

## Volatility Exposure for Strategic Asset Allocation

**Marie Brière**, Amundi, Paris Dauphine University, Université Libre de Bruxelles

**Alexandre Burgues**, Amundi

**Ombretta Signori**, Amundi



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**Marie Brière**

*Head of Investor Research Center - Amundi*

*Associate Professor - Paris Dauphine University*

*Associate Researcher - Université Libre de Bruxelles*

Solvay Brussels School of Economics and Management - Centre Emile Bernheim, Belgium

[marie.briere@amundi.com](mailto:marie.briere@amundi.com)

**Alexandre Burgues**

*Absolute Return Fund Manager - Amundi*

**Ombretta Signori**

*Senior Strategist - Amundi*

## About the authors



**Marie Brière**, PhD, Head of Investor Research Center - Amundi and associate researcher with the Centre Emile Bernheim at Université Libre de Bruxelles.

A graduate of the ENSAE School of economics, statistics and finance and a PhD in Economics, Marie Brière worked from 1998 to 2002 as a quantitative researcher at the proprietary trading desk at BNP Paribas. She joined Amundi (*previously Credit Agricole Asset Management*) in 2002 as a fixed income strategist, then a Head of Fixed Income, Forex and Volatility Strategy. She also teaches empirical finance, asset allocation and investment strategies at Paris I and II Universities. Marie Brière is the author of a book on anomalies in the formation of interest rates, and a number of her scientific articles have been published in books and leading academic journals, including *The Journal of Portfolio Management*, *The Journal of Fixed Income*, and *European Economic Review*.



**Alexandre Burgues**, CFA, CAIA, Absolute Return Fund Manager

Alexandre Burgues joined Amundi in 2003 as a Global Balanced Portfolio Manager and ran for several years a wide range of asset allocation investment solutions dedicated to Institutional Clients. In 2010, Alexandre joined Amundi London's Global Fixed Income & Absolute Return team to develop global volatility strategies. Since 2012 he is also in charge of managing the range of global macro products of Amundi's London.

Alexandre holds a Master's degree in Economics and International Finance from the University of Bordeaux and a Postgraduate degree in Asset Management from University of Paris IX Dauphine. He is also a Chartered Financial Analyst and a Chartered Alternative Investment Analyst.



**Ombretta Signori**, Senior Strategist - Amundi

She holds a B.A. in Economics from the University of Cà Foscari and a Master Degree in Economics from Bocconi University. She is the author of a number of scientific articles published in academic and practitioners' journals, including *Journal of Portfolio Management* and *European Financial Management*.

She joined Amundi (*previously Credit Agricole Asset Management*) in 2006 as Fixed Income, Forex and Volatility Strategist, before she was strategist at Nextra Investment Management SGR (2002-2005) and economist at Rasfin SIM (2001).

## Abstract

This paper examines the advantages of incorporating strategic exposure to equity volatility into the investment-opportunity set of a long-term equity investor. We consider two standard volatility investments: implied volatility and volatility risk premium strategies. To calibrate and assess the risk/return profile of the portfolio, we present an analytical framework offering pragmatic solutions for long-term investors seeking exposure to volatility. The benefit of volatility exposure for a conventional portfolio is shown through a mean / modified Value-at-Risk portfolio optimization. Pure volatility investment makes it possible to partially hedge downside equity risk, thus reducing the risk profile of the portfolio. Investing in the volatility risk premium substantially increases returns for a given level of risk. A well calibrated combination of the two strategies enhances the absolute and risk-adjusted returns of the portfolio.

**Keywords:** variance swap, volatility risk premium, higher moments, portfolio choice, Value at Risk

**JEL codes:** G11, G12, G13

## **1. Introduction**

Direct exposure to volatility has been made easier, for a wide range of underlyings, by the creation of standardized instruments. The widespread use and increasing liquidity of volatility products, such as index futures and variance swaps, clearly show that investors are taking a keen interest in volatility. In addition to short-term trading ideas, some investors have been seeking structural exposure to volatility because they consider it either as a well identified asset class or, at the very least, a set of strategies with strong diversifying potential for their portfolios. Basically, standard exposure to volatility can be achieved through two complementary strategies: on the one hand, long exposure to implied volatility and on the other hand, exposure to the volatility risk premium. Both strategies are consistent with the traditional rationales required by investors to invest in an asset class: potential for return enhancement and risk diversification.

Being long implied volatility is compelling to investors for diversification purposes (Daigler and Rossi (2006), Dash and Moran (2005)). The remarkably strong negative correlation between implied volatility and equity prices during market downturns offers timely protection against the risk of capital loss. It has been well-documented in the academic literature (Turner et al. (1989), Haugen et al. (1991), Glosten et al. (1993)), and has led to two theoretical explanations. The first one is the “leverage effect” (Black (1976), Christie (1982), Schwert (1989)): equity downturn increases the leverage of the firm and thus the risk of the stock. Another alternative explanation (French et al. (1987), Bekaert and Wu (2000), Wu (2001), Kim et al. (2004)) is the “volatility feedback effect”: assuming that volatility is incorporated in stock prices, a positive volatility shock increases the future required return on equity and stock prices are expected to fall simultaneously.

Historically, exposure to the volatility risk premium has delivered very attractive risk-adjusted returns (Egloff et al. (2007), Hafner and Wallmeier (2008)). As documented by Bakshi and Kapadia (2003), Bondarenko (2006), and Carr and Wu (2009), implied variance is higher on average than ex-post realized variance. This can be explained by the risk asymmetry between a short volatility position (a net seller of options faces an unlimited potential loss), and a long volatility position (where the loss is capped at the premium). Moreover, going long the variance swap contract can be seen as a hedge against the risks associated with the random arrival of discontinuous price movements. To make up for the uncertainty on the future level of realized volatility, sellers of implied volatility demand compensation in the form of a premium over the expected realized

volatility<sup>1</sup>. Qualitatively, the variance risk premium is consistent with the Capital Asset Pricing Model (CAPM) framework and the well documented negative correlation between stock index returns and their variance. But Carr and Wu (2009) show that this negative correlation does not fully account for the negative variance risk premium. Other traditional equity risk factors such as size, book-to-market and momentum cannot explain it either. The majority of the premium is thus explained by an independent variance risk factor, which relates to the willingness of investors to receive an excess return not only because volatility hikes are seen as signals of equity market downturns, but also because these hikes by themselves are seen as unfavorable shocks on investors' portfolios (through the reduction of Sharpe ratios for instance).

The scope of this paper is to analyze investors' portfolio choices when volatility is added to their investment-opportunity set. Taking the perspective of a long-term US equity investor, we first design standard volatility strategies and then build efficient frontiers within a mean / modified Value-at-Risk framework. Our study is related to the strand of the literature that examines the asset allocation problem in the presence of derivatives. Carr and Madan (2001) study how to choose options at different strikes to span the random jump risk in the stock price, while Liu and Pan (2003) look at spanning the variance risk. In the same vein, Egloff et al. (2007) use variance swaps at different maturities to span the variance risk and benefit from large variance risk premia. Furthermore, volatility as an investment theme is often associated with the universe of alternative strategies, identified as a source of "alternative beta" (Kuenzi (2007)), that is to say a source of returns that is linked to systematic exposure to a risk factor but is not directly investable through conventional asset classes. Another strand of investment research related to our paper analyzes the interest of having different sources of alternative beta, such as hedge funds (Amin and Kat (2003), Amenc et al. (2005)), in a portfolio.

We believe this research is original for two reasons. First, it offers a framework for analyzing the inclusion of volatility strategies into a portfolio, departing from most of the previous literature which have in common the use of the mean-variance optimization. Adding volatility strategies to the investment-opportunity set raises the issue of how to measure a portfolio's expected utility when returns are non-normal. This issue is crucial, since returns to volatility strategies are asymmetric and leptokurtic. Accordingly, the goal of minimizing risk through a conventional mean-variance optimization framework can be misleading because extreme risks are not properly captured (Sornette *et al.* (2000), Goetzmann et al. (2002)). For volatility premium strategies, low volatility of

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<sup>1</sup> Other components can provide partial explanations of this premium: the convexity of the P&L of the variance swap, and the fact that investors tend to be structural net buyers of volatility to hedge equity exposure or to meet risk constraint requirements (Bollen and Whaley (2004), Carr and Wu (2008)).

returns is generally countered by higher negative skewness and higher kurtosis, which could prove costly for investors if not properly taken in account (Amin and Kat (2003)). Thus, appropriate optimization techniques must be used in order to assess risk through measures which capture higher-order moments of the return distribution. For our purposes, modified Value-at-Risk meets that requirement (Favre and Galeano (2002), Agarwal and Naik (2004), Martellini and Ziemann (2007)). Another aspect that needs to be addressed is the practical implementation of exposing a portfolio to volatility. Because these strategies are implemented through derivative products, they require limited capital, leverage being the key factor. In this case, the amount of risk to be taken in the portfolio needs to be properly calibrated.

Second, our research combines two complementary sets of volatility exposures, which have been so far examined separately. Daigler and Rossi (2006) analyzed the effect of adding a long volatility strategy to an equity portfolio, Dash and Moran (2005) to a fund of hedge funds. Egloff et al. (2007) and Hafner and Wallmeier (2008) examined the contribution to an equity portfolio of volatility risk premium strategies. Since investing in the volatility premium is a strategy similar to selling insurance premium, which exhibits very high downside risk, Egloff et al. (2007) highlight the need to hedge such an investment, at least partially. They show that under a two-factor risk dynamics, two distinct variance swap contracts of any maturity span the variance risk; and they propose a partial hedge of the short-term volatility risk premium through a short position in the stock index and a long position in the long-term volatility risk premium. From this point of view, our research is related to the authors' work and adds to this literature by demonstrating the usefulness of approaching volatility by combining two standard, complementary strategies. Long implied volatility strategy is an excellent hedge against the risks engendered by investing in the volatility risk premium. All in all, adding long implied volatility leads to lower levels of risk since this strategy typically hedges downside equity risk. Meanwhile, investing in the volatility risk premium enhances returns for a given level of risk. Finally, a combination of the two strategies is very appealing because they tend to hedge each other in adverse events. This combination can deliver compelling results for the portfolio, not just in terms of enhanced returns but also in terms of risk diversification.

## **2. Investing in volatility**

We begin by presenting two standard and complementary approaches to investing in volatility. We then address the practical issues of risk measurement and calibration in order to properly add the two designed strategies to the investment-opportunity set of an equity investor.

### **Long exposure to volatility**

One approach to volatility investing is to expose a portfolio to implied volatility changes in an underlying asset. The rationale for this kind of investment is primarily the diversification benefits arising from the strong negative correlation between performance and implied volatility of the underlying, particularly during market downturns (Daigler and Rossi (2006)). Tracking implied volatility for a specific underlying requires the computation of a synthetic volatility indicator. A volatility index, expressed in annualized terms, prices a portfolio of available options across a wide range of strikes (volatility skew) and with constant maturity (interpolation on the volatility term structure). Within the family of volatility indices, the Volatility Index (VIX) of the Chicago Board Options Exchange (CBOE) is widely used as a benchmark by investors. The VIX is the expression of the 30-day implied volatility generated from S&P 500 traded options. The details of the index calculation are given in a *White Paper* published by the CBOE (2004)<sup>2</sup>. As it reflects a consensus view of short-term volatility in the equity market, the VIX (see the time series plotted in Figure 1 of Appendix 1) is widely used as a measure of market participants' risk aversion (the "investor fear gauge").

Although the VIX index itself is not a tradable product, the Chicago Futures Exchange<sup>3</sup> launched futures contracts on VIX in March 2004. Thus investors now have a direct way of exposing their portfolios to variations in the short-term implied volatility of the S&P 500. VIX futures provide a better alternative to achieving such exposure than traditional approaches relying on the use of delta-neutral combinations of options such as straddles (at-the-money call and put), strangles (out-of-the-money call and put) or more complex strategies (such as volatility-weighted combinations of calls and puts). On short maturities (less than 3 months), the impact of neutralizing the delta exposure of these portfolios can easily dominate the impact of implied volatility variations.

The approach we take in establishing a structurally long investment in implied volatility tries to take advantage of the mean-reverting nature of volatility<sup>4</sup> (Dash and Moran (2005)). This is achieved by

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<sup>2</sup> VIX's method of calculation changed in September 2003. The current method (applied retroactively to the index since 1990) takes into account S&P 500 traded options at all strikes, unlike the previous VXO index, which was based solely on at-the-money S&P 100 options.

<sup>3</sup> The Chicago Futures Exchange is part of the Chicago Board of Options Exchange (CBOE).

<sup>4</sup> Empirical tests have shown that having an exposure inversely proportional to the observed level of implied volatility markedly increases the profitability of the strategy.



calibrating the exposure according to the absolute levels of the VIX, with the highest exposure when implied volatility is at historical low levels, and reducing such exposure as volatility rises. Implementing the long volatility (LV) strategy then consists in buying the number of VIX futures such that the impact of a 1-point variation in the price of the future is equal to  $\frac{1}{F_{t-1}} * 100\%$ <sup>5</sup>. The

P&L generated between  $t-1$  (contract date) and  $t$  (maturity date) can then be written as:

$$PL_t^{VIX} = \frac{1}{F_{t-1}}(F_t - F_{t-1}) \quad (1)$$

Where  $F_t$  is the price of the future at time  $t$ .

In practice, VIX futures prices are available only since 2004. They represent the 1-month forward market price for 30-day implied volatility. This forward-looking component is reflected in the existence of a term premium between the VIX future and VIX index. This premium tends to be positive when volatility is low (it represents a cost of carry for the buyer of the future) and negative when volatility peaks. We approximated VIX futures prices prior to 2004 using the average relationship between VIX futures and the VIX index, estimated econometrically over the period from March 2004 to August 2008. Finally, we consider a rolling 1-month investment in VIX futures, on a buy-and-hold basis, with the dynamically adjusted exposure explained above.

### **Capturing the volatility risk premium**

The second volatility strategy involves taking exposure to the difference between implied and realized volatility. As already stressed in the introduction, this difference has historically been strongly positive on average for equity indices (Bakshi and Kapadia (2003), Bondarenko (2006)), delivering very attractive risk-adjusted returns (Egloff et al. (2007), Carr and Wu (2009)).

The VRP (see Figure 2 in Appendix 1 for a historical time series) is captured by entering into a variance swap, i.e. a swap contract on the spread between implied and realized variance. Through an over-the-counter transaction, the two parties agree to exchange a specified implied variance level for the level of variance realized over a specified period. The implied variance at inception is the level that gives the swap a zero net present value. Variance swaps on major equity indices are today actively traded<sup>6</sup>.

<sup>5</sup> For instance, the 1-point impact is 5% when VIX is 20.

<sup>6</sup> The most widely traded indices include the S&P 500 in US, the EuroStoxx 50 in EMU, and the Nikkei 225 in Japan.

We consider a short variance swap strategy on the S&P 500 held over a one month period. The P&L of a short variance swap position between the start date ( $t-1$ ) and end date ( $t$ ) can be written as:

$$PL_t^{VARSWAP} = \frac{N_{VEGA}}{2K_{t-1}} * [K_{t-1}^2 - RV_{t-1,t}^2] \quad (2)$$

Where  $K_{t-1}$  is the volatility strike of the variance swap contract entered at date  $t-1$ ,  $RV_{t-1,t}$  is the realized volatility between  $t-1$  and  $t$ , and  $N_{VEGA}$  is the vega notional of the contract (see Appendix 2 for further details on variance swaps).

One of the main interests of the variance swap investment is that it is a linear contract in variance risk, so the investor does not create additional delta exposure to the underlying, as he would with strategies involving vanilla options. From a theoretical point of view, the strike level of the variance swap is computed from the price of the portfolio of options that is used to calculate the volatility index itself. Thus, the theoretical strike of a 1-month variance swap on the S&P 500 is the value of the VIX (Carr and Wu (2006, 2009)). In practice, there are a number of difficulties in replicating a volatility index, and synthetic variance swap rates are exposed to measurement errors due to the no-price jump hypothesis, bid-ask spreads and state dependencies. Carr and Wu (2009) conducted a large panel of robustness checks and concluded that the errors generated are usually small. According to Standard & Poor's (2008), the VIX level has to be reduced by 1 point to fully reflect the average replication costs borne by arbitrageurs. Furthermore, computing realized volatility also depends on the type of returns used (log-returns vs. standard returns), the data frequency (high-frequency vs. daily data), and the annualization method, as outlined by Wu (2005), Carr and Wu (2009), Bollerslev et al. (2008). Referring to the most liquid and standardized variance swap contracts, where realized volatility is computed with daily-log-returns annualized on a 252-business-day-basis (JPMorgan (2006), Standard and Poor's (2008)), we have used this market standard for our computations.

### **Adequate risk measure**

A key issue in implementing volatility strategies is the non-normality of return distributions, as shown in the next section. When returns are not normally distributed, the mean-variance criterion of Markowitz (1952) is no longer adequate. To compensate for this, many authors have sought to include higher-order moments of the return distribution in their analysis. Lai (1991) and Chunhachinda et al. (1997), for example, introduce the third moment of the return distribution (ie skewness) and show that this leads to significant changes in the optimal portfolio construction.

Extending portfolio selection to a four-moment criterion brings a further significant improvement (Jondeau and Rockinger (2006, 2007)).

Within the proposed volatility investment framework, the main danger for the investor is the risk of substantial losses in extreme market scenarios (left tail of the return distribution). As returns on volatility strategies are not normally distributed, we choose “modified Value-at-Risk” as our reference measure of risk. Value-at-Risk (VaR) is defined as the maximum potential loss over a time of period given a specified probability  $\alpha$ . Within normally distributed returns, VaR can be written:

$$VaR(1 - \alpha) = -(\mu + z_\alpha * \sigma) \quad (3)$$

Where  $\mu$  and  $\sigma$  are, respectively, the mean and standard deviation of the return distribution and  $z_\alpha$  is the  $\alpha$ -quantile of the standard normal distribution  $N(0,1)$ .

To capture the effect of non-normal returns, we replace the quantile of the standard normal distribution with the “modified” quantile of the distribution  $w_\alpha$ , approximated by the Cornish-Fisher expansion based on a Taylor series approximation of the moments (Stuart et al. (1999)). This enables us to correct the distribution  $N(0,1)$  by taking skewness and kurtosis into account. Modified VaR is accordingly written as:

$$ModVaR(1 - \alpha) = -(\mu + w_\alpha * \sigma) \quad (4)$$

$$\text{Where } w_\alpha = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1) * S + \frac{1}{24}(z_\alpha^3 - 3z_\alpha) * EK - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha) * S^2$$

$w_\alpha$  is the modified percentile of the distribution at threshold  $\alpha$ ,  $S$  is the skewness and  $EK$  is the excess kurtosis of the portfolio.

Two portfolios that offer the same expected return for a given level of volatility will have the same “normal” VaR but different modified VaR if their returns present different skewness or/and excess kurtosis. In particular, modified VaR will be greater for the portfolio that has negative skewness (left-handed return distribution) and/or higher excess kurtosis (leptokurtic return distribution). In addition to being simple to implement in the context of constructing the investor’s risk budget, modified VaR explicitly takes into account how his utility function changes in the presence of returns that are not normally distributed. A risk-averse investor will prefer a return distribution where the odd moments (expected return, skewness) are positive and the even moments (variance, kurtosis) are low.

### Calibrating volatility strategies

In practice, when volatility strategies are implemented, cash requirements are very limited as the required exposure is achieved through listed or OTC derivatives. Capital requirements are limited to the collateral needed when entering swap contracts, and futures margin deposits for listed products. A key step in the process of investing in the volatility asset class is the proper calibration of the strategies.

Based on raw computations of modified VaR for each asset, each volatility exposure is normalized by setting its monthly modified 99% VaR to the targeted level of the equity asset class (see Table 2, Appendix 3). Once calibrated, the return of each volatility strategy is thus the risk-free-rate return plus a fixed proportion (called “degree of leverage” for simplicity) of the strategy’s P&L, this leverage being determined ex ante by our calibration methodology:

$$r_t^{LV} = r_t^f + L_1 * PL_t^{VIX} \quad (5)$$

$$r_t^{VRP} = r_t^f + L_2 * PL_t^{VARSWAP} \quad (6)$$

Where  $r_t^{LV}$  (resp.  $r_t^{VRP}$ ) is the monthly return of the LV strategy (resp. VRP),  $r_t^f$  is the cash return,  $L_1$  (resp.  $L_2$ ) is the degree of leverage calibrated on the LV strategy (resp. VRP).

### 3. Building an efficient portfolio with volatility

We now turn our discussion to the investment case of adding the two volatility strategies to the opportunity set of a long-term investor whose portfolio is fully invested in US equities.

#### Data

The S&P 500 and CBOE VIX indices and the 1-month US LIBOR rate<sup>7</sup> time series are used for the equity asset class, the volatility strategies and the risk-free-rate respectively. The study covers the period between February 1990 and August 2008. To gauge the stability of our results, the data sample is divided into two equal sub-periods (approximately 9 years of data), where the first sub-period is used to calibrate the risk exposure to volatility strategies, and the second is used to measure out-of-sample performances. Moreover, we also controlled for the stability of our results depending of the choice of the starting date. Four different time series of non-overlapping one-month returns are compared by starting each monthly investment at four different dates during the

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<sup>7</sup> Data provided by Datastream (S&P 500 index) and Bloomberg (CBOE VIX index and 1-month US LIBOR rate).

month<sup>8</sup>. The reported results exhibit average summary statistics computed on each of the four series of returns.

### Summary statistics

Tables 1-3 (Appendix 3) present the descriptive statistics of equities, LV and VRP strategies during the whole sample period and the two sub-sample periods. Looking at Sharpe ratios and annualized returns on the whole sample, the VRP strategy appears to be the most attractive, with a global Sharpe ratio of 2.3 and an annualized return of 37.1%. Equities (0.4 and 7.9%) and LV strategy (0.2 and 7.03%) follow in that order. Although the LV strategy is last in this ranking, it holds considerable interest during market downturns. While annualized equity returns decreased from 18.9% to 0.8% during the second sub-sample marked by the bursting of the Internet bubble, the LV strategy preserved almost all the returns at 6.7% vs. 8.3%. The VRP is the most consistent strategy. It delivered a relatively high performance, except during periods of rapidly increasing realized volatility (onset of crises, unexpected market shocks), when returns are strongly negative<sup>9</sup> and much greater in amplitude than for traditional asset classes. Annualized returns decreased from 54.5% to 24.0% during the second sub-sample.

Modified VaR increased from a calibrated 7.5% for the three strategies to 11.8%, 7.7% and 11.8% respectively for equities, LV and VRP strategies, during the second sub-sample period. Nevertheless, the risk-calibration of volatility strategies can still be considered as adequate, given that out-of-sample VaR of volatility strategies (calibrated in sample to be the same than equities) does not exceed out-of-sample that of equities.

Analysis of extreme returns (minimum and maximum monthly returns) highlights the asymmetry of the two volatility strategies: the LV strategy offers the highest maximum return at 31.3% (with a minimum return at -12.5%), whereas the VRP strategy posts the worst monthly performance at -16.3% (with a best month at 11.7%). The higher-order moments clearly highlight that returns are not normally distributed<sup>10</sup>, especially for the two volatility strategies. While skewness of equity returns is negative over the global period at -0.2, there is a remarkable difference between the two sub-periods: skewness becomes negative after 1999 at -0.3 from 0.2, due to adverse markets. The VRP strategy shows very strong negative skewness (-1.7), remaining fairly stable over the two sub-

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<sup>8</sup> The 7, 14, 21 and 28 of each month. When this date is a non-business day, we use quotes from the previous business day. We thank an anonymous referee for having made this suggestion.

<sup>9</sup> Realized volatility rises above implied volatility.

<sup>10</sup> For equity returns and returns on the two volatility strategies, the null hypothesis of a normality test is significantly rejected.

sample periods. These empirical results are consistent with the evidence of a strong significant jump component in the variance rate process in addition to a significant jump component in the stock index return (Eraker et al. (2003), Wu (2005)). The only strategy showing positive skewness is LV (1.6, remaining fairly stable): being long implied volatility provides a partial hedge of the leftward asymmetry of the other assets. All three assets exhibit kurtosis in excess of 3, with 4.4 for equities, and 8.9 and 10.2 for the LV and VRP strategies respectively.

## Codependencies

The multivariate characteristics of returns are likewise of great interest. Correlation matrices for all three study periods are provided in Tables 4-6 of Appendix 3. For the whole period, the LV strategy always offers strong diversification power relative to the equities with a -44% correlation coefficient, a phenomenon already well publicized by other studies (Daigler and Rossi (2006)). Interestingly this behavior is even stronger during a less favorable equity market environment (-51%, from 1999 to 2008). The VRP strategy shows different characteristics: it offers little diversification to equity exposure with a 43% correlation coefficient. But, the two volatility strategies are mutually diversifying with a -44% correlation coefficient, a result even stronger in the 2<sup>nd</sup> sample period (-50%), which, as we will see, is of great interest for portfolio construction.

The importance of extreme risks also requires the analysis of the coskewness and cokurtosis matrices provided in Tables 7-9 and 10-12<sup>11</sup> in Appendix 3. With the sole exception of equity skewness, codependences display a certain degree of stability between the two sub periods. Positive coskewness value  $sk_{ij}$ <sup>12</sup> suggests that asset  $j$  has a high return when volatility of asset  $i$  is high, i.e.,  $j$  is a good hedge against an increase in the volatility of  $i$ . This is particularly true for the LV strategy, which offers a good hedge for the VRP strategy and equities. In contrast, the VRP strategy does not hedge the other assets efficiently because it tends to perform poorly when their volatility increases.

Positive cokurtosis value  $ku_{ij}$ <sup>13</sup> means that the return distribution of asset  $i$  is more negatively “skewed” when the return on asset  $j$  is lower than expected, ie  $i$  is a poor hedge against a decrease

<sup>11</sup> We give a summary presentation of these matrices. For  $n=3$  assets, it suffices to calculate 10 elements for the coskewness matrix of dimension (3,9) and 15 elements for the cokurtosis matrix of dimension (3, 27).

<sup>12</sup> The general formula for coskewness is:  $sk_{ijk} = \frac{E[(r_i - \mu_i)(r_j - \mu_j)(r_k - \mu_k)]}{\sigma_i \sigma_j \sigma_k}$ ,

where  $r_i$  is the return on asset  $i$  and  $\mu_i$  its mean.

<sup>13</sup> The general formula for cokurtosis is:  $ku_{ijkl} = \frac{E[(r_i - \mu_i)(r_j - \mu_j)(r_k - \mu_k)(r_l - \mu_l)]}{\sigma_i \sigma_j \sigma_k \sigma_l}$

in the value of  $j$ . Here again, we find that the LV strategy is an excellent hedge against equities, unlike the VRP strategy. However, the two volatility strategies hedge each other very well.

Positive cokurtosis  $ku_{ijk}$  is a sign that the covariance between  $j$  and  $k$  increases when the volatility of asset  $i$  increases. We find that in periods of rising equity volatility, the VRP/LV correlation is negative. Thus, during periods of stress in the equity market, VRP and equities perform badly at the same time, while LV does better.

Lastly, positive cokurtosis  $ku_{ijij}$  means that volatilities of  $i$  and  $j$  tend to increase at the same time. This is the case for all four assets. Here again, all coskewness and cokurtosis values are respectively significantly different from 0 and 3, a sign that the structure of dependencies between these strategies differs significantly from a multivariate normal distribution.<sup>14</sup>

This initial analysis already allows us to highlight different advantages of the two volatility strategies within an equity portfolio: the LV strategy delivers excellent diversification relative to equities; the VRP strategy allows for very substantial increase in returns, at the expense of a broadly increased risk profile (extreme risks and codependencies with equities). A combination of the two volatility strategies appears particularly attractive since they tend to hedge each others', especially in extreme market scenarios.

## **Efficient portfolios**

As previously described, volatility strategies have been collateralized in our analytical framework. Also, while long and short positions are permitted with volatility strategies, net short exposure to the equity asset class is restricted. Moreover, optimal portfolios are designed under an additional budget constraint, where the sum of the percentage shares in the three assets must equal 100%.

To determine the interest for an equity investor of adding a systematic exposure to volatility we consider four investment cases: (1) equities-only, (2) equities and LV strategy, (3) equities and VRP strategy (4) equities and the two volatility strategies. We focus on portfolios minimizing modified VaR on the first half of the sample and backtest the results in the second half to gauge their out-of-

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<sup>14</sup> The null hypothesis of a multivariate normality test (Kotz et al. (2000)) is significantly rejected.

sample performances. Tables 13 -14 (Appendix 4) report the portfolios' compositions and performances in the two sub-periods<sup>15</sup>.

In sample, the addition of the LV strategy (37%) to an equity investment reduces risk and improves risk-adjusted performances. The modified VaR of the portfolio decreases to 3.0% from 7.5% and the Sharpe ratio increases to 1.2 from 1.0. The resulting allocation delivers a slightly smaller performance (15.9% vs. 18.9%) but overall the portfolio is less sensitive to extreme events, with a monthly maximum loss more than halved at -4.0% vs. -9.9%, and a distribution of returns offering much higher positive skewness at 0.8 vs. 0.2, and smaller kurtosis at 4.1 vs. 4.9. Adding the VRP strategy (37%) to the equity-only-portfolio, makes it possible to achieve significantly higher returns at 27.4% vs. 19.0%, along with a lower modified-VaR at 6.3%. The success rate of the portfolio is improved to 83%, and the Sharpe ratio rises to 2.0. But overall, the portfolio's return distribution shows a more pronounced leftward asymmetry at -0.1 versus 0.2, and higher kurtosis at 6.0 vs. 4.9, which demonstrates higher sensitivity to extreme events and thus reduces its appeal for the most risk-averse investors.

The most interesting risk/return profile is obtained by adding a combination of the two volatility strategies. Adding both the LV (29%) and the VRP (44%) strategies, makes it possible to achieve the smallest modified-VaR at 1.7% and the highest Sharpe ratio at 2.7. Overall, the portfolio shows a significant decreased sensibility to extreme risks as measured by the worst monthly loss: -2.9% against -9.9% for the equity-only-portfolio. Also, higher-order moments of its return distribution are improved with higher positive skewness at 0.7 vs. 0.2, and an almost equivalent kurtosis at 4.7 vs. 4.9.

As already stated, the turbulence of equity markets during the second part of our study period affects the returns and Sharpe ratios of all portfolios. Nevertheless, all the results confirm the strong interest of introducing a systematic exposure to volatility strategies for equity investors. We retrieve in the out-of-sample performances all the positive characteristics previously underlined for the volatility strategies: the LV strategy's capacity to reduce risks, the VRP strategy's capacity to boost returns, and the reciprocal hedge of LV/VRP strategies. With the part of the portfolio dedicated to the LV strategy, we observe that the worst monthly loss and the modified VaR are roughly halved with respect to a simple equity investment at -7.5% vs -13.4% and 5.9% vs. 11.8%, respectively. The Sharpe ratio turns slightly positive at 0.1, instead of being negative at -0.1 for the equity-only-portfolio. The portfolio invested in both equities and the VRP strategy continues to achieve strong

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<sup>15</sup> We ran portfolio optimizations and computed performances for each of the four starting dates considered in the month. Averages summary statistics are reported.



annualized return at 8.2% and a Sharpe ratio of 0.4. Furthermore, the risks of this portfolio are smaller than those of a simple equity investment with a modified VaR of 9.6% vs. 11.8% and a maximum monthly loss of -12.0% vs. -13.4%, even if skewness and kurtosis are higher in absolute terms (respectively -0.7 vs -0.3 and 4.3 vs 3.5). Finally, the most attractive portfolio for investors proves to be the combination of the two volatility strategies with equities. It achieves the best performance at 11.6% with the lowest volatility at 6.9%, obtaining the highest Sharpe ratio at 1.2. It is also the only portfolio with a success rate higher than 75%. The good news for risk adverse investors is that these results are not obtained at the expense of extreme risks; the portfolio returns distribution has a leftward asymmetry comparable to that of equities (-0.3), even though kurtosis is still high, at 6.1, and maximum monthly loss is -6.7% vs. -13.4%.

#### **4. Conclusion**

Recent literature has begun to show the merit of including long exposure to implied volatility in a pure equity portfolio (Daigler and Rossi (2006)), in a portfolio of funds of hedge funds (Dash and Moran (2006)) or to investigate the interest of volatility risk premium strategies (Egloff et al. (2007), Hafner and Wallmeier (2008)). The purpose of this paper was to examine a classic equity allocation with a combination of volatility strategies, since little has been written so far on the subject. Among the standardized strategies for adding volatility exposure to the investment-opportunity set, we identified not only buying implied volatility but also investing in the volatility risk premium. While these strategies have attracted considerable interest on the part of some market professionals, especially hedge fund managers (and, more recently, more sophisticated managers of traditional funds), the academic literature to date has paid little attention to them.

We explored an exposure to two very simple types of volatility strategy added to an equity portfolio. Our results from a historical analysis of the past twenty years show the great interest of including these volatility strategies in such a portfolio. Taken separately, each of the strategies displaces the efficient frontier significantly outward, but combining the two produces even better results. A long exposure to volatility is particularly valuable for diversifying a portfolio holding equities: because of its negative correlation to the asset class, its hedging function during equity market downturns is clearly interesting. For its part, a volatility risk premium strategy boosts returns. It provides little diversification to equities (it loses significantly when equities fall) but good diversification with respect to implied volatility. Combining the two strategies offers the major advantage of fairly effective reciprocal hedging during periods of market stress, significantly

improving portfolio return for a given level of risk. With the presence of volatility strategies, investors radically change their portfolio composition, giving less weight to equity investments and replacing them with volatility exposure. We show that doing this would have improved investment performance, both in-sample and out-of-sample.

One of the limits of our work relates to the period analyzed. Although markets experienced several major crises over the period from 1990 to 2008, with significant volatility spikes, there is no assurance that, in the future, crises will not be more acute than those experienced over the testing period and that losses on variance swap positions will not be greater, thereby partly erasing the high reward associated with the volatility risk premium. An interesting continuation of this work would be to explore the extent to which long exposure to volatility is a satisfactory hedge of the volatility risk premium strategy during periods of stress and sharp increases in realized volatility. It would also be important to analyze the dynamics of the volatility risk premium and its determinants, as envisaged by Bollerslev et al. (2008).

Like fixed-income and equity, volatility as an asset class can be approached not only in terms of directional volatility strategies but also in terms of inter-class arbitrage strategies (relative value, correlation trades, etc.). Tactical strategies can also be envisaged. The possibilities are numerous, and they deserve further investigation to precisely measure both the benefits and the risks to an investor who incorporates such strategies in an existing portfolio.

### **Acknowledgements**

The authors are grateful to E. Bourdeix, T. Bulger, F. Castaldi, B. Drut, L. Dynkin, C. Guillaume, B. Houzelle, S. de Laguiche, A. Szafarz, K. Topeglo and an anonymous referee for helpful comments and suggestions.

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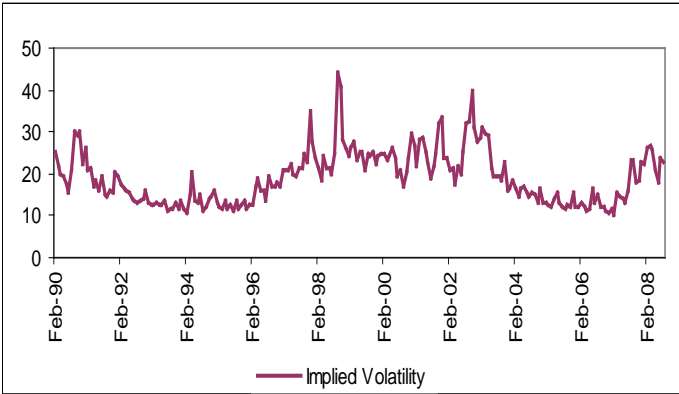
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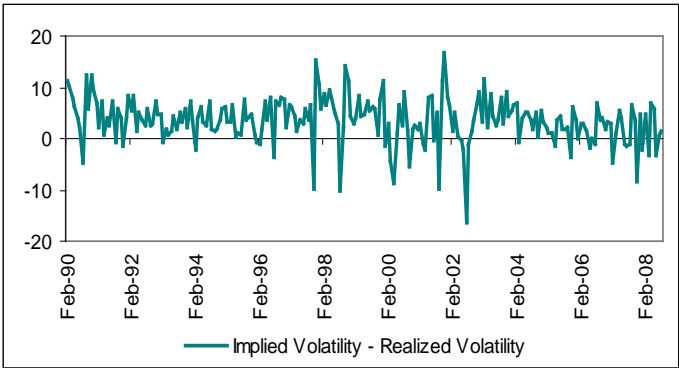
**Appendices**

**Appendix 1**

**Figure 1: Implied Volatility (VIX), February 1990 – August 2008**



**Figure 2: Implied Volatility – Realized Volatility, February 1990 – August 2008**



## Appendix 2

From a theoretical standpoint, a variance swap can be seen as a representation of the structure of implied volatility (the volatility “smile”) since the strike price of the swap is determined by the prices of options of the same maturity and different strikes (all available calls/puts in, at, or out of the money) that make up a static portfolio replicating the payoff at maturity. The calculation methodology for the VIX volatility index represents the theoretical strike of a variance swap on the S&P 500 index with a maturity of one month (interpolated from the closest maturities so as to keep maturity constant).

From a practical standpoint, the two markets are closely linked through the hedging activity of market-makers: to a first approximation, a market-maker that sells a variance swap will typically hedge the *vega* risk on its residual position by buying the 95% out-of-the-money put on the listed options market.

The P&L of a variance swap is expressed as follows (Demeterfi et al. (1999)):

$$P \& L = N_{\text{variance}} * [RV_{0,T}^2 - K_T^2]$$

Where  $K_T$  is the volatility strike of a variance swap of maturity  $T$  ( $K_T^2$  is the delivery price of the variance),  $RV_{0,T}$  is the realized volatility of the asset underlying the variance swap over the term of the swap, and  $N_{\text{variance}}$  is the “variance notional.”

Realized volatility  $RV_{0,T}$  is calculated from closing prices of the S&P 500 index according to the following formula:

$$RV_{0,T} = \sqrt{\frac{252}{T} \sum_{t=1}^T \left[ \ln \left( \frac{SP500_t}{SP500_{t-1}} \right) \right]^2}$$

In terms of the Greek-letter parameters popularized by the Black-Scholes-Merton option pricing model, the notional of a variance swap is expressed as a vega notional, which represents the mean P&L of a variation of 1% (one vega) in volatility. Although the variance swap is linear in variance, it is convex in volatility (a variation in volatility has an asymmetric impact). The relationship between the two notionals is the following:

$$N_{\text{vega}} = N_{\text{variance}} * 2K$$

Where  $N_{\text{vega}}$  is the vega notional.

## Appendix 3: Descriptive Statistics

**Table 1**  
**Descriptive Statistics**  
**Monthly Returns, US, February 1990 – August 2008**

\*Downside Deviation is determined as the sum of squared distances between the returns and the cash return series

	<b>Geometric Mean</b>	<b>Ann. Geometric Mean</b>	<b>Median</b>	<b>Min</b>	<b>Max</b>	<b>Ann. Std. dev.</b>	<b>Skewness</b>	<b>Kurtosis</b>	<b>Ann. Down. dev.*</b>	<b>Mod. VaR</b>	<b>Sharpe Ratio</b>	<b>Success Rate</b>
<b>Equity</b>	0.63%	7.87%	0.98%	-13.37%	12.66%	14.46%	-0.19	4.40	8.34%	10.64%	0.40	60.67%
<b>LV</b>	0.57%	7.03%	0.32%	-12.54%	31.29%	20.28%	1.65	8.91	9.07%	7.79%	0.25	62.44%
<b>VRP</b>	2.54%	37.13%	2.98%	-16.32%	11.73%	11.78%	-1.72	10.19	6.37%	11.42%	2.31	85.56%

**Table 2**  
**Descriptive Statistics**  
**Monthly Returns, US, February 1990 – July 1999**

\*Downside Deviation is determined as the sum of squared distances between the returns and the cash return series

	<b>Geometric Mean</b>	<b>Ann. Geometric Mean</b>	<b>Median</b>	<b>Min</b>	<b>Max</b>	<b>Ann. Std. dev.</b>	<b>Skewness</b>	<b>Kurtosis</b>	<b>Ann. Down. dev.*</b>	<b>Mod. VaR</b>	<b>Sharpe Ratio</b>	<b>Success Rate</b>
<b>Equity</b>	1.46%	18.95%	1.42%	-9.92%	14.27%	12.48%	0.24	4.87	5.53%	7.50%	1.04	68.20%
<b>LV</b>	0.67%	8.32%	0.43%	-11.96%	30.59%	20.47%	1.74	9.46	9.20%	7.50%	0.23	63.82%
<b>VRP</b>	3.45%	54.48%	3.67%	-9.71%	12.05%	10.22%	-1.59	11.43	4.05%	7.50%	3.33	92.11%

**Table 3**  
**Descriptive Statistics**  
**Monthly Returns, US, August 1999 – August 2008**

\*Downside Deviation is determined as the sum of squared distances between the returns and the cash return series

	<b>Geometric Mean</b>	<b>Ann. Geometric Mean</b>	<b>Median</b>	<b>Min</b>	<b>Max</b>	<b>Ann. Std. dev.</b>	<b>Skewness</b>	<b>Kurtosis</b>	<b>Ann. Down. dev.*</b>	<b>Mod. VaR</b>	<b>Sharpe Ratio</b>	<b>Success Rate</b>
<b>Equity</b>	0.07%	0.84%	0.68%	-13.37%	12.03%	15.81%	-0.30	3.51	9.73%	11.82%	-0.10	55.73%
<b>LV</b>	0.54%	6.68%	0.24%	-11.19%	26.35%	19.58%	1.46	7.50	8.87%	7.66%	0.24	62.84%
<b>VRP</b>	1.77%	24.04%	2.47%	-15.50%	9.45%	12.60%	-1.61	8.10	7.74%	11.82%	1.56	80.96%



**Table 4**  
**Correlation matrix**  
**Monthly Returns, US, February 1990–August 2008**

	Equity	LV	VRP
Equity			
LV	-0.44		
VRP	0.43	-0.44	

**Table 5**  
**Correlation matrix**  
**Monthly Returns, US, February 1990–July 1999**

	Equity	LV	VRP
Equity			
LV	-0.40		
VRP	0.41	-0.41	

**Table 6**  
**Correlation matrix**  
**Monthly Returns, US, August 1999–August 2008**

	Equity	LV	VRP
Equity			
LV	-0.51		
VRP	0.41	-0.50	

**Table 7**  
**Co-Skewness matrix**  
**Monthly Returns, US, February 1990 – August 2008**

	Equity ^2	LV^2	VRP^2	Equity*LV
Equity	<b>-0.19</b>	-0.68	-0.61	
LV	0.42	<b>1.65</b>	0.83	
VRP	-0.37	-0.78	<b>-1.72</b>	0.39

**Table 8**  
**Co-Skewness matrix**  
**Monthly Returns, US, February 1990 – July 1999**

	Equity ^2	LV^2	VRP^2	Equity*LV
Equity	<b>0.24</b>	-0.90	-0.43	
LV	0.56	<b>1.74</b>	0.98	
VRP	-0.05	-0.94	<b>-1.59</b>	0.48

**Table 9**  
**Co-Skewness matrix**  
**Monthly Returns, US, August 1999 – August 2008**

	Equity ^2	LV^2	VRP^2	Equity*LV
Equity	<b>-0.30</b>	-0.56	-0.64	
LV	0.33	<b>1.46</b>	0.74	
VRP	-0.46	-0.72	<b>-1.61</b>	0.32

**Table 10**  
**Co-kurtosis matrix**  
**Monthly Returns, US, February 1990 – August 2008**

	Equity ^3	LV^3	VRP^3	Equity^2*LV	Equity*VRP^2	LV^2*VRP	LV*VRP^2
Equity	<b>4.40</b>	-3.94	4.10		4.10	2.08	
LV	-1.83	<b>8.91</b>	-4.29	2.57	-1.87		3.60
VRP	2.18	-4.05	<b>10.19</b>	-1.41			

**Table 11**  
**Co-kurtosis matrix**  
**Monthly Returns, US, February 1990 – July 1999**

	Equity ^3	LV^3	VRP^3	Equity^2*LV	Equity*VRP^2	LV^2*VRP	LV*VRP^2
Equity	<b>4.87</b>	-5.04	4.09		2.91	2.59	
LV	-2.43	<b>9.46</b>	-5.34	3.55	-2.18		4.28
VRP	2.56	-4.59	<b>11.43</b>	-1.77			

**Table 12**  
**Co-kurtosis matrix**  
**Monthly Returns, US, August 1999 – August 2008**

	Equity ^3	LV^3	VRP^3	Equity^2*LV	Equity*VRP^2	LV^2*VRP	LV*VRP^2
Equity	<b>3.51</b>	-3.32	3.42		2.52	2.01	
LV	-1.66	<b>7.50</b>	-3.88	2.21	-1.85		3.63
VRP	1.54	-4.04	<b>8.10</b>	-1.31			

**Table 13**  
**Portfolio allocation: Minimum Modified VaR**  
**US, February 1990 – July 1999**  
**In sample performances**

	Equity	Equity + LV	Equity + VRP	Equity + LV + VRP
Ann. Geometric Mean	18.95%	15.88%	27.39%	24.54%
Ann. Std. Dev.	12.48%	8.64%	9.68%	6.32%
Skewness	0.24	0.81	-0.12	0.73
Kurtosis	4.87	4.08	5.96	4.74
Max Monthly Loss	-9.92%	-3.95%	-7.75%	-2.94%
Max Monthly Gain	14.27%	9.41%	12.40%	8.26%
Mod. VaR(99%)	7.51%	3.04%	6.32%	1.71%
Sharpe Ratio	1.04	1.15	2.00	2.68
Success Rate	68.20%	67.76%	83.11%	86.62%
<i>Equity</i>	100%	63%	63%	26%
<i>LV</i>	-	37%	-	29%
<i>VRP</i>	-	-	37%	44%

**Table 14**  
**Portfolio allocation: Minimum Modified VaR**  
**US, August 1999 – August 2008**  
**Out of sample performances**

	Equity	Equity + LV	Equity + VRP	Equity + LV + VRP
Ann. Geometric Mean	0.84%	4.11%	8.22%	11.64%
Ann. Std. Dev.	15.81%	9.34%	12.06%	6.86%
Skewness	-0.30	0.38	-0.69	-0.32
Kurtosis	3.51	4.79	4.34	6.05
Max Monthly Loss	-13.37%	-7.47%	-12.03%	-6.73%
Max Monthly Gain	12.03%	9.11%	8.73%	6.49%
Mod. VaR(99%)	11.82%	5.92%	9.63%	5.00%
Sharpe Ratio	-0.10	0.10	0.42	1.23
Success Rate	55.73%	56.42%	63.07%	76.61%
<i>Equity</i>	100%	63%	63%	26%
<i>LV</i>	-	37%	-	29%
<i>VRP</i>	-	-	37%	44%



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## Amundi Working Paper

WP-004-2009

Revised: February 2011



Written by Amundi.

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