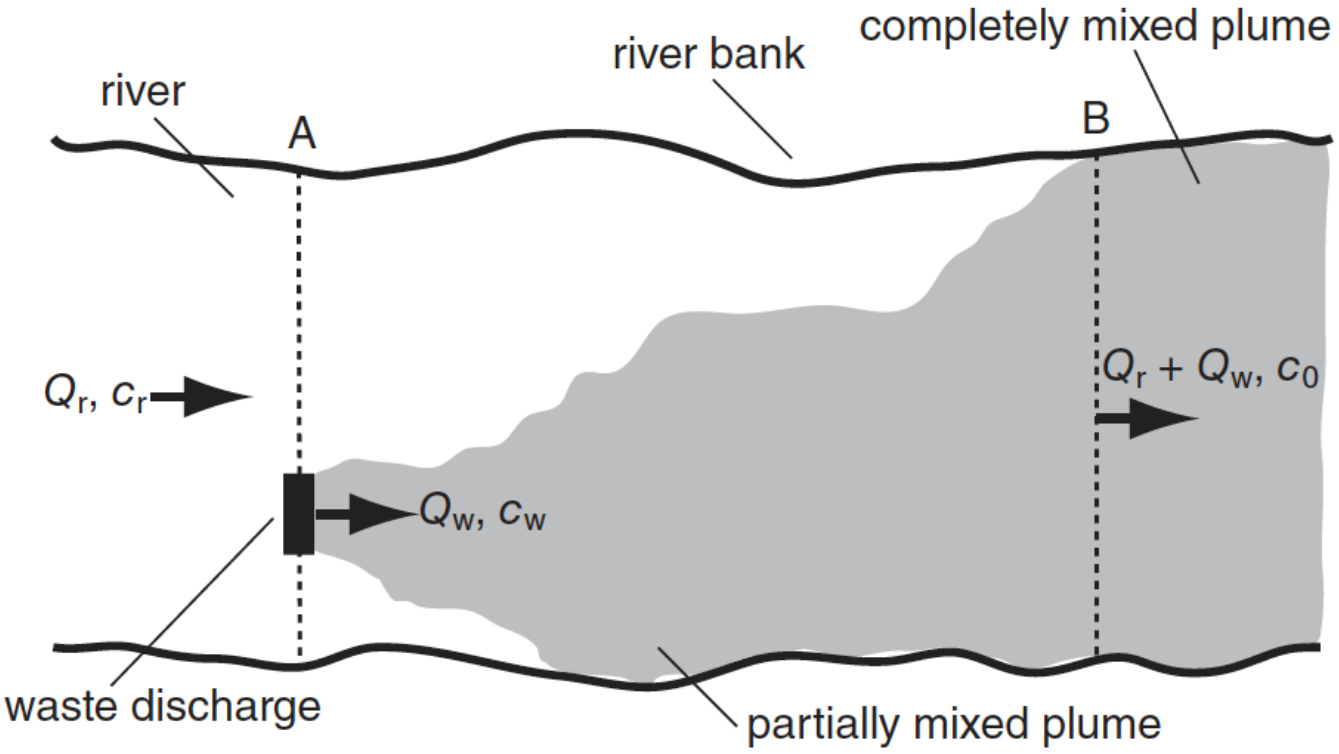
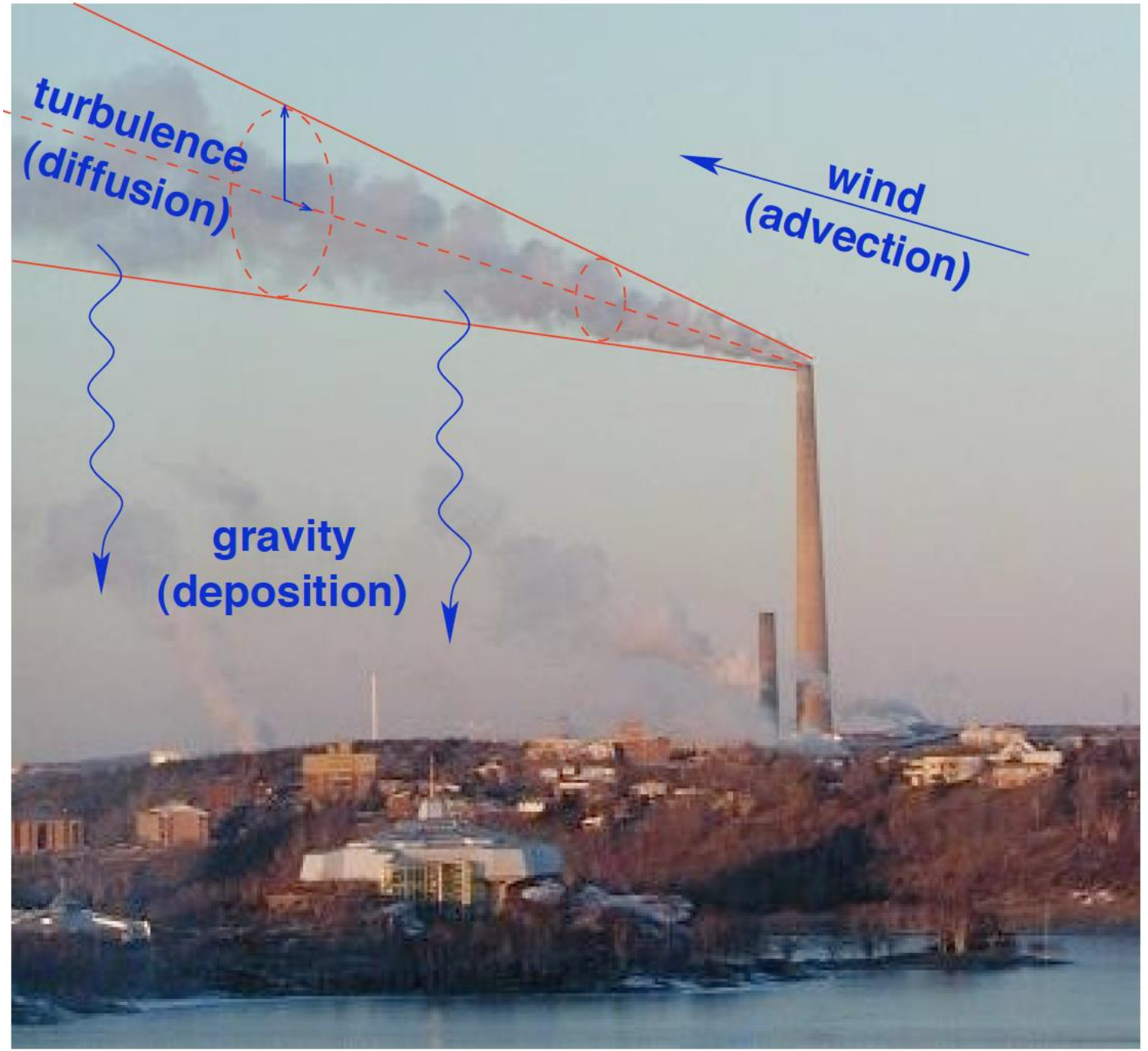
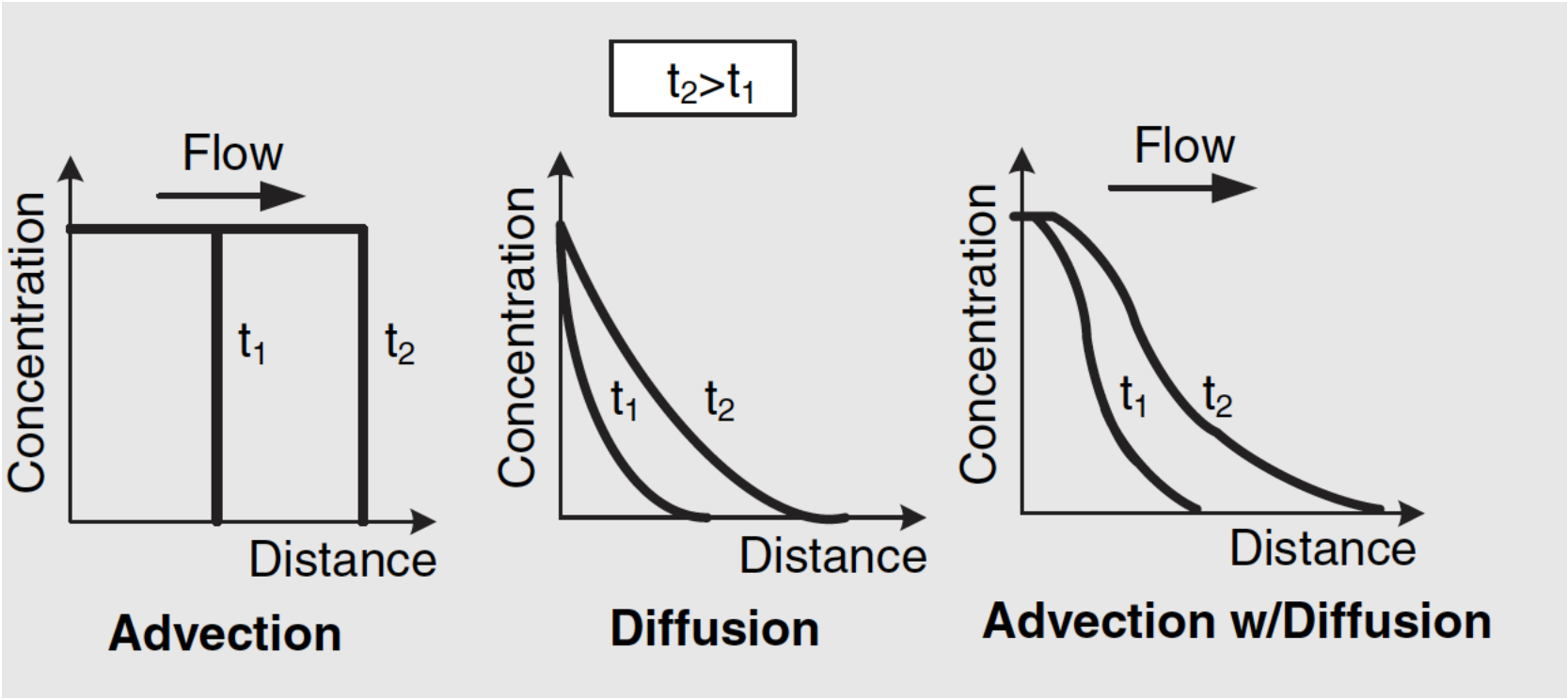
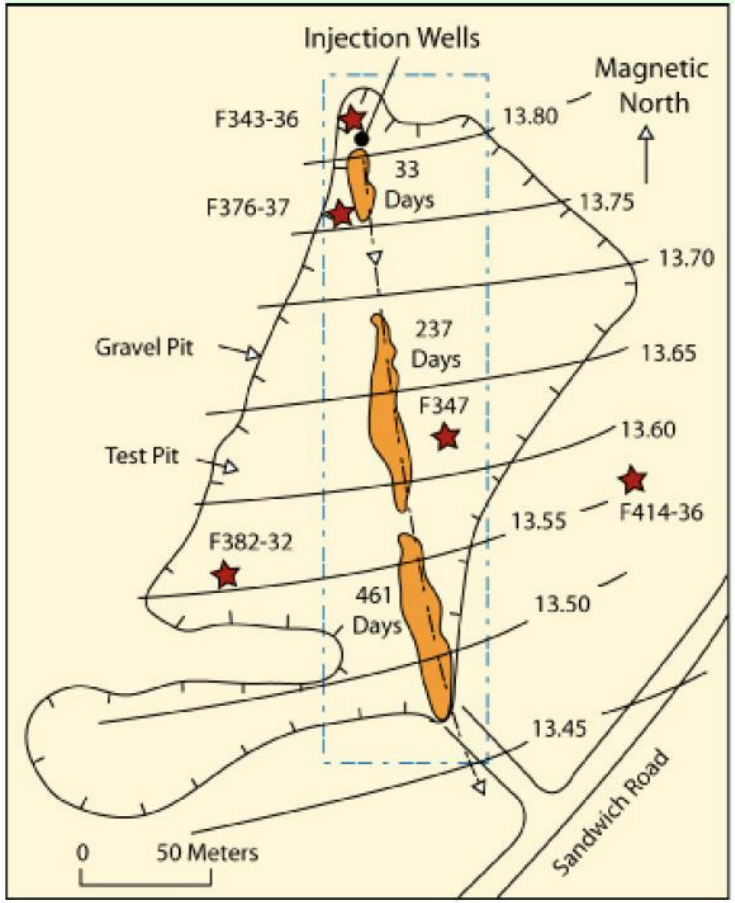

river
river bank
completely mixed plume
A
B
Q_r, c_r
$Q_r + Q_w$, c_0
Q_w, c_w
waste discharge
partially mixed plume


turbulence
(diffusion)
wind
(advection)
gravity
(deposition)


$t_2 > t_1$
Flow
Concentration
t_1
t_2
Distance
Advection
Concentration
t_1
t_2
Distance
Diffusion
Flow
Concentration
t_1
t_2
Distance
Advection w/Diffusion



LEGEND

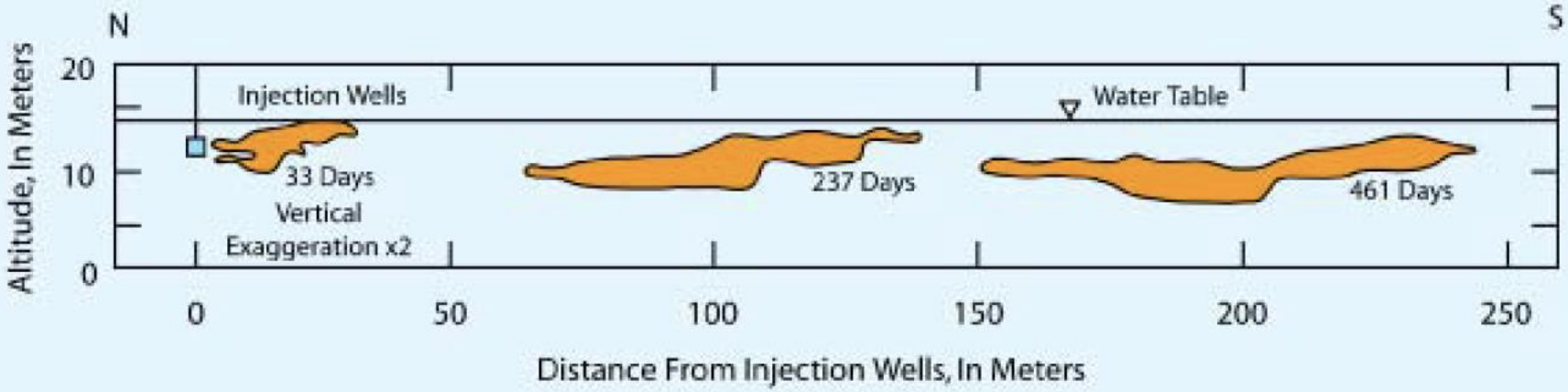
Area of tracer cloud in which bromide concentrations exceeded 1 milligram per liter

Area of multilevel sampler array

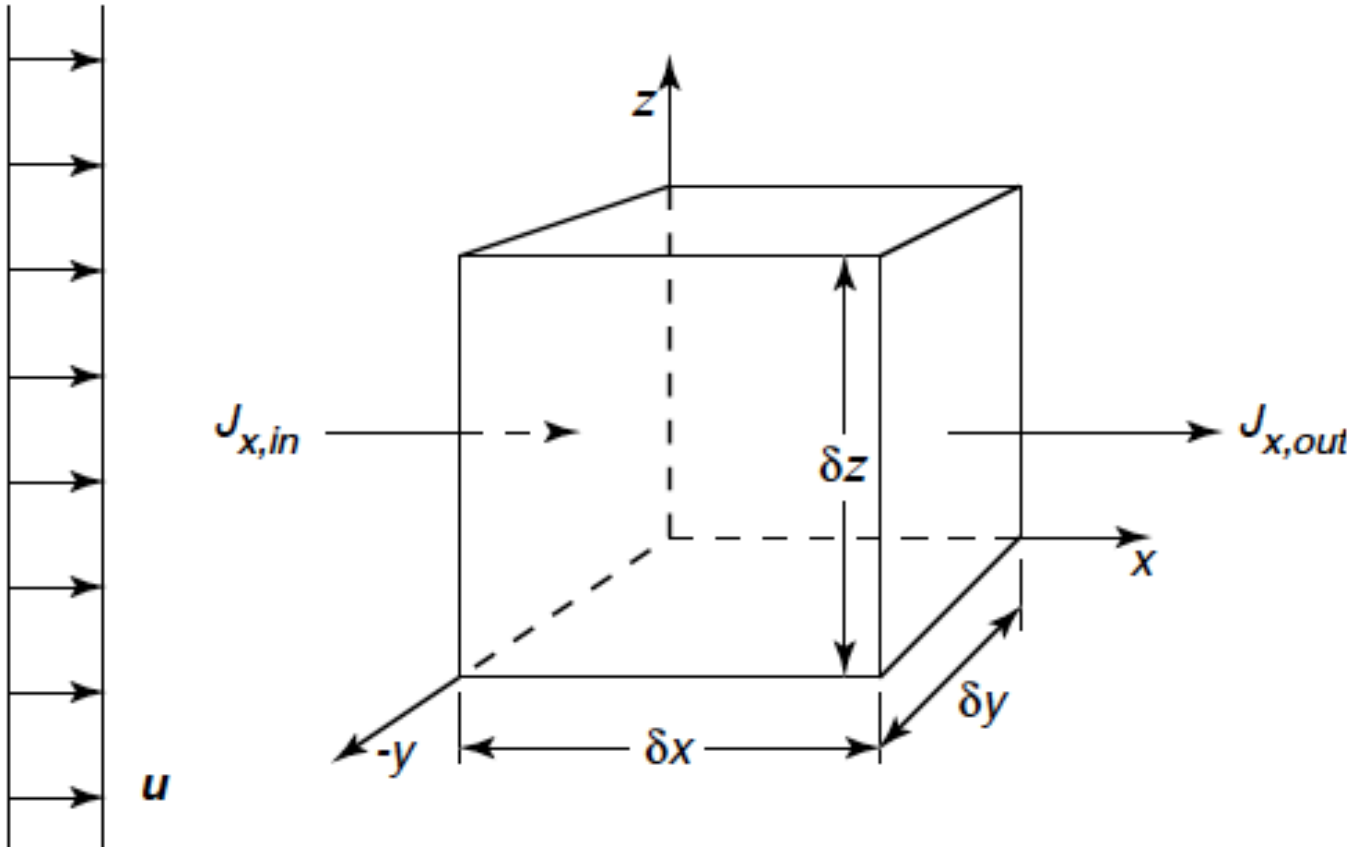
—13.45— Water-table contour, August 2, 1985 -- Shows altitude of water table. Contour interval .05 meters. Datum is sea level.

Predicted path of tracer cloud

Monitoring wells



Vertical location of bromide tracer cloud at 33, 237, and 461 days after injection. Cloud locations defined by zones in which bromide concentration exceeded 1 mg/L.


z
$J_{x,in}$
$J_{x,out}$
δz
x
-y
δx
δy
u

$$\frac{\partial M}{\partial t} = \sum mass_{in} - \sum mass_{o} \pm S$$

$$\frac{dC}{dt} + \nabla . \vec{J} = \pm S$$

Where,

C= concentration of compound

t= time

J= Flux (mass/area-time)

S= source or sinks

∇ = Del operator $\left(\frac{\partial}{\partial x} , \frac{\partial}{\partial y} , \frac{\partial}{\partial z} \right)$

$$J_{advection} = uC$$

$$J_{difussion} = -D \nabla . C$$

Where,
u = velocity of fluid
D= diffusion co-efficient

$$\frac{dC}{dt} + \nabla.(uC) = D\,\nabla^2.C \pm S$$

Considering flux due to diffusion to be negligible in the case of surface waters and considering reactions that lead to formation or loss of compound:

$$\frac{dC}{dt} + \nabla.(uC) = r_F - r_L$$

r_F =rate of formation of compound

r_L =rate of loss of compound

Considering flow only along X direction and constant velocity of flow of fluid.

$$\frac{dC}{dt} + u\frac{dC}{dx} = r_F - r_L$$

Simplifying it for macroscopic systems :-

$$\frac{dC}{dt} + u\frac{A\Delta C}{A\Delta x} = r_F - r_L$$

$$\frac{\partial M}{\partial t} = \sum mass_{in} - \sum mass_o \pm S$$

$$\frac{\partial M}{\partial t} = \sum mass_{in} - \sum mass_{o} \pm S$$

$$V * \frac{dC}{dt} = QinCin - Q_{out}C_{out} + V\,(r_F - r_L)$$

Where,

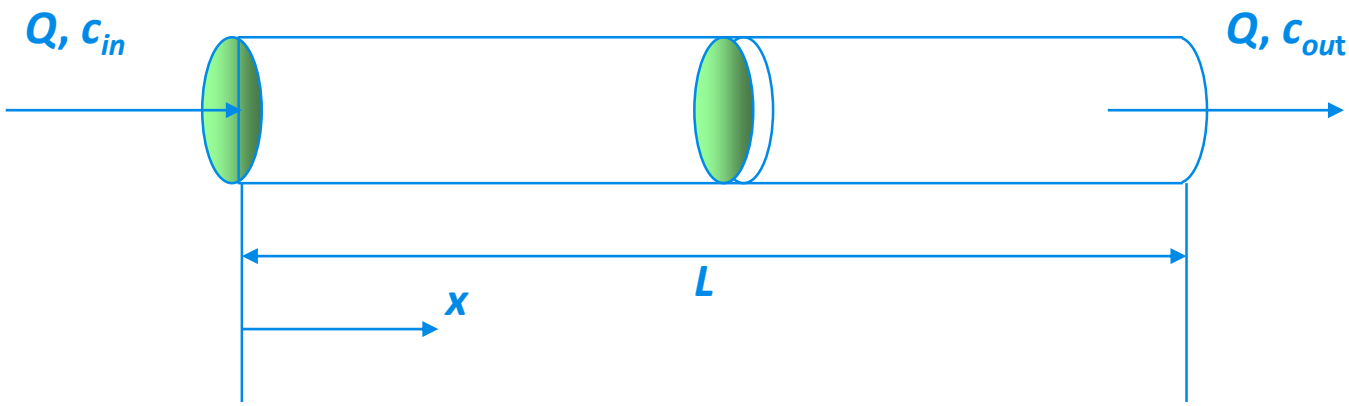
Q = Discharge, $m^3 / \sec$

= u x A

A = cross-sectional area

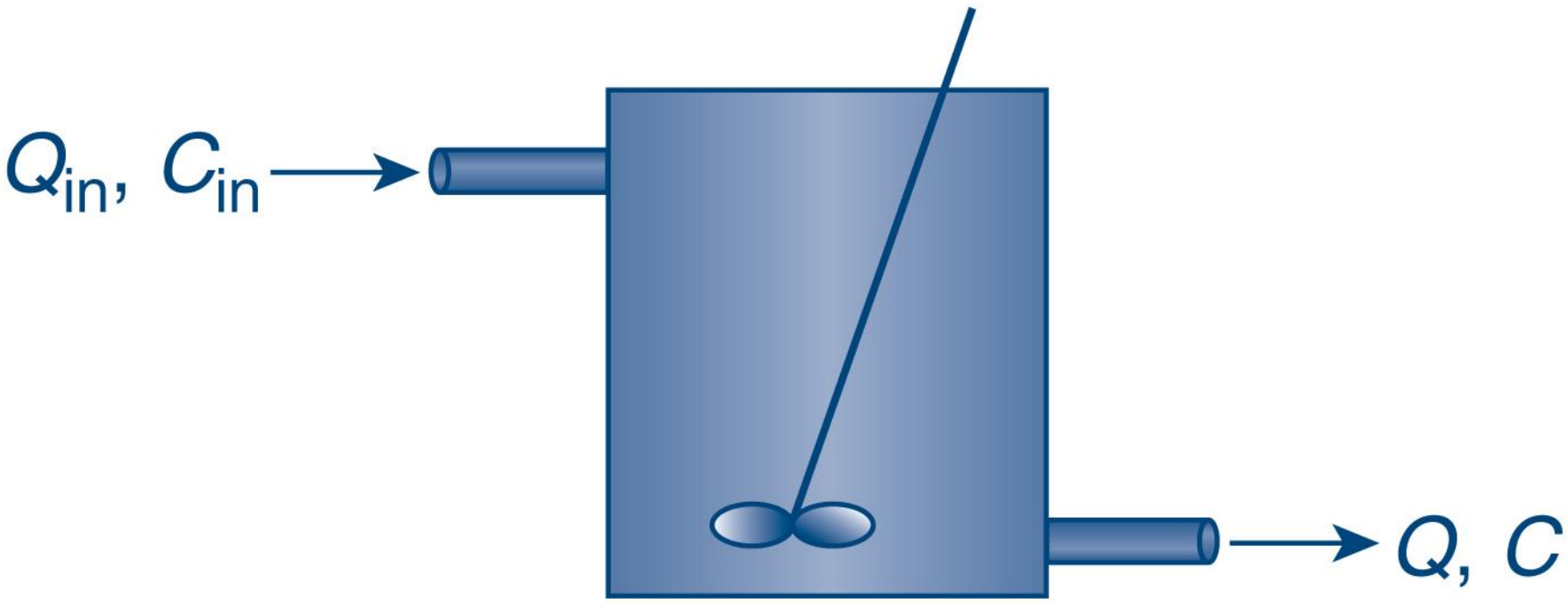
u = velocity of fluid

Plug flow reactor



CSTR or CMFR

Continuously stirred tank reactor or continuously mixed flow reactor

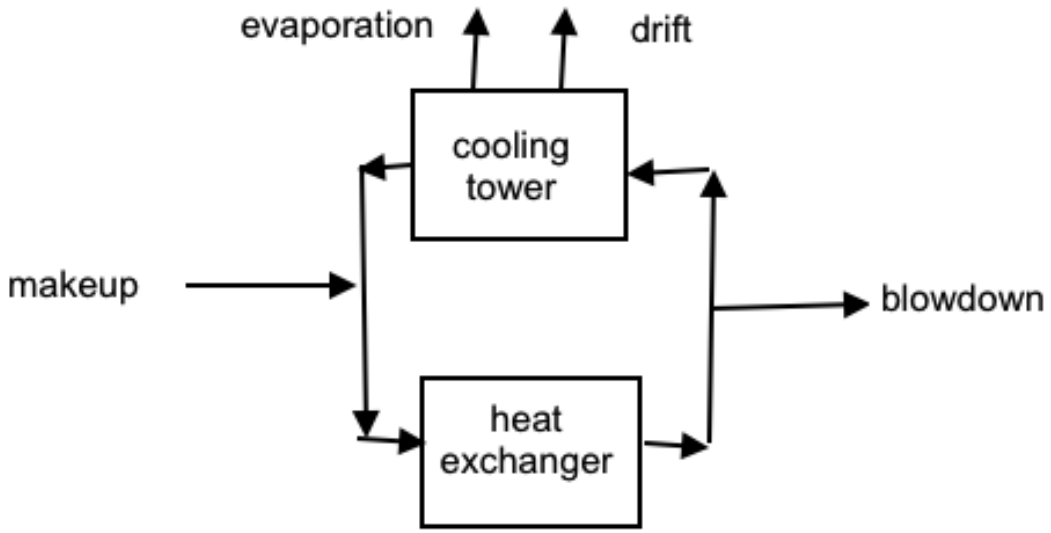


- A cooling tower is operating at steady-state as indicated below. In the cooling tower, water is lost as water vapor in the "evaporation" stream and as small droplets of liquid in the "drift" stream. Water is added as a liquid to the system in the "makeup" stream and removed in the "blowdown" stream. Total dissolved solids (TDS) include all compounds that remain in a test after the water is evaporated. Assume that drift and the recirculating water contains the same concentration of TDS as does the blowdown. Assume that TDS is not destroyed or produced in the system.
- a) If the makeup volumetric flow were set at 0.002 m^3/s, what would be the concentration of total dissolved solids in the blowdown/recirculating water?
- b) What would be the minimum possible makeup flow and what would be the concentration of TDS in blowdown/recirculating water at that makeup flow?

makeup TDS = 500 mg/L

evaporation flow = 0.001 m^3/s

drift flow = 1 E -4 m^3/s



a)Steady-state conservative balance around tower on water:

$Q_m = Q_e + Q_d + Q_b$

$Q_b = Q_m - Q_d - Q_b = 0.002 – 0.0001 – 0.001 = 9.0$ E-4 m^3/s

Steady-state conservative balance around tower on TDS:

$Q_m C_m = Q_d C_r + Q_b C_r = (Q_d + Q_b)C_r$

$C_r = Q_m C_{m/}(Q_d + Q_b) = (0.002\ m^3/s)(500\ mg/L)/(0.0001 + 0.0009)$ = **1000 mg/L**

b) The water balance shows that as the makeup flow decreases the blowdown flow decreases. The physical limit to reducing makeup flow would occur when the blowdown flow reaches zero.

$Q_{m,min} = Q_e + Q_d = 0.001 + 0.0001 =$ **1.1 E-3 m^3/s**

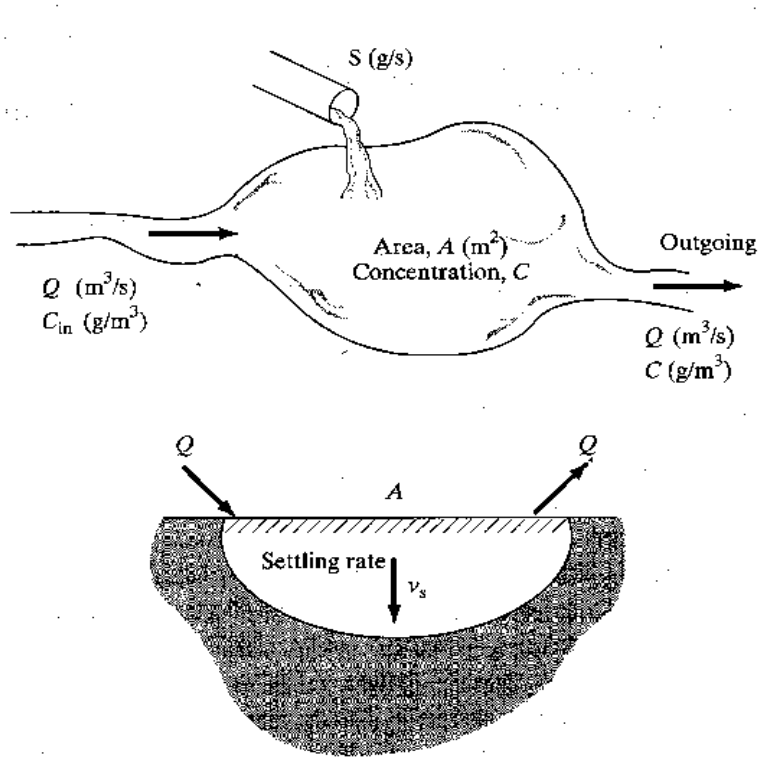
applying the result from part (a):

$C_r = Q_m C_{m/}(Q_d + Q_b) = (0.0011\ m^3/s)(500\ mg/L)/(0.0001) =$ **5,500 mg/L**

Derive the steady state Phosphorous concentration in a lake with the following conditions:

Any assumptions you make need to be clearly specified.

vs = Phosphorous removal rate (m/s)



$$V\frac{dC}{dt} = Q_{in}C_{in} - Q_oC_o + V\left(r_F - r_L\right)$$

$$0 = Q\left(C_{in} - C_o\right) + S - V_s X \;\; AC$$

$$C\left(Q + V_s A\right) = S + QC_{in}$$

$$C = \frac{S + QC_{in}}{Q + V_s A}$$

An experiment is run in the lab (batch reactor) that shows that a toxic compound (‘tri-methyl double death, or TMDD) is removed by a chemical treatment process. The experiment was conducted in a batch system (beaker in the lab) with a solution that initially contained 10 mg/L TMDD. Many samples were taken at different times, but all but one were lost due to poor storage conditions. That sample was taken after 30 minutes of reaction and showed a TMDD concentration of 2 mg/L.

Use this data to make preliminary calculations of the hydraulic retention time ($\theta = V/Q$) that would be required to achieve an effluent concentration of 1 mg/L TMDD from an initial concentration of 10 mg/L, at steady state, in treatment systems that are a) plug flow, and b) completely mixed flow reactor.

Assume that TMDD is known to be removed by a process that can be described by a first-order rate equation ($r_{loss} = k_1 C_{tmdd}$).

a) First, a value for the rate constant (k_1) must be obtained from the laboratory data. Since the lab data was obtained in a batch system, the result for a batch system with first-order decay can be applied.

$$d(VC)/dt = \Sigma Q_{in}C_{in} - \Sigma Q_{out}C_{out} + V\,(r_f - r_l)$$

Applying the above to a batch system:

$$dC/dt = r_F - r_L = 0 - k_1 C$$

Integrating,

$$C = C_{t0}\exp(-k_1 t)$$

$$k_1 = -\ln(C/C_{t0})/t = -\ln\,(2/10)/(30\ \text{min}) = 0.0536\ \text{min}^{-1}$$

$$\frac{dC}{dt} + u\frac{dC}{dx} = r_F - r_L$$

Applying the above mass balance equation for the plug flow system, at steady state:

$dC/d\theta=(r_f - r_l)=-k_1C$

The result obtained for a plug flow system with first-order decay can be applied.

$C = C_{t0}\exp(-k_1\theta)$
$\theta = -\ln(C/C_{t0})/k_1 = -\ln (1/10)/(0.0536\ \text{min}^{-1}) =$ **42.9 min**

b) The result obtained for a completely mixed system with first-order decay can be applied.

$d(VC)/dt = \Sigma Q_{in}C_{in} - \Sigma Q_{out}C_{out} + V\,(r_f - r_l)$

At steady state,

$0 = Q_{in}C_{in} - Q_{out}C_{out} + V\,(r_f - r_l)$

$Q(C_{in} - C_{out}) = V\,(0 - r_l)$

$Q(C_{in} - C_{out}) = V\,(0 - kC)$

For completely mixed flow reactor (CMFR or CSTR) $C_{out} = C$

$C = C_0/(1 + k_1\,\theta)$

$\theta = (C_0 - C)/(k_1 C) = (10 - 1\ \text{mg/L})/[(0.0536\ \text{min}^{-1})(1\ \text{mg/L}) =$ **168 min.**