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Modeling the Volatility Risk Premium Term Structure in Swaptions[†]

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in

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by

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Abstract

Despite the large body of evidence of a volatility risk premium embedded in option prices, little is known about its behavior over the cross-section of option maturities. This study introduces maturity term structure modeling in the literature of the volatility premium. My findings are consistent with a concave, upward sloping maturity term structure, constructed on the basis of delta-hedged swaption straddle portfolios in two major currency markets (USD and GBP). I extend the application of the Nelson-Siegel model to volatility premium term structure modeling using both linear estimation techniques and state space optimization by means of a Kalman filter. The model is found to be capable of accurately fitting the premium term structure in-sample through three latent factors that correspond to the structure's level, slope and curvature. The factors are largely driven by movements in swaption implied volatility and to a lesser extent by dynamics in the forward swap rate and macroeconomic influences. Due to large randomness in the premia dynamics, out-of-sample forecasts of the term structure are inaccurate. The shape of the term structure is furthermore robust to alternative premium valuations, indicating that the Nelson-Siegel model can be successfully applied to a range of alternatively constructed volatility risk premium term structures.

Keywords: Volatility risk premium, Term structure, Nelson-Siegel curve, Swaptions, Straddles, Interest rate derivatives, Asset pricing

JEL classification: C58, G12, G13, G17

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Contents

1 Introduction

The tendency of option-implied volatilities to systematically exceed the ex-post realized volatilities of their underlying assets is widely observed in many option markets [Jackwerth and Rubinstein (1996), Rennison and Pedersen (2012)]. A possible explanation for this persistent bias originates from investor risk aversion. Option sellers demand a premium as compensation for the risk of losses during sudden increases in market volatility, which buyers are willing to pay as options provide protection against unfavorable market movements.[1] This "volatility risk premium" increases option prices and in turn causes option-implied volatility to exceed realized volatility, on average. Options, however, can vary in a range of aspects, of which one is maturity. Yet, most questions on the behavior of volatility risk premia over option maturity remain unanswered. This thesis attempts to fill this gap in the literature.

A substantial body of literature provides convincing evidence that the volatility premium embedded in a wide range of option types is non-zero. Early studies, such as Goodman and Ho (1997) and Bakshi and Kapadia (2003), examine the presence and sign of the volatility risk premium in different option markets by analyzing the returns of a delta-hedged investment strategy. The latter study constructs a theoretical framework that relates the volatility risk premium to option returns by mathematically proving that expected average delta-hedged option portfolio returns must be non-zero if volatility risk is priced. A number of following publications on volatility premia build on this theoretical framework. An example is Fornari (2010), who examines the volatility premium in swaptions, and finds the premium to be negative. A negative premium implies that volatility investments through options on average earn less than the risk free rate, which relates to the implied-realized volatility mismatch. Other studies that document the presence of negative volatility risk premia in a range of derivative markets are Duarte, Longstaff and Yu (2007), Almeida and Vicente (2009), Trolle (2009), and Aït-Sahalia *et al.* (2015), among others. These publications conclude that simple trading strategies that aim to capture the premium deliver significant risk-adjusted returns. None of these strategies however covers trading on another component of this premium: its maturity term structure. In fact, so far only very limited attention is payed to the cross-sectional behavior of volatility premia over option maturity.

This study complements the literature on volatility premia by introducing a new niche that aims to comprehend and model the volatility premium term structure embedded in options using a specifically chosen modeling technique. Increased knowledge on the term structure can possibly have valuable implications for various market participants. Investors might be able to improve their volatility premium trading strategies whereas premium awareness in risk management can increase hedging profitability.

One of the few studies that does make inferences on the term structure in volatility risk premia

[1] Options are essentially instruments used for hedging purposes against downward moving markets. Literature confirms that these movements coincide with high market volatility [French, Schwert and Staumbaugh (1987); Glosten, Jagannathan and Runkle (1993)]. Premia therefore tend to be highest during and post periods of turbulent markets [Rennison and Pedersen (2012)].

is conducted by Low and Zhang (2005). Also based on the framework of Bakshi and Kapadia (2003), they find the premium in currency options to be negative and increasing towards zero over option maturity. Duyvesteyn and De Zwart (2015) build on Low and Zhang (2005) but extend their approach to swaption straddles. Furthermore, they consider a 1-month holding period instead of a hold-to-expiry set-up as applied by Low and Zhang (2005), yielding a different view upon the term structure in volatility risk premia. Their obtained term structure, which they refer to as the "swaption curve", is upward sloping in line with Low and Zhang (2005) but contains positive premia for longer maturity swaption straddles in contrast to most earlier studies. Moreover, the shape of the obtained term structure resembles the shape of another well-known and related term structure: the yield curve. Duyvesteyn and De Zwart (2015) however focus mainly on exploiting the curve using tailor made trading strategies rather than modeling it, leaving this subject open for further research.

In this thesis I focus on modeling and determining the drivers of the volatility premium term structures in swaptions, or, the swaption curves. I construct the curves on the basis of the fundamental delta-hedged portfolio theory from Bakshi and Kapadia (2003) and the rebalancing framework introduced by Duyvesteyn and De Zwart (2015). The dataset that underlies the curves consists of at-the-money (ATM) swaption and forward swap rate data over a period ranging from January 1998 through August 2016 for swaption maturities ranging from 1 month up to 5 years, in the USD and GBP currency markets. The use of swaptions appeals to the goals of this thesis, as their relation with yield curve dynamics naturally gives rise to a maturity term structure.

In particular, the contribution of this study is two fold. As a first contribution I employ an assessment of the term structure in the volatility risk premium which covers a number of perspectives that are not yet described in existing literature. This part serves as the foundation of this research and includes an intra-sample analysis of the curve, individual premia assessments and various sensitivity analyses. The second and main contribution of this study is the introduction of term structure modeling in the volatility risk premium literature. Motivated by the little knowledge on the term structure and its observed similarities with the yield curve, I introduce a new approach that extends the application of yield curve modeling techniques to volatility risk premium term structure modeling. The modeling segment of the second contribution in particular comprises three parts. First, I examine the ability of the renowned Nelson-Siegel term structure model to in-sample fit the swaption curve. I estimate the model using linear estimation and state space optimization by means of a Kalman filter. Second, I relate the explaining factors of this model to observable factors in the swaption market to identify the main drivers of the curve by applying linear regressions. These observable factors include swaption underlying pricing variables and macroeconomic variables. Third, I assess the predictability of the curve using a specifically constructed set of forecasting methods.

The latent factor term structure model based on the contribution of Nelson and Siegel (1987) is a popular yield curve modeling framework that is frequently applied by central banks. Diebold and Li (2006) adjust the initial representation and show that the model is able to fit the yield curve in-sample and outperforms many other models when it comes to out-of-sample forecasting.

The Nelson-Siegel framework as proposed by Diebold and Li (2006) explains the variety in a multivariate dataset by means of three dynamically evolving factors, commonly referred to as level, slope and curvature. Since volatility premium series corresponding to different maturities are usually highly correlated, this aspect of the Nelson-Siegel model appeals. Furthermore, the model has the substantial flexibility required to match the changing shape of the swaption curve while its imposed structure on the factor loadings facilitates estimation.

Diebold and Li (2006) specifically describe a 2-step estimation approach that first estimates the model factors over the cross-section for each point in time and subsequently models the dynamics of the factors using autoregressive processes. Diebold, Rudebusch and Aruoba (2006) build on the work of Diebold and Li (2006) but merge the model into a unified state-space framework. Using a Kalman filter, this modeling approach allows to simultaneously fit the yield curve at each point in time and optimally estimate the underlying dynamics of the factors. They argue that this 1-step approach improves upon the 2-step approach in terms of theoretical soundness and accuracy. Since the Nelson-Siegel term structure has not yet been applied to term structures in volatility premia this study examines both estimation techniques. For the same reason, I employ both univariate and multivariate autoregressive processes to model the factor dynamics.

This study provides a number of new results. First, convincing evidence is found for the capability of the considered yield curve modeling technique to accurately fit the volatility premium term structure in-sample. The results imply that the Nelson-Siegel model, which purpose originally lies in modeling the term structure in interest rates, can be used to in-sample model the cross-section of volatility premia over maturity. The estimated Nelson-Siegel factors relate to the level, slope and curvature of the term structure in line with the factors obtained by Diebold and Li (2006), but differ completely in character and shape. The latent factors generally behave like typical stock returns and can largely be explained by dynamics of swaption implied volatilities and to a lesser extent by movements in the forward swap rate. Macroeconomic variables also tend to influence the shape of the swaption curve, although to a limited extent. Furthermore, the obtained randomness in the premia causes low autocorrelation in the returns and, hence, in the estimated Nelson-Siegel factors, making forecasting difficult. Alternative forecasting methods improve on the autoregressive Nelson-Siegel forecasting framework, but still provide unreliable forecasts in the largest part of the forecasting sample. Lastly, the particular concave upward sloping pattern in the swaption curve remains when the term structure is exposed to a number of alternative construction approaches. These approaches include the application of different holding periods and an alternative pricing method, among others. Hence, the results of this analysis indicate that the Nelson-Siegel model can probably be successfully applied to in-sample model alternatively constructed term structures as well.

The remainder of this study is organized as follows. Section 2 describes preliminary background theory required to understand the concepts of this thesis. Section 3 thereafter describes the data used in this research and maps the dynamics of the volatility risk premium embedded in my dataset. Section 4, 5 and 6 cover the the three different angles of the swaption curve modeling approach of this study. Section 4 introduces the yield curve modeling techniques considered, and subsequently describes their results. Section 5 focuses on identifying the driving factors behind the estimated term structures and extends the standard Nelson-Siegel model. Thereafter, section 6 examines the forecastability of both the basic and the extended Nelson-Siegel representation. Eventually, section 7 assesses the sensitivity of the results to particular assumptions made throughout this study.

2 Preliminaries

This section describes essential theory underlying the content of this research. Subsection 2.1 contains background theory on the definitions and concepts of swaps and swaptions. Subsection 2.2 subsequently describes pricing theory of swaptions with regard to the Black model. Lastly, subsection 2.3 introduces the volatility risk premium and mathematically relates it to delta-hedged swaption portfolio returns following Bakshi and Kapadia (2003).

2.1 Swaps and Swaptions

A swap is a derivative in which two counterparties exchange one stream of future cash flows for another based on a specified notional amount. This research considers plain-vanilla interest rate swaps, which exchange a fixed rate for a floating rate upon the same notional. The market distinguishes two types of swaps, the payer and the receiver swap. The holder of a payer swap is obliged to pay a pre-determined fixed rate, the swap rate, to the seller of the payer swap and in turn receives a floating rate. This floating rate is often an interbank lending rate such as the LIBOR. The receiver swap obliges the holder to pay the floating rate and in turn receive the fixed swap rate. This swap rate is set in such a way that the value of the swap at the moment of initiation is zero. That means that the expected discounted value of the future cash flows from the fixed and floating leg are equal. The frequency of cash flow exchanges can vary and is agreed on in the contract. The underlying swaps of the considered swaptions in this research make use of a semi-annual cash settlement. Besides settlement frequency and the swap rate, another pre-specified characteristic of the swap contract is the tenor, which is the maturity of the swap.

A swaption contract is an OTC derivative that gives the right to enter into a pre-specified underlying swap on a pre-specified expiration date. This research considers European swaptions on interest rate swaps. Like options on stocks, swaptions have a put and call side. The put side gives the buyer the right to enter into a payer swap. Therefore, the put swaption is better known as a payer swaption. The call side on the other hand corresponds to a receiver swaption. Swaptions are typically used by financial institutions and banks for managing interest rate risk arising from their core business or financial agreements with other institutions. Hedge funds however often use swaptions to speculate on future interest rate movements.

Swaptions have a number of pre-specified aspects that determine the price of the derivative. Its maturity, tenor of the underlying swap and discount rate of future cash flows are examples of important factors that influence the price of these products. Furthermore, swaptions also have a strike price. For swaptions however, this price is a rate referred to as the strike or exercise rate. It is the swap rate of the fixed leg of the underlying swap that will be demanded in case the swaption is exercised at its maturity. Hence, future changes in the interest rate environment of the underlying swap can among others determine the profitability of the swaption. Subsection 2.2 further elaborates on the valuation of swaptions.

2.2 Pricing Theory: The Black Model

In contrast to stock options, swaption prices are generally expressed in implied volatility quotes and in particular in Black implied volatility. The Black (1976) option pricing model is the most commonly used formula to convert the implied volatility quotes to actual market prices. The formula is similar to the Black–Scholes formula for valuing stock options except that the spot price of the underlying is replaced by a discounted futures price, denoted by F. The model is based on the assumption that this futures price, for swaptions the forward swap rate, follows a geometric Brownian motion where the volatility is constant, described as

$$dF = \mu F dt + \sigma F dW, \tag{1}$$

where W is a Wiener process and F is the forward swap rate with drift μ and volatility σ.

Consider a specific payer swaption giving the right to pay the fixed swap rate K and receive the floating rate with a tenor of N years, starting in T years, with m coupon payments per year and principal P. Furthermore, let $t_i = T+i/2$ be the moments of each of the coupon payments expressed in years. Applying Black (1976) leads to the following contribution of the value V_{cf} of each individual cash flow (coupon payment) of the underlying swap to the the swaption

$$V_{cf,i} = \frac{P}{m} e^{-r_i t_i}[F\Phi(d_1) - K\Phi(d_2)], \tag{2}$$

with

$$d_1 = \frac{ln(F/K) + \sigma^2(T/2)}{\sigma\sqrt{T}}, \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T}, \tag{4}$$

where F is the forward swap rate with the same maturity and tenor as the swaption, Φ is the cumulative normal distribution function and r_i is the spot rate that corresponds to the maturity of cash flow i at t_i. The sum of individual cash flows results in the total value of the payer swaption V_P given by

$$V_P = PA[F\Phi(d_1) - K\Phi(d_2)], \tag{5}$$

where A represents the annuity factor which is calculated as

$$A = \frac{1}{m}\Sigma_i e^{-r_i t_i}. \tag{6}$$

The value of the corresponding receiver swaption with same strike price K is given by

$$V_R = PA[-F\Phi(-d_1) + K\Phi(-d_2)]. \tag{7}$$

Following Duyvesteyn and De Zwart (2015) and the general swaption market, the Black model is considered as the main pricing model throughout this research.

2.3 Delta-Hedged Straddle Returns and the Volatility Risk Premium

Options are in general products that are often used to either hedge against or bet on a particular directional movement of the underlying. If one is however interested in trading volatility, straddles are a widely used approach. A long straddle only pays off if the price of the underlying moves heavily, either down- or upward, making their prices very sensitive to volatility. Therefore, if volatility risk is priced in the swaption market, straddles are good instruments through which to observe the risk premium.

The central idea in analyzing the volatility risk premium is that if swaption prices incorporate a non-zero volatility risk premium, its existence can be inferred from the returns of a swaption straddle portfolio that has dynamically hedged all risks except volatility risk. Bakshi and Kapadia (2003) prove this argument by mathematically relating the return of a delta-hedged call-option on a stock index to the volatility risk premium. Following their steps I derive the same relation for a delta-hedged portfolio of swaption straddles.

First of all, of key importance in this relation is the definition of the return on a delta-hedged option, or in this case, swaption straddle portfolio, expiring in τ periods from t, which is denoted by $\Pi_{t,t+\tau}$ and expressed as

$$\Pi_{t,t+\tau} \equiv S_{t+\tau} - S_t - \int_t^{t+\tau} \Delta_u dF_u - \int_t^{t+\tau} r_t\big(S_u - \Delta_u F_u\big)du, \tag{8}$$

where S represents the price of the straddle, F is the underlying forward swap rate and r denotes the risk free rate. The first part of the right-hand side of the equation defines the return on the straddle over a period τ, whereas the second part embodies the gains or losses on the continuous delta hedge due to movements of the forward swap rate. The Δ represents the sensitivity of the swaption price to changes in the underlying, which for swaption straddles is defined as

$$\Delta = \frac{\delta S}{\delta F} = PA[\Phi(d_1) - \Phi(-d_1)], \tag{9}$$

with Φ representing the cumulative normal distribution function. The third part of equation (8) eventually defines the net cash investment that earns or pays the risk free rate. In case of a delta-hedged equity option portfolio, entering the hedge position will involve costs. Entering a forward swap however is an unfunded investment which doesn't involve a premium. Hence, the net cash investment part of a delta-hedged swaption straddle portfolio reduces to the interest that is paid to borrow money to buy the swaption straddle at the beginning of the holding period.

In deriving the relation between returns on the delta-hedged portfolio and the volatility risk premium one applies Ito's lemma on the price process of the underlying, which is in case of the

Black model the geometric Brownian motion given in equation (1). First however, the practically seen questionable assumption of volatility being constant is relaxed and is instead assumed to follow a stochastic process characterized by

$$d\sigma_t = \theta_t dt + \eta_t dV_t, \tag{10}$$

with V a Wiener process. By combining the definition of returns on a delta-hedged straddle portfolio given in equation (8), the stochastic process of the volatility given in (10) and the outcome of Ito, the expectation of delta-hedged gains can be deduced to[2]

$$E(\Pi_{t,t+\tau}) = \int_t^{t+\tau} E\big(\lambda_u \frac{\delta S_u}{\delta \sigma_u}\big) du, \tag{11}$$

where λ is a term that represents the price of volatility risk and $\frac{\delta S}{\delta \sigma}$ is known as the option's vega. Equation (11) implies that the expected returns on the portfolio depend on whether or not volatility risk is priced through λ. Hence, significant nonzero average returns would support the existence of a volatility risk premium in swaption straddles. Therefore it is theoretically sound to infer the existence and sign of the premium from returns on dynamically delta-hedged option portfolios. Furthermore, the equation shows that the premium is proportional to the option's vega, which is always positive and increases over option maturity.

The result of this derivation also shows that the volatility risk premium can only be obtained at the end of the holding period. Essentially, the risk premium can be seen as the return on an option due to over- or underestimation of the actual volatility of the underlying by the option seller, resulting in a negative or positive premium, respectively. Note furthermore that the price of volatility risk λ is time-varying, allowing various movements of the premium during the lifetime of the option. The sensitivity of this premium to option maturity is examined in the next section.

3 Data and the Volatility Risk Premium

This section describes the data used for this thesis and visualizes the volatility risk premium and its term structure embedded in the swaption prices. Subsection 3.1 describes the characteristics of the data whereas subsection 3.2 presents the volatility risk premium and its term structure.

3.1 Data

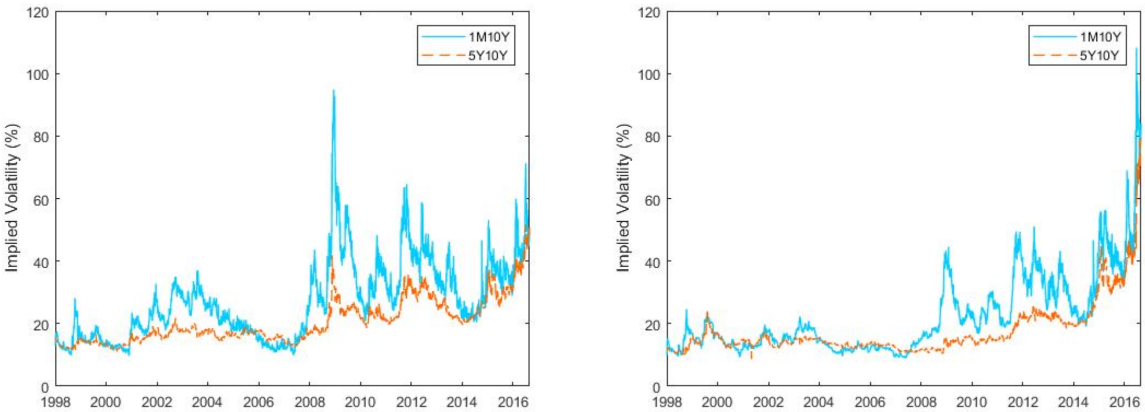
This thesis considers swaption implied volatility and swap forward rate data in the currencies USD and GBP, which represent the most and fourth most traded currency, respectively, obtained from Bloomberg. The Bloomberg data are based on the most recent trades and quotes from multiple pricing sources such as ICAP, which is the largest inter-dealer broker in the swap derivative market [Trolle and Schwartz (2014)].

The sample covers a period ranging from January 1998 to August 2016. Daily close Black implied volatility quotes are available for at-the-money swaptions with maturities of 1, 3, 6, 12,

[2]The full derivation of the expected value of the delta-hedged option portfolio is provided in appendix A.

24, 36, 48 and 60 months on swaps with a tenor of 10 years. Moreover, the floating rate of the underlying swaps is the 6-month LIBOR rate which is updated continuously after each settlement. The tenor and moneyness combination is chosen because swaption contracts with these features are found to be the most liquid [Duyvesteyn and De Zwart (2015)].

Figure 1a and 1b plot the time series behavior of implied volatilities of the shortest and longest maturity swaptions in my dataset for the USD and GBP, respectively. The figures show that the implied volatilities of both currencies varied heavily over the course of the period, where the USD volatilities seem to behave the most volatile. Furthermore, the 5-year implied volatilities tend to be lower and move much more calm than the 1-month implied volatilities for both currencies, indicating that current events affect short-term volatilities more than long-term volatilities and that the effects diminish gradually over time. In general both markets seem to co-move with some common spikes such as during the Lehman Brothers default in 2008 and in the period 2011-2012 which can be related to the Eurozone Debt Crisis and the Fed's "Operation Twist". A somewhat more country specific spike is found in the last months of the GBP sample, which relates to the Brexit vote in June 2016.



(a) USD (b) GBP

Figure 1: Swaption Implied Volatilities.

The figures show the implied volatilities corresponding to the shortest and longest maturity swaption in the dataset over the full sample period January 1998 through August 2016 for both currencies. 1M10Y denotes the swaption with a maturity of 1 month and a tenor of 10 years, whereas the 5Y10Y swaption has a maturity of 5 years and the same tenor.

Table 1 reports the summary statistics of the swaption data across the 8 considered maturities. Like indicated by figure 1, the implied volatilities exhibit a decreasing pattern in the mean and standard deviation over increasing maturity in both currencies. Furthermore, the table shows a decreasing spread between minimum and maximum values over maturity. These observations illustrate the higher risk embedded in short maturity swaption contracts. Hence, it is reasonable to expect that swaption buyers pay a higher volatility premium for shorter maturity swaptions as a compensation to sellers for bearing higher volatility risks.

Comparing over currency markets, it can be concluded that the average implied volatilities

Table 1: Summary statistics of swaption implied volatilities.

	Mat. (m)	Mean (%)	St. Dev. (%)	Skew	Kurt	Min (%)	Max (%)
USD	1	27.4	12.7	1.10	1.90	9.7	94.8
	3	27.2	11.7	0.78	0.47	10.2	80.6
	6	26.7	10.9	0.64	-0.17	11.0	67.4
	12	25.7	9.9	0.62	-0.32	11.6	61.8
	24	24.0	8.7	0.81	0.22	12.0	58.9
	36	22.6	7.9	0.99	0.79	11.4	56.9
	48	21.5	7.3	1.12	1.15	11.3	54.1
	60	20.5	7.1	1.21	1.37	10.9	51.8
GBP	1	22.3	12.9	1.92	5.15	9.1	108.2
	3	22.6	12.7	1.94	5.24	9.3	97.5
	6	22.3	12.4	2.02	5.68	9.4	96.1
	12	21.5	11.9	2.22	6.85	9.7	92.1
	24	20.2	11.0	2.54	8.85	10.0	93.6
	36	19.2	10.1	2.81	10.71	10.3	96.7
	48	18.3	9.4	3.32	12.09	9.6	81.1
	60	17.6	8.7	3.16	13.35	8.8	79.4

Note: This table reports summary statistics of annualized, at-the-money implied volatilities on the 10-year swap forward yield for the USD and GBP markets. The table shows the sample mean (Mean), standard deviation (St. Dev.), skewness (Skew), kurtosis (Kurt), minimum (Min) and maximum (Max) of the implied volatility quotes for maturities ranging from 1 month to 5 years. The sample runs from January 1998 through August 2016 and counts 224 observations.

are higher in the USD market. Contrary to the intuition from the first figure is the standard deviation in the volatilities, which is higher in the GBP market. This is mainly caused by the volatile last year of the sample related to the Brexit. The kurtosis and maximum values of the GBP volatilities support this argument as they imply a peaked distribution where the variance is mostly caused by a small number of large outliers.

Besides the swap forward rate and implied volatility data, the discount rates used to price the swaptions are a combination of LIBOR and swap par rates. The LIBOR rates are used to price swaptions with a time to expiry of less than 12 months, as LIBOR rates are only available up to this maturity. The longer maturity swaptions are priced using the swap par rates.[3] At last, throughout this research macroeconomic and financial variables are included in several of the methodologies. The macroeconomic indicator variables are the revised versions, if applicable, and are obtained from Bloomberg.

3.2 The Premium and its Term Structure

As observed in subsection 2.3, the volatility risk premium essentially is an intangible value in the price of an option, which can only be derived at the end of the holding period. The decreasing

[3] The volatility risk premia are identical when swap par rates are used for the whole cross-section of maturities.

implied volatility structure observed in table 1 forms the first conjectures that the demanded compensation of risk decreases when option maturity lengthens. Most literature on such volatility premia in options find them to be negative over a wide range of underlying products. This includes studies of Fornari (2010) and Mueller *et al.* (2013) who investigate the premium in options on interest rate related products. Duyvesteyn and De Zwart (2015) find an upward sloping return structure for delta-hedged swaption portfolios, implying that returns related to volatility are lower for short-term maturity contracts due to a relatively higher demanded compensation of risk. However, up to a maturity of 12 months the earlier publications report only negative returns, whereas Duyvesteyn and De Zwart (2015) find positive returns for most maturities longer than 3 months, in various currencies. They relate this difference to the 1-month rebalancing frequency they use against the hold-to-expiration set-up in the earlier two studies. This choice of holding period is motived by the fact that hold-to-expiration returns for maturities larger than 1 month overlap and might bias any analysis on the relationship between the swaption maturity and the volatility risk premium.

I construct the compensation for volatility risk following the method proposed by Bakshi and Kapadia (2003) based on gains or losses of a delta-hedged strategy. As derived in subsection 2.3, these delta-hedged returns consist of returns on the swaption straddles, hedging returns and borrowing costs to enter the straddle position. Furthermore, I follow the rebalancing framework introduced by Duyvesteyn and De Zwart (2015). That means that a long position in an ATM swaption straddle is entered at the last day of a particular month and is closed at next month-end. Delta-hedging will be performed on a daily basis.

Table 2 reports the summary statistics of returns on a daily delta-hedged swaption straddle portfolio with maturities ranging from 1 month to 5 years, using a 1-month holding period. The second column shows the average portfolio returns as a ratio of the notional. This way of scaling is in line with Duyvesteyn and De Zwart (2015) and is common practice when it comes to fixed income derivatives.[4] In accordance with the literature are the negative average returns for the lower 1 and 3 month maturities. For maturities larger than 3 months my returns strongly match with those found by Duyvesteyn and De Zwart (2015). The concave upward average return structure they describe is also found in the my dataset, even for maturities exceeding 12 months, as visualized by figure 2. This observed shape of the swaption curve is crucial to the remainder of this study, since it is the most important similarity with the average yield curve. The returns generally imply that option sellers demand a relatively high price for short-term delta-hedged swaption straddle portfolios. Buying these portfolios and selling just before maturity on average entails a loss due to a very low vega near maturity, which results in a negative return and thus a negative premium. Longer maturity straddles however seem to be quoted against relatively low prices, implying a low demanded compensation for risk which increases over the holding

[4]Returns scaled by the swaption price at inception reveal a similar pattern. However, these returns appear to be far more extreme and somewhat unreliable due to the fact that, especially for the short-term, far in- or out-of-the-money options can reach very large or small values relative to the value at initiation, causing extreme returns.

period when the straddles get closer to maturity, resulting in a positive return at the end of the holding period. Hence, premia tend to become positive if a position in a portfolio is held for a limited amount of time and is closed when it still has considerable life time left. Throughout this research I apply the 1-month holding period in line with Duyvesteyn and De Zwart (2015). However, the sensitivity of the term structure to different holding periods is examined in the robustness subsection 7.1.

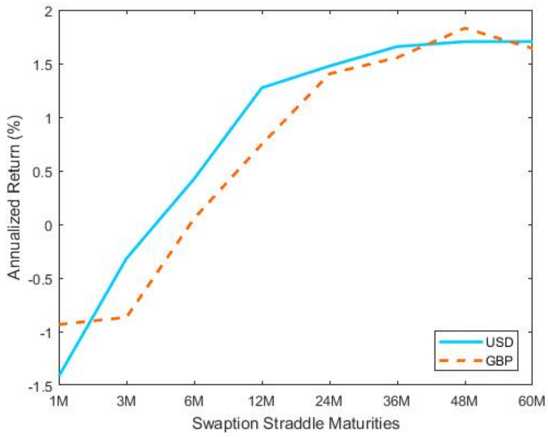


Figure 2: The swaption curve.
This figure presents the average delta-hedged swaption straddle portfolio returns for all considered maturities, or, the swaption curves, over the full sample period in the USD and GBP market as reported in table 2. These are monthly annualized returns which are scaled by the notional of the underlying swaps.

Also in accordance with the results of Duyvesteyn and De Zwart (2015) are the increasing standard deviations in the volatility premium over maturity. This seems counterintuitive, since prices of near-term ATM options generally tend to be very volatile when reaching expiry due to their large sensitivity to value changes of the underlying. However, the straddles in this study are delta hedged, making them relatively insensitive to these changes.[5] The t-statistics of the sample mean furthermore show that almost all average returns in both currencies differ significantly from zero on a 1% significance level. At last, the table reports small autocorrelations in the returns that convert towards zero over maturity and even go negative in the GBP market, suggesting low persistence in the monthly straddle returns.

Table 3 reports the cross-correlations of the straddle portfolio returns for the full set of maturities. In general the cross-correlations are relatively large, especially in the long-end of the curve. The correlations among short maturities decay on a higher pace, where the 1-month returns show the least co-movement with the other return series. These results suggest that long-term returns are driven by a more equal set of factors than short-term returns. Moreover, option theory tells that short-term options are more sensitive to gamma and theta exposures,

[5]The word "relatively" is used here since delta-hedging is done discretely on a daily basis instead of continuously. Hence, intra-day open delta-exposures can influence the returns, although marginally, making the straddles not completely insensitive to the underlying.

Table 2: Pricing of volatility risk.

	Maturity (m)	Mean (%)	St. Dev. (%)	t-stat	ρ_1 (%)
USD	1	-1.42	1.57	-13.55	4.74
	3	-0.32	1.99	-2.40	16.05
	6	0.43	2.28	2.79	20.19
	12	1.28	2.65	7.22	18.47
	24	1.49	3.29	6.76	9.18
	36	1.67	3.70	6.75	4.63
	48	1.72	4.04	6.36	0.45
	60	1.73	4.28	6.01	0.34
GBP	1	-0.94	1.32	-10.43	11.84
	3	-0.86	1.39	-9.28	19.70
	6	0.05	1.67	0.49	15.33
	12	0.75	2.00	5.60	10.22
	24	1.41	2.52	8.41	1.68
	36	1.57	2.75	8.52	-2.05
	48	1.85	2.92	9.45	1.43
	60	1.65	3.03	8.17	-1.17

Note: This table reports the summary statistics of average annualized monthly returns of the daily delta-hedged swaption straddle portfolios, scaled by the notional of the underlying swap. The table shows the sample mean (Mean), standard deviation (St. Dev.), t-statistic (t-stat) and first order autocorrelation (ρ_1) of the returns. The sample runs from January 1998 through August 2016 and counts 224 observations.

that means exposures to actual volatility and time to maturity, respectively, whereas long-term options are more affected by vega exposures, that is exposures to movements in implied volatility. These different underlying exposures cause differences in the return structure of the options and could explain the lower correlation between the short- and long-end returns.

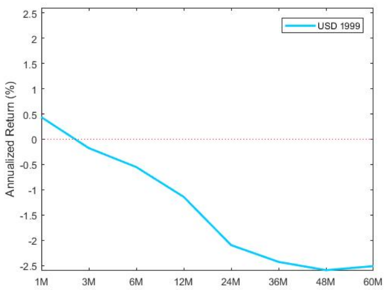
Throughout the sample the swaption curve not only adopts a concave increasing shape. Figure 3 zooms in on the average swaption curve in yearly subsamples. In the USD market in 1999 for instance, the swaption curve takes on the complete opposite shape of the full sample average, namely a decreasing convex pattern. The 2015 USD swaptions market shows an "inverted hump" shape. In other subsamples, such as in the GBP market in 2004 and 2015, the curves are relatively flat and move around different return levels. Remarkable is further the very different return distribution between the USD and GBP market in 2015. Since the implied volatilities of both currencies seemed to co-move in this year, this observation suggests that implied volatility is not the only factor that influences the return distribution.

So far, this assessment of the swaption curve gives an insight in its characteristics and behavior over a period of almost 19 years in two major distinct currency markets. The results in Duyvesteyn and De Zwart (2015) advocate for significant influences of implied volatility dynamics on the curve. First signs of such a relation are also visible in my dataset, recalling the decreasing pattern of implied volatility over maturity coinciding with an increasing volatility premium. Conjectures

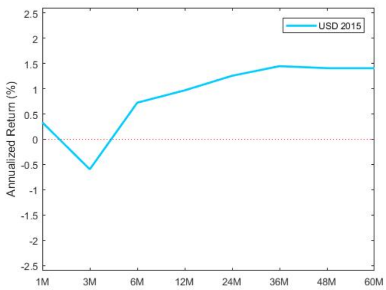
Table 3: Cross-correlations in volatility risk premia.

USD	GBP 1	3	6	12	24	36	48	60
1	-	0.82	0.70	0.55	0.41	0.36	0.32	0.30
3	0.75	-	0.95	0.85	0.71	0.65	0.60	0.58
6	0.61	0.95	-	0.95	0.84	0.79	0.74	0.72
12	0.46	0.84	0.95	-	0.94	0.91	0.86	0.85
24	0.33	0.70	0.85	0.95	-	0.97	0.92	0.91
36	0.29	0.64	0.79	0.92	0.99	-	0.97	0.97
48	0.26	0.60	0.75	0.88	0.97	0.99	-	0.97
60	0.24	0.56	0.71	0.85	0.95	0.98	0.99	-

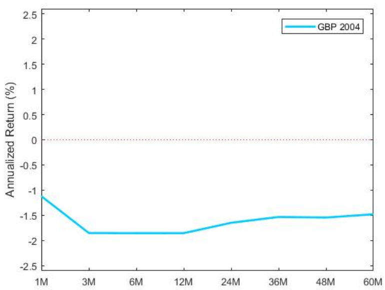
Note: This table reports the cross-correlations among the delta-hedged swaption straddle portfolio returns corresponding to different swaption maturities.



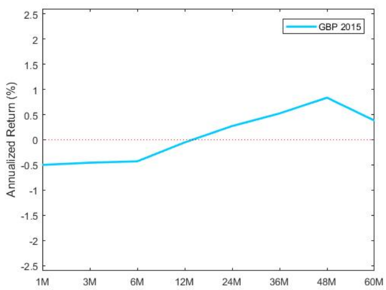
(a) USD 1999



(b) USD 2015



(c) GBP 2004



(d) GBP 2015

Figure 3: Swaption curve in subsamples.

These figures show the notional scaled average annualized delta-hedged swaption straddle portfolio returns in four yearly subsamples in the USD and GBP market.

on this relation are strengthened by Figure 7 in appendix B, which suggests that the behavior of implied volatility not only strongly contributes to the level of the curve but also to its slope. Duyvesteyn and De Zwart (2015) do however also state that this factor cannot explain the full return distribution by itself, supporting the discrepancies in the USD and GBP market in 2015.

The characteristics of the swaption curves in the considered currencies exhibit both similarities and contradictions to characteristics of the yield curve in interest rates. On the one hand, the increasing standard deviation in returns over maturity and the low autocorrelations are two observations that do not match with stylized facts of the yield curve, indicating that both curves have their differences. On the other hand however, an on average concave upward sloping term structure that takes on a variety of shapes over time and large cross-correlations among the “premium yields” do suggest that swaption curve dynamics are comparable to yield curve dynamics. Besides, swaption returns are naturally related to interest rate dynamics and, hence, to a maturity term structure.

The fact that Duyvesteyn and De Zwart (2015) found the underlying price components of swaptions to be incapable of fully capturing the term structure in the volatility risk premium increases the motivation for a different approach. The matching features of the curve with the yield curve inspires the idea of applying yield curve modeling techniques to the swaption return structure. Moreover, the large cross-correlations in the returns advocate for the use of a factor model that is able to explain the full term structure in terms of a small number of key explanatory factors. For these reasons the next section introduces a prominent term structure model from yield curve literature that has the ability to be framed into a factor model.

4 In-Sample Curve Modeling

This research aims to get a better understanding of driving factors of the swaption curve by applying methods stemming from interest rate modeling literature. As a start of the whole swaption curve modeling framework, this section focuses on in-sample modeling of the curve. To this end, I first introduce the main method of this thesis known as the Nelson-Siegel term structure model and subsequently motivate and describe two considered estimation approaches. Thereafter, the second part of this section covers the in-sample modeling results using these methods. I end this section with a short conclusion on the obtained results.

4.1 The Nelson-Siegel Model

The representation of the cross-section of yields in fixed income markets as proposed by Nelson-Siegel (1987) is a very popular model among market participants such as central banks. The construction of the curve allows a variety of shapes and is therefore able to properly replicate the dynamics of the yield curve. This framework of exponential components is later modified by Diebold and Li (2006) to the following formulation

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\Big(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\Big) + \beta_{3,t}\Big(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\Big), \tag{12}$$

where $\beta_t = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t})$ and λ_t are parameters and τ represents maturity. λ_t is commonly referred to as the loading parameter and governs the exponential decay rate; small values of λ_t produce slow decay and better fit the long maturities of the curve, while large values of λ_t have the opposite effect. λ_t also determines the maximum of the third component. Therefore, this parameter is often fixed at the value for which this loading reaches its maximum.[6]

[6]Throughout this research, and for reasons that will be discussed subsequently in detail, I set $\lambda_t = 0.1494$, for

Diebold and Li (2006) show that the three time-varying parameters β_t may be interpreted as factors. An advantage of this Nelson-Siegel framework over factor analysis however is the imposed structure on the factor loadings. They argue that this characteristic not only facilitates the factor estimation, but that it also lets the factors be directly interpreted as level, slope and curvature. The factor loading on β_1 equals 1 for all maturities, a constant that does not decay to zero in the limit meaning that this factor influences the yields of all maturities equally. Hence, this factor can be seen as a long-term level factor. The loading on β_2 is a function that starts at one but decays monotonically towards zero over maturity. This means that short-term yields load on β_2 more heavily than long-term yields causing differences between the short- and long-end of the curve. This factor is therefore referred to as the short-term, or, slope factor. The last loading converges to zero for τ moving either to zero or infinity, but adopts a concave structure over maturities in between. This component thus mostly affects medium-term yields and increases the yield curve curvature. Hence, the third factor is called the curvature factor. Figure 8 in Appendix C illustrates these dynamics in Nelson-Siegel factor loadings.

4.1.1 Nelson-Siegel 2-step

Diebold and Li (2006) notice in their yield curve examination that the estimated factors in the Nelson-Siegel equation are strongly correlated over time. This means that these factors are forecastable and that the Nelson-Siegel framework can be used for forecasting in this way. In particular, Diebold and Li (2006) create a more dynamic version of the Nelson-Siegel representation which comprises two steps. First they estimate β_t by applying least squares over the cross section of maturities on the curve representation of equation (12) for all t, $t = 1, ..., T$. Secondly, they model the time series of the factors using an autoregressive set-up.

Instead of interest rate yields, this research tries to capture and model the delta-hedged swaption straddle return dynamics. By replacing the yields in equation (12) by the observed return series $\Pi_t(\tau_i)$ for a set of n maturities $\{\tau_i\}_{i=1}^n$, I obtain a customized version of the Nelson-Siegel representation described as

$$\Pi_t(\tau_i) = \beta_{1,t} + \beta_{2,t}\Big(\frac{1 - e^{-\lambda_t \tau_i}}{\lambda_t \tau_i}\Big) + \beta_{3,t}\Big(\frac{1 - e^{-\lambda_t \tau_i}}{\lambda_t \tau_i} - e^{-\lambda_t \tau_i}\Big), \tag{13}$$

for $t = 1, ..., T$. To assess whether the Nelson-Siegel factors in the customized Nelson-Siegel return equation also contain predictive content on their future value, I follow the 2-step method from Diebold and Li (2006). First, I estimate the 3x1 vector β_t by applying least squares in equation (13) for $t = 1, ..., T$, such that a time series of T estimates β_t is obtained. Subsequently, I model the time series process for β_t using first order autoregressive processes. Both univariate [AR(1)] and multivariate [VAR(1)] processes are considered, which are given by

$$\beta_{j,t} - \mu_j = \phi_j(\beta_{j,t-1} - \mu_j) + \eta_{j,t} \tag{14}$$

and

$$\beta_{j,t} - \mu_j = \Sigma_{k=1}^3 \phi_{j,k}(\beta_{k,t-1} - \mu_k) + \eta_{j,t}, \tag{15}$$

all t.

respectively, where μ_j denotes the unconditional mean of factor β_j with $j = 1, 2, 3$. Furthermore, error terms $\eta_{j,t}$ are assumed to be normally distributed with mean zero and variance σ_j^2 and to be mutually and serially independent over all time periods.

To simplify the estimation in the first step I fix λ_t at a pre-specified value λ for each month t, which allows me to use trivial ordinary least squares instead of potentially challenging numerical optimizations. Following Diebold and Li (2006), λ is fixed at the value for which the loading of the curvature factor achieves its maximum at a chosen medium term. Given the maturity range of 1 month up to 5 years, I set this term at 1 year, resulting in $\lambda = 0.1494$. Robustness checks on the choice of the loading parameter are executed and conclude that modeling results are optimal under the chosen $\lambda = 0.1494$. The complete results of this analysis are covered in robustness subsection 7.2.

Diebold and Li (2006) furthermore create empirical proxies for the three factors using specifically chosen combinations of yield series. These combinations stem from the earlier described identities of the three factor loadings and the behavior of the Nelson-Siegel representation in the maturity limits. As these proxies function as a helpful tool in examining the behavior of the estimated factors, I fabricate a similar set of proxies for the swaption curve using motives from Diebold and Li (2006). The first proxy considers the level factor. From equation (13) it can easily be verified that $\Pi_t(\infty) = \beta_{1,t}$. Hence, Diebold and Li (2006) treat the long-term 120-month yield as a proxy for level. Accordingly, I define the longest maturity straddle return in my dataset as the level proxy, denoted by L_t for $t = 1, .., T$. Next, the short-term factor β_2 is closely related to the slope of the curve and therefore the slope proxy is defined as the longest maturity returns minus the shortest maturity returns, denoted by S. In particular, $S_t \equiv \Pi_t(60) - \Pi_t(1) = -0.82\beta_{2,t} + 0.04\beta_{3,t}$. Finally, the proxy for the curvature factor is defined as twice the 12-month return, which represents the medium term of the term structure, minus the long- and short-term returns, denoted by C. In particular, $C_t \equiv 2\Pi_t(12) - \Pi_t(1) - \Pi_t(60) = -0.12\beta_{2,t} + 0.42\beta_{3,t}$.

In the result sections of this thesis I denote the 2-step method by OLS, referring to the estimation technique. If applicable, suffixes '-AR' and '-VAR' are added to indicate the time-series model specification for the state equations.

4.1.2 A State-Space Framework

Diebold, Rudebusch and Aruoba (2006) recognize that the dynamic Nelson-Siegel model can also be presented as a state space model, where the factors β_t are again treated as latent factors. They introduce a unified state-space modeling approach that simultaneously fits the yield curve at each point in time and estimates the underlying dynamics of the factors using an autoregressive process. They suggest that this 1-step approach, which applies a Kalman filter, improves upon the 2-step estimation of Diebold and Li (2006) in terms of accuracy and theoretical soundness. Besides, it provides a solid framework that allows for inclusion of external informative factors such as macroeconomic variables, making this method very applicable to the research questions in this thesis.

A state space representation is a system of equations that models an observed time series in terms of latent state variables and describes how these variables evolve over time using autoregressive processes. Following Diebold *et al.* (2006) the state transition equation, which governs the dynamics of the state vector with state variables, is written as

$$\begin{pmatrix} \beta_{1,t}-\mu_1 \\ \beta_{2,t}-\mu_2 \\ \beta_{3,t}-\mu_3 \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \begin{pmatrix} \beta_{1,t-1}-\mu_1 \\ \beta_{2,t-1}-\mu_2 \\ \beta_{3,t-1}-\mu_3 \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \end{pmatrix}, \tag{16}$$

$t = 1, ..., T$. As for the 2-step model, both vector and univariate autoregressive constructions of order one are applied. For the univariate transition equation the off-diagonal coefficients of the state variables are set equal to zero. The corresponding measurement equation, which relates a set of n returns to the three unobservable factors, is given by

$$\begin{pmatrix} \Pi_t(\tau_1) \\ \Pi_t(\tau_2) \\ \vdots \\ \Pi_t(\tau_n) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} & \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} - e^{-\lambda\tau_n} \end{pmatrix} \begin{pmatrix} \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_n) \end{pmatrix}, \tag{17}$$

$t = 1, ..., T$. Here the yields of Diebold *et al.* (2006) are replaced by the delta-hedged straddle portfolio returns Π. In vector-matrix notation, the state-space system is rewritten as

$$\Pi_t = \Lambda(\lambda)\beta_t + \varepsilon_t, \tag{18}$$

$$(\beta_t - \mu) = \Phi(\beta_{t-1} - \mu) + \eta_t, \tag{19}$$

$t = 1, ..., T$, with nx1 return vector Π_t, nx3 loading matrix $\Lambda(\lambda)$, 3x1 state vector β_t, 3x1 mean vector μ and 3x3 coefficient matrix Φ. The disturbances from both equations are assumed to be orthogonal to each other and distributed as

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim NID\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_\varepsilon & 0 \\ 0 & \Sigma_\eta \end{bmatrix} \right), \tag{20}$$

with Σ_ε and Σ_η the nxn and 3x3 covariance matrices of ε and η, respectively. In line with Diebold *et al.* (2006) I assume the Σ_ε to be diagonal, implying that the disturbances from the measurement equation corresponding to different maturities are uncorrelated. This assumption is often used to reduce the number of coefficients and to obtain computational tractability. The diagonality of the Σ_η matrix depends on the type of autoregressive process used. In case of the AR variant I assume the matrix to be diagonal and the error terms in the factors to be uncorrelated. In case of a vector autoregressive process the off-diagonals of Σ_η are nonzero. Furthermore, I consider the initial condition $\beta_1 \sim N(\mu, \Sigma_\beta)$ where variance matrix Σ_β is chosen such that $\Sigma_\beta - \Phi\Sigma_\beta\Phi' = \Sigma_\eta$, following Koopman, Mallee and Van Der Wel (2012).

Diebold *et al.* (2006) argue that the Nelson-Siegel model framed into a state-space form is particularly useful because application of the Kalman filter then delivers maximum likelihood estimates and optimal filtered and smoothed estimates of the underlying factors. This recursive estimation method conditions on past and current observations of the data. Jagadeesh and

Pennacchi (1996) already showed in the past that applying Kalman filter estimation techniques produce more accurate parameter estimates than those obtained from other empirical methods. Therefore, I also apply the Kalman filter in estimating the factors and do this in a way described by Durbin and Koopman (2012).[7] The estimation of the unknown parameters is based on the numerical maximization of the log-likelihood function which is constructed via the prediction error decomposition. I maximize the likelihood by iterating a quasi-newton optimization method. Furthermore, the startup parameter values for the Kalman filter are obtained from the 2-step estimation method. The state space method results are denoted by SS with corresponding suffixes '-AR' and '-VAR'.

To close the methodology part of this section, it is important to emphasize the relevance of the Nelson-Siegel method in light of the research question. This method allows to explain the swaption curve in terms of unobserved factors that account for most of the variety in the curve. As this approach has never been applied before, it can yield new insights in volatility premium term structure modeling.

4.2 In-Sample Modeling Results

The following part of this section describes, analyzes and evaluates the main outcomes of the curve modeling methods discussed in the previous subsections. First subsection 4.2.1 covers the characteristics of the estimated factors from the three Nelson-Siegel representations, respectively. Thereafter, subsection 4.2.2 elaborates on the in-sample fit of the models.

4.2.1 Factor Interpretations

Descriptive statistics of the estimated factors in the three different representations are given in table 4.[8] For the USD market, the model representations return similar estimation results. The mean and standard deviations are roughly at the same level, where the means of the third factor are the highest with values between 0.35 and 0.39, and the means of the second factors are negative. Recall that during the proxy initiation a negative relation was obtained between the second factor and the slope proxy. Hence, a negative mean of β_2 means that returns tend to increase as maturity lengthens. The first order autocorrelations are furthermore very similar among the different model representations with small negative correlations for the first two factors and small positives for the third factor. The low autocorrelation in the factors is in line with the little persistence found in the monthly straddle returns.

The estimates in the GBP market generally show much similarities with those from the USD market. The means of the second factor are again negative however now also largest in absolute value. Autocorrelations in the factors reveal a similar structure as those from the USD market again implying low persistence in the factors. The standard deviations are in general lower than

[7] A more detailed description and derivation of the Kalman filter is provided in appendix D.

[8] The nature of the autoregressive process in the second step of the 2-step approach is irrelevant for the in-sample estimation of the factors, as this happens in the first step. Hence, both AR and VAR representations of the 2-step approach provide the same in-sample estimation results, which are presented below *OLS*.

those of the USD, which is also observable in the spread between the maximum and the minimum in both currencies. The latter observation agrees with the generally lower standard deviation in average straddle returns of the GBP in comparison with the USD returns.

Table 4: Descriptive statistics Nelson-Siegel factors.

	USD					GBP				
	Mean	St. Dev.	Max.	Min.	ρ_1	Mean	St. Dev.	Max.	Min.	ρ_1
OLS										
β_1	0.129	1.400	11.055	-3.378	-0.077	0.173	0.998	6.005	-3.942	-0.037
β_2	-0.282	1.412	3.583	-10.299	-0.111	-0.290	1.003	4.074	-5.322	-0.048
β_3	0.352	1.768	9.798	-4.353	0.145	0.070	1.280	5.177	-3.898	0.192
SS-AR										
β_1	0.104	1.491	11.508	-3.698	-0.096	0.144	0.970	5.793	-3.794	-0.050
β_2	-0.202	1.471	4.171	-9.271	-0.156	-0.304	0.976	4.046	-5.287	-0.079
β_3	0.391	1.420	6.778	-2.902	0.137	0.265	0.956	4.412	-2.454	0.203
SS-VAR										
β_1	0.109	1.500	11.675	-3.779	-0.097	0.153	0.986	5.789	-3.920	-0.064
β_2	-0.212	1.471	4.204	-9.426	-0.153	-0.312	0.980	4.062	-5.246	-0.080
β_3	0.383	1.413	7.360	-2.913	0.121	0.203	0.959	4.706	-2.315	0.176

Note: This table presents the descriptive statistics for the three estimated factors β_1, β_2 and β_3 after fitting the Nelson-Siegel model using the monthly return data 1998:01-2016:08, with λ_t fixed at 0.1494. The table reports the mean, standard deviation, maximum, minimum and first order autocorrelation of the factors in that order.

Table 5 presents the correlations of the estimated Nelson-Siegel factors with the empirical proxies. The results in the table confirm the assertion that the three factors in the models correspond to the level, slope and curvature aspects of the curve. Especially the Nelson-Siegel OLS factors show large correlations with the proxied factors, which is visualized in figure 4 for the USD market. These figures suggest that the factors and proxies, and thereby the actual returns, move somewhat randomly, resembling typical stock returns. The figures also reveal the large co-movement between the first and second factor, indicating that the slope might be largely determined by the long-end of the curve, which in turn suggest that this end is more turbulent than the short-end. This conjecture is supported by the larger standard deviations in long-term returns observed earlier. Furthermore, the small differences in correlations suggest that the factors from the different representations are very but not completely similar, inspiring questions on the cause of these differences. All in all, these observations strongly resemble the high correlations between Nelson-Siegel factors and the empirical level, slope and curvature, found in famous yield curve literature such as Diebold and Li (2006). Hence, first indications of the capability of interest rate term structure models to capture the term structure in volatility risk premia are found.

Both the stepwise dynamic Nelson-Siegel and the Nelson-Siegel in state space form consider an autoregressive set-up to model the factor time series. Panel A of table 6 presents the coefficients of

Table 5: Nelson-Siegel Factor Correlations.

	USD			GBP		
	β_1 & L	-β_2 & S	β_3 & C	β_1 & L	-β_2 & S	β_3 & C
OLS	0.987	0.987	0.971	0.982	0.983	0.936
SS-AR	0.979	0.940	0.857	0.980	0.957	0.924
SS-VAR	0.981	0.944	0.886	0.977	0.956	0.875

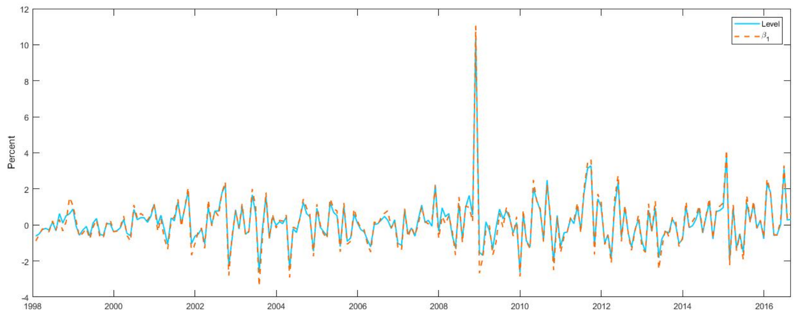
Note: This table presents the correlations between the different Nelson-Siegel factors and the empirical proxies introduced in subsection 4.1.1. OLS, AR and VAR represent the Nelson-Siegel 2-step, state space AR and state space VAR representations, respectively.

the transition matrices Φ from equations (14), (15) and (19). The low autocorrelations among the factors reported in table 4 are reflected in the little significance of autoregressive coefficients. In the 2-step approach using OLS, I observe only convincing significant relations between consecutive values of the third factor, both using the VAR and AR processes. The coefficient, which is of similar magnitude across currencies but induces an opposite effect, is however small and probably not able to fully explain future values of the third factor. Looking at the state space variants, the AR in the GBP yields similar results as the 2-step approach. In the USD sample I now observe a small but significant negative relation between consecutive factors of the second factor. In the VAR set-up, cross-factor dynamics again appear unimportant, with the exception of a statistically significant effect of $\beta_{1,t-1}$ on $\beta_{3,t}$ in both currencies, and $\beta_{2,t-1}$ on $\beta_{3,t}$ in the USD market. Although the cross-factor dynamics seem negligible, the likelihood ratio tests reported in panel B of the table clearly rejects the diagonality of the transition matrix Φ for both currencies, and consequently the variance matrix Σ_η, advocating for the use of the VAR representation of the state space model over the AR variant. The means denoted by μ furthermore appear sensible for most of the factors, as they lie close to the means observed in table 4.

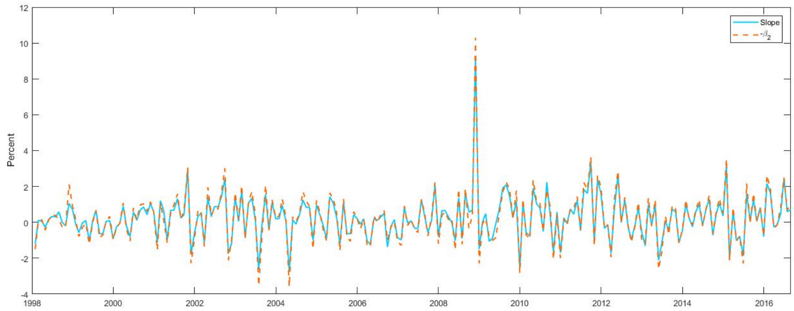
At last, in the maximum likelihood estimation of the Kalman filter all parameters of the Nelson-Siegel representation are optimized. This also holds for the decay factor λ for which I considered a start-up value of 0.1494. After optimization I find estimates of 0.0938 and 0.1016 in the USD currency and 0.1278 and 0.1483 in the GBP currency for the AR and VAR models, respectively. These levels of λ imply that the curvature factor is maximized at maturities of around 20 months for the USD market and slightly above the chosen 12 months for the GBP market. This is displayed in figure 9 in appendix C.

4.2.2 In-sample Model Fit

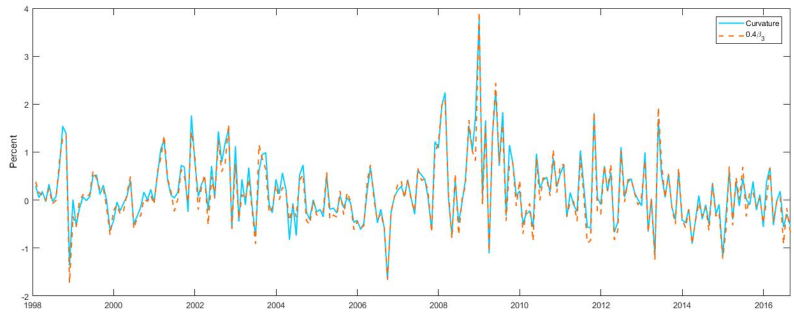
To really draw conclusions on whether yield curve models are able to accurately model the maturity cross-section of straddle returns, one cannot only rely on particular correlations. Hence, a second examination is executed which focuses on the models' in-sample fit. Table 7 reports the estimated absolute means and standard deviations of the measurement equation residuals, expressed in basis points, for all 8 maturities in both currency markets. The means of the absolute errors behave in general quite similar over currencies. Over maturity however, the errors seem



(a) Level



(b) Slope



(c) Curvature

Figure 4: Nelson-Siegel factors & empirical proxies.
The figures show the estimated Nelson-Siegel factors, estimated using the 2-step approach, on the USD data plotted against the timeseries of the empirical proxies introduced in subsection 4.1.1. The figures cover the full sample 1998:01-2016:08.

to behave more or less randomly. The 2-step OLS model returns the most constant absolute errors with values ranging from around 4 bps up to around 8 bps. The actual magnitude of these errors is however rather hard to interpret since it is not possible to directly label them as large or small. The mean relative error measure, denoted by "MRE", gives more insight in this matter. This measure scales the mean absolute errors by the average absolute returns, to obtain the relative fitting error proportional to the magnitude of the actual returns. The results show that when the average is taken over all maturity MRE's per model, the estimated returns deviate between 12.6% and 15.7% from the actual returns in absolute terms. The figures in the table

Table 6: Nelson-Siegel transition coefficients and Diagonality test.

Panel A: Transition coefficients

	USD					GBP				
	$\beta_{1,t-1}$	$\beta_{2,t-1}$	$\beta_{3,t-1}$	μ	λ	$\beta_{1,t-1}$	$\beta_{2,t-1}$	$\beta_{3,t-1}$	μ	λ
OLS-AR					0.1494					0.1494
$\beta_{1,t}$	-0.077			0.143		-0.038			0.178***	
	(0.067)			(0.094)		(0.067)			(0.068)	
$\beta_{2,t}$		-0.111*		-0.321***			-0.048		-0.305***	
		(0.067)		(0.098)			(0.067)		(0.070)	
$\beta_{3,t}$			0.146**	0.296***				0.196***	0.054	
			(0.067)	(0.123)				(0.067)	(0.085)	
OLS-VAR					0.1494					0.1494
$\beta_{1,t}$	0.131	0.207	0.025	0.165		0.158	0.193	0.041	0.196***	
	(0.200)	(0.194)	(0.057)	(0.100)		(0.179)	(0.175)	(0.055)	(0.071)	
$\beta_{2,t}$	-0.106	-0.188	-0.059	-0.307***		-0.052	-0.068	-0.090	-0.294***	
	(0.200)	(0.195)	(0.057)	(0.101)		(0.180)	(0.175)	(0.056)	(0.072)	
$\beta_{3,t}$	0.418*	-0.033	-0.270***	0.219*		0.329	0.210	0.236***	0.056	
	(0.236)	(0.230)	(0.067)	(0.119)		(0.225)	(0.220)	(0.070)	(0.090)	
SS-AR					0.0938					0.1278
$\beta_{1,t}$	-0.095			0.104		-0.048			0.143**	
	(0.067)			(0.092)		(0.068)			(0.063)	
$\beta_{2,t}$		-0.156***		-0.202***			-0.079		-0.304***	
		(0.067)		(0.085)			(0.068)		(0.062)	
$\beta_{3,t}$			0.138*	0.390***				0.210***	0.264***	
			(0.077)	(0.122)			(0.087)	(0.098)		
SS-VAR					0.1016					0.1483
$\beta_{1,t}$	0.049	0.168	-0.017	0.110		0.198	0.271	0.033	0.152***	
	(0.181)	(0.183)	(0.077)	(0.092)		(0.193)	(0.198)	(0.078)	(0.063)	
$\beta_{2,t}$	0.020	-0.137	0.006	-0.213***		0.001	-0.041	-0.110	-0.311***	
	(0.177)	(0.178)	(0.076)	(0.085)		(0.191)	(0.197)	(0.079)	(0.061)	
$\beta_{3,t}$	0.672***	0.371**	0.243***	0.380***		0.442**	0.313	0.213***	0.201***	
	(0.178)	(0.185)	(0.077)	(0.120)		(0.222)	(0.228)	(0.091)	(0.096)	

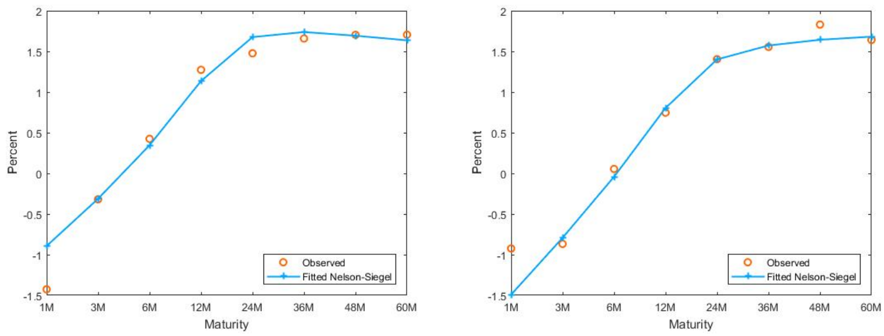
Panel B: Test for diagonality of transition matrix SS-models

	USD	GBP
LR-statistic	482.96	473.98
P-value	0.00	0.00

Note: Panel A of the table represents the estimated transition coefficients from equations (14), (15) and (19). OLS refers to the the linear 2-step approach, whereas SS refers to the state space set-up using the Kalman filter. Standard errors appear in parentheses. Entries with one asterisk (*) denote a significant parameter estimate at the 10% significance level, two asterisks (**) at the 5% level and three asterisks (***) at the 1% level. Panel B reports the Likelihood ratio statistic of the SS-AR against the SS-VAR model, where SS-AR represents the restricted model. The statistic follows a Chi-square distribution with 12 degrees of freedom, equal to the number of restrictions in the SS-AR model.

furthermore show that a lot of the MRE's are even below 10%. These relatively small errors suggest that all considered models generally fit the swaption curve remarkably well. Figures 5a and 5b emphasize this, as they show that the fitted Nelson-Siegel curve provided by the SS-VAR neatly approximates the average curve in both currencies.

Across models and maturity there are however also some large outliers in the mean absolute errors. On model level, the smallest deviations are on average found when applying the 2-step approach on the USD swaptions, whereas the highest deviations are found for the SS-AR model on GBP data. The larger error means of the state space models are mainly caused by the large errors in the shortest maturity. When adjusting for the 1-month maturity the average MAE's reduce to a range of 7 to 14 percent, with the lowest percentages for the state space models. Taking another look at the fitted curves from figures 5a and 5b, note that the 1-month returns deviate much from the pattern in the other maturities. The Nelson-Siegel models estimated using the Kalman filter rely on a maximum likelihood estimation that generates a smooth curve, which adopts the most likely shape considering all maturities. Series that suddenly deviate from the rest of the data series can therefore be missed to a large extent by these kind of models. The deviating returns of 1-month maturity straddles can in turn be related to their different pay-off scheme compared to the other maturities. Positions in 1-month maturity straddles are held until expiry implying that the swaptions either pay off or are completely worthless, while the longer-term swaptions that do not expire yet still hold value while they are out-of-the-money.



(a) USD **(b)** GBP

Figure 5: Fitted Nelson-Siegel curve.
The figures show the actual average swaption curves against the average curves obtained by evaluating the Nelson-Siegel representation at the estimated factor means from the state-space VAR model given in table 4, in the USD and GBP markets.

4.3 Conclusions: In-Sample Modeling

Throughout this section I introduced the Nelson-Siegel factor model and examined its ability to capture and fit the observed term structure in swaption straddle volatility premia by means of a small set of factors. The estimated latent factors of the model, split up in three distinct representations, confirmed their appointed identity of level, slope and curvature, according to

Table 7: Summary statistics for measurement errors of returns.

	OLS			SS-AR			SS-VAR		
Maturity	MAE	MRE	Std.	MAE	MRE	Std.	MAE	MRE	Std.
(months)	(bps)	(%)	(bps)	(bps)	(%)	(bps)	(bps)	(%)	(bps)
Panel A: USD									
1	7.02	20.3	7.67	21.66	62.7	25.32	21.19	61.3	24.74
3	8.27	20.5	9.24	0.00	0.0	0.00	0.81	2.0	0.80
6	6.12	13.9	5.87	6.68	15.1	6.53	5.97	13.5	5.86
12	5.67	11.3	5.61	5.48	10.9	5.76	4.55	9.0	4.70
24	7.21	11.7	6.59	4.62	7.5	4.36	5.46	8.8	5.21
36	6.28	8.8	6.22	5.53	7.8	5.96	6.05	8.5	6.51
48	4.07	5.2	3.82	1.57	2.0	1.57	0.96	1.2	0.94
60	7.40	9.0	6.50	7.35	8.9	6.72	7.88	9.5	7.22
Mean	6.51	12.6	6.44	6.61	14.4	7.03	6.61	14.2	7.00
Panel B: GBP									
1	5.12	17.4	4.53	14.34	48.6	11.81	13.02	44.2	11.01
3	6.00	19.8	5.11	0.59	2.0	0.56	1.57	5.2	1.48
6	4.70	14.2	4.41	3.52	10.6	3.69	2.69	8.1	2.74
12	5.27	13.8	5.78	3.50	9.2	3.47	4.22	11.1	4.33
24	8.27	17.7	11.34	8.81	18.8	12.78	8.90	19.0	12.62
36	5.87	11.3	6.04	3.87	7.5	3.89	2.56	4.9	2.62
48	5.14	9.3	11.76	6.85	12.3	14.38	7.58	13.7	15.36
60	6.83	12.0	8.51	9.45	16.5	9.82	10.84	19.0	11.06
Mean	5.90	14.4	7.19	6.37	15.7	7.55	6.42	15.6	7.65

Note: This table reports the mean (MAE) and standard deviation (Std.) of the absolute measurement errors, expressed in basispoints, for the returns of various maturities measured in months. Furthermore, a relative measure is added denoted by MRE, which is defined as the mean absolute errors divided by the mean absolute returns of the corresponding maturity.

the high correlations with the empirical proxies. The relatively small in-sample measurement errors furthermore demonstrate that the considered models are able to decently fit the average volatility premium term structure of both currencies. Hence, the capability of the Nelson-Siegel model to in-sample model the swaption curve is confirmed, advocating for the application of yield curve modeling techniques to the volatility premium term structure. Significant improvement of the state space estimation approach over the 2-step approach is not found. Lastly, prediction of the estimated factors using the Nelson-Siegel framework is only possible by iterating estimated autoregressive processes, since the factors themselves are unobservable. The observed low persistence in these factors and little significance in the state transition equations however probably entails very poor forecasts. Yet, if the factors can be defined in terms of particular observable variables, forecastability might be enhanced using these variables' time series. This lies in scope of the next section.

5 Explaining the Factors

This section of the study focuses on identifying the driving factors behind the term structure. To this end, I apply linear regressions that resemble the regression approach applied by Duyvesteyn and De Zwart (2015). They conclude that their set of regressors is not able to accurately explain differences in volatility premia of long- and short-maturity straddles. I aim to improve on their regressions by adjusting their approach in two ways. First, since the estimated factors β_1, β_2 and β_3 together largely explain the straddle return distribution in-sample according to the small measurement errors described in subsection 4.2.2, I treat these factors as dependent variables. Second, I relate them to a more extensive set of specifically chosen observable factors. The regressions can be seen as so-called "kitchen sink" regressions, in which a large set of regressors is included. With these regressions I asses the ability of the included variables to explain the Nelson-Siegel factors, and produce a selection of the key drivers among this pool of variables. Diagnosed relations between the yield factors and the considered variables might be useful for curve forecasting purposes, given that the yield factors embed little predictive content on themselves. Eventually, relevant variables are integrated in an extended Nelson-Siegel model in line with Diebold *et al.* (2006), to asses their additional content on curve fitting to the original factors.

Subsection 5.1 describes the shape of the regressions and selects a set of regressors related to underlying price components of swaptions. Subsection 5.2 introduces a set of macroeconomic variables and includes those as regressors in the regressions from subsection 5.1. Subsequently, subsection 5.3 describes the extension of the Nelson-Siegel representation. Subsection 5.4 eventually concludes on this section.

5.1 Underlying Drivers

As a first attempt to expose the identity of the yet unobserved factors from the set of term structure models, I apply simple univariate OLS regressions on the factors of these models. The regressions are described as

$$\beta_{p,q,t} = \alpha_{p,q} + \sum_{j=1}^{J} \gamma_{p,q,j} I_{j,t} + \varepsilon_{p,q,t}, \tag{21}$$

where $\beta_{p,q,t}$ denotes factor $q = \{1, 2, 3\}$ of model $p = \{OLS, SS\text{-}AR, SS\text{-}VAR\}$ at t, and I_t the Jx1 vector of monthly changes in underlyings, or, internal regressors from $t-1$ to t, for $t = 2, ..., T$. These individual regressions are performed for the three components of each of the models.

To select a set of internal regressors, I refer back to the swaption pricing formulas defined in subsection 2.2. These formulas clearly show that daily changes in the value of swaptions are subject to changes in a whole set of underlying components. Hence, the internal regressors are selected by means of a decomposition of swaption value changes to changes of its underlyings. Starting point here is writing the swaption value S as a function of the components; the swap forward swap rate F, implied volatility σ, interest rate r, and time to maturity τ, which is given by

$$S = f(F, \sigma, r, \tau). \tag{22}$$

Now, it is possible to approximate the change in the swaption value for small changes in the function arguments by using a Taylor series expansion [Stewart (2006)]. This expansion boils down to the following equation

$$\Delta S = \frac{\delta S}{\delta F}\Delta F + \frac{\delta S}{\delta \sigma}\Delta \sigma + \frac{\delta S}{\delta r}\Delta r + \frac{\delta S}{\delta \tau}\Delta \tau + \frac{1}{2}\frac{\delta^2 S}{\delta F^2}\Delta F^2. \tag{23}$$

The partial derivatives in this equation with respect to the forward rate, implied volatility and interest rate are known as delta, vega and rho, respectively. For these Greek sensitivities I include regressors that relate to changes in their corresponding underlying factors. Whether these changes are absolute or relative depends on the character of the variable. For all variables that represent a rate I take absolute changes. This choice is motivated by the fact that rates essentially already express a form of relative return and that they can turn negative, causing problems in calculation of relative changes. Changes in variables other than rates are relative.[9]

The last term of the Taylor expansion is a second order partial derivative with respect to the swap forward rate and is known as the gamma of the swaption. The gamma represents the rate of change of the delta to the forward rate and essentially measures the realized volatility of the underlying. Trading strategies that focus on open gamma exposure pay off if realized volatility is higher than implied volatility during the holding period. This might also be an explaining factor in the term structure of swaption straddle returns. Therefore, to represent this element of the Taylor expansion as an internal regressor, I include the difference between the realized volatility of the 1-month forward swap rate calculated at the end of the holding period and the implied volatility of the swaption on this forward swap rate at the moment the position is entered. This particular combination of realized and implied volatility is chosen for two reasons. First and quite straightforward, because they represent the time period that equals the considered holding period in this research. Second, estimates of longer term implied volatilities also include or, are influenced by, shorter term implied volatilities. Hence, in case realized volatilities over the short-term deviate from estimated short-term implied volatilities, longer term implied volatilities will be affected, in turn influencing returns of the longer term swaptions as well.

The goal of this subsection is to relate the factors to the discussed underlyings. However, in the previous results section the three factors were already assigned a particular identity, namely, level, slope or curvature. Therefore the internal regressors representing delta, vega and rho are customized in a same manner as the earlier introduced empirical proxies of level, slope and curvature. That means for instance that the level factor is regressed on changes in the implied volatility and forward swap rate of the longest maturity swaption, which I refer to as level

[9] Regressions are also performed considering either only absolute or only relative changes of all variables. Results show that regardless the nature of the change, similar regressors are found to be relevant. However, due to different magnitudes of absolute and relative changes, their coefficients differ in size.

variables. Slope factors will among others be regressed on relative changes in the slope of the implied volatility, a slope variable. Table 15 in appendix E contains an overview of the variables and their exact definition. Since the first result section exposed large co-movement between level and slope factors, I also include level variables in the slope factor regressions and vice versa.

Table 8 summarizes regression results from the three different representations in both currencies. The left column below each of the factors $\{\beta_1, \beta_2, \beta_3\}$ denoted by "Internal" reports the regression results using solely internal swaption underlying variables. In particular, the table shows the R-squared value, the Akaike Information Criterion value and the variables of which the coefficients are found to be statistically significant on a 5% significance level in each of the regressions. The complete regression results including estimated coefficients and standard errors are reported in Table 16, 17 and 18 in appendix E.

Expected from the large correlations between the estimated factors from the different model representations observed earlier, are the generally similar regression fits according to the R-squared values and the largely overlapping explanatory factors in the regressions across representations. The columns below β_1 consider the regressions on the level factor consisting of level and slope variables. The R-squared values in the USD market are all equal to 0.97, whereas in the GBP market they range from 0.92 to 0.94, suggesting that the level factor can be largely explained by the considered set of regressors. The natural relation of implied volatility dynamics and the level in volatility premia is confirmed by the significant coefficient of the implied volatility level variable in every model for both currencies. Excluding this regressor reduces the model fit considerably. For the USD currency market the slope in implied volatilities is also significantly related to the first factor from the three different models, which is not the case for the state space models in the GBP market, indicating that the implied volatility slope is not captured by the first factor using the Kalman filter estimation in this market. Furthermore, disregarding the implied volatility factors, the level factor in the USD market shows to have a different set of drivers than the level factor in the GBP. The level and slope of the US forward swap rates and the long and short US interest rates show to have significant explanatory content on the level factor in the three models, whereas this generally only holds for the short rate and to a limited extend for the gamma factor, denoted by "Real-IV", in the GBP market.

For the regressions on the slope factor β_2 an equivalent set of regressors is considered as for the level factor. I again find strong significance for the implied volatility terms. In contrast with the results for the first factor, the forward rate terms now also seem to have significant influence in the GBP market. The interest rate slope factor and the gamma term show some significance here and there, however not convincing. The R-squared values for the regressions on the second factor are again substantial and slightly higher in the USD market.

The R-squared values of the curvature factor regressions reported below β_3 are considerably lower than those observed for the previous factors in both currencies, ranging from 0.63 to 0.68. Significant coefficients for almost all included implied volatility variables once again confirm the explanatory content of this variable on different aspects of the swaption curve. The gamma term shows for the first time some convincing significant coefficients in the USD market, however, not in the GBP market, making its relevance questionable. Given that an extended set of regressors is used compared to the other factor regressions in combination with the lower R-squared values, it seems that the latent curvature factor is much harder to explain than the level and slope factor.

In general this subsection confirmed the informational content of implied volatilities on the delta-hedged swaption returns. Given the fact that swaption prices are essentially quoted in implied volatilities, this makes much sense. What might be counterintuitive is the significance of the delta term in most of the regressions, given that the straddle portfolios were delta-hedged. An argument for this observation is that the delta-exposures are hedged discretely on a daily basis, which means that intra-day movements of the forward swap rate can cause profits or losses of the portfolio. However, when excluding the forward rate terms from the regressions the results differences in terms of R-squared are negligible, indicating their small relevance.

5.2 Including Macroeconomic Information

Literature on interest rate dynamics that include macroeconomic information confirm the explanatory power of macroeconomic and financial indicators in interest rate term structure dynamics [Estrella and Mishkin (1997), Evans and Marshall (1998)]. The internal regressors are able to capture a substantial part of the variety in the factors, but especially in explaining the second and third factor they leave room for improvement. This research therefore also relates the timeseries of unobserved factors to dynamics in macroeconomic variables. This subsection introduces a set of macroeconomic variables that are added to the regressions described in the previous subsection.[10] This extended regression is expressed as

$$\beta_{p,q,t} = \alpha_{p,q} + \sum_{j=1}^{J} \gamma_{p,q,j} I_{j,t} + \sum_{k=1}^{K} \theta_{p,q,k} M_{k,t} + \varepsilon_{p,q,t}, \tag{24}$$

where $M_{k,t}$ represents the Kx1 vector of monthly changes in macroeconomic variables from $t-1$ to t. The rest of the parameters are similar to those in equation (21).

The macroeconomic variables selected are generally indicators that have proven to contain predictive and explanatory content on economic business cycles [Stock and Watson (1989), De Pooter *et al.* (2010)]. The dataset in particular includes the US and UK country specific effective banking rate (BR), 10-years government bond yield (GY), consumer confidence index (CCI), consumer price index (CPI), industrial production (IP), money supply (MS) and stock index (SI). Panel B of table 15 in appendix E describes the complete set of macro variables more extensively.

The columns denoted by "Internal+Macro" in table 8 contain the summarized results for the regressions including macroeconomic variables. The complete results are again presented in tables 16, 17 and 18 in appendix E.

Table 8 shows that for the first factor β_1 most of the AIC values decrease, although marginally, when including macroeconomic variables in the regressions. In the USD market this change can

[10]Regressions consisting of solely macroeconomic variables are also considered but perform considerably worse than the regressions from subsection 5.1 Hence, I only report the combined regressions.

Table 8: Regression results summary

		β_1		β_2		β_3	
		Internal	Internal+Macro	Internal	Internal+Macro	Internal	Internal+Macro
Panel A: USD	*OLS*	$R^2 = 0.97$ AIC = -0.05 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{120M}	$R^2 = 0.98$ AIC = -0.04 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{120M}, IP	$R^2 = 0.93$ AIC = 1.00 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, Real-IV	$R^2 = 0.93$ AIC = 0.98 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, Real-IV, IP, SI	$R^2 = 0.66$ AIC = 2.97 σ_{slope}, σ_{curv}, F_{lvl}, F_{slope}, F_{curv}, Real-IV	$R^2 = 0.71$ AIC = 2.86 σ_{slope}, σ_{curv}, F_{lvl}, F_{curv}, Real-IV, BR, IP
	SS-AR	$R^2 = 0.97$ AIC = 0.35 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{3M}, r_{120M}	$R^2 = 0.98$ AIC = 0.33 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{120M}, IP	$R^2 = 0.92$ AIC = 1.11 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{slope}	$R^2 = 0.93$ AIC = 1.06 σ_{lvl}, σ_{slope}, F_{lvl}, Real-IV, IP, SI	$R^2 = 0.63$ AIC = 2.64 σ_{lvl}, σ_{slope}, σ_{curv}, F_{lvl}, F_{slope}, Real-IV	$R^2 = 0.67$ AIC = 2.57 σ_{slope}, σ_{curv}, Real-IV, BR
	SS-VAR	$R^2 = 0.97$ AIC = 0.28 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{3M}, r_{120M}	$R^2 = 0.97$ AIC = 0.27 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{120M}, IP	$R^2 = 0.92$ AIC = 1.10 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{slope}	$R^2 = 0.93$ AIC = 1.05 σ_{lvl}, σ_{slope}, F_{lvl}, Real-IV, IP	$R^2 = 0.66$ AIC = 2.54 σ_{slope}, σ_{curv}, F_{lvl}, F_{slope}, Real-IV	$R^2 = 0.71$ AIC = 2.45 σ_{slope}, σ_{curv}, Real-IV, BR, IP
Panel B: GBP	*OLS*	$R^2 = 0.94$ AIC = 0.17 σ_{lvl}, σ_{slope}, r_{3M}, Real-IV	$R^2 = 0.94$ AIC = 0.09 σ_{lvl}, σ_{slope}, Real-IV, GY, BR	$R^2 = 0.86$ AIC = 0.95 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{slope}, Real-IV	$R^2 = 0.86$ AIC = 0.99 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{slope}, Real-IV	$R^2 = 0.63$ AIC = 2.41 σ_{lvl}, σ_{slope}, σ_{curv}, F_{curv}	$R^2 = 0.70$ AIC = 2.27 σ_{lvl}, σ_{slope}, σ_{curv}, F_{lvl}, GY, BR, CPI
	SS-AR	$R^2 = 0.93$ AIC = 0.19 σ_{lvl}, r_{3M}, Real-IV	$R^2 = 0.94$ AIC = 0.14 σ_{lvl}, r_{3M}, GY, BR	$R^2 = 0.85$ AIC = 0.96 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{slope}, Real-IV	$R^2 = 0.85$ AIC = 0.98 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{slope}, GY	$R^2 = 0.67$ AIC = 1.73 σ_{slope}, σ_{curv}, F_{slope}, F_{curv}	$R^2 = 0.73$ AIC = 1.67 σ_{lvl}, σ_{slope}, σ_{curv}, F_{lvl}, GY, BR
	SS-VAR	$R^2 = 0.92$ AIC = 0.32 σ_{lvl}, r_{3M}	$R^2 = 0.93$ AIC = 0.29 σ_{lvl}, r_{3M}, GY	$R^2 = 0.84$ AIC = 1.03 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}	$R^2 = 0.84$ AIC = 1.06 σ_{lvl}, σ_{slope}, F_{lvl}, F_{slope}, r_{slope}	$R^2 = 0.68$ AIC = 1.70 σ_{lvl}, σ_{slope}, σ_{curv}, F_{slope}, F_{curv}	$R^2 = 0.75$ AIC = 1.52 σ_{lvl}, σ_{slope}, σ_{curv}, F_{lvl}, GY, BR

Note: This table summarizes the regressions results from models (21), below "Internal", and (24), below "Internal+Macro". For the regressions of each of the three factors ($\beta_1, \beta_2, \beta_3$) from every model (OLS, SS-AR, SS-VAR) the following results are reported: (1) R-squared value, (2) Akaike Information Criterion value, (3) Variables with significant coefficients on a 5% significance level. The number of observations included in all regressions is 224 [1998:01-2016:08].

mainly be assigned to the significant coefficient for the industrial production variable, which seems to capture some explanatory content from the short-term interest rate. In the GBP market I observe some significant effects from the banking rate, but especially the long-term government bond yields catch the eye with statistically significant coefficients for every model. Most coefficients of the internal regressors kept their significance after adding the macro's.

The results for the second factor β_2 do not improve in the GBP market according to the Akaike Information Criterion. This can also be inferred from the little significance in coefficients of the macro's in this market. Hence, the considered macroeconomic factors do not add value in defining the slope of the GBP swaption curve. This does not apply to the USD market where the results show improvements in model fit. The IP again seems to contain relevant content, whereas the coefficient of the S&P500 index also shows some significance in the models. The forward rate and spot interest rate slope variables lose their explanatory power when the macro's are included.

Lastly, results below β_3 reveal that the macro factors also have their added value in explaining the curvature factor. Most R-squared values increase with about 10%, whereas all AIC values decrease. In both currencies monthly changes in the effective banking rate have a significant negative effect on the degree of curvature of the slope. Furthermore, in the USD market IP again plays its part in two of the models. In the GBP market we find such a repeating pattern for the 10-years government yield, which also has a negative effect on the curvature factor.

Summarizing this subsection, evidence of significant correlations between the yield factors and macroeconomic variables is found, yielding improvements in model fit when adding the macro's to the regressions from subsection 5.1. In the USD market in particular changes in industrial production, bank rate and stock index seem to contain relevant information on dynamics of the estimated factors and indirectly on the swaption curve. In the GBP market this applies to a combination of the bank rate and the 10-years government bond yield. A note is however placed on the informational content of monthly changes in the effective bank rate in GBP market. Due to little changes in the Bank of England's interest rate policy in the last few years, this rate is rather static. Therefore, when using estimation samples that describe recent periods, one should be careful using this variable.

5.3 Extending Nelson-Siegel

The R-squared values of the linear regressions of subsection 5.1 and 5.2 state that the factors of all three models could be defined to a considerable extent. The underlying drivers seem to be the most correlated variables with the latent factors. However, including macroeconomic variables in the regressions improves the fit, indicating their informative content on the swaption curve.

Joslin, Priebsch and Singleton (2012) assess how variation in economic indicators in the US influences premia of level, slope and curvature risks in its Treasury markets. They dispute the implication of macro-finance affine term structure models that macro factors are fully spanned by the current yield curve. Together with a large body of literature they find strong evidence against this implication and show that the principal components of the curve only span these macro factors to a very limited extent. Furthermore, they prove that the unspanned macro factors have large effects on risk premia.

Where this research meets mine is in the inclusion of unspanned factors in a pricing model. The underlying factors from the first set of regressions discussed in subsection 5.1 have a strong bidirectional relation with the swaption curve, and can be considered as spanned factors. Predictive content of the swaption curve on the macroeconomic variables however is highly unlikely and also rejected in my dataset. Since the regressions in subsection 5.2 showed that some of these variables contain explanatory content on the yield factors, they can be considered as unspanned factors. Motivated by the results of Joslin *et al.* (2012), I include the macro variables that have proved to contain information on the swaption curve to the state vector of the Nelson-Siegel model, such that these variables can have significant predictive content on the swaption returns above the information in the model factors.

This idea of including macroeconomic factors in the state vector of Nelson-Siegel model is equivalent to the approach of Diebold *et al.* (2006). Following this paper, equations (18)-(20) are replaced with

$$\begin{pmatrix} \Pi_t \\ M_t \end{pmatrix} = \begin{pmatrix} \Lambda(\lambda) & 0_{nxm} \\ 0_{mx3} & I_{mxm} \end{pmatrix} \beta_t + \begin{pmatrix} \varepsilon_t \\ \nu_t \end{pmatrix}, \tag{18'}$$

$$(\beta_t - \mu) = \Phi(\beta_{t-1} - \mu) + \eta_t, \tag{19'}$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \\ \nu_t \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_\varepsilon & 0 & 0 \\ 0 & \Sigma_\eta & 0 \\ 0 & 0 & 0 \end{bmatrix} \right), \tag{20'}$$

where M_t an mx1 vector denoting the set of macroeconomic regressors at t with $M'_{t,USD} = (IP_t, BR_t, SI_t)$ and $M'_{t,GBP} = (GY_t, BR_t)$. Furthermore, $\beta'_{t,USD} = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t}, M_{t,USD})$, whereas $\beta'_{t,GBP} = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t}, M_{t,GBP})$.[11] Note that the yields still load only on the yield curve factors. This is consistent with Joslin *et al.* (2012), who argue that three factors are enough to explain most of the cross-sectional variation in the yield curve. For the Kalman filter it is however required to include the macroeconomic variables in the measurement equation. Since these are already known values, they are only related to themselves through identity matrix I, with an independent disturbance term ν_t for which the mean and variance are set equal to zero. Lastly, same assumptions are made on the variance matrices of the disturbances as for the VAR variant of the yields-only model, namely a non-diagonal Σ_η matrix which now also includes macro-yield disturbance covariances, and a diagonal Σ_ε matrix.

Table 9 displays the estimates of the transition coefficients, which contains the macro and term structure interactions. First of all, the interactions among the yield curve factor lags are close to the interactions observed in the state space VAR without macro factors described in table 6, meaning low persistence and low mutual predictive power in the factors. In the USD market, more predictive content is found in the macro variables. In particular, monthly changes in the industrial production significantly influence the future first and second factor, considering a 5% significance level, whereas changes in the effective bank rate and the stock index contain significant predictive content on the first en third factor. The interaction between the included macro variables in the GBP market and the yield factors is only moderate, with only significant influence on the future value of the third factor. Given the fact that the third factor explains a less fundamental part of the yield curve compared to the other two factors, the GBP macro

[11] Bikbov and Chernov (2010) document that recursive frameworks in macro-yield models are possibly sensitive to the ordering of the variables. When including the macroeconomic factors in front of the yield factors in the VAR structure I obtain almost equivalent transition estimates and in-sample results.

Table 9: Macro-yield model transition coefficients.

Panel A: USD							
	$\beta_{1,t-1}$	$\beta_{2,t-1}$	$\beta_{3,t-1}$	IP_{t-1}	BR_{t-1}	SI_{t-1}	μ
$\beta_{1,t}$	-0.080	0.115	-0.059	0.362**	-1.371**	-0.051**	0.110
	(0.188)	(0.182)	(0.081)	(0.151)	(0.627)	(0.024)	(0.092)
$\beta_{2,t}$	0.097	-0.122	0.025	-0.434***	1.153*	0.042*	-0.217***
	(0.183)	(0.177)	(0.079)	(0.147)	(0.611)	(0.023)	(0.082)
$\beta_{3,t}$	0.416**	0.181	0.142*	-0.270*	-1.315**	-0.047**	0.372**
	(0.185)	(0.181)	(0.080)	(0151)	(0.619)	(0.024)	(0.145)
IP_t	-0.109	-0.026	-0.010***	0.151**	0.660**	0.009	0.085
	(0.079)	(0.077)	(0.034)	(0.064)	(0.264)	(0.010)	(0.071)
BR_t	-0.035**	-0.033**	-0.009	0.037***	0.611***	0.003	-0.021
	(0.015)	(0.147)	(0.007)	(0.012)	(0.051)	(0.002)	(0.029)
SI_t	0.183	-0.198	0.116	1.422***	2.827	0.109	0.474
	(0.547)	(0.531)	(0.235)	(0.441)	(1.830)	(0.069)	(0.370)
Panel B: GBP							
	$\beta_{1,t-1}$	$\beta_{2,t-1}$	$\beta_{3,t-1}$	GY_{t-1}	BR_{t-1}		μ
$\beta_{1,t}$	0.211	0.313	-0.019	0.074	-0.686		0.154**
	(0.191)	(0.197)	(0.089)	(0.300)	(0.424)		(0.064)
$\beta_{2,t}$	-0.036	-0.094	-0.055	-0.194	0.797*		-0.315***
	(0.188)	(0.195)	(0.089)	(0.296)	(0.421)		(0.062)
$\beta_{3,t}$	0.388*	0.468**	0.016	-0.910***	-1.559***		0.201**
	(0.214)	(0.219)	(0.100)	(0.328)	(0.485)		(0.101)
GY_t	-0.095*	-0.103**	-0.008	0.047	-0.007		-0.018
	(0.050)	(0.052)	(0.023)	(0.079)	(0.112)		(0.018)
BR_t	-0.023	-0.021	-0.032	0.039*	0.517***		-0.031
	(0.027)	(0.029)	(0.013)	(0.043)	(0.061)		(0.023)

Note: This table represents the estimated transition coefficients for the macro-yield model from equation (19'). Standard errors appear in parentheses. Entries with one asterisk (*) denote a significant parameter estimate at the 10% significance level, two asterisks (**) at the 5% level and three asterisks (***) at the 1% level. The model for the USD market includes the macroeconomic variables IP, BR and SI, whereas the model for the GBP market includes GY and BR.

variables seem to contain little information on future shapes of the swaption curve.

The in-sample measurement errors of the extend Nelson-Siegel model, reported in table 10, lie very close to errors obtained in the VAR yields-only model. This is caused by the fact that the estimated yield factors of the two approaches are very similar. The same holds for the estimated decay loadings, which are 0.1047 and 0.1493 for the USD and GBP, respectively. Hence, in-sample fit is not directly improved by including macroeconomic information in the Nelson-Siegel model.

5.4 Conclusions: Explaining the Factors

The goal of this section is to identify the factors that largely explain the term structure in swaptions, by relating them to variables stemming from different backgrounds. I conclude that, as expected, movements in implied volatility form the largest driver of the behavior of the swaption

Table 10: Summary statistics for the macro-yield model measurement errors of returns.

	USD			GBP		
Maturity (months)	MAE (bps)	MRE (%)	Std. (bps)	MAE (bps)	MRE (%)	Std. (bps)
1	21.25	61.5	24.71	13.11	44.5	11.39
3	0.57	1.4	0.56	1.40	4.6	1.30
6	5.96	13.5	5.82	2.88	8.7	2.89
12	4.21	8.4	4.33	4.13	10.9	4.07
24	5.88	9.5	5.58	8.92	19.1	12.72
36	6.28	8.8	6.74	2.48	4.8	2.54
48	0.74	1.0	0.74	7.74	13.9	15.45
60	8.13	9.8	7.46	10.99	19.2	11.22
Mean	6.63	14.2	6.99	6.46	15.7	7.70
λ		0.1047			0.1493	

Note: This table reports the mean (MAE) and standard deviation (Std.) of the absolute measurement errors, expressed in basispoints, for the returns of various maturities measured in months. Furthermore, a relative measure is added denoted by MRE, which is defined as the mean absolute errors divided by the mean absolute returns of the corresponding maturity.

curve, in all its aspects. Moreover, the considered set of underlying variables and their empirical modifications are able to explain most of the variety in the factors over the sample according to the obtained R-squared values. In the second set of regressions described in subsection 5.2, the macro factors also showed their added value by improving the AIC values of the regressions from subsection 5.1. Hence, after concluding that the swaption curve can neatly be explained in-sample by three latent factors using the Nelson-Siegel model, the second part of the curve modeling segment of this thesis largely succeeds in exposing their identity. Subsequently including relevant unspanned macroeconomic variables in the Nelson-Siegel representation did not directly lead to an improvement in in-sample fit. It did however reveal some predictive content in the variables on the level, slope and curvature factors of the USD curve, which might be useful for forecasting purposes.

All in all, this section revealed the driving factors behind the volatility premium term structure in swaptions. Time series of these influential variables can possibly be useful in the eventual purpose of many modeling techniques in financial literature: forecasting. Examining the forecastability of the term structure forms the last part of the curve modeling segment and is described in the following section.

6 Out-of-Sample Forecasting

Being able to forecast the swaption curve can have valuable implications for trading strategies on this curve. If the shape of the curve is forcastable, riding strategies as employed by Duyvesteyn and De Zwart (2015) can for instance be enhanced by determining optimal straddle combinations and moments to enter them.

Subsection 4.2.2 showed that the swaption curve can in-sample be accurately explained by the three estimated factors $\beta = \{\beta_1, \beta_2, \beta_3\}$. Therefore, swaption curve forecasts can be obtained by forecasting the factors and plugging them into the Nelson-Siegel measurement equation. To this end, I use a moving window of 60 months of observations to estimate the factors and their individual and mutual time series dynamics. This means that the forecast sample starts at 2003:01 and that in every step ahead the first of 60 in-sample observations is excluded from the estimation sample, whereas the actual first out-of-sample observation is added. This particular size of the estimation sample is chosen as it corresponds to the forward looking life time of the longest maturity swaption. The large variety in the implied volatilities over the course of the full sample as displayed by figure 1 in section 3 furthermore advocates for a moving window rather than an expanding one. Lastly, since the different Nelson-Siegel model representations generate quite similar results in-sample and given the outcome of the likelihood ratio test described in subsection 4.2.1, I only evaluate the VAR variant of the state space model.

6.1 Forecasting Frameworks

The dynamic Nelson-Siegel model adopts a framework which straightforwardly allows forecasting through the autoregressive transition equation. The observed low autocorrelations however also urge for the use of alternative approaches that anticipate on this feature. Let me now describe and motivate the considered approaches.

1. VAR(1) on factors [AR]:

   $\hat{\beta}_{t+h|t} = \mu + \Phi^h(\beta_t - \mu)$.

   The first approach constructs the h-period ahead factors using the transition matrix Φ from the dynamic Nelson-Siegel model which is estimated using the Kalman filter as described in section 4. This approach is denoted by AR, referring to its autoregressive nature.

2. Average approach [AVG]:

   $\hat{\beta}_{t+h|t} = \mu$.

   Given the little significance of the coefficients in the in-sample VAR transition matrix Φ reported in table 6, including an autoregressive term to forecast $\hat{\beta}_{t+h|t}$ might bias the results. Hence, the second approach treats the estimated sample mean itself as a forecast for the next period. This model is denoted by AVG.

3. Moving average approach [MA]:

   $\hat{\beta}_{t+h|t} = \mu$.

   $\hat{\Pi}_{t+h|t} = \Lambda(\lambda)\hat{\beta}_{t+h|t} + \gamma\epsilon_t$.

   Figure 4 shows that the estimated factors and thereby the returns themselves are quite vulnerable to random shocks in their environment. Such shocks can for instance be unexpected movements in the market's volatility. This approach extends the average approach by including the last unexpected deviation ϵ in the straddle returns in the measurement equation, to capture the effects of a previous shock. γ is linearly estimated in-sample. This approach is denoted by MA, referring to its similarities with typical moving average models.

4. Underlying factor dynamics [UF]:
   $\hat{x}_{t+h|t} = \Psi^h x_t,$
   $\hat{\beta}_{t+h|t} = \theta \Delta \hat{x}_{t+h|t}.$
   The regressions of subsection 5.1 showed that the estimated factors largely depend on particular underlying components of swaptions. In particular the changes in implied volatility and the forward swap rate account for a substantial part in the variety of the factors. The levels of these variables show considerable monthly autocorrelation over time - 0.93 and 0.97 for implied volatility and swap rate in the USD market, respectively - meaning that their first-differences might be predictable to a certain extent. Using the above described set of equations I first estimate the autoregressive transition matrix Ψ of the considered set of underlying variables x. Secondly, I estimate in-sample relations between changes in x from $t-1$ to t, denoted by Δx_t, and β_t in a similar way as the regressions from 5.1. This model is denoted by UF, referring to the underlying factors.

The approaches discussed above are described in terms of yield-only models that do not consider any macroeconomic input as seen in subsection 5.3. Table 9 however showed the significant interactions between lagged macroeconomic variables and the estimated factors in the USD currency market. Therefore I also consider macro-yield versions of the Nelson-Siegel VAR model, but only for this market. The models are estimated as described in 5.3.

Lastly, I evaluate the forecasts in two ways. First, 1-period (h=1) ahead forecasts are constructed to examine the forecastability of individual monthly returns per maturity. As measures of accuracy I use the mean absolute error measure, denoted by MAE, and a relative version of this measure, denoted by RMAE, which scales the MAE by the absolute size of the actual return to give insight in the relative magnitude of the forecast error. Second, at the start of each year included in the forecasting sample, I forecast the full year ahead and compare the shape of the predicted swaption curve with the actual curve of that year. Hence, I consider forecast horizons of 1 to 12 months. Furthermore, I only apply the best performing forecasting model according to the 1-period ahead forecasts and do this on both currencies. The essence of this second part is to test the ability of the model to forecast different shapes of the curve as observed in the yearly subsamples, and to assess if the model can predict the shape over a 12-month period such that trading strategies can be constructed at the start.

6.2 Forecasting Results

Panel A of table 11 presents the forecasting results and statistics for the four introduced forecasting methods in the USD market. The models do all forecast an upward sloping average term structure with negative premia for the shorter maturities and positive premia for the longer maturities. The peek of the term structure is however mostly predicted between the 36- and 48-month maturity, instead of the actual peek at the 60-month maturity, given in the last column of the table. The only model that does predict a continuously upward sloping structure is the UF-model. This model also predicts most accurately according to the MAE and RMAE.

On average its forecasts are however a bit more conservative, with relatively small premia in absolute terms, and more negatively skewed. Furthermore, in general the prediction errors are substantial and often of a similar magnitude as the returns themselves. Especially the standard autoregressive Nelson-Siegel set-up provides inaccurate forecasts, with RMAE's ranging between approximately 104 and 110 percent. Including macro-economic variables in the models does not improve the forecasting accuracy and, hence, it can be concluded that macroeconomic input does not significantly enhance the considered models.

For the GBP model no macro-yield models are considered due to the little predictive content in the macro factors observed in-sample. Panel B of table 11 reports the results for this currency which are generally similar to the results in the USD currency. The AVG-model seems to do best on the short-end of the curve, whereas it is again the UF-model that does best on the long-end. The AR-model is also in the GBP currency the worst performing forecasting model.

The results show that the three alternative forecasting approaches AVG, MA and UF all improve on the standard autoregressive Nelson-Siegel framework in terms of out-of-sample forecasting. The average approach does best in the short-end, which yields for the UF-model in the long-end. Overall, the latter model provides relatively seen the most accurate 1-month ahead forecasts, on average. Therefore, this model is applied in the second part of the forecasting section.

Results of this second forecasting framework are displayed by figures 6a and 6b for the USD and GBP currency markets, respectively. They show the actual average term structures for yearly subsamples 2004 to 2015 together with the predicted curves. The figures visualize the relative inaccuracy of the models already observed in the 1-period ahead forecasts. The predicted curves deviate considerably from the actual versions in shape and level in both currencies. Only if the subsample curves resemble the full-sample average curve, the predictions seem able to follow, such as in 2004 and 2010 in the USD market and 2010, 2012 and 2015 in the GBP market. This suggests that deviating behavior of the curve from its generally expected moves are hard to capture. Deviating behavior is commonly caused by particular shocks in the market. To improve the forecasts, more focus should be put on the character of these disturbances.

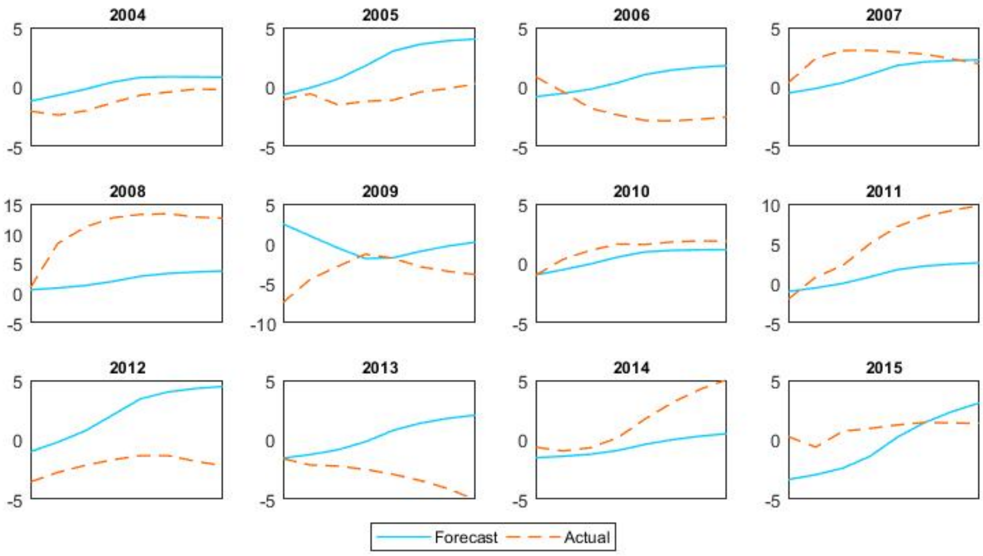
6.3 Conclusions: Forecasting

The results of this section generally reveal the difficulty of accurately forecasting the term structure in the volatility risk premium. As observed earlier in this study, individual premia behave somewhat randomly and remind of stock returns, which are also vulnerable to shocks in their markets. Focusing on the underlying factors and their time series already improved the standard autoregressive Nelson-Siegel forecasting framework, but still leaves a lot of room for improvement. These improvements might be found in the modeling framework of the underlying factors and their shocks. Other solutions might be found in literature that aims to forecast stock returns. Further enhancing the forecasting methods however lies beyond the scope of this study and would serve as a subject for further research.

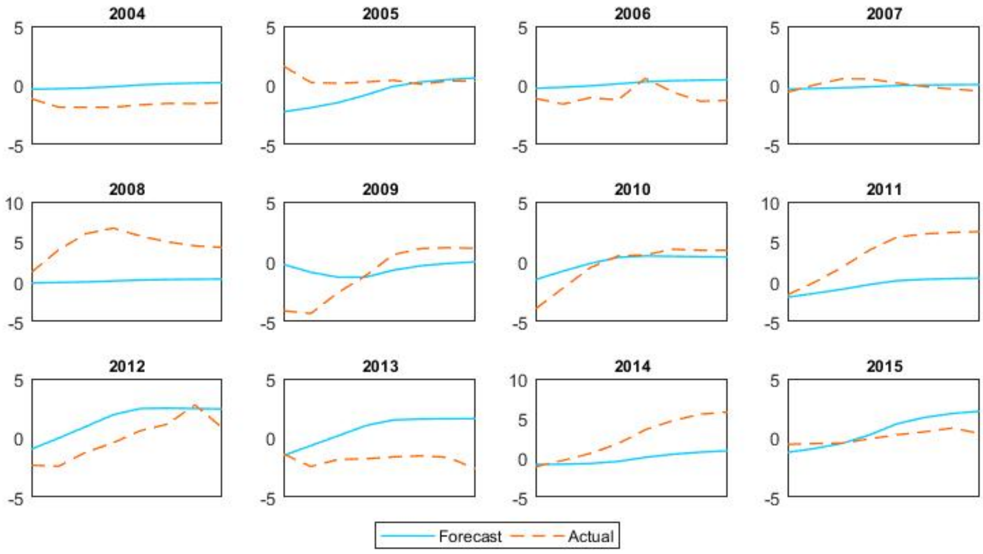
Table 11: Out-of-sample forecast results.

		Yield-Only				Macro-Yield					Yield-Only				Macro-Yield				Actual
	Maturity	Mean	Std. Dev.	MAE	RMAE	Mean	Std. Dev.	MAE	RMAE	Maturity	Mean	Std. Dev.	MAE	RMAE	Mean	Std. Dev.	MAE	RMAE	Avg. Return
Panel A: USD	*AR-VAR*									*AVG-VAR*									
	1M	-0.69	0.67	0.38	109.1	-0.93	0.96	0.42	119.9	1M	-0.73	0.26	0.35	100.4	-0.80	0.25	0.35	100.2	-1.50
	3M	-0.03	0.75	0.46	105.7	-0.32	1.00	0.47	110.6	3M	-0.17	0.29	0.44	102.1	-0.21	0.27	0.44	102.1	-0.31
	6M	0.69	0.88	0.50	107.2	0.32	1.10	0.53	113.4	6M	0.47	0.34	0.49	104.3	0.45	0.31	0.49	104.5	0.39
	12M	1.49	1.02	0.58	107.3	0.99	1.30	0.62	114.1	12M	1.24	0.39	0.57	104.4	1.23	0.37	0.57	105.0	1.15
	24M	1.92	1.10	0.73	107.2	1.31	1.60	0.79	116.3	24M	1.79	0.40	0.70	103.2	1.75	0.40	0.70	103.7	1.57
	36M	1.86	1.12	0.82	105.5	1.27	1.81	0.88	112.3	36M	1.84	0.39	0.79	101.0	1.79	0.40	0.79	101.6	1.83
	48M	1.71	1.13	0.91	105.9	1.14	1.95	0.97	113.4	48M	1.77	0.36	0.87	101.9	1.71	0.39	0.88	102.6	1.89
	60M	1.56	1.15	0.96	104.9	1.04	2.06	1.02	111.0	60M	1.67	0.34	0.93	101.0	1.62	0.38	0.93	101.6	1.92
	MA-VAR									*UF-VAR*									
	1M	-1.60	0.35	0.34	96.8	-0.83	0.39	0.35	101.7	1M	-0.78	0.79	0.37	106.7	-1.14	0.95	0.39	111.7	
	3M	-0.02	0.43	0.45	105.1	-0.13	0.51	0.45	104.3	3M	-0.64	0.72	0.45	104.0	-0.99	0.90	0.45	104.3	
	6M	0.71	0.72	0.50	106.7	0.54	0.69	0.50	106.2	6M	-0.47	0.72	0.47	101.0	-0.80	0.91	0.48	102.8	
	12M	1.53	0.57	0.57	105.4	1.25	0.68	0.57	104.8	12M	-0.19	0.83	0.51	93.7	-0.52	1.00	0.53	96.7	
	24M	1.65	0.69	0.71	104.8	1.95	0.72	0.72	106.7	24M	0.17	1.02	0.58	85.1	-0.14	1.12	0.60	88.5	
	36M	1.63	1.23	0.85	108.5	1.73	1.18	0.85	108.5	36M	0.39	1.14	0.64	81.8	0.11	1.20	0.72	84.77	
	48M	1.68	1.12	0.90	105.3	1.69	1.22	0.93	108.1	48M	0.54	1.23	0.69	80.5	0.29	1.27	0.72	83.4	
	60M	1.60	0.87	0.96	104.7	1.54	0.88	0.96	104.5	60M	0.65	1.29	0.72	78.9	0.43	1.33	0.75	81.8	
Panel B: GBP	*AR-VAR*									*AVG-VAR*									
	1M	-2.42	0.59	0.29	102.5					1M	-1.68	0.16	0.26	93.2					-1.05
	3M	-1.59	0.69	0.28	93.8					3M	-0.92	0.13	0.29	95.3					-0.73
	6M	-0.74	0.85	0.34	101.0					6M	-0.16	0.19	0.34	100.4					0.22
	12M	0.13	0.97	0.42	105.3					12M	0.60	0.27	0.41	103.0					0.96
	24M	0.65	0.94	0.52	105.1					24M	1.00	0.33	0.51	102.8					1.70
	36M	0.75	0.92	0.58	104.6					36M	1.05	0.36	0.56	101.0					1.90
	48M	0.77	0.92	0.60	102.7					48M	1.05	0.38	0.58	99.0					2.13
	60M	0.77	0.93	0.63	102.9					60M	1.04	0.39	0.61	99.4					1.95
	MA-VAR									*UF-VAR*									
	1M	-1.28	0.29	0.27	97.0					1M	-0.26	0.52	0.30	106.8					
	3M	-0.97	0.52	0.29	97.1					3M	-0.12	0.49	0.33	109.4					
	6M	-0.06	0.21	0.34	100.9					6M	0.03	0.46	0.36	108.1					
	12M	0.63	0.36	0.42	104.3					12M	0.19	0.41	0.42	105.0					
	24M	1.07	0.33	0.51	102.9					24M	0.35	0.36	0.49	98.5					
	36M	1.05	0.48	0.56	101.9					36M	0.43	0.38	0.52	94.8					
	48M	1.06	0.62	0.58	98.8					48M	0.48	0.42	0.54	92.0					
	60M	0.93	0.45	0.62	100.7					60M	0.52	0.45	0.56	91.1					

Note: This table presents the forecasting results using the 4 introduced forecasting models denoted by AR, AVG, MA and UF. The VAR affix denotes the Nelson-Siegel representation used to estimate the factors. The table reports the mean and standard deviations of the forecasted returns in percentages. Furthermore, the mean absolute forecasting errors denoted by MAE are reported in basispoints whereas the relative mean absolute errors RMAE, defined as the MAE divided by the mean absolute return of the corresponding straddle maturity, are reported in percentages. Lastly, the last columns describe the actual delta-hedged straddle returns corresponding to the list of maturities over the full forecasting sample [2003:01-2017:08]. For the USD market both the Yield-Only as the Macro-Yield Nelson-Siegel model is considered, for the GBP market solely the Yield-only.



(a) USD



(b) GBP

Figure 6: Out-of-sample predicted swaption curves.
The subfigures show the yearly predicted swaption curves, forecasted at the beginning of the year, against the actual curves of that year. The years 2004 until 2015 are included for both USD and GBP currency markets.

7 Robustness and Sensitivity Analysis

Throughout this research particular choices and assumptions have been made that evidently have their effect on the obtained results. To strengthen the reliability of conclusions made on the basis of these results and to examine the possible applicability of the Nelson-Siegel model to deviating term structures, this section covers some robustness tests. In particular, I focus on assumptions made in three distinct segments of the thesis, including the curve construction, the curve modeling segment, and the swaption pricing part. More specifically, I first inspect the volatility premium term structure considering different holding periods. Second, I examine the sensitivity of the Nelson-Siegel estimation results to different values of the initial decay parameter. Third, I address the concern that the assumptions behind the Black model can be violated in practice and might affect the effectiveness of my hedges. After these sensitivity analyses, I also assess the implications of a new swaption valuation framework, introduced after the Global Financial Crisis. Since implied volatility data rising from this framework is only available since 2011, no literature has yet applied methods concerning volatility premia on this data.

7.1 The Holding Period

The main focus of this research lies in modeling the volatility risk premium term structure that arises from the rebalancing framework initially constructed by Duyvesteyn and De Zwart (2015). This particular framework yields a different term structure than obtained by earlier studies that apply a hold-to-expiry strategy. Hence, differences in rebalancing strategies can yield different term structures. In this subsection I examine the behavior of the term structure for different holding periods. I consider holding periods of 3-, 6- and 12 months and compare them with the 1-month period returns used throughout this study. Positions are entered in the same manner as the 1-month set-up. That is, I enter a delta-hedged swaption straddle position at the beginning of the month, starting in January 1998 and close it 3, 6 or 12 months later at month-end. At that same day, a new position is entered. I do not consider overlapping samples, which means that for the 3-month set-up I obtain 75 returns, for 6-month I obtain 38 returns, and for the 12-month I obtain 19 returns.

The annualized average volatility premia over maturity using the 3 different rebalancing frequencies are reported in table 12. The column below “1M HP” reports the returns using 1-month holding periods as observed earlier in table 2. In general the longer holding periods replicate the concave upward sloping term structure observed in the 1-month holding period returns, indicating that the modeling approach from this study might also be relevant for returns corresponding to different holding periods. It is difficult to say whether it is more profitable to consider longer holding periods. For longer maturity straddles it seems so, with higher returns for these maturities compared to their 1-month holding period returns. For medium term maturities it does not seem so, since the 1-month holding periods are highest for these maturities. The returns for portfolios that are held till maturity, that is the 1-month straddle for 1M HP, the 3-month straddle for the 3M HP, the 6-month straddle for the 6M HP and the 12-month straddle for the 12M HP, are in line with findings from Low and Zhang (2005) who report negative returns for hold-to-expiry portfolios that increase towards zero for larger maturities. Furthermore, in subsection 3.2 a conjecture was drawn that the premia tend to become positive when delta-hedged straddles are held for a limited amount of time and closed when there is considerable life time left. It can be observed that for straddles getting relatively close to maturity longer holding periods generate lower returns than shorter holding periods. This implies that the market’s demanded compensation of risk starts to decline at some point during the lifetime of the options, yielding decreasing option prices as of this moment and, hence, decreasing returns. Finding the optimal moment to close the position in the delta-hedged option during its lifetime can therefore be very valuable in optimizing volatility premia trading strategies. Essentially, this is finding the moment when the market's demanded compensation for risk is highest.

This limited analysis on holding period sensitivity considers non-overlapping holding periods which all start at the last day of the same months in different years. This results in a relatively small sample of returns that cover similar periods in a year and start each time at the same day of the month. Overlapping samples can broaden the result sample and different starting days throughout the month might account for day-of-the-month sensitivity. The construction of a more reliable rebalancing frequency analysis lies however out of scope of this study. Given the generally higher returns for longer term straddles when the holding period lengthens, future research that more extensively examines the term structure's sensitivity to the holding period is strongly supported.

Table 12: Holding period sensitivity.

Maturity	USD				GBP			
(m)	1M HP	3M HP	6M HP	12M HP	1M HP	3M HP	6M HP	12M HP
1	-1.42	-	-	-	-0.94	-	-	-
3	-0.32	-0.66	-	-	-0.86	-0.65	-	-
6	0.43	0.09	-0.27	-	0.05	-0.13	-0.09	-
12	1.28	0.93	0.84	-0.11	0.75	0.57	0.73	-0.03
24	1.49	1.31	1.50	1.35	1.41	1.16	1.30	1.04
36	1.67	1.55	1.86	1.76	1.57	1.36	1.67	1.60
48	1.72	1.64	2.02	2.05	1.85	1.59	1.98	1.88
60	1.73	1.69	2.17	2.19	1.65	1.47	1.90	1.87

Note: This table presents the average annualized volatility premia over maturity for 3 different holding periods in the USD and GBP currency. 1M HP, 3M HP and 6M HP denote the 1-, 3- and 6-month holding period, respectively. The premia are given in percentages.

7.2 The Loading Parameter

In the methodology section that described the Nelson-Siegel 2-step approach I chose to set the medium term of the maturity cross-section at 12 months, resulting in a λ of 0.1494 when maximizing the curvature loading at this term. The estimated λ's in the Kalman filter were all lower than the chosen λ, indicating that the true medium term in swaptions might be somewhat longer than 12 months. To test whether results of the basic Nelson-Siegel model are sensitive to the initially chosen λ, I examine the average in-sample fit of the models, when considering a range of λ's that are obtained when maximizing the curvature loadings for the terms: 6, 14, 16, 18, 20, 22, 24, 30 and 36 months.

Results of the robustness analysis on λ sensitivity are reported in table 13. Both the 2-step and the VAR variant estimated using the Kalman filter are considered. First thing that stands out is the complete insensitivity of the Kalman filter to the initial λ in both currencies. The 2-step approach who's linear regressions naturally depend on the inserted λ shows that the mean absolute errors for the full range of initial λ's is quite similar. The chosen λ of 0.1494 seems very reasonable in the sense that it results in the lowest MAE in the USD market, and little above the lowest MAE in the GBP market. All in all, the Nelson-Siegel estimation seems robust to choices for the initial loading parameter.

Table 13: Initial λ sensitivity.

Implied		USD			GBP		
Term	Initial λ	2-step MAE	Estimated λ	SS-VAR MAE	2-step MAE	Estimated λ	SS-VAR MAE
12	0.1494	6.51	0.1016	6.61	5.90	0.1483	6.42
6	0.2989	7.13	0.1016	6.61	6.43	0.1483	6.42
14	0.1281	6.52	0.1016	6.61	5.87	0.1483	6.42
16	0.1121	6.56	0.1016	6.61	5.86	0.1483	6.42
18	0.0996	6.59	0.1016	6.61	5.86	0.1483	6.42
20	0.0897	6.64	0.1016	6.61	5.88	0.1483	6.42
22	0.0815	6.71	0.1016	6.61	5.92	0.1483	6.42
24	0.0747	6.81	0.1016	6.61	5.99	0.1483	6.42
30	0.0598	6.93	0.1016	6.61	6.07	0.1483	6.42
36	0.0498	7.12	0.1016	6.61	6.19	0.1483	6.42

Note: This table reports the mean absolute measurement errors (MAE) averaged over all maturities, expressed in basispoints, under different values of initial loading parameter λ. In particular, the 2-step approach and the state space VAR set-up is considered. The initial λ's from the second column are obtained when the curvature factor of the Nelson-Siegel model is maximized at the implied term, given in the first column. The first row of the table reports the results under initial $\lambda = 0.1494$, which is used throughout this study.

7.3 A New Pricing Model

The assumption of lognormality in interest rates has been the standard in a lot of literature on rate dynamics during the 1980s and 1990s. This is also an important assumption underlying the Black model. However, recent literature questions the soundness of the model as rates tend to move more in absolute values than in relative values, which advocates for normality rather than lognormality. Levin (2004) argues that if market participants believe in lognormality, there would be little reason for the implied volatilities to change with the option's strike. Levin's diagnosed volatility skew however testifies against the lognormality assumption and proposes the assumption of normality.

Due to these speculations about the Black model, market participants have started to valuate swaptions differently. Since swaption prices are still mainly quoted in Black implied volatility, prices calculated using different pricing models are still transformed to implied volatilities using the Black model. This entails that market prices might be correct, but that the Greeks calculated through the Black model might be different than actual sensitivities in the market, as they are constructed relying on a set of assumptions that might not match with practice.

Possible incorrectly calculated Greeks can affect the hedges in my delta-hedged swaption straddle portfolios and influence the results of this thesis. As part of the robustness section, I will therefore examine the return distribution of swaption straddles using the Gaussian Black model, which is a model that relates to the Black model but assumes the by Levin (2004) suggested normal distribution for interest rate changes. Normally distributed forward swap rates can be modeled by means of an arithmetic Brownian motion, given by

$$dF = \mu dt + \sigma dW, \tag{25}$$

with W a Wiener process. As this price process implies that prices can become negative, pricing methods using this process have been found unreasonable for a long time. However, recently we have witnessed the first negative rates on government bonds of a number of European countries.

By following Hull (2006) and using Alexander *et al.* (2012) I derive the closed form pricing formulas for swaptions according to this model.[12] The value of the payer leg is described as

$$V_{P,abm,t} = PA[\sigma\sqrt{\tau}\phi(d_\tau) - (K - F_t)\Phi(-d_\tau)], \tag{26}$$

and the price of receiver leg as

$$V_{R,abm,t} = PA[\sigma\sqrt{\tau}\phi(d_\tau) + (K - F_t)\Phi(d_\tau)], \tag{27}$$

with

$$d_\tau = \frac{K - F_t}{\sigma\sqrt{\tau}}, \tag{28}$$

and a corresponding swaption straddle delta given by

$$\Delta_{abm,t} = \frac{\delta S_{abm,t}}{\delta F_t} = PA[\Phi(-d_\tau) - \Phi(d_\tau)], \tag{29}$$

where F_t denotes the forward swap rate at t, K the strike rate, Φ the cumulative normal distribution function, P the swaption principal and A the annuity factor as introduced in subsection 2.2.

An essential difference between the Gaussian and regular Black model besides the stochastic process of the underlying, is the character of the included implied volatility. The Black implied volatility is essentially interpreted as a standard deviation in future relative returns. The Gaussian Black model assumes that changes in rates are independent of their level and hence considers an implied volatility that is interpreted as the standard deviation in absolute changes of the swap rate. This implied volatility is referred to as normal or basis point volatility and is approximately equal to the Black implied volatility times the forward swap rate corresponding to the same time period.

The volatility risk premia in USD swaptions under the Gaussian Black model, denoted by ABM referring to the assumed arithmetic Brownian motion, are presented in the second column of table 14. The decremental upward sloping pattern in the premia resembles the pattern in the premia under the Black model - described in table 2 in section 3 - but is somewhat more negatively skewed, with a lower maximum value for the 60-month maturity and positive premia

[12]The full derivation is provided in appendix F.

as of the 12-month maturity instead of the 6-month maturity observed for the Black model. In general these results suggest that the shape of the swaption curve is not affected by assumptions on the underlying, confirming the strong presence of a volatility risk premium term structure in swaptions.

Table 14: Pricing and data sensitivity.

Maturity (m)	USD LIBOR ABM (%)	GBP OIS GBM (%)	GBP OIS ABM (%)	GBP LIBOR GBM (%)
1	-1.44	-0.09	-1.16	-0.12
3	-0.66	-0.70	-1.41	-0.69
6	-0.21	0.46	-0.78	0.38
12	0.26	1.44	-0.57	1.56
24	0.37	3.09	0.46	2.90
36	0.54	3.65	0.80	3.65
48	0.62	4.33	1.39	4.38
60	0.70	4.22	1.26	3.97

Note: This table presents the average annualized delta-hedged swaption straddle portfolio returns using different underlying assumptions and data compared to the main research. The table reports the USD risk premia using the Gaussian Black model over the period 1998:01-2016:08, the GBP risk premia using OIS implied volatilities in the Black model over the period 2011:01-2016:08 and the GBP risk premia using OIS implied volatility in the Gaussian Black model over the period 2011:01-2016:08, in that order. As a reference, the last column contains premia over the sample 2011:01-2016:08 using LIBOR rates and the Black model.

7.4 OIS-based Implied Volatilities

The Global Financial Crisis of 2008 initiated a lot of adjustments in regulations and pricing methods in various corners of the financial world. As well in the swaption market. Prior to the crisis, swaption dealers relied on a single curve to forecast rates, depending on an underlying index such as the LIBOR, and to discount cash flows. After changes in market conditions and regulations a new multi-curve pricing framework is introduced, that uses separate forecasting and discounting curves. The market in particular evolved towards cash flow discounting using Overnight Index Swap (OIS) rates. One of the leading international swaption dealers ICAP for instance started publishing OIS-based implied volatilities rates in 2010, next to the LIBOR-based volatilities.

To test whether this new and fast growing way of swaption implied volatility valuation also embeds a volatility premium term structure as observed in the LIBOR-based volatilities, I create similar delta-hedged swaption portfolios and examine their return dynamics. I do this using data over the period January 2011 through August 2016 for GBP swaptions. I furthermore consider both the Black and Gaussian Black pricing models.

The third and fourth column of table 14 report the average volatility premia in the GBP market embedded in OIS-based implied volatilities under the Black model, denoted by GBM, and the Gaussian Black model, again denoted by ABM. As a point of reference the premia using LIBOR-based volatilities under the Black model for the same sample are added in the last column. The premium under the Black model using OIS rates moves in line with the premium using LIBOR rates over the same period. In this particular period the curve moves however less smooth with decreases in the 3- and 60-month maturity premium with respect to their shorter maturity predecessors. This can be related to the smaller sample. Earlier we have seen that for LIBOR-based volatilities the curve can also vary considerably in smaller subsamples. The little difference and same shape of the curve under the different implied volatility types suggest that volatility risk premia are present in both datasets. Furthermore, using the Gaussian Black model I again find a similar pattern in returns which is more negative skewed than the returns under the Black model, repeating the earlier observed curve behavior under the former model.

7.5 Conclusions: Robustness and Sensitivity Analysis

This section mainly examined the behavior of the curve under alternative underlying assumptions and a different swaption market environment. The results showed that irrespective of underlying pricing assumptions, nature of implied volatility estimation or holding period, the estimated term structures generally mirror the concave upward sloping term structure found using the 1-month holding period in combination with the Black model and LIBOR based implied volatilities. It therefore seems likely that the Nelson-Siegel term structure model that has been positively tested on the latter data combination is also capable of capturing alternative term structures. However, since conclusions on especially the returns using different holding periods and OIS-based implied volatilities are based on a limited amount of returns and data, further research on these subjects and in particular the applicability of the Nelson-Siegel model on their curves is supported.

8 Conclusion

Studies of Low and Zhang (2005) and Duyvesteyn and De Zwart (2015) are the first to document and analyze the maturity effect in volatility risk premia. These studies focus mainly on trading strategies applied on the premium term structure and pay only limited attention to defining the factors that drive the curve, nor attempt to model it. My study contributes to the literature by introducing a framework that extensively maps, models and examines the term structure in swaption volatility risk premia by means of a yet unconsidered approach.

The framework in particular consists of three parts. The first part describes a preliminary analysis on particular stylized facts of the term structure that yields new insights in its behavior over time. The second and core segment of this study anticipates on the similarities between the observed average term structure and the yield curve by extending the application of the Nelson-Siegel model as described by Diebold and Li (2006) to volatility premium term structure modeling. This curve modeling part comprises in-sample fitting, factor identification and out-of-sample forecasting. To the purpose of model estimation I consider linear estimation techniques and a state space optimization using a Kalman filter. The third part assesses the term structure's sensitivity to different assumptions and market environments that could possibly have implications on the applicability of the Nelson-Siegel model.

I find that the for yield curve modeling purposes constructed Nelson-Siegel term structure model is able to accurately fit the average volatility premium term structure over a sample of almost 19 years, advocating for the use of this method when it comes to in-sample modeling of the volatility risk premium term structure. Both considered estimation techniques yield similar results. The three estimated latent factors of the model relate to the level, slope and curvature of the term structure, resembling results obtained by Diebold and Li (2006). The dynamics of these unobservable factors are found to have a strong link to dynamics in swaption underlying pricing factors such as implied volatility and the forward swap rate. Macroeconomic information seems to form an additional driver of the factors, but only to a limited extent. When extending the standard Nelson-Siegel model with macroeconomic input, in- and out-of-sample results do not improve.

The obtained term structure using the rebalancing scheme as introduced by Duyvesteyn and De Zwart (2015) furthermore seems to represent the general shape of the volatility risk premium over maturity, that is concave, upward sloping. Sensitivity analyses that apply different holding periods, use an alternative pricing model or consider a dataset of implied volatilities rising from a newly introduced swaption valuation framework, did not result in deviating shapes of the average curve. Hence, it seems likely that the Nelson-Siegel model is able to model the cross-section of premia from these alternative curves as well.

The new angle of volatility premia research proposed by this study leaves a lot of room for further research. Given that the alternative curves received limited attention in this research, deeper assessments on these curves are supported. Another extension lies in option moneyness. This research considered datasets consisting of ATM implied volatilities only. Besides time to maturity, the moneyness of an option changes over its lifetime as well. The fact that implied volatilities tend to differ over option moneyness - volatility smile or smirk - might affect the swaption curve. Data availability of implied volatilities that are not ATM is however very limited, forming a limitation to this research. Furthermore, another limitation to this study lies in the randomness and low persistence in the volatility premia, resulting in little autocorrelation in the Nelson-Siegel factors. Auto-regressive set-ups such as the dynamic Nelson-Siegel model become unable to accurately forecast under these circumstances. Premia predictions are improved when focus shifts to modeling the exposed swaption curve drivers instead of the factors themselves, but still do not form proper approximations of future swaption curve shapes. Since accurately forecasting the swaption curve can result in improving volatility trading strategies, I strongly encourage future research on this topic. In particular, modeling techniques that are known for their ability to capture return timeseries dynamics might deliver valuable improvements on the models applied in this study.

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A Delta-Hedged Option Portfolio Derivation

To prove and understand this mathematical relation between delta-hedged returns and the volatility risk premium I first recall the geometric Brownian motion given in equation (1). Furthermore, I relax the assumption of constant volatility, which is practically seen a questionable assumption, and let the volatility follow a stochastic process. This system of equations is now given by

$$dF_t = \mu F_t dt + \sigma_t F_t dW_t, \tag{30}$$

$$d\sigma_t = \theta_t dt + \eta_t dV_t, \tag{31}$$

with W and V Wiener processes.

Let S_t denote the price of a European swaption straddle with underlying forward swap rate F_t. By Ito's lemma, S_t follows a diffusion process expressed as:

$$dS_t = \frac{\delta S_t}{\delta F_t} dF_t + \frac{\delta S_t}{\delta \sigma_t} d\sigma_t + \Big(\frac{\delta S_t}{\delta t} + \frac{1}{2}\sigma_t^2 F_t^2 \frac{\delta^2 S_t}{\delta F_t^2} + \frac{1}{2}\eta_t^2 \frac{\delta^2 S_t}{\delta \sigma_t^2} + \rho \eta_t \sigma_t F_t \frac{\delta^2 S_t}{\delta F_t \delta \sigma_t}\Big) dt, \tag{32}$$

where ρ denotes the correlation coefficient between the random Wiener innovations. By integrating both sides over a period τ the price change dS_t can be written as a stochastic integral equation

$$\begin{aligned} S_{t+\tau} = S_t + \int_t^{t+\tau} \frac{\delta S_u}{\delta F_u} dF_u + \int_t^{t+\tau} \frac{\delta S_u}{\delta \sigma_u} d\sigma_u + \\ \int_t^{t+\tau} \Big(\frac{\delta S_u}{\delta u} + \frac{1}{2}\sigma_u^2 F_u^2 \frac{\delta^2 S_u}{\delta F_u^2} + \frac{1}{2}\eta_u^2 \frac{\delta^2 S_u}{\delta \sigma_u^2} + \rho \eta_u \sigma_u F_u \frac{\delta^2 S_u}{\delta F_u \delta \sigma_u}\Big) du. \end{aligned} \tag{33}$$

Standard arbitrage arguments further restrict the straddle price S_t to satisfy the following partial differential equation:

$$\frac{1}{2}\sigma^2 F^2 \frac{\delta^2 S}{\delta F^2} + \frac{1}{2}\eta^2 \frac{\delta^2 S}{\delta \sigma^2} + \rho \eta \sigma F \frac{\delta^2 S}{\delta F \delta \sigma} + rF\frac{\delta S}{\delta F} + (\theta - \lambda)\frac{\delta S}{\delta \sigma} + \frac{\delta S}{\delta t} - rS = 0, \tag{34}$$

where r denotes the risk free interest rate. The term λ in this equation represents the market price of risk, or, the volatility risk premium. Now by combining equations (33) and (34) I obtain:

$$S_{t+\tau} = S_t + \int_t^{t+\tau} \frac{\delta S_u}{\delta F_u} dF_u + \int_t^{t+\tau} \frac{\delta S_u}{\delta \sigma_u} d\sigma_u + \int_t^{t+\tau} \Big(rS_u - rF_u \frac{\delta S_u}{\delta F_u} - (\theta_u - \lambda_u)\frac{\delta S_u}{\delta \sigma_u}\Big) du. \tag{35}$$

Substituting equation (31) in equation (35) now results in:

$$S_{t+\tau} = S_t + \int_t^{t+\tau} \frac{\delta S_u}{\delta F_u} dF_u + \int_t^{t+\tau} r\Big(S_u - F_u \frac{\delta S_u}{\delta F_u}\Big) du + \int_t^{t+\tau} \lambda_u \frac{\delta S_u}{\delta \sigma_u} du + \int_t^{t+\tau} \eta_u \frac{\delta S_u}{\delta \sigma_u} dV_u. \tag{36}$$

Note here the term $\frac{\delta S}{\delta F}$, which is the delta of the straddle and will hereafter be denoted by Δ. The delta of a swaption straddle is defined as follows:

$$\Delta = PA[\Phi(d_1) - \Phi(-d_1)]. \tag{37}$$

In this research I consider a dynamically delta-hedged portfolio consisting of a long swaption straddle position and a spot position in the underlying forward rate. The position in the forward rate is rebalanced on a daily basis to hedge the delta exposure of the long straddle. Literature defines the excess return of a dynamically delta-hedged portfolio as:

$$\Pi_{t,t+\tau} \equiv S_{t+\tau} - S_t - \int_t^{t+\tau} \Delta_u dF_u - \int_t^{t+\tau} r\big(S_u - \Delta_u F_u\big) du. \tag{38}$$

The first part of the return equation defines the return on the straddle over period τ, where the second part represents the dynamic delta-hedged returns. The third part expresses the net cash investment which either pays or earns the interest rate, depending on the values of the swaption and the hedge throughout the holding period.

From equations (36) and (38) it can be deduced that the portfolio returns can also be expressed as:

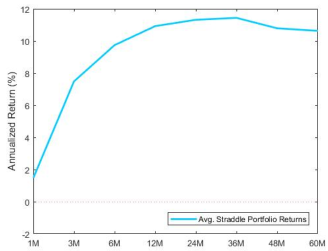
$$\Pi_{t,t+\tau} = \int_t^{t+\tau} \lambda_u \frac{\delta S_u}{\delta \sigma_u} + \int_t^{t+\tau} \eta_u \frac{\delta S_u}{\delta \sigma_u} dV_u. \tag{39}$$

The portfolio returns are now described in terms of a volatility risk premium part and a term called the Ito integral. From the martingale property of the Ito integral we know that its expected value equals zero. The expected value of the delta-hedged returns is therefore given by:

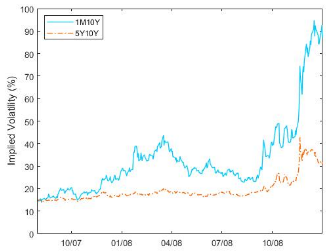
$$E(\Pi_{t,t+\tau}) = \int_t^{t+\tau} E\big(\lambda_u \frac{\delta S_u}{\delta \sigma_u}\big) du. \tag{40}$$

Equation (40) implies that the expected returns on the portfolio depend on whether or not volatility risk is priced through λ. Hence, significant non-zero average returns would support the existence of a volatility risk premium in swaption straddles.

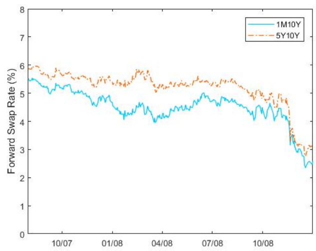
B Swaption Curve Behavior



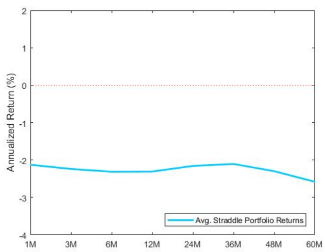
(a) Increasing IV - Swaption Curve



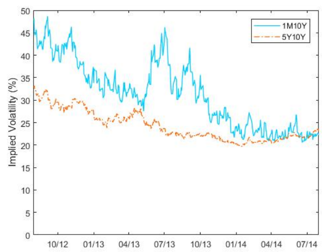
(b) Increasing IV - IV



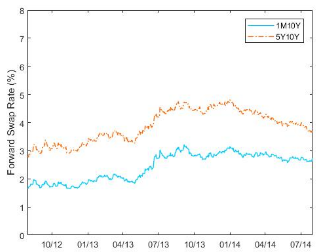
(c) Increasing IV - FSR



(d) Decreasing IV - Swaption Curve



(e) Decreasing IV - IV



(f) Decreasing IV - FSR

Figure 7: Swaption curve in subsamples.

These figures show the notional scaled average annualized delta-hedged swaption straddle portfolio returns in two specific samples in the USD together with the implied volatility and forward swap rate series over the same samples. The first subsample spans the period 2007:08-2008:12 and represents a period of increasing volatility. The second subsample spans the period 2012:08-2014:07 and represents a decreasing volatility regime.

C Swaption Curve Modeling

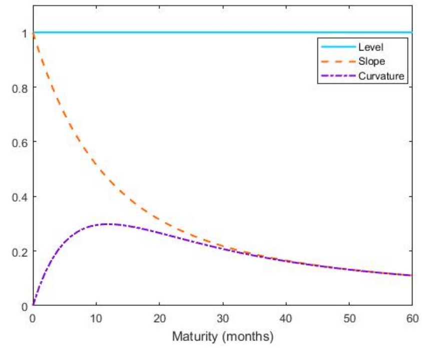
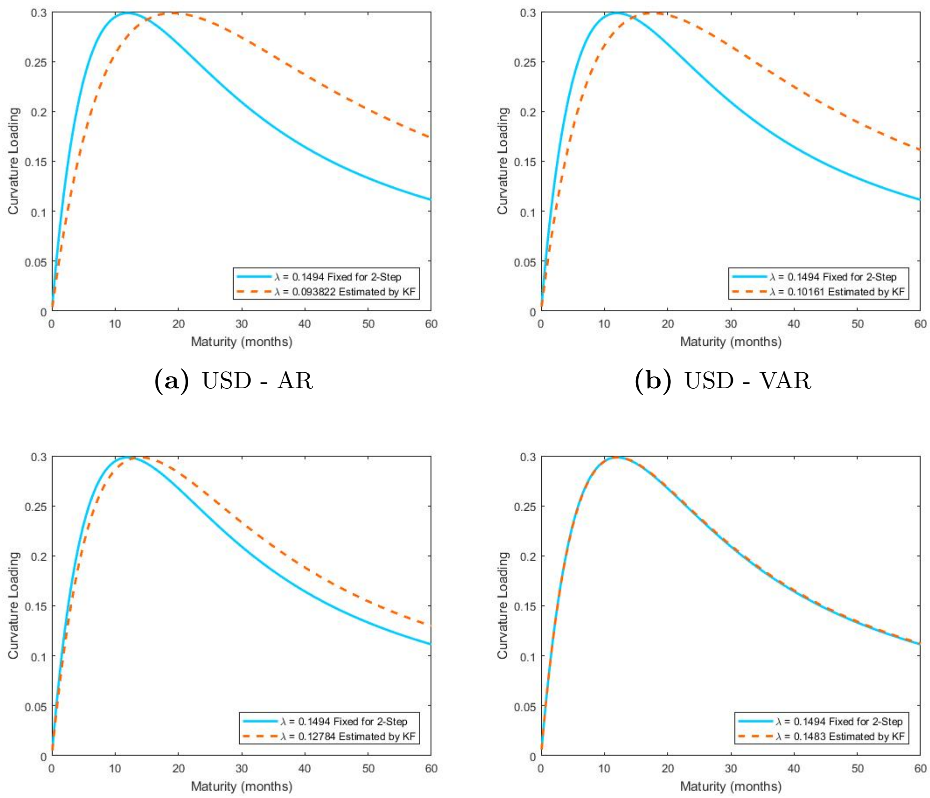


Figure 8: Nelson-Siegel factor loadings.
The associated loadings of β_1, β_2 and β_3 in the Nelson-Siegel model are equal to 1, $(1 - e^{-\lambda_t \tau})/\lambda_t \tau$, and $(1 - e^{-\lambda_t \tau})/\lambda_t \tau - e^{-\lambda_t \tau}$, respectively, where τ denotes maturity and λ_t is fixed at 0.1494.



(a) USD - AR **(b)** USD - VAR

(c) GBP - AR **(d)** GBP - VAR

Figure 9: Nelson-Siegel Curvature loadings.
These figures show the Nelson-Siegel curvature loading under the initial loading parameter of 0.1494 against the loading under the estimated λ by the Kalman filter in the USD and GBP market, for the AR and VAR representation.

D Kalman Filter Derivation and Estimation

To start, recall the state space set of equations (18), (19) and (20), described in section 4.1.2:

$$\Pi_t = \Lambda(\lambda)\beta_t + \varepsilon_t, \tag{41}$$

$$\beta_t = (I - \Phi)\mu + \Phi\beta_{t-1} + \eta_t, \tag{42}$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim NID\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_\varepsilon & 0 \\ 0 & \Sigma_\eta \end{bmatrix} \right), \tag{43}$$

for $t = 1, ..., T$, where equation (42) is rewritten by relocating μ. Furthermore, recall the initial condition $\beta_1 \sim N(\mu, \Sigma_\beta)$ with $\Sigma_\beta - \Phi\Sigma_\beta\Phi' = \Sigma_\eta$. Besides the unobservable factors, the parameters in $\Lambda, \Phi, \mu, \Sigma_\varepsilon$ and Σ_η are unknown. Since the Kalman filter requires the parameters to be known to construct estimates of the latent factors, which I refer to as b_t, I initialize them following Koopman *et al.* (2012). These initialization steps will be described later in this appendix.

The Kalman filter is a recursive algorithm that provides an optimal forecast of β_t given the information known at time $t-1$. The filter is based on the following property of variables z_1 and z_2 that have a joint normal distribution:

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \sim N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \right). \tag{44}$$

Now, the distribution of z_2 conditional on z_1 is $N(m, \Sigma)$, where

$$m = \mu_2 + \Omega_{21}\Omega_{11}^{-1}(z_1 - \mu_1), \tag{45}$$

$$\Sigma = \Omega_{22} - \Omega_{21}\Omega_{11}^{-1}\Omega_{12}. \tag{46}$$

The Kalman filter essentially applies this result to each observation $t = 1, 2, ...$ in a recursive manner, where Π_t and b_t adopt the role of z_1 and z_2, respectively.

Now consider the optimal forecast of β_t given all information up to time $t-1$, described as

$$b_{t|t-1} = E[\beta_t | I_{t-1}], \tag{47}$$

with I_{t-1} representing the information set at $t-1$. The corresponding conditional variance of β_t, denoted by B_t, is defined as

$$B_{t|t-1} = E[(\beta_t - b_{t|t-1})(\beta_t - b_{t|t-1})']. \tag{48}$$

Given the information set I_{t-1} we are able to obtain optimal forecast $b_{t|t-1}$ and the associated variance $B_{t|t-1}$. To construct the next forecast $b_{t+1|t}$ we use the state equation (42). It follows that

$$\begin{aligned} b_{t+1|t} &= E[\beta_{t+1}|I_t] \\ &= E[(I-\Phi)\mu + \Phi\beta_t + \eta_{t+1}|I_t] \\ &= (I-\Phi)\mu + \Phi E[\beta_t|I_t], \end{aligned}$$

where in the last step I assume Φ and μ to be known at t and the expected value of η_t to be zero for all t. The results suggest that to construct forecast $b_{t+1|t}$ I first need to obtain $b_{t|t} \equiv E[\beta_t|I_t]$.

Using the measurement equation (41), it follows that the optimal forecast for Π_t given I_{t-1} is given by

$$E[\Pi_t|I_{t-1}] = \Lambda b_{t|t-1}.$$

The corresponding forecast error v_t is equal to

$$\begin{aligned} v_t &= \Pi_t - E[\Pi_t|I_{t-1}] \\ &= (\Lambda\beta_t + \varepsilon_t) - (\Lambda b_{t|t-1}) \\ &= \Lambda(\beta_t - b_{t|t-1}) + \varepsilon_t. \end{aligned} \tag{49}$$

The variance of the forecast error is subsequently written as

$$F_t = \Lambda B_{t|t-1}\Lambda' + \Sigma_\varepsilon. \tag{50}$$

Note that this is also the variance of Π_t conditional on I_{t-1}.

With the information gathered I am able to construct the fundamental joint distribution of the Kalman filter of Π_t and β_t. This distribution is described as

$$\begin{pmatrix} \Pi_t \\ \beta_t \end{pmatrix} \sim N\left(\begin{bmatrix} \Lambda b_{t|t-1} \\ b_{t|t-1} \end{bmatrix}, \begin{bmatrix} F_t & \Lambda B_{t|t-1} \\ B_{t|t-1}\Lambda' & B_{t|t-1} \end{bmatrix} \right). \tag{51}$$

Now using the results on the joint normal distribution given in (45) and (46), it follows that $\beta_t|\Pi_t, I_{t-1} \sim N(b_{t|t}, B_{t|t})$, where

$$b_{t|t} = b_{t|t-1} + B_{t|t-1}\Lambda' F_t^{-1} v_t, \tag{52}$$

$$B_{t|t} = B_{t|t-1} - B_{t|t-1}\Lambda' F_t^{-1}\Lambda B_{t|t-1}. \tag{53}$$

This recursive step illustrated by the set of equations is known as the filtering step. Now we obtained $b_{t|t}$ and $B_{t|t}$ through this filtering step the $t+1$ forecast can be constructed by

$$b_{t|t+1} = (I-\Phi)\mu + \Phi b_{t|t}, \tag{54}$$

$$B_{t|t+1} = \Phi B_{t|t}\Phi' + \Sigma_\eta, \tag{55}$$

and so the range of unobservable factors β_t, $t = 1, .., T$, can be estimated.

This brings me to the initialization step. As described in section 4.1.2, I use the results of the 2-step approach as initialization parameters for the state space approach. In particular, I set $b_{1|0} = \mu$, with μ the mean vector of the 3 estimated factors from the 2-step approach, and

$B_{1|0} = \Sigma_\beta$. Σ_β is not directly observable, but can be derived by solving $\Sigma_\beta - \Phi\Sigma_\beta\Phi' = \Sigma_\eta$, where Σ_η is estimated in the 2-step.

Besides μ and Σ_η, the parameters in the coefficient matrix Φ, Σ_ε and λ are also initialized using estimated or introduced values from the 2-step approach and collected in parameter vector ψ. Estimation of ψ is based on a numerical optimization of the log-likelihood function that is constructed via the conditional distribution of the prediction errors and given by

$$l(\psi) = -\frac{nT}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' F_t^{-1} v_t. \tag{56}$$

After initialization of ψ, $l(\psi)$ can be evaluated by the Kalman filter. To find the optimal parameter estimates of ψ that optimize the log-likelihood function, a quasi-newton optimization method is employed.

E Explaining the Factors: Linear Regressions

Table 15: Regressor definitions.

Symbol	Name	Definition
Panel A: Underlying Variables		
σ_{lvl}	Implied volatility level	5Y10Y implied volatility.
σ_{slope}	Implied volatility slope	5Y10Y - 1M10Y implied volatility.
σ_{curv}	Implied volatility curvature	[1Y10Y - 1M10Y] - [5Y10Y - 1Y10Y] implied volatility
F_{lvl}	Forward swap rate level	5Y10Y forward swap rate.
F_{slope}	Forward swap rate slope	5Y10Y - 1M10Y forward swap rate.
F_{curv}	Forward swap rate curvature	[1Y10Y - 1M10Y] - [5Y10Y - 1Y10Y] forward swap rate
r_{3M}	Short-term interest rate	3M interest rate (LIBOR).
r_{120M}	Long-term interest rate	10Y interest rate (swap rate).
r_{slope}	Interest rate slope	10Y - 3M interest rate.
r_{curv}	Interest rate curvature	[1Y - 3M] - [10Y - 1Y] interest rate.
Real-IV	Realized minus implied volatility	1M10Y implied volatility - 1M10Y realized volatility at closing. This variable represents the gamma term of the Taylor expansion described in equation (23).
Panel B: Macroeconomic Variables		
BR	Bank Rate	Country specific lending rates (USD: Effective federal funds rate, GBP: Bank of England official bank rate).
CCI	Consumer Confidence Index	Indicator that measures the degree of optimism on the state of the economy that consumers are expressing through their activities of savings and spending (USD: Conference board consumer confidence, GBP: GFK UK CCI, revised).
CPI	Consumer Price Index	Measure of prices paid by consumers for a market basket of consumer goods and services (USD: US CPI Urban Consumers, GBP: UK CPI EU Harmonized).
GY	10-years Government Bond Yield	Annualized return on the country specific 10-years government bond.
IP	Industrial Production	Measure of the output of the industrial sector of the economy (revised).
MS	Money Supply	Measure of the entire stock of currency and other liquid instruments circulating in a country's economy (USD: M2, GBP: M4).
SI	Stock Index	Country specific stock index (USD: S&P500 Index, GBP: FTSE100 Index).

Note: This table reports the definitions of the variables used in the linear regressions of subsection 5.1 and 5.2. Panel A describes the variables stemming from the swaption pricing formula, Panel B describes the considered macroeconomic variables.

Table 16: Regression coefficients; dependent variable β_1.

	β_1				Internal								External					
		c	$\Delta\sigma_{lvl}$	ΔF_{lvl}	Δr_{3M}	Δr_{120M}	Real-IV	$\Delta\sigma_{slope}$	ΔF_{slope}	ΔGY	ΔBR	ΔCCI	ΔCPI	ΔIP	ΔMS	ΔSI	R^2	AIC
Panel A: USD	$NS-OLS$	-0.008	0.125***	4.942**	0.141*	-6.462***	0.000	0.007***	-5.708***								0.973	-0.048
		(0.026)	(0.003)	(2.119)	(0.076)	(2.142)	(0.001)	0.001	(2.132)									
		-0.051	0.126***	5.754***	0.120	-7.626***	-0.001	0.007***	-6.494***	0.378	0.130	0.002	0.035	0.064**	0.044	-0.003	0.975	-0.041
		(0.039)	(0.003)	(2.142)	(0.094)	(2.205)	(0.001)	(0.001)	(2.154)	(0.257)	(0.123)	(0.002)	(0.059)	(0.025)	(0.046)	(0.004)		
	$NS-AR$	-0.019	0.131***	5.873**	0.224**	-7.464***	0.001	0.007***	-7.330***								0.965	0.346
		(0.032)	(0.003)	(2.581)	(0.092)	(2.610)	(0.001)	(0.001)	(2.597)									
		-0.072	0.131***	6.373**	0.118	-8.241***	0.000	0.006***	-7.806***	0.301	0.278*	0.004*	0.070	0.065**	0.058	-0.008	0.968	0.331
		(0.047)	(0.004)	(2.581)	(0.113)	(2.656)	(0.001)	(0.001)	(2.596)	(0.310)	(0.148)	(0.002)	(0.071)	(0.030)	(0.055)	(0.005)		
	$NS-VAR$	-0.021	0.132***	6.329**	0.240***	-7.952***	0.000	0.007***	-7.623***								0.968	0.282
		(0.031)	(0.003)	(2.499)	(0.089)	(2.527)	(0.001)	(0.001)	(2.515)									
		-0.069	0.132***	6.903***	0.158	-8.833***	0.000	0.007***	-8.167***	0.323	0.222	0.004*	0.061	0.070**	0.048	-0.007	0.970	0.268
		(0.046)	(0.003)	(2.500)	(0.110)	(2.573)	(0.001)	(0.001)	(2.515)	(0.300)	(0.143)	(0.002)	(0.069)	(0.029)	(0.053)	(0.005)		
		c	$\Delta\sigma_{lvl}$	ΔF_{lvl}	Δr_{3M}	Δr_{120M}	Real-IV	$\Delta\sigma_{slope}$	ΔF_{slope}	ΔGY	ΔBR	ΔCCI	ΔCPI	ΔIP	ΔMS	ΔSI	R^2	AIC
Panel B: GBP	$NS-OLS$	-0.005	0.105***	-0.320	0.398***	-0.704	-0.003**	0.005***	0.230								0.935	0.165
		(0.027)	(0.003)	(2.261)	(0.094)	(2.277)	(0.001)	(0.001)	(2.272)									
		-0.006	0.107***	0.330	0.166	-1.697	-0.003**	0.004***	-0.396	0.423***	0.421***	-0.001	-0.50	0.026	0.024	0.002	0.944	0.086
		(0.030)	(0.003)	(2.165)	(0.139)	(2.181)	(0.001)	(0.001)	(2.176)	(0.098)	(0.160)	(0.005)	(0.048)	(0.019)	(0.019)	(0.005)		
	$NS-AR$	-0.028	0.102***	1.258	0.477***	-2.304	-0.003**	0.002	-1.279								0.930	0.191
		(0.027)	(0.003)	(2.290)	(0.095)	(2.307)	(0.001)	(0.002)	(2.301)									
		-0.023	0.104***	1.786	0.294**	-3.171	-0.002*	0.002	-1.749	0.406***	0.339**	0.003	-0.021	0.023	0.007	0.002	0.937	0.144
		(0.031)	(0.003)	(2.229)	(0.143)	(2.245)	(0.001)	(0.002)	(2.240)	(0.101)	(0.165)	(0.005)	(0.049)	(0.019)	(0.020)	(0.005)		
	$NS-VAR$	-0.019	0.103***	1.897	0.475***	-3.000	-0.003*	0.002	-1.791								0.923	0.315
		(0.029)	(0.003)	92.436)	(0.101)	(2.454)	(0.002)	(0.002)	(2.448)									
		-0.008	0.105***	2.269	0.334**	-3.706	-0.002	0.002	-2.094	0.396***	0.270	0.004	-0.029	0.024	-0.001	0.001	0.929	0.291
		(0.034)	(0.003)	(2.398)	(0.154)	(2.416)	(0.002)	(0.002)	(2.411)	(0.109)	(0.177)	(0.006)	(0.053)	(0.021)	(0.021)	(0.005)		

Note: This table presents the results from regression models (21) and (24): $Z_{p,q,t} = \alpha_{p,q} + \sum_{j=1}^{J} \gamma_{p,q,j} I_{j,t} + \varepsilon_{p,q,t}$, and $Z_{p,q,t} = \alpha_{p,q} + \sum_{j=1}^{J} \gamma_{p,q,j} I_{j,t} + \sum_{k=1}^{K} \theta_{p,q,k} M_{k,t} + \varepsilon_{p,q,t}$, for $p = (NS-OLS, NS-AR, NS-VAR)$ and $q = \beta_1$. Furthermore, $Z_{p,q,t}$ represents the estimated factor value at t corresponding to model p and factor q, I_t the Jx1 vector of monthly changes in internal regressors and $M_{k,t}$ the Kx1 vector of monthly changes in macroeconomic variables. For every model the first row in the table represents the estimated coefficients of regression model (21), whereas the second row represents model (24). The corresponding estimated standard errors appear in parentheses. The number of observations included in all regressions is 224 [1991:06-2016:08].

Table 17: Regression coefficients; dependent variable β_2.

	β_2			Internal								External				R^2	AIC
		c	$\Delta\sigma_{slope}$	ΔF_{slope}	Δr_{slope}	Real-IV	$\Delta\sigma_{lvl}$	ΔF_{lvl}	ΔGY	ΔBR	ΔCCI	ΔCPI	ΔIP	ΔMS	ΔSI		
Panel A: USD	$NS-OLS$	-0.038	-0.025***	-1.515***	0.179	0.008***	-0.120***	1.358***								0.925	0.997
		(0.045)	(0.002)	(0.224)	(0.123)	(0.002)	(0.005)	(0.175)									
		-0.078	-0.026***	-1.282***	0.197	0.009***	-0.115***	1.083***	0.349	0.193	-0.005	0.022	-0.088**	0.096	0.018***	0.930	0.984
		(0.065)	(0.002)	(0.441)	(0.149)	(0.002)	(0.005)	(0.406)	(0.421)	(0.205)	(0.003)	(0.099)	(0.041)	(0.076)	(0.007)		
	$NS-AR$	-0.055	-0.035***	-0.610**	0.402***	0.003	-0.115***	1.295***								0.922	1.113
		(0.047)	(0.002)	(0.237)	(0.130)	(0.002)	(0.005)	(0.185)									
		-0.067	-0.035***	-0.745	0.225	0.005**	-0.113***	1.390***	0.111	-0.253	-0.005	-0.041	-0.139***	0.090	0.015**	0.931	1.059
		(0.068)	(0.002)	(0.458)	(0.155)	(0.002)	(0.005)	(0.422)	(0.437)	(0.212)	(0.003)	(0.103)	(0.043)	(0.079)	(0.007)		
	$NS-VAR$	-0.056	-0.035***	-0.804***	0.396***	0.004*	-0.116***	1.315***								0.924	1.096
		(0.047)	(0.002)	(0.235)	(0.129)	(0.002)	(0.005)	(0.183)									
		-0.071	-0.034***	-0.924**	0.247	0.005**	-0.113***	1.388***	0.112	-0.194	-0.005	-0.028	-0.138***	0.090	0.014*	0.931	1.051
		(0.068)	(0.002)	(0.456)	(0.154)	(0.002)	(0.005)	(0.420)	(0.435)	(0.212)	(0.003)	(0.102)	(0.043)	(0.079)	(0.007)		
		c	$\Delta\sigma_{slope}$	ΔF_{slope}	Δr_{slope}	Real-IV	$\Delta\sigma_{lvl}$	ΔF_{lvl}	ΔGY	ΔBR	ΔCCI	ΔCPI	ΔIP	ΔMS	ΔSI	R^2	AIC
Panel B: GBP	$NS-OLS$	-0.100	-0.022***	-1.291***	0.517***	0.005**	-0.091***	0.886***								0.858	0.949
		(0.040)	(0.002)	(0.231)	(0.137)	(0.002)	(0.004)	(0.197)									
		-0.102**	-0.021***	-1.526***	0.548**	0.005**	-0.093***	1.059***	-0.249	0.024	-0.003	0.026	-0.027	-0.008	0.000	0.861	0.993
		(0.048)	(0.002)	(0.312)	(0.220)	(0.002)	(0.004)	(0.292)	(0.155)	(0.248)	(0.008)	(0.075)	(0.030)	(0.030)	(0.007)		
	$NS-AR$	-0.139***	-0.024***	-1.121***	0.529***	0.004**	-0.089***	0.732***								0.848	0.964
		(0.040)	(0.002)	(0.233)	(0.138)	(0.002)	(0.004)	(0.199)									
		-0.163***	-0.024***	-1.548***	0.475**	0.004*	-0.091***	1.066***	-0.339**	-0.107	-0.006	0.064	-0.040	0.010	0.000	0.854	0.984
		(0.048)	(0.002)	(0.310)	(0.219)	(0.002)	(0.004)	(0.291)	(0.154)	(0.247)	(0.008)	(0.075)	(0.030)	(0.030)	(0.007)		
	$NS-VAR$	-0.145***	-0.022***	-1.292***	0.499	0.004*	-0.090***	0.797***								0.839	1.031
		(0.042)	(0.002)	(0.241)	(0.143)	(0.002)	(0.004)	(0.206)									
		-0.172***	-0.021***	-1.638***	0.516**	0.003	-0.091***	1.045***	-0.311*	0.005	-0.005	0.059	-0.037	0.017	0.000	0.844	1.060
		(0.049)	(0.002)	(0.322)	(0.227)	(0.002)	(0.004)	(0.302)	(0.160)	(0.257)	(0.009)	(0.078)	(0.031)	(0.031)	(0.008)		

Note: This table presents the results from regression models (21) and (24): $Z_{p,q,t} = \alpha_{p,q} + \sum_{j=1}^{J} \gamma_{p,q,j} I_{j,t} + \varepsilon_{p,q,t}$, and $Z_{p,q,t} = \alpha_{p,q} + \sum_{j=1}^{J} \gamma_{p,q,j} I_{j,t} + \sum_{k=1}^{K} \theta_{p,q,k} M_{k,t} + \varepsilon_{p,q,t}$, for $p = (NS-OLS, NS-AR, NS-VAR)$ and $q = \beta_2$. Furthermore, $Z_{p,q,t}$ represents the estimated factor value at t corresponding to model p and factor q, I_t the Jx1 vector of monthly changes in internal regressors and $M_{k,t}$ the Kx1 vector of monthly changes in macroeconomic variables. For every model the first row in the table represents the estimated coefficients of regression model (21), whereas the second row represents model (24). The corresponding estimated standard errors appear in parentheses. The number of observations included in all regressions is 224 [1998:01-2016:08].

Table 18: Regression coefficients; dependent variable β_3.

	β_3		Internal								External								
		c	$\Delta\sigma_{curv}$	ΔF_{curv}	Δr_{curv}	Real-IV	$\Delta\sigma_{lvl}$	ΔF_{lvl}	$\Delta\sigma_{slope}$	ΔF_{slope}	ΔGY	ΔBR	ΔCCI	ΔCPI	ΔIP	ΔMS	ΔSI	R^2	AIC
Panel A: USD	$NS-OLS$	-0.061	0.133***	2.833***	-0.384	-0.024***	0.017	1.635***	-0.130***	2.580**								0.662	2.968
		(0.119)	(0.012)	(1.038)	(0.390)	(0.005)	(0.013)	(0.373)	(0.009)	(1.251)									
		0.153	0.130***	2.030**	-0.482	-0.020***	-0.003	2.257**	-0.124***	0.734	-0.813	-1.933***	0.001	-0.189	-0.310***	-0.224	-0.019	0.714	2.863
		(0.170)	(0.011)	(1.027)	(0.388)	(0.005)	(0.013)	(1.018)	(0.008)	(1.509)	(1.078)	(0.457)	(0.008)	(0.254)	(0.105)	(0.199)	(0.018)		
	$NS-AR$	0.054	0.132***	-0.819	0.364	-0.020***	0.027**	0.883***	-0.091***	2.108**								0.625	2.636
		(0.101)	(0.010)	(0.879)	(0.330)	(0.005)	(0.011)	(0.316)	(0.007)	(1.060)									
		0.206	0.131***	-1.560*	0.272	-0.019***	0.015	0.341	-0.089***	1.547	0.502	-1.599***	-0.004	-0.075	-0.147	-0.255	-0.006	0.669	2.573
		(0.147)	(0.010)	(0.888)	(0.336)	(0.005)	(0.011)	(0.881)	(0.007)	(1.305)	(0.933)	(0.396)	(0.007)	(0.219)	(0.090)	(0.173)	(0.016)		
	$NS-VAR$	0.064	0.131***	-0.285	0.317	-0.019***	0.014	1.075***	-0.096***	2.033**								0.656	2.539
		(0.096)	(0.010)	(0.837)	(0.315)	(0.004)	(0.010)	(0.301)	(0.007)	(1.010)									
		0.202	0.129***	-1.053	0.207	-0.019***	0.002	0.659	-0.093***	1.285	0.396	-1.595***	-0.006	-0.096	-0.177**	-0.200	-0.006	0.705	2.448
		(0.138)	(0.009)	(0.834)	(0.315)	(0.004)	(0.010)	(0.827)	(0.007)	(1.226)	(0.876)	(0.372)	(0.007)	(0.206)	(0.085)	(0.162)	(0.015)		
		c	$\Delta\sigma_{curv}$	ΔF_{curv}	Δr_{curv}	Real-IV	$\Delta\sigma_{lvl}$	ΔF_{lvl}	$\Delta\sigma_{slope}$	ΔF_{slope}	ΔGY	ΔBR	ΔCCI	ΔCPI	ΔIP	ΔMS	ΔSI	R^2	AIC
Panel B: GBP	$NS-OLS$	0.074	0.096***	3.454**	0.358	-0.001	-0.033***	0.144	-0.088***	3.521								0.631	2.412
		(0.082)	(0.008)	(1.615)	(0.339)	(0.004)	(0.008)	(0.395)	(0.006)	(1.462)									
		0.028	0.096***	-2.139	-0.483	0.000	-0.041***	1.456***	-0.087***	-3.024*	-0.679**	-2.163***	0.009	0.339**	-0.043	-0.060	0.000	0.700	2.266
		(0.090)	(0.007)	(1.881)	(0.365)	(0.004)	(0.008)	(0.456)	(0.005)	(1.809)	(0.294)	(0.390)	(0.016)	(0.140)	(0.056)	(0.057)	(0.014)		
	$NS-AR$	0.258***	0.085***	3.821***	0.422*	0.000	-0.007	0.049	-0.065***	3.832***								0.665	1.730
		(0.059)	(0.006)	(1.148)	(0.241)	(0.003)	(0.006)	(0.280)	(0.004)	(1.040)									
		0.238***	0.084***	-0.889	-0.251	0.000	-0.013**	1.042***	-0.064***	-1.550	-0.424**	-1.779***	-0.004	0.101	-0.007	-0.028	0.001	0.731	1.674
		(0.064)	(0.005)	(1.331)	(0.258)	(0.003)	(0.006)	(0.323)	(0.004)	(1.281)	(0.208)	(0.276)	(0.011)	(0.099)	(0.040)	(0.040)	(0.010)		
	$NS-VAR$	0.212***	0.074***	4.755***	0.333	0.000	-0.023***	0.153	-0.069***	3.991***								0.676	1.702
		(0.058)	(0.006)	(1.132)	(0.238)	(0.003)	(0.006)	(0.277)	(0.004)	(1.025)									
		0.170***	0.073***	0.180	-0.302	0.000	-0.030***	1.187***	-0.068***	-1.334	-0.495**	-1.767***	-0.010	0.135	-0.023	-0.011	0.002	0.748	1.515
		(0.062)	(0.005)	(1.292)	(0.251)	(0.003)	(0.005)	(0.313)	(0.004)	(1.243)	(0.202)	(0.268)	(0.011)	(0.096)	(0.038)	(0.039)	(0.009)		

Note: This table presents the results from regression models (21) and (24): $Z_{p,q,t} = \alpha_{p,q} + \sum_{j=1}^{J} \gamma_{p,q,j} I_{j,t} + \varepsilon_{p,q,t}$, and $Z_{p,q,t} = \alpha_{p,q} + \sum_{j=1}^{J} \gamma_{p,q,j} I_{j,t} + \sum_{k=1}^{K} \theta_{p,q,k} M_{k,t} + \varepsilon_{p,q,t}$, for $p = (NS-OLS, NS-AR, NS-VAR)$ and $q = \beta_3$. Furthermore, $Z_{p,q,t}$ represents the estimated factor value at t corresponding to model p and factor q, I_t the Jx1 vector of monthly changes in internal regressors and $M_{k,t}$ the Kx1 vector of monthly changes in macroeconomic variables. For every model the first row in the table represents the estimated coefficients of regression model (21), whereas the second row represents model (24). The corresponding estimated standard errors appear in parentheses. The number of observations included in all regressions is 224 [1998:01-2016:08].

F Gaussian Black Swaption Price Derivation

This appendix describes the derivation of the Black model under an arithmetic Brownian motion. In this derivation I consider a stock S that forms the underlying of the option. In a later phase, when the closed form pricing formulas are derived, I replace the stock price with the forward swap rate. In this derivation I make use of Hull (2006) and Alexander *et al.* (2012).

Starting point of the derivation is the stock price process given by

$$dS_t = \mu dt + \sigma dW_t, \tag{57}$$

and the process for a discount bond B which is only influenced by risk-free rate r by

$$dB_t = B_t r dt. \tag{58}$$

Secondly, I consider the discounted stock price $Z_t = B_t^{-1} S_t$, with a corresponding process described as

$$\begin{aligned} dZ_t &= dB_t^{-1} S_t + B_t^{-1} dS_t + dB_t^{-1} dS_t \\ &= -rB_t^{-1} St + B_t^{-1}(\mu dt + \sigma dW_t) \\ &= B_t^{-1}[(\mu - rS_t)dt + \sigma dW_t]. \end{aligned} \tag{59}$$

Now using the Cameron-Martin-Girsanov (CMG) theorem the discounted stock price process under the P-measure can be converted to a process under the risk-neutral Q-measure, such that the stock price Z_t is a martingale. Applying CMG with $\gamma = \frac{\mu - rS_t}{\sigma}$ yields

$$dZ_t = B_t^{-1}(\sigma dW_t'), \tag{60}$$

$$\sigma dW_t' = (\mu - rS_t)dt + \sigma dW_t, \tag{61}$$

with W_t' the Wiener process under the Q-measure.

To derive analytical formulas for the call and put options, the distribution of S_t under risk neutral measure Q needs to be determined. First I rewrite dZ_t to the following form

$$\begin{aligned} dZ_t &= d(B_t^{-1} S_t) \\ &= -rS_t B_t^{-1} dt + B_t^{-1} dS_t. \end{aligned} \tag{62}$$

Now combining equation (60) and (62) results in

$$dS_t = rS_t dt + \sigma dW_t'. \tag{63}$$

This function can be written as a Ornstein-Uhlenbeck process with $\alpha = 0$ and $\beta = -r$, given by

$$dS_t = -\beta(S_t - \alpha)dt + \sigma dW_t'. \tag{64}$$

The analytical solution of the Ornstein-Uhlenback process is given by

$$S_t = \alpha + (s_0 - \alpha)e^{-\beta t} + \sigma \int_0^t e^{-\beta(t-s)} dW_s' \tag{65}$$

with $S_0 = s_0$, which results in

$$S_T = e^{r\tau} S_t + \sigma e^{rT} \int_t^T e^{-rs} dW'_s, \tag{66}$$

with $\tau = T - t$ denoting time to maturity. With equation (66) we have an analytical solution for S_t under the Q-measure. Its distribution depends on the last term of the equation, which is known as the Ito integral. This term is normally distributed with mean zero and a variance term that is obtained using Ito's isometry:

$$\begin{aligned} V\Big[\int_t^T e^{-rs} dW'_s | S_t = s_t\Big] &= E\Big[(\int_t^T e^{-rs} dW'_s)^2\Big] \\ &= E\Big[\int_t^T e^{-2rs} ds\Big] \\ &= E\Big[\int_t^T \frac{1}{-2r} de^{-2rs}\Big] \\ &= \frac{e^{-2rt} - e^{-2rT}}{2r}. \end{aligned} \tag{67}$$

Hence, S_T is also normally distributed with mean μ_τ and variance Σ_τ^2 defined as

$$E[S_T | S_t = s_t] = e^{r\tau} s_t \equiv \mu_\tau, \tag{68}$$

$$V[S_T | S_t = s_t] = e^{2rT} \sigma^2 V\Big[\int_t^T e^{-rs} dW'_s | S_t = s_t\Big] = \frac{\sigma^2}{2r}(e^{2r\tau} - 1) \equiv \Sigma_\tau^2. \tag{69}$$

Now the characteristics of S_T are known, option prices can determined. The price of a call option c_t with exercise price K under the risk-neutral measure can be derived through

$$\begin{aligned} c_t &= e^{-r\tau} E_Q[\max(S_T - K, 0) | S_t = s_t] \\ &= e^{-r\tau} \int_K^\infty (S_T - K) f(S_T | S_t) dS_T, \end{aligned} \tag{70}$$

where $f(S_T|S_t)$ is the probability density function of a normal random variable with mean μ_τ and variance Σ^2. Since $\frac{S_T - \mu_\tau}{\Sigma_\tau} \sim N(0,1)$, I can change variable $S_T = \Sigma_\tau z + \mu_\tau$ and rewrite equation (70) in terms of a standard normal variable z, provided by

$$\begin{aligned} c_t &= e^{-r\tau} \int_{\frac{K - \mu_\tau}{\Sigma_\tau}}^\infty (\Sigma_\tau z + \mu_\tau - K) \phi(z|S_t) dz \\ &= e^{-r\tau} \Sigma_\tau \int_{d_\tau}^\infty (z - d_\tau) \phi(z|S_t) dz, \end{aligned} \tag{71}$$

with $d_\tau = \frac{K - \mu_\tau}{\Sigma_\tau}$ and ϕ the standard normal density function. Equation (71) can subsequently be rewritten and solved as follows

$$\begin{aligned} c_t &= e^{-r\tau} \Sigma_\tau \Big[\int_{d_\tau}^\infty z\phi(z|S_t) dz - \int_{d_\tau}^\infty d_\tau \phi(z|S_t) dz\Big] \\ &= e^{-r\tau} \Sigma_\tau \big[\phi(d_\tau) - d_\tau \Phi(-d_\tau)\big] \\ &= e^{-r\tau} \big[\Sigma_\tau \phi(d_\tau) - (K - e^{r\tau} S_t) \Phi(-d_\tau)\big], \end{aligned} \tag{72}$$

with Φ the normal cumulative density function. In the same way the value of a put option can be derived, which is given by

$$p_t = e^{-r\tau}\left[\Sigma_\tau \phi(d_\tau) + (K - e^{r\tau}S_t)\Phi(d_\tau)\right] \tag{73}$$

The prices of options on a stock that is assumed to follow an arithmetic Brownian motion are now derived. To convert these prices to payer and receiver swaption prices I need to adjust two things. First, the stock price S_t can be replaced by forward swap rate F_t, whereas strike price K now represents a strike rate. Secondly, remember that in contrast to stocks the swaption payoff depends on a number of cash flows in the future based on the agreed principal P. The option prices that are currently derived are essentially expected payoff values at expiry. Hence, for the swaptions I need to consider multiple expected cash flows and discount them the using annuity factor A, like described in section 2.2. Discounting through r in equations (69), (72), (73) and the formula for d_τ can be neglected as all relevant discounting in swaptions is now performed by A. Therefore, the discount factor r is set to zero. This eventually results in the following price of a payer and receiver swaption under the assumed arithmetic Brownian motion:[13]

$$V_{P,abm,t} = PA[\sigma\sqrt{\tau}\phi(d_\tau) - (K - F_t)\Phi(-d_\tau)], \tag{74}$$

$$V_{R,abm,t} = PA[\sigma\sqrt{\tau}\phi(d_\tau) + (K - F_t)\Phi(d_\tau)], \tag{75}$$

with

$$d_\tau = \frac{K - F_t}{\sigma\sqrt{\tau}}. \tag{76}$$

Lastly, the delta of the swaptions can be obtained by differentiating equation (74) and (75) to the underlying. This yields a payer delta equal to

$$\frac{\delta V_{P,abm,t}}{\delta F_t} = PA\Phi(-d_\tau), \tag{77}$$

and a receiver delta equal to

$$\frac{\delta V_{R,abm,t}}{\delta F_t} = -PA\Phi(d_\tau), \tag{78}$$

resulting in a swaption straddle delta equal to

$$\Delta_{abm,t} = \frac{\delta S_{abm,t}}{\delta F_t} = PA[\Phi(-d_\tau) - \Phi(d_\tau)]. \tag{79}$$

[13]To obtain this result the one should take the limit of r in Σ^2 towards zero, which yields τ.