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Master thesis in Mathematics-Economics

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Swaptions pricing

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Abstract

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In this thesis we will investigate swaptions pricing.

2 Swaptions as a missing link in asset allocation

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3 Mathematics of pricing swaptions

Look at Swaption pricing and isolating volatility exposure.

To determine swaptions prices, it is important to understand which things there affects the price of the swaption. This chapter simplifies these concepts by explaining interest rates, bonds, swaps, and options, and then shows how they come together to determine the price of a swaption.

3.1 Time value of money

Understanding the concept of interest rates begins with the fundamental idea that a dollar today holds more value than the same dollar in the future. To understand these concept, a discount factor is introduce

$$B(t, T) = \text{value at time t of a dollar received at time T}$$

$B(t, T)$ refer to a contract that pays one dollar maturity, T, which can be illustrated as below

$$t < T \rightarrow B(t, T) < 1$$
$$t = T \rightarrow B(t, T) = 1$$

The concept of the "time value of money" it asserts that the value of money today is worth more than the same amount in the future due to its potential earning capacity, inflation, and risk. This principle underpins various financial decisions, including investing, borrowing, and pricing financial instruments. Essentially, it recognizes that a dollar received today can be invested and earn interest over time, thereby increasing its value. Conversely, a dollar received in the future is subject to uncertainty and may not retain its purchasing power due to inflation or other factors. The discount factor represents the present value of future cash flows, taking into account the time value of money. It reflects the idea that receiving a certain amount of money in the future is less valuable than receiving the same amount today.

3.2 Zero coupon bonds

3.3 The yield curve

Where the concept "time value of money" and the discount factor are fundamental concepts used to assess the present value of future cash flows, while the yield curve provides insights into market expectations regarding future interest rates. Understanding the interplay between these concepts is crucial for making informed investment decisions and pricing financial instruments.

The yield curve is a graphical representation illustrating the interest rates (bond yields) for various maturities. Yield curve can provide a intuition about future interest rates and give insight in the bond market today. The general intuition is that longer-term rates is higher then short-term rates, which in other words means that a lager premium is expect for lending money over a longer period of time. This case sketches a yield cure with a positive slope. It is important to know that the yield curve for a given interest change over time. This is supported by the yield curves illustrated below, where the same interest rate is displayed for the same maturities, but the data is from different days.

Make yield curve

3.4 Interest rates

3.4.1 Spot rates

The spot rate represents the yield-to-maturity of a zero coupon bond, while the forward rate refers to the anticipated interest rate in the future. Where we have that the definition for determined spot is listed below

Definition 1. *The simple spot rate for $[S, T]$, henceforth referred to as the LIBOR spot rate, is defined ad [1]*

$$L(t; S, T) = -\frac{p(t, T) - p(t, S)}{(T - s)p(t, T)}$$

3.4.2 Forward rates

Forward rates play a crucial role in financial markets, particularly in the realm of interest rate analysis and derivative pricing. They represent the interest rate applicable to a future period, agreed upon today. Understanding forward rates requires grasping the concept of forward contracts and the expectations theory of interest rates. Forward rates can be derived from the yield curve. The yield curve plots the yields of bonds with different maturities. By analyzing the yield curve, one can infer the implied forward rates for future periods. For example, the forward rate between year 1 and year 2 is the rate at which an investor can borrow or lend money for the period between year 1 and year 2, starting at year 1.

Lets consider three time points on the yield curve $t = 0, 1, 2$, where it is assumed that $t_0 < t_1 < t_2$. At time t_0 we have the spot rates $r(t_0, t_1)$ and $r(t_1, t_2)$, which represent the yields for bonds maturing at time t_1 and t_2 respectively. Hence the forward rate, $F(t_1, t_2)$, can med determined using the equation below

$$F(t_1, t_2) = \frac{(1 + r(t_0, t_1))^2}{(1 + r(t_0, t_1))} - 1$$

Imagine investing one dollar in a one-year zero-coupon bond, $B(t_0, t_1)$, and instantly reinvesting the money received at time t_1 in a new one-year zero-coupon bond, $B(t_1, t_2)$, at rate $F(t_1, t_2)$. This strategy should yield the same return as investing one dollar in a two-year zero coupon bond $B(t_0, t_2)$ and holding it for two years. This strategy illustrated the idea of forward rates. Let us then look a the general formula for forward rates.

Definition 2. *The continuously compounded forward rate for $[S, T]$ contracted at t is defined as [1]*

$$F(t; S, T) = -\frac{\log r(t, T) - \log r(t, S)}{(T - S)}$$

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References

[1] Björk, Arbitrage Theory in Continuous Time, Oxford, fourth edition, 2020