

PH 212: PHYSICS II

Lecture PowerPoints

CAPACITANCE

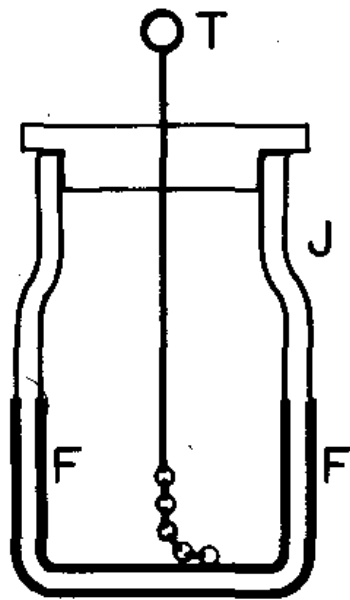
Topics Covered

- Capacitor
- Charging and Discharging Capacitor
- Charging and Discharging Processes
- Capacitors and A.C. Circuits
- Capacitance Definition and Units
- Parallel Plate Capacitor
- Capacitance of Isolated Sphere
- Capacitors in series and parallel
- Charging and discharging a capacitor in C-R circuit
- Energy stored in a capacitor
- Energy and Q — V Graph, Heat Produced in Charging
- Connected Capacitors, Loss of Energy

Capacitor

Capacitor

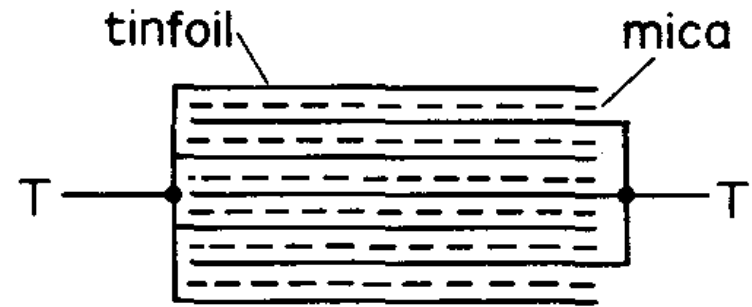
- A capacitor is a device for storing charge.
- The earliest capacitor was invented almost accidentally by van Musschenbroek of Leyden, in about 1746 and became known as a Leyden jar.
- One form of it is shown in Figure 1(i);
 - J is a glass jar, FF are tin-foil coatings over the lower parts of its walls, and T is a knob connected to the inner coating.
 - Modern forms of capacitor are shown at (ii) and (iv) in the figure.
- Essentially *all capacitors consist of two metal plates separated by an insulator.*
 - The insulator is called the dielectric; in some capacitors it is polystyrene, oil or air.
- Figure 1 (iii) shows the circuit symbol for such a capacitor; T T are terminals joined to the plates.



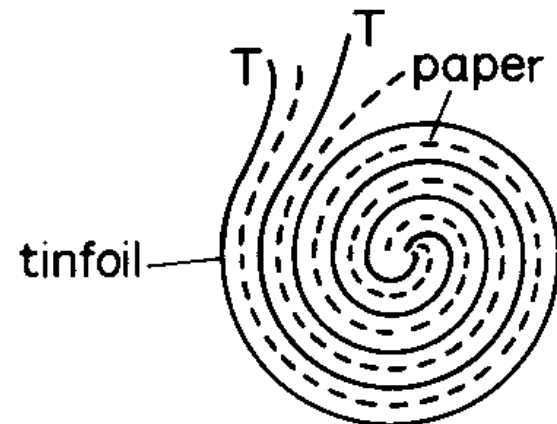
(i) Leyden jar



(iii) conventional symbol



(ii) mica dielectric



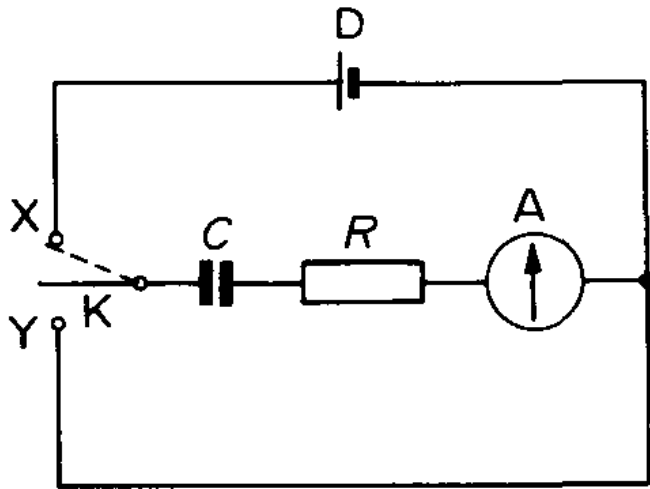
(iv) paraffin-waxed
paper dielectric

Figure 1: Types of capacitor

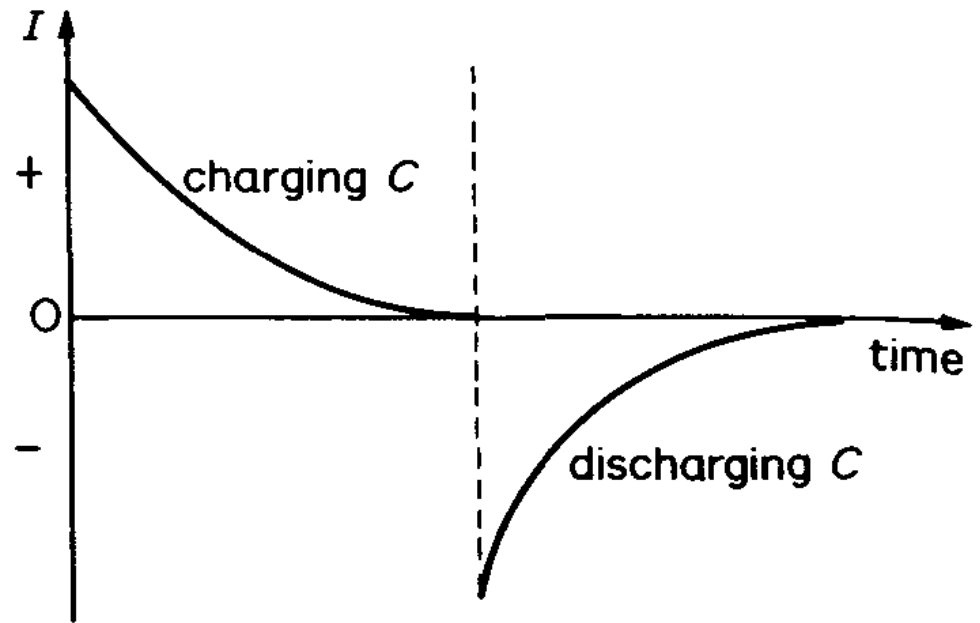
Charging and Discharging Capacitor

Charging and Discharging Capacitor (1)

- Figure 2 (i) shows a circuit which may be used to study the action of a capacitor.
 - C is a large capacitor such as 500 microfarad,
 - R is a large resistor such as 100 kiloOhms ($10^5 \Omega$),
 - A is a current meter reading up to 100 microamperes (100 μA),
 - K is a two-way key, and D is a 6 V d.c. supply.
- When the battery is connected to C by contact at X,
 - The current I in the meter A is seen to be initially about 60 μA
 - Then as shown in Figure 2 (ii), it slowly decreases to zero.
- So current flows to C for a short time when the battery is connected to it, even though the capacitor plates are separated by an insulator.
- We can disconnect the battery from C by opening X.



(i)



(ii)

Figure 2: Charging and discharging capacitor

Charging and Discharging Capacitor (2)

- If contact with Y is now made, so that in effect the plates of C are joined together through R and A ,
 - the current in the meter is observed to be about $60\ \mu\text{A}$ initially in the opposite direction to before and then slowly decreases to zero, Figure 2 (ii).
- This current flow shows that C stored charge when it was connected to the battery originally.
- Generally, a capacitor is charged when a battery or p.d. is connected to it.
- When the plates of the capacitor are joined together, the capacitor becomes discharged.
- Large values of C and R in the circuit of Figure 2 (i) help to slow the current flow,
 - so that we can see the charging and discharging which occurs. ¹⁰

Charging and Discharging Capacitor (3)

- We can also show that a charged capacitor has stored energy by connecting the terminals by a piece of wire.
- A spark, a form of light and heat, passes just as the wire makes contact.

Charging and Discharging Processes

Charging and Discharging Processes (1)

- When we connect a capacitor to a battery,
 - electrons flow from the negative terminal of the battery on to the plate A of the capacitor connected to it (Figure 3).
- At the same rate, electrons flow from the other plate B of the capacitor towards the positive terminal of the battery.
 - Equal positive and negative charges therefore appear on the plates, and opposite the flow of electrons which causes them.
- As the charges increase, the potential difference between the plates increases,
 - and the charging current falls to zero when the potential difference becomes equal to the battery voltage V_0 .
- The charges on the plates B and A are now $+Q$ and $-Q$, and the capacitor is said to have stored a charge Q in amount.

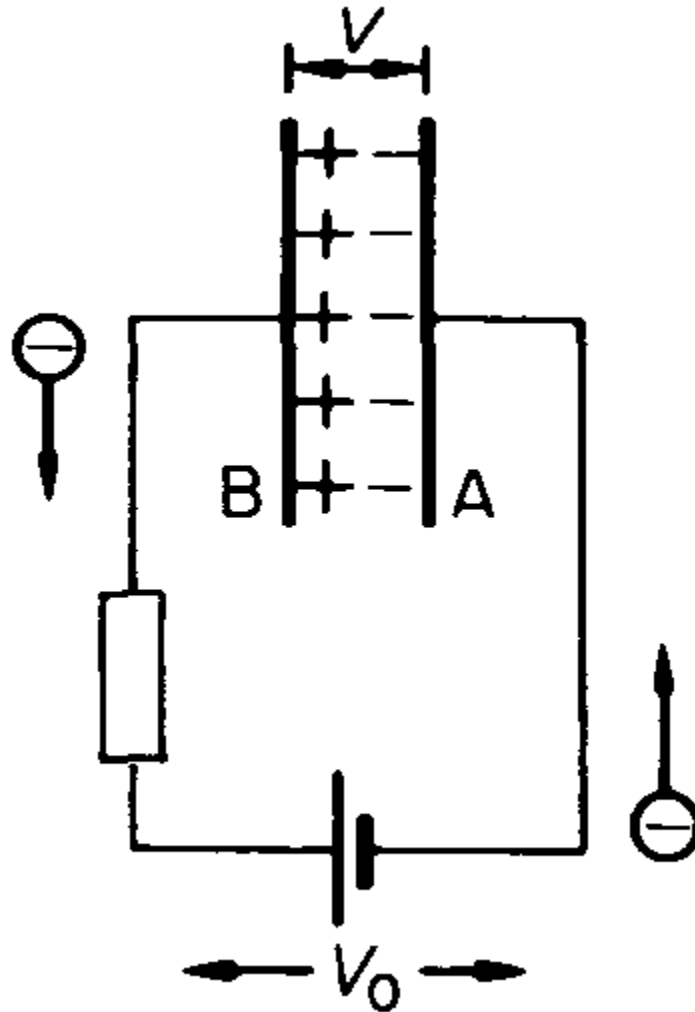


Figure 3: A capacitor charging (resistance is shown because some is always present, even if only that of the connecting wires)

Charging and Discharging Processes

(2)

- Alternatively, if we think of the battery as an ‘electric pump’, it pushes electrons or negative charge from B round to A.
 - So when the electrons stop moving, A has a negative charge $-Q$ and B is left with an equal positive charge $+Q$.
- When the battery is disconnected and the plates are joined by a wire,
 - electrons flow back from plate A to plate B until the positive charge on B is completely neutralized.
- So a current flows for a time in the wire, and at the end of the time the charges on the plates become zero.
 - So the capacitor is discharged.
 - Note that a charge Q flows from one plate to the other during the discharge.

Capacitors and A.C. Circuits

Capacitors and A.C. Circuits (1)

- Capacitors are widely used in alternating current and radio circuits,
 - because they can transmit alternating currents.
- To see how they do so, let us consider circuit of Figure 4, in which the capacitor D may be connected across either of the batteries X, Y.
- When the key is closed at A, current flows from battery X, and charges the plate D of the capacitor positively.
- If the key is now closed at B instead, current flows from the battery Y;
 - the plate D loses its positive charge and becomes negatively charged.

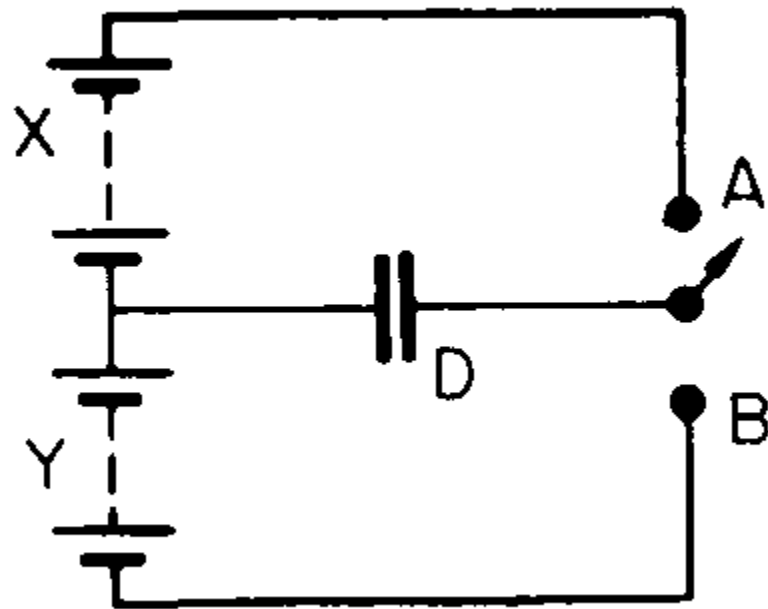


Figure 4: Reversals of voltage applied to capacitor

Capacitors and A.C. Circuits (2)

- So if the key is rocked rapidly between A and B,
 - current surges backwards and forwards along the wires connected to the capacitor.
- An alternating voltage, is one which reverses many times a second.
- When such a voltage is applied to a capacitor, therefore, an alternating current flows in the connecting wires,
 - even though the capacitor has an insulator between its plates.

Capacitance Definition and Units

Capacitance Definition and Units (1)

- Since $Q \propto V$, the ratio Q/V is a constant for the capacitor.
- The ratio of the charge on either plate to the potential difference between the plate is called the capacitance, C , of the capacitor:
 $C = Q/V$, so $Q = CV$ and $V = Q/C$
- When Q is in coulombs (C) and V in volts (V), the capacitance C is in farads (F).
 - One farad (1F) is the capacitance of a very large capacitor.
- In practical circuits, such as in radio receivers, the capacitance of capacitors used are therefore expressed in microfarads (μF).
- One microfarad is one millionth part of a farad, i.e., $1 \mu\text{F} = 10^{-6} \text{ F}$.

Capacitance Definition and Units (2)

- It is also quite usual to express small capacitors, such as those used on hi-fi systems, in picofarads (pF).
- A picofarad is one millionth part of a microfarad, i.e., $1\text{ pF} = 10^{-6} \mu\text{F} = 10^{-12} \text{ F}$.
- A nanofarad (nF) = 10^{-9} F . $1 \mu\text{F} = 10^{-6} \text{ F}$, $1 \text{ nF} = 10^{-9} \text{ F}$,
 $1 \text{ pF} = 10^{-12} \text{ F}$

Parallel Plate Capacitor

Parallel Plate Capacitor (1)

- We now obtain a formula for the capacitance of a parallel-plate capacitor which is widely used.
- Suppose two parallel plates of a capacitor each have a charge numerically equal to Q , Figure 5.
- The surface density σ is then Q/A where A is the area of either plate and the field-strength between the plates, E , is given by,

$$E = \sigma / \epsilon = Q / (\epsilon A)$$

- Now E is numerically equal to the potential gradient V/d

$$V/d = Q / (\epsilon A)$$

$$Q/V = \epsilon A / d$$

$$\text{Therefore } C = \epsilon_0 A / d$$

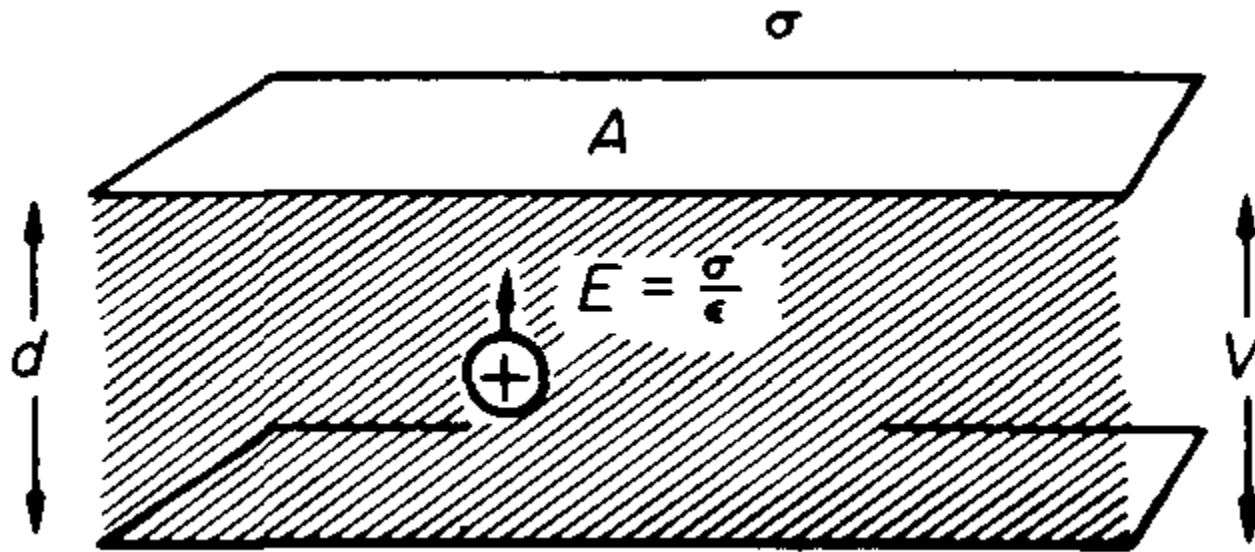


Figure 5: Parallel-plate capacitor

Parallel Plate Capacitor (2)

- So a capacitor with parallel plates, having a vacuum (or air, if we assume the permittivity of air is the same as a vacuum) between them, has a capacitance given by

$$\mathbf{C = \epsilon_0 A / d}$$

where

C = capacitance in farads (F),

A = area of overlap of plates in metre²,

d = distance between plates in metres and

$\epsilon_0 = 8.854 \times 10^{-12}$ farads metre⁻¹.

Capacitance of Isolated Sphere

Capacitance of Isolated Sphere

- Suppose a sphere of radius r metres situated in air is given a charge of Q coulombs.
- We assume, that the charge on a sphere gives rise to potentials on and outside the sphere as if all the charge were concentrated at the centre.
- The surface of the sphere thus has a potential relative to that 'at infinity' (or, in practice, to that of the earth) given by

$$V = Q / (4\pi\epsilon_0 r)$$

$$\text{Therefore } Q/V = 4\pi\epsilon_0 r$$

$$\text{Capacitance, } C = 4\pi\epsilon_0 r$$

- The other 'plate' of the capacitor is the earth.
- Suppose $r = 10 \text{ cm} = 0.1 \text{ m}$.
 - Then $C = 4\pi\epsilon_0 r = 4\pi \times 8.85 \times 10^{-12} \times 0.1 \text{ F} = 11 \times 10^{-12} \text{ F}$
(approx.) = 11 pF

Capacitors in series and parallel

Capacitors in series and parallel (1)

- In radio circuits, capacitors are used in arrangements whose total capacitance C must be known.
- To find C , we need the equations
 $C = Q/V$, $V = Q/C$, $Q = CV$
- *Capacitors In Parallel.*
 - Figure 6 shows three capacitors, having all their left-hand plates connected, and also all their right-hand plates.
 - They are said to be connected in parallel across the same potential difference V .
 - The charges on the individual capacitors are respectively
 $Q_1 = C_1 V$, $Q_2 = C_2 V$, $Q_3 = C_3 V$.
 - The total charge on the system of capacitors is
 $Q = Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3) V$

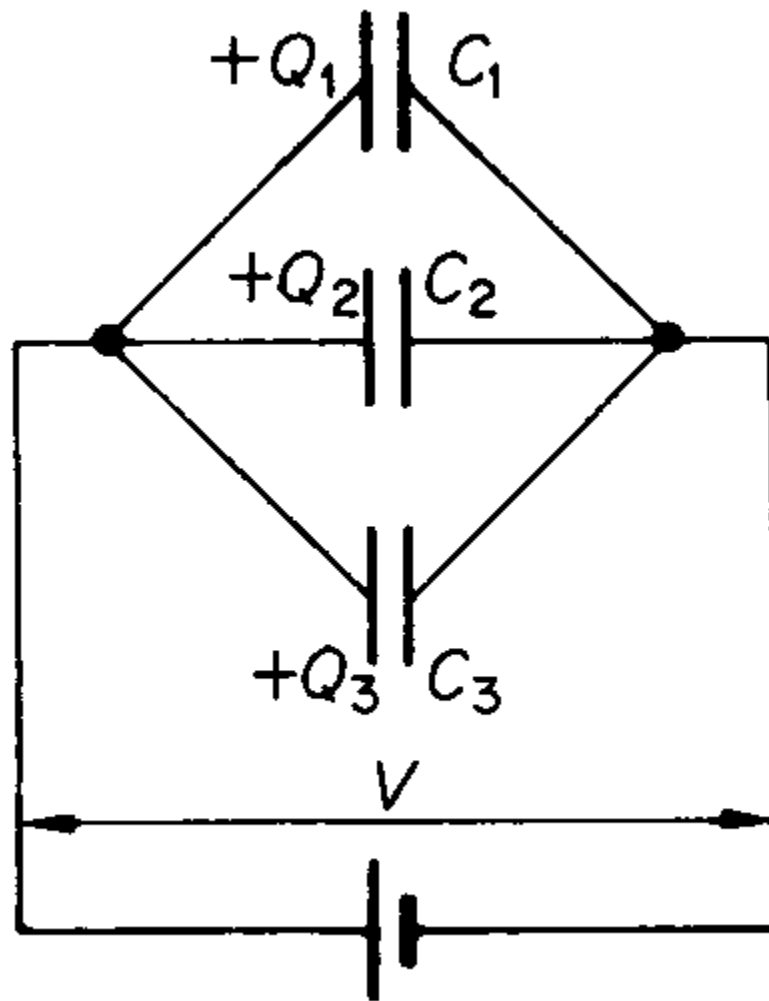


Figure 6: Capacitors in parallel

Capacitors in series and parallel (2)

- The system is equivalent to a single capacitor, of capacitance
$$C = Q/V = C_1 + C_2 + C_3$$
- So when capacitors are connected in parallel, their resultant capacitance C is the *sum* of their individual capacitances.
- C is greater than the greatest individual one.
- *Capacitors In Series*
 - Figure 7 shows three capacitors having the right-hand plate or one connected to the left-hand plate of the next, and so on – connected in *series*.
- When a cell is connected across the ends of the system,
 - a charge Q is transferred from the plate H to the plate A, a charge $-Q$ being left on H.
 - This charge induces a charge $+Q$ on plate G; similarly, charges appear on all the other capacitor plates, as shown in the figure.

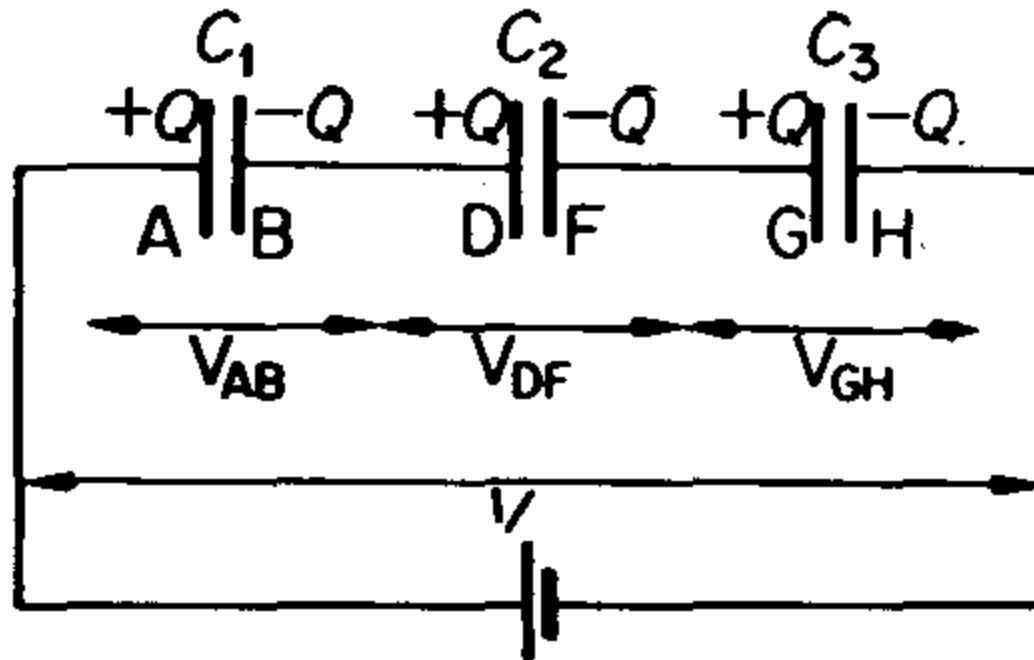


Figure 7: Capacitors in series

Capacitors in series and parallel (3)

- The induced and inducing charges are equal because the capacitor plates are very large and very close together; in effect, either may be said to enclose the other.
- The potential differences across the individual capacitors are, therefore, given by

$$V_{AB} = Q/C_1, V_{DF} = Q/C_2, V_{GH} = Q/C_3$$

- The sum of these is equal to the applied potential difference V
 - because the work done in taking a unit charge from H to A is the sum of the work done in taking it from H to G, from F to D, and from B to A.
- So $V = V_{AB} + V_{DF} + V_{GH}$
$$= Q (1/C_1 + 1/C_2 + 1/C_3)$$
- The resultant capacitance of the system is the ratio of the charge stored to the applied potential difference, V .
- The charge stored is equal to Q , because, if the battery is removed, and the plates HA joined by a wire, a charge Q will pass through that wire, and the whole system will be discharged.

Capacitors in series and parallel (4)

- The resultant capacitance is therefore given by

$$\mathbf{C = Q/V \text{ or } 1/C = V/Q}$$

- So

$$\mathbf{1/C = 1/C_1 + 1/C_2 + 1/C_3}$$

- So, to find the resultant capacitance C of capacitors in series,
 - we must add the reciprocals of their individual capacitances.
- C is less than the smallest individual.

Comparison of Series and Parallel Arrangements

- Let us compare Figures 6 and 7.
- In Figure 7, where the capacitors are in *series*,
 - all the capacitors have the same charge, which is equal to the charge carried by the system as a whole Q .
- So to find the charge Q on each capacitor, use **$Q = CV$**
 - where C is the resultant or total capacitance given by the $1/C$ in the equation for the resultant capacitance.
- The potential difference applied to the system is divided amongst the capacitors, in inverse proportion to their capacitances.
- In Figure 6, where the capacitors are in *parallel*,
 - they all have the same potential difference. The charge stored is divided amongst them, in direct proportion to the capacitances. ³⁶

Examples on Capacitors in Series and Parallel (1)

1. In Figure 8(i), $C_1(3 \mu\text{F})$ and $C_2(6 \mu\text{F})$ are in series across a 90 V d.c. supply. Calculate the charges on C_1 and C_2 and the p.d. across each.
 - Total capacitance C is given by $1/C = 1/C_1 + 1/C_2$
 $1/C = 1/3 + 1/6 = 3/6$
Thus $C = 2 \mu\text{F}$
 - The charges on C_1 and C_2 are the same and equal to Q on C .
 - So $Q = CV = 2 \times 10^{-6} \times 90 = 180 \times 10^{-6} \text{ C}$
 - Then $V_1 = Q/C_1 = 180 \times 10^{-6} / 3 \times 10^{-6} = 60\text{V}$
 - and $V_2 = Q/C_2 = 180 \times 10^{-6} / 6 \times 10^{-6} = 30\text{V}$

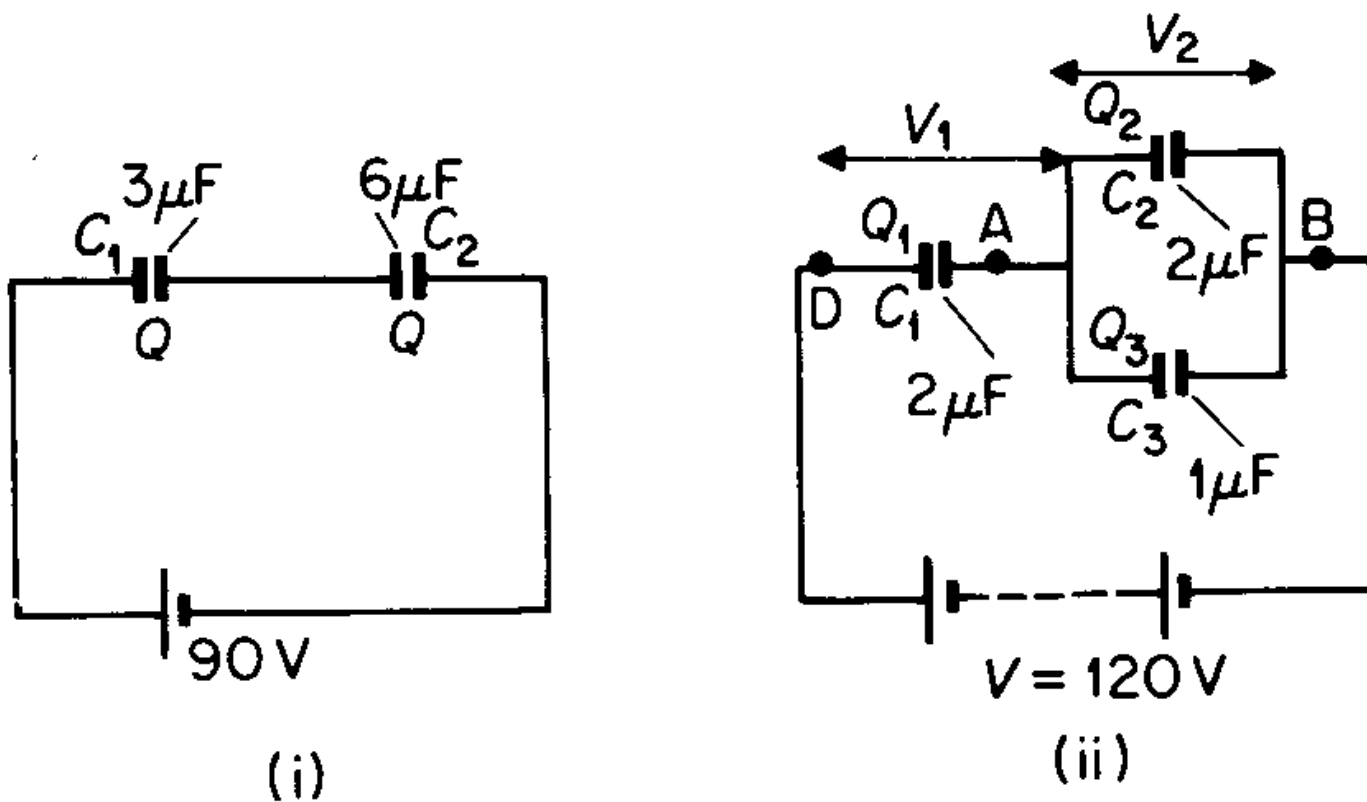


Figure 8: Examples on capacitors

Examples on Capacitors in Series and Parallel (2)

2. Find the charges on the capacitors in Figure 8 (ii) and the potential differences across them.

- Capacitance between A and B,

$$C' = C_2 + C_3 = 3\mu F$$

- Overall capacitance B to D, since C_1 and C' are in series, is, from $1/C = 1/C_1 + 1/C'$,

$$C = C_1 C' / (C_1 + C') = (2 \times 3) / (2 + 3) = 1.2\mu F$$

- Charge stored in this capacitance $C = Q_2 + Q_3 = CV = 1.2 \times 10^{-6} \times 120 = 144 \times 10^{-6} C$

$$\text{Therefore } V_1 = Q_1 / C_1 = 144 \times 10^{-6} / (2 \times 10^{-6}) = 72 V$$

- So $V_2 = V - V_1 = 120 - 72 = 48 V$

$$Q_2 = C_2 V_2 = 2 \times 10^{-6} \times 48 = 96 \times 10^{-6} C$$

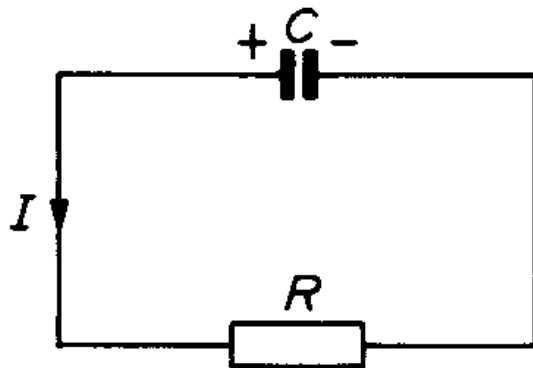
$$Q_3 = C_3 V_2 = 10^{-6} \times 48 = 48 \times 10^{-6} C$$

Charging and discharging a capacitor in C-R circuit

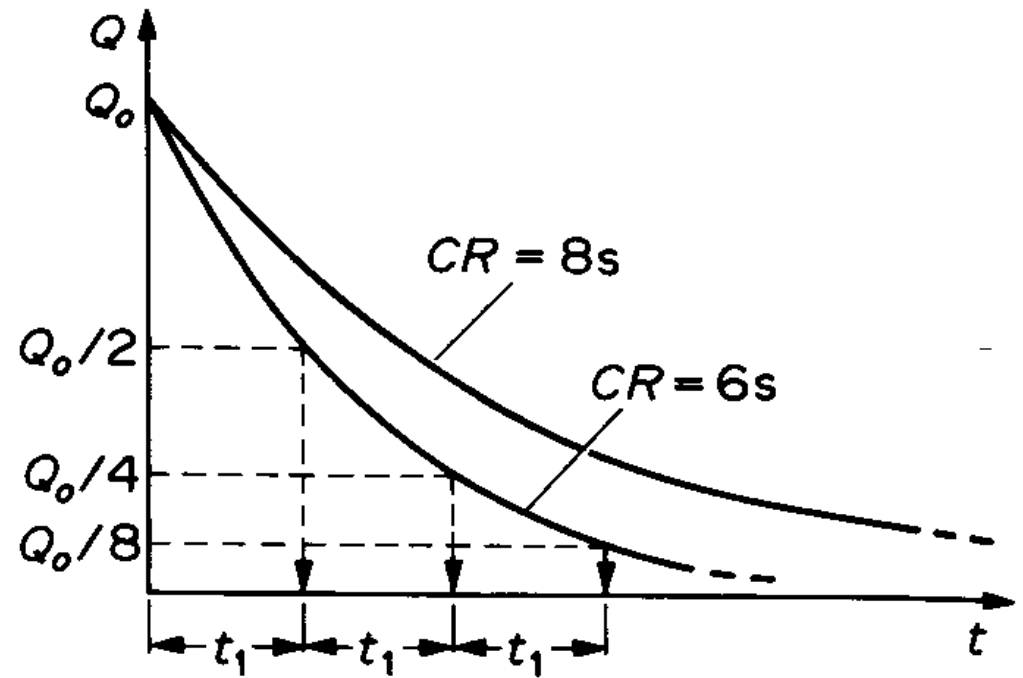
Discharge in C –R Circuit (1)

- We now consider in more detail the *discharge* of a capacitor C through a resistor R which is widely used in electronic circuits.
- Suppose the capacitor is initially charged to a p.d. V_0 so that its charge is then $Q = CV_0$.
- At a time t after the discharge through R has begun, the current I flowing = V/R_0 where V is the p.d. across C , Figure 9(i).
- Now $V = Q/C$ and $I = -dQ/dt$ (the minus shows Q decreases with increasing t).
- So, from $I = V/R$, we have

$$-\frac{dQ}{dt} = \frac{1}{CR}Q$$



(i)



(ii)

Figure 9: Discharge in C—R circuit

Discharge in C –R Circuit (2)

- Integrating,

$$\int_{Q_0}^Q \frac{dQ}{Q} = -\frac{1}{CR} \int_0^t dt$$

$$\therefore \ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{CR} \quad \therefore Q = Q_0 e^{-t/CR}$$

- So Q decreases exponentially with time t , Figure 9(ii).
- Since the p.d. V across C is proportional to Q ,
 - it follows that $V = V_0 e^{-t/CR}$.
- Further, since the current I in the circuit is proportional to V , then $I = I_0 e^{-t/CR}$,
 - where I_0 is the initial current value, V_0/R .

Discharge in C –R Circuit (3)

- From the equation for Q , Q decreases from Q_0 to half its value, $Q_0/2$, in a time t given by

$$e^{-t/(CR)} = \frac{1}{2} = 2^{-1}$$

- Taking logs to the base e , $-t/CR = -\ln 2$
- So $t = CR \ln 2$
- Similarly, Q decreases from $Q_0/2$ to half this value, $Q_0/4$, in a time $t = CR \ln 2$.
 - This is the same time from Q_0 to $Q_0/2$.
- So the time for a charge to diminish to half its initial value, no matter what the initial value may be, is always the same. See Figure 9(ii).

Discharge in C –R Circuit (4)

- This is true for fractions other than one-half.
- It is typical of an exponential variation or ‘decay’ which also occurs in radioactivity.

Time Constant

- The *time constant* T of the discharge circuit is defined as CR seconds, where C is in farads and R is in ohms.
- So if $C = 4 \mu F$ and $R = 2 M\Omega$,
 - then $T = (4 \times 10^{-6}) \times (2 \times 10^6) = 8 \text{ seconds}$.
- Now, from the equation $Q = Q_0 e^{-t/CR}$,
 - if $t = CR$, then, using $e = 2.72$ (approx)
- $Q = Q_0 e^{-1} = 1/e Q_0 = 0.37 Q_0$ (approx.)

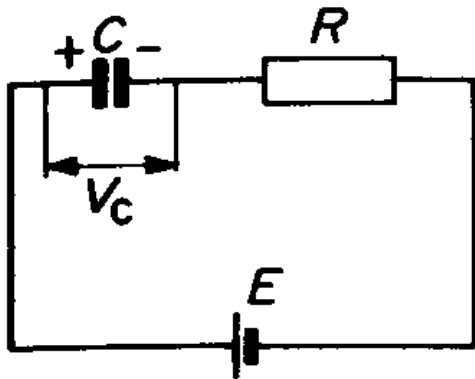
Charging C through R (1)

- Consider now the *charging* of a capacitor C through a resistance R in series,
 - and suppose the applied battery has an emf E and a negligible internal resistance, Figure 10(i).
- At the instant of making the circuit, there is no charge on C and so no p.d.
 - then the initial current flowing, $I_0 = E/R$.
- Suppose I is the current flowing after a time t . Then, V_C is the p.d. now across C . So

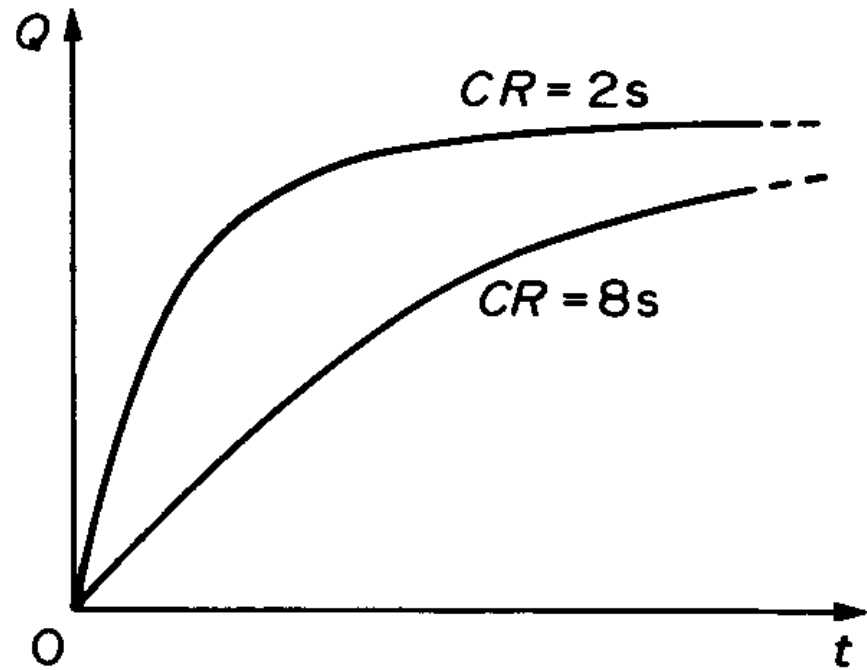
$$I = \frac{E - V_C}{R}$$

- Now $I = dQ/dt$ and $V_C = Q/C$. Substituting in the above equation and simplifying.

$$CR \frac{dQ}{dt} = CE - Q = Q_0 - Q$$



(i)



(ii)

Figure 10: Charging in C—R circuit

Charging C through R (2)

- Where $Q_0 = CE$ = final charge on C, when no further current flows. Integrating,

$$\frac{1}{CR} \int_0^t dt = \int_0^Q \frac{dQ}{Q_0 - Q}$$
$$\therefore \frac{t}{CR} = -\ln\left(\frac{Q_0 - Q}{Q_0}\right) \quad \therefore Q = Q_0(1 - e^{-t/(CR)})$$

- In the case of the discharge circuit,
 - the *time constant* T is defined as CR in seconds with C in farads and R in ohms.
- If T is high, it takes a long time for C to reach its final charge, i.e, C charges slowly.
 - If T is small, C charges rapidly. See Figure 10 (ii).
- The voltage V follows the same variation as Q since V is proportional to Q .

Rectangular Pulse Voltage and C-R Circuit (1)

- Let us find how the voltages across a capacitor C and resistor R vary when a *rectangular pulse voltage*, shown in Figure 11(i) is applied to a C R series circuit.
 - This type of circuit is used in analogue computers.
- On one half of a cycle, the p.d. is constant along AB at a value E say.
- We can therefore consider that this is similar to the case of charging a C-R circuit by a battery of e.m.f. E .
- The p.d. V_C across the capacitor hence rises along an exponential curve, Figure 11(ii).
- During the same time, the p.d. across R , V_R falls exponentially as shown in Figure 11(iii).

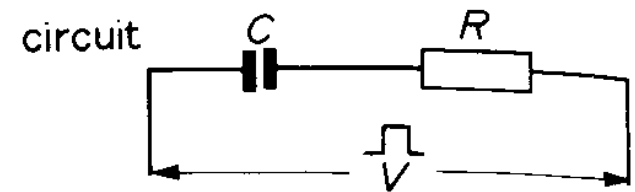
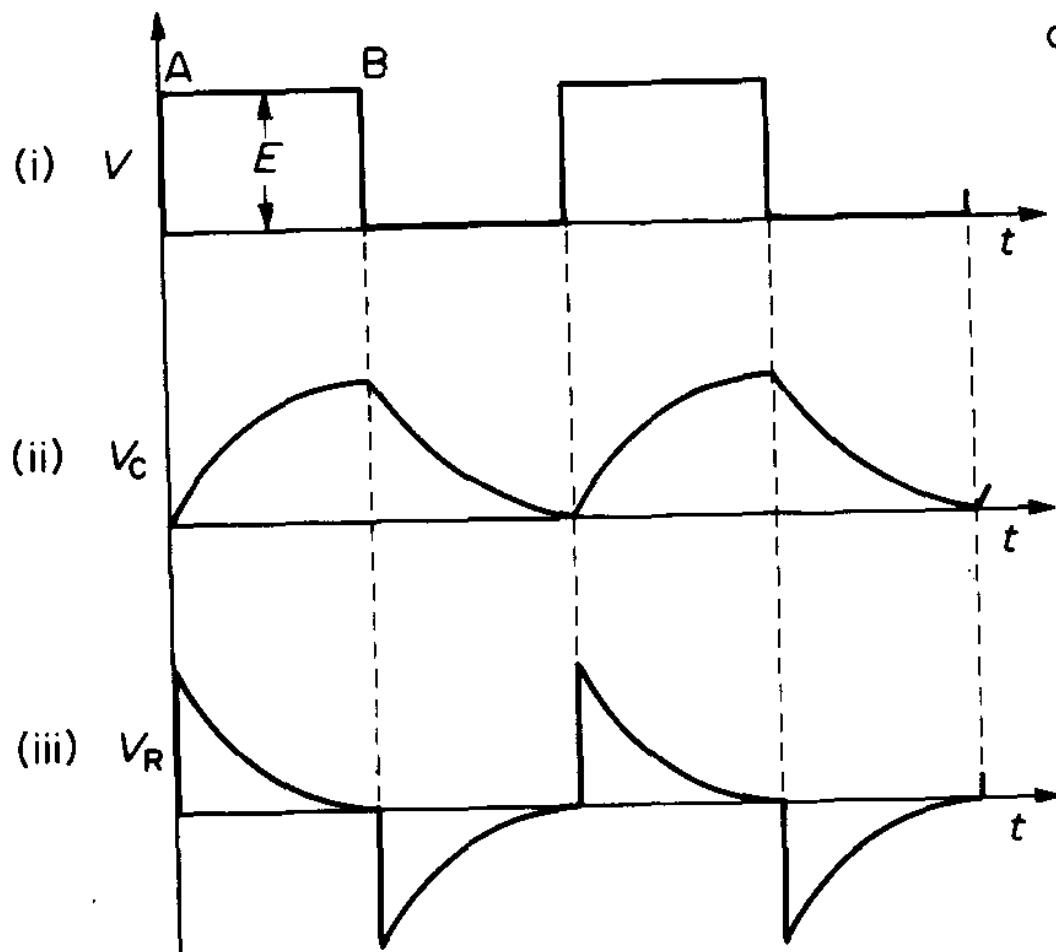


Figure 11: Rectangular pulse voltage and C—R circuit

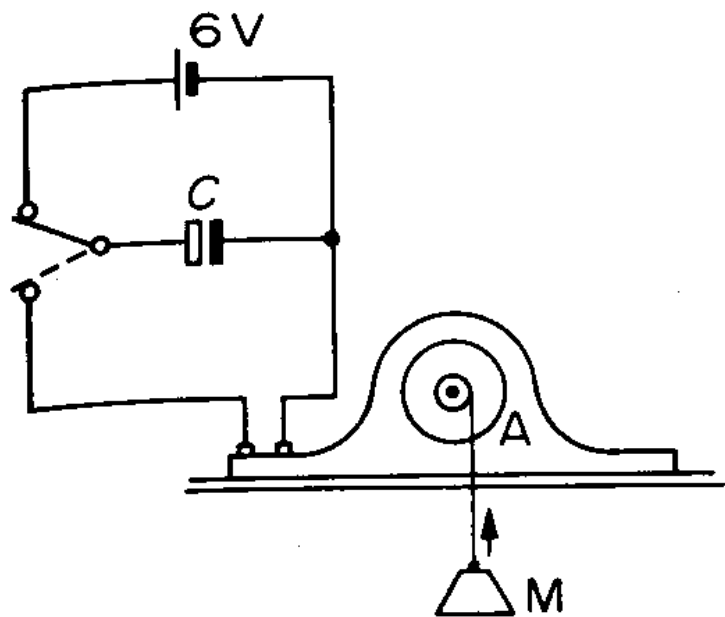
Rectangular Pulse Voltage and C—R Circuit (2)

- Since $V_R = E - V$, the curves for V_R and V together add up to the straight line graph AB in Figure 11(i).
- Similarly, during the time when $V = 0$, the curves for V_C and V_R add up to zero.
- This helps to check the drawings of the two curves.

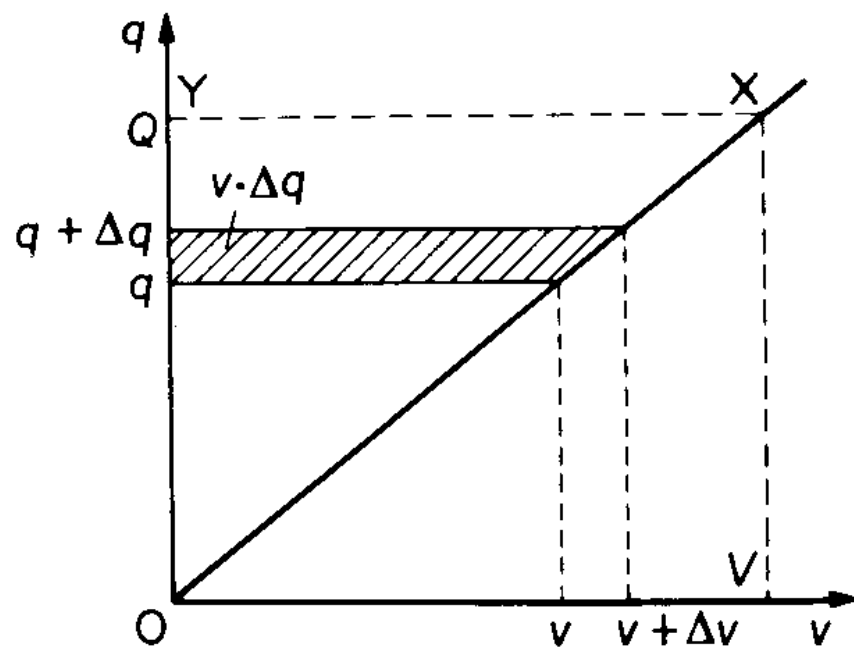
Energy stored in a capacitor

Energy stored in a capacitor (1)

- A charged capacitor is a store of electrical energy,
 - as we may see from the vigorous spark it can give on discharge.
- This can also be shown by charging a large electrolytic capacitor C , such as $10000\ \mu\text{F}$, to a p.d. of $6\ \text{V}$,
 - and then discharging it through a small or toy electric motor A , Figure 12(i).
- A small mass M such as $10\ \text{g}$, suspended from a thread tied round the motor wheel, now rises as the motor functions.
- Some of the stored energy in the capacitor is then transferred to gravitational potential energy of the mass;
 - the remainder is transferred to kinetic energy and heat in a motor



(i)



(ii)

Figure 12: Energy in charged capacitor

Energy stored in a capacitor (2)

- To find the energy stored in the capacitor,
 - since q (charge) $\propto v$ (pd. across the capacitor) at any instant, the graph OX showing how q varies with v is a straight line, Figure 12(ii).
- We may therefore consider that the final charge Q on the capacitor moved from one plate to the other through an *average* p.d. equal to $\frac{1}{2} (0 + V)$,
 - since there is zero p.d. across plates at the start and a p.d. V at the end.
- So work done, $W = \text{energy stored} = \text{charge} \times \text{p.d.} = Q \times \frac{1}{2} V$
- So $W = \frac{1}{2} Q V$
- From $Q = CV$, other expressions for energy stored are

Energy stored in a capacitor (3)

$$W = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

$$\text{Energy } W = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV$$

- If C is measured in farads, Q in coulombs and V in volts, then the formulae will give the energy W in joules.

Alternative Proof of Energy Formulae (1)

- We can also calculate the energy stored in a charged capacitor by a calculus method.
- At any instant of the charging process, suppose the charge on the plates is q and the p.d. across the plates is then v .
- If an additional tiny charge Δq now flows from the negative to the positive plate, we may say that the charge Δq has moved through a p.d. equal to v .
- So work done in displacing the charge $\Delta q = v \Delta q$

$$\text{total work done} = \text{energy stored} = \int_0^Q v dq$$

– where the limits are $q = Q$, final charge, and $q = 0$, as shown.

- To integrate, we substitute $v = q/C$.

Alternative Proof of Energy Formulae (2)

- Then

$$\text{Energy stored } W = \int_0^Q \frac{q dq}{C} = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q = \frac{Q^2}{2C}$$

- Using $Q = CV$, other expressions for W are

$$W = \frac{1}{2} CV^2 \text{ or } W = \frac{1}{2} QV$$

Energy and Q — V Graph, Heat Produced in Charging

Energy and Q-V Graph, Heat Produced in Charging (1)

- Figure 12 (ii) shows the variation of the charge q on the capacitor and its corresponding p.d. v while the capacitor is charged to a final value Q .
- The small shaded area shown = $v \cdot \Delta q$.
- So the area represents the small amount of work done or energy stored during a change from q to $q + \Delta q$.
- It therefore follows that the total energy stored by the capacitor is represented by the area of the triangle OXY.
- This area = $\frac{1}{2} QV$, as previously obtained.
- If a high resistor R is included in the charging circuit,
 - the rate of charging is slowed.

Energy and Q- V Graph, Heat Produced in Charging (2)

- When the charging current ceases to flow, the final charge Q on the capacitor is the same as if negligible resistance was present in the circuit,
 - since the whole of the applied p.d. V is the p.d. across the capacitor when the current in the resistor is zero.
- So the energy stored in the capacitor is $\frac{1}{2} Q V$ whether the resistor is large or small.
- It is important to note that the energy in the capacitor comes from the battery.
- This supplies an amount of energy equal to QV during the charging process.
- Half of the energy, $\frac{1}{2} QV$, goes to the capacitor.
- The other half is transferred to heat in a circuit resistance.

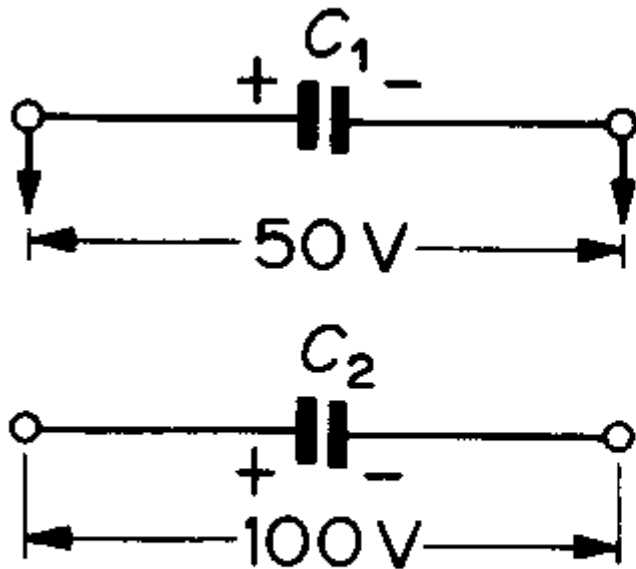
Energy and Q- V Graph, Heat Produced in Charging (3)

- If this is a high resistance, the charging current is low and the capacitor gains its final charge after a long time.
- If it is a low resistance, the charging current is higher and the capacitor gains its final charge in a quicker time.
- In both cases, however, the total amount of heat produced is the same, $\frac{1}{2} Q V$.

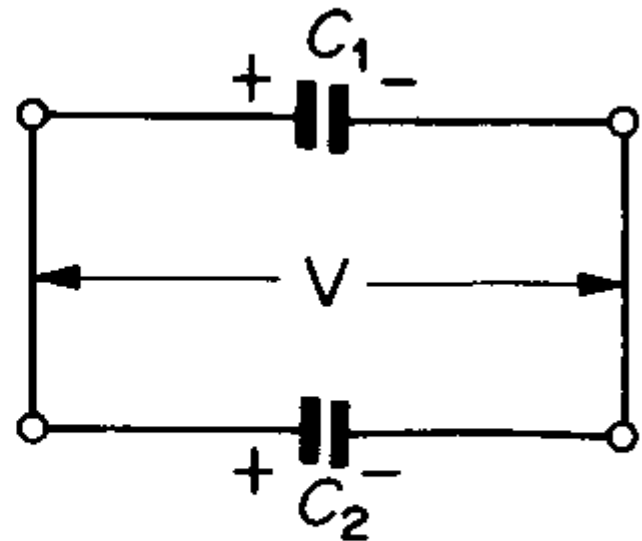
Connected Capacitors, Loss of Energy

Connected Capacitors, Loss of Energy (1)

- Consider a capacitor C_1 of $2\text{ }\mu\text{F}$ charged to a p.d. of 50 V , and a capacitor C_2 of $3\text{ }\mu\text{F}$ charged to a p.d. of 100 V , Figure 13(i).
- Then charge Q_1 on $C_1 = C_1 V_1 = 2 \times 10^{-6} \times 50 = 10^{-4}\text{ C}$ and charge Q_2 on $C_2 = C_2 V_2 = 3 \times 10^{-6} \times 100 = 3 \times 10^{-4}\text{ C}$.
- Therefore total charge $= 4 \times 10^{-4}\text{ C}$
- Suppose the capacitors are now joined with plates of like charges connected together, Figure 13(ii).
- Then some charges will flow from C_1 to C_2 until the p.d across each capacitor becomes equal to some value V .
- Further, since charge is conserved, the total charge on C_1 and C_2 after connection $=$ the total charge before connection.



(i)



(ii)

Figure 13: Loss of energy in connected capacitors

Connected Capacitors, Loss of Energy (2)

- Now after connection,
 - total charge = $C_1 V + C_2 V = (C_1 + C_2)V = 5 \times 10^{-6} \times V$
- From total charge = $4 \times 10^{-4} \text{C}$, $5 \times 10^{-6} V = 4 \times 10^{-4}$.
 - Therefore $V = 80 \text{V}$.
- Total energy of C_1 and C_2 after connection
 $= \frac{1}{2} (C_1 + C_2)V^2 = \frac{1}{2} \times 5 \times 10^{-6} \times 80^2 = 0.016 \text{J}$
- Total energy of C_1 and C_2 before connection
 $= \frac{1}{2}C_1 V_1^2 + \frac{1}{2}C_2 V_2^2 = \frac{1}{2} \times 2 \times 10^{-6} \times 50^2 + \frac{1}{2} \times 3 \times 10^{-6} \times 100^2$
 $= 0.0025 + 0.015 = 0.0175 \text{ J}$
- Comparing the energies, we can see that a loss of energy occurs when the capacitors are connected.
- This loss of energy is transferred to heat in the meeting wires.