26-02-24

Assignment 3

7.30 Given $n = 25$

$E[X] = \mu = \bar{x} = 106$

$\sigma = 12$.

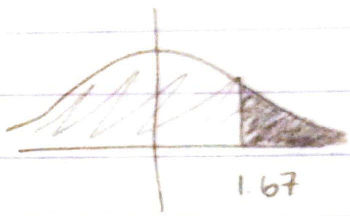
a)

The mean is 106.

The standard deviation, $\sigma = \frac{12}{\sqrt{25}} = 2.4$

b) $z = \frac{x - \mu}{\sigma/\sqrt{n}} = \frac{110 - 106}{2.4} = 1.67$

Using Table 3

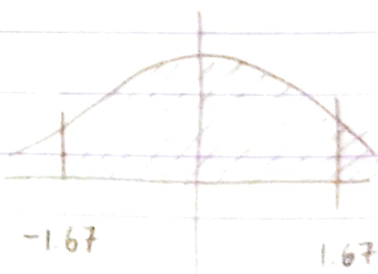


$P(Z > 1.67) = 1 - P(Z < 1.67)$

$= 1 - 0.95254$

$= 0.04746$

c) The sample mean deviates by no more than 4



$P(106 - 4 < X < 106 + 4)$

$= P(102 < X < 110)$

$= P\left(\frac{102 - 106}{2.4} < Z < \frac{110 - 106}{2.4}\right)$

$= P(-1.67 < Z < 1.67)$

$= P(Z < 1.67) + P(Z > -1.67)$

$= 2(P(Z < 1.67)$

$= 2 \times 0.45254$

$= 0.90508$

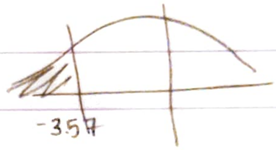
7.36 a) The approximate sampling distribution of the sample mean is it is a normal curve that is right skewed.

b) Given, * $\mu = 1450$, $n = 10$

$\sigma = 140$

$SE = \frac{\sigma}{\sqrt{n}} = \frac{140}{10} = 14$

$P(X < 1400) = P\left(Z < \frac{1400-1450}{14}\right)$



$= P(Z < -3.57)$

$= 0.00018$

c) $P(X < 1400) = P\left(Z < \frac{1400-\mu}{14}\right) = 0.001$

$= \frac{1400-\mu}{14} = -3$

$= 1400-\mu = -3 \times 14$

$= -\mu = -42-1400$

$= \mu = 1442.$

7.58 Given $\bar{\bar{x}} = 155.9$

$s = 4.3$

a) The upper control limit for an $\bar{x}$ chart is $\mu + 3\frac{\sigma}{\sqrt{n}}$

$= 155.9 + \left(3 \times \frac{4.3}{\sqrt{5}}\right)$

$= 161.7$ (1 decimal place)

The lower control limit for an $\bar{x}$ chart is $\mu - 3\frac{\sigma}{\sqrt{n}}$

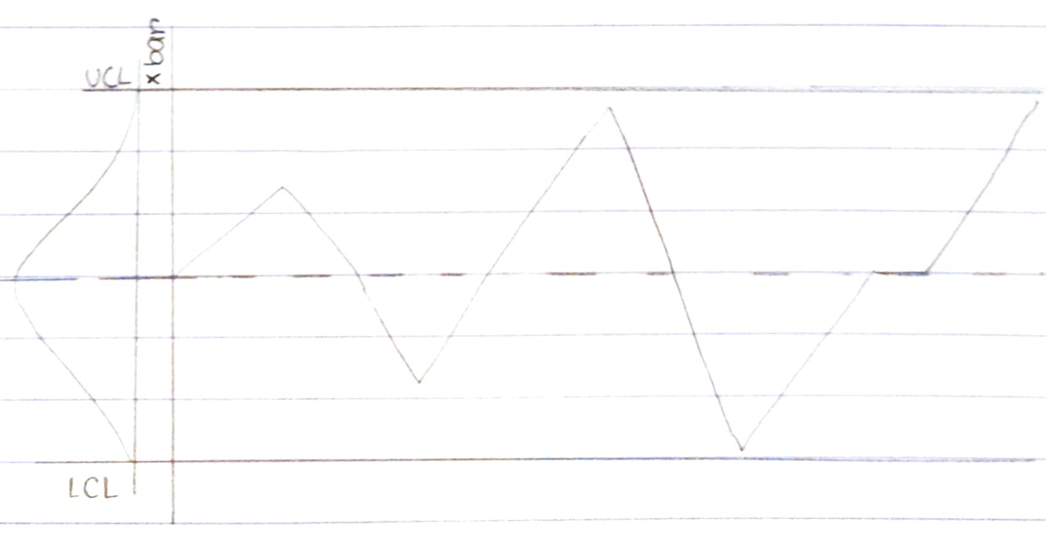
$= 155.9 - \left(3 \times \frac{4.3}{\sqrt{5}}\right)$

$= 150.1$ (1 decimal place)

b) Centreline : $\bar{\bar{x}} = 155.9$

Lower limit $= 150.1$

Upper limit $= 161.7$



As per given in the question and we can see from the $\bar{x}$ chart, the sampling process was controlled. We can see the upper control limit UCL 161.7 and lower control limit, LCL, 150.1, and the centreline, 155.9. We can easily conclude that the distribution is normal due to the sampling size being 200. We can use the $\bar{x}$-chart for range and the mean of the sample.

8.54. Given,

Major	Mean	SD
Education ($)	40,554	2225
Social Sciences ($)	38,348	2375

$n = 50$

a) Point estimate for the difference in average starting salaries of university students majoring in education and the social sciences is

$\mu_1 = 40554 = \bar{x}_1$

$\mu_2 = 38348 = \bar{x}_2$

$\mu_1 - \mu_2 = \bar{x}_1 - \bar{x}_2 = 40554 - 38348 = 2206$

Margin of error for the estimate $= 2206 \pm 1.96\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

here $S_1 = 2225$, $S_2 = 2375$, $n_1 = n_2 = 50$

$= 2206 \pm 1.96 \times \sqrt{\frac{(2225)^2}{50} + \frac{(2375)^2}{50}}$

$= 2206 \pm 1.96 \times 460.2$

$= 2206 \pm 901.80$

So the margin of error is 1304.2 to 3107.8

b) We can conclude that the average salary of education major is higher than the average salary of social sciences majors from the point of estimate. We can also say that the range within the majors for salary varies a lot by checking the margin of error.

8.63. a) Given $n_1 = 56$ $n_2 = 32$

$P_1 = \frac{12}{56}$ $P_2 = \frac{8}{32}$

$= 0.21$ $= 0.25$

95% confidence interval

$$(\hat{p}_1 - \hat{p}_2) \pm z_{a/2} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$= (0.21-0.25) \pm 1.96 \times \sqrt{\left(\frac{0.21 \times 0.79}{56}\right) + \left(\frac{0.25 \times 0.75}{32}\right)}$$

$$= -0.04 \pm 0.18$$

$$-0.22 < \hat{p}_1 - \hat{p}_2 < 0.14$$

d) We can conclude that there is a difference in the proportions for red candies in plain M&Ms and in peanut M&Ms. A higher proportion of red candies are in peanut M&M's packet than in plain M&Ms packet

8.87. Given, propability near 0.95

std, $6 = 0.5$

estimate to lie within 0.1

$$1.96\ SE \leq 0.1$$

$$\Rightarrow 1.96 \times \frac{6}{\sqrt{n}} \leq 0.1$$

$$\Rightarrow 1.96 \times \frac{0.5}{0.1} \leq \sqrt{n}$$

$\Rightarrow n \geqslant (9.8)^2$

$\Rightarrow n \geqslant 96.04$

$\therefore n = 97$

Approximately 97 rainfalls must be included in my sample (one pH reading per rainfall).

No, it would not be valid to select all of the rainfall water specimens from a single rainfall, because there would not be any variance in pH, therefore in this case it would become an all controlled sampling.