Permutations.

The different arrangements which can be made out of a given set of things, by taking some or all of them at a time is called Permutations. The number of permutations of n different things taken $r (\leq n)$ at a time is denoted by $P(n, r)$ or nP_r.

$$nP_r = \frac{n!}{(n-r)!}$$

1. How many different strings of length 4 can be formed by using the letters of word flower.

Soln:- $6P_4 = \frac{6!}{(6-4)!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 360$

2. find the number of permutations of the letters of the word success.

Total − 9 letters
(S − repeated − 3 times
C − two times

Soln:- $\frac{7!}{(3!)(2!)} = 420$

3. How many 9 letter word can be formed by using the letters of the word difficult.

(Total − 9 letters.
f − 2 (repeated), i − 2 (repea)

Soln:- $\frac{9!}{2!\,2!}$

4. find the number of permutations of letters of word MASSASAUGA. In how many of these

all four A's are together? how many of them begin with S.

Soln:- A – 4 times, S – 3 times repeated.

Total 10 letters.

∴ Total number of permutations are $\frac{10!}{4!\,3!}$

all A's together. AAAA SSSMUG.

no of permutations are $\frac{7!}{3!} = 840$

begin with S   S _ _ _ _ _ _ _ _ _

no of permutations are $\underbrace{\frac{9!}{4!\,2!}}_{4! \to A's,\ 2! \to S'} = \frac{9 \times 8 \times 7 \times \cancel{6}^{3} \times 5 \times \cancel{4 \times 3 \times 2}}{\cancel{4 \times 3 \times 2} \times 1 \times \cancel{2} \times 1}$

$= 7560$.

5) How many positive integers 'n' can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000?

Soln:- to exceed 5 lakhs, first digit must be 5 or 6 or 7.   _ _ _ _ _ _ _

5 as first digit — $1 \times \frac{6!}{2!} = 360$ (since 4's repeated twice; remain 6 in 6! way)

(or)

6 as first digit — $1 \times \frac{6!}{2!\,2!} = 180$ (4 and 5 are repeated twice.

(or)

7 as first digit — $1 \times \frac{6!}{2!\,2!} = 180$.

∴ number of Positive integers n formed are

$360 + 180 + 180 = \underline{\underline{720}}$

6. In how many ways can n persons be seated at a round table if arrangements are considered the same when one can be obtained from the other by rotation? (12)

Soln:- If one person is seated anywhere, the remaining n-1 persons can be seated in $(n-1)!$ ways. This is the total number of ways of arranging n persons in a circle.

7. It is required to seat 5 men and 4 women in a row so that women occupy odd places. How many such arrangements are possible.

Soln:- $5! \times 4!$

_ M M M _ M _ M _
w w w w w

5 women can be seated in 5! ways odd places and 4 men can be operated in 4! ways

∴ no of arrangements are $5! \times 4!$

8. In how many ways can 6 men and 6 women be seated in a row 1) if any person may sit next to any other. 2) If men and women occupy alternate seats.

Soln:- 1) any person may sit next to any other person (neither women, nor men). ∴ There are 12 member 12! ways.

2) men occupy odd women occupy rest of the

(or) men occupy odd, w - even.

$(6! \times 6!) + (6! \times 6!)$

$518400 + 518400 = 1,036800$

9. four different mathematics books, five different computer science books and two different control theory books are to be arranged in a shelf. How many different arrangements are possible if (13) pg-350

a) the books in each particular subject (book) must all be together.

b) only the mathematics books must be together.

Soln:- a) $\boxed{4M}$, $\boxed{5CS}$ $\boxed{2CT}$ — 3! ways.

4M — among themselves in 4! ways; 5CS — 5! ; 2CT — 2!

∴ Total no of different arrangements are

$3!\,4!\,5!\,2! = 34,560$.

b) (4M) 5CS, 2CT $= 8! \times 4!$ (4 mathematics among themselves).

$1+5+2$

10. Find the number of positive integers that can be formed from the digits 1, 2, 3, 4 if no digit is repeated in any one integer.

Soln:- — — — — ; $\underset{4}{—}\ \underset{3}{—}\ \underset{2}{—}$; $\underset{4w}{—}\ \underset{3w}{—}$; $\underset{4way}{—}$

one digit no's — 4

two digit no's — 12 ; 3 digit — 24 ; 4 digit — 4!

∴ no of positive integers that can be formed from the digits 1, 2, 3, 4 are $4+12+24+24=64$

11. How many 8-digit telephone numbers have one or more digits repeated?

Soln:- Total no of 8 digit telephone numbers are

10^8 (— — — — — — — —, 10 way — (0, 1 — 9))

The number of digits in which

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The number of digits in which.

repetitions are not allowed is $10P_8$

$\therefore$ require number is $10^8 - 10P_8$

12. find the value of n so that $2P(n,2) + 50 = P(2n,2)$

Soln:- $2P(n,2) + 50 = P(2n,2)$

$$2\,{}^nP_2 + 50 = {}^{2n}P_2$$

$$\frac{2\,n!}{(n-2)!} + 50 = \frac{(2n)!}{(2n-2)!}$$

$$\frac{2\times(n)(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} + 50 = \frac{(2n)(2n-1)\cancel{(2n-2)!}}{\cancel{(2n-2)!}}$$

$$2n^2 - \cancel{2n} + 50 = 4n^2 - \cancel{2n}$$

$$2n^2 = 50$$

$$n^2 = 25$$

$$\boxed{n = 5}$$ ($\because n$ cannot be negative)

13) Prove that for all integers n, $r \geq 0$, if $n+1 > r$ then $P(n+1, r) = \frac{(n+1)}{(n+1-r)} P(n,r)$.

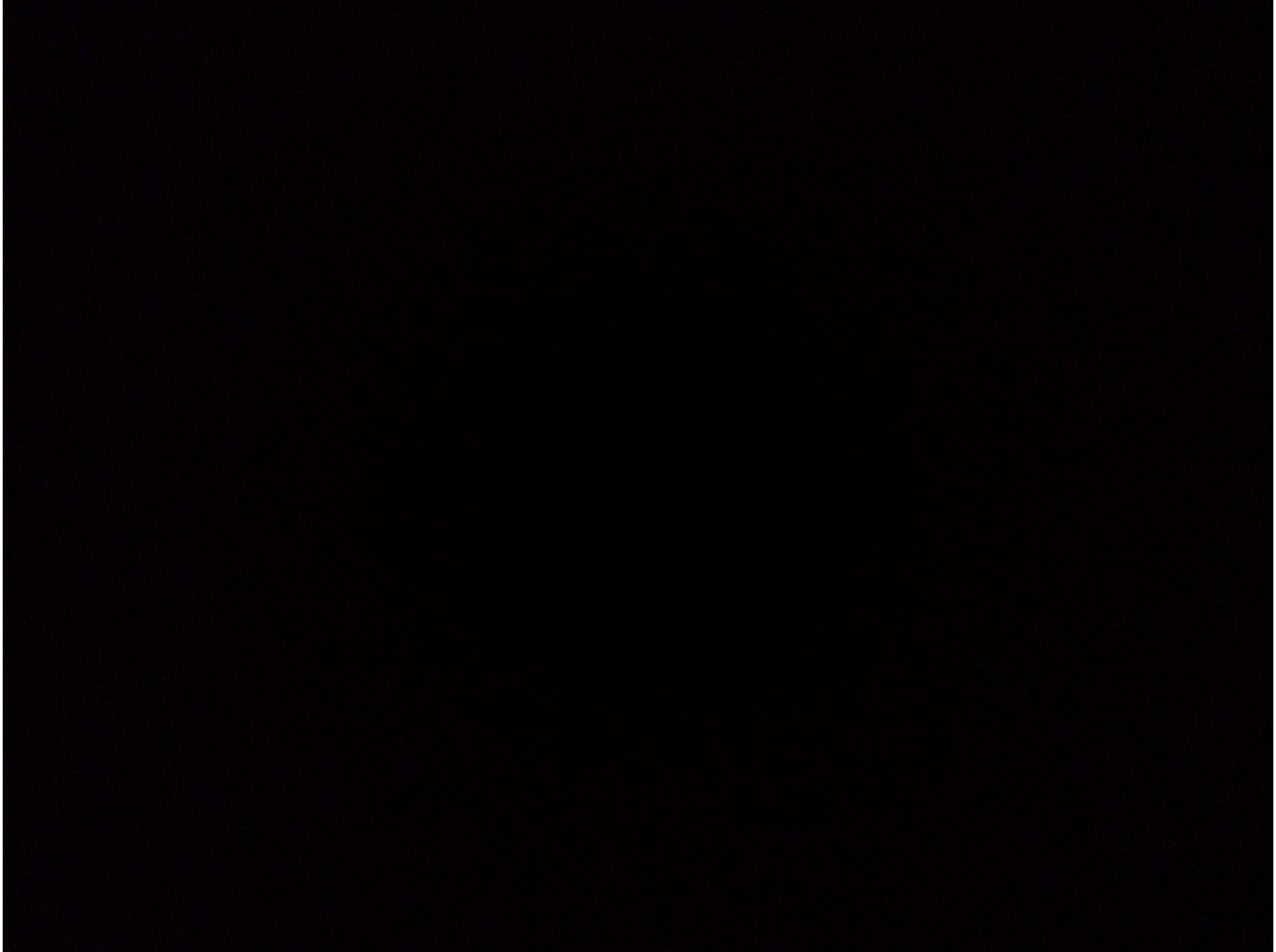
Soln:- $$\frac{(n+1)!}{(n+1-r)!} = \frac{(n+1)}{(n+1-r)} \frac{n!}{(n-r)!} = \frac{(n+1)!}{(n-r+1)!}$$

14) If k is a positive integer and $n = 2k$; P.T $\frac{n!}{2^k}$ is a Z^+.

Soln:- Consider the symbols $x_1, x_2 \ldots x_k$, 2 in each number in which each of $x_1, x_2 \ldots x_k$ is 2 in number. Evidently, the number of these symbols is $2k$.

$\therefore$ no of Permutations of these $2k$ symbol is

$$\frac{(2k)!}{\underbrace{2!\,2!\ldots 2!}_{k\text{-factors}}} = \frac{(2k)!}{2^k} = \frac{n!}{2^k}$$ is a positive integer.



exercise:-

1. find the value of n in each of the following cases:-

1) $P(n,2) = 30 = nPr$

$$\frac{n!}{(n-r)!} = 30$$

$$n(n-1) = 6 \times 5$$

$$\therefore n = 6$$

2) $P(n,3) = 3P(n,2)$

$$\frac{\cancel{n!}}{(n-3)!} = 3\frac{\cancel{n!}}{(n-2)!}$$

$$\frac{1}{(n-3)!} = \frac{3}{(n-2)!}$$

$$(n-2)! = 2(n-3)(n-2)$$

3) $P(n,4) = 42P(n,2)$

$$\frac{\cancel{n!}}{(n-4)!} = 42\frac{\cancel{n!}}{(n-2)!}$$

$$\frac{1}{(n-4)!} = \frac{42}{(n-2)!}$$

$$\frac{1}{(n-4)!} = \frac{42}{(n-4)!(n-3)(n-2)}$$

$$(n-3)(n-2) = 7 \times 6$$

$$n = 7.$$

2) Ten different paintings are to be put in n rooms so that no room gets more than one painting find the number of ways of accomplishing this : if 1) $n=12$ 2) $n=5$.

Soln: 1) $P(12, 10)$

2) $P(10, 5)$.

3) find the number of distinguishable permutations of letters in the following words:

no of words

1) BASIC — 5 — $5!$

2) PASCAL — 6 — A - repeated twice — $\frac{6!}{2!}$ — An twi

$= \frac{6 \times 5 \times 4 \times 3 \times \cancel{2} \times 1}{\cancel{2}} = 360$.

3) BANANA $= \frac{6!}{3!\,2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2} = 60$

4) PEPPER $= \frac{6!}{3!\,2!} = 60$

5) CALCULUS $= \frac{8!}{2!\,2!\,2!} = \frac{\cancel{8} \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{\cancel{2} \times \cancel{2} \times \cancel{2}}$

6) DISCRETE $= \frac{8!}{2!}$

7) STRUCTURES $= \frac{10!}{\underbrace{2!}_{T}\,\underbrace{2!}_{R}\,\underbrace{2!}_{S}\,\underbrace{2!}_{U}}$

8) ENGINEERING $= \frac{11!}{3!\,3!\,2!\,2!}$

9) MATHEMATICS $= \frac{11!}{2!\,2!\,2!}$

4) How many different arrangements of letters (17).
in the word Bought can be formed if the vowels must be kept next to each other.

Soln:- Vowels are OU

other letters (consonants) are BGHT — 4

$2! 5! = 2 \times 5 \times 4 \times 3 \times 2 \times 1 = 240$.

$\boxed{OU}$ BGHT — 5 term

$\downarrow 2!$

5) find the number of permutations of all letters of the word Baseball if the words are to begin and end with vowel.

Soln:- 6 places in 6! ways

Vowels in word Baseball are a, e, a

ae, ea, a,a

begin and end can be filled in 3 ways

$\therefore$ no of permutations are $\frac{6! \times 3}{2! \times 2!} = 540$.

6. find the number of permutations of the letters of the word MISSISSIPPI. How many of those begin with an I. How many of these begin and end with S.

Soln:- 1) no of permutations are $\frac{11!}{4!\,4!\,2!}$

(11-words, S-4, I-4; P-2)

$= \frac{11 \times 10 \times 9 \times \cancel{8} \times 7 \times \cancel{6} \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{\cancel{4 \times 3 \times 2 \times 1} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{2}} = 34650$.

2) Begin with I — first place can be filled in 4 ways, remaining 10 in 10! ways, (4-I, 4-S, 2-P)

$\therefore$ no of permutations $\frac{4 \times 10!}{4! \times 4! \times 2} = 12600$.

(68)

3) begin and end with S.

$\underline{S}$ _ _ _ _ _ _ _ _ $\underline{S}$. $\dfrac{9!\times 2!}{2!\times 4!\times 2!}$ → twos

$= \dfrac{9\times \cancel{8}^{4}\times 7\times 6\times 5}{2} = \underline{7560}$ ↳ 2P's ↳ 4S ↳ 2's rep

7. How many different signals, each consisting of six flags hung in a vertical line, can be formed from four identical red flags and two identical blue flags.

Soln:- 4 Red flags, 2 blue flags — Total 6 of flags

$= \dfrac{6!}{4!\,2!} = \dfrac{\cancel{6}^{3}\times 5\times \cancel{4\times 3\times 2\times 1}}{\cancel{4\times 3\times 2\times 1}\times \cancel{2}} = \underline{\underline{15}}.$

8. In how many ways can the symbols a, b, c, d, e, e, e, e, e be arranged so that no e is adjacent to another 'e'.

Soln:- e $\underset{4\,way}{\_}$ e $\underset{3\,wa}{\_}$ e $\underset{2}{\_}$ e $\underset{1}{\_}$ e $= 4\times 3\times 2\times 1$ ways

$= 4! = \underline{\underline{24}}$

9. Consider the permutations of the letters a, c, f, g, i, t, w, x. How many of these start with t? How many start with t and end with c.

Soln:- $\underline{t}$ $\underset{7\,ways}{\_}$ _ _ _ _ _ _ $= 7! = 5040$

$\underline{t}$ _ _ _ _ _ _ $\underline{c}$ $= 6! = 720.$

10. five Red pens, two black pens and three blue pens are arranged in a row. If the pens of the same colour are not distinguishable, how many different arrangements are possible. | 5R, 2Bl, 3 Blue.

Soln:- Total arrangements are $\dfrac{10!}{5!\,2!\,3!} = \dfrac{\cancel{10}^{4}\times 9\times 8\times 7\times \cancel{6}}{\cancel{5}\times \cancel{2}\times \cancel{6}}$

$= 2520.$

(19)

11. In how many ways can seven books be arranged on a shelf if a) any arrangement is allowed.
b) three particular books must always be together.
c) two particular books must occupy the ends?

Soln a) $7! = 5040$

b) $\boxed{1\ 2\ 3}\ 4\ 5\ 6\ 7 = 5! \times 3! = 120 \times 6 = 720$

c) $\boxed{1}$ _ _ _ _ _ $\boxed{7}$
$\boxed{7}$ _ _ _ _ _ $\boxed{1}$ $\}$ $= 5! \times 2$ ways $= 120 \times 2 = 240$

12. A student has three books on C^{++} and four books on Java. In how many ways can he arrange ~~three~~ books on a shelf 1) if there are no restrictions. 2) If the languages should alternate 3) If all C^{++} books must be next to each other?
4) If all C^{++} books must be next to each other and all Java must be next to each other.

Soln:- 3 C^{++}, 4 Java — Total 7 books.

1) $7!$ ways.

2) $\underset{4 \text{ ways}}{J}\ C\ \underset{3 \text{ wa}}{J}\ C\ \underset{2 \text{ way}}{J}\ C\ \underset{1 \text{ way}}{J}$, with 3w, 2way, 1way over the C places $= 4! \times 3! = 24 \times 6 = 144$ ways (only one arrangement)

3) $(C^{++} = 3)$ _ _ _ _ $= 5! \times 3!$ (C^{++} among themselves)
1 2 3 4 5

4) $(C^{++} = 3)$ $(J = 4)$ $= 2! \times 3! \times 4!$

13. How many four digit numbers can be formed with 10 digits 0, 1, 2, -- 9. if a) repetitions are allowed b) repetitions are not allowed. c) The last digit must be zero and repetitions are not allowed. (20)

Soln:- 1) $\underset{9\text{ ways}}{\underline{\ \ }}\ \underset{10}{\underline{\ \ }}\ \underset{10}{\underline{\ \ }}\ \underset{10}{\underline{\ \ }}$ $= 9 \times 10 \times 10 \times 10 = 9000$

(0 cannot be in unit place)

2) $\underset{9}{\underline{\ \ }}\ \underset{9}{\underline{\ \ }}\ \underset{8}{\underline{\ \ }}\ \underset{7}{\underline{\ \ }}$ remaining $= 9 \times 9 \times 8 \times 7 = 81 \times 56 = 4536$

3) $\underset{9\text{ ways}}{\underline{\ \ }}\ \underset{8\text{ way}}{\underline{\ \ }}\ \underset{7\text{ ways}}{\underline{\ \ }}\ \underset{0}{\underline{\ \ }}$ $= 9 \times 8 \times 7 = 72 \times 7 = 504$

14. How many different three digit numbers can be formed with 3 fours, 4 two's and 2 threes.

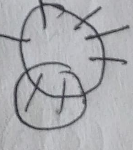
Soln:-

15. In how many ways can 7 women and 3 men be arranged in a row, if 3 men must always stand next to each other.

Soln:- (3M) _ _ _ _ _ _ _ $= 8! \times 3! = 2,41,920$

16) In how many ways can 7 people be seated at a round table if two particular persons sit next to each other

6 can sit around a round table in $(6-1)!$



Soln:- (remaining 5) — $5!$ ways

2 together — $2!$ ways

$5! \times 2! = 240$