# Stochastic Cellular Automata model of Stock Market Dynamics with Heterogeneous Agents

Ernani Hazbolatow, Alexis Ledru, Marcell Szegedi,
Michelangelo Grasso

January 31, 2025

# Introduction


- Understanding behavior of market dynamics in terms of complex system theory is a worthwhile goal.
- “Self-Organised Criticality” (SOC) is the property to check for.
- The critical points are characterized by fat-tailed fluctuations and long-memory correlations.

# Introduction
0

- A stock, also known as equity, is a security that represents the ownership of a fraction of the issuing corporation.
- Log return measures the percentage change in the price of a stock and is calculated using the natural logarithm of the ratio of consecutive prices. Properties: additive over time, symmetric for price increases and decreases. $R(t) = lnP(t+1) - lnP(t)$, volatility $v(t) = |R(t)|$

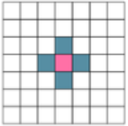
# Table of Contents

# Base Model

## 1 Background Knowledge

Characteristics and Assumptions:

Figure: A 2-dimensional grid, each cell is a trader, Von Neumann neighborhood.



# Percolation Dynamics

## 1 Background Knowledge

The number of active traders in the market is determined through percolation dynamics. The decision to be active in the market depends on the state of your neighbors.

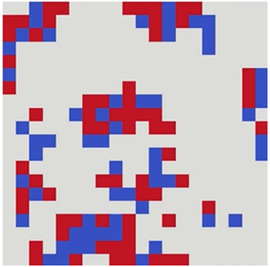


Figure: Example of CA of a stock at arbitrary time step $t$.

A stochastic exchange of information organizes the active traders in a cluster by acting upon what is occurring within, aligning their decisions to buy or sell.

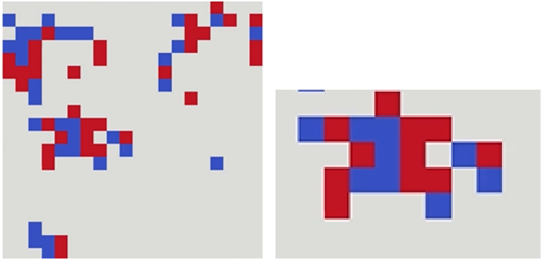


Figure: Grid level, Cluster level and Individual level.

# Log return and Price



- Once all active traders decide to buy or sell, a weighted sum based on the size of clusters is calculated
- Traders in larger clusters represent hedge funds, which tend to have higher buying power and resources than individual traders.
- Normalizing the result provides the log return, which allows us to get the price of the stock at time $t + 1$.
- Cycle repeats.

# Table of Contents

## 2 Methodology

# Proposed Solution



- Two coupled stochastic CAs modelling stocks. Second model (C2) introduces dependence on stock market. The first model (C1) is a commodity market upon which C2 is dependent in one directional causality. (e.g. C1 is oil and C2 is British petrol)
- In C2 same percolation dynamics as C1, but different stochastic dynamics. Every trader decides to either follow base strategy (as in C1) or the coupling strategy (buy or sell deterministically).
- Heterogeneity in agents introduced using an individual threshold criterion, which represents their individual idea of the market and risk appetite.

# Procedure



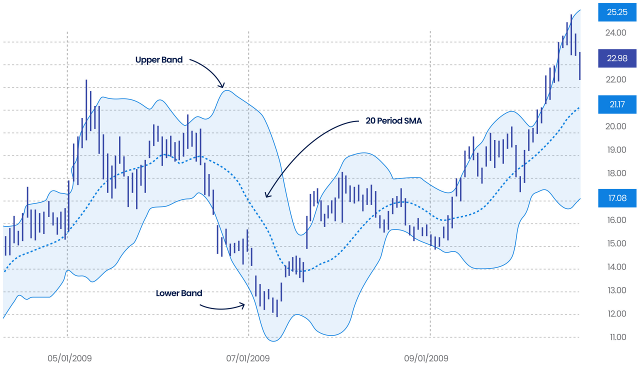
- Take time series of 100 steps of C1.
- Calculate average and standard deviation of price from C1.
- Calculate normalized difference of the latest price from the mean.
- All trader base their threshold range upon this value. If normalized difference is above or below the threshold in absolute value, then it represents a significant enough difference for a trader to switch from base strategy to coupled strategy.
- Coupled strategy: If above threshold, the traders own prediction is undervalued, so buy. If below threshold, the traders own prediction is overvalued, so sell.

# Information Delay

| | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 5 | 4 | 3 | 7 | 4 | 4 | 7 | 6 | 5 | 4 | 7 | 6 | 5 | 3 | 7 | 8 | 4 | 3 | 6 | 1 | 2 | 5 | 3 | 4 | 5 |
| Big Clusters | | | | | | | | | | | | | | | | | | 8 | 4 | 3 | 6 | 1 | 2 | 5 | 3 | 4 | 5 |
| Medium Clusters | | | | | | | | | | | | | | | 5 | 3 | 7 | 8 | 4 | 3 | 6 | 1 | 2 | 5 | | | |
| Small Clusters | | | | | | | | | | | | 4 | 7 | 6 | 5 | 3 | 7 | 8 | 4 | 3 | 6 | | | | | | |
| Individual Traders | | | | | | | | | 7 | 6 | 5 | 4 | 7 | 6 | 5 | 3 | 7 | 8 | | | | | | | | | |

# Procedure

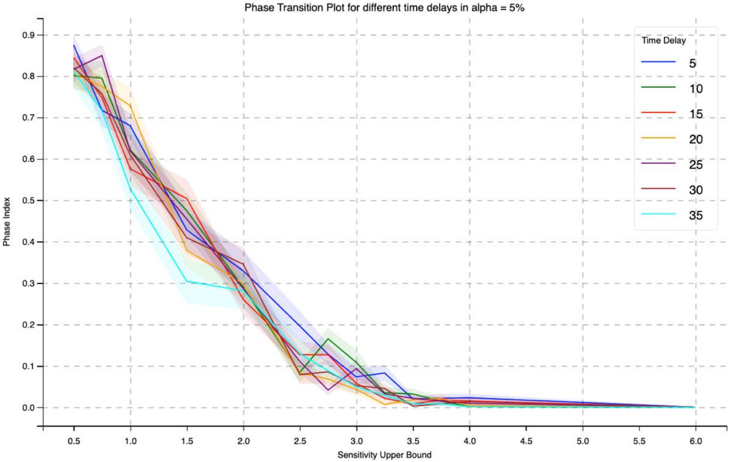
## 2 Methodology



# Table of Contents

# Varying Latency has no dampening effect on market volatility

## 3 Results and Discussion



# Table of Contents

4 Conclusion

# Conclusions



- **Hypothesis:** Information delay between traders does not have a dampening on the (dis)organization of the market. $\longrightarrow$ **Result** Yes, the (dis)organization curve is the same for all values of latency.

# *Q&A*

*Thank you for listening!*
*Your feedback will be highly appreciated!*

# Table of Contents

# The effect of $A$



- Whether an active trader in the $k$th cluster, $\sigma(t)_k = \pm 1$, sells or buys in the next time step is determined by $p_i^k$.

$$p_i^k = \frac{1}{1 + e^{-2I_i^k(t)}}$$

- The local field interaction, $I_i^k(t)$, can be written as follows

$$I_i^k(t) = \frac{1}{N^k(t)} \sum_{j=1}^{N^k(t)} A_{ij}^k \sigma_j^k(t)$$

$$A_{ij}^k = A(\zeta^k(t) + 2\eta_{ij}^k)$$

So when $A << 1$, $p_i^k$ is essentially random and we would expect little allignment within the clusters.