ktunotes

the learning companion.

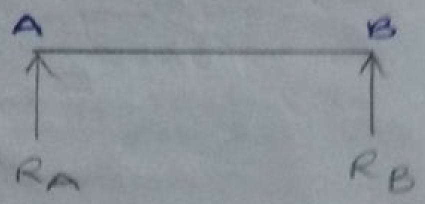
MODULE - III

SHEAR FORCE DIAGRAM & BENDING MOMENT DIAGRAM

SFD & BMD

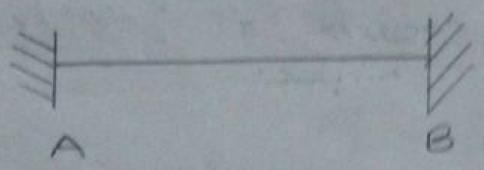
* Types of support.

1. Simple Support (Knife edge support)



Rean is always upward

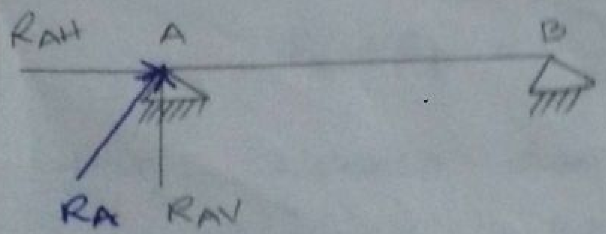
2. Fixed support : The beam rigidly fixed on support.



The beam is fixed so that is also a moment.

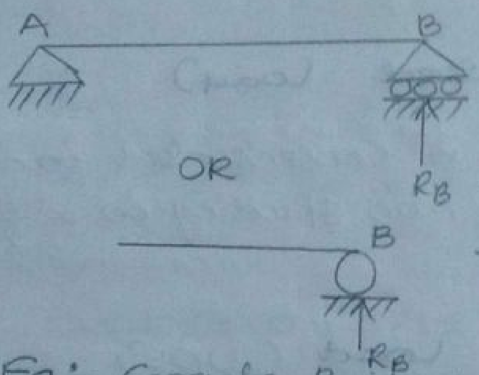
3. Hinged support : (Eg : Door)

Beam is placed on a hinge so the beam can rotate. (ie, no moment in hinged support)



i) Inclined load → Rean inclined.

4. Roller support ÷ a small cylindrical roller.



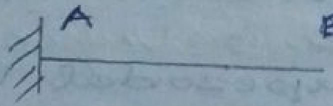
For prevent Expand during rain is drop we use roller support.

Rolling is permitted in one dir^n. so there is no rea^n. But the rea^n will always $\perp^r$ to the ~~load~~ surface of rolling load

Eg ÷ Concrete Bridges

✻ Types of beams ÷

1. Cantilever beam

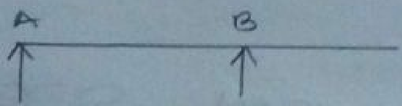


one end fixed & other end free

2. Simply supported beams.

Both ends of beam are provided in Simple support

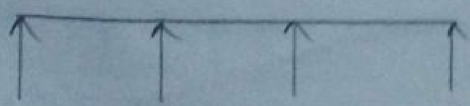
3. Overhanging beams



The beams extends beyond the support

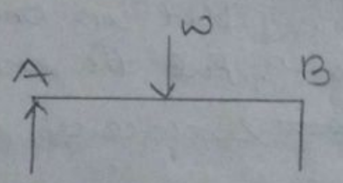
4. Fixed ~~supp~~ beams

5. Continuous beams → beams are resting on more than 2 support



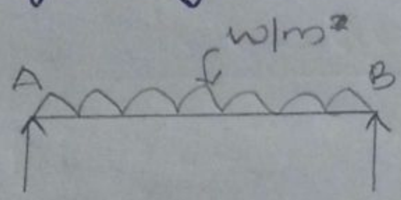
* Types of load

1. point load (Concentrated load)



load is Concentrated on a point
Eg ÷ man standing on a floor

2. Uniformly distributed Load (UDL)



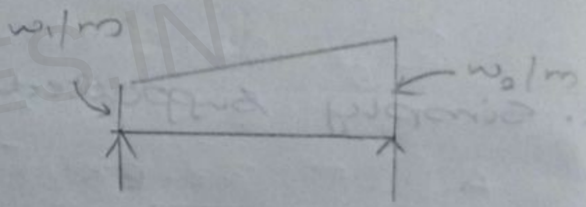
load is distributed uniformly over load.

Eg ÷ water collected in a water tank.

3. Uniformly Varying load ÷ load is not uniformly distributed. but variation is uniform

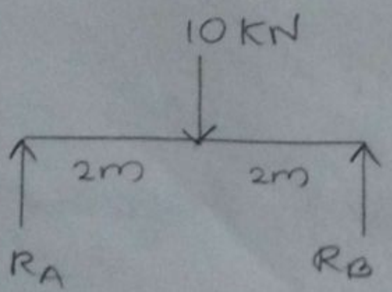
a) Triangular load

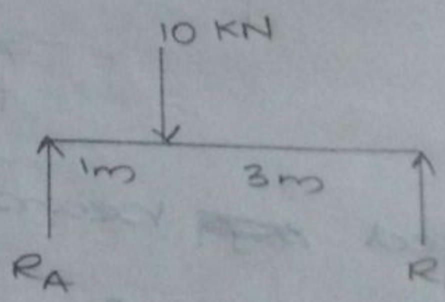
b) Trapezoidal load



Eg ÷

*) Support Reaction

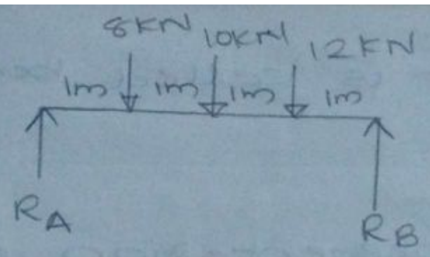




$R_A + R_B = 10$

the load is symmetrical. ∴ the reactions are equal

*)



$R_A + R_B = 30$ KN

$\Sigma m = 0,$

$1 \times 8 + 2 \times 10 + 3 \times 12 - 4 \times R_B = 0$

$8 + 20 + 36 = 4R_B$

$R_B = \frac{64}{4} = 16$

$R_A = \cancel{16}\ 30 - 16 = 14$ //

*) SHEAR FORCE → Unbalanced vertical force acting tangentially cross-section

* Shear force :-

when a beam is subjected to a load the unbalanced vertical force acting tangential to the cross section of beam at any section of beam.

* SFD

The graphical representation of the variation of shear force along the length is called shear force diagram.

* Bending moment.

The ~~as~~ bending moment at the section of the beam is the algebraic sum of the moments of right & left section. This moment causes bending of beam. ∴ It is called bending moment. The analysis

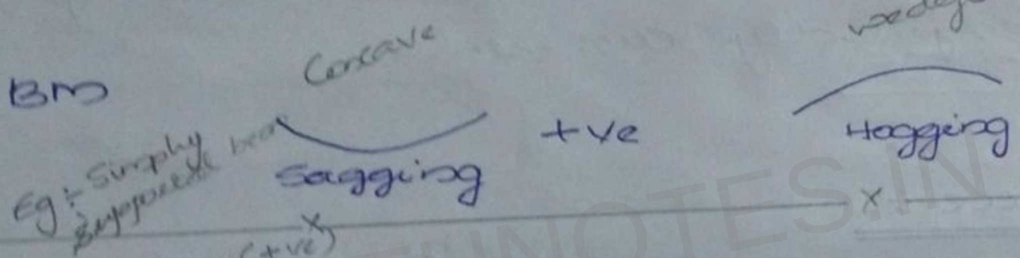
The graphical representation of the variation of the bending moment along a beam is called Bending moment diagram (BMD)

*) Sign Convention.

* SF ↑↓ +ve (clock wise)

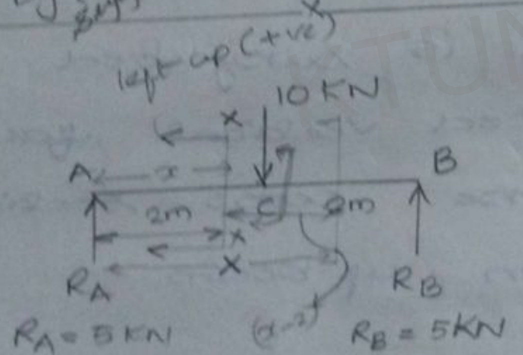
↓↑ −ve (Anti-clock wise)

* BM



+ve

−ve



For a simply supported beam, the BM is always +ve for everywhere on the beam

1) AC, we are looking towards left at xx

shear force at section xx, $F_x = 5$

$F_A = 5$

just before) F_C (left) = 5

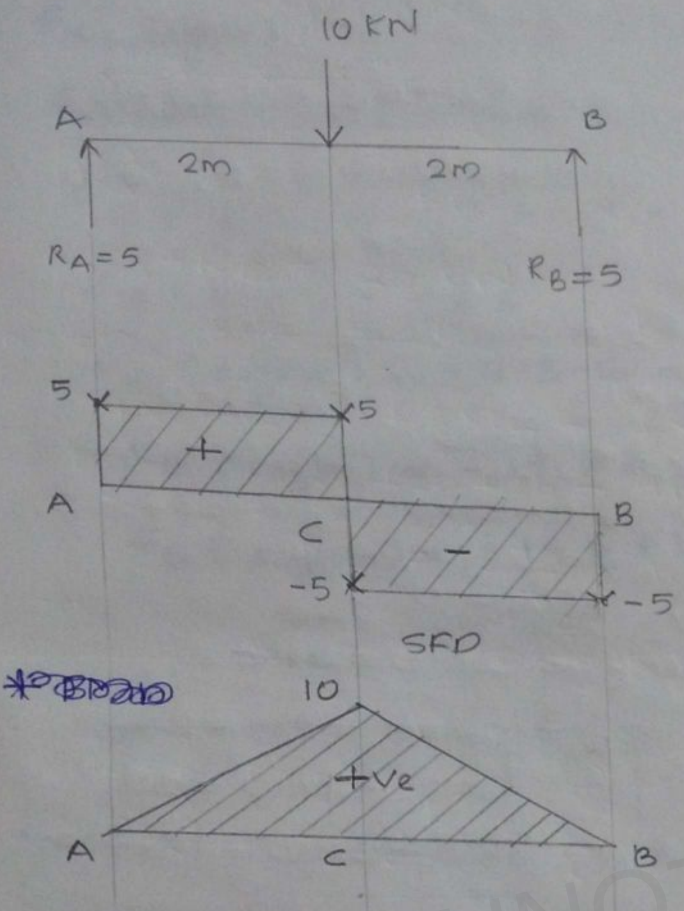
crosses the 'c' F_C (right) = 5−10 = −5

2) CB

$F_x = 5-10 = -5$

F_B(left) = 5−10 = −5

F_B (right) = 5−10+5 = 0



→ For Simply supported beam, Only have sagging bending moment (always +ve).

→ For simple support beam always zero moment at loads.

* BMD

moment = load × ⊥ᴸ distance

AC

moment about A

$$M_x = R_A \cdot x = 5x$$

$$M_A = R_A \times x = 5 \times 0 = 0$$

$$M_C = R_A \times x = 5 \times 2 = 10$$

CB

$$M_x = R_A \cdot x - R_C(x-2)$$

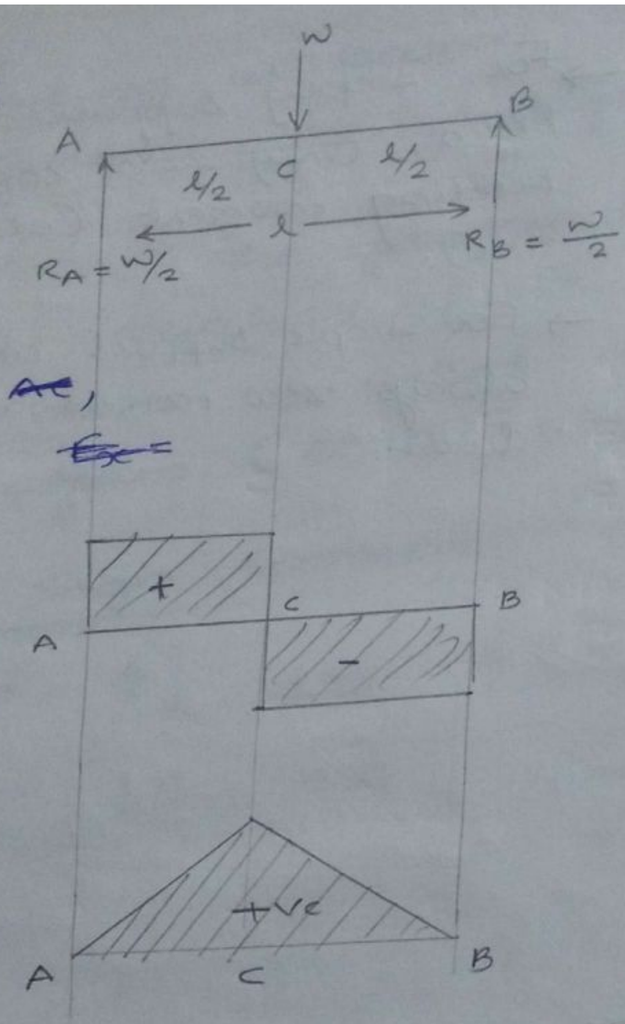
$$= 5x - 10x + 20$$

$$= -5x + 20$$

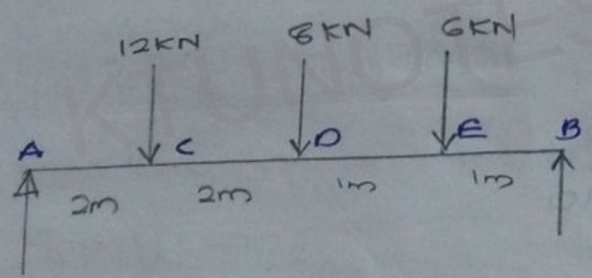
$$M_C = 5 \times 2 - 10(2-2) = 10$$

$$M_B = 5 \times 4 - 10(4-2) = 20 - 20 = 0.$$

*



*



$R_A + R_B = 12 + 8 + 6 = 26\ \text{KN}$

moment about A,

$0 \times R_A + 2 \times 12 + 4 \times 8 + 5 \times 6 - 6R_B = 0$

$24 + 32 + 30 = 6R_B$

$R_B = \frac{86}{6} = 14.3\ \text{KN}$

$R_A = 26 - 14.3 = 11.7\ \text{KN}$

SFD

$f_A = 11.7$

$f_C\ (\text{left}) = 11.7$

$f_C (right) = 11.7 - 12 = 0.3$

$f_D (\cancel{tor}\ left) = -0.3\ kN$

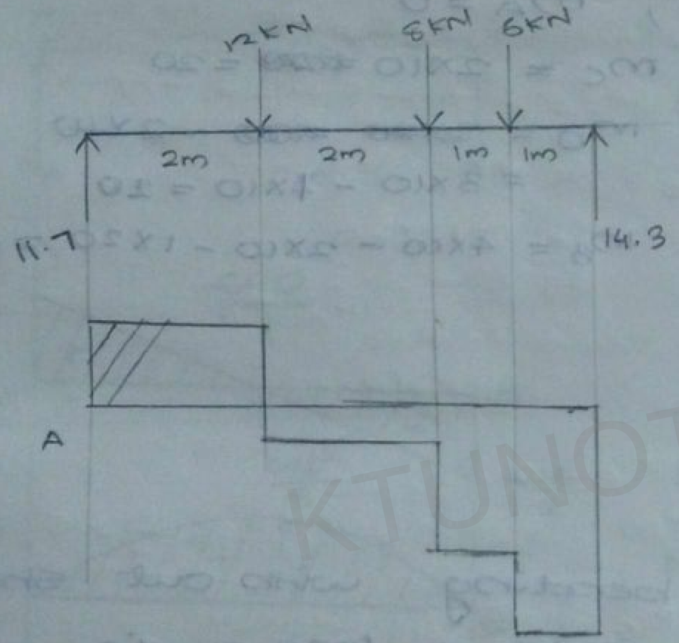
$f_D (right) = -0.3 - 8 = -8.3$

$f_E (left) = -8.3$

$f_E (right) = -8.3 - 6 = -14.\cancel{8}3$

$f_B (left) = -14.\cancel{6}3$

$f_B (right) = -14.\cancel{6}3 + 14.3$



$M_A = 0$, $M_C = 11.7 \times 2 = 23.4$, $M_B = 0$

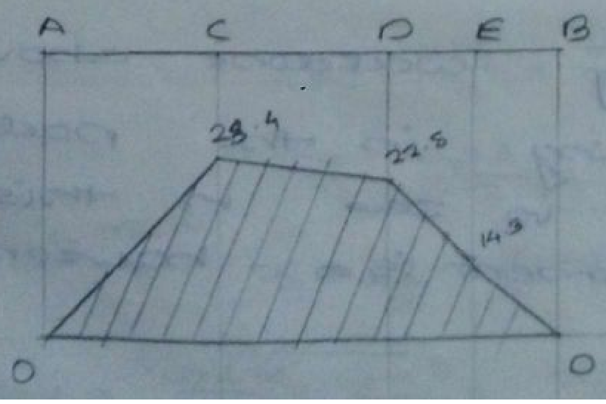
$M_D = 4 \times 11.7 - 12 \times 2 = 46.8 - 24 = 22.8$

$(14.3 \times 2 - 6 \times 1) = 22.8$

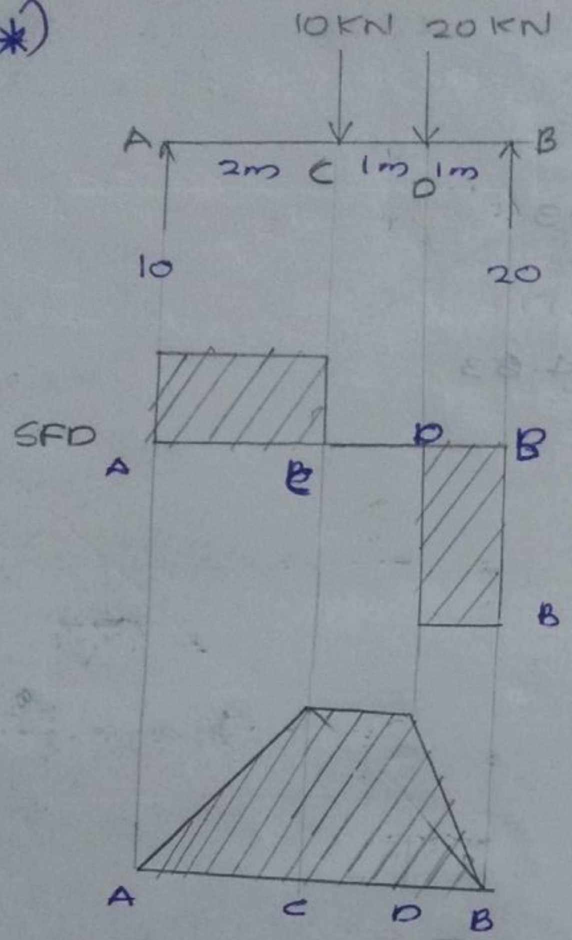
$M_E = 5 \times 11.7 - 3 \times 12 - 1 \times 8 =$ ~~26.55~~ 14.3

$M_B =$

BMD



*)



$R_A + R_B = 30\ kN$

$2\times10 + 3\times20 - 4\times R_B = 0$

$80 = 4R_B$

$R_B = 20\ ,\ R_A = 10$

$F_A = 10$

$F_{C(L)} = 10$

$F_{C(R)} = 10 - 10 = 0$

$F_{D(L)} = 0$

$F_{D(R)} = 0 - 20 = -20$

$F_B = -20 + 20 = 0$

BMD, $M_A = 0$

$M_C = 2\times10 = 20$

$M_D = \cancel{3\times20}\ \cancel{\ }\ \cancel{-2\times10}$

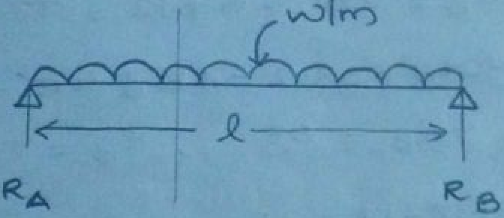
$= 3\times10 - 1\times10 = 20$

$M_B = 4\times10 - 2\times10 - 1\times20 = 0$

* PURE BENDING

It is the bending with out shear force. Some times when a beam is subjected to loading shear force will be zero for a certain part of a beam & Bending moment should be constant at this portion. This is called pure bending.

In the fig workdone above there is pure bending in the portion CD. b/w shear force is zero in this portion & Bending moment is a horizontal line.

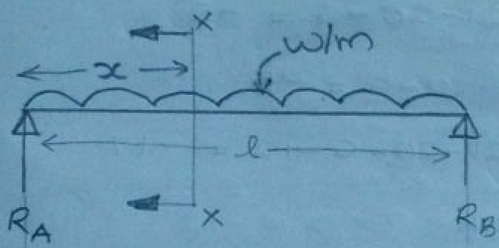
(*) 

① Find the reaction.

Shape of curve is parabola due to the fun^n x^2.

$R_A + R_B = wl$

$R_A = \frac{wl}{2}$, $R_B = \frac{wl}{2}$

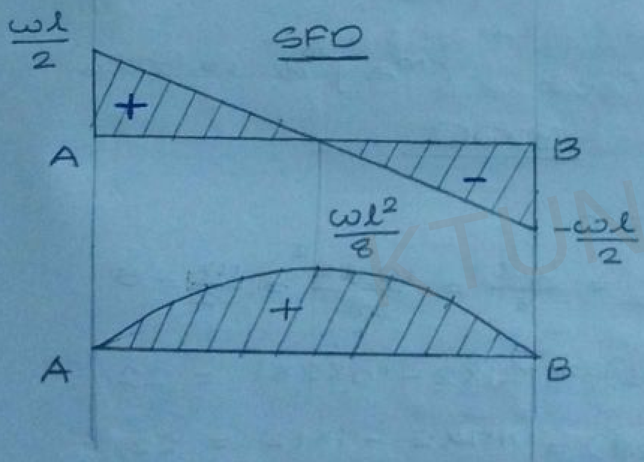


SFD

$F_x = \frac{wl}{2} - wx$

$F_A = \frac{wl}{2} - 0 = \frac{wl}{2}$

$F_B = \frac{wl}{2} - wl = -\frac{wl}{2}$



BMD

$M_x = \frac{wl}{2}x - wx \cdot \frac{x}{2}$

$= \frac{wlx}{2} - \frac{wx^2}{2}$

$M_A = 0$ (x at A is zero)

$M_B = \frac{wl}{2} \cdot l - \frac{wl^2}{2}$

$= 0$ ($\because x = l$ at B)

→ if a fun^n $f(x)$ is max or min at $x = a$ then $f'(x)$ is zero at that point.

max Value of Bending,

$$\frac{dM_x}{dx} = \frac{d}{dx}\left(\frac{wlx}{2} - \frac{wx^2}{2}\right) = 0$$

$$= \frac{wl}{2} - \frac{w}{2} \cdot 2x = 0$$

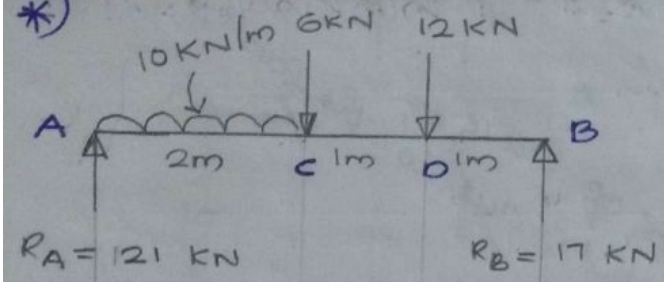
$$= \frac{wl}{2} - wx = 0$$

BM is max

$x = \frac{l}{2}$ (The max Value

$BM_{max} = \frac{wlx}{2} - \frac{wx^2}{2} = \frac{wl}{2} \cdot \frac{l}{2} - \frac{w}{2}\frac{l^2}{4} = \frac{wl^2}{4} - \frac{wl^2}{4 \cdot 2}$

*)



$R_A + R_B = 2\times10 + 6 + 12 = 38$

$1\times20 + 2\times6 + 3\times12 - 4R_B = 0$

$4R_B = 68$, $R_B = 17$

$R_A = 38 - 17 = 21$

<u>SFD</u>

$F_A = 21$

$F_{C(L)} = 21 - 10\times2 = 21 - 20 = 1$

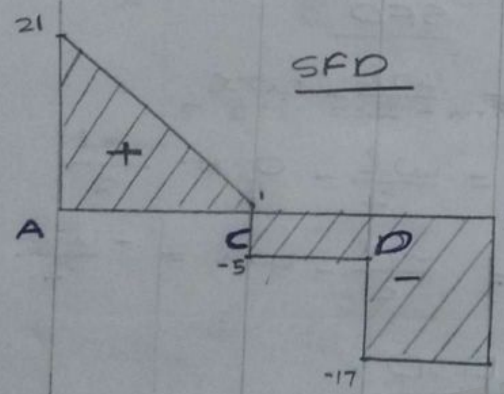
$F_{C}(R) = 21 - 20 - 6 = -5$

$F_D(L) = 21 - 20 - 6 = -5$

$F_D(R) = 21 - 20 - 6 - 12 = -17$

$F_B(L) = -17$

$F_B(R) = -17 + 17 = 0$



Bending moments at supports is zero. (M_A & M_B is zero)

<u>BMD</u>

$M_A = 0$

$BM_{max} = \frac{wl^2}{8} = \frac{10\times2^2}{8} = \frac{10}{2} = 5$

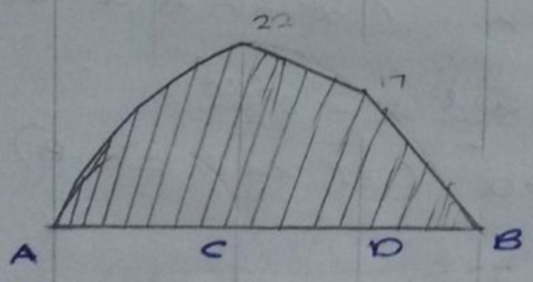
$M_C(L) = 21\times2 - 10\times2\times1 = 22$

$M_C(R) = 17\times2 - 1\times12 = 22$

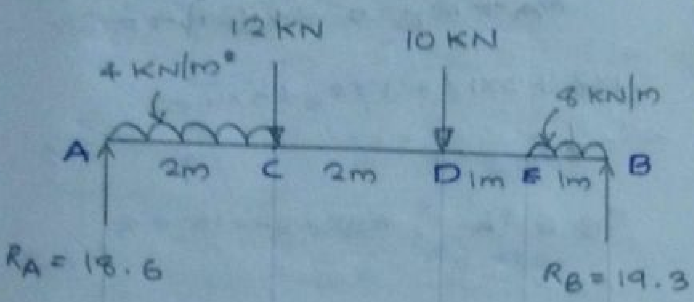
$M_D(R) = 1\times17 = 17$

$M_D(L) = 3\times21 - 2\times10\times2 - 6 = 17$

$M_B = 0$



*



$R_A + R_B = 2\times 4 + 12 + 10 + 1\times 8$

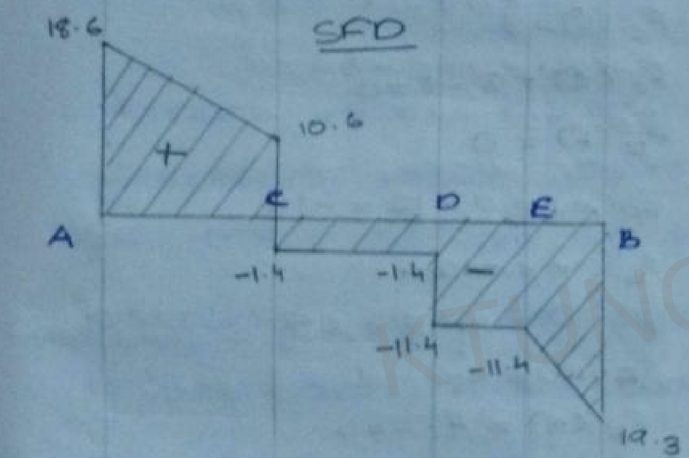
$= 38$

moment,

$2\times 4\times 1 + 2\times 12 + 4\times 10 + 1\times 8\times 5.5 - 6R_B = 0$

$6R_B = 116$

$R_B = 19.33$, $R_A = 38 - 19.3 = 18.6$



SFD

$F_A = 18.6$

$F_C(L) = 18.6 - 2\times 4 = 10.6$

$F_C(R) = 10.6 - 12 = -1.4$

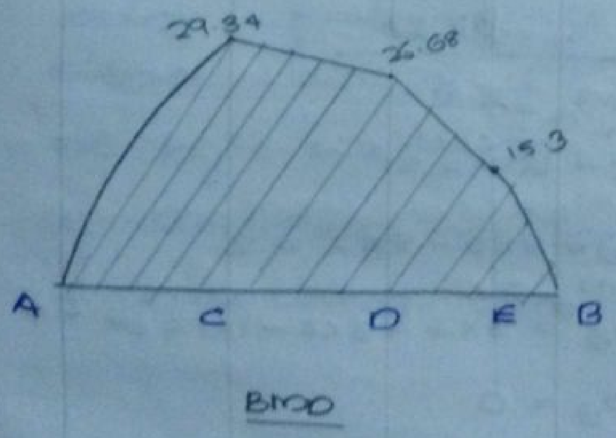
$F_D(R) = -1.4 - 10 = -11.4$

$F_D(L) = -1.4$

$F_B(R) = -19.3 + 19.3 = 0$

$F_B(L) = \cancel{-19.33}\ -11.4 - 1\times 8 = -19.4$

$F_E = -11.4$



BMD

$M_A = 0$

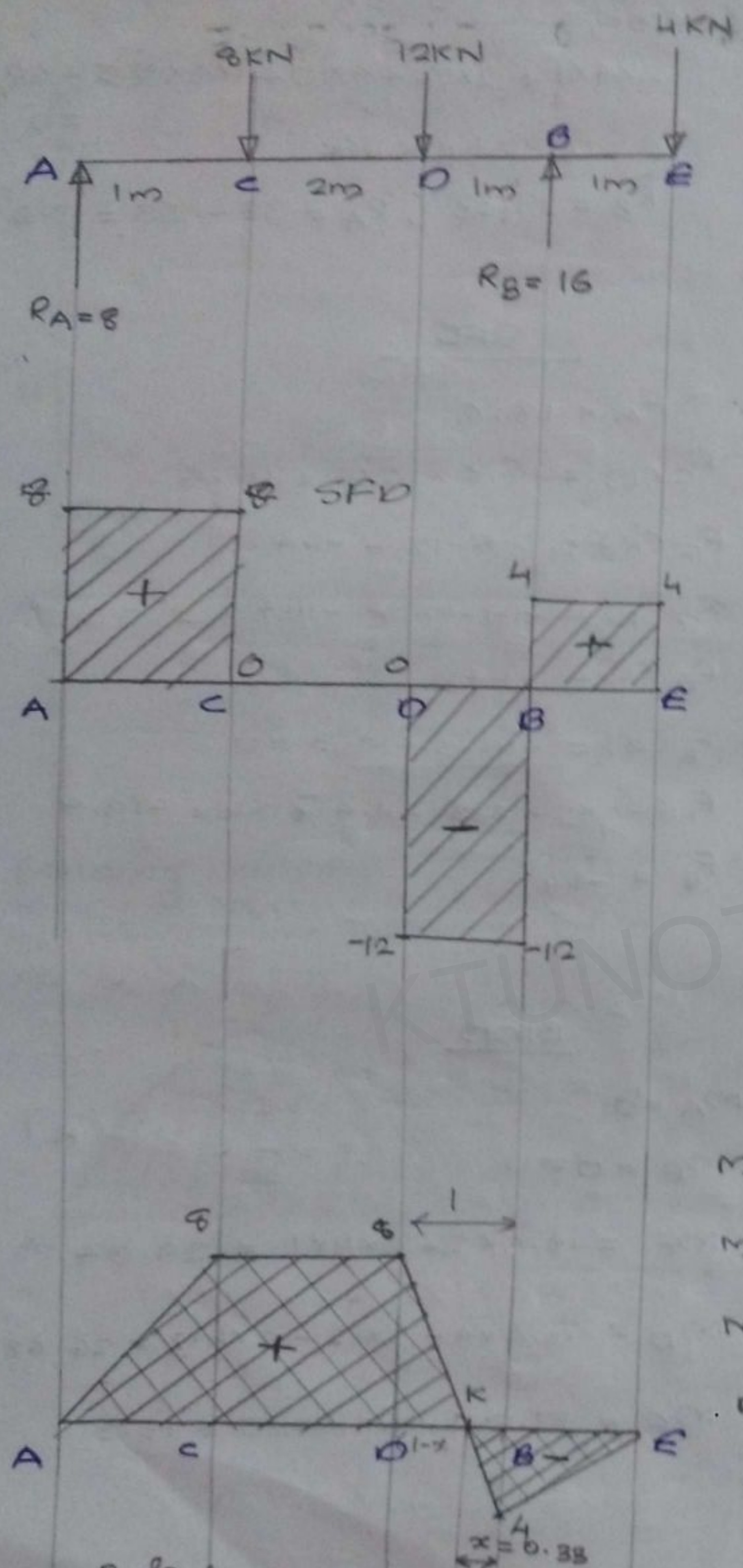
$M_B = 0$

$M_C = 18.6\times 2 - 2\times 4\times 1 = 29.34$

$M_D = 18.6\times 4 - 2\times 12 - 2\times 4\times 3 = 26.68$

$M_E = 19.3\times 1 - 8\times 1\times 0.5 = 15.3$

* OVERHANGING BEAM



$R_A + R_B = 8 + 12 + 4 = 24$

$1 \times 8 + 3 \times 12 - 4 \times R_B + 5 \times 4 = 0$

$4R_B = 64$

$R_B = 16$, $R_A = 24 - 16 = 8$

SFD

$F_A = 8$

$F_C(L) = 8$

$F_C(R) = 8 - 8 = 0$

$F_D(L) = 0$

$F_D(R) = 0 - 12 = -12$

$F_B(L) = -12$

$F_B(R) = -12 + 16 = 4$

$F_E(L) = 4$

$F_E(R) = 4 - 4 = 0$

BMD

$M_A = 0$

$M_C = 1 \times 8 = 8$

$M_D = 8 \times 3 - 8 \times 2 = 8$

$M_B = 8 \times 4 - 3 \times 8 - 1 \times 12 = -4$

$M_E = 0$

$\frac{8}{4} = \frac{1-x}{x} \Rightarrow$ $8x = 4 - 4x$

$4 = 12x$

$x = \frac{4}{12} = \frac{1}{3} = 0.33$

B to find x → By using similar triangle principle

K → Point of contraflexure

The bending moment changes its sign

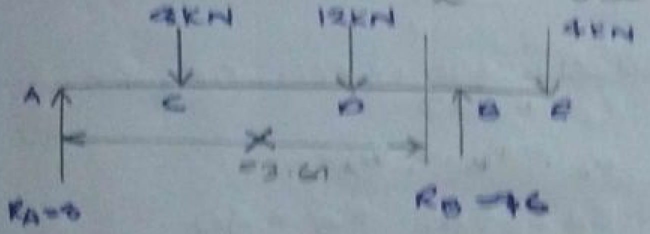
of Bending moment at the point of Contraflexure is zero,

$M_x = 8x - 8(x-1) - 12(x-3) = 0$

$8x - 8x + 8 - 12x + 36 = 0$

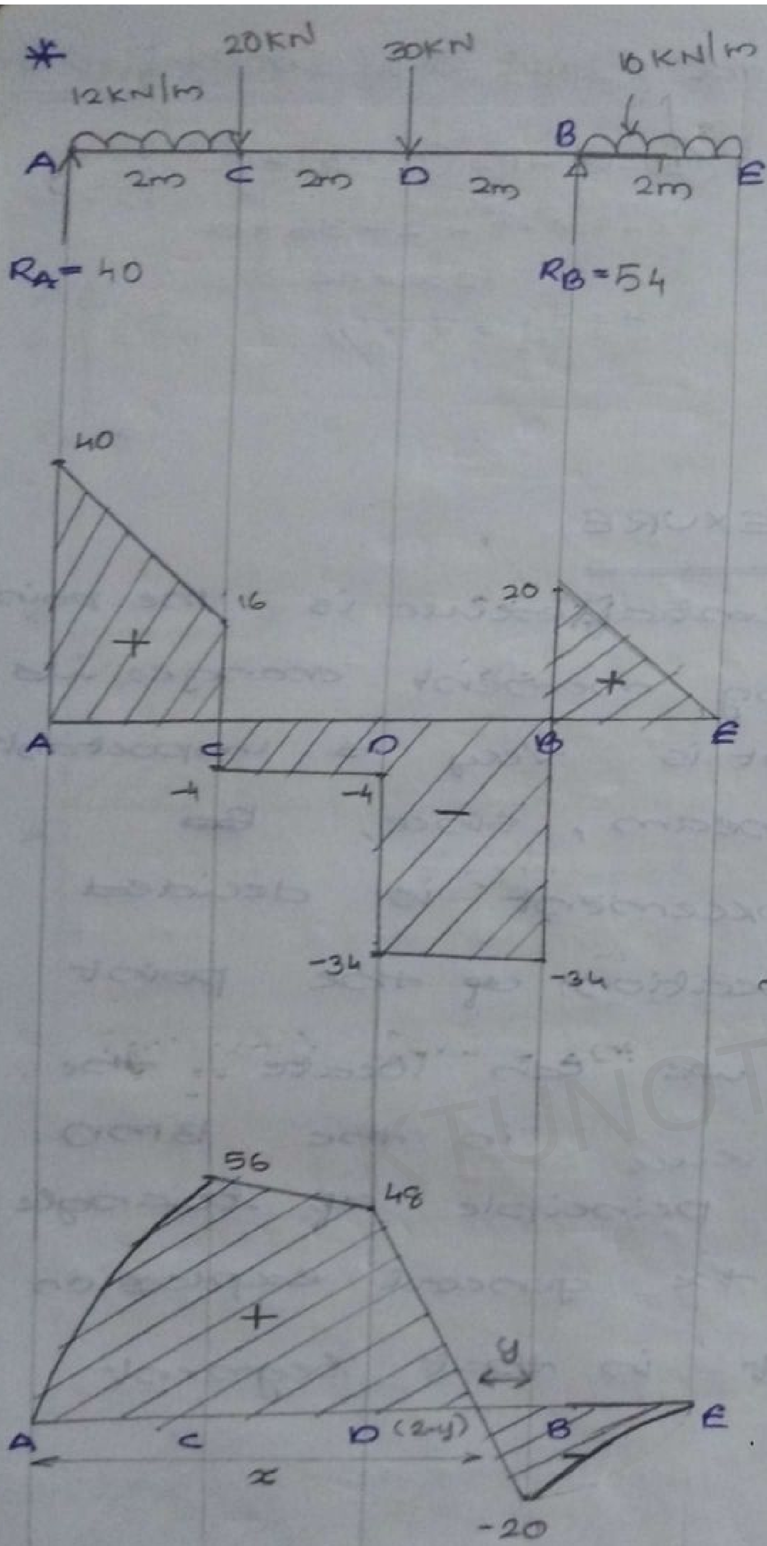
$12x = 44$

$x = \frac{44}{12} = 3.67$



* POINT OF CONTRAFLEXURE

Point of contraflexure is the point where the bending moment changes its direction. This point is very important for the design of beam, since, the placing of reinforcement is decided based on the location of the point of contraflexure. We can locate the point of contraflexure in the BMD either using the principle of triangle or by writting the general expression of Bending moment in that segment & equating zero.



$R_A + R_B = 2\times12 + 20 + 30 + 2\times10$
$= 94$

$2\times12\times1 + 2\times20 + 4\times30 - 6R_B + 2\times10\times7 = 0$

$6R_B = 324$

$R_B = 54, \quad R_A = 40$

SFD.

$F_A = 40$

$F_C(L) = 40 - 2\times12 = 16$

$F_C(R) = 16 - 20 = -4$

$F_D(L) = -4$

$F_D(R) = -4 - 30 = -34$

$F_B(L) = -34$

$F_B(R) = -34 + 54 = 20$

$F_E = 0$ (if there is a point load then we take left of right force)

BMD

$M_A = 0$

$M_C = 40\times2 - 2\times12\times1 = 56$

$M_D = 40\times4 - 2\times12\times3 - 2\times20 = 48$

$M_B = 6\times40 - 2\times12\times5 - 4\times20 - 2\times30 = -20$

$M_E = 0$

$M_x = 40x - 2\times12\times(x-1) - 20\times(x-2) - 30\times(x-4) = 0$

$40x - 24x + 24 - 20x + 40 - 30x + 120 = 0$

$-34x + 184 = 0$

$34x = \cancel{324}\ 184$

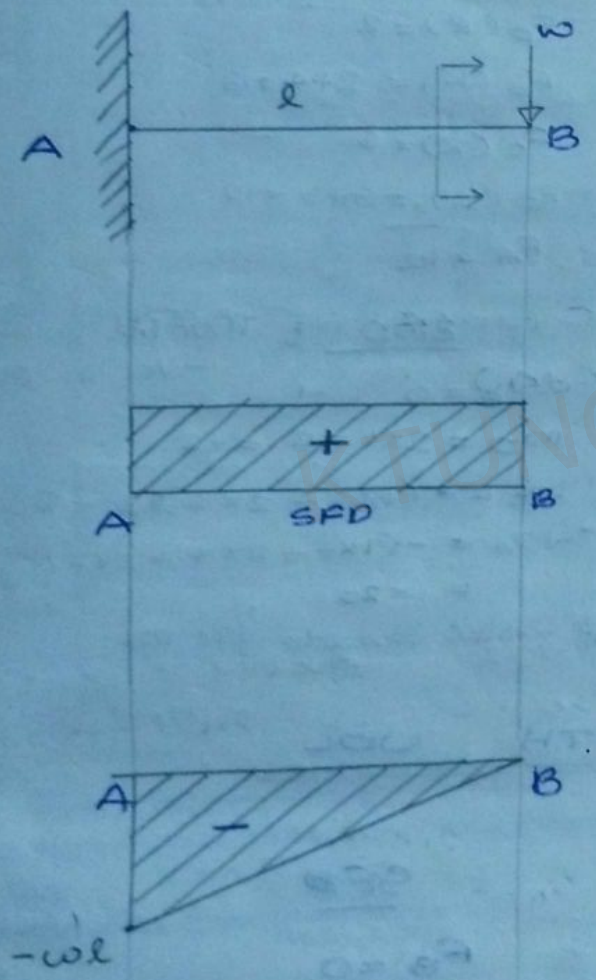
point of contraflexure, $x = \frac{184}{34} = \frac{184}{34} = \underline{5.41\,m}$

$\frac{20}{48} = \frac{y}{2-y}$

$40 - 20y = 48y \Rightarrow 40 = 68y \Rightarrow y = \frac{40}{68} = \underline{0.58\,m}$

*) CANTILIVER BEAM

→ First start from free end



SFD

$F_B = w$

$F_A = w$

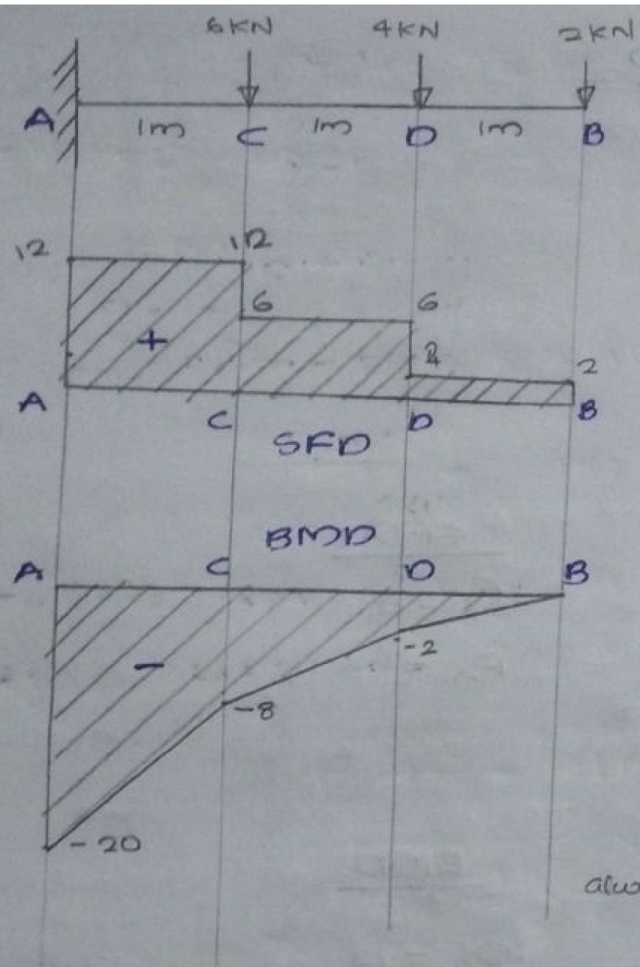
BMD

$M_B = 0$

so −ve

$M_A = -wl$ (hogging)

*)



SFD.

$F_B = 2$

$F_D(R) = 2$

$F_D(L) = 2+4 = 6$

$F_C(R) = 6$

$F_C(L) = 6+6 = 12$

$F_A = 12$

BMD (Moment is always -ve in Cantilever beam)

$M_B = 0$

$M_D = 2\times1 = 2 = -2$

$M_C = -(1\times4 + 2\times2) = -8$

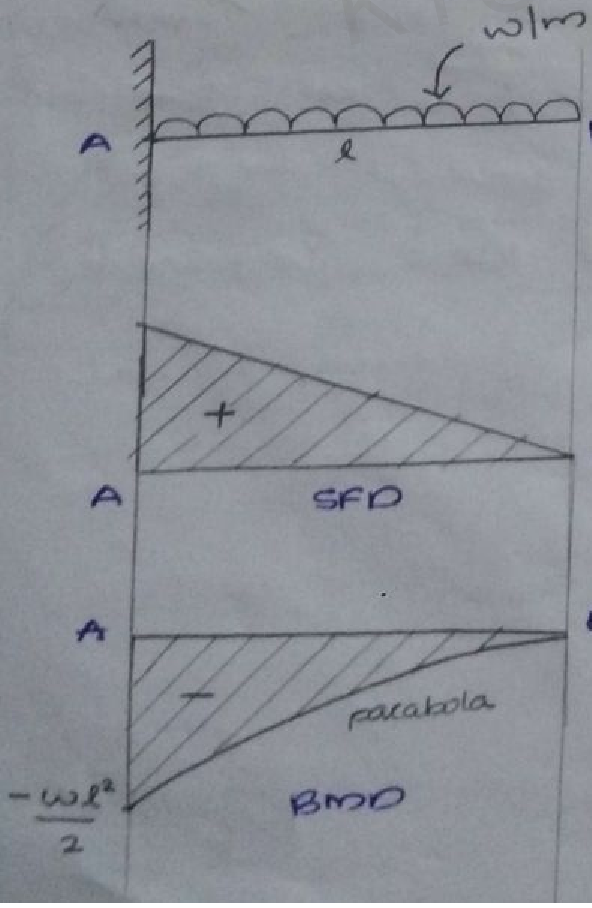
$M_A = -(1\times6 + 2\times4 + 3\times2)$

$= -20$

always watch towards free end.

$-(1\times4 + 2\times2)$

* CANTILIVER BEAM WITH UDL



SFD.

$F_B = 0$

$F_A = wl$

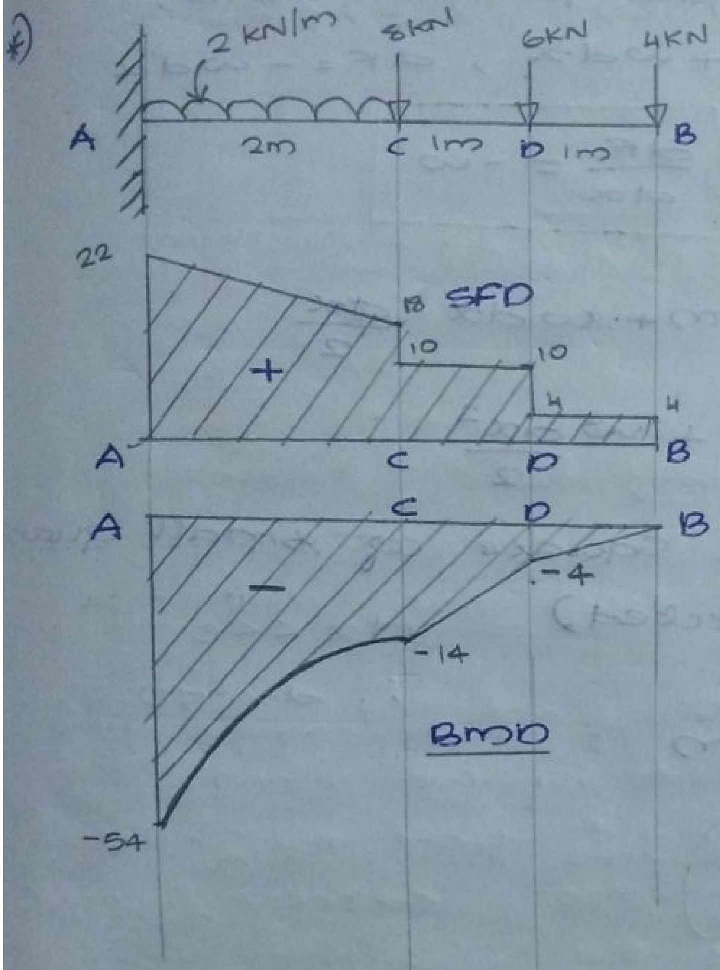
BM

$M_B = 0$

$M_A = wl \cdot \frac{l}{2}$

$= -\frac{wl^2}{2}$

*)



SFD.

$F_B = 4kN$

$F_D(R) = 4$

$F_D(L) = 4+6 = 10$

$F_C(R) = 10$

$F_C(L) = 10+8 = 18$

$F_A = 18 + 2\times 2 = 22$

BMD.

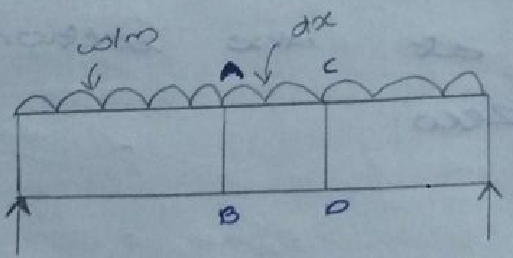
$M_B = 0$

$M_D = -(1\times 4) = -4$

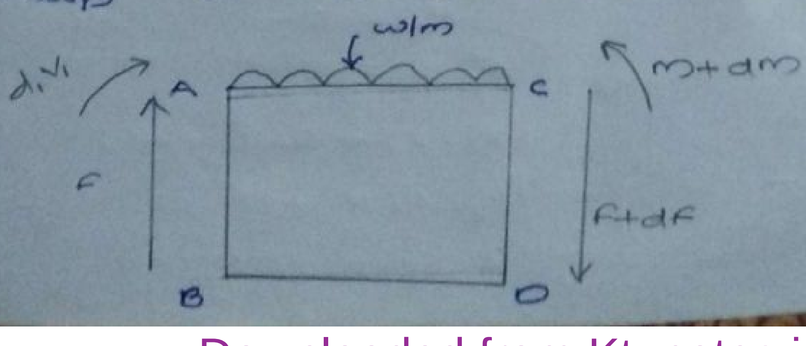
$M_C = -(1\times 6 + 2\times 4) = -14$

$M_A = -(4\times 4 + 3\times 6 + 2\times 8 + 2\times 2\times 1) = -54$

*) Relationship b/w SF, BM & Intensity of Loading.



Consider an elementary strip ABCD of a beam subjected to UDL w/m. Let dx be the length of the strip. The shear force & bending moment ~~...~~ acting this elementary strip is mark below.



considering the eq^n of the portion ABCD,

$\Sigma V = 0$ $\quad F = F + dF + w\,dx$, $dF = -w\,dx$

$\Sigma M = 0 \Rightarrow$

$$\boxed{\frac{dF}{dx} = -w}$$

$\Sigma M_{CD} = 0$

$$M + F\,dx = M + dM + w\,dx \cdot \frac{dx}{2}$$

$$F\,dx = dM + w\frac{dx^2}{2}$$

(dx^2 being a square of small quan-tity, can be neglected) $\quad dx << 0$

$\therefore dx^2 \approx 0$

$$F\,dx = dM$$

$$\boxed{F = \frac{dM}{dx}}$$

if F is zero, then M is max.

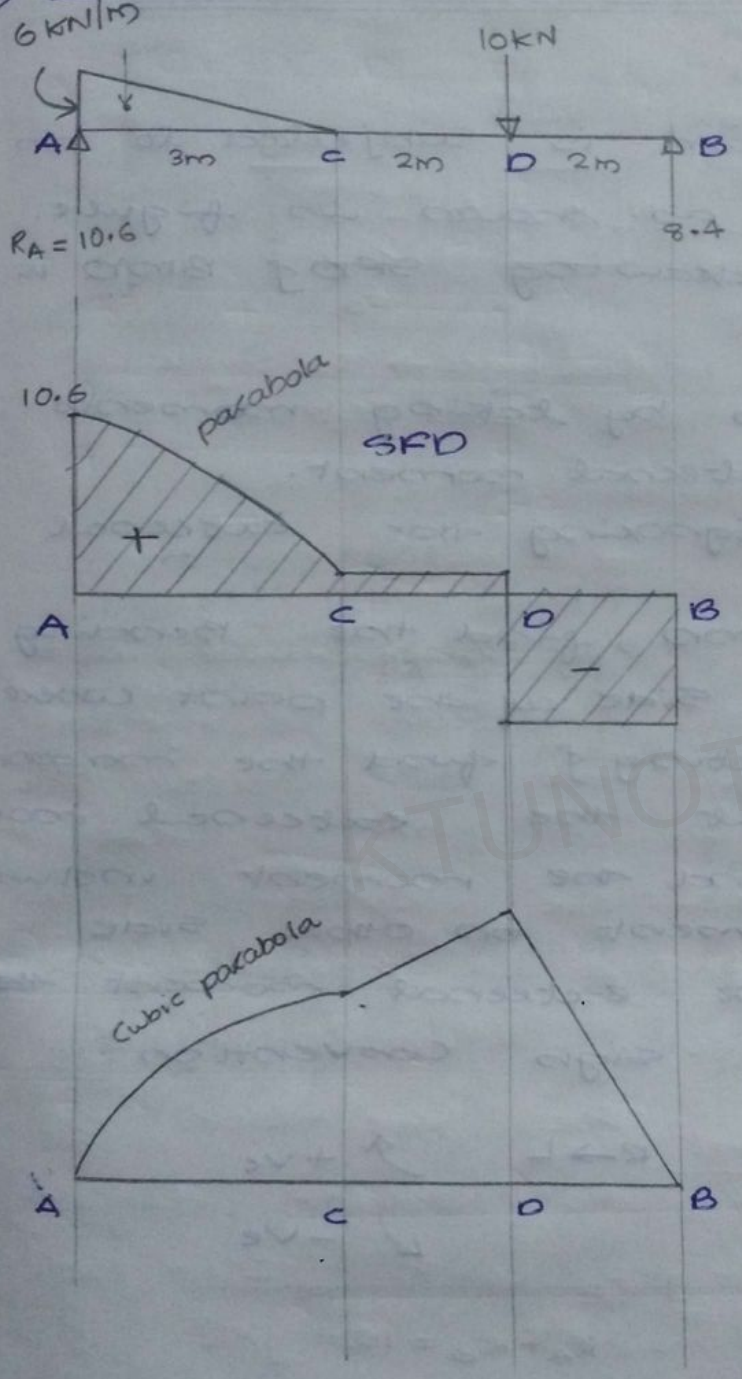
→ when shear force is zero $\frac{dM}{dx}$ is zero hence M is maximum. Hence we get the max. BM at the section where the shear force is zero.

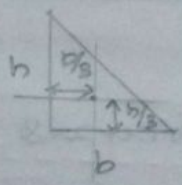
if $SF = 0$

$$\frac{dM}{dx} = 0 \Rightarrow M = max$$

*) SIMPLE SUPPORT WITH TRIANGLE LOAD.

In triangle load, the load is acting at the centroid of triangle.





$\Sigma M_B = 0$

$R_A \times 7 = 10 \times 2 + \frac{1}{2} \times 3 \times 6 \times 6 = 20 + 54$

$= 74$ $(R_A + R_B = \frac{1}{2} \times 3 \times 6 + 10$

$R_A = 10.6$

$R_B = 8.4$

SFD

$F_A = 10.6$

$F_{C(L)} = 10.6 - \frac{1}{2} \times 3 \times 6 = 1.6$

$F_C(R) = 1.6$

$F_D(L) = 1.6$

$F_D(R) = 1.6 - 10 = -8.4$

$F_B(L) = -8.4$

$F_B(R) = -8.4 + 8.4 = 0$

BMD

$M_A = 0$

$M_C = 8.4 \times 4 - 10 \times 2$

$= 10.6$

$M_D = 8.4 \times 2 = 16.8$

$M_B = 0$

$M_C = 8.4 \times 4 - 10 \times 2$

$= 23.4$

$M_D = 8.4 \times 2$

$M_B = 0$

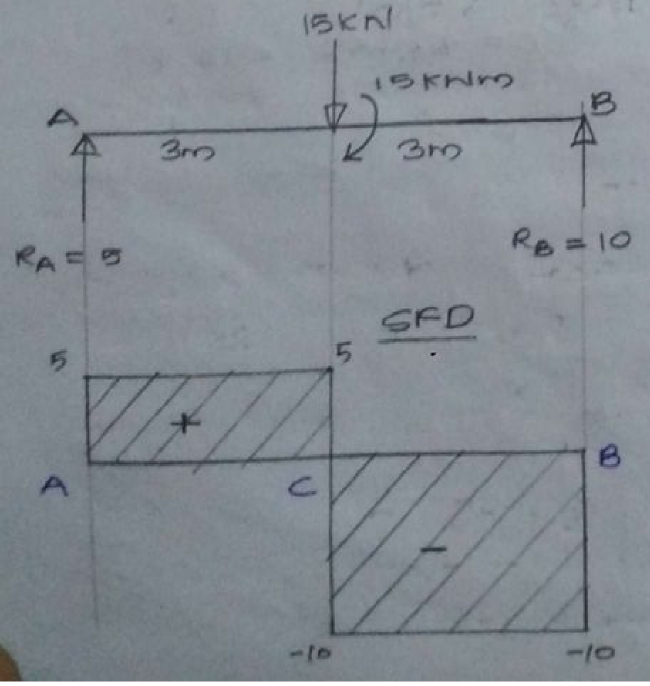
* BEAM SUBJECTED TO EXTERNAL MOMENTS

When a beam is subjected to an external moment as shown in figure. The procedure of drawing SFD & BMD is as follows.

1. Find the reaction by taking moments including the external moment.
2. Draw SFD by ignoring the external moment.
3. While drawing BMD, find the Bending moment on either side of the point where the moment is acting & find the moment ~~including~~ with out the external moment on side & find the moment including external moment on other side. While adding the external moment use the following sign convention.

L → R   ↻ +ve   R → L   ↺ +ve

↺ −ve   ↻ −ve



$R_A + R_B = 15$

$M_B = 0;\ R_A \cdot 6 + 15 = 15 \times 3$

$6R_A = 45 - 15$

$R_A = 5$, & $R_B = 10$

SFD

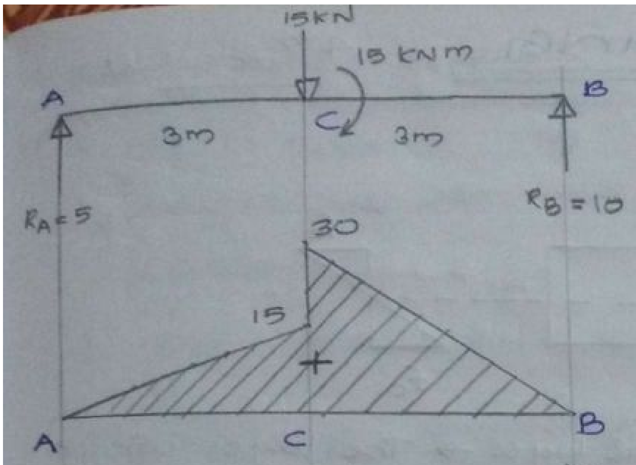
$F_A = 5$

$F_C(L) = 5$

$F_C(R) = 5 - 15 = -10$

$F_B(L) = -10$

$F_B(R) = -10 + 10 = 0$



BMD

$M_A = 0$

$M_C(L) = 5\times3 = 15$

$M_C(R) = 15+15 = 30$

$M_B = 0$

---

*)

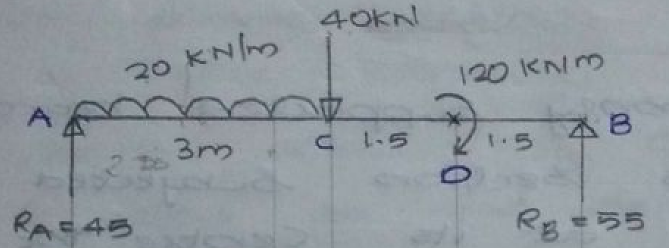
$R_A + R_B = 40 + 3\times20 = 100$

$M_B = 0, \quad 6R_A + \cancel{} + 120 = 40\times3 + 20\times3\times4.5$

$6R_A = 120 + \cancel{540}^{270} - 120$

$R_A = \dfrac{\cancel{540}}{6} = \dfrac{270}{6} = 45$

$R_B = 100 - 45 = 55$



SFD

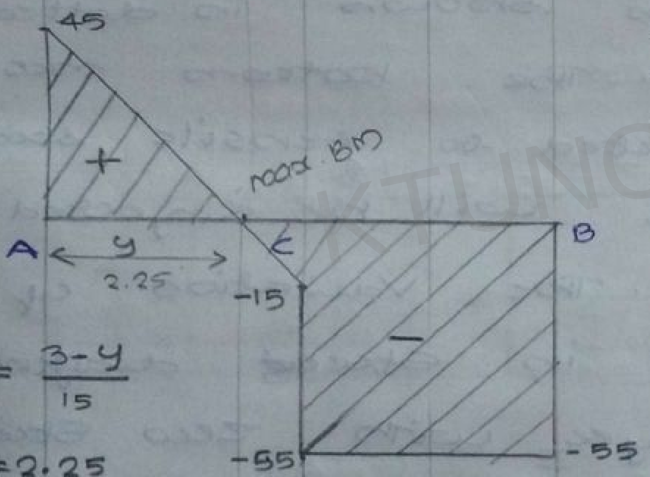
$F_A = 45$

$F_C(L) = 45 - 3\times20 = -15$

$F_C(R) = -15 - 40 = -55$

$F_B(L) = -55$

$F_B(R) = -55 + 55 = 0$



$\dfrac{y}{45} = \dfrac{3-y}{15}$

$y = 2.25$

BMD

$M_A = 0$

$M_C = 3\times45 - 3\times20\times1.5 = 45$

$M_D(L) = 4.5\times45 - 3\times20\times3 - 1.5\times40$

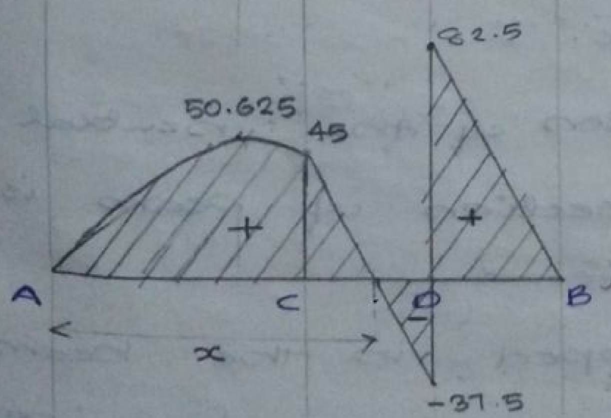
$= -37.5$

$M_D(R) = -37.5 + 120 = 82.5$

$M_B = 0.$

$M_{max} = M_y = 45\times2.25 - \dfrac{20\times2.25\times2.25}{2}$ (UDL)

$= 50.625$



If the shear force is zero then there is max. BM

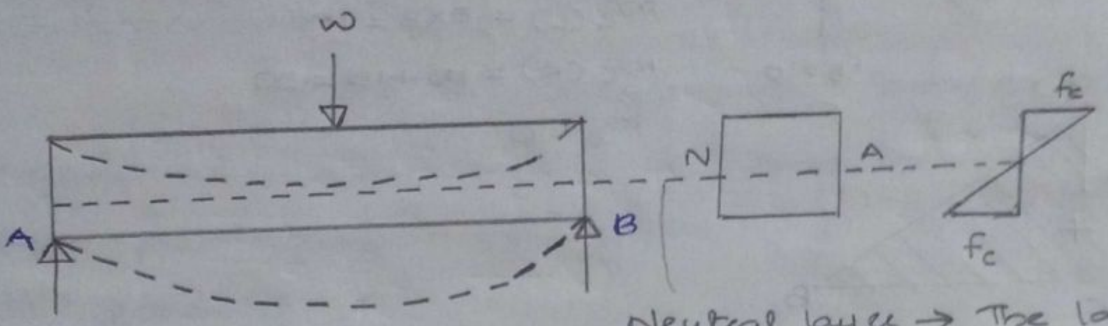
$M_x = 45\times x - 3\times20\times(x-1.5) - 40\times(x-3) = 0$

$45x - 60x + 90 - 40x + 120 = 0$

$210 = 55x$

$x = 3.81\ m$

* THEORY OF BENDING   4th module



Neutral layer → The layer which have zero stress.

Neutral Axis →

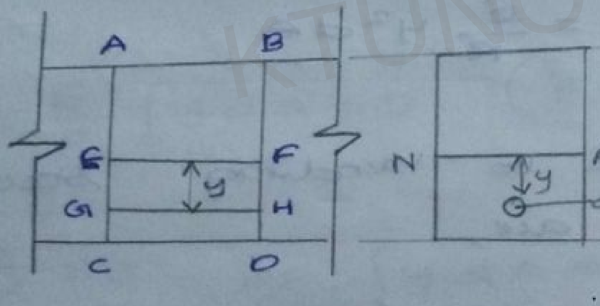
Consider a simply supported beam of rectangular cross section subjected to a point load W at its centre. The beam will bend as shown in dotted lines in the figure. The bottom most layer will be subjected to tensile stress & top most layer will be subjected to compressive stress. The variation of stresses is shown in stress diagram. There will be a layer with zero stress in b/w top & bottom. This layer is called neutral layer.

The line of intersection of the neutral layer with cross section of beam is called neutral axis.

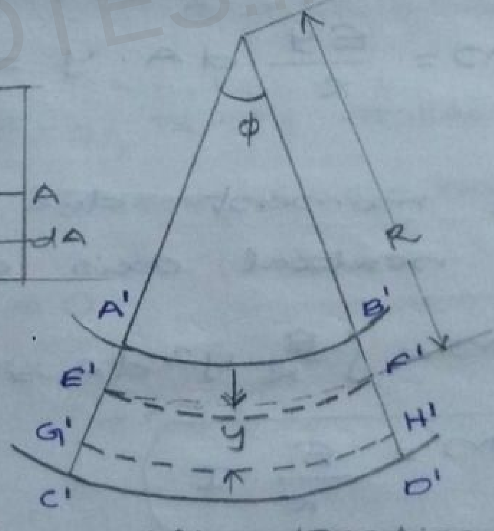
The stresses developed in the beam due to bending are called bending stresses.

Assumptions in theory of bending

1. The beam is initially straight & every layer of it is free to expand or contract.
2. The material is homogeneous & isotropic.
3. Youngs modulus is same in tension & compression.
4. Stresses are with in elastic limit.
5. plane sections remains plane even after bending. (ie, c/s is same)
6. The radius of curvature is large compared to the depth of beam.



Before bending



After bending

$l = r\theta$

$GH = EF = E'F' = R\phi$

$G'H' = (R+y)\phi$

$G'H' - GH = (R+y)\phi - R\phi$

$= R\phi + y\phi - R\phi = y\phi$

Strain in $GH = \dfrac{G'H' - GH}{GH} = \dfrac{y\phi}{EF} = \dfrac{y\phi}{R\phi} = \dfrac{y}{R}$

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Strain} = \frac{\text{Stress}}{E} = \frac{y}{R}$$

$f \rightarrow$ Stress

$$\text{Stress} = E \cdot \frac{y}{R} = f$$

$$\frac{f}{y} = \frac{E}{R} \quad \text{--- (1)}$$

let 'dA' will be an elementary area on the c/s of the beam at a distance 'y' from neutral axis. The force on this area is,

$$F = \text{Stress} \cdot \text{area}$$

$$= f \cdot dA$$

$$F = \frac{Ey}{R} \cdot dA$$

moment about neutral axis

$$M = \frac{Ey}{R} \cdot dA \cdot y = \frac{E}{R} y^2 dA$$

total moment due to bending stress about neutral axis are,

$$M = \int \frac{E}{R} y^2 dA = \frac{E}{R} \int y^2 dA$$

$$\boxed{M = \frac{E}{R} \cdot I}$$

; I - moment of Inertia of c/s about neutral axis.

it is balanced by Bending

since this is resisting moment of bending moment at section (M)

$$M = \frac{E}{R} \cdot I$$

$$\frac{M}{I} = \frac{E}{R} \quad \text{--- (2)}$$

comparing eq^n (1) & (2) ;

$$\boxed{\frac{f}{y} = \frac{E}{R} = \frac{M}{I}}$$

This eq^n is known as <u>bending eq^n</u>.

→ <u>To proove that neutral axis passes through the centroid of the c/s area.</u>

we have the force on the elimentary area dA,

$$\frac{f}{y} = \frac{E}{R}$$

$$F = dA \cdot f$$
$$= dA \cdot \frac{E}{R} \cdot y$$

total force on neutral axis;

$$F = \int \frac{E}{R} y \, dA = \frac{E}{R} \int y \, dA$$

$$\frac{E}{R} \int y \, dA = 0$$ ($F = 0$, since there is no stress in neutral axis).

$$\frac{E}{R} \neq 0 \quad \therefore \int y \, dA = 0$$

(The first moment of area about any axis there is centroid, then it is zero)

since, $\int y \, dA = 0$, the neutral axis passes through the centroid of the cross - sectional area.

* Moment Resisting Capacity of a given Section (moment of resistance)

If 'f' is the max permissible stress of the material of the beam then $f \cdot \frac{I}{y}$ is the max. moment the section can resist.

ie, moment of resistance ;

$$M = f_{permissible} \cdot \frac{I}{y}$$

A beam resist the max value of moment $\{ M = f_{per} \cdot \frac{I}{y}$

We have to find the moment of resistance on top & bottom of beam & min. value will be moment of resistance

$\frac{I}{y}$ = modulus of section. (Z)

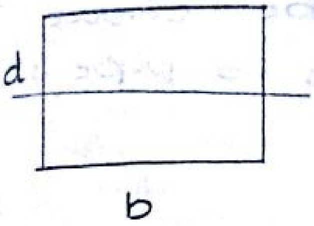
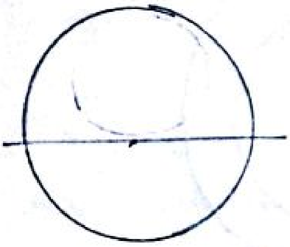
$$\boxed{M = f \cdot Z}$$

*

max. bending moment of a point load = $\frac{wl}{4}$ $\left(\frac{w}{2} \cdot \frac{l}{2}\right)$

max. bending moment of a UDL = $\frac{wl^2}{8}$

* MOMENT OF INERTIA

Figure	I	y	Z
	$\frac{bd^3}{12}$	$\frac{d}{2}$	$\frac{bd^2}{6}$ $\frac{\frac{bd^3}{12}}{d/2} = \frac{bd^2}{6}$
	$\frac{\pi d^2}{64}$ or $\frac{\pi r^2}{4}$	$\frac{d}{2}$	$\frac{\pi d^3}{32}$ $\frac{\frac{\pi d^4}{64}}{d/2} = \frac{\pi d^3}{32}$

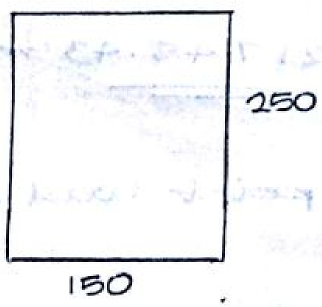
* A simply supported beam of span 5m has a c/s 150 x 250 mm. If the permissible stress is 10 N/mm². Find the max. intensity of UDL it can carry?

always നീളം x വീതി

breadth x width (bxd)

Ans) $l = 5m$

max. Intensity of UDL $= \frac{wl^2}{8}$



$$Z = \frac{I}{y}$$

$$Z = \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{bd^2}{6}$$

$$= \frac{150 \times 250^2}{6} = 1562500$$

$$M = f \cdot Z = 10 \times 1562500 = 15625000 \text{ Nmm}$$

$$M = 15625 \text{ Nm}$$

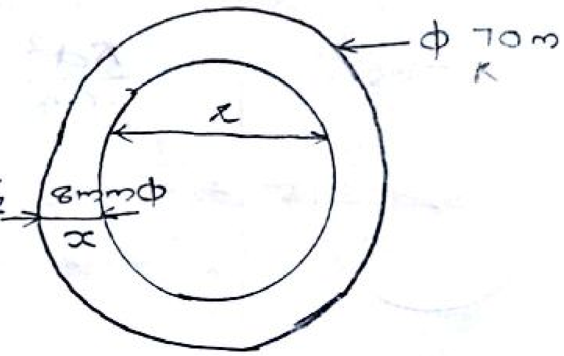
$$\frac{wl^2}{8} = M \Rightarrow w = \frac{M8}{l^2} = \frac{15625 \times 8}{5^2} = \underline{5000} \text{ N/m}$$

*) A circular pipe of external dia 70 mm & thickness 8 mm is used as a simply supported beam over an effective span 2.5 m. Find the max. concentrated load that can be applied at the centre of span if the permissible stress in pipe is 150 ~~mm~~ N/mm^2.

Ans). moment of resistance, $M = f \cdot Z$

$$Z = \frac{I}{y} =$$

$$Z = \frac{\pi d^3}{32}$$



~~Z = Z of outer circle − Z of inner circle~~

$$\cancel{= \frac{\pi d_1^3}{32} - \frac{\pi d_2^3}{32}}$$

$$\cancel{= \frac{\pi}{32}(70^3 - 54^3)}$$

$$\cancel{= 18214.95}$$

$x = R - (x - x) = 70 - 16$
$= 54$ mm

$I = I$ of outer circle $-$ I of inner circle.

$$= \frac{\pi}{64}(70^4 - 54^4) = 761195.3336 \ mm^4$$

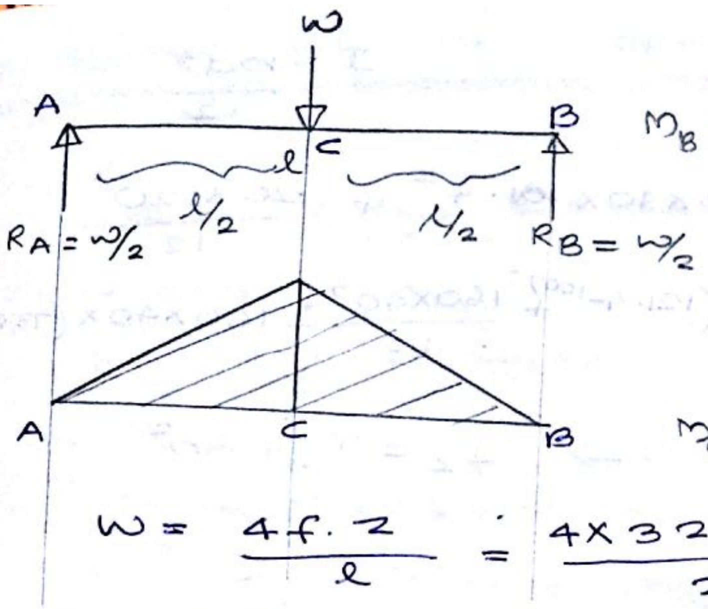
$$Z = \frac{I}{y} = \frac{761195.33}{\left(\frac{70}{2}\right)} = 21748.43 \ mm^3$$

max. bending moment of a point load $= \frac{wl}{4}$

$$M = f \cdot Z = 150 \times 21748.43 = 3262265.715$$

$$f \cdot Z = \frac{wl}{4}$$

$l = 2.5 = 2500$ mm



$R_A + R_B = W$

$M_B = 0$; $W \times \frac{l}{2} - l R_A = 0$

$\frac{Wl}{2} = l R_A$

$R_A = W/2$ & $R_B = W/2$

BMD

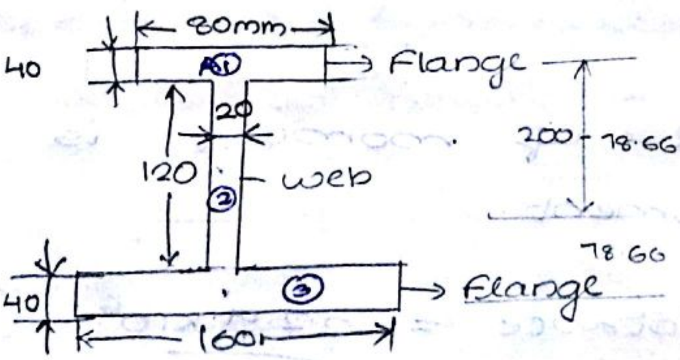
$M_C = \frac{W}{2} \cdot \frac{l}{2} - 0 = \frac{Wl}{4}$

$W = \frac{4 f . Z}{l} = \frac{4 \times 3262265.715}{2500}$

$= \underline{\underline{5219.625 \text{ N}}}$

* A cast iron beam has an I section with top ~~plased~~ flange 80 mm x 40 mm, web 120 mm x 20 mm & bottom flange 160 mm x 40 mm if tensile stress is note to exceed 30 N/mm² & compressive stress 40 N/mm². what is the max UDL the beam can carry over a simply supported span of 6 m if the larger flange is in tension.

Ans)



$A_1 = 80 \times 40 = 3200 \text{ mm}^2$

$A_2 = 120 \times 20 = 2400 \text{ mm}^2$

$A_3 = 40 \times 160 = 6400 \text{ mm}^2$

$\bar{Y}_3 = \frac{40}{2} = 20$

$\bar{Y}_2 = 40 + \frac{120}{2} = 100$

$\bar{Y}_1 = 160 + \frac{40}{2} = 180$

$A\bar{Y} = A_1\bar{y}_1 + A_2\bar{Y}_2 + A_3\bar{y}_3$

$\bar{Y} = \frac{A_1Y_1 + A_2Y_2 + A_3Y_3}{A_1 + A_2 + A_3}$

$= \frac{3200 \times 180 + 2400 \times 100 + 6400 \times 20}{3200 + 2400 + 6400} = \underline{\underline{78.66}}$

$$I = \frac{bd^3}{12}$$

$$I = I_1 + I_2 + I_3$$

$$= \frac{80\times40^3}{12} + 80\times40\times121.4^2 + \frac{20\times120^3}{12}$$

$$+ 20\times120\times(121.4-100)^2 + \frac{160\times40^3}{12} + 160\times40\times(78.6$$

Parallel Axis theorem $\Rightarrow$ $I_z = I_G + Ah^2$

$$= \underline{\underline{60.13\times10^6}}$$

$$Z_t = \frac{60.13\times10^6}{78.6} = 765.01\times10^3$$

$$Z_c = \frac{60.13\times10^6}{121.4} = 495.3\times10^3$$

$$M_t = Z_t\times f_t = Z_t\times 30$$

$$M_t = 22950381.68 = 22.9\times10^6$$

$$M_c = Z_c\cdot f_c = Z_c\times 90 = \underline{\underline{44.57}}\times10^6$$

The min. value of moment is value resistance of moment.

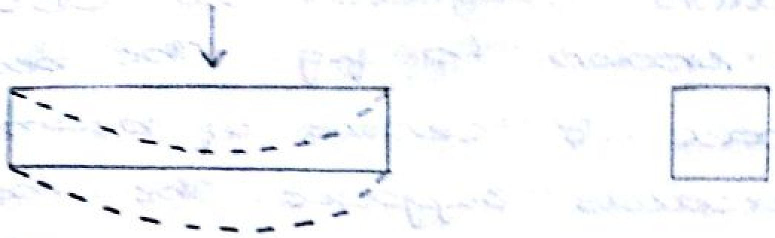
Moment of resistance $= \underline{\underline{22.9\times10^6}}$

$$\frac{wl^2}{8} = 22.95\times10^3 \quad , \quad l=6$$

$$w = \frac{8\times22.95\times10^3}{6^2} = \underline{\underline{5.1\times10^3}}\ N$$

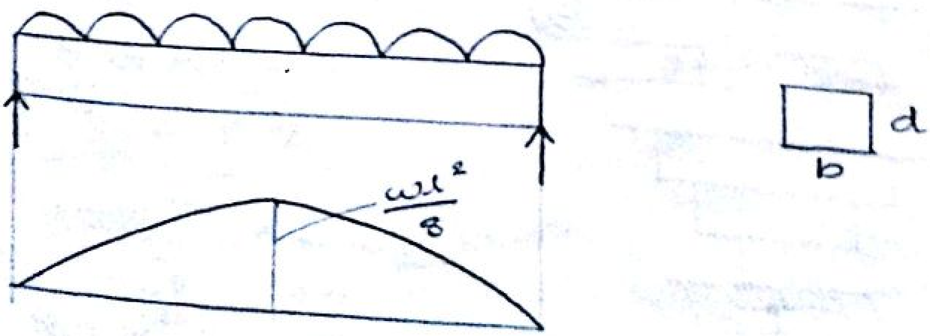
max. load of UDL $= 5100\ N/m$

* LIMITATIONS OF BENDING THEOREM



- In deriving the bending eq^n we have assumed that bending takes place in a vertical plane. It is possible only if the c/s of the beam is symmetrical about vertical axis. If the c/s is not symmetrical about 'y' axis the bending can takes place in any plane inclined vertical plane. Such bending is called Unsymmetrical bending. ∴ bending eq^n can be applied only in symmetrical bending. ie, only if c/s of beam is symmetrical about vertical axis. This is the main limitations of bending theory.

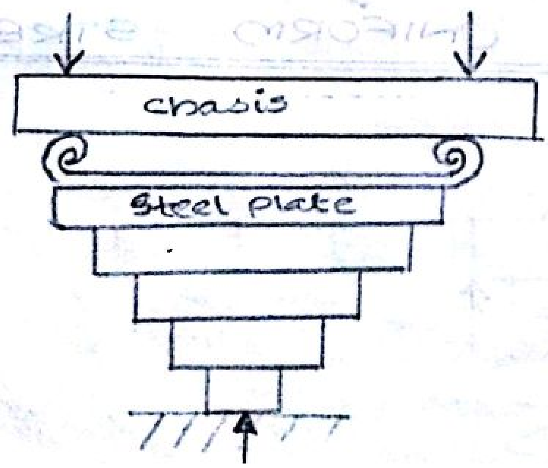
* BEAMS OF UNIFORM STRENGTH.



$M = f \cdot Z$

Consider a simply supported beam of rectangular section subjected to UDL for full length as shown in fig. The bending moment is max at centre of beam & get reduce towards support. The max bending moment is $\frac{wl^2}{8}$. We usually design the beam for max bending moment & provide the design section for the full length of beam. This is uneconomical since there is waste of material, since full section is provided at places where bending moment is less. To overcome this waste we can provide reduce section towards the support based on the BM at each section. Such beam of varying cross section with max size at centre & getting towards support is called Beam of Uniform Strength. Reduction can be made in width or depth of beam.

*) Eg :- Shock absorber or leaf spring.



A practical example of Beam of uniform strength is shock absorber used in railway waggon, truck etc. Steel plates of diff length are placed one over other in a shape of a curve.

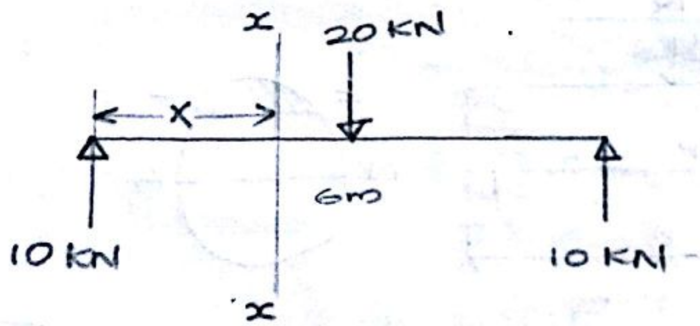
Since the BM is max at centre, max thickness is provided at the centre. Since the BM is zero at ends the min. thickness is provided at the support.

Q) Determine the c/s of a rectangular beam of uniform strength for a singly supported beam of 6m span subjected to a central concentrated load of 20 kN.

a) By keeping a depth of 300mm throughout.
b) By keeping a width of 200 throughout.

The permissible stress is 8 N/mm².

Ans)



$$Z = \frac{I}{y} = \frac{\frac{bd^3}{12}}{d/2}$$

$M_x = 10x$ $\qquad$ $Z = \frac{I}{y}$

$M = f \cdot Z$

$10x = 8 \times \frac{1}{6} bd^2$

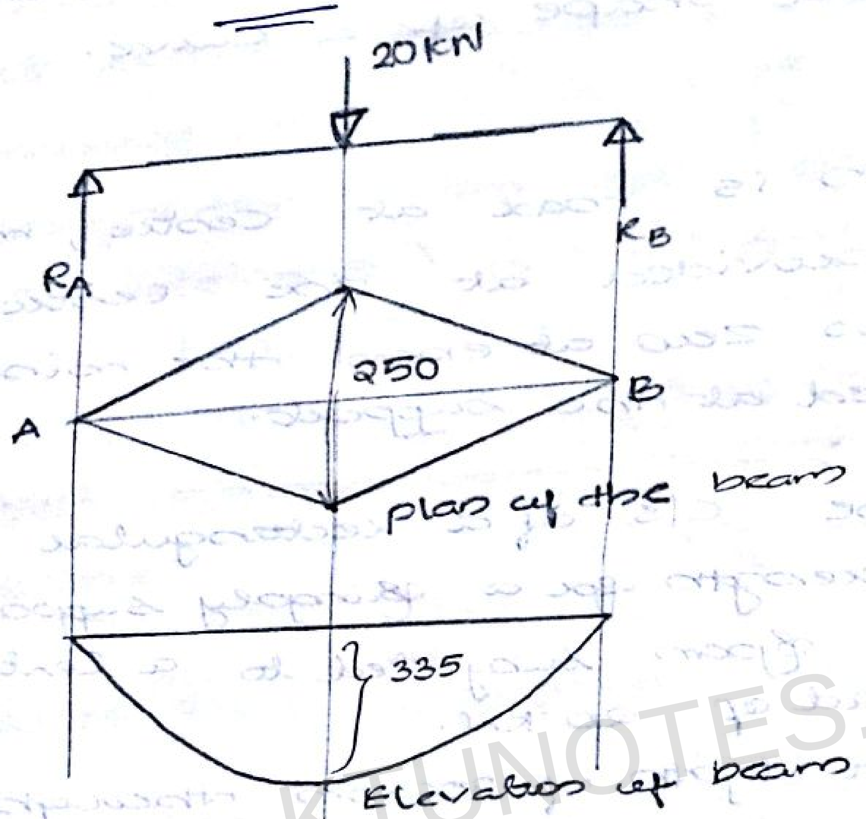
a) $10 \times 10^3 \times 3000 = 8 \times \frac{1}{6} \times b \times 300^2 \Rightarrow b = 250$

b) $b = 200$

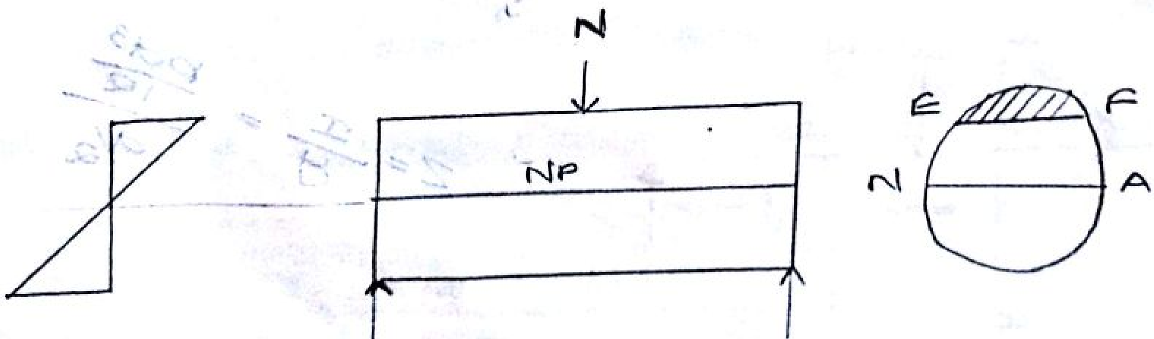
$10 \times 10^3 \times 3000 = 8 \times \frac{1}{6} \times 200 \times d^2$

$d^2 = 112500$

$d = \underline{\underline{335.4}}\ mm$



* Shear stress distribution in beams :-



shear stress on any layer, $q = \dfrac{F A \bar{y}}{I b}$

F - shear force

A - Area of C/S above the layer

$\bar{y}$ - Distance of centroid of area above layer from neutral axis.

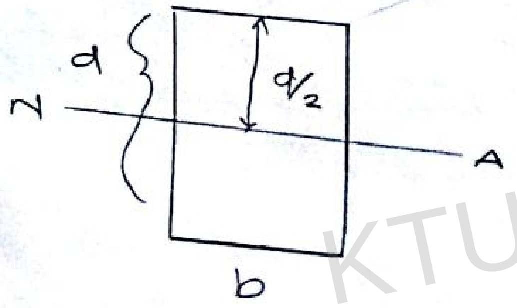
I – Moment of Inertia of c/s about NA.

b → width of section (EF)

From the above expression can be noticed that the shear stress is zero at top & bottom level & max at neutral axis.

Since $A\bar{y}$ is a product of 2 variables the variation of shear stress is parabolic.

→ Rectangular beam

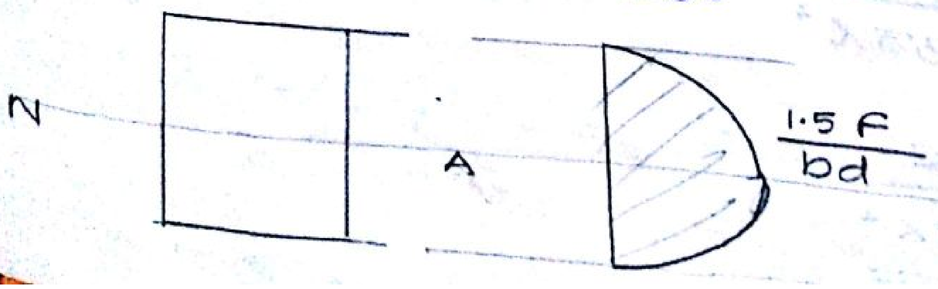


Shear stress at NA,

$$q_{max} = \frac{F \cdot b \cdot \frac{d}{2} \cdot \frac{d}{4}}{\frac{bd^3}{12} \cdot b}$$

$$= \frac{3F}{2bd}$$

$$= \frac{1.5\,F}{bd}$$

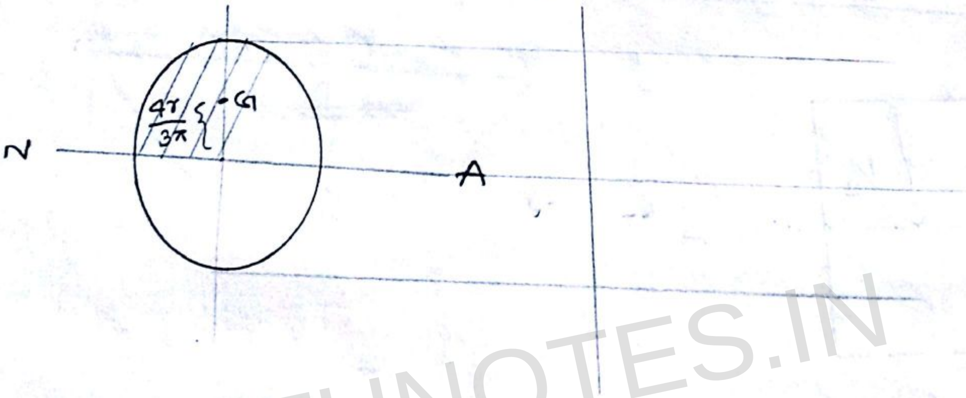


~~avg~~ shear stress ratio $= \dfrac{\text{max shear stress}}{\text{mean shear stress}}$

$$= \frac{\frac{(1.5)F}{bd}}{\frac{F}{bd}} = 1.5$$

$$q = \frac{FA\bar{y}}{Ib}$$

b) CIRCULAR SECTION



$$q = \frac{FA\bar{y}}{Ib}$$

$$= \frac{F \cdot \frac{\pi r^{\cancel{2}}}{\cancel{2}} \cdot \frac{4\cancel{r}}{3\pi}}{\frac{\cancel{\pi} r^{\cancel{4}\,2}}{\cancel{4}\,2} \cdot \cancel{2}\cancel{r}}$$

$$\frac{\pi r^4}{4}$$

$= ~~16~~$

$$= \frac{8F}{2 \times 3\pi r^2}$$

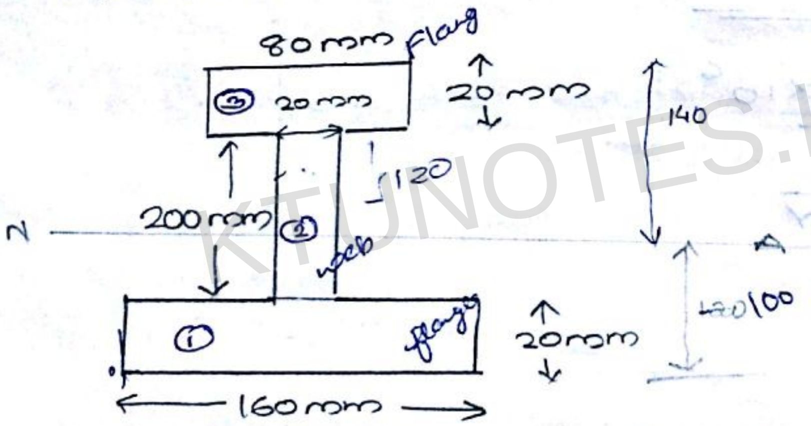
$$= \frac{4F}{3\pi r^2}$$

$$\frac{\text{max shear stress}}{\text{avg shear stress}} = \frac{\frac{4F}{3\pi r^2}}{\frac{F}{\pi r^2}} \quad \left(\frac{F}{A}\right)$$

exam Question → $= \frac{4}{3} = 1.33$

ST max shear stress in a circular section is 1.33 times avg shear stress

* Find the max shear stress and draw the shear stress distribution diagram for the I section given below if it is subjected to a shear ~~force~~ ~~stress~~ stress of 40 kN/mm²



Ans) First we find neutral axis (By finding centroid).

$A_1 = 160 \times 20 = 3200$

$A_2 = 200 \times 20 = 4000$

$A_3 = 80 \times 20 = 1600$

$y_1 = \frac{20}{2} = 10$

$y_2 = 20 + \frac{200}{2} = 120$

$y_3 = 220 + \frac{20}{2} = 230$

$$\bar{Y} = \frac{A_1Y_1 + A_2Y_2 + A_3Y_3}{A_1 + A_2 + A_3} = \frac{3200 \times 10 + 4000 \times 120 + 1600 \times 230}{3200 + 4000 + 1600}$$

$= 100$ from bottom

$$I_G = \frac{bd^3}{12}$$

$$I = I_G + A y^2$$

$$I_1 = \frac{160 \times 20^3}{12} + (160 \times 20) \times (100 - 10)^2 =$$

$$= 26026666.67$$

$$I_2 = \frac{20 \times 200^3}{12} + (20 \times 200) \times (100 - 120)^2$$

$$=$$

$$I_3 = \frac{80 \times 20^3}{12} + (80 \times 20) \times (100 - 230)^2$$

$$=$$

$$I = I_1 + I_2 + I_3$$

$$= 68053333.33$$

$$= 68.05 \times 10^6$$

$$q = \frac{FA\bar{y}}{Ib}$$

$$=$$

Bottom of top flange

$$q = \frac{FA\bar{y}}{Ib} = \frac{40 \times 10^3 \times 80 \times 20 \times (230 - 100)}{68.05 \times 10^6 \times 80}$$

$$= 1.528 = 1.53$$

top of web

$$q = \frac{FA\bar{y}}{Ib} = \frac{40 \times 10^3 \times 80 \times 20 \times (230 - 100)}{68.05 \times 20 \times 10^6}$$

$$= 6.11$$

$q_{max} =$

at neutral axis, q is max

$$= \frac{40\times10^3 (80\times20\times(230-100) + 120\times20\times60)}{68.05\times10^6 \times 20}$$

$= 10.35 \ N/mm^2$

Top of bottom flange

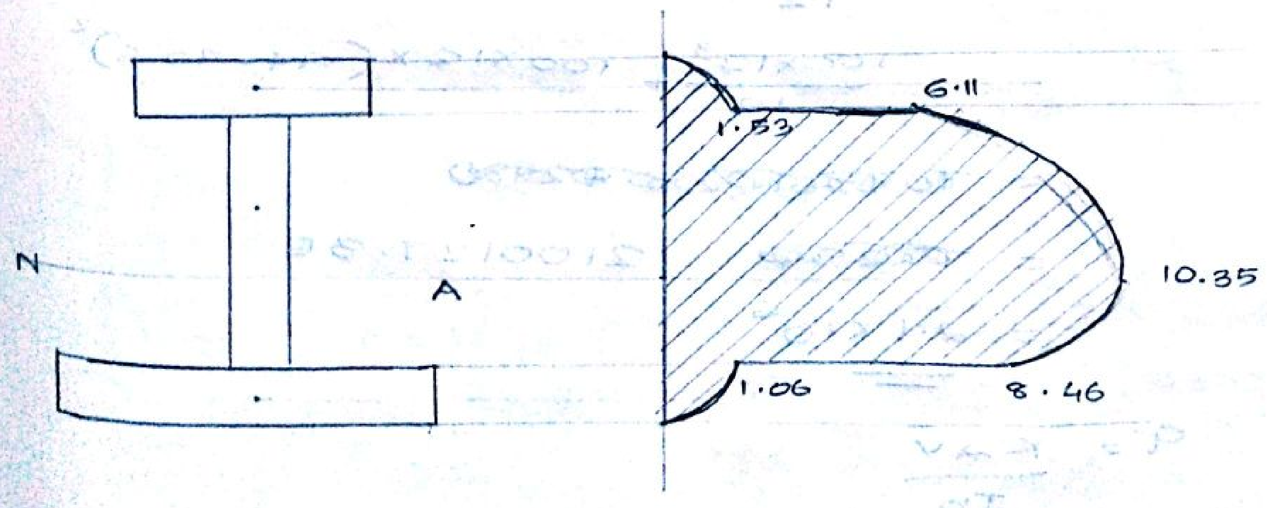
$$q = \frac{FA\bar{y}}{Ib} = \frac{40\times10^3 \times 160\times20\times90}{68.05\times10^6 \times 160}$$

$= 1.058 = 1.06 \ N/mm^2$

Bottom of web.

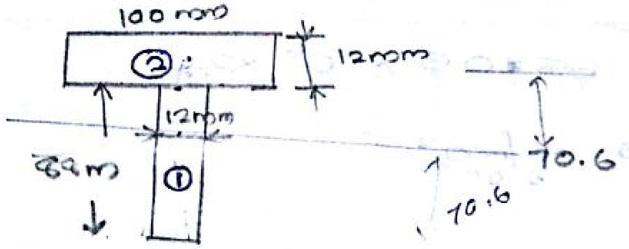
$$q = \frac{FA\bar{y}}{Ib} = \frac{40\times10^3 \times 160\times20\times90}{68.05 \times 20}$$

$= 8.46 \ N/mm^2$



SSDD

*) Draw the shear stress distribution diagram shown the below T section if it is subjected to a shear force of 20 kN.



Ans)
$A_1 = 88 \times 12 = 1056 \text{ mm}^2$ $\quad Y_1 = \frac{88}{2} = 44 \text{ mm}$

$A_2 = 100 \times 12 = 1200 \text{ mm}^2$ $\quad Y_2 = 88 + \frac{12}{2} = 94 \text{ mm}$

$$\bar{Y} = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2} = \frac{1056 \times 44 + 1200 \times 94}{1056 + 1200}$$

$= 70.59 = 70.6 \text{ mm}$

$I = I_G + Ah^2$ $\quad I_G = \frac{bd^3}{12}$

$$I = I_1 + I_2 = \frac{88 \times 12^3}{12} + 88 \times 12 \times (70.6 - 44)^2 + \frac{100 \times 12^3}{12} + 100 \times 12 \times (94 - 70.6)^2$$

$= 2100127.36$

$= 2.1 \times 10^6$

$$q = \frac{FA\bar{Y}}{Ib}$$

a) Bottom of Flange

$$q = \frac{FA\bar{y}}{Ib} = \frac{F\cancel{A}\bar{y}}{Ib}$$

$$= \frac{20\times10^3 \times 100\times12 \times (94-70.6)}{2.1\times10^6 \times 100}$$

$$= 2.67 \text{ N/mm}^2$$

b) ~~Bottom~~ Top of web

$$q = \frac{FA\bar{y}}{Ib}$$

$$= \frac{20\times10^3 \times 12\times100 \times (94-70.6)}{2.1\times10^6 \times 12}$$

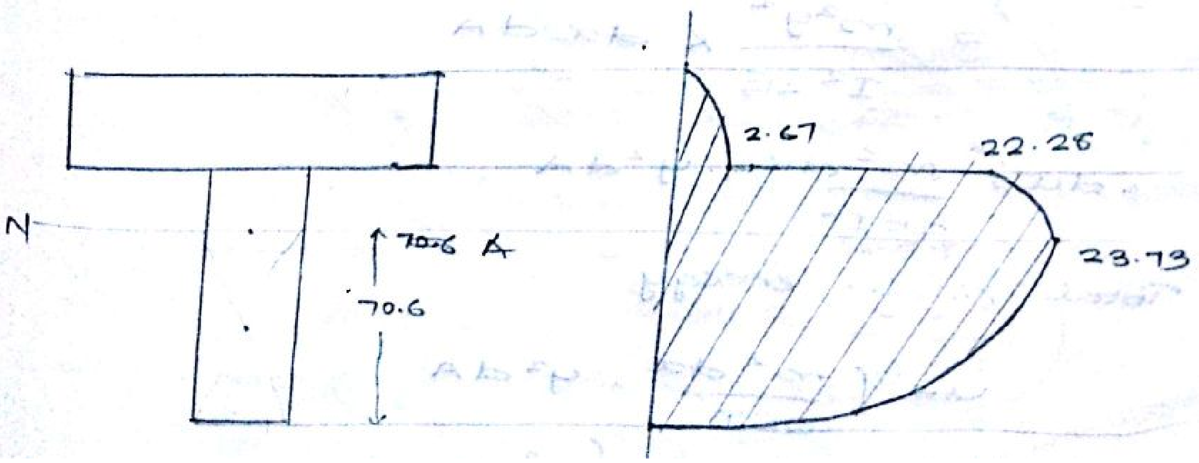
$$= 22.28 \text{ N/mm}^2$$

c) max q at Neutral axis

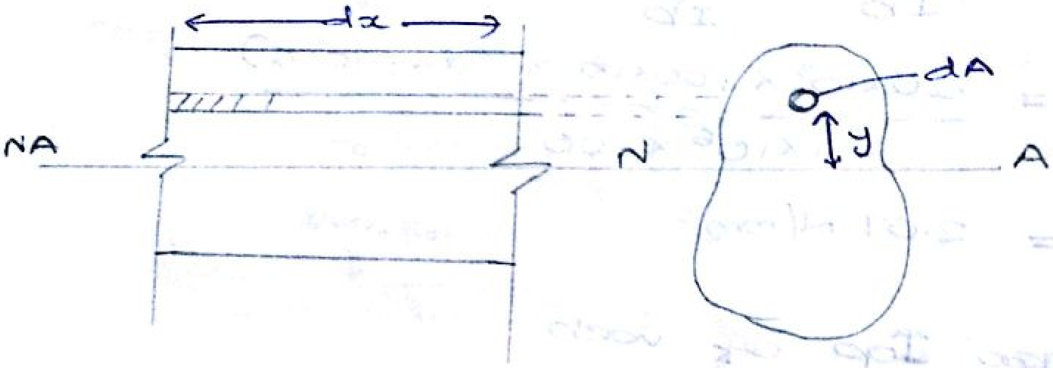
$$q_{max} = \frac{FA\bar{y}}{Ib}$$

$$= \frac{20\times10^3 \times 70.6\times12 \times \left(\frac{70.6}{2}\right)}{2.1\times10^6\times12}$$

$$= 23.73 \text{ N/mm}^2$$



* STRAIN ENERGY DUE TO BENDING



Consider an elementary strip of length dx & c/s area dA at a distance 'y' from neutral axis of a beam subjected to bending. This strain energy is store in the elementary strip du,

$$du = \frac{f^2}{2E} \cdot Volume \qquad \frac{f}{y} = \frac{M}{I}$$

$$f = \frac{M}{I} \cdot y$$

$$du = \frac{\left(\frac{M}{I}\right)^2 y^2}{2E} \cdot Volume$$

$$= \frac{M^2 y^2}{I^2 2E} \times dx\, dA$$

$$du = \frac{M^2 dx}{2EI^2} \cdot y^2 dA$$

Total strain energy

$$U = \int \frac{M^2 dx}{2EI} \cdot y^2 dA$$

$$= \frac{M^2 dx}{2EI^2} \int y^2 dA$$

$$u = \frac{M^2\,dx}{2EI^2}\, I$$

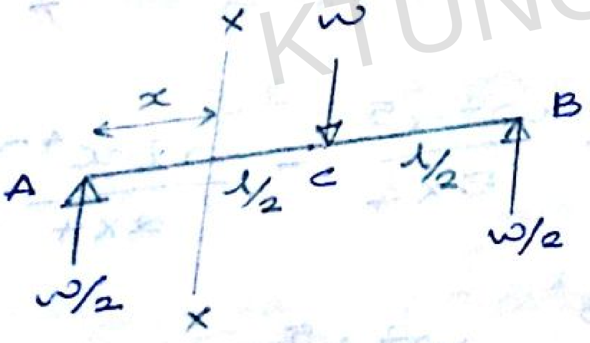
$$= \frac{M^2\,dx}{2EI}$$

$$\boxed{SE \text{ in beam} = \int_0^l \frac{M^2\,dx}{2EI}}$$

$$= \left.\frac{M^2 x}{2EI}\right]_0^l$$

$$\boxed{u = \frac{M^2 L}{2EI}}$$

* STRAIN ENERGY IN SIMPLY SUPPORTED BEAM WITH CENTRAL POINT LOAD



$$M_x = \frac{W}{2}x$$

$$u = 2\int_0^{l/2} \frac{M^2}{2EI}\,dx$$

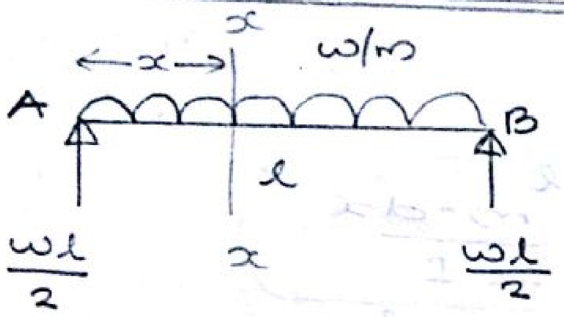
$$u = \frac{1}{EI}\int_0^{l/2} \left(\frac{W}{2}x\right)^2 dx$$

$$= \left.\frac{W^2}{4EI}\,\frac{x^3}{3}\right]_0^{l/2}$$

$$= \frac{W^2}{12EI}\left(\frac{l}{2}\right)^3$$

$$\boxed{u = \frac{W^2 l^3}{96EI}}$$

* Strain energy in simply supported beam with UDL for full length



$$M_x = \frac{wl}{2}x - wx.\frac{x}{2} = \frac{wlx}{2} - \frac{wx^2}{2}$$

$$U = \int_0^l \frac{M^2}{2EI}\,dx$$

$$= \int_0^l \frac{\left(\frac{wlx}{2} - \frac{wx^2}{2}\right)^2}{2EI}\,dx$$

$$w^2lx^3$$

$$= \frac{1}{2EI}\int_0^l \frac{w^2l^2x^2}{4} + \frac{w^2x^4}{4} - \cancel{2}\,\frac{wlx}{\cancel{2}}\cdot\frac{wx^2}{2}\,dx$$

$$= \frac{1}{2EI}\left(\frac{w^2l^2x^3}{3\times4} + \frac{w^2x^5}{5\times4} - \frac{wlx^4}{2\times4}\right)_0^l$$

$$= \frac{1}{2EI}\left(\frac{wl^2\,l^3}{12} + \frac{w^2l^5}{20} - \frac{wl.\,l^4}{8}\right)$$

$$= \frac{w^2l^5}{2EI}\left(\frac{1}{12} + \frac{1}{20} - \frac{1}{8}\right)$$

$$= \frac{w^2l^5}{2EI}\cdot\frac{1}{120}$$

$$= \frac{w^2l^5}{240EI}$$

Hw) Find SE

$$\frac{w^2 l^3}{6EI}$$

$$\frac{w^2 l^5}{40EI}$$

* strain energy in cantilever beam with point load.

$F_A = w$

$F_B = w$

$M_B = 0$

$M_A = -wl$

$M_x = wl = wx$

$$M_x = wx$$

$$u = \int_0^l \frac{M^2}{2EI}\,dx$$

$$= \int_0^l \frac{w^2 x^2}{2EI}\,dx$$

$$= \frac{w^2}{2EI}\int_0^l x^2\,dx$$

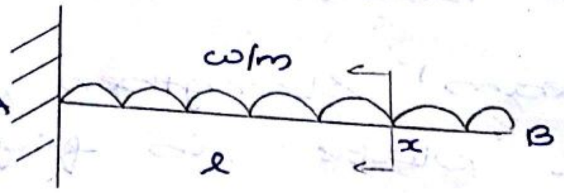
$$= \frac{w^2}{2EI}\left(\frac{x^3}{3}\right)_0^l$$

$$= \frac{w^2 \cdot l^3}{6EI}$$

$$\cancel{= \frac{w^2}{2EI} \times l^5 \cdot \frac{8}{15}}$$

* strain energy in Cantilever beam with UDL for full length

*



$$M = \frac{wx^2}{2} \quad \left(\because \; w \cdot x \times \frac{x}{2} \right)$$

$$U = \int_0^l \frac{M^2}{2EI} \, dx$$

$$= \frac{1}{2EI} \int_0^l \frac{w^2 x^4}{4} \, dx$$

$$= \frac{w^2}{8EI} \left. \frac{x^5}{5} \right)_0^l$$

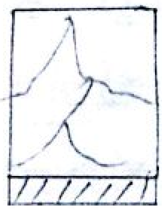
$$\boxed{U = \frac{w^2 l^5}{40EI}}$$

2016 ; FRIDAY

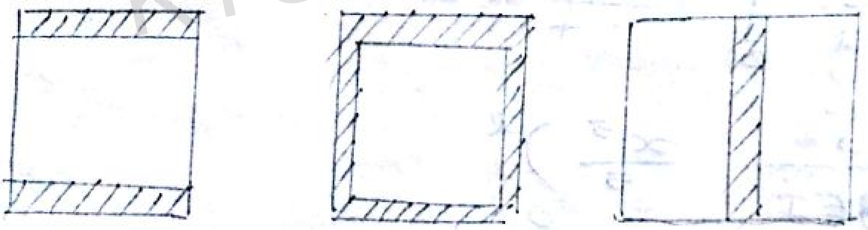
COMPOSITE BEAMS (Flichted beams)

A beam of one material strengthen with another stronger material to increase its moment of resistance is called composite beam.

eg:- A wooden beam strengthen by providing a steel plate in fig ①



→ strengthening the beam by providing steel plates as shown;



Let f_s be the stress in steel & f_w is stress in wood in the composite beam. Since steel is rigidly fixed with wood the strain in steel will be equal to strain in wood.

ie, $e_s = e_w$

$$\frac{f_s}{E_s} = \frac{f_w}{E_w}$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{f_s}{e_s}$$

$$f_s = \frac{E_s}{E_w} \cdot f_w$$

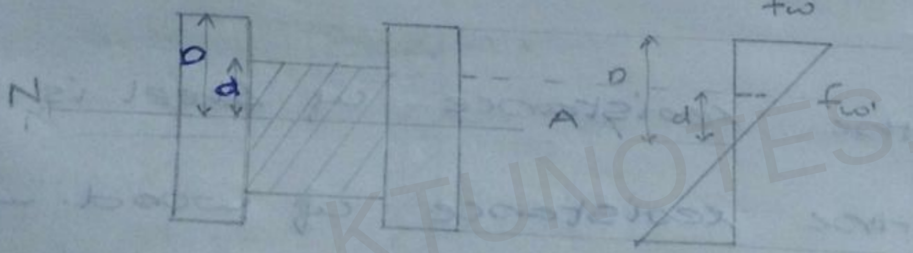
larger young modulus / smaller young modulus

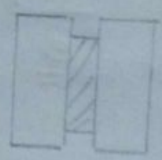
The ratio $\frac{E_s}{E_w}$ is called modular ratio (m)

$$f_s = m \cdot f_w$$

ie, The stress in steel is 'm' times the stress in wood.

* Consider a composite beam with wood & steel as shown in figure.





Let f_w be the max stress in wood & $f_{w'}$ be the stress in wood at depth 'd' & f_s be the max stress in steel,

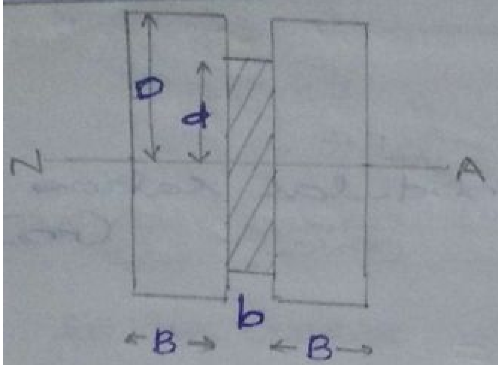
$$\frac{f_w}{f_{w'}} = \frac{D}{d} \qquad\qquad f_s = m \cdot f_w$$

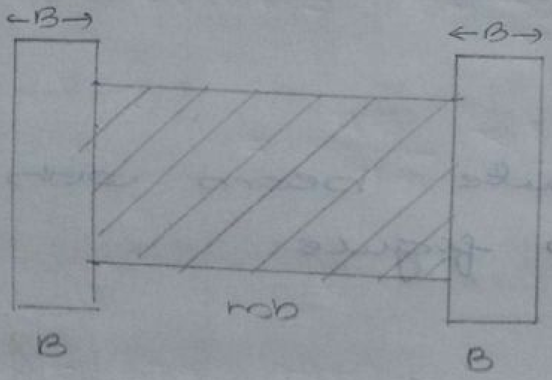
$$f_w = \frac{D}{d} f_{w'} \qquad\qquad f_w = \frac{f_s}{m}$$

$$f_w = \frac{D}{d} \frac{f_s}{m}$$

$$\cancel{f_s = m \frac{D}{d} f_w} \qquad\qquad f_s = m \frac{d}{D} f_w$$

- TRANSFORMED SECTION





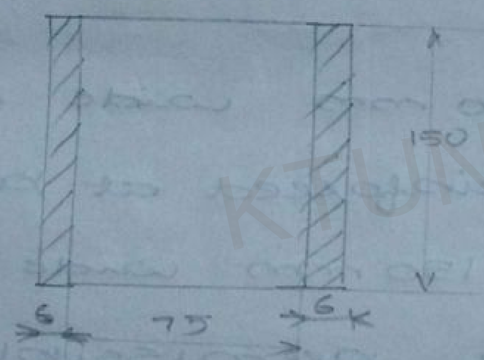
Since the resistance of steel is 'm' times the resistance of wood. if c/s of composite section can be transform in to c/s of wood alone by increasing the width of steel 'm' times as shown above. and stresses can be found by treating the section is section of wood alone. such a section is called transformed section. Calculation of stresses becomes easy if transformed section is used to find stresses.

moment of resistance of any

composite section is equal to the sum of the moment of resistance of the two sections are used.

*) A timber beam of width 75 ~~cm~~ mm & ~~breadth~~ depth 150 mm is strengthen with a 6 mm of steel plate along the longer sides. If the bending stresses in composite beams are to be limited to 115 N/mm² in steel & 9.5 N/mm² in timber. Find the MR of the beam. given $E_S = 20 \times E_T$

Ans)



$E_S = 20 \times E_T$, $E_S = m \cdot E_T$

$m = 20$

$f_S = 115 \text{ N/mm}^2$

$f_w = 9.5 \text{ N/mm}^2$

$f_S = m \cdot f_w = 20 \cdot f_w$

$f_w = \frac{f_S}{20} = \frac{115}{20} = 5.75$ $\Rightarrow f_w < f_w'$ (safe)

If $f_w = 9.5$

$f_S = m \cdot f_w = 20 \times 9.5 = 190$ (Not safe)

MR for Timber = $f \cdot \frac{I}{y}$

$$= \frac{5.75}{75} \times \frac{75 \times 150^3}{12} \quad \frac{bd^3}{12}$$

$$=$$

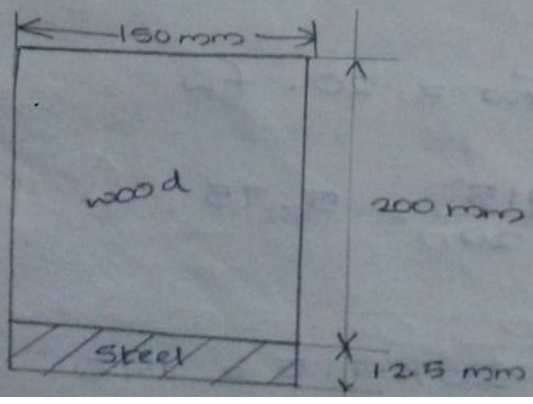
MR for steel = $f \cdot \frac{I}{y}$

$$= \frac{115}{75} \times \frac{12 \times 150^3}{12}$$

MR = 6792 Nm

* A wooden beam 150 mm wide and 200 mm deep is reinforced at bottom by a steel plate 150 mm wide & 1.25 mm deep. If the permissible stress in wood & steel are 9 N/mm² & 120 N/mm² resp. Find the moment of resistance of the section. Given E_s = 210 GPa & E_w = 14 GPa

Ans).

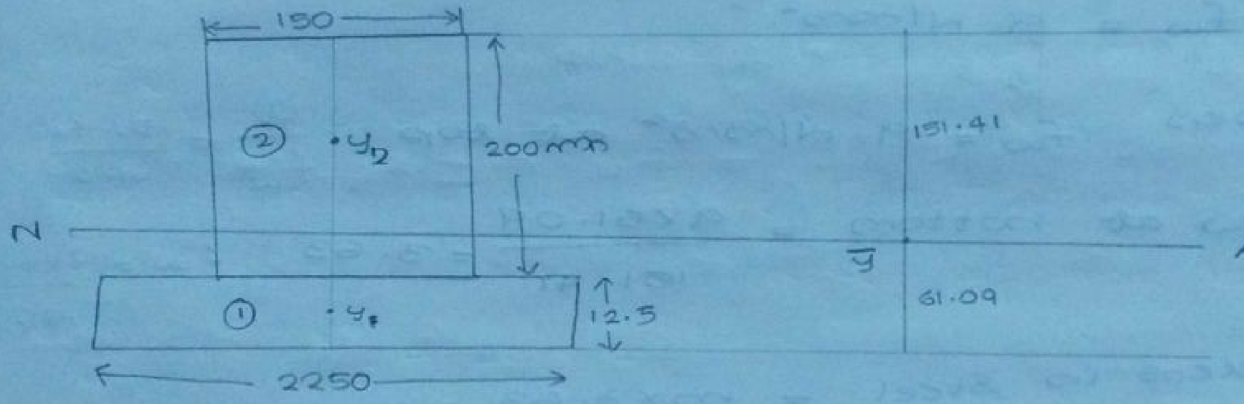


→ By using transformed section method,

$E_s = 210$ GPa , $E_w = 14$ GPa

$E_s = m\,E_w$

$m = \frac{E_s}{E_w} = \frac{210}{14} = 15$ $\therefore mb = 15 \times 150 = 2250$ mm



Moment of resistance, $M = f \cdot \frac{I}{\bar{y}}$

$A_1 = 2250 \times 12.5 = 28125$ mm²

$A_2 = 200 \times 150 = 30000$ mm²

$y_1 = \frac{12.5}{2} = 6.25$ mm

$y_2 = 12.5 + \frac{200}{2} = 112.5$ mm

$\bar{y} = \frac{A_1 Y_1 + A_2 Y_2}{A_1 + A_2}$

$= 61.09$ from bottom.

From top, $\bar{y}_T = 200 + 12.5 - 61.09 = 151.41$ mm

$I_1 = I_G + A h_1^2$ $\frac{bd^3}{12} +$

$= \frac{150 \times 200^3}{12} + 150 \times 200 \times 51.41^2$

$= 174334040.3$ mm⁴.

$$I_2 = \frac{2250 \times 12.5^3}{12} + 30000 \times 54.84^2$$

$$= 90588978.94 \, mm^4$$

$$I = I_1 + I_2 = 2649231019.2 \, mm^4 = 265 \times 10^6 \, mm^4$$

$$f_s = 120 \, N/mm^2$$

$$f_w = 9 \, N/mm^2.$$

when $f_w = 9 \, N/mm^2$ at top $\quad \frac{9}{f'_w} = \frac{151.41}{61.09}$

$\therefore \quad f'_w = \frac{9 \times 61.09}{151.41}$

$$f_w \text{ at bottom} = \frac{9 \times 61.09}{151.41} = 3.63$$

stress in steel $= m \times 3.63 = 15 \times 3.63$

$= 54.45 < 120$ } safe

$$\therefore \; M = f \cdot Z = f \cdot \frac{I}{y}$$

$$= \frac{9 \times 265 \times 10^6}{151.41}$$

$$= \underline{\underline{15751.93}} \, Nm$$