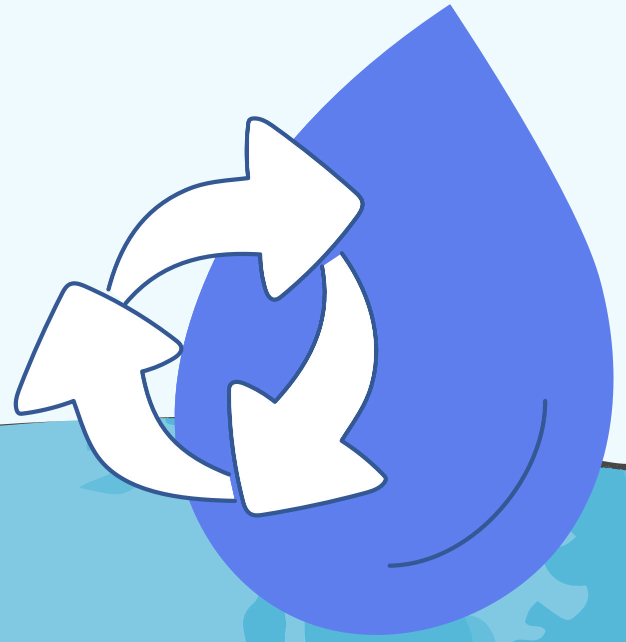
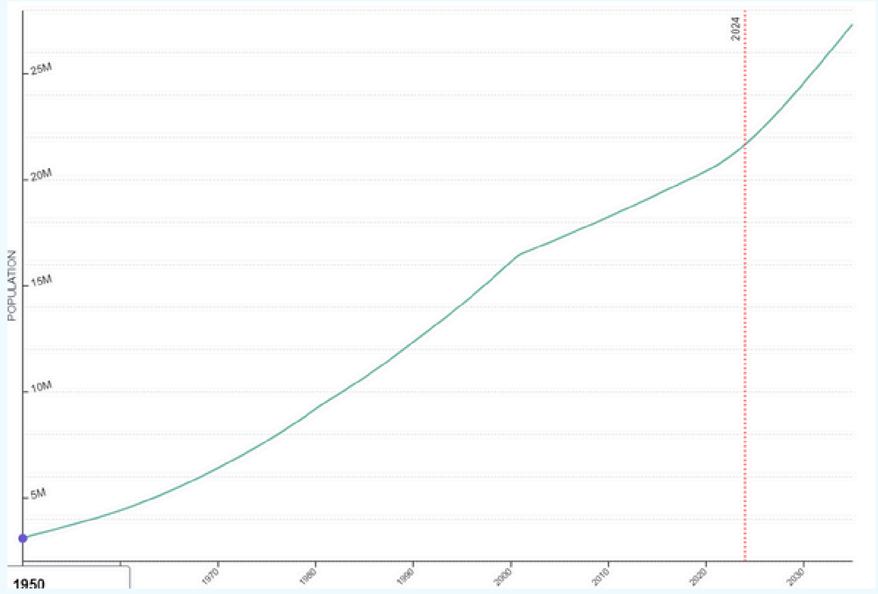
ILP MODELS FOR WATER DISTRIBUTION SYSTEM OPTIMISATION

A PRESENTATION BY
SPARSH AMARNANI



Motivation

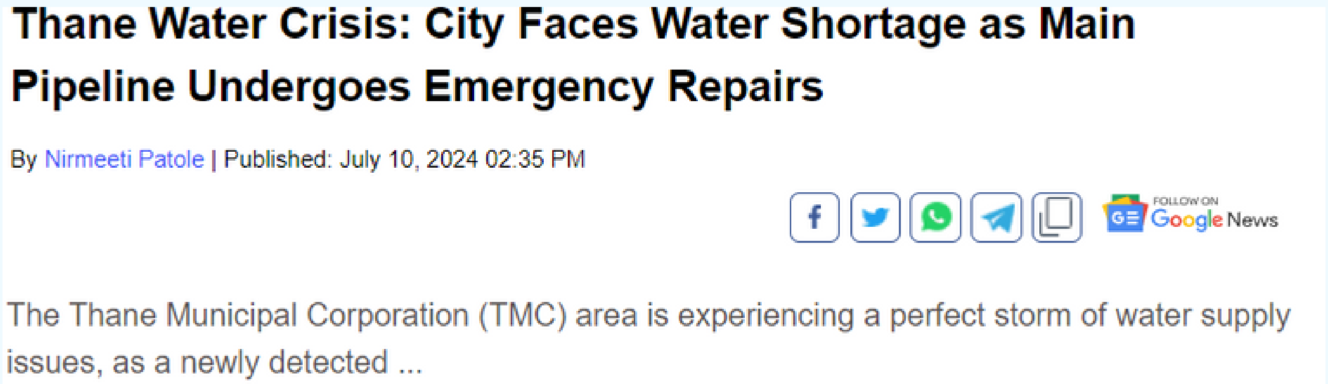


Rapid Urbanisation

+

Variable Water Supply

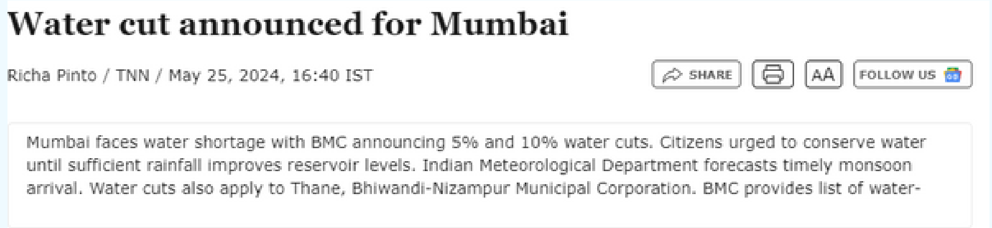
=



Thane Water Crisis: City Faces Water Shortage as Main Pipeline Undergoes Emergency Repairs

By Nirmeeti Patole | Published: July 10, 2024 02:35 PM

The Thane Municipal Corporation (TMC) area is experiencing a perfect storm of water supply issues, as a newly detected ...



Water cut announced for Mumbai

Richa Pinto / TNN / May 25, 2024, 16:40 IST

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Mumbai faces water shortage with BMC announcing 5% and 10% water cuts. Citizens urged to conserve water until sufficient rainfall improves reservoir levels. Indian Meteorological Department forecasts timely monsoon arrival. Water cuts also apply to Thane, Bhiwandi-Nizampur Municipal Corporation. BMC provides list of water-

Water

Tanker economy revealed: Rajasthan's groundwater level takes a dip as water mafias emerge

Water shortages in Rajasthan need no introduction; but the extent of the crisis covers a lot more than what meets the eye

Water problems

WHAT IS WDS OPTIMISATION ?

DECISION VARIABLE

Hydralic Components that we can choose like ,pipes, pumps and valves

OBJECTIVE FUNTION

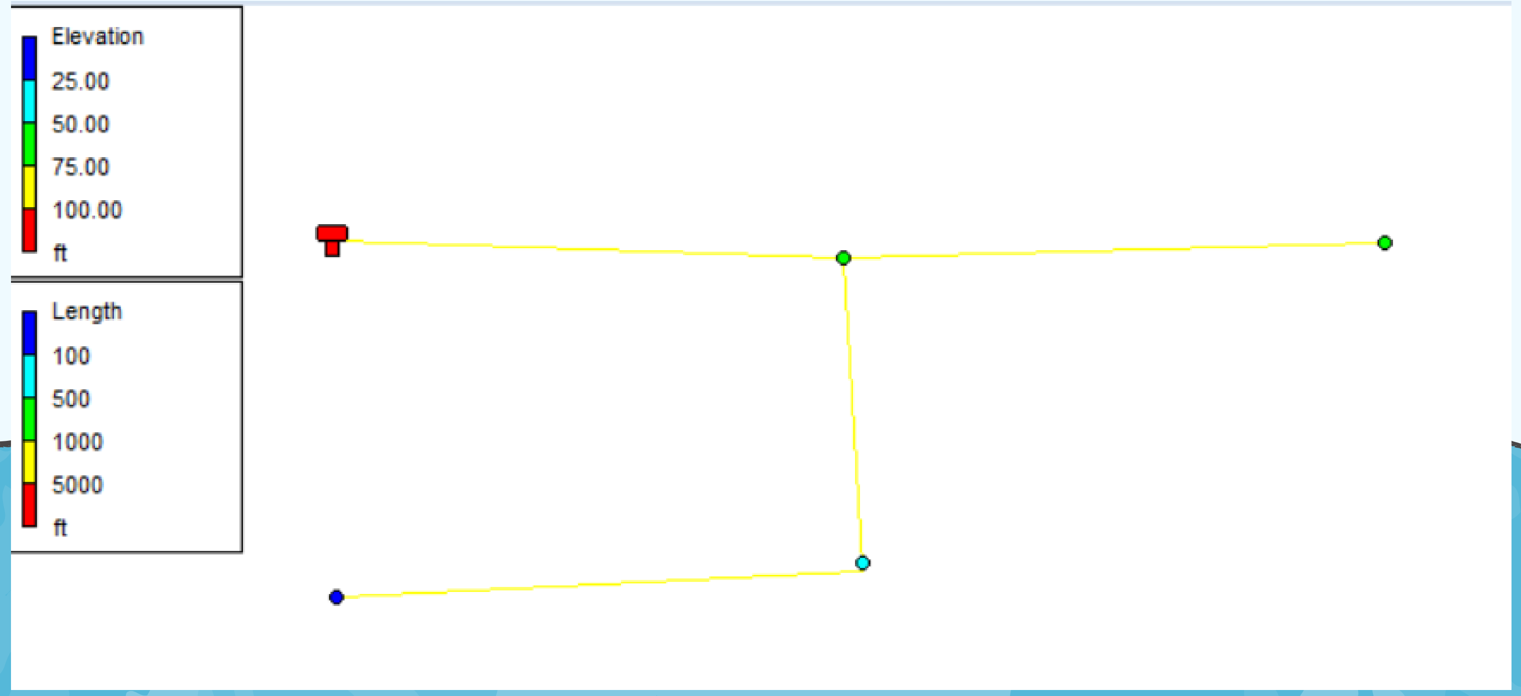
Variables that are dependent on our choices that we want to minimize or maximize, like cost, water quality or greenhouse gas emmisions.

CONSTRAINTS

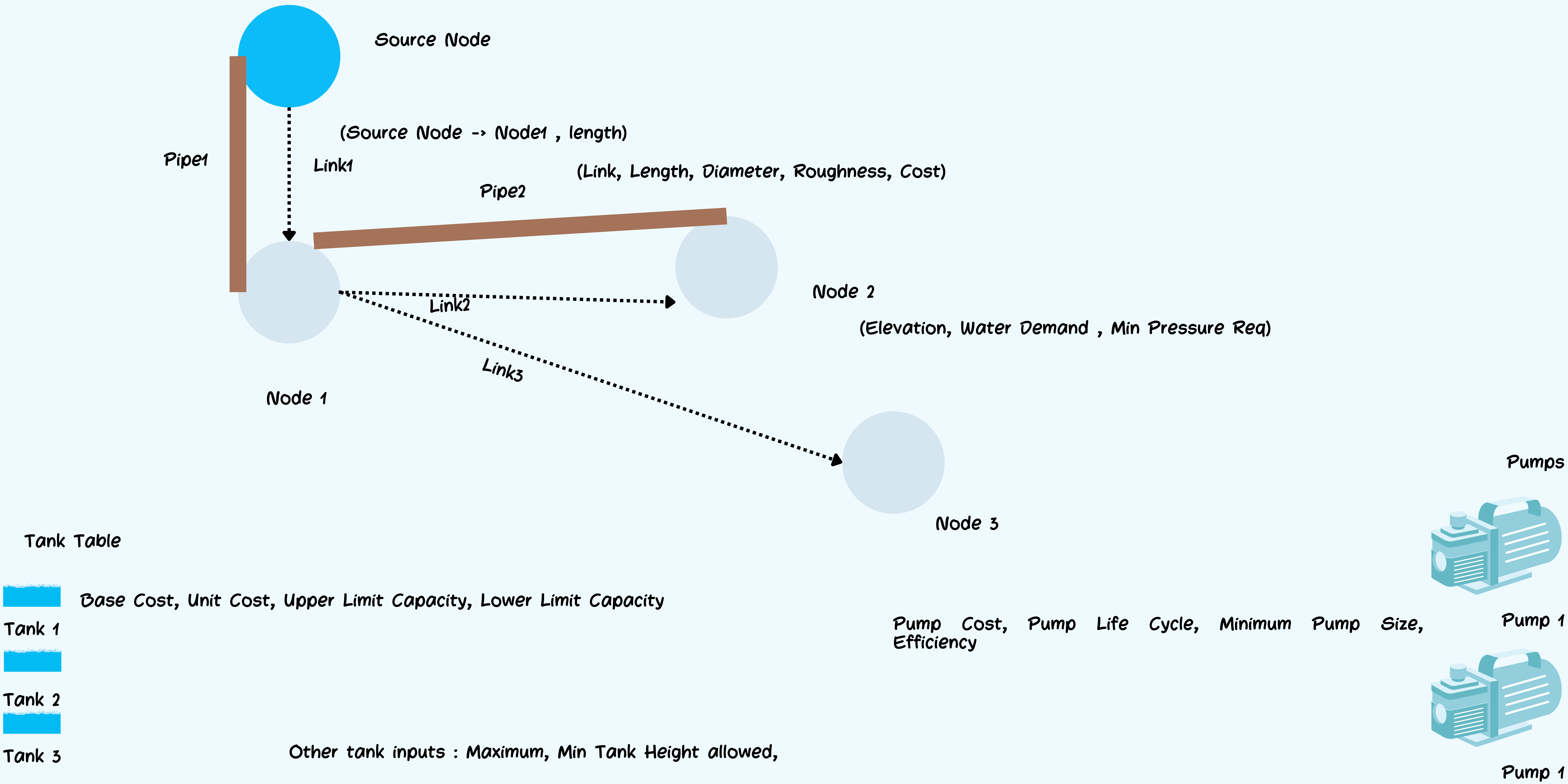
Hydraulic constraints are given by physical laws governing the fluid flow in a pipe network.

System Constraints like water demands and water quality requirement

THE GOAL IN DESIGING A SOFTWARE WHICH CAN OPTIMISE THIS IS THAT WE CAN UPDATE OUR WATER NETWORKS WITH INCREASING WATER DEMANDS OR CHANGE IN WATER SUPPLY



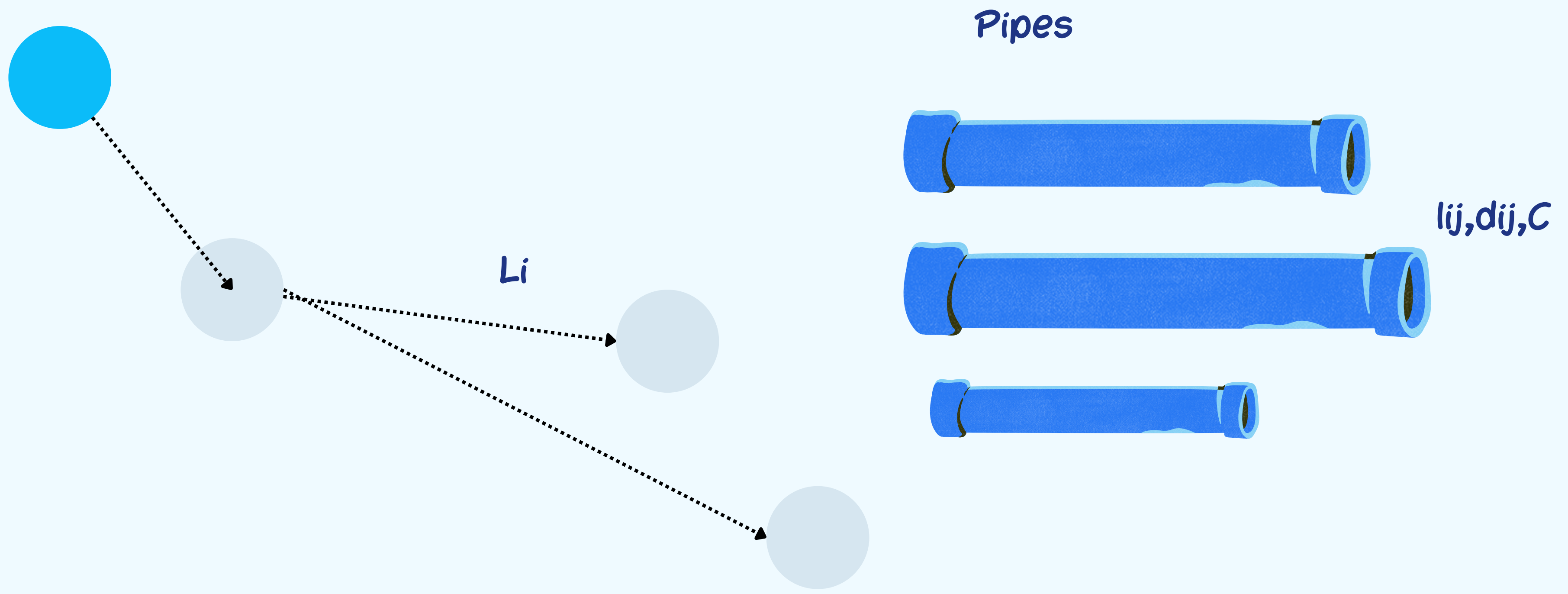
Inputs of the Optimisation Model



A SYSTEM FOR OPTIMAL DESIGN OF PRESSURE CONSTRAINED BRANCHED PIPED WATER NETWORKS

N. Hoodaa , O. Damanib

PIPE ONLY MODEL OPTIMISATION



OUR GOAL IS TO PUT PIPES IN EACH LINK WHICH WILL MINIMIZE THE COST OF THE NETWORK

OBJECTIVE FUNCTION

minimize L : C(Di,j) * Lij sum over i and j

CONSTRAINTS

Length of Pipes

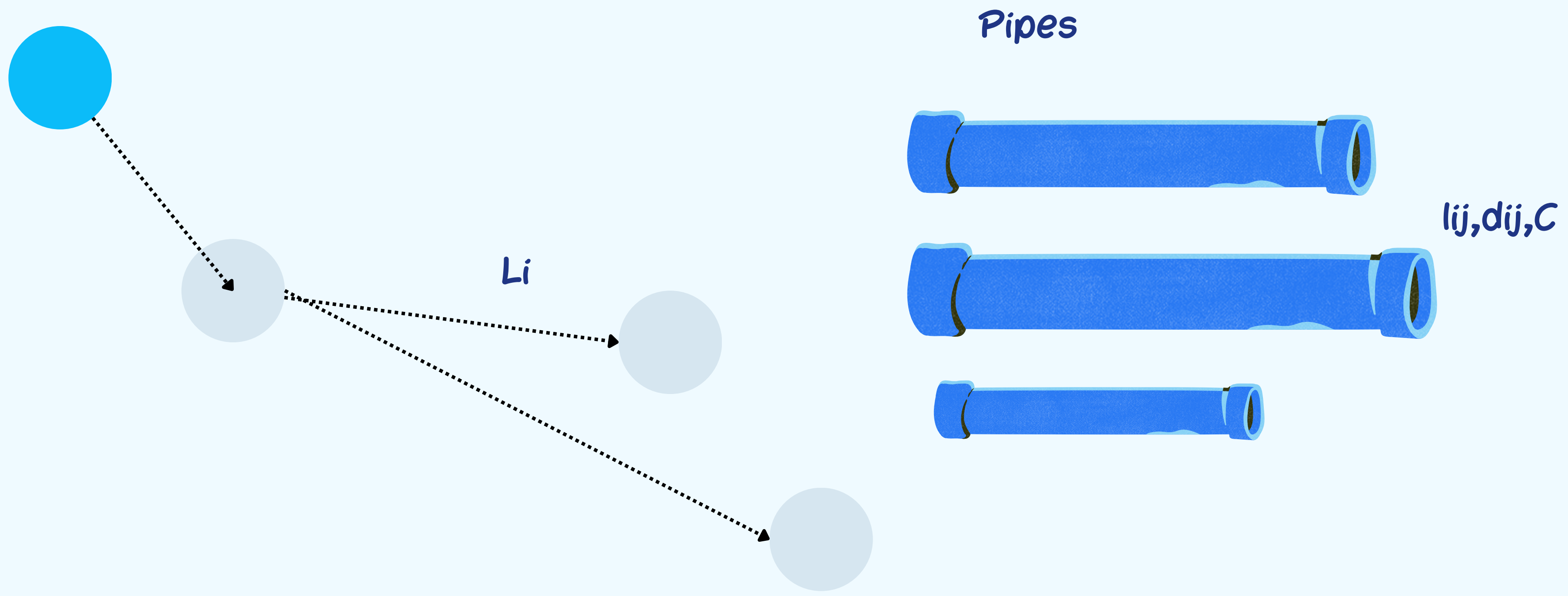
lij sum over j = Li

lij>=0

Pressure requirment at each node

Pn<= Hr -En-(HLij*Li) sum over j and i which are parent links

HAZEN WILLIAMS EQUATION



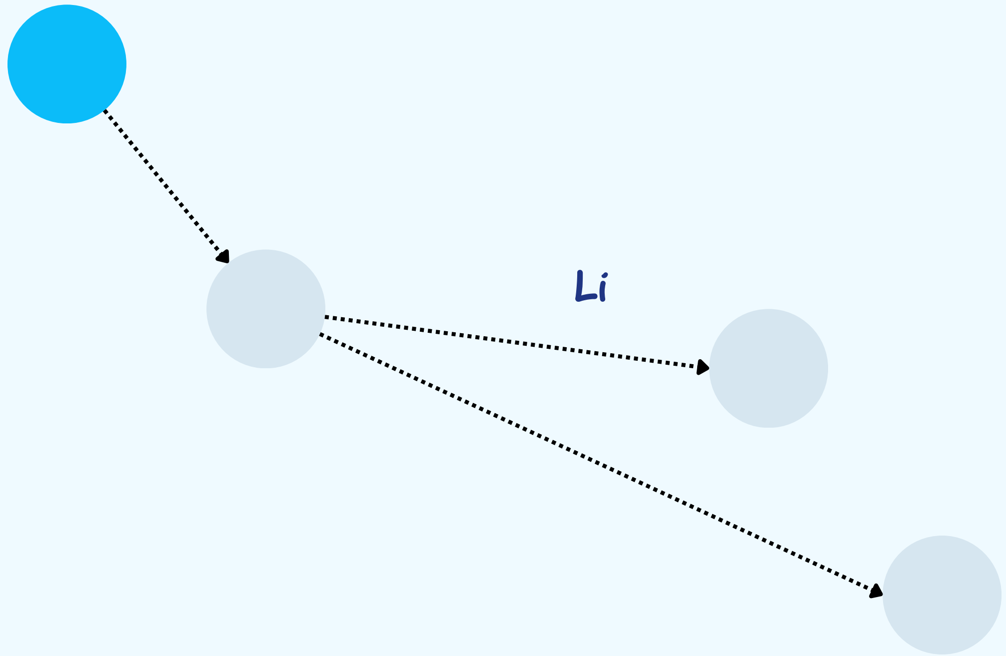
OUR GOAL IS TO PUT PIPES IN EACH LINK WHICH WILL MINIMIZE THE COST OF THE NETWORK

$$HL'_{ij} = \frac{10.68 * \left(\dfrac{flow_i}{roughness_j}\right)^{1.852}}{diameter_j^{\,4.87}}$$

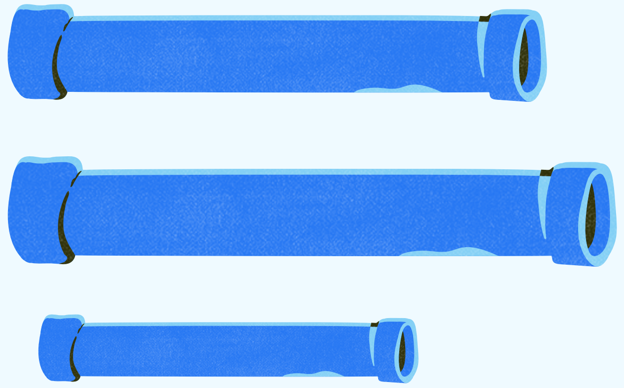
A SERIES OF ILP MODELS FOR THE OPTIMIZATION OF WATER DISTRIBUTION NETWORKS

NIKHIL HOODA1,*
, ASHUTOSH MAHAJAN2 and OM DAMANI

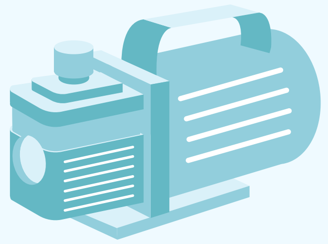
PIPES AND PUMP MODEL OPTIMISATION



Pipes



Pump



What power pump should be applied at which link i

Efficiency of a pump, Max and minimum power allowed by a pump, Capital and Operational Cost of a Pump

OBJECTIVE FUNCTION

minimize L,p : Pipe Cost + (Cp * pi) + (EF * DF * SH *pi) for all i

CONSTRAINTS

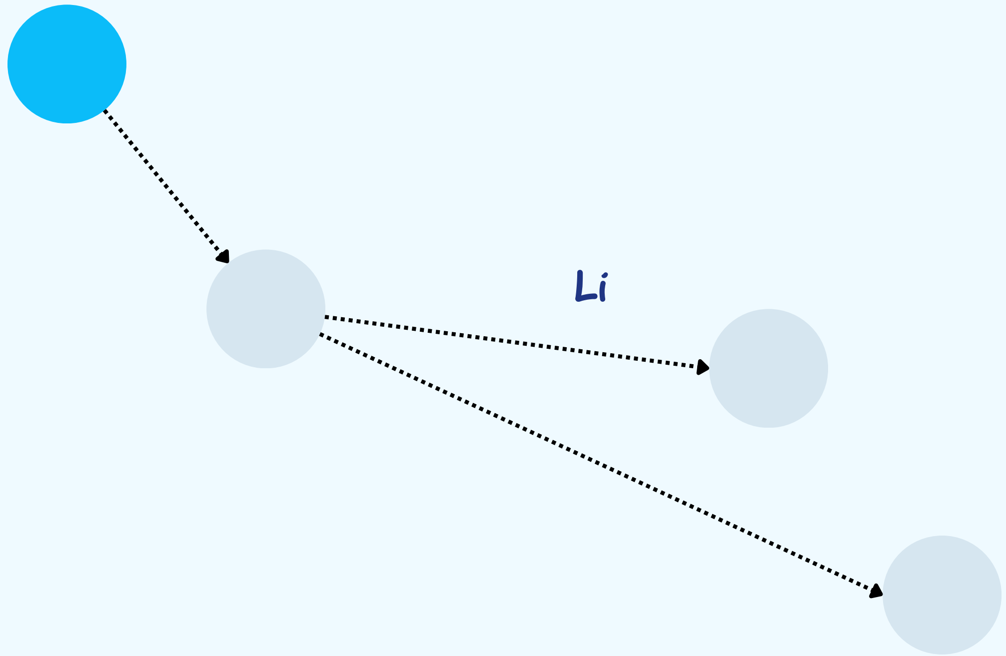
Presure on Link

phi= (eff * pi)/wtden * g*FLi

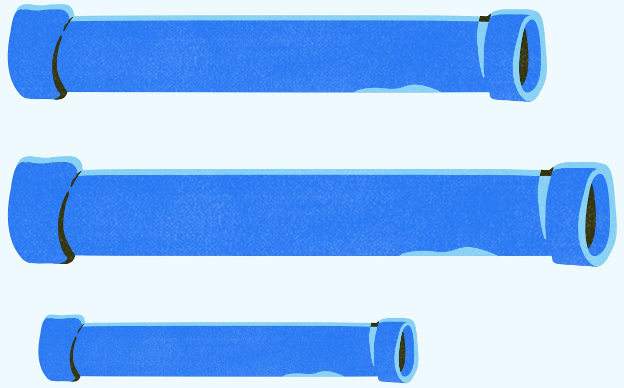
Pressure requirment at each node

Pn<= Hr -En-(HLij*Li) +phi sum over j and i which are parent links

PIPES AND PUMP MODEL OPTIMISATION

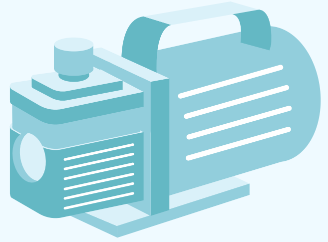


Pipes



lij,dij,C

Pump



What power pump should be applied at which link i

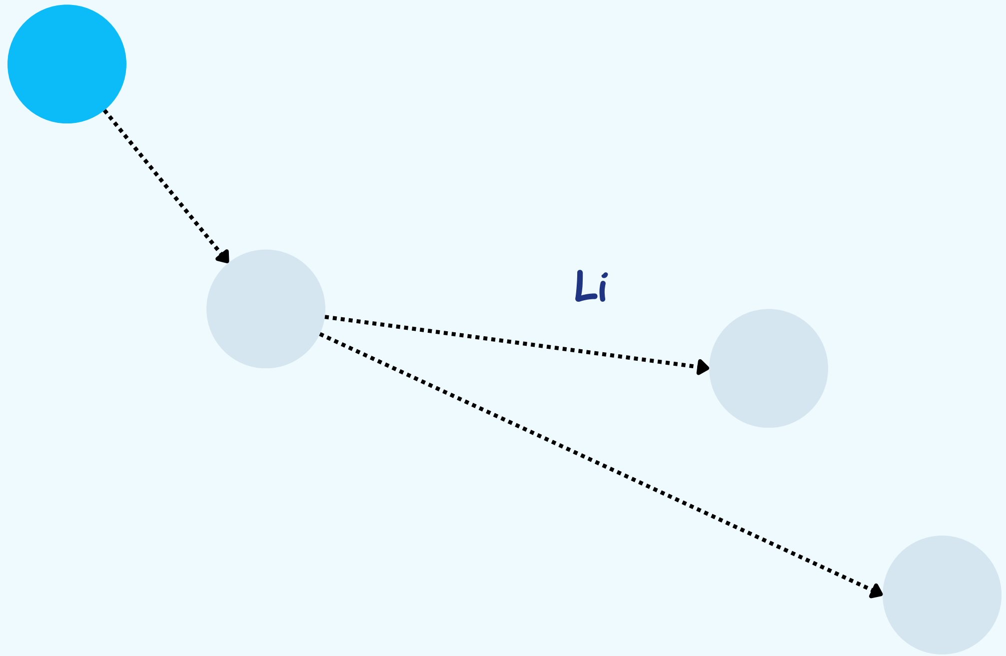
Efficiency of a pump, Max and minimum power allowed by a pump, Capital and Operational Cost of a Pump

CONSTRAINTS CONTINUED

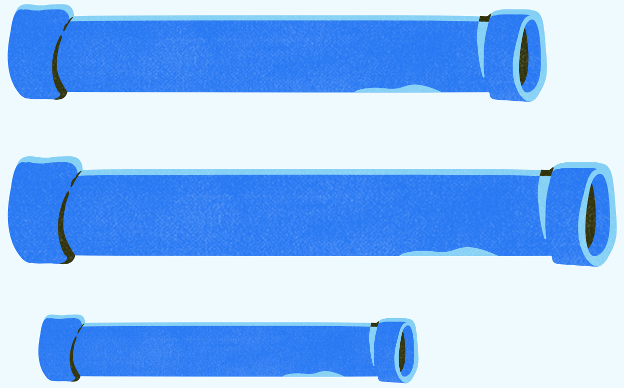
Power Constraints

PPmin<=pi<=PPmax

FINAL MODEL INPUTS

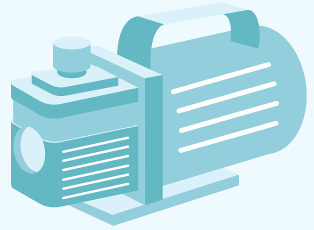


Pipes



lij,dij,C

Pump



Efficiency of a pump, Max and minimum power allowed by a pump, Capital and Operational Cost of a Pump

Tank Cost Table

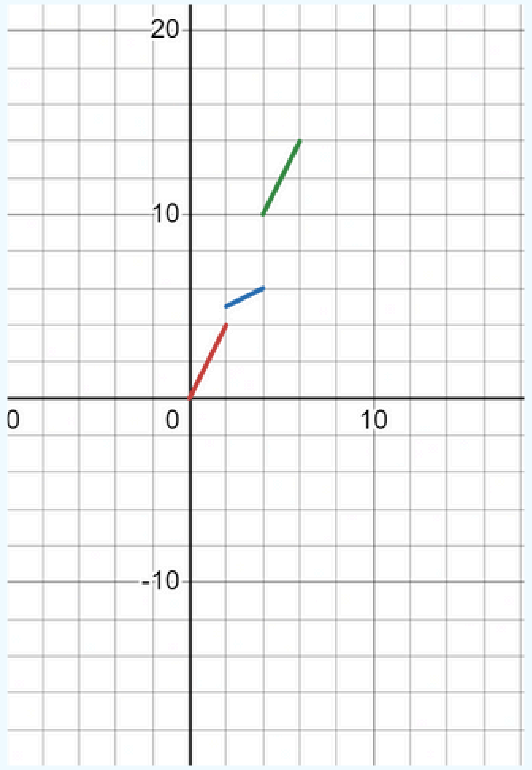
Base Cost, Unit Cost, Upper Limit Capacity, Lower Limit Capacity

Tank 1

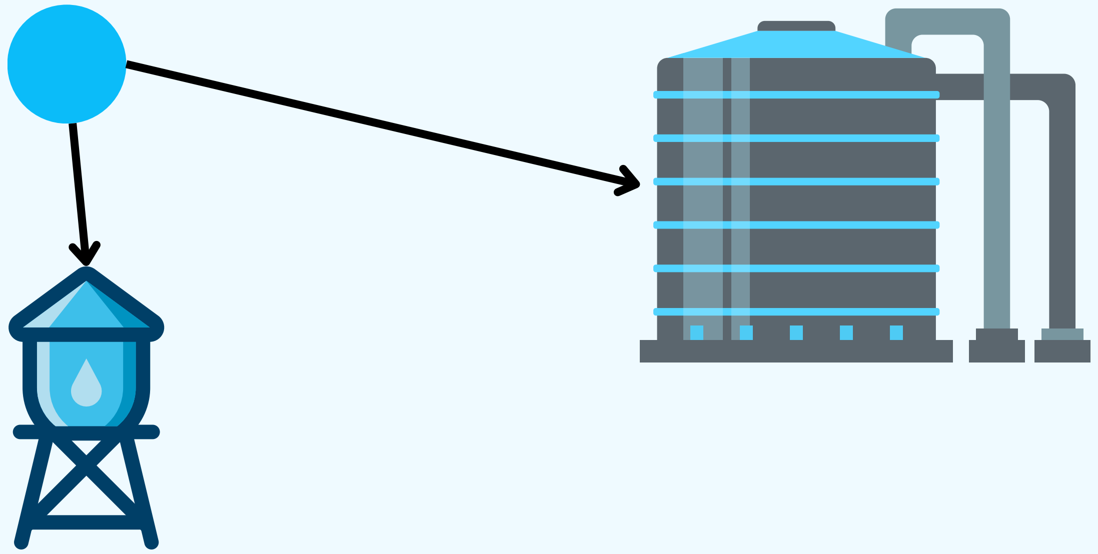
Tank 2

Tank 3

Other tank inputs : Maximum, Min Tank Height allowed,

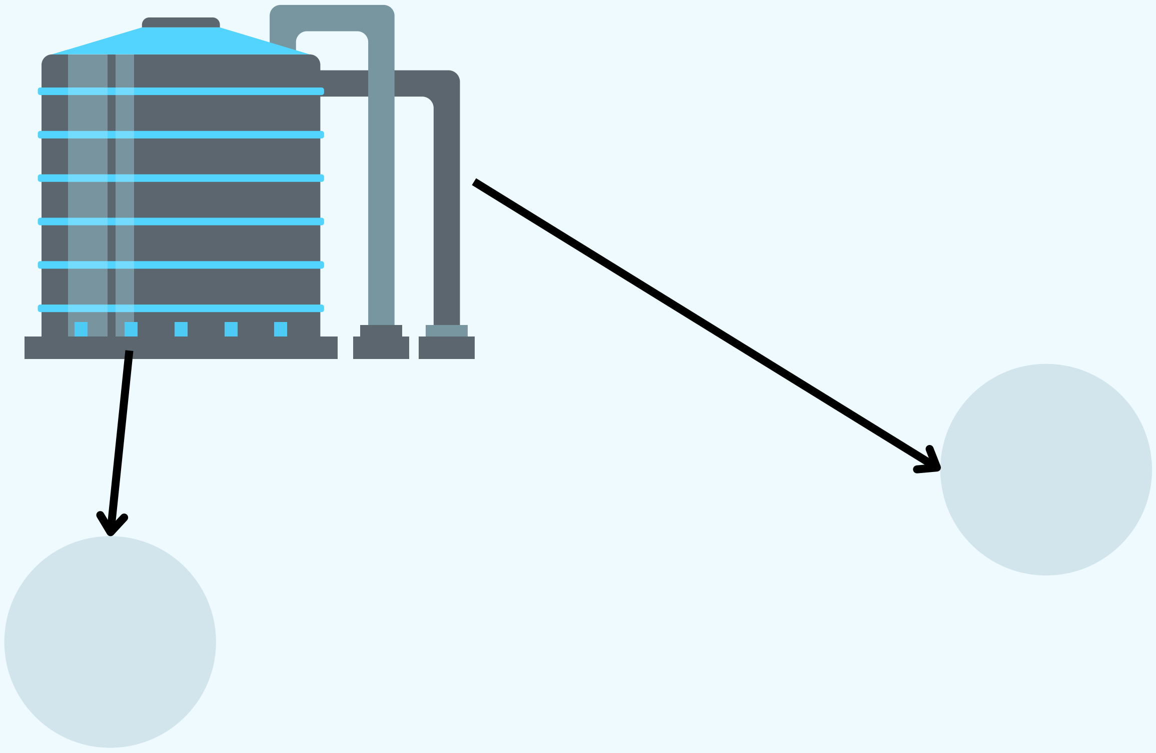


PRIMARY



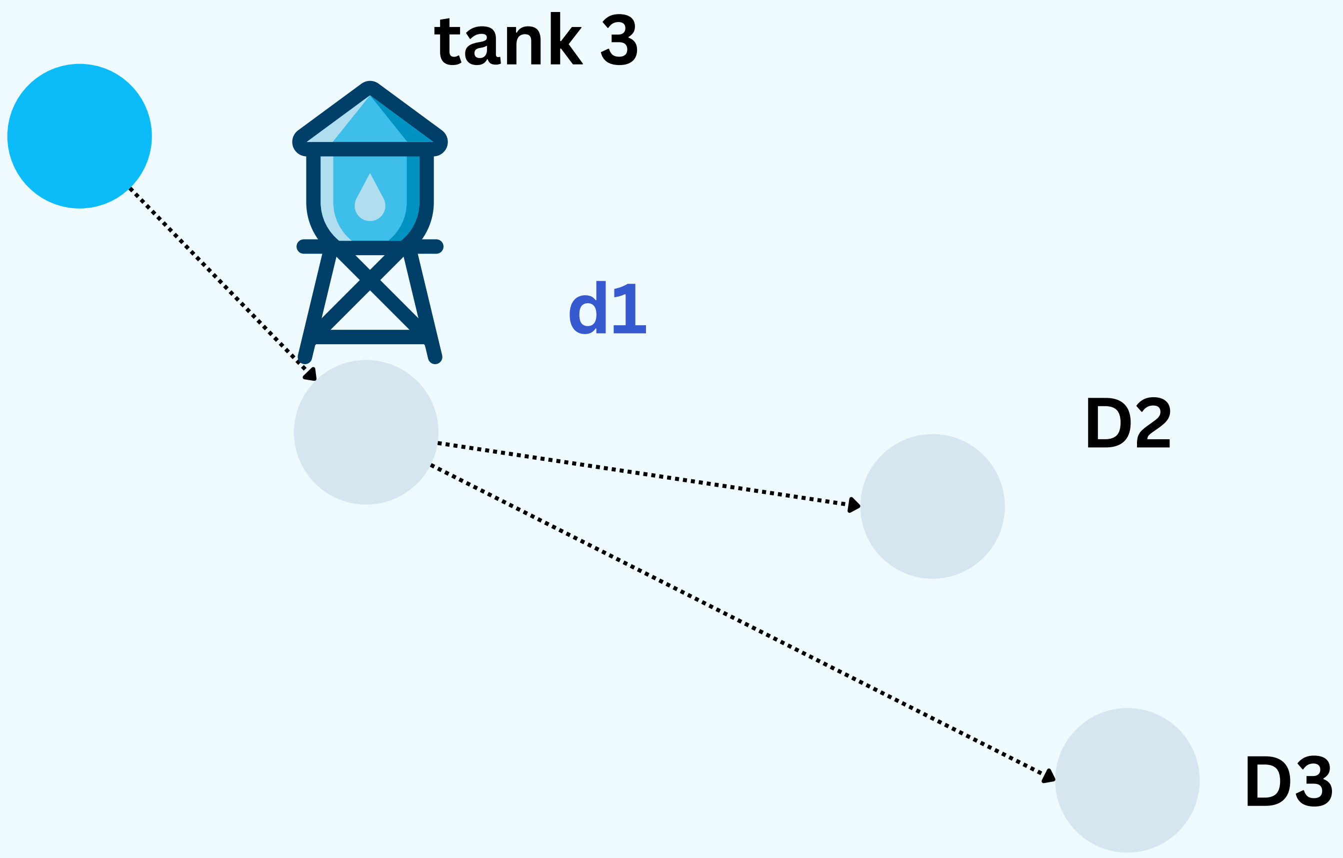
The primary network is the main system of pipes and infrastructure that transports water from the source (such as a water treatment plant or reservoir) to various storage facilities

The secondary network is the subsystem that distributes water from the primary network (or storage tanks) to individual demand nodes, such as households, businesses, and public facilities.



SECONDARY

OBJECTIVE FUNCTION WITH TANK



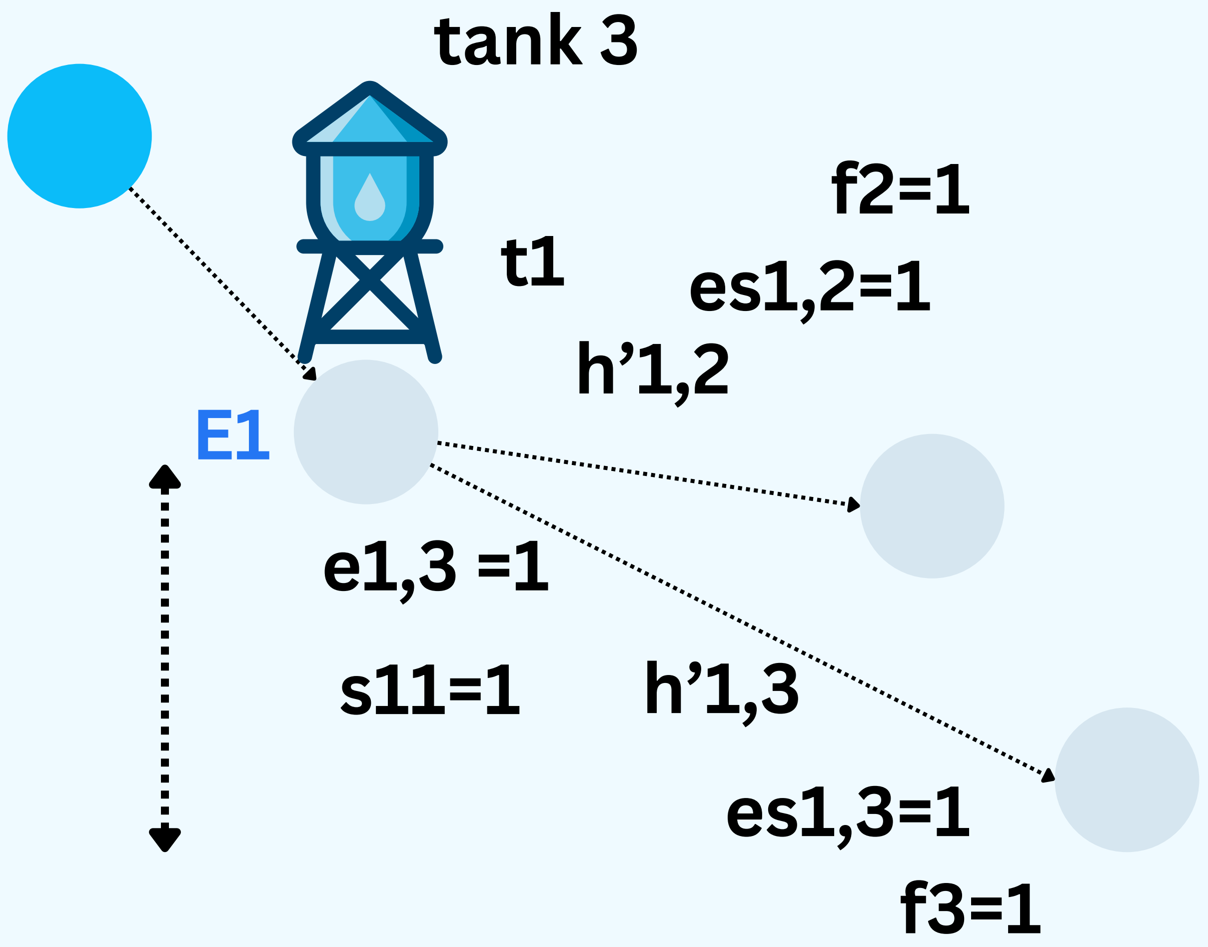
Head on link if tank is its source

d1=D2+D3

znk=dn*enk (linearlisation of the)

min l,p Cost of pipes + Cost of Pumps + enk * (Bk + UnitCk*(dn-LOk)) over all nodes (n) of the network and rows of the tank table (k)

PRESSURE CONSTRAINT WITH TANK



Head on link if tank is its source

$h'ni=(tn+En)*esn + hn*(1-esn)$

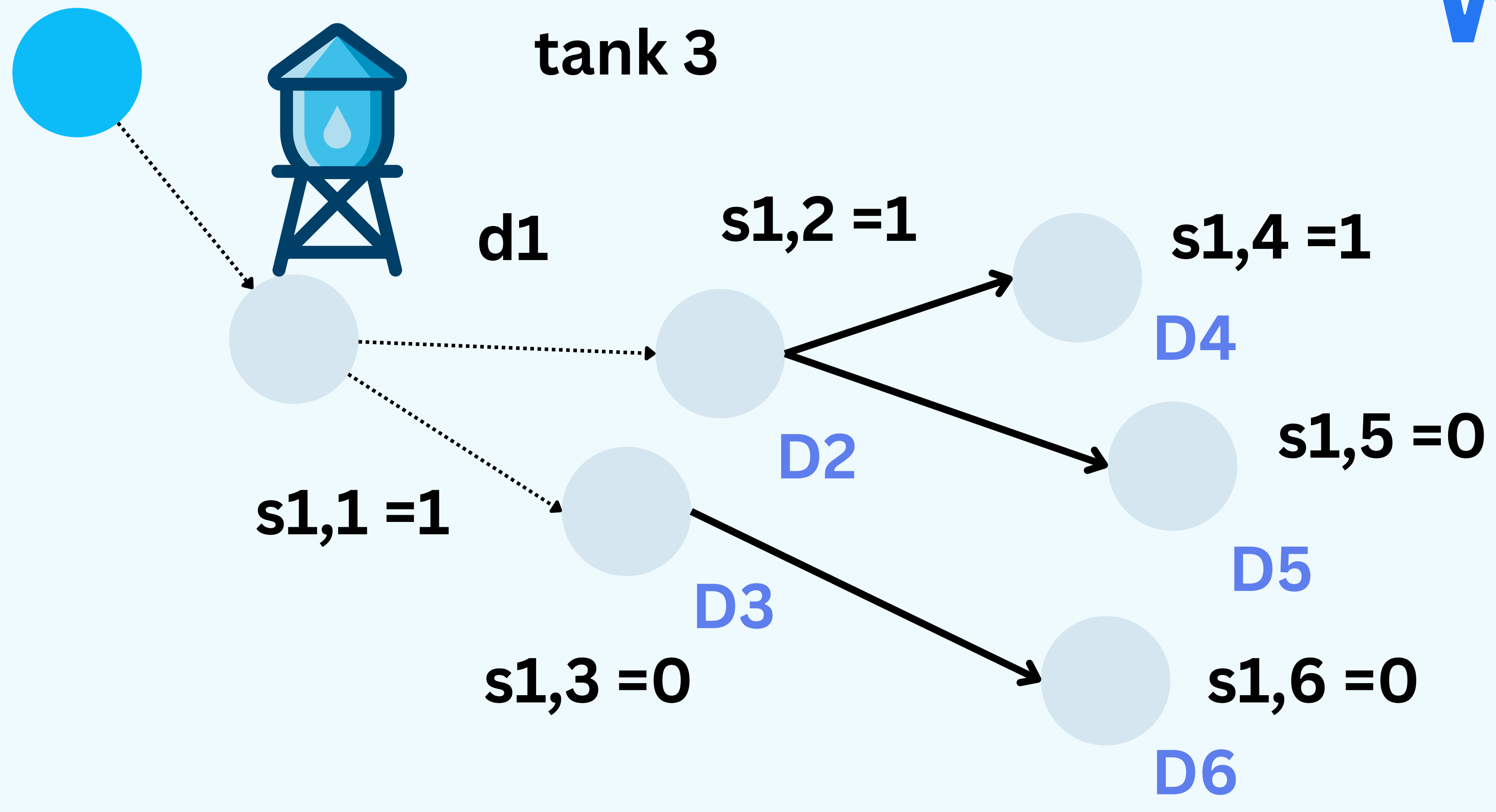
$h'1,2=t1+E1$

$esni=snn*(1-fi)$

$pn<=h'mi \; -hli$

If the starting node is a source node then the head loss equation changes

CONSTRAINTS ON TANK AND THEIR DECENDANTS



Where m is a descendant of n

smm<=snn

snm<=snn

sum over all Ancestors of m snm=1

dn = snm*DEm

snm<=1-soo for all o in path of n,m

Nodes can serve their decendants if they serve their own

OTHER CONSTRAINTS

Only 1 tank per row:

sum over k enk =1

Capacity-demand Constraint:

dn>=LOk*enk

UPk+DE(1-enk)>=dn

EFFECT ON OTHER CONSTRAINTS

Splitting headloss calculation:

hli = sum over all pipe segments (HLPij*lpij+HLSij*[lij-lpij])

Primary and Secondary Flows:

FLPi/FLSi = SH/PH

Primary and Secondary Pumps:

pi = ppi+psi

CONCLUSION

The paper goes on to create tighter relaxed feasible set of the model established along with giving proofs as to why are they tighter

The first ILP model took 45 mins to solve a 150 nodes network while this one took 5sec

Future research opportunities with creating a pump scheduling algorithm and designing a model for looped networks

OPTIMAL SCHEDULING OF WATER DISTRIBUTION SYSTEMS

By Manish K. Singh Student Member, IEEE
and Vassilis Kekatos, Senior Member, IEEE

OPTIMAL WATER FLOW PROBLEM

DECISION VARIABLE

Water Flow in pumps and Pressure given out by pumps for a given time period

OBJECTIVE FUNTION

Minimize operational cost of pumps, which in our case mainly include electricity cost

CONSTRAINTS

Hydraulic constraints are given by physical laws governing the fluid flow in a pipe network.

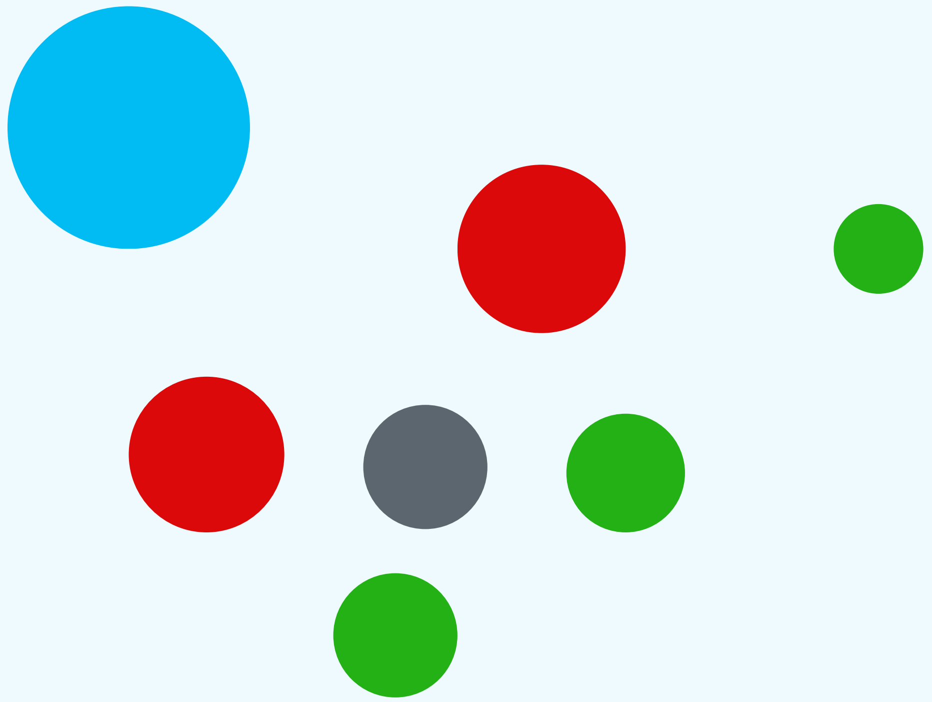
System Constraints like water demands and water quality requirement

THE GOAL IN DESIGING SOFTWARE WHICH CAN COME UP WITH AN OPTIMAL SCHEDULE FOR A GIVEN WATER DISTRIBUTION NETWORK , CONSISTING OF PIPES ,TANKS AND PUMPS

HOW THE PAPER IS DIVIDED

1. WDS Modelling
2. Optimal Water Flow
3. Convex Relaxation
4. Penalty Term Tests and Results

NODES (M)



Mr = Resevior Nodes

Mb = Tank Nodes

Mc = Consumer Nodes

Junction Node

Flow (d) for each node (m) at time (t)

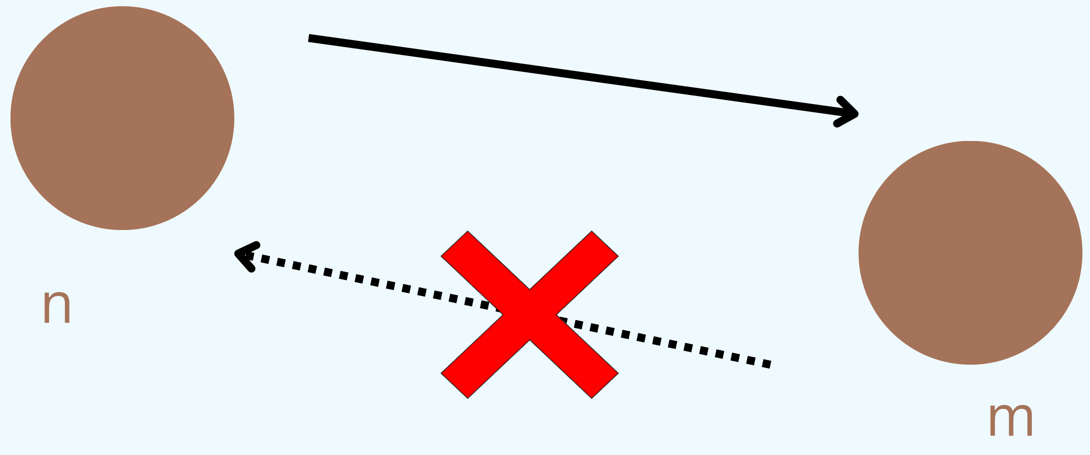
dm>=0

dm belongs to R

dm<=0

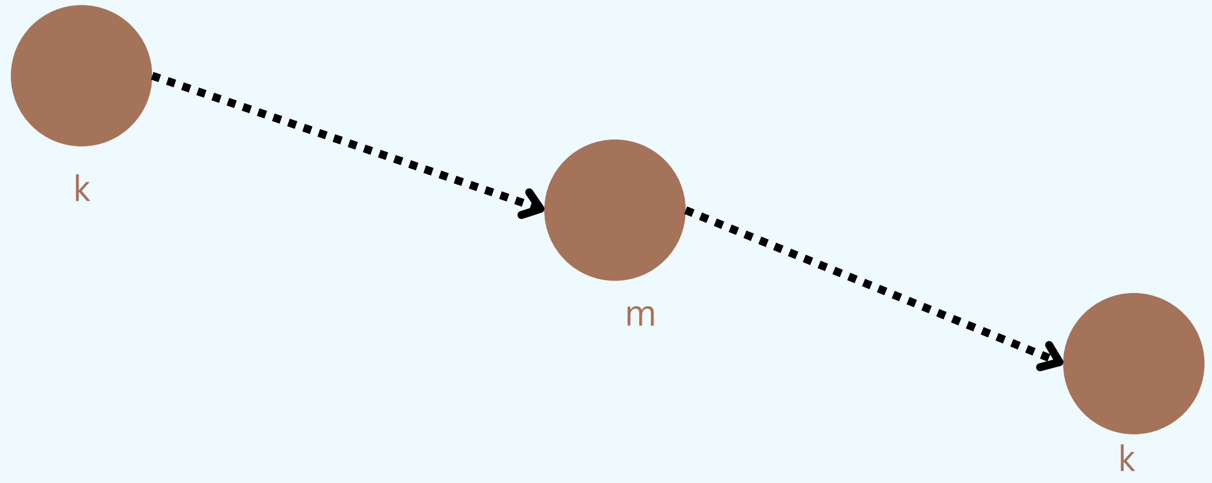
dm=0

PIPES (P)



if (n,m) in P then (m,n) not in

Flow between nodes (dmkt)



dmt = sum(dkmt) (for all incoming pipes) - sum(dkmt)(for all outgoing pipes)

PRESSURE IN OUR NETWORK (PIPES)

Pressure at each node is refered to as pressure head, a quantity that is linearly correlated with pressure

For water to flow each node requires to be above a certain pressure level

hmt>=hmmin

When water flows through a pipe, it loses some pressure due to frictional losses proportional to **square(dnm)**

$$h_m^t - h_n^t = c_{mn}\,\mathrm{sign}(d_{mn}^t)(d_{mn}^t)^2$$

We express this using **Darcy-Wesbach** equation

$$c_{mn} := \frac{\ell_{mn} f_{mn}}{4\pi^2 r_{mn}^5 \tilde{g}}$$

Linearize to remove **sign**

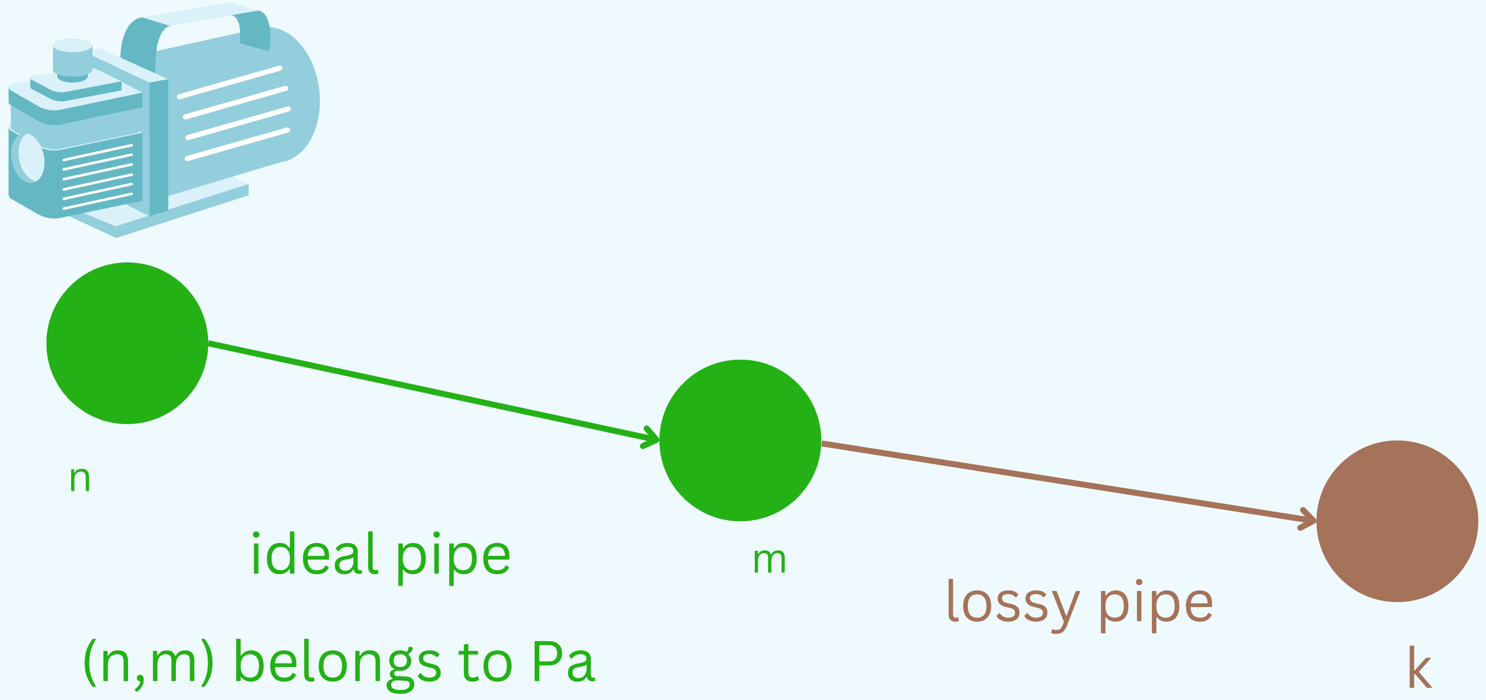
$$-M(1 - x_{mn}^t) \le d_{mn}^t \le M x_{mn}^t \tag{4a}$$

$$-M(1 - x_{mn}^t) \le h_m^t - h_n^t - c_{mn}(d_{mn}^t)^2 \le M(1 - x_{mn}^t) \tag{4b}$$

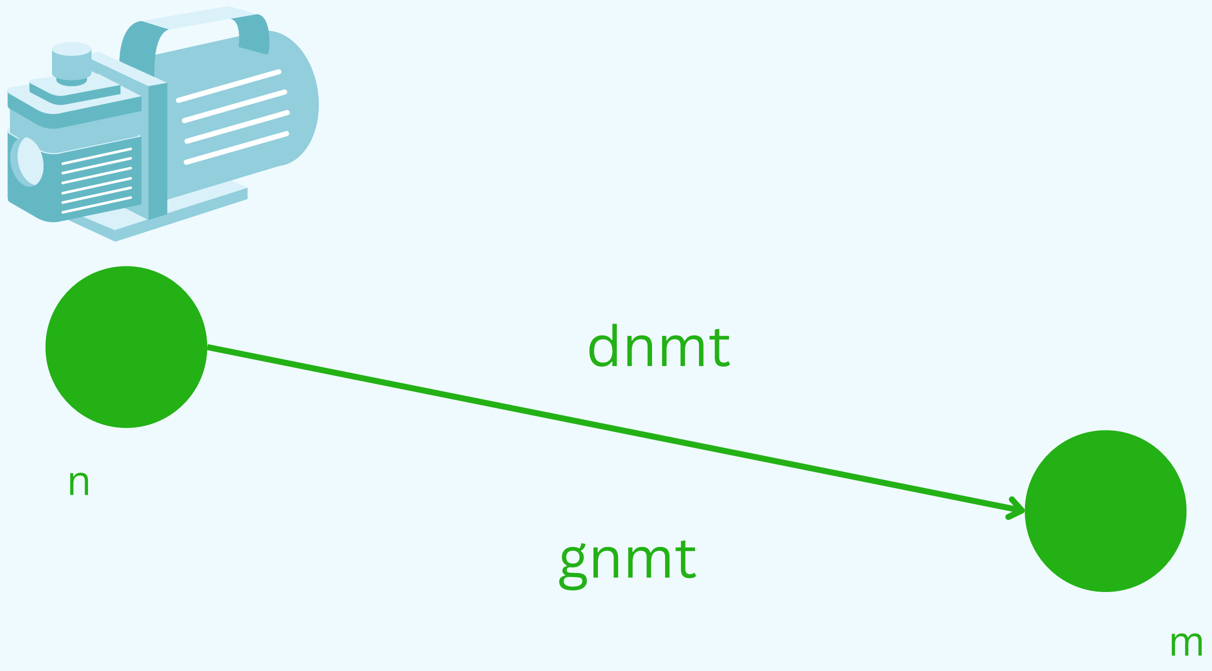
$$-M x_{mn}^t \le h_m^t - h_n^t + c_{mn}(d_{mn}^t)^2 \le M x_{mn}^t \tag{4c}$$

$$x_{mn}^t \in \{0, 1\} \tag{4d}$$

PRESSURE IN OUR NETWORK (PUMPS)



Pumps over a pipe are modelled as two pipe



Relation between pressure gain (gnmt) and water flow (dnmt)

if the pump is operational
xmnt=1

dnmmin<=dnmt<=dnmmax

hnt-hmt=hnmt

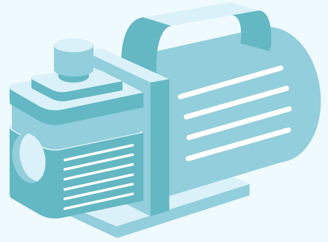
if the pump is not operational
xmnt=0

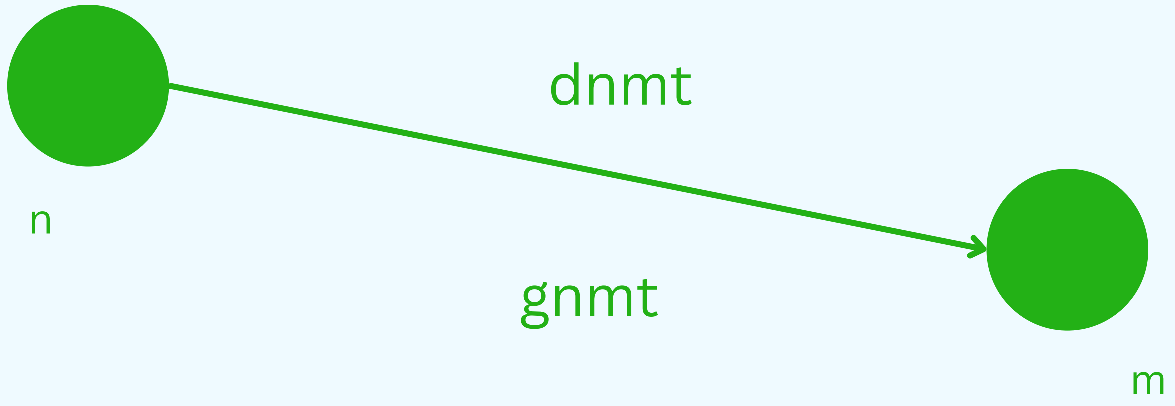
dnmt belongs to R

hnt=hmt

Pressure gain (gnmt) is considered constant for a tank

PRESSURE IN OUR NETWORK (PUMPS)





Relation between pressure gain (gnmt) and water flow (dnmt)

if the pump is operational

xmnt=1

dnmmin<=dnmt<=dnmmax

hnt-hmt=hnmt

if the pump is not operational

xmnt=0

dnmt belongs to R

hnt=hmt

Pressure gain (gnmt) is considered constant for a tank

Linearize our relationship

d’tmn is an auxilary variable

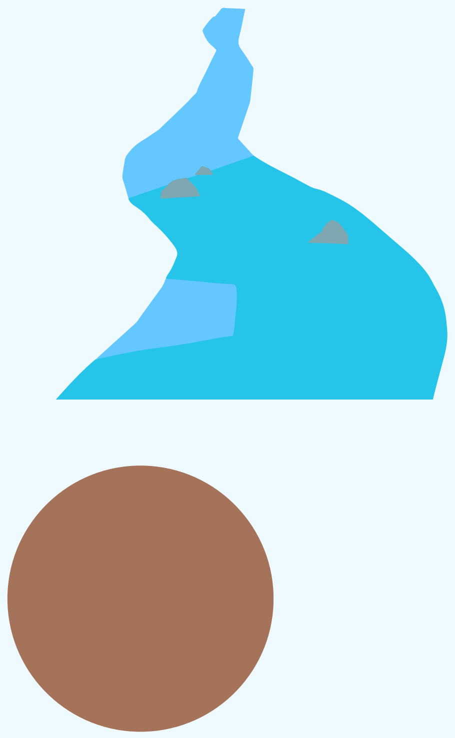
$$h_m^t - h_n^t = -g_{mn} x_{mn}^t \tag{6a}$$

$$-M(1 - x_{mn}^t) \leq d_{mn}^t - \tilde{d}_{mn}^t \leq M(1 - x_{mn}^t) \tag{6b}$$

$$\underline{d}_{mn} x_{mn}^t \leq \tilde{d}_{mn}^t \leq \overline{d}_{mn} x_{mn}^t \tag{6c}$$

$$x_{mn}^t \in \{0, 1\}. \tag{6d}$$

PRESSURE IN OUR NETWORK (RESEVIORS)



h’m does not change over time

hmt is the pressure drawn to the node from the resevior

When the reservoir is connected
Alpha(mt)=1

0<=hmt<=h’m

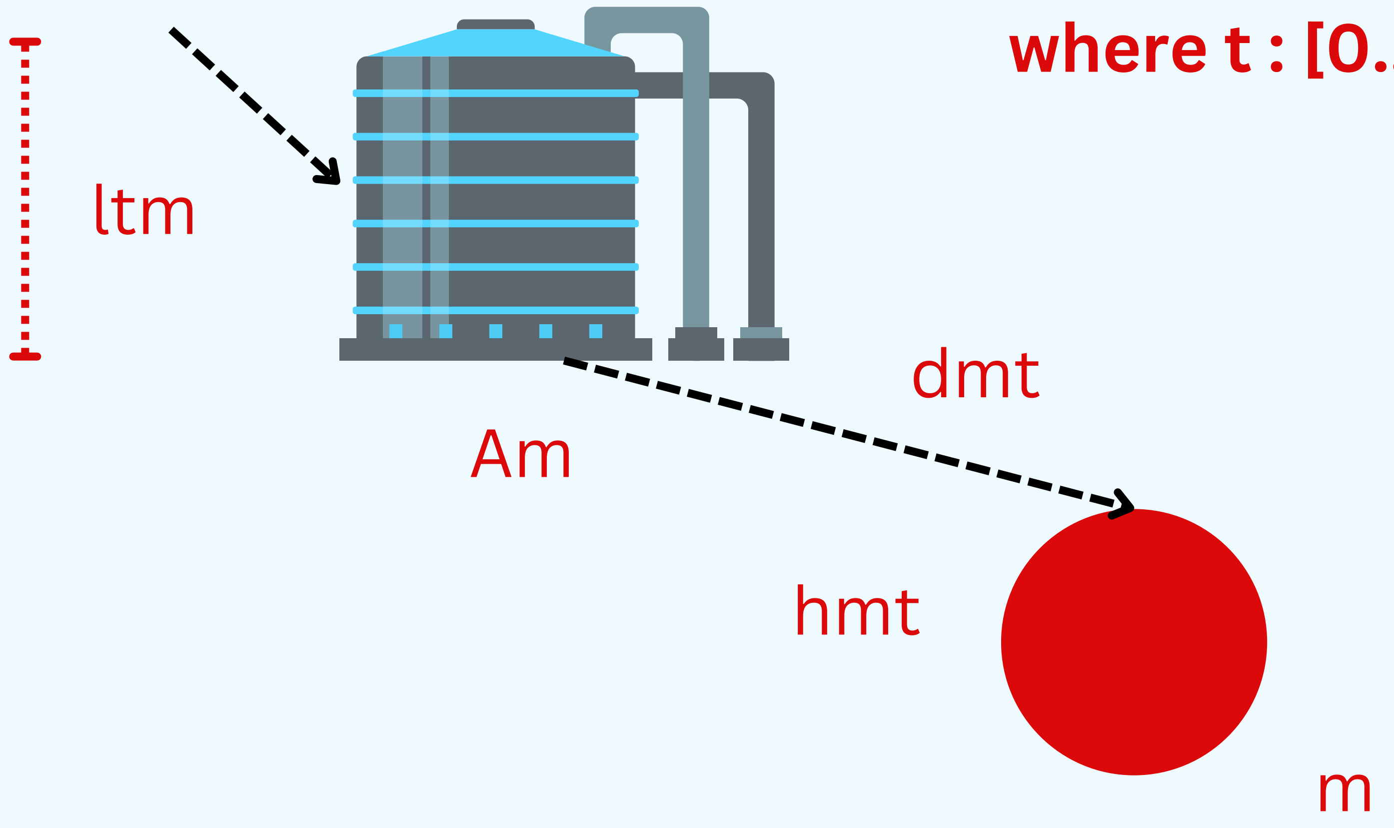
Linearize this relationship

$$0 \leq d_m^t \leq M\alpha_m^t \quad (7a)$$

$$h_m^t \leq \bar{h}_m + M(1 - \alpha_m^t) \quad (7b)$$

$$\alpha_m^t \in \{0, 1\} \quad (7c)$$

PRESSURE IN OUR NETWORK (TANKS)



where t : [0.....T]

Water Level after a given time interval **ltm=lt-1m-dmt*Del/Am**

Tank should be between a min and max level **lmmin<=lltm<=lmmax**

Net water exchange is kept at zero **l0m=lTml**

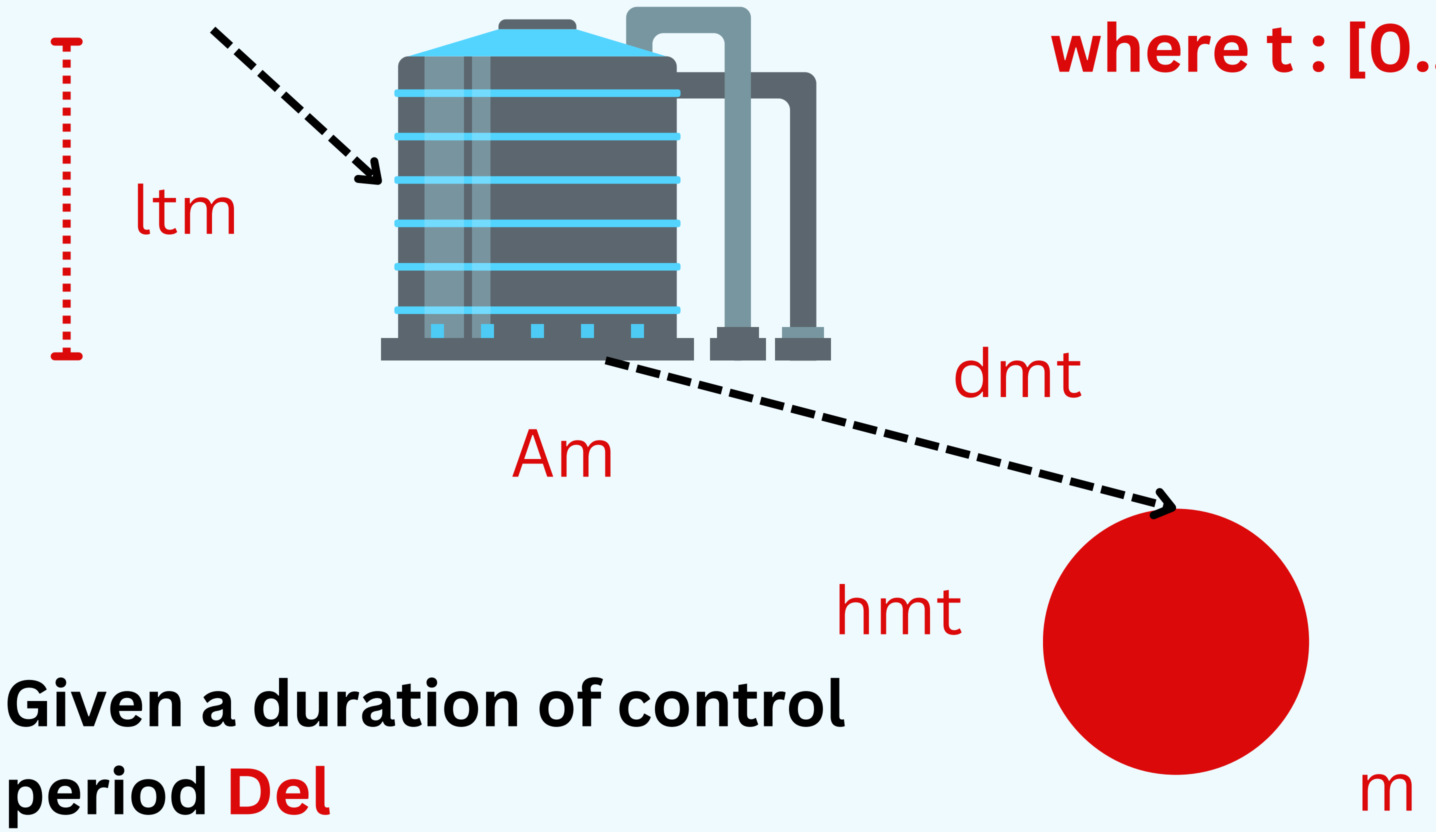
When tanks is being filled
Betamt=1 and Alphamt =1 **htm<=lmmax**

When tank is not being filled
Betamt=0 and Alphamt =1 **htm<=lmt**

When tank is not connected
Alphamt=0 **htm is not related to lmt**

Given a duration of control period Del

PRESSURE IN OUR NETWORK (TANKS)



where t : [0.....T]

When tanks is being filled
Betamt=1 and Alphamt =1

htm<=lmmax

When tank is not being filled
Betamt=0 and Alphamt =1

htm<=lmt

When tank is not connected
Alphamt=0

htm is not related to lmt

Given a duration of control period Del

Linearizing our tank level and pressure relationship

$$-M(1-\alpha_m^t) \leq \tilde{h}_m^t - h_m^t \leq M(1-\alpha_m^t) \tag{11a}$$

$$-M\alpha_m^t \leq d_m^t \leq M\alpha_m^t \tag{11b}$$

$$-M\beta_m^t \leq d_m^t \leq M(1-\beta_m^t) \tag{11c}$$

$$\bar{\ell}_m - M(1-\beta_m^t) \leq \tilde{h}_m^t \leq \ell_m^t + M\beta_m^t \tag{11d}$$

$$\alpha_m^t, \beta_m^t \in \{0,1\}. \tag{11e}$$

HOW WE MODEL CONTROL PERIODS

- We discretize our continuous system into time chunks **t=0,1,2,.T**
- **0** represents the start of our system, while **T** represents the end
- All of these chunks are of a constant duration **Del**

HOW CAN WE CALCULATE ELECTRICITY CONSUMPTION

- The overall energy consumed by a pump in **Del** is proportional to the **pressure (gnmt)** and the **flow-induced (d'nmt)** at a pipe

- Energy Consumption Coefficient for **Del**

$$c_{mn} := \frac{\delta \rho \tilde{g} g_{mn}}{\eta_{mn}}, \quad \forall (m,n) \in \mathcal{P}_a.$$

- **Emn(t)=cmn*d'mn(t)**

PROBLEM FORMATION

COST AND OBJECTIVE FUNCTION

- Our electricity cost is **pi mn (t)** for pump located at **(m,n)**

- We assume this cost to be invariant for all time for simplicity

Objective Function

$$f(\tilde{\mathbf{d}}) := \sum_{t=1}^{T} \sum_{(m,n)\in\mathcal{P}_a} c_{mn}\pi_t \tilde{d}_{mn}^t$$

DECISION VARIABLE AND CONSTRAINTS

- Lemma1 = The variable **htm ,dtmn and d’tmn** are sufficient to represent any feasible point in our constraints

- Other variables are really there to avoid, discontinous equations and multiplication of binary and continuous variables

$$\begin{aligned} \text{over} \quad & \{h_m^t\}_{m\in\mathcal{M}}, \{d_m^t\}_{m\in\mathcal{M}_b\cup\mathcal{M}_r}, \{d_{mn}^t\}_{(m,n)\in\mathcal{P}}, \\ & \{\tilde{h}_m^t\}_{m\in\mathcal{M}_b}, \{\ell_m^t\}_{m\in\mathcal{M}_b}, \{\tilde{d}_{mn}^t\}_{(m,n)\in\mathcal{P}_a}, \\ & \{x_{mn}^t\}_{(m,n)\in\mathcal{P}}, \{\alpha_m^t\}_{m\in\mathcal{M}_r\cup\mathcal{M}_b}, \{\beta_m^t\}_{m\in\mathcal{M}_b}, \quad \forall t \end{aligned}$$

- Our Constraints were the ones we discussed before

PROBLEM FORMATION

MAKING OUR RELAXATION CONVEX

- The equations 4b and 4c make our relaxation non convex

$$-M(1-x_{mn}^t) \leq d_{mn}^t \leq M x_{mn}^t \tag{4a}$$
$$-M(1-x_{mn}^t) \leq h_m^t - h_n^t - c_{mn}(d_{mn}^t)^2 \leq M(1-x_{mn}^t) \tag{4b}$$
$$-M x_{mn}^t \leq h_m^t - h_n^t + c_{mn}(d_{mn}^t)^2 \leq M x_{mn}^t \tag{4c}$$
$$x_{mn}^t \in \{0,1\} \tag{4d}$$

P1

- We get rid of the righside inequality for 4b and the left side inequality for 4c

- This makes our relaxation convex (MISOCP), but since P2 is a relaxation of P1, they only share a minimizer when 13b and 13c's equalities are satisfied

$$-M(1-x_{mn}^t) \leq d_{mn}^t \leq M x_{mn}^t \tag{13a}$$
$$-M(1-x_{mn}^t) \leq h_m^t - h_n^t - c_{mn}(d_{mn}^t)^2 \tag{13b}$$
$$h_m^t - h_n^t + c_{mn}(d_{mn}^t)^2 \leq M x_{mn}^t. \tag{13c}$$

P2

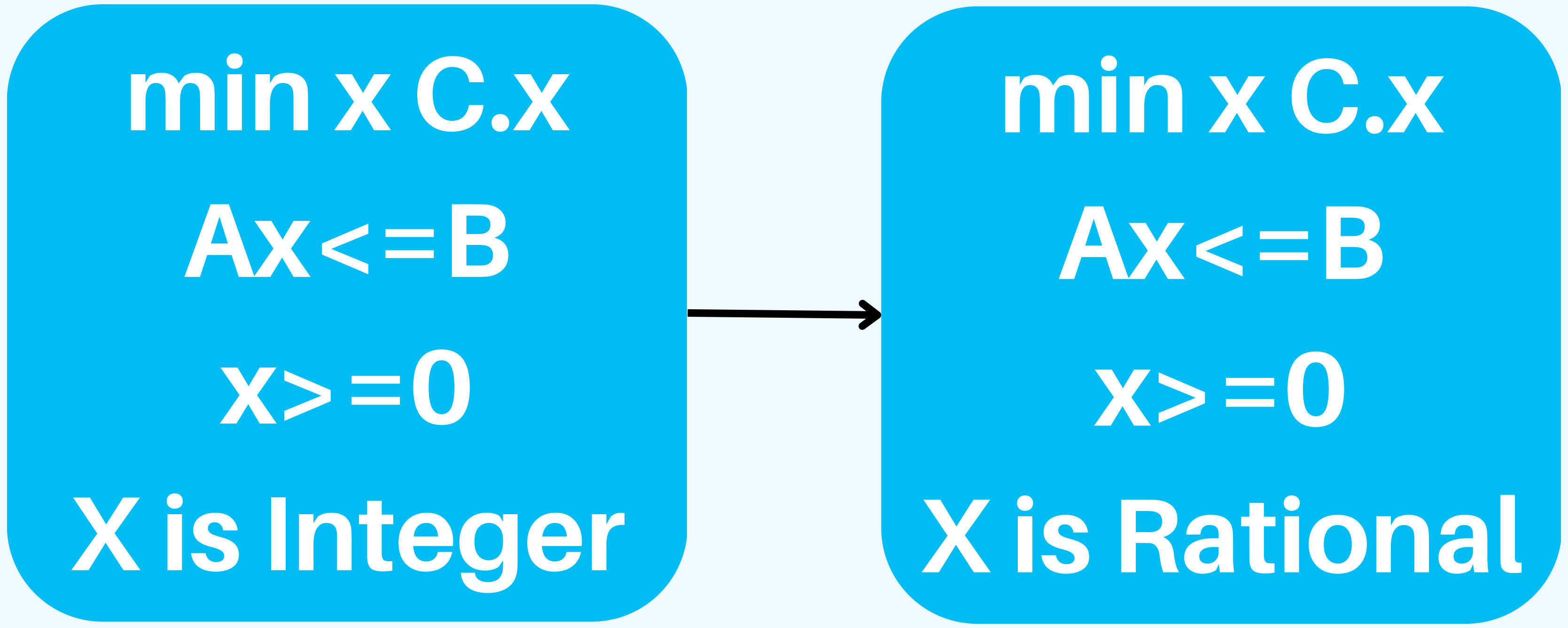
CONCLUSION

We can solve this model with off the shelfs MISOCP solvers, but the paper creates a penalty term for our objective to help minimizers

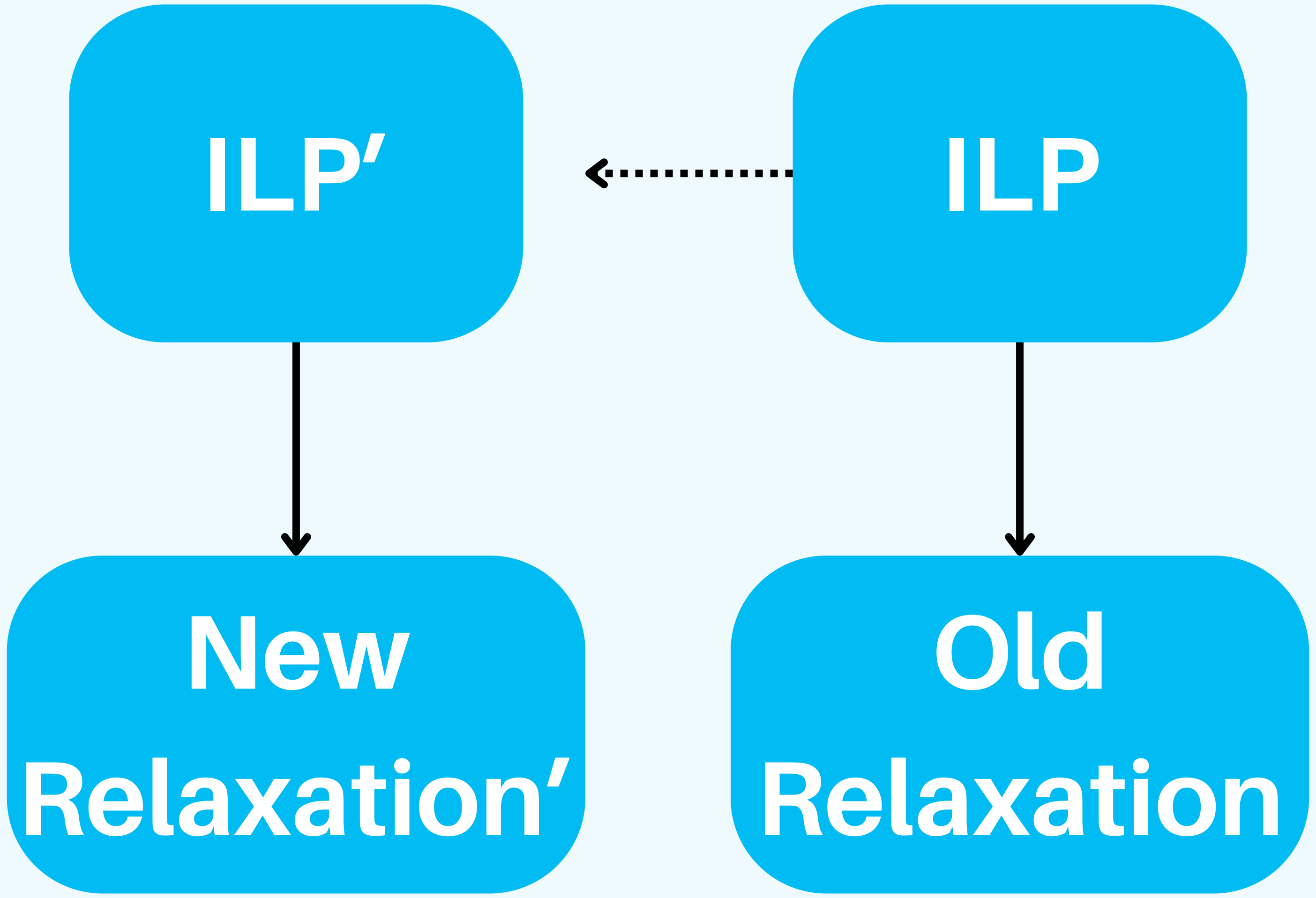
The numerical tests show how this formulation is implemented in MATLAB and Gurobi and simulated in EPANET.

Future research directions suggest extensions of the formulation to variable speed pumps and electricity prices

HOW IS OUR MODEL IMPROVED



SOLVING A LINEAR INTEGER PROGRAM INVOLVES LINEAR RELATION



IF NEW RELAXATION IS A STRICT SUBSET OF THE OLD RELAXATION THEN IT S A SMALLER SOLUTION SPACE FOR THE SAME PROBLEM

HEADLOSS IMPROVEMENT

Old model

lijp=lij*fi

Linearize

lijp>=0

lijp<=Li*fi

lij-Li*(1-fi)<=lijpj

lijp<=lij

New model

We introduce lijs which represents length of link for the secondary network

lijp+lijs=lij

sum over a link lijp = Li*fi

sum over a link lijs = Li*(1-fi)

HEADLOSS IMPROVEMENT STRICT SUBSET PROVE

Consider the point :

Link Length = L

Primary pipe segment =[L/2,L/2] ,f = 0.5

Secondary pipe segment =[0,0]

Link/Pipe Length = [L/2,L/2]

Old model Satisfied

lijp=lij*fi

L/2=L*0.5

New model Unsatisfied

lijp+lijs=lij — L/2=L/2

sum over a link ljp = L*fi — L/2=L/2

sum over a link lijs = Li*(1-fi) — 0=L/2

TANK COST IMPROVEMENT

Old model

znk=enk*dn eq0

Linearize

znk>=0

znk<=DE*enk

znk-DE*(1-enk)<=dn

dn<=znk

znk is the water demand of tank k at node n

DE is total wated demand of the network

New model

eq1 LOk*enk<=znk

eq2 znk<=UPk*enk

eq3 sum for all row for n enk =1

eq4 sum for all row for n znk =dn

lets say for node n we have 4 tanks

applying eq1&2 to enk

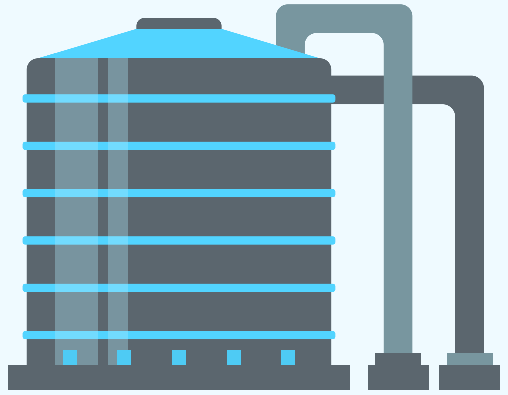
applying eq4 to znk

enk : 0 1 0 0

znk : 0 zn2 0 0

zn2=dn

TANK COST IMPROVEMENT : STRICT SUBSET PROOF



z1k : [d,d]
e1k:[1/2,1/2]
d1=d

LOk : [0,d]
UPk:[d,2d]
DE=2d

Old model is satisfied

znk>=0

znk<=DE*enk

znk-DE*(1-enk)<=dn

dn<=znk

New model is violated

eq1	LOk*enk<=znk
eq2	znk<=UPk*enk
eq3	sum for all row for n enk =1
eq4	sum for all row for n znk =dn

TANK COST IMPROVEMENT : BEST RELAXATION PROOF

z1k : [1,1]
e1k:[1/2,1/2]
d1=1

P1

t=1/2
n’=1
k’=1

z1k : [0,2]
e1k:[0,1]
d1=0

P2

LOk : [2,0]
UPk:[1,0]
DE=2

P3

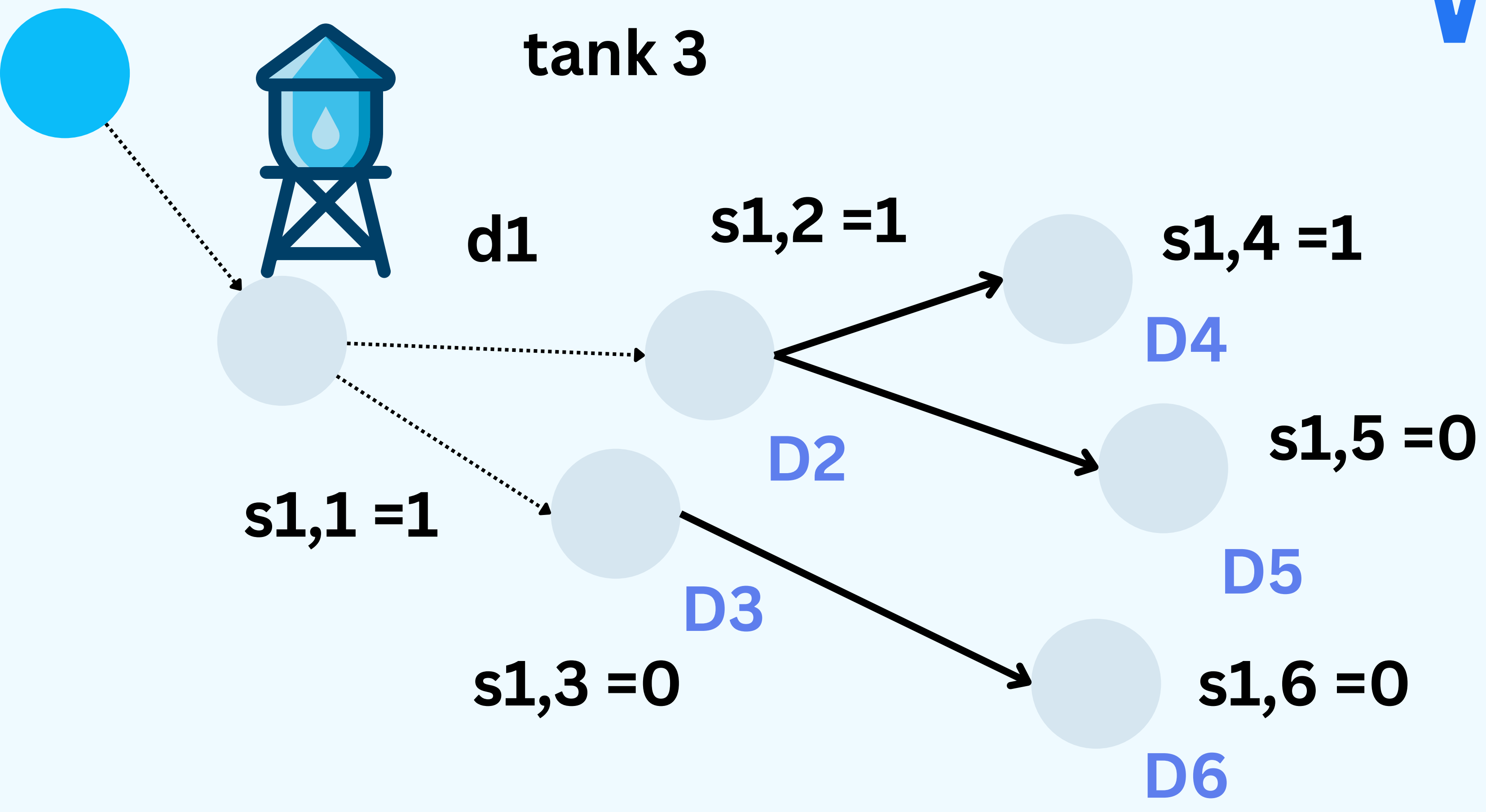
New model is Satisfied

eq1	LOk*enk<=znk
eq2	znk<=UPk*enk
eq3	sum for all row for n enk =1
eq4	sum for all row for n znk =dn

P1 = P2/2 +P3/2

Since P can be represented as a linear combination of two other points belonging to R2, P cannot be a corner point of R2.

TANK CONFIGURATION IMPROVEMENT : INITIAL MODEL



Where m is a descendant of n

smm<=snn

snm<=snn

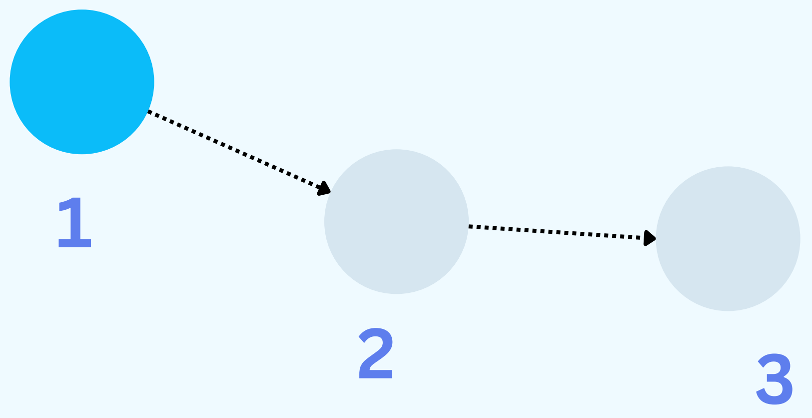
sum over all Ancestors of m snm=1

dn = snm*DEm

snm<=1-soo for all o in path of n,m

Nodes can serve their decendants if they serve their own

TANK CONFIGURATION IMPROVEMENT : TIGHTNESS PROOF



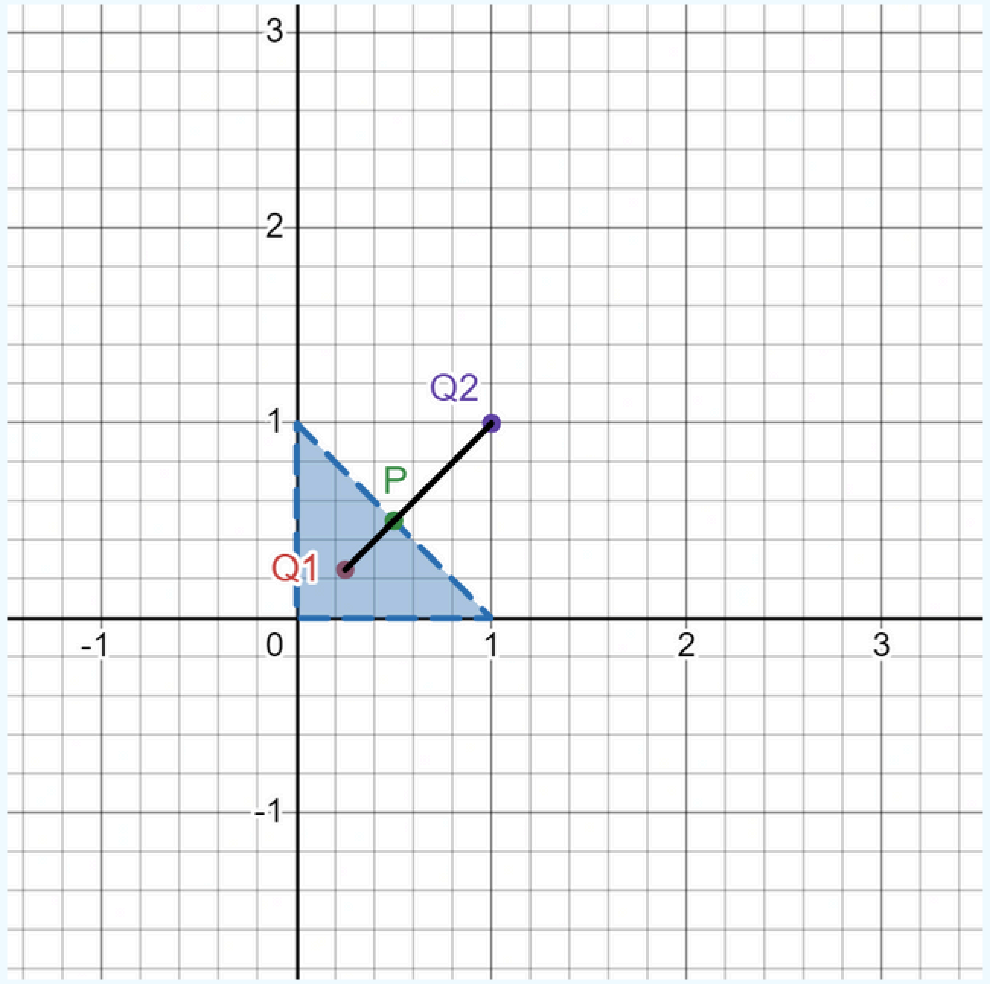
P : {1,0.5,0,0.5,0.5,0.5}

Q1 : {s11,s12,s13,s22,s23,s33}

Q2 : {s11,s12,s13,s22,s23,s33}

To prove

P
cannot be described as a linear combination of two distinct points that belong to R.



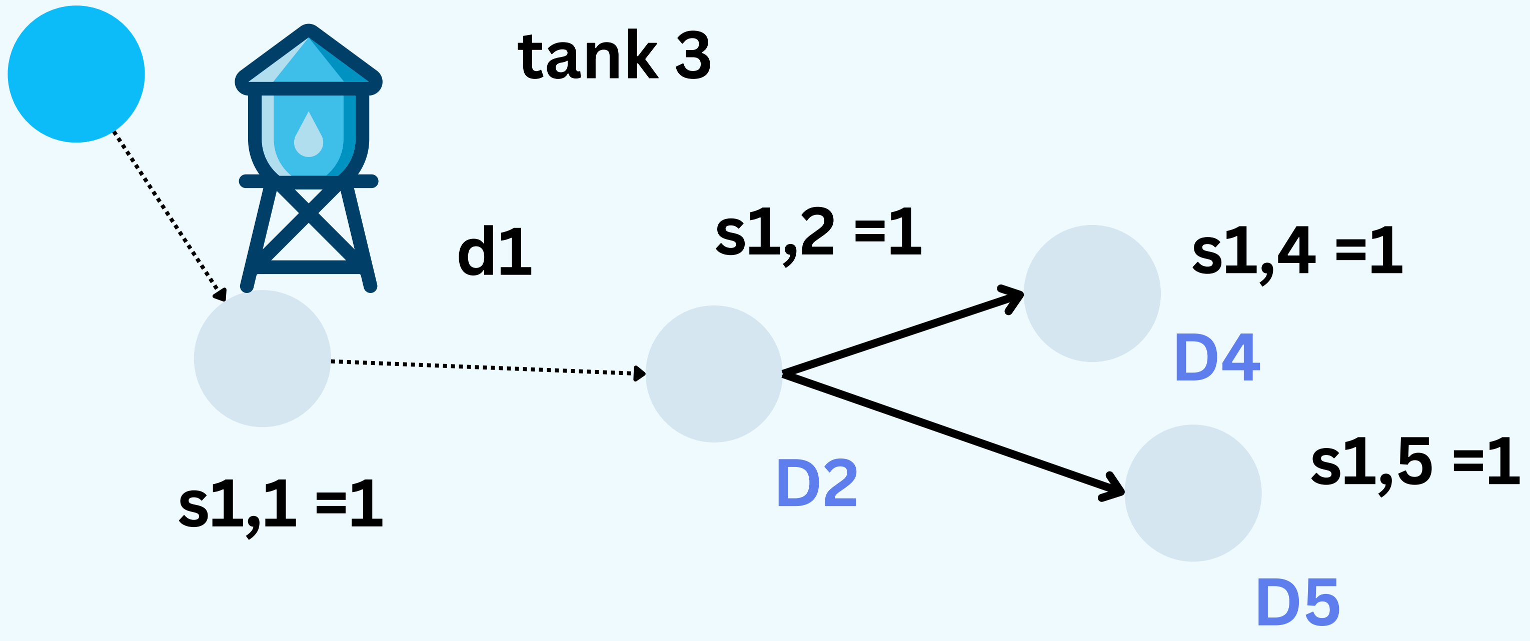
P = Q1+t(Q2-Q1)

P,Q1,Q2 belong in R2

The papers proof shows that this condition is only possible when Q1=Q2=P

Thus P is a corner point with fractional value

TANK CONFIGURATION IMPROVEMENT : NEW MODEL



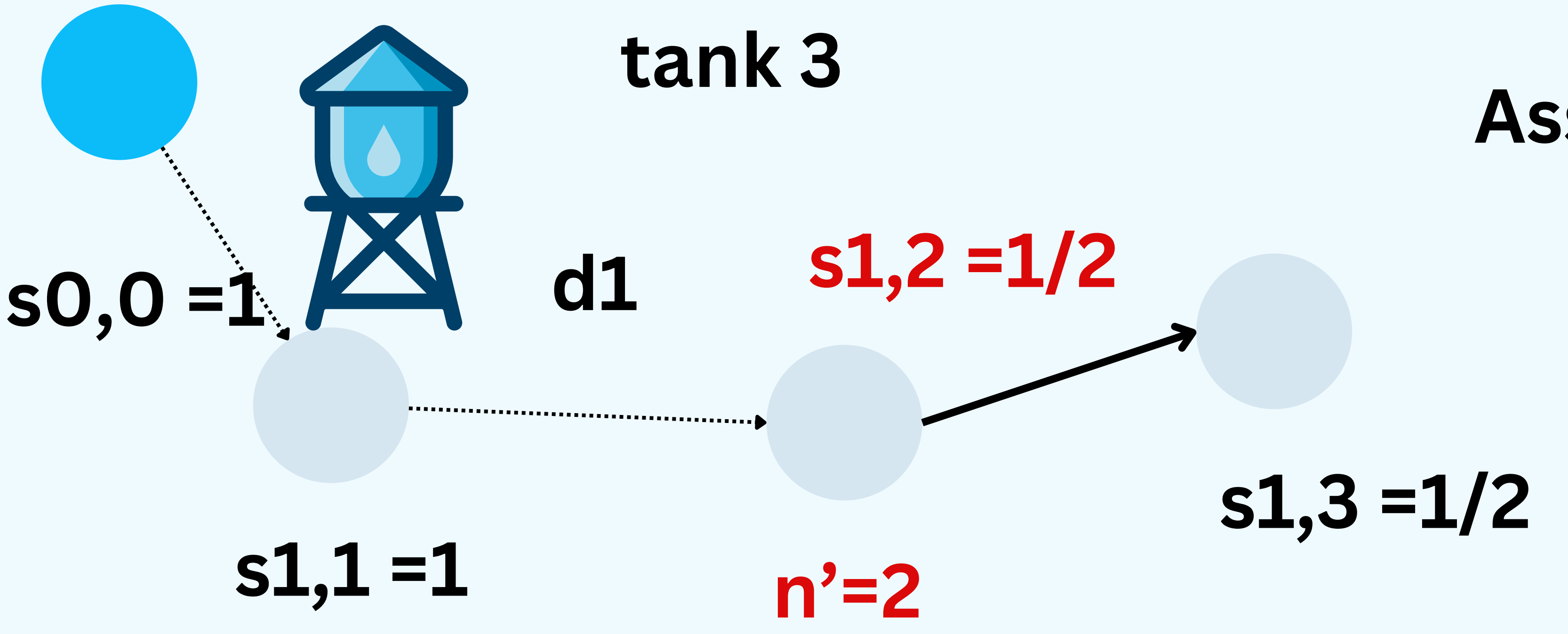
New Constraints

sum over n snm =1 for all ancestors of m

snm=snk where m is the child node and k is the descendant node

0<=snm<=1

TANK CONFIGURATION IMPROVEMENT : NEW MODEL TIGHTNESS



Assume this allocation of tanks is P

Claim 1

s0,0 =1, root node can never be fractional or zero

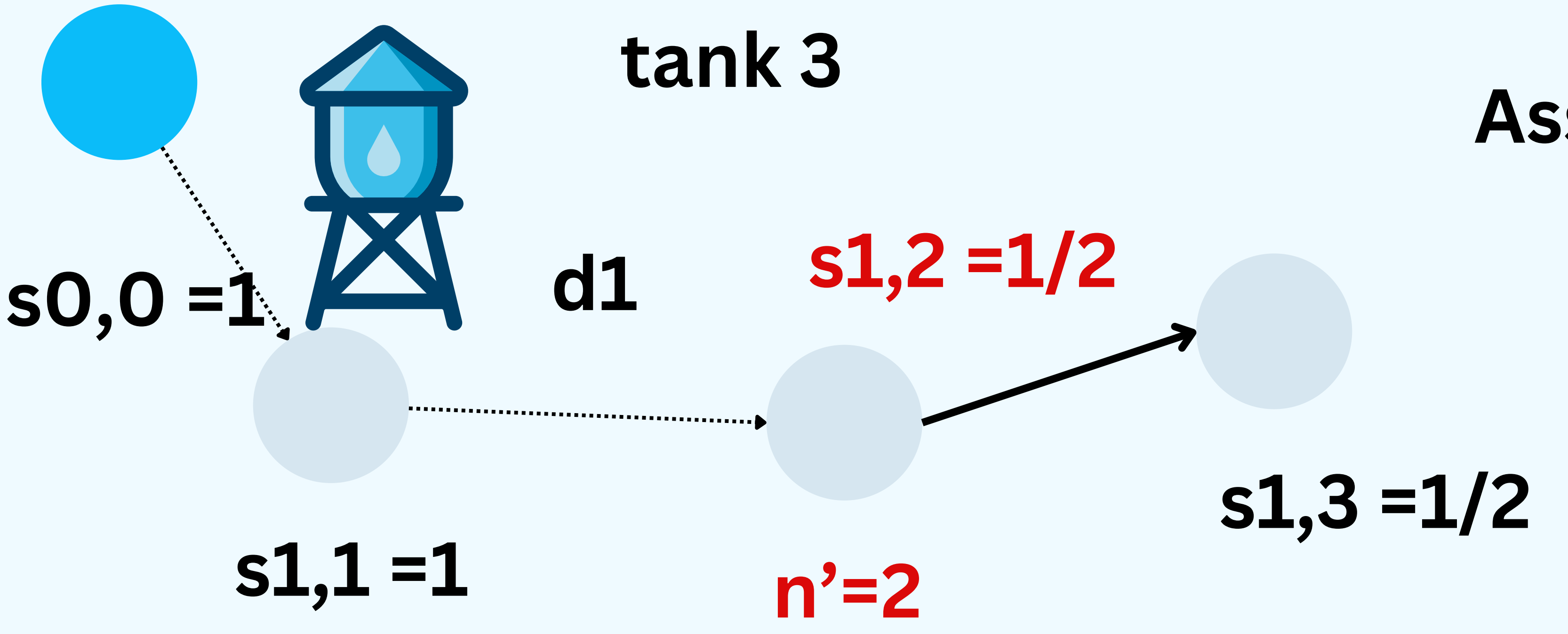
Claim 2

s0,2 =0 , descendants of secondary networks do not depend on nodes above their source node

Claim 3

- Q1 is a valid allocation of our tank
- snm=snm of P for all m not in 2’s path
- snm = snm /1-s1,2 for all n which are ancestors of 2
- s2m=0 for all m which are descendants of 2

TANK CONFIGURATION IMPROVEMENT : NEW MODEL TIGHTNESS



Assume this allocation of tanks is P

Claim 4

- Q2 is a valid allocation of our tank
- snm=snm of P for all m not in 2's path
- snm = snm /s1,2 for all n which are ancestors of 2
- s2m=0 for all m which are descendants of 2

Claim 5

P=s1,2*Q1+(1-s1,2)*Q2