
```

```
In [17]: 1 # Function to perform Huffman Encoding
2 def huffman_encode(image_array):
3     # Calculate symbol frequencies
4     frequencies = defaultdict(int)
5     for value in image_array.flatten():
6         frequencies[value] += 1
7
8     # Build Huffman Tree and generate codes
9     huffman_tree = build_huffman_tree(frequencies)
10    huffman_codes = generate_codes(huffman_tree)
11
12    # Encode the image
13    encoded_image = "".join(huffman_codes[value] for value in image_array.flatten())
14    return encoded_image, huffman_codes, len(image_array.flatten()) * 8
15
```

```
In [18]: 1 # Function to decode Huffman Encoded image
2 def huffman_decode(encoded_image, huffman_codes, original_shape):
3     reverse_codes = {v: k for k, v in huffman_codes.items()}
4     current_code = ""
5     decoded_values = []
6
7     for bit in encoded_image:
8         current_code += bit
9         if current_code in reverse_codes:
10            decoded_values.append(reverse_codes[current_code])
11            current_code = ""
12
13    return np.array(decoded_values).reshape(original_shape)
```

```
In [19]: 1 # Function to calculate Compression Ratio
2 def calculate_compression_ratio_huffman(original_bits, encoded_bits):
3     return original_bits / len(encoded_bits)
4
```

```
In [20]: 1 # Load the image and convert to a grayscale numpy array
2 image = Image.open('Image_compres1.jpg').convert('L')
3 image_array = np.array(image)
4
5 # Perform Huffman Encoding and Decoding
6 encoded_image, huffman_codes, original_bits = huffman_encode(image_array)
7 decoded_image_array = huffman_decode(encoded_image, huffman_codes, image_array.shape)
8
```

```
In [21]: 1 # Calculate Compression Ratio
2 compression_ratio = calculate_compression_ratio_huffman(original_bits, encoded_image)
3 print(f"Compression Ratio: {compression_ratio:.2f}")
4
5 # Calculate RMSE
6 rmse = np.sqrt(np.mean((image_array - decoded_image_array) ** 2))
7 print(f"Root Mean Square Error (RMSE): {rmse:.2f}")
8
```

Compression Ratio: 1.06  
Root Mean Square Error (RMSE): 0.00

```
In [22]: 1 plt.figure(figsize=(10, 5))
2
3 # Display the original image
4 plt.subplot(1, 2, 1)
5 plt.imshow(image_array, cmap='gray')
6 plt.title("Original Image")
7 plt.axis('off')
8
9 # Display the decoded image
10 plt.subplot(1, 2, 2)
11 plt.imshow(decoded_image_array, cmap='gray')
12 plt.title("Decoded Image (Huffman)")
13 plt.axis('off')
14
15 plt.show()
```

Original Image



Decoded Image (Huffman)



### 3. LZW Encoding for Image Compression

LZW (Lempel-Ziv-Welch) Encoding is a lossless compression technique that builds a dictionary of input patterns and replaces sequences of data with shorter codes. It's efficient for images with repetitive patterns or areas of uniform color and is commonly used in formats like GIF.

```
In [23]: 1 import numpy as np
2 from PIL import Image
3 import matplotlib.pyplot as plt
```

```
In [29]: 1 def lzw_encode(image_array):
2     data = image_array.flatten()
3     dictionary = {tuple([i]): i for i in range(256)} # Initialize dictionary with single pixel values
4     current_sequence = []
5     encoded_data = []
6     code = 256 # Next available code for new sequences
7
8     for symbol in data:
9         current_sequence.append(symbol)
10        if tuple(current_sequence) not in dictionary:
11            dictionary[tuple(current_sequence)] = code
12            encoded_data.append(dictionary[tuple(current_sequence[:-1])])
13            current_sequence = [symbol] # Start new sequence
14            code += 1
15
16    # Encode the last sequence
17    if current_sequence:
18        encoded_data.append(dictionary[tuple(current_sequence)])
19
20    original_bits = len(data) * 8 # Assuming 8 bits per pixel
21    return encoded_data, dictionary, original_bits
22
```

```
In [30]: 1 def lzw_decode(encoded_data, dictionary):
2     reverse_dictionary = {v: k for k, v in dictionary.items()}
3     current_sequence = list(reverse_dictionary[encoded_data[0]])
4     decoded_data = current_sequence.copy()
5
6     for code in encoded_data[1:]:
7         if code in reverse_dictionary:
8             entry = list(reverse_dictionary[code])
9         elif code == len(reverse_dictionary):
10            entry = current_sequence + [current_sequence[0]]
11        else:
12            raise ValueError("Invalid LZW code encountered")
13
14        decoded_data.extend(entry)
15        current_sequence.append(entry[0])
16        reverse_dictionary[len(reverse_dictionary)] = current_sequence
17        current_sequence = entry
18
19    return np.array(decoded_data)
```

```
In [31]: 1 image = Image.open('Image_compres1.jpg').convert('L')
2 image_array = np.array(image)
3
4 # Perform LZW Encoding and Decoding
5 encoded_data, dictionary, original_bits = lzw_encode(image_array)
6 decoded_image_array = lzw_decode(encoded_data, dictionary).reshape(image_array.shape)
7
8
```

```
In [32]: 1 # Calculate Compression Ratio
2 compressed_size = len(encoded_data) * 16 # Assuming 16 bits per encoded symbol
3 compression_ratio = original_bits / compressed_size
4 print(f"Compression Ratio: {compression_ratio:.2f}")
5
6 # Calculate RMSE
7 rmse = np.sqrt(np.mean((image_array - decoded_image_array) ** 2))
8 print(f"Root Mean Square Error (RMSE): {rmse:.2f}")
```

Compression Ratio: 1.38  
Root Mean Square Error (RMSE): 0.00

```
In [33]: 1 plt.figure(figsize=(10, 5))
2
3 # Display the original image
4 plt.subplot(1, 2, 1)
5 plt.imshow(image_array, cmap='gray')
6 plt.title("Original Image")
7 plt.axis('off')
8
9 # Display the decoded image
10 plt.subplot(1, 2, 2)
11 plt.imshow(decoded_image_array, cmap='gray')
12 plt.title("Decoded Image (LZW)")
13 plt.axis('off')
14
15 plt.show()
16
```

Original Image



Decoded Image (LZW)



#### 4. Run-Length Encoding (RLE) for Image Compression

Run-Length Encoding (RLE) is a simple lossless compression technique used to encode sequences of repeated pixel values. It is particularly effective for images that have large areas of uniform color, such as icons or binary images. RLE works by storing the value of a pixel and the number of times it repeats consecutively, which reduces the amount of data needed to represent the image.



```
In [34]: 1 import numpy as np
2 from PIL import Image
3 import matplotlib.pyplot as plt
```

```
In [35]: 1 def rle_encode(image_array):
2     data = image_array.flatten()
3     encoded_data = []
4     count = 1
5
6     for i in range(1, len(data)):
7         if data[i] == data[i - 1]:
8             count += 1
9         else:
10            encoded_data.append((data[i - 1], count))
11            count = 1
12
13     # Append the last run
14     encoded_data.append((data[-1], count))
15
16     original_bits = len(data) * 8 # Assuming 8 bits per pixel
17     return encoded_data, original_bits
```

```
In [36]: 1 def rle_decode(encoded_data, shape):
2     decoded_data = []
3     for value, count in encoded_data:
4         decoded_data.extend([value] * count)
5     return np.array(decoded_data).reshape(shape)
```

```
In [37]: 1 def calculate_compression_ratio_rle(original_bits, encoded_data):
2     compressed_size = len(encoded_data) * (8 + 8) # 8 bits for value and 8 bits for count
3     return original_bits / compressed_size
```

```
In [38]: 1 image = Image.open('Image_compres1.jpg').convert('L')
2 image_array = np.array(image)
3
4 # Perform Run-Length Encoding and Decoding
5 encoded_data, original_bits = rle_encode(image_array)
6 decoded_image_array = rle_decode(encoded_data, image_array.shape)
```

```
In [39]: 1 compression_ratio = calculate_compression_ratio_rle(original_bits, encoded_data)
2 print(f"Compression Ratio: {compression_ratio:.2f}")
3
4
5 rmse = np.sqrt(np.mean((image_array - decoded_image_array) ** 2))
6 print(f"Root Mean Square Error (RMSE): {rmse:.2f}")
```

Compression Ratio: 0.54  
Root Mean Square Error (RMSE): 0.00



```
In [40]: 1 plt.figure(figsize=(10, 5))
2
3 # Display the original image
4 plt.subplot(1, 2, 1)
5 plt.imshow(image_array, cmap='gray')
6 plt.title("Original Image")
7 plt.axis('off')
8
9 # Display the decoded image
10 plt.subplot(1, 2, 2)
11 plt.imshow(decoded_image_array, cmap='gray')
12 plt.title("Decoded Image (RLE)")
13 plt.axis('off')
14
15 plt.show()
16
```

Original Image



Decoded Image (RLE)



## 5. Arithmetic Coding for Image Compression

Arithmetic Coding is a more advanced lossless compression technique compared to Huffman and LZW encoding. Instead of using distinct codes for each symbol, Arithmetic Coding represents the entire message as a single floating-point number in the range  $[0, 1)$ . It is particularly efficient when dealing with data with skewed probabilities.

```
In [1]: 1 import numpy as np
2 from PIL import Image
3 import matplotlib.pyplot as plt
```

```
In [2]: 1 from collections import defaultdict
2
3 def calculate_frequencies(data):
4     """Calculate frequency of each symbol in the data."""
5     frequency = defaultdict(int)
6     for symbol in data:
7         frequency[symbol] += 1
8     return frequency
9
10 def cumulative_frequency(freq):
11     """Calculate cumulative frequency from symbol frequencies."""
12     total = sum(freq.values())
13     cum_freq = {}
14     cum_sum = 0
15     for symbol in sorted(freq.keys()):
16         cum_sum += freq[symbol]
17         cum_freq[symbol] = cum_sum / total
18     return cum_freq
19
20 def arithmetic_encode(data, cumulative_freq):
21     """Encode data using arithmetic coding."""
22     low = 0.0
23     high = 1.0
24
25     for symbol in data:
26         range_ = high - low
27         high = low + range_ * cumulative_freq[symbol]
28         low = low + range_ * (cumulative_freq[symbol] - (1 / len(cumulative_freq)))
29
30     return (low + high) / 2 # Return the final code
31
32 def arithmetic_decode(code, data_length, cumulative_freq):
33     """Decode a given code using arithmetic coding."""
34     low = 0.0
35     high = 1.0
36     decoded_data = []
37
38     # Reverse the cumulative frequencies for decoding
39     reverse_cum_freq = {v: k for k, v in cumulative_freq.items()}
40
41     for _ in range(data_length):
42         range_ = high - low
43         value = (code - low) / range_
44
45         for symbol, cum_freq in cumulative_freq.items():
46             if value < cum_freq:
47                 decoded_data.append(symbol)
48                 high = low + range_ * cum_freq
49                 low = low + range_ * (cumulative_freq[reverse_cum_freq[cum_freq]] - (1 / len(cumulative_freq)))
50                 break
51
52     return decoded_data
53
54
```

```
In [3]: 1 # Example usage
2 if __name__ == "__main__":
3     data = "ABABAC" # Input string
4     freq = calculate_frequencies(data)
5     cum_freq = cumulative_frequency(freq)
6
7     encoded_value = arithmetic_encode(data, cum_freq)
8     print("Encoded Value:", encoded_value)
9
10    # Decode
11    decoded_data = arithmetic_decode(encoded_value, len(data), cum_freq)
12    print("Decoded Data:", ''.join(decoded_data))
```

Encoded Value: 0.3758573388203018

Decoded Data: ABABAB

Here i got the output as manual coding but not with the image so i perform both.

```
In [1]: 1 import numpy as np
2 from PIL import Image
3 from collections import defaultdict
4 import math
5
6 def read_image(image_name):
7     """Read the image from the given name (assumes it's in the same directory)."""
8     image = Image.open(image_name).convert('L') # Convert to grayscale
9     return np.array(image)
10
11 def calculate_frequencies(image):
12     """Calculate the frequency of each pixel value."""
13     freq = defaultdict(int)
14     for pixel in image.flatten():
15         freq[pixel] += 1
16     total_pixels = image.size
17     probabilities = {k: v / total_pixels for k, v in freq.items()}
18     return probabilities
19
20 def arithmetic_encode(image, probabilities):
21     """Encode the image using Arithmetic Coding."""
22     low = 0.0
23     high = 1.0
24     for pixel in image.flatten():
25         range = high - low
26         cumulative_prob = 0.0
27
28         for value in sorted(probabilities.keys()):
29             cumulative_prob += probabilities[value]
30             if pixel == value:
31                 high = low + range * cumulative_prob
32                 break
33             low += range * (cumulative_prob - probabilities[value])
34
35     return (low + high) / 2
36
37
38
```

```
In [2]: 1 def arithmetic_decode(encoded_value, probabilities, image_shape):
2         """Decode the encoded value back to an image."""
3         image = np.zeros(image_shape, dtype=np.uint8)
4         low = 0.0
5         high = 1.0
6
7         for i in range(image_shape[0] * image_shape[1]):
8             range = high - low
9             value = (encoded_value - low) / range
10
11             cumulative_prob = 0.0
12             for pixel in sorted(probabilities.keys()):
13                 cumulative_prob += probabilities[pixel]
14                 if value < cumulative_prob:
15                     image[i // image_shape[1], i % image_shape[1]] = pixel
16                     high = low + range * cumulative_prob
17                     low = low + range * (cumulative_prob - probabilities[pixel])
18                     break
19
20         return image
21
22
```

```
In [3]: 1 def calculate_rmse(original, reconstructed):
2         """Calculate the Root Mean Square Error (RMSE)."""
3         mse = np.mean((original - reconstructed) ** 2)
4         return math.sqrt(mse)
5
6 def calculate_compression_ratio(original_size, compressed_size):
7     """Calculate the Compression Ratio."""
8     return original_size / compressed_size
9
10
```

```
In [4]: 1 # Example Usage
2 image_name = 'Image_Compression.jpg' # Replace with your image filename
3 image = read_image(image_name)
4
5
```

```
In [5]: 1 # Step 1: Calculate frequencies
        2 probabilities = calculate_frequencies(image)
        3
        4 # Step 2: Encode the image
        5 encoded_value = arithmetic_encode(image, probabilities)
        6
        7 # Step 3: Decode the image
        8 reconstructed_image = arithmetic_decode(encoded_value, image.shape)
        9
        10
```

```
-----
TypeError                                Traceback (most recent call last)
C:\Users\BAROTN~1\AppData\Local\Temp\ipykernel_19060\1362716680.py in <module>
      6
      7 # Step 3: Decode the image
----> 8 reconstructed_image = arithmetic_decode(encoded_value, image.shape)
      9

TypeError: arithmetic_decode() missing 1 required positional argument: 'image_shape'
```

```
In [ ]: 1 original_size = image.size # Number of pixels
        2 compressed_size = 8 # Size in bytes for float representation
        3 compression_ratio = calculate_compression_ratio(original_size, compressed_size)
        4 rmse = calculate_rmse(image, reconstructed_image)
        5
        6 print(f"Compression Ratio: {compression_ratio}")
        7 print(f"RMSE: {rmse}")
```

```
In [ ]: 1
```