Optics Formula Sheet

September 21, 2025

Sign Convention (Cartesian)

- Light travels from left to right.
- Object distance (u): Negative (-)
- Image distance (v): Positive for real image, Negative for virtual image.
- Focal length (f): Positive for converging lens/surface, Negative for diverging.
- Radius of Curvature (R): Positive if center is to the right of the vertex.
- Heights: Positive above the principal axis.

Geometrical Optics

Spherical Refracting Surface

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Lens Maker's Formula (Thin Lens)

$$\frac{1}{f} = (n_{\text{lens}} - n_{\text{med}}) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Magnification

$$m = \frac{h_i}{h_o} = \frac{v}{u}$$

Newton's Lens Equation

$$x_o \cdot x_i = f^2$$

Wave Optics

Wave Equation & Solutions

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

Traveling Harmonic Wave Solutions:

$$\begin{aligned} \psi &= A \sin(k(x \mp vt)) \\ \psi &= A \sin(2\pi(kx \mp \nu t)) \\ \psi &= A \sin(kx \mp \omega t) \end{aligned}$$

Where: $k = \frac{2\pi}{\lambda}$, $\omega = 2\pi\nu$, $v = \nu\lambda$.

Phase & Group Velocity

$$\begin{aligned} v_p &= \frac{\omega}{k} \\ v_g &= \frac{d\omega}{dk} = \frac{\omega_1 - \omega_2}{k_1 - k_2} \end{aligned}$$

In a dispersive medium ($n(\omega)$):

$$k(\omega) = \frac{\omega}{c} n(\omega)$$

$$v_g = \frac{c}{n(\omega) + \omega \frac{dn}{d\omega}}$$

Polarization

Jones Vectors & Matrices

$$\begin{aligned} &(\text{Linear } x) && \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &(\text{Linear } y) && \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &(\text{Linear } 45^\circ) && \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &(\text{RCP}) && \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \\ &(\text{LCP}) && \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} \end{aligned}$$

Jones Matrices

$$\begin{aligned} &(\text{Linear polarizer } || \ x) && \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ &(\lambda/4 \text{ plate, fast } || \ x) && \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \\ &(\text{Linear polarizer at } 45^\circ) && \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

Malus's Law

$$I = I_0 \cos^2 \theta$$

Waveplates

Phase Retardation ($\Delta\phi$):

$$\Delta\phi = \frac{2\pi}{\lambda} d |n_e - n_o|$$

Maxwell's Equations & EM Waves

In a linear, isotropic, homogeneous medium ($\rho = 0, \vec{J} = 0$):

$$\begin{aligned} \nabla \cdot \vec{D} &= 0 \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} &= \frac{\partial \vec{D}}{\partial t} \end{aligned}$$

With $\vec{D} = \epsilon \vec{E}$, $\vec{B} = \mu \vec{H}$.

Wave Solution

$$\vec{E} = \vec{E_0} e^{i(\vec{k} \cdot \vec{r} - \omega t)}, \quad \vec{B} = \vec{B_0} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$k = \frac{\omega}{v} = \frac{n\omega}{c}, \quad n = \sqrt{\frac{\epsilon\mu}{\epsilon_0 \mu_0}}$$

$$\vec{S} = \vec{E} \times \vec{H} \quad (\text{Poynting Vector})$$

Reflection & Refraction

Snell's Law

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

Fresnel's Equations

Parallel Polarization (p):

$$r_p = \frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$
$$t_p = \frac{2n_1\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t}$$

Perpendicular Polarization (s):

$$r_s = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$
$$t_s = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t}$$

Reflectance & Transmittance

$$R = |r|^2$$
$$T = \frac{n_2\cos\theta_t}{n_1\cos\theta_i}|t|^2$$

Brewster's Angle (No r_p)

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

Total Internal Reflection (TIR)

Critical Angle:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Evanescent Wave decay constant ($\theta_i > \theta_c$):

$$\alpha = \frac{2\pi}{\lambda} n_2 \sqrt{\sin^2\theta_i - \sin^2\theta_c}$$

Waveguides

V-Number

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

Single-mode operation: $V < 2.405$.

Propagation Constant

$$b = \frac{\beta^2 - k_0^2 n_2^2}{k_0^2(n_1^2 - n_2^2)}$$

A mode is guided if $0 < b < 1$. Cut-off occurs at $b = 0$.