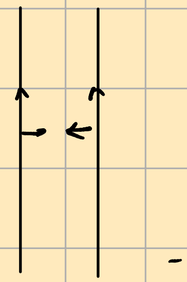
General Physics.

磁学.

1. 安培定律:

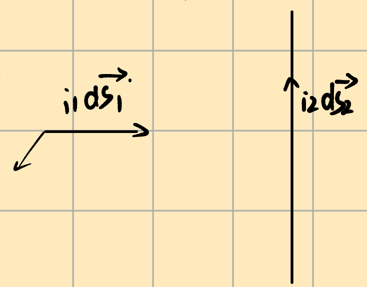
$$dF_{1,2} = \frac{\mu_0}{4\pi} \frac{i_2 d\vec{s}_2 \times (i_1 d\vec{s}_1 \times \hat{r}_{12})}{r_{12}^2}$$



两电流元平行. 互相吸引

$F_{12} = -F_{21}$

2.



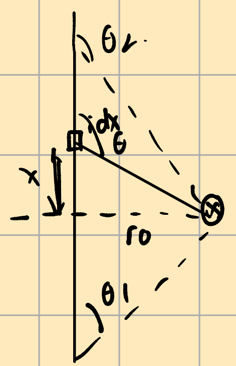
$$dF_{12} = \frac{\mu_0}{4\pi} \frac{i_2 ds_2 \times (i_1 ds_1 \times \hat{r}_{12})}{r_{12}^2} = 0$$

$$dF_{21} = \frac{\mu_0}{4\pi} \frac{i_1 ds_1 \times (i_2 ds_2 \times \hat{r}_{21})}{r_{21}^2}$$

3. Biot-Savart law

$$B = \frac{\mu_0}{4\pi} \int_L \frac{i ds \times \hat{r}}{r^2}.$$

3.1 长直导线



$$B = \frac{\mu_0}{4\pi} \int_L \frac{i ds \times \hat{r}}{r^2}.$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i dx \sin^3\theta}{r_0^2}.$$

$$x = \frac{r_0}{\tan\theta}.$$

$$dx = r_0 \frac{1}{\sin^2\theta} d\theta$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i \sin\theta}{r_0} d\theta$$

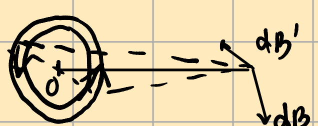
$$B = \frac{\mu_0}{4\pi} \cdot \frac{i}{r_0} [\cos\theta_1 - \cos\theta_2]$$

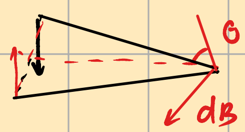
长直导线:

$\theta_1 = 0 \quad \theta_2 = \pi$

$$B = \frac{\mu_0 i}{2\pi r_0}$$

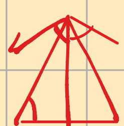
3.2 圆环.





$$dB_{net} = \frac{\mu_0}{4\pi} \frac{ids \cos\alpha \cdot 2 \cdot \sin^2\alpha}{r_0^2}$$

$$= \frac{\mu_0}{2\pi r_0^2} ids \cos\alpha \sin^2\alpha.$$



$$B = \frac{\mu_0 R i}{2 r_0^3} \cos\alpha \sin^2\alpha.$$

$$B=\frac{\mu_0 R i}{2r_0^2}\cdot\frac{r_0^2 R}{(r_0^2+R^2)^{\frac{3}{2}}}=\frac{\mu_0 i R^2}{2(r_0^2+R^2)^{\frac{3}{2}}}.$$

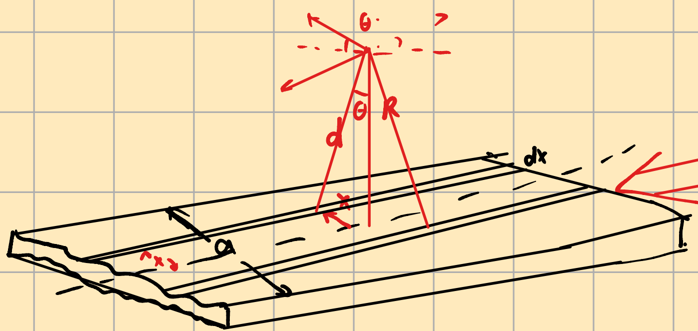
$r_0\to 0 \quad B\to\frac{\mu_0 i}{2R} \qquad r_0\to\infty \quad B\to\frac{\mu_0 i R^2}{2r_0^3}.$

定义磁偶极矩

$\mu=iA=i\pi R^2.$

若有N匝线圈，则 $\mu=NiA=Ni\pi R^2.$

3.3 电流平面产生的磁场



$$dB_{net}=2\cdot\cos\theta\cdot\frac{\mu_0\frac{i}{a}dx}{2\pi d}$$

$$=\frac{\mu_0 i}{\pi a d}\cos\theta\,dx$$

$$=\frac{\mu_0 i}{\pi a R}\quad d\theta$$

$$B=\oint dB_{net}$$

$$=\frac{\mu_0 i}{\pi a}\arctan\frac{a}{2R}$$

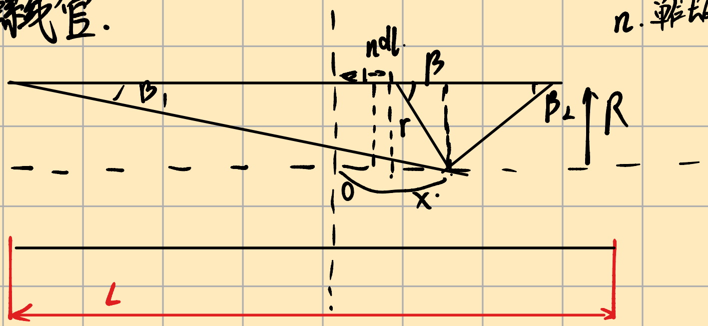
point is far...

$d=\frac{a}{2R} \qquad B=\frac{\mu_0 i}{2\pi R}.$

$R\to 0 \qquad B=\frac{\mu_0 i}{2a}$

3.4 螺线管.

n. 单位长度匝数



$L\to\infty,\ B\simeq\mu_0 n i$

$$dB=\frac{\mu_0}{2}\frac{indl\,R^2}{(R^2+(x-l)^2)^{\frac{3}{2}}}.$$

$$r^2=\frac{R^2}{\sin\beta^2}.$$

$$\frac{R}{x-l}=\tan\beta.$$

$$x-l=\frac{R}{\tan\beta}$$

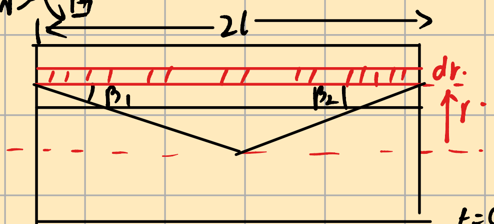
$$dl=\frac{R\,d\beta}{\sin^2\beta}$$

$$dB=\frac{ni\mu_0}{2}\cdot\frac{\sin^3\beta}{R^3}\frac{R^2R}{\sin^2\beta}d\beta$$

$$=\frac{ni\mu_0}{2}\sin\beta\,d\beta$$

$$B=\frac{\mu_0 n i}{2}\left[\cos\beta_1-\cos\beta_2\right]$$

多层螺线管



$t=\cos\theta.$

$$\frac{1}{\sqrt{1+t^2}}dt=\frac{\cos\theta d\theta}{\cos^2\theta}$$

$$dB=\frac{\mu_0}{2}\frac{N}{2l}\frac{i}{(R_2-R_1)}\cdot\frac{2l}{\sqrt{l^2+r^2}}dr.$$

$$B=\mu_0\quad j\quad l\int_{R_1}^{R_2}\frac{1}{\sqrt{l^2+r^2}}dr.$$

$$=\mu_0 j l\ln\frac{R_2+\sqrt{R_2^2+l^2}}{R_1+\sqrt{R_1^2+l^2}}.$$

磁场高斯定理：$\oiint_S \vec{B}\cdot d\vec{S}=0$

回路定理 $\oint \vec{B}\cdot d\vec{l}=\mu_0\sum i$ i的符号由右手定则决定

└ 适用于高对称性.

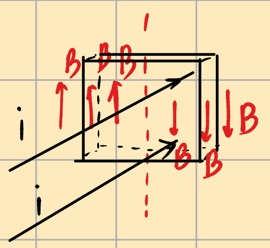
e.g. 1. 电线.

$r>R$ $B\cdot 2\pi r=\mu_0 i$

$r<R$时 $B\cdot 2\pi r=\mu_0 \frac{ir^2}{R^2}$

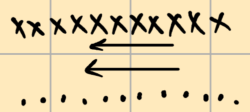
$B=\frac{\mu_0 i r}{2\pi R^2}$

2. 电流板.



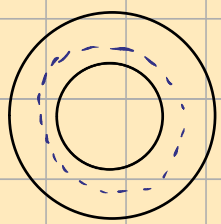
取一个边长为w的框 → $\oint B\cdot dl=2Bw$
$=\mu_0 n w i$
↑单位长度.

3. 螺线管



$B=\mu_0 n i$

4. 螺绕环

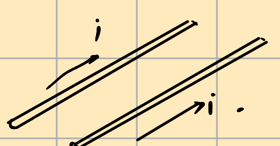


$2\pi r B=\mu_0 N i$

$B=\mu_0 n i$

安培力.

1. 两通相同电流的导线

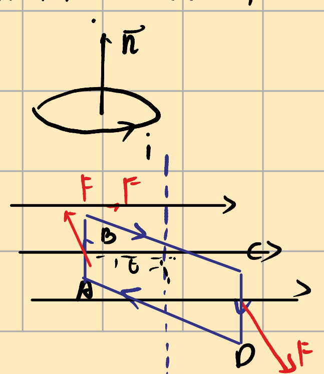


$f=\frac{\mu_0 i^2}{2\pi d}$

$i=\sqrt{\frac{2\pi f d}{\mu_0}}$ ⇒定义1A.

2. 力矩.

$\mu=iA\hat{n}$ （$\vec{n}$方向由右手定则确定）



$\tau=2F\cdot\frac{AD}{2}\cdot\sin\theta$

$=iBA\sin\theta$

$=\mu\times B$

3. 磁偶极子

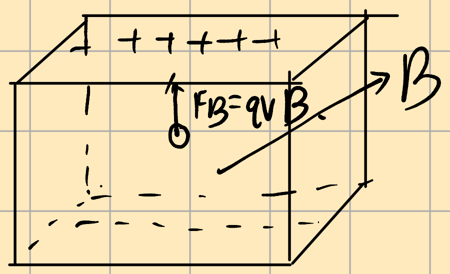
环形电流与磁偶极矩是等价的

$\tau=\mu\times B$

在B中，它会受到磁场作用，

势能 $U=-\mu\cdot B$

Hall 效应



$$V_{AA'} = E \cdot b = vBb = \frac{j}{nq} Bb = \frac{iB}{nqd} = k\frac{iB}{d}.$$

Hall 电阻：$R_H = \frac{V_{AA'}}{i} = \frac{B}{nqd}.$

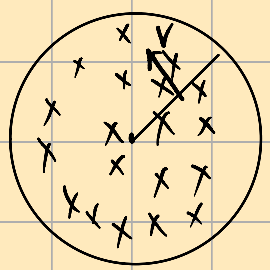
$i = jBd.$

电磁感应

动生电动势

非静电力 $K = \frac{f}{e} = v \times B$

$$\varepsilon = \int_-^+ K \cdot dl = \int_C^D (v \times B) \cdot dl$$



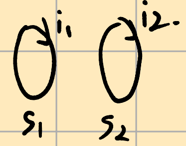
$$d\varepsilon = (v \times B) \cdot dr = -vB\,dr$$

$$\varepsilon = -\int_0^R Bv\,dr = -\int_0^R B\omega r\,dr = -\frac{1}{2}B\omega R^2$$

感生电动势：

$$\oint E\,dl = \varepsilon_0 = -\frac{d\Phi_B}{dt}. = -\iint \frac{\partial B}{\partial t} \cdot dA$$

互感.



$\propto N_2A_2B_1 \propto N_2\Phi_{12}$.

S_1对S_2的磁通匝链数 $\psi_{12}=M_{12}i_1$

S_2对S_1 : $\psi_{21}=M_{21}i_2$.

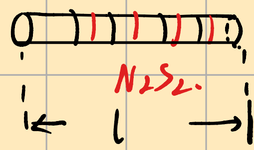
$M_{12}=\frac{\psi_{12}}{i_1}=\frac{N_2\Phi_{12}}{i_1}$ $\varepsilon_2=-\frac{d\psi_{12}}{dt}=-M_{12}\cdot\frac{di_1}{dt}$

$M_{21}=\frac{\psi_{21}}{i_2}=\frac{N_1\Phi_{21}}{i_2}$ $\varepsilon_1=-\frac{d\psi_{21}}{dt}=-M_{21}\frac{di_2}{dt}$

一般情况 $M_{12}=M_{21}=M$.

计算: 设 $i\to B\to\psi\to M$.

e.g.



计算S_2的互感系数, ε_2.

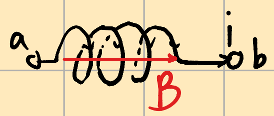
$B=\mu_0 ni$ $B_1=\frac{\mu_0N_1}{l}i_1$

$\psi_{12}=\frac{\mu_0N_1N_2}{l}i_1A$.

$M_{12}=\frac{\psi_{12}}{i_1}=\frac{\mu_0N_1N_2A}{l}$

$\varepsilon_2=-M\frac{di}{dt}$

自感.

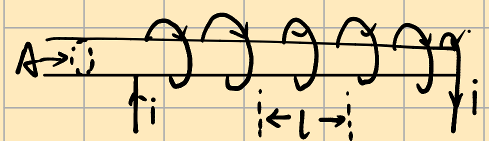


$\psi=NBA=Li$

$\varepsilon_L=-\frac{d\psi}{dt}=L\frac{di}{dt}$.

$V_b-V_a=-L\frac{di}{dt}$

e.g.

1. 

$B=\mu_0 ni$

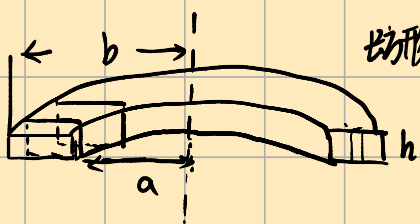
$\psi=nl\mu_0 niA$.

$L=\frac{\psi}{i}=n^2l\mu_0A$

$=\mu_0n^2V$.

单位体积自感: $L_V=\frac{L}{V}=\mu_0n^2$

单位长度自感: $L_V=\frac{L}{l}=\mu_0n^2A$

2. 

长方形螺线环.

$\oint B\,dl=\mu_0Ni$

$B=\frac{\mu_0Ni}{2\pi r}$

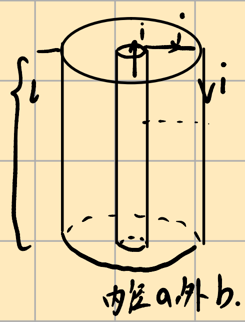
$\psi_B=\iint B\cdot dA=\iint\frac{\mu_0Ni}{2\pi r}Nh\,dr$

$=\frac{\mu_0N^2ih}{2\pi}\int_a^b\frac{1}{r}dr$

$=\frac{\mu_0N^2ih}{2\pi}\ln\frac{b}{a}$

$L=\frac{\psi_B}{i}=\frac{\mu_0N^2h}{2\pi}\ln\frac{b}{a}$

3. 同轴电缆



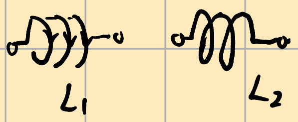
$$B=\frac{\mu_0 i}{2\pi r}$$

$$\Phi_B=\int_a^b \frac{\mu_0 i}{2\pi r}\cdot l\,dr=\frac{\mu_0 i l}{2\pi}\ln\frac{b}{a}$$

$$L=\frac{\Phi_B}{i}=\frac{\mu_0 L}{2\pi}\ln\frac{b}{a}$$

互感与自感关系.

两线圈无磁通量泄漏 $M_{12}=M_{21}=M=\sqrt{L_1L_2}$.



同向串联总线 $L=(\sqrt{L_1}+\sqrt{L_2})^2$.

反向 $L=(\sqrt{L_1}-\sqrt{L_2})^2$

磁化材料.

向线圈插入磁材料 $L=k_m L_0$.
↓磁导率

k_m由材料决定，微观上.

$$\mu=iA=\frac{e}{T}\cdot\pi r^2=\frac{ev}{2\pi r}\cdot\pi r^2=\frac{e}{2}vr=\frac{e}{2m}l.$$

又 $L=p\times r=mvr$

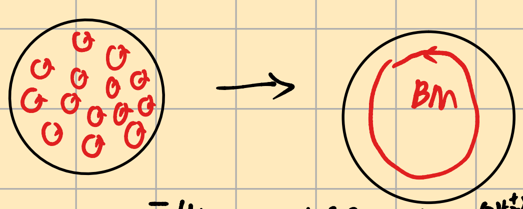
$$\mu_L=-\frac{e}{2m}l.$$

磁性质由总磁矩 $\sum\mu_j=\sum-\frac{e}{2m}(L+2S)$ 决定

轨道角 自旋量子数.

磁化强度M.

给磁材料外加一个B, 内部磁矩排列整齐, 形成一个B_M.(等效了一个表面电流i')

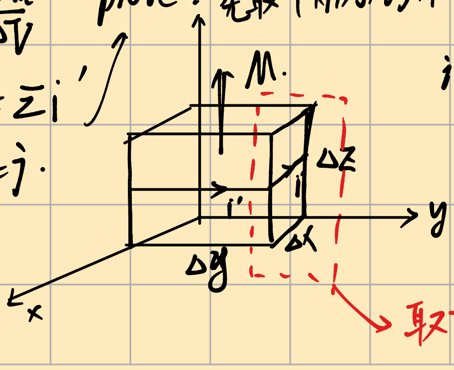


定义 $M=\frac{\sum\mu_m}{\Delta V}$

性质: $\oint M\cdot dl=\sum i'$

$M\times n=j'$.

prove: 先取一个特例, 长方体:



设 $j'=\frac{i'}{\Delta z}$

则 $\Delta m=i'\Delta A=j'\Delta x\Delta y\Delta z$

$M=\frac{\Delta m}{\Delta V}=j'$

$M\cdot\Delta z=i'$

取一个loop $\oint M\cdot dl=M\Delta z=i'$ $M\times n=j'$

非均匀的情况可分割为很小的均匀磁化之和

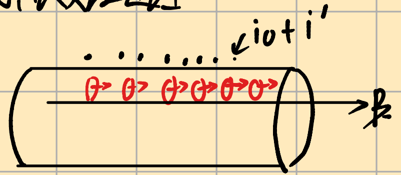
$\oint M\cdot dl = \Sigma i'$ (束缚电流)

$M\times n = j'$ (束缚电流密度)

$M\times\vec{n} = j'$.

$\oint M\,dl = \Sigma i$

扩展安培定理

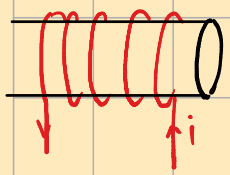


$$\oint B\cdot dl = \mu_0\Sigma i$$
$$= \mu_0\Sigma i_0 + \mu_0\Sigma i'$$
$$= \mu_0\Sigma i_0 + \mu_0\oint M\,dl$$
$$\oint\left(\frac{B}{\mu_0} - M\right)dl = \Sigma i_0$$
$$\Downarrow$$

磁场强度 $H \triangleq \dfrac{B}{\mu_0} - M$. $\rightarrow$ $\oint H\cdot dl = \Sigma i_0$.

在真空中，$M=0$ $H=\dfrac{B}{\mu_0}$

e.g. 1. 置入磁材料的螺线管.



$$\oint H\cdot dl = \Sigma i_0$$
$$H\Delta l = N i_0$$
$$H = n i_0$$

原磁场 $B_0 = \mu_0 n i_0$

$$B = (H+M)\mu_0 = \mu_0 M + B_0$$

$$B = \mu_0 k_m H = k_m B_0$$
$$\frac{B}{B_0} = k_m.$$

此时 $\dfrac{\Phi}{\Phi_0} = k_m$ 提电感增大: $\dfrac{L}{L_0} = k_m$

$\vec{M}, \vec{B}, \vec{H}$ 关系. 详见 Chapter3

同性材料: $M = \chi_m H$.

$$B = \mu_0(H+M)$$
$$= \mu_0(1+\chi_m)H$$
$$= \mu_0 k_m H.$$

$\mu_m\neq 0$ 外加B磁化,与[illegible]相反

$\mu_m = 0$,但磁导致 $\Delta\mu$ 变化

顺磁、抗磁、铁磁材料

$\chi_m>0$ $k_m>1$ $\chi_m<0$ $k_m<1$

$k_m\approx 1$, $|\chi_m|$很小

$k_m \approx 10^2\sim10^3$

磁场能

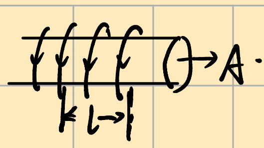
一个线圈: $dW = -\varepsilon_L dq$
$$= -\varepsilon_L i\,dt$$
$$= L i\frac{di}{dt}dt$$
$$W = \frac{1}{2}LI^2$$

2个: $W = W_1 + W_2$
$$= -\int_0^\infty(\varepsilon_1 i_1 dt + \varepsilon_2 i_2 dt)$$
$$= -\int_0^\infty(-M_{21}i_1 di_2 - M_{12}i_2 di_1)$$
$$\overset{若M_{12}=M_{21}=M}{=} M\int_0^{I_1I_2} d(i_1i_2)$$
$$= MI_1I_2.$$

$$U = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + \frac{1}{2}M_{12}I_1I_2 + \frac{1}{2}M_{21}I_2I_1$$

k个 $U = \frac{1}{2}\sum_{i=1}^{k}L_iI_i^2 + \frac{1}{2}\sum_{i,j=1}^{k}M_{ij}I_iI_j$

磁场能量密度： 对螺线管

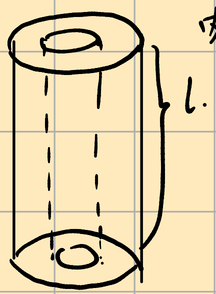


$$u_B=\frac{U}{V}=\frac{\frac{1}{2}LI^2}{lA}$$

$$=\frac{1}{2}\frac{\mu_0 n^2 lAI^2}{lA}=\frac{1}{2}\mu_0 n^2 I^2=\frac{B^2}{2\mu_0}$$

而 $B=\mu_0 nI$

e.g. 同轴电缆



内径a, 外b

$$\oint B\cdot dl=\mu_0 i$$

$$B=\frac{\mu_0 i}{2\pi r}$$

$$u_B=\frac{B^2}{2\mu_0}=\frac{\mu_0 i^2}{8\pi^2 r^2}$$

$$U=\int_a^b u_B\, dV$$

$$=\int_a^b \frac{\mu_0 i^2}{8\pi^2 r^2}\cdot 2\pi r l\, dr$$

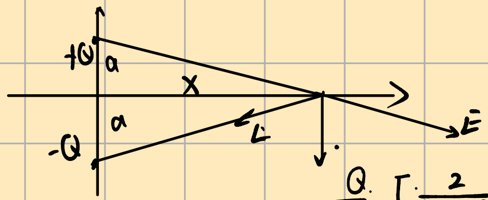
$$=\int_a^b \frac{\mu_0 i^2 l}{4\pi r}\, dr$$

$$=\frac{\mu_0 i^2 l}{4\pi}\ln\frac{b}{a}=\frac{1}{2}Li^2$$

$$L=\frac{\mu_0 l}{2\pi}\ln\frac{b}{a}.$$

电学.

电偶极矩: $p=2Qa=Ql$.



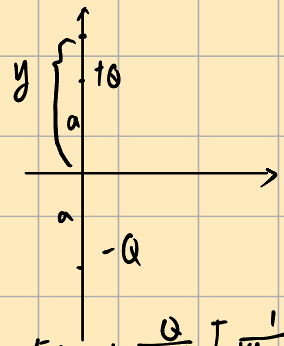
$$E_{(x,0)} = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{2}{x^2+a^2}\cdot\frac{a}{\sqrt{x^2+a^2}}\right]$$

$$= \frac{-Q}{2\pi\varepsilon_0}\cdot\frac{a}{(x^2+a^2)^{\frac{3}{2}}}.$$

for $x>>a$ $\quad E_{(x,0)} = \frac{-Q}{2\pi\varepsilon_0}\frac{a}{x^3[1+(\frac{a}{x})^2]^{\frac{3}{2}}}$

$$\approx \frac{-Qa}{2\pi\varepsilon_0 x^3}$$

$$= \frac{p}{4\pi\varepsilon_0(x^2+a^2)^{\frac{3}{2}}}.$$



$$E(0,y) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(y-a)^2}-\frac{1}{(y+a)^2}\right]$$

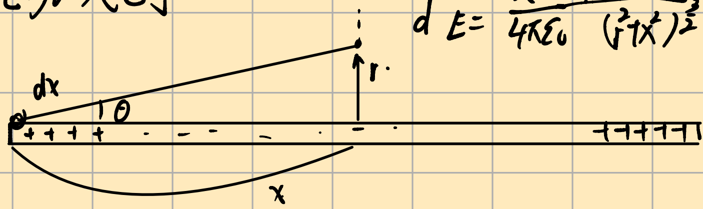
$$= \frac{Q}{4\pi\varepsilon_0}\cdot\frac{4ay}{(y^2-a^2)^2}.$$

$$= \frac{Qa}{\pi\varepsilon_0}\frac{y}{y^4[1-(\frac{a}{y})^2]^2}$$

for $y>>a$ $\quad E(0,y) \approx \frac{1}{\pi\varepsilon_0}\frac{Qa}{y^3}.$

为了描述连续体的电场分布, 引入线, 面, 体电荷密度的概念.

1. 无穷长线电荷

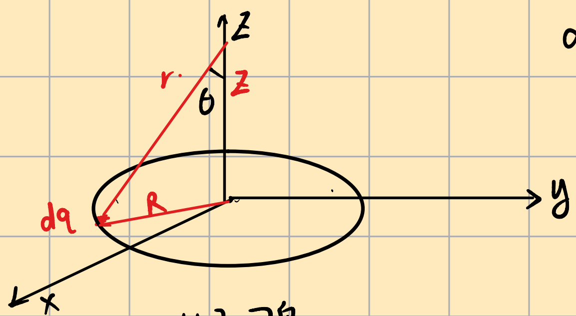


$$dE = \frac{\lambda dx}{4\pi\varepsilon_0}\frac{r}{(r^2+x^2)^{\frac{3}{2}}}. \quad E = \frac{\lambda r}{4\pi\varepsilon_0}\int_{-l}^{l}\frac{1}{(r^2+x^2)^{\frac{3}{2}}}dx = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$x=\frac{r}{\tan\theta}.$

$$\frac{1}{(r^2+\frac{r^2}{\tan^2\theta})^{\frac{3}{2}}} \qquad \int\frac{-\sin\theta}{r^2}d\theta. = \frac{\lambda}{4\pi\varepsilon_0 r}\cos\theta\Big|_{\theta_1}^{\theta_2}$$

2. 均匀带电圆环

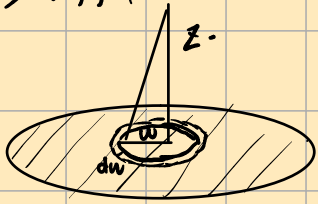


$$dE_z = \frac{dq}{4\pi\varepsilon_0(R^2+z^2)}\cdot\cos\theta$$

$$= \frac{\lambda ds}{4\pi\varepsilon_0}\frac{z}{(R^2+z^2)^{\frac{3}{2}}}.$$

$$E_z = \frac{zq}{4\pi\varepsilon_0(R^2+z^2)^{\frac{3}{2}}}. \quad z>>R \Rightarrow \frac{q}{4\pi\varepsilon_0 z^2}.$$

3. 均匀带电圆盘.



$$dE_z = \frac{dq\, z}{4\pi\varepsilon_0(z^2+\omega^2)^{\frac{3}{2}}}.$$

$$= \frac{\sigma 2\pi\omega\, d\omega\, z}{4\pi\varepsilon_0(z^2+\omega^2)^{\frac{3}{2}}}$$

$$= \frac{\sigma z}{4\varepsilon_0}\frac{d(\omega^2+z^2)}{(\omega^2+z^2)^{\frac{3}{2}}}$$

$$E = -\frac{\sigma z}{2\varepsilon_0}\frac{1}{\sqrt{\omega^2+z^2}}\Big|_0^R$$

$$= \frac{-\sigma z}{2\varepsilon_0\sqrt{R^2+z^2}}+\frac{\sigma}{2\varepsilon_0}$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{1+\frac{R^2}{z^2}}}\right)$$

$R>>z$时

$E\approx\frac{\sigma}{2\varepsilon_0}$ (和无穷大板相同)

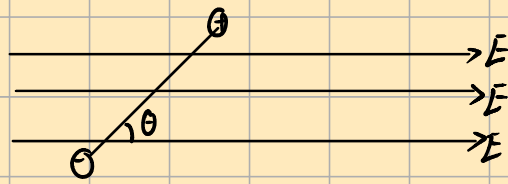
$z>>R$时

$E\approx\frac{\sigma}{2\varepsilon_0}\cdot\frac{1}{2}\frac{R^2}{z^2}$

$\approx\frac{q}{4\pi\varepsilon_0 z^2}$ (与点电荷相同)

电偶极矩在电场中 $p=qd$（方向由负电荷→正电荷）

力矩



$\tau = p\times E$

$\Delta U=-W$

$W=\int dW=\int_{\theta_0}^{\theta}\tau\cdot d\theta$

↑ τ与dθ相反

$=-\int_{\theta_0}^{\theta}pE\sin\theta d\theta$

$=pE(\cos\theta-\cos\theta_0)$

$\Delta U=-W=U(\theta)-U(\theta_0)$

取 $\theta_0=90^\circ$，$U(\theta_0)=0$

$U(\theta)=-pE\cos\theta=-p\cdot E$

高斯定理：

导体表面电场 $E\,dA=\frac{\sigma}{\varepsilon_0}dA$ $E=\frac{\sigma}{\varepsilon_0}$

通量：$\Phi=v\cdot A$ v（场矢量） A（面积矢量）

电通量 $\Phi_E=\oint E\cdot dA$ 方向朝外

Gauss定理：$\oint E\cdot dA=\Phi_E=\frac{q_{enclosed}}{\varepsilon_0}$

微分形式

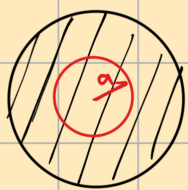
$\oint E\cdot dA=\iiint_V\left(\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}\right)dxdydz$

$=\frac{1}{\varepsilon_0}\iiint_V\frac{\rho\text{（体电荷密度）}}{\varepsilon_0}dxdydz$

即 $\nabla\cdot E=\frac{\rho}{\varepsilon_0}$

应用：高对称性

1. 均匀带电球体



$a>r$.

$E\cdot 4\pi a^2=\frac{q}{\varepsilon_0}$

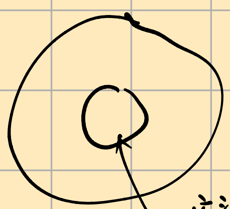
$E=\frac{q}{4\pi\varepsilon_0 a^2}$

$a<r$

$E\cdot 4\pi a^2=\frac{q\cdot\frac{4}{3}\pi a^3}{\frac{4}{3}\pi r^3\varepsilon_0}$

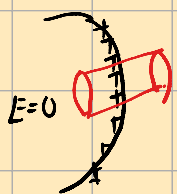
$E=\frac{q}{4\pi\varepsilon_0}\frac{a}{r^3}$

2. 孤立导体



空洞，无论洞内是否有电荷，取高斯面，里面电场处处为0，而外表面电场不受影响。

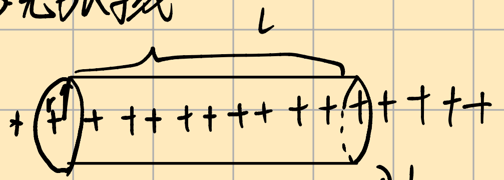
外表面电场计算



$E\Delta A=\frac{\sigma\Delta A}{\varepsilon_0}$

$E=\frac{\sigma}{\varepsilon_0}$

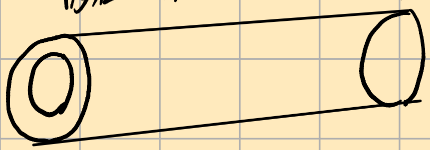
3.无穷长导线



$$E\cdot 2\pi r l=\frac{\lambda l}{\varepsilon_0}$$

$$E=\frac{\lambda}{2\pi r\varepsilon_0}$$

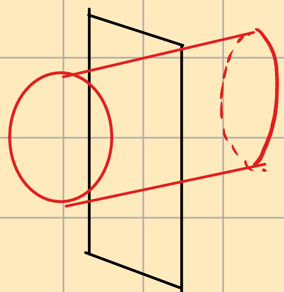
内径a 外径b，轴线上放一线电荷，线电荷密度为λ，求外表面σ.



$$E=\frac{\sigma}{\varepsilon_0}=\frac{\lambda}{2\pi b\varepsilon_0}$$

$$\sigma=\frac{\lambda}{2\pi b}$$

4.无穷大板(薄板)

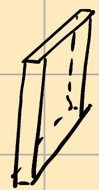


$$E\cdot 2A=\frac{\sigma A}{\varepsilon_0}$$

$$E=\frac{\sigma}{2\varepsilon_0}$$

对于厚为d的板

设两外表面 $\sigma_1=\sigma_2=\frac{\sigma}{2}$

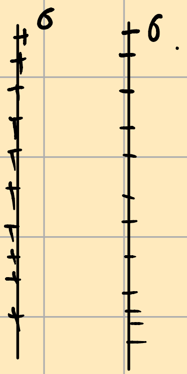


取高斯面，
包裹一个表面和内部

$$EA=\frac{\sigma A}{2\varepsilon_0}$$

$$E=\frac{\sigma}{2\varepsilon_0}$$

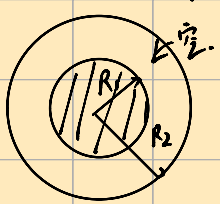
5.两平行无穷大薄板



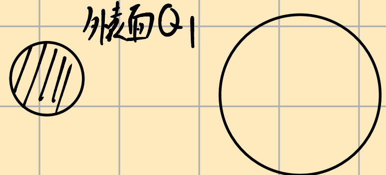
$$E=\frac{\sigma}{\varepsilon_0}\ \left(2\times\frac{\sigma}{2\varepsilon_0}\right).$$

6. 实心导体球，外套导体球壳，$-3Q_1$

Q_1



电荷分布.



外表面Q_1

内表面$-Q_1$（取高斯面至球壳内，$q=0$）

外表面$-2Q_1$

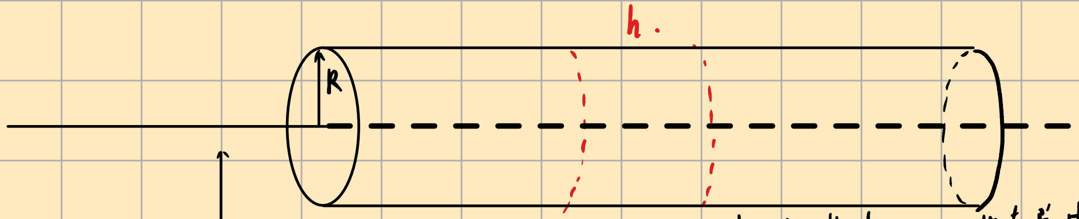
电场：

$$E=\begin{cases}0, & r<R_1\\ \frac{Q_1}{4\pi\varepsilon_0 r^2} & R_1<r<R_2\\ \frac{-2Q_1}{4\pi\varepsilon_0 r^2}. & r>R_2\end{cases}$$

$\sigma=\varepsilon_0 E$ for conductor.

导线连接内外表面，则 $r<R_2$ $E=0$　$r>R_2$ $E=-\frac{2Q_1}{4\pi\varepsilon_0 r^2}$

7. 圆柱体



无限长线电荷，外套一个无限长导体圆柱薄壳，已知薄壳总电荷密度为 σ_{total}

求圆柱壳 内表面密度 σ_{inner}

外表面密度 σ_{outer}

$\sigma_{inner} = \frac{-\lambda}{2\pi R}$

$\frac{\lambda}{2\pi R} + \sigma_{total} = \sigma_{outer}$

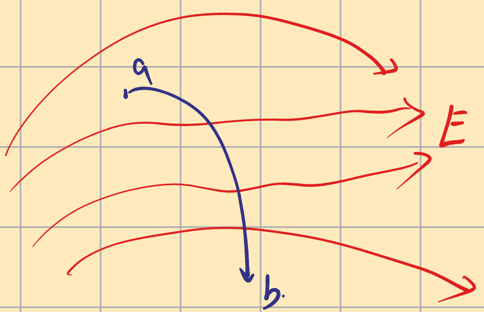
$r<R$ 时，$2\pi r h E = \frac{\lambda h}{\varepsilon_0}$　$E = \frac{\lambda}{2\pi r\varepsilon_0}$

$r>R$ 时，$2\pi r h E = \frac{\sigma_{total}\cdot 2\pi R h + \lambda h}{\varepsilon_0}$

$E_r = \frac{\lambda}{2\pi r} + \frac{\sigma_{total}R}{\varepsilon_0 r}$.

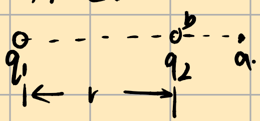
势能

静电力是保守力，做功只与起点、终点有关；可以定义势能差：



$U_b - U_a = -q\int_a^b E\cdot dl$.

对于点电荷：$U_b - U_a = -\int_0^b F\cdot dl$



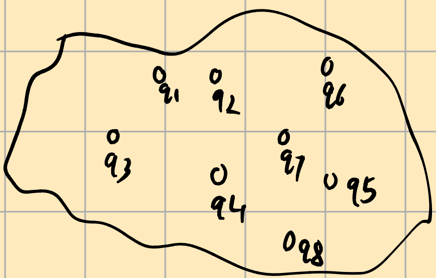
$= -\int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}dr$

$= \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$

取 $r_a=\infty$，定义此点 $U_\infty = 0$,

$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$. 注意：电势能是系统所具有的能量.

比如下例，多个点电荷系统的势能



$U = \sum_{i\neq j}\frac{1}{2}U_{ij}$（每对系统能量和，$\frac{1}{2}$ 消去重复 (U_{12}, U_{21})）

静电场环路定律

$\oint E\cdot dl = 0$ ⟶ 微分形式 $\nabla\times E = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = 0$

电势：$V = \frac{U}{q_0} = \frac{q}{4\pi\varepsilon_0 r}$ ⟶ 场源电荷

定义电势能差，电势差，

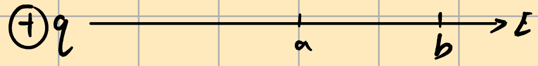
在 E 中，想将 q_0 从 a 至 b，至少施加与 Eq_0 等大反向的力

$W_{AB} = \int_A^B F_{ext}\, dl = -\int_A^B$ 电场力 $dl = -\int_A^B q_0 E\cdot dl$

$$V_b - V_A = \frac{W_{AB}}{q} = -\int_A^B E\cdot dl$$

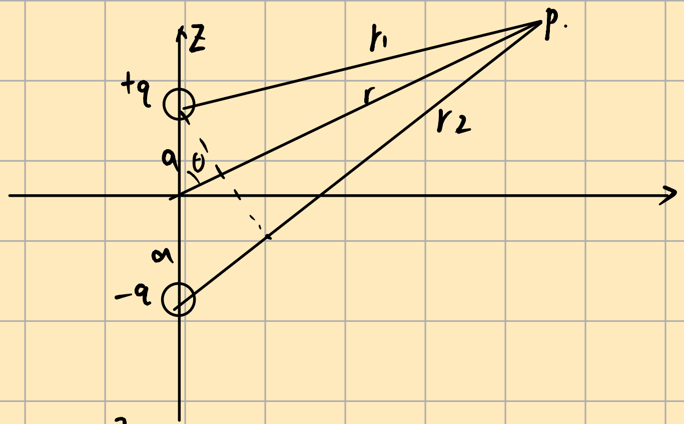
我们一般选取 $V_\infty=0$，那么E中点P势能为 $V_p - V_\infty = -\int_\infty^p E\cdot dl = \int_p^\infty E\cdot dl$

对于点电荷：



$$V_b - V_a = -\int_a^b E\cdot dl = -\int_a^b \frac{q}{4\pi\varepsilon_0 r^2}dr = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_b}-\frac{1}{r_a}\right)$$

电偶极矩.

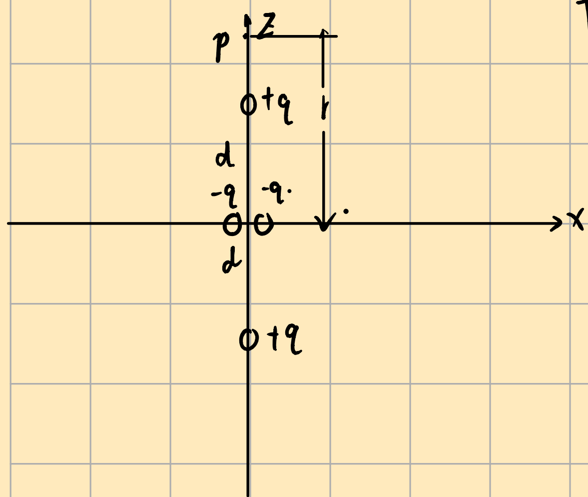


$$V_p = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

for $r \gg a$

$$V_p \simeq \frac{q}{4\pi\varepsilon_0 r^2}\cdot 2a\cos\theta = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cdot\hat{r}}{4\pi\varepsilon_0 r^2}$$

电四偶极矩



$$V_p = \sum_i (V_i P)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r-d}+\frac{q}{r+d} - \frac{2q}{r}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{2r}{r^2-d^2} - \frac{2}{r}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{2}{r(1-\frac{d^2}{r^2})} - \frac{2}{r}\right)$$

$$= \frac{2q}{4\pi\varepsilon_0 r}\left[\frac{d^2}{r^2}+o()\right]$$

for $r \gg d$ $\simeq \frac{2qd^2}{4\pi\varepsilon_0 r^3} \equiv \frac{Q}{4\pi\varepsilon_0 r^3}$ $Q=2qd^2$(电四偶极矩)

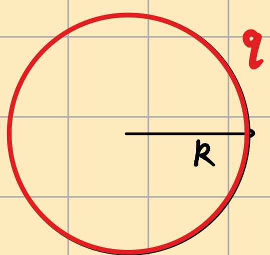
连续分布带电体的电势能：

线电荷密度 $dq=\lambda dx$

面电荷密度 $dq=\sigma dA$

体电荷密度 $dq=\rho dV$.

e.g. 1. 充电薄壳的电势能和电势



$E=\frac{q}{4\pi\varepsilon_0 r^2}$ $(r\geq R)$. $E=0$, $(r<R)$.

$r_p>R$, $V_p=\int_p^\infty E\cdot dl$.

$$= \int_{r_p}^\infty \frac{q}{4\pi\varepsilon_0 r^2}dr = \frac{q}{4\pi\varepsilon_0 r_p}.$$

$r_p<R$ $V_p = \left(\int_p^R + \int_R^\infty\right)E\cdot dl$.

$$= 0+\frac{q}{4\pi\varepsilon_0 R}.$$

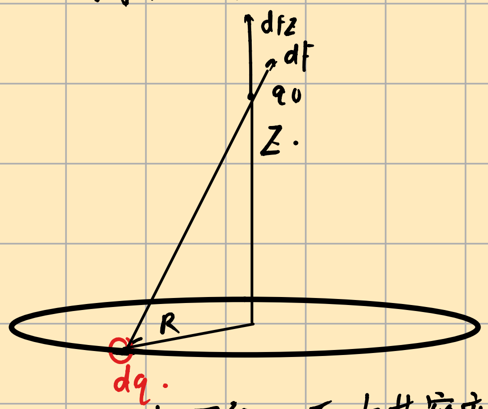
电荷自能（球壳具有的电势能，即从∞把电荷一个个搬运过来组装好消耗的能量）

$$U=\sum_{\substack{i,j=1\\(j>i)}}^{n}\frac{q_iq_j}{4\pi\varepsilon_0 r_{ij}}=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_iq_j}{4\pi\varepsilon_0 r_{ij}}$$

$$=\frac{1}{2}\sum_{i=1}^{n}q_iV_i$$

$$=\frac{1}{2}\int V\,dq=\frac{1}{2}\frac{q}{4\pi\varepsilon_0 R}\cdot q=\frac{q^2}{8\pi\varepsilon_0 R}.$$

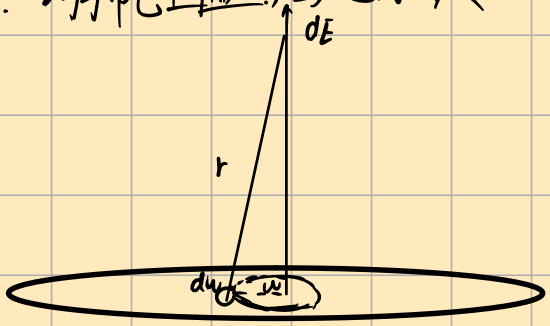
2. 均匀带电圆环 总电量 q.



$$dV=\frac{dq}{4\pi\varepsilon_0 r}=\frac{\lambda ds}{4\pi\varepsilon_0 r}$$

$$V=\int dV=\frac{q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}.$$

3. 均匀带电圆盘，面电荷密度 σ.



$$dq=\sigma\cdot 2\pi w\,dw.$$

$$dV=\frac{dq}{4\pi\varepsilon_0\sqrt{w^2+z^2}}=\frac{\sigma\pi\,dw^2}{4\pi\varepsilon_0\sqrt{w^2+z^2}}$$

$$V=\int_0^R\frac{\sigma\pi\,d(w^2+z^2)}{4\pi\varepsilon_0\sqrt{w^2+z^2}}$$

$$=\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2}-z\right)$$

for $z\gg R$,

$$=\frac{\sigma}{2\varepsilon_0}z\left(\sqrt{1+\frac{R^2}{z^2}}-1\right)$$

$(1+x)^{\frac{1}{2}}=1+\frac{1}{2}x-\frac{1}{8}x^2.$ $\approx\frac{\sigma R^2}{4\varepsilon_0 z}=\frac{q}{4\pi\varepsilon_0 z}.$ （类似点电荷）

绝缘体的介电强度（若电场强度超过这个值，绝缘体被击穿变成导体）

球对称均匀带电体 $E_{surface}=\frac{V}{r}$ （r为半径）

（无论是否导体）

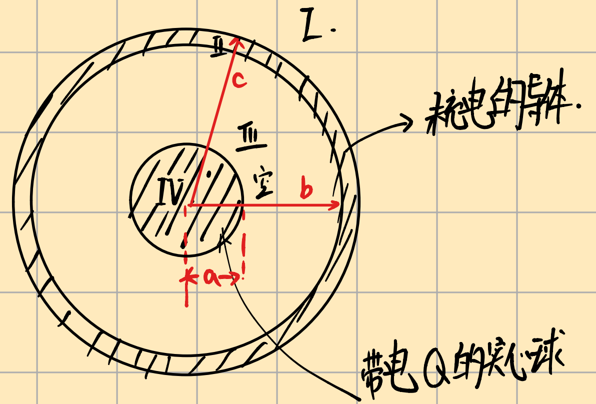
4. Appendix B，计算各区域的电势 距圆心 r 处：

I: $r>c$: $V=\int_p^\infty E\cdot dl$

$$=\int_p^\infty\frac{Q}{4\pi\varepsilon_0 r^2}\cdot dr$$

↓ Gauss

$$=\frac{Q}{4\pi\varepsilon_0 r}$$



II: $b<r<c$ $V=\left(\int_p^{r=c}+\int_{r=c}^\infty\right)E\cdot dl$

$$=\frac{Q}{4\pi\varepsilon_0 c}.$$

III: $V=\left(\int_{p}^{r=b}+\int_{r=b}^{r=c}+\int_{r=c}^{\infty}\right)E\cdot dl$

$=\int_{r}^{b}\frac{Q}{4\pi\varepsilon_0 r^2}dr+\frac{Q}{4\pi\varepsilon_0 c}$

$=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{c}+\frac{1}{r}-\frac{1}{b}\right)$

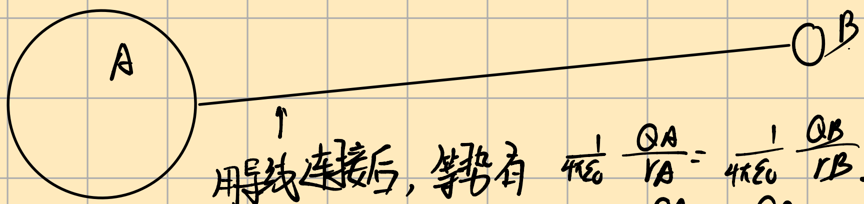
IV: $V=\left(\int_{p}^{r=a}+\int_{r=a}^{r=b}+\int_{r=b}^{r=c}+\int_{r=c}^{\infty}\right)E\,dl$

for $r<a$,

$E=\frac{\frac{Q r^3\cdot\frac{4}{3}\pi}{\frac{4}{3}\pi a^3}}{4\pi\varepsilon_0 r^2}=\frac{Qr}{4\pi\varepsilon_0 a^3}$

$V=\frac{Q}{4\pi\varepsilon_0 a^3}\left(\frac{a^2}{2}-\frac{r^2}{2}\right)+\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}+\frac{1}{c}\right)$

5. 等势面 (E垂直于等势面)



用导线连接后，等势有 $\frac{1}{4\pi\varepsilon_0}\frac{Q_A}{r_A}=\frac{1}{4\pi\varepsilon_0}\frac{Q_B}{r_B}$.

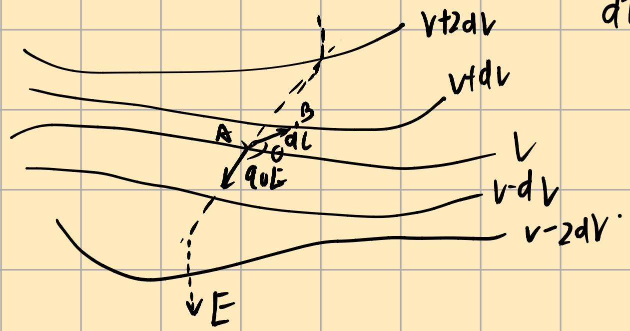
$\frac{Q_A}{r_A}=\frac{Q_B}{r_B}$

对于导体，内部 $E=0$ 表面E与表面垂直

尖端放电：

$\frac{\sigma_A}{\sigma_B}\approx\frac{Q_A/r_A^2}{Q_B/r_B^2}=\frac{r_B}{r_A}$

从V计算E



$dW_{AB}=-q_0 dV$

$=F\cdot dl=q_0E\cdot dl$

$=q_0 E\,dl\cos\theta$. (θ为dl与F夹角)

$-q_0 dV=q_0 E\,dl\cos\theta$.

$E\cos\theta=-\frac{dV}{dl}$.

即 $E_{(沿l)}=E_l=-\frac{dV}{dl}$

而 $E=-\left(\frac{dV}{dl}\right)_{max}$ (此时称这个值为 $\frac{dV}{dl}$ 的梯度)

由上： $E_x=-\frac{\partial V}{\partial x}$ $E_y=-\frac{\partial V}{\partial y}$ $E_z=-\frac{\partial V}{\partial z}$.

即 $E=-\nabla V$

在笛卡尔坐标系 $E=\nabla V=\frac{\partial V}{\partial x}\hat{x}+\frac{\partial V}{\partial y}\hat{y}+\frac{\partial V}{\partial z}\hat{z}$

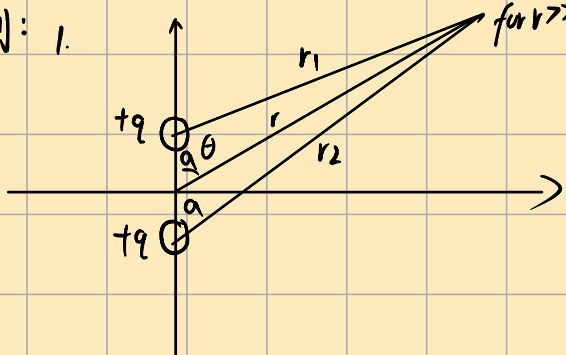
柱坐标系：

$\nabla V=\frac{\partial}{\partial r}(Vr)$

$+\frac{1}{r}\frac{\partial V}{\partial \varphi}\hat{\varphi}+\frac{\partial V}{\partial z}\hat{z}$

球坐标系 $-E=\nabla V=\frac{\partial V}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\hat{\varphi}$

例：1.



for $r\gg a$, $V\approx\frac{2aq\cos\theta}{4\pi\varepsilon_0 r^2}$.

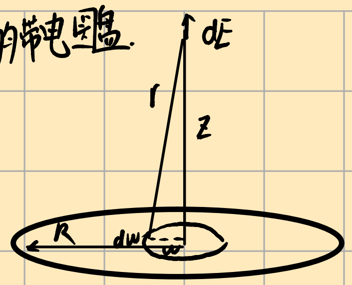
$E=-\frac{\partial V}{\partial r}\hat{r}-\frac{1}{r}\frac{\partial V}{\partial\theta}\hat{\theta}$

$=\frac{qa\cos\theta}{\pi\varepsilon_0 r^3}\hat{r}+\frac{1}{r}\frac{2qa\sin\theta}{4\pi\varepsilon_0 r^2}\hat{\theta}$.

$=\frac{qa\cos\theta}{\pi\varepsilon_0 r^3}\hat{r}+\frac{qa\sin\theta}{2\pi\varepsilon_0 r^3}\hat{\theta}$

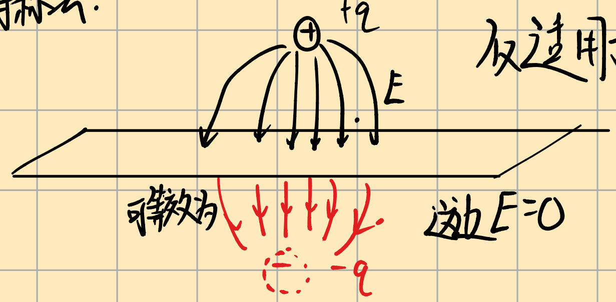
$=\frac{2aq}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{r}+(\sin\theta)\hat{\theta}\right)$

2. 均带电圆盘.



$$V=\int \frac{\sigma\, 2\pi w\,dw}{4\pi\varepsilon_0\sqrt{w^2+z^2}}$$
$$=\int_0^R \frac{\sigma}{4\varepsilon_0}\frac{d(w^2+z^2)}{\sqrt{w^2+z^2}}$$
$$=\frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2}-z\right)$$
$$E=-\frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{R^2+z^2}}-1\right)$$
$$=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{1+(\frac{R}{z})^2}}\right)$$

对称法:



仅适用于靠近场源电荷那一侧区域

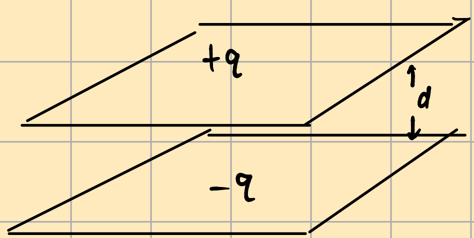
电容器和电介质

1. $C=\frac{Q}{\Delta V}$

求C: 假设q, $\rightarrow E \rightarrow \Delta V \rightarrow C$

e.g.

1. 平行板电容器



$q=\sigma\cdot A$

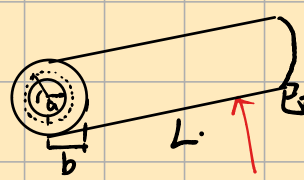
$E=\frac{\sigma}{\varepsilon_0}$

$\Delta V=E\,d=\frac{\sigma d}{\varepsilon_0}$

$C=\frac{q}{\Delta V}=\frac{\varepsilon_0 A}{d}$

2. 圆柱形电容器

设 内径a,带-Q.
外径b,带+Q 电势差V.
已知 长为L.



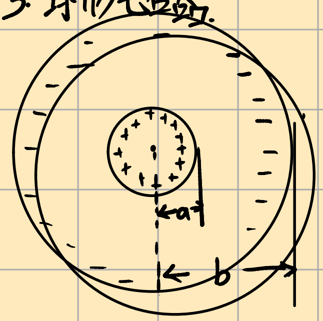
中空,两个半径为a,b的圆柱壳

Gauss: $E\cdot 2\pi r L=\frac{-Q}{\varepsilon_0}$ $E=\frac{-Q}{2\pi L\varepsilon_0 r}$

$$V=-V_{ab}=\int_a^b \frac{Q}{2\pi L\varepsilon_0}\cdot\frac{1}{r}\,dr$$
$$=\frac{Q}{2\pi L\varepsilon_0}\ln\frac{b}{a}$$

$$C=\frac{Q}{\Delta V}=\frac{2\pi L\varepsilon_0}{\ln\frac{b}{a}}$$

3. 球形电容器



实心球, 径a, +Q

外球壳, 径b, −Q, 中空.

$$4\pi r^2 E = \frac{+Q}{\varepsilon_0}.$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

$$\Delta V = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

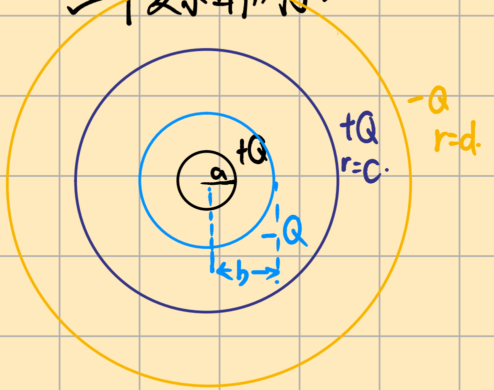
$$C = \frac{Q}{\Delta V} = \frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}} = \frac{4\pi\varepsilon_0 \cdot ab}{b-a}$$

对于孤立球形导体, 可设为 $b=\infty$, $C = 4\pi\varepsilon_0 a = 4\pi\varepsilon_0 R$.

电容并联 $C = C_1 + C_2$

串联 $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Leftrightarrow C = \frac{C_1 C_2}{C_1 + C_2}$

一个复杂的例子:



求ad间电容,

可看作 C_{ab} 与 C_{cd} 串联.

$$C_{ab} = \frac{+Q}{\int_a^b \frac{Q}{4\pi r^2 \varepsilon_0}\,dr} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

$$C_{cd} = \frac{4\pi\varepsilon_0 cd}{d-c}$$

$$\frac{1}{C_{ad}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)$$

$$C_{ad} = \frac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{c}-\frac{1}{b}-\frac{1}{a}}$$

若为圆柱形:

$$C_{ab} = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$$

$$C_{cd} = \frac{2\pi\varepsilon_0 L}{\ln\frac{d}{c}}$$

$$\frac{1}{C_{ad}} = \frac{1}{2\pi\varepsilon_0 L}\ln\frac{bd}{ac}$$

$$C_{ad} = \frac{2\pi\varepsilon_0 L}{\ln\frac{bd}{ac}}$$

2. 电容器储能

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

能量密度.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}.$$

$$u = \frac{W}{Ad} = \frac{\frac{Q^2}{2C}}{Ad} = \frac{\frac{1}{2}E^2\varepsilon_0 A d}{Ad} = \frac{1}{2}\varepsilon_0 E^2.$$

电介质.

插入绝缘体.

若不与电源相连

$C = \kappa_e C_0$

$V = \frac{V_0}{\kappa_e}$.

$E = \frac{V}{d} = \frac{E_0}{\kappa_e}$ → 能量变化，因为电场力对介质做功了

与电源相连.

$C = \kappa_e C_0$

V不变

$Q = \kappa_e Q_0$

微观解释: 感生极化 (无极分子)

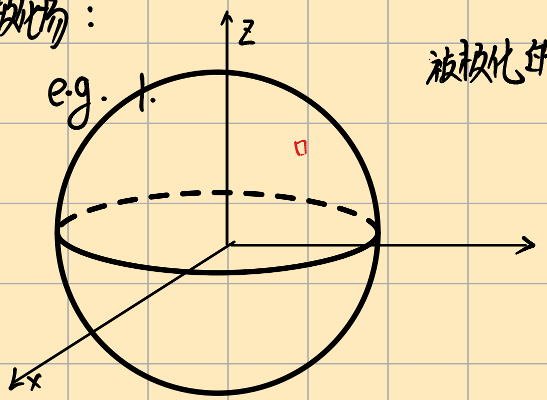
位移极化 (有极分子)

极化强度 P: ≜: $P = \frac{\sum P_{molecular}}{\Delta V} = nql$ (对位移极化)

$$\oiint P \cdot dA = \sum_{out} q' \qquad dq' = P \cdot dA. \qquad \sigma' = \frac{dq'}{dA} = P\cos\theta = P \cdot n = P_n$$

退极化场:

e.g. 1.

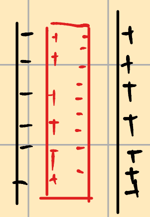


被极化的球的退极化场.

$$E_z \overset{?}{=} \oiint dE' \cos(\pi-\theta) = -dE'\cos\theta = -\frac{P}{3\varepsilon_0}$$

$$dE' = \frac{Rd\theta \cdot R\sin\theta d\varphi \cdot \sigma'}{4\pi\varepsilon_0 R^2}.$$

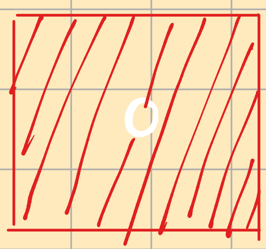
2.



$$E' = \frac{\sigma'}{\varepsilon_0} = \frac{P}{\varepsilon_0}$$

对各向同性材料: $P = \chi_e \varepsilon_0 E$.

$\kappa_e = 1 + \chi_e$



$$\oiint E \cdot dA = \frac{\sum q}{\varepsilon_0}$$

$$\varepsilon_0 \oiint E \cdot dA = \sum_{in} (q_0 + q')$$

$$\oiint P \cdot dA = -\sum_{in} q'.$$

$$\oiint \varepsilon_0 E \cdot dA = \sum_{in} q_0 - \oiint P \cdot dA$$

$$\oiint (\varepsilon_0 E + P) dA = \sum_{in} q_0$$

$$\oiint D \cdot dA = \sum_{in} q_0.$$

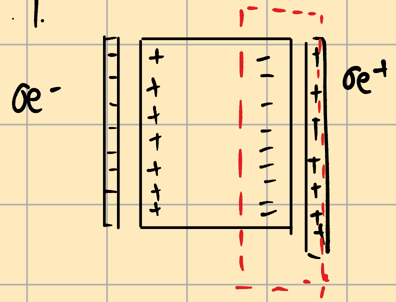
定义 $D = \varepsilon_0 E + P$

若线性 $= \varepsilon_0 E + \chi_e \varepsilon_0 E$

$= (1+\chi_e)\varepsilon_0 E$

$= \kappa_e \varepsilon_0 E$

e.g. 1.

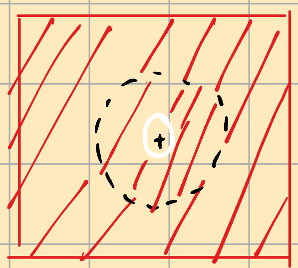


$$\iint D\cdot dA = \sigma e^{+} dA$$

$$k_e \varepsilon_0 E' A = \sigma e^{+} A$$

$$E' = \frac{\sigma e'}{\varepsilon_0 k_e} = \frac{E_0}{k_e}.$$

2.



$$\underset{4\pi r^2 D}{\underbrace{\iint D\,dA}} = \Sigma q_0$$

$$E = \frac{D}{k_e \varepsilon_0} = \frac{q_0}{4\pi r^2 \varepsilon_0 k_e} = \frac{E_0}{k_e}.$$

$$j = \frac{di}{dA} \qquad \iint j\cdot dA = -\frac{dq}{dt}$$

欧姆定律微分形式 $j = \sigma E$. 给题 $j = neu$.

迁移速率

几何光学

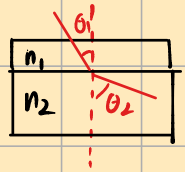
三定律：折、反、直线传播

1. 折射率

$$v=\frac{1}{\sqrt{k_e k_m \varepsilon_0 \mu_0}}=\frac{c}{\sqrt{k_e k_m}}\triangleq\frac{c}{n}.$$

$n\simeq\sqrt{k_e}>1$ （$k_m\simeq 1$，对大多数材料）

2. 全反射



$n_1>n_2$

$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{n_1}{n_2}>1$$

但 $\theta_2\le 90^\circ$

$$\sin\theta_1=\frac{n_2}{n_1}\sin\theta_2<\frac{n_2}{n_1}$$

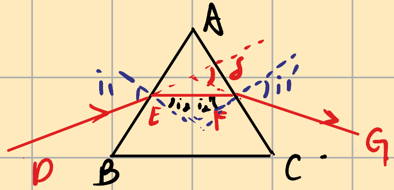
$\sin\theta_1=\frac{n_2}{n_1}$ 时，发生全反射

3. 色散

折射率随 ω 变化 $\quad n(\omega)=1+\frac{A}{\omega^2-\omega_0^2}$

ω_0 由介质决定

4. 棱镜



$\delta=(i_1-i_2)+(i_1'-i_2')$

$\quad=(i_1+i_1')-(i_2+i_2')$

$\alpha=i_2+i_2'$

$\delta=(i_1+i_1')-\alpha$

$$\frac{d\delta}{di_1}=1+\frac{di_1'}{di_1}$$

$\sin i_1=n\sin i_2$

$\sin i_1'=n\sin i_2'$

$$\frac{\sin i_1}{\sin i_1'}=\frac{\sin i_2}{\sin i_2'},\quad \cos i_1=\frac{-\sin i_2}{\sin i_2'}\cos i_1'$$

$i_1'=i_1$ 为极值条件，此时 $i_2=i_2'$

$$n=\frac{\sin\left(\frac{\alpha+\delta_{min}}{2}\right)}{\sin\frac{\alpha}{2}}$$

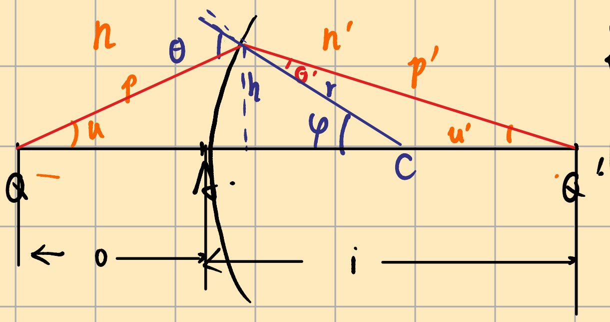
5. 惠更斯原理

6. 费马定理

光选择路径满足 $\delta(QP)=\delta\left[\int_Q^P n\,dl\right]=0$

即光程取最小值/最大值/常数

球面镜成像.



$$\begin{cases} \dfrac{\sin\theta}{\sin\theta'} = \dfrac{n'}{n} & ① \\ \theta - u = \theta' + u' & ② \\ \dfrac{o+r}{\sin\theta} = \dfrac{r}{\sin u} = \dfrac{p}{\sin\varphi} & ③ \\ \dfrac{i-r}{\sin\theta'} = \dfrac{r}{\sin u'} = \dfrac{p'}{\sin\varphi} & ④ \end{cases}$$

$$\frac{③}{④} \quad \frac{o+r}{i-r}\,\frac{n}{n'} = \frac{p}{p'}$$

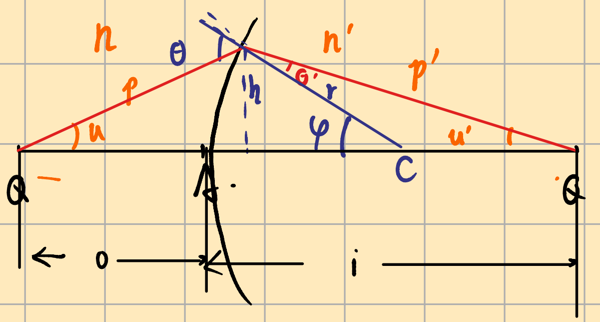
$$\frac{p}{n(o+r)} = \frac{p'}{n'(i-r)} \quad ⑤$$

$$p^2 = r^2 + (o+r)^2 - 2(o+r)r\cos\varphi = o^2 + 4\sin^2\frac{\varphi}{2}(o+r)r$$
$$p'^2 = r^2 + (i-r)^2 + 2(i-r)r\cos\varphi = i^2 - 4\sin^2\frac{\varphi}{2}(i-r)r.$$

代入 $⑤^2$ 可得: $\dfrac{o^2}{n^2(o+r)^2} - \dfrac{i^2}{n'^2(i-r)^2} = -4r\sin^2\dfrac{\varphi}{2}\left[\dfrac{1}{n^2(o+r)} + \dfrac{1}{n'^2(i-r)}\right]$

fixed o, 对不同的 φ, i 不同, 所以球面镜是不能成像的.

但对于 $h^2 << o^2, i^2, r^2$, 可以作傍轴近似



$u^2, u'^2, \varphi^2 << 1 \Rightarrow \theta^2, \theta'^2 << 1$

$\Rightarrow \sin^2\frac{\varphi}{2} \simeq (\frac{\varphi}{2})^2 \to 0$

$$\frac{o^2}{n^2(o+r)} = \frac{i^2}{n'^2(i-r)}$$

$$\frac{n(o+r)}{o} = \frac{n'(i-r)}{i}$$

$i \to \infty$, $f = o = \dfrac{nr}{n'-n}$ 第一焦距

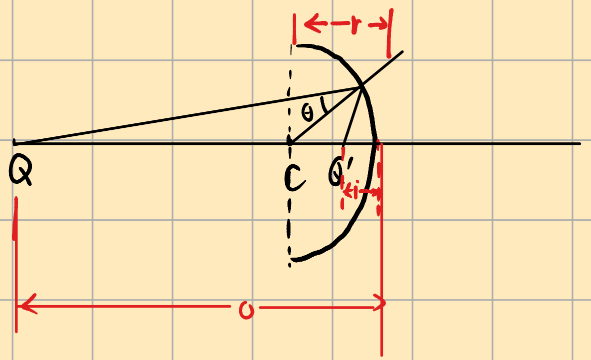
$$n + \frac{nr}{o} = n' - \frac{n'r}{i}$$

$o \to \infty$, $f' = i = \dfrac{n'r}{n'-n}$ 第二焦距.

$$\frac{nr}{o} + \frac{n'r}{i} = n' - n.$$

$$\Rightarrow \frac{f}{o} + \frac{f'}{i} = 1$$

球面反射成镜. (这里的正负相反, 成在左边为正, 右边为负)



$$\frac{f}{o} + \frac{f'}{i} = 1$$

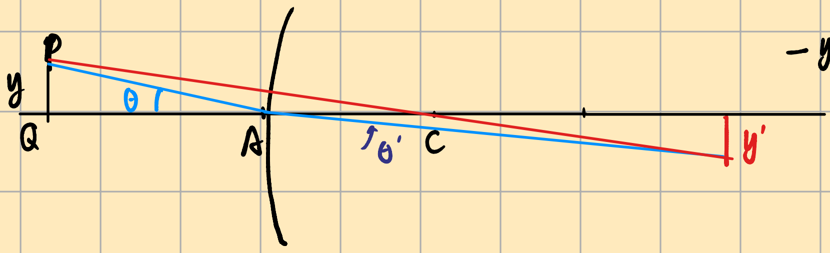
$f = \dfrac{n}{n'-n} r$. (这里 $n' = -n$)
$= -\dfrac{r}{2}$

$f' = \dfrac{n'}{n'-n} = \dfrac{r}{2}$

$$\frac{-\frac{r}{2}}{o} + \frac{\frac{r}{2}}{(-i)} = 1$$

$$\frac{1}{o} + \frac{1}{i} = -\frac{2}{r}.$$

放大倍数



$m=\frac{y'}{y}$

$-y'\approx i\theta'$

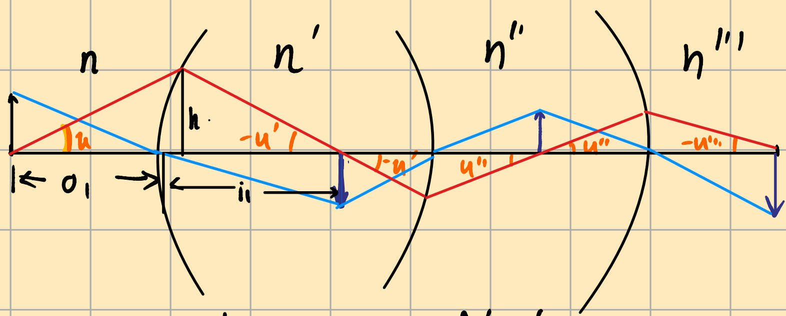
$y\approx o\theta$

$m=\frac{y'}{y}\approx\frac{-i\theta'}{o\theta}\approx\frac{-i}{o}\frac{\sin\theta'}{\sin\theta}\approx\frac{-ni}{n'o}$

对于镜：$m=\frac{-i}{o}$

$$\frac{nr}{o}+\frac{n'r}{i}=n'-n$$

复合光学系统



$$\begin{cases}\frac{nr}{o}+\frac{n'r}{i}=n'-n\\ \frac{n'r}{o}+\frac{n''r}{i}=n''-n'\\ \frac{n''r}{o}+\frac{n'''r}{i}=n'''-n''\end{cases}$$

$$\begin{cases}\frac{f_1'}{i_1}+\frac{f_1}{o_1}=1\\ \frac{f_2'}{i_2}+\frac{f_2}{o_2}=1\\ \frac{f_3'}{i_3}+\frac{f_3}{o_3}=1\end{cases}$$

$$\begin{cases}m_1=\frac{-n\,i_1}{n'o_1}\\ m_2=\frac{-n'i_2}{n''o_2}\\ m_3=\frac{-n''i_3}{n'''o_3}\end{cases}$$

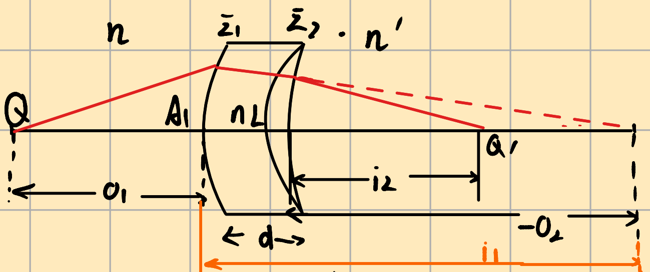
$$\begin{cases}u\approx\frac{h}{QA_1}=\frac{h}{o_1}\\ -u'\approx\frac{h}{i_1}\\ \frac{u}{u'}=-\frac{i_1}{o_1}\end{cases}$$

总放大倍数：$m=m_1m_2m_3$

$m=\frac{-ni}{n'o}=\frac{nu}{n'u'}$

Lagrange-Helmholtz law：　$ynu=y'n'u'=y''n''u''=\cdots$

薄透镜式



$$\begin{cases}-o_2=i_1-d\\ o_2=d-i_1\end{cases}$$

$$\begin{cases}\frac{f_1}{o_1}+\frac{f_1'}{i_1}=1\\ \frac{f_2}{o_2}+\frac{f_2'}{i_2}=1\end{cases}$$

$$\begin{cases}f_1=\frac{n}{n_L-n}r_1\qquad f_1'=\frac{n_L}{n_L-n}r_1\\ f_2=\frac{n_L}{n'-n_L}r_2.\quad f_2'=\frac{n'}{n'-n_L}r_2\end{cases}$$

$$\begin{cases} \dfrac{f_1'f_2}{i_1} + \dfrac{f_1f_2}{o_1} = f_2 \\ \dfrac{f_1'f_2'}{i_2} + \dfrac{f_2f_1'}{o_2} = f_1' \end{cases}$$

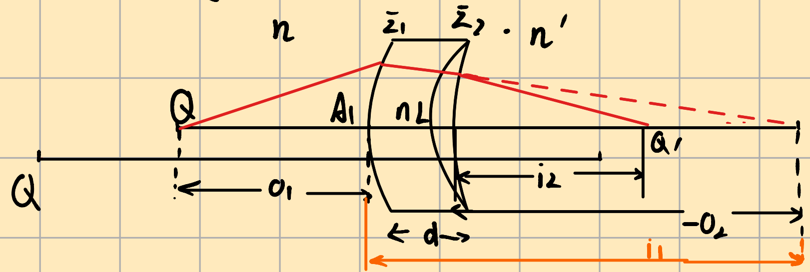
$o_2 = d - i_1 \xrightarrow{d \approx 0} o_2 \approx -i_1$

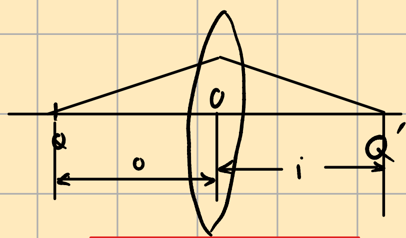
$$\Rightarrow \begin{cases} \dfrac{f_1f_2}{o_1} + \dfrac{f_1'f_2}{i_1} = f_2 \\ \dfrac{f_1'f_2}{-i_1} + \dfrac{f_2'f_1'}{i_2} = f_1' \end{cases}$$

$$\Rightarrow \frac{f_1f_2}{o_1} + \frac{f_1'f_2'}{i_2} = f_1' + f_2.$$

$$\frac{f}{o} + \frac{f'}{i} = 1 \qquad f = \frac{f_1f_2}{f_1'+f_2} \quad f' = \frac{f_1'f_2'}{f_1'+f_2}$$

磨镜者公式





$$\boxed{\begin{aligned} o &= OQ \approx o_1 \\ i &= OQ' \approx i_2 \end{aligned}}$$

$f_1 = \dfrac{n}{n_L - n} r_1 \quad f_1' = \dfrac{n_L}{n_L - n} r_1$

$f_2 = \dfrac{n_L}{n' - n_L} r_2 \quad f_2' = \dfrac{n'}{n' - n_L} r_2$

$$\begin{cases} \dfrac{f_1}{o_1} + \dfrac{f_1'}{i_1} = 1 \\ \dfrac{f_2}{o_2} + \dfrac{f_2'}{i_2} = 1 \end{cases} \Rightarrow \begin{cases} \dfrac{f_1f_2}{o_1} + \dfrac{f_1'f_2}{i_1} = f_2 \\ \dfrac{f_1'f_2}{o_2} + \dfrac{f_1'f_2'}{i_2} = f_1' \end{cases} \Rightarrow \frac{f_1f_2}{o_1} + \frac{f_1'f_2'}{i_2} = f_1' + f_2.$$

$$\frac{f}{o} + \frac{f'}{i} = 1$$

$$f' = \frac{f_1'f_2'}{f_1'+f_2} = \frac{\frac{n_L}{n_L-n} r_1 \cdot \frac{n'}{n'-n_L} r_2}{\frac{n_L}{n_L-n} r_1 + \frac{n_L}{n'-n_L} r_2} = \frac{n' r_1 r_2}{(n'-n_L) r_1 + (n_L - n) r_2} = \frac{n'}{\frac{n'-n_L}{r_2} + \frac{n_L-n}{r_1}}$$

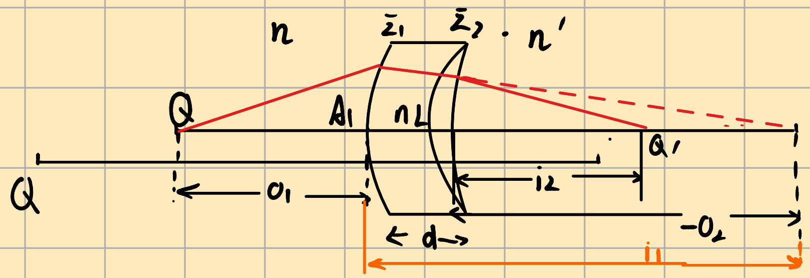
$$f = \frac{f_1f_2}{f_1'+f_2} = \frac{\frac{n}{n_L-n} r_1 \cdot \frac{n_L}{n'-n_L} r_2}{\frac{n_L}{n_L-n} r_1 + \frac{n_L}{n'-n_L} r_2} = \frac{n\, r_1 r_2}{(n'-n_L) r_1 + (n_L-n) r_2} = \frac{n}{\frac{n_L-n}{r_1} + \frac{n'-n_L}{r_2}}$$

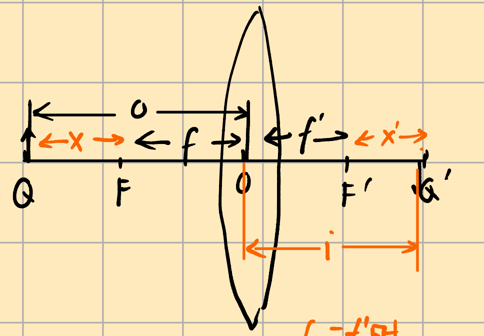
$$\frac{f'}{f} = \frac{n'}{n}.$$

$n = n'$, $\quad f = f' = \dfrac{n}{\frac{n_L-n}{r_1} + \frac{n-n_L}{r_2}} = \dfrac{1}{\left(\frac{n_L-n}{n}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$

$n = n' = 1 \quad = \dfrac{1}{(n_L - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$

薄透镜放大倍数





$f=f'$时

$\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$

$\frac{1}{f+x}+\frac{1}{f+x'}=\frac{1}{f}$

$xx'=f^2$

$$m=m_1 m_2=-\frac{n\, i_1}{n_L\, o_1}\cdot -\frac{n_L\, i_2}{n'\, o_2}$$

$$=\frac{n}{n'}\frac{i_1 i_2}{o_1 o_2}$$

$$=\frac{-n\, i}{n' o}$$

$$=-\frac{f\, i}{f' o}$$

$$\boxed{\begin{aligned} o&=o_1\\ i_1&=-o_2\\ i&=i_2.\end{aligned}}$$

$f=f'$时

$$m=-\frac{f+x'}{f+x}$$

$$=-\frac{f+\frac{f^2}{x}}{f+x}$$

$$=-\frac{f}{x}$$

$$=-\frac{x'}{f}$$

应用：近视，远视，老花；放大镜 → 角放大率 M 处于 $\frac{N}{f}\sim\frac{N}{f}+1$

像成在∞　像成在近点