# Stochastic Cellular Automata model of Stock Market Dynamics with Heterogeneous Agents

Ernani Hazbolatow, Alexis Ledru, Marcell Szegedi, Michelangelo Grasso

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- Whether an active trader in the $k$th cluster, $\sigma(t)_k = \pm 1$, sells or buys in the next time step is determined by $p_i^k$.

$$p_i^k = \frac{1}{1 + e^{-2I_i^k(t)}}$$

- The local field interaction, $I_i^k(t)$, can be written as follows

$$I_i^k(t) = \frac{1}{N^k(t)} \sum_{j=1}^{N^k(t)} A_{ij}^k \sigma_j^k(t)$$

$$A_{ij}^k = A(\zeta^k(t) + 2\eta_{ij}^k)$$

where $\eta_{ij}^k, \zeta^k(t)$ are randomly sampled from $U(-1, 1)$. So when $A << 1$, $p_i^k$ is essentially random and we would expect little alignment within the clusters.

# Quantifying The effect of $A$ in Base Model



A cluster can be seen as a dynamic, bound-by-activity Ising model. Thus, we compute the net magnetization of a cluster as

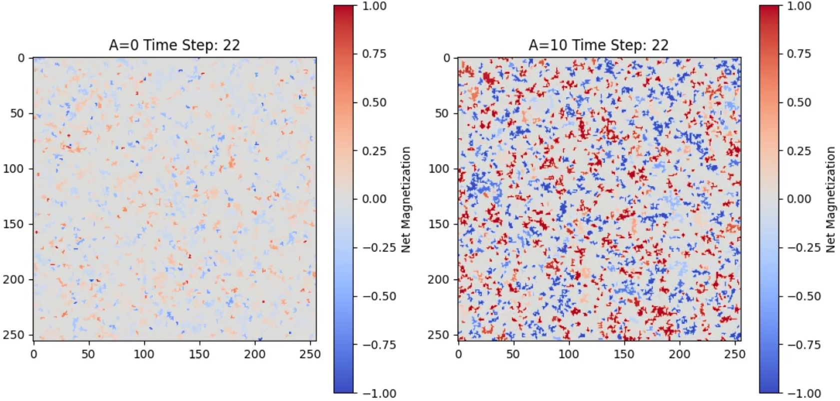
$$M_C = \frac{\#\text{Buyers} - \#\text{Sellers}}{\#\text{Buyers} + \#\text{Sellers}}$$

where # refers to the count within a cluster. The total signal that determines the price given by the market is the weighted sum of $M_C$ across clusters.

# Quantifying The effect of $A$ in Base Model



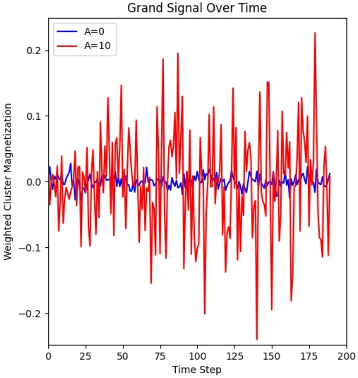
For $A = 0$ and $A = 10$, we observe more intense magnetization. Clusters are more aligned to be all buy or all sell.

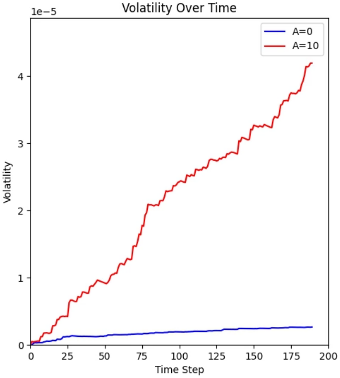


# Quantifying The effect of $A$ in Base Model



Both signals do not have positive or negative drift and fluctuate around 0. However, the signal is more volatile around 0 for $A = 10$. Since signal dictates price, the cumulative volatility of log returns, $\sigma$, show stronger price swings.



# Quantifying The effect of $A$ in Base Model



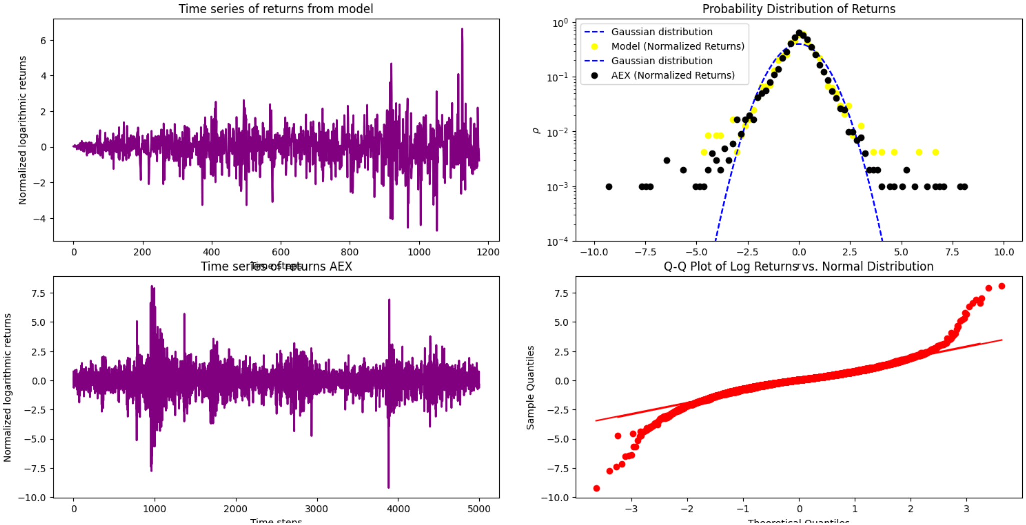
Now, we can conclude the following about $A$ and the volatility in the market:

- In the base model, volatility is directly tied to $A$. As $A$ increases, so does the volatility of the market. $\longrightarrow$ **Future research should investigate the relationship between $\sigma$ and $A$**
- Temporary up- or downtrends (”Booms and Crashes”) in the market are modeled through consequent probabilities.
- Taking the above point together necessarily implies that the market will always eventually revisit its initial value ($P_0 = 100$)

This implies the need for external forcing in the base model; the base model can never show SOC otherwise.

# Base Model follows Stylized Market Facts





# Power Law Distribution

5 Appendix

The probability of observing of a cluster size depends on the probability of herding, $p_h$, and approximately follows a power-law distribution. The fit to the power-law distribution depends on $p_h$. Although an exponential cutoff is observed for both probabilities due to the finite system size ($\sim 256^2$ agents), it is stronger for $p_h = 0.0475$.