普通物理学Ⅱ(H).

个人期末笔记整理.

电学

1. Coulomb's law. 库仑定律

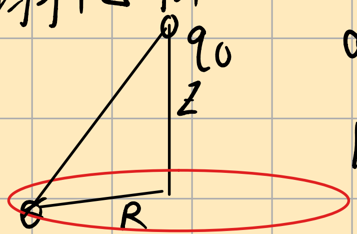
$$F_{12}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\cdot\hat{r}_{12}$$,对点电荷成立.

电荷密度: 线: $\lambda=\frac{dq}{dx}$

面: $\sigma=\frac{dq}{dA}$

体: $\rho=\frac{dq}{dV}$.

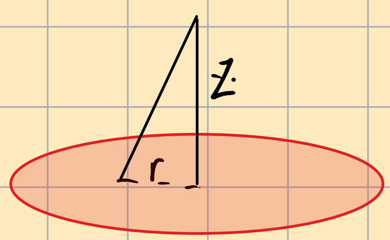
1)均匀带电圆环



$$dF_z=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0\cdot\lambda dx\,z}{(R^2+z^2)^{\frac{3}{2}}}$$

$$F_z=\frac{1}{4\pi\varepsilon_0}\frac{q_0Qz}{(R^2+z^2)^{\frac{3}{2}}}$$

2)均匀带电圆盘



看作许多圆环:

$$dF_z=\frac{1}{4\pi\varepsilon_0}\frac{q_0\,\sigma\cdot 2\pi r dr\,z}{(r^2+z^2)^{\frac{3}{2}}}$$

$$=\frac{q_0\sigma z}{2\varepsilon_0}\frac{rdr}{(r^2+z^2)^{\frac{3}{2}}}.$$

$$F_z=\frac{q_0\sigma z}{2\varepsilon_0}\int_0^R\frac{rdr}{(r^2+z^2)^{\frac{3}{2}}}$$

$$=\frac{q_0\sigma z}{4\varepsilon_0}\int_0^R\frac{d(r^2+z^2)}{(r^2+z^2)^{\frac{3}{2}}}$$

$$=\frac{q_0\sigma z}{4\varepsilon_0}\cdot(-2)\cdot(r^2+z^2)^{-\frac{1}{2}}\Big|_0^R$$

$$=\frac{q_0\sigma z}{2\varepsilon_0}\left[\frac{1}{z}-\frac{1}{\sqrt{R^2+z^2}}\right]$$

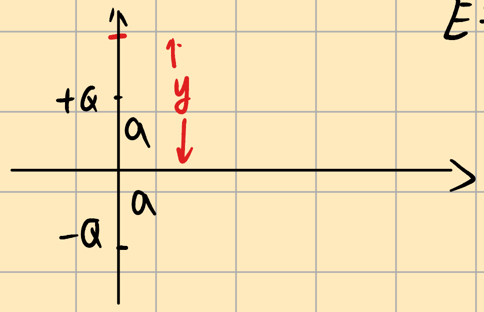
$$=\frac{q_0Q}{2\pi R^2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$$

2. 电场

点电荷电场: $\vec{E}=\frac{\vec{F}}{q_0}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$.

满足叠加原理.

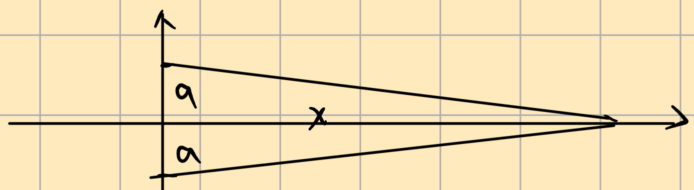
电偶极矩: $p = q\vec{l}$ （l 从负电荷指向正电荷）



$$E = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{(y-a)^2} - \frac{1}{(y+a)^2}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\,\frac{4ay}{(y^2-a^2)^2}$$

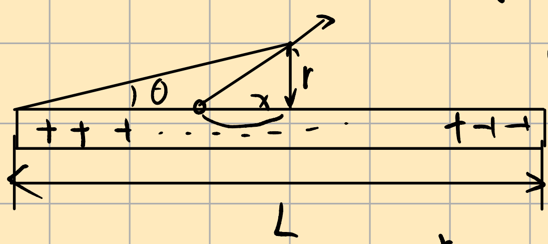
$$= \frac{Q}{4\pi\varepsilon_0}\cdot\frac{4a}{y^3\left(1-\frac{a^2}{y^2}\right)^2} \approx \frac{Qa}{\pi\varepsilon_0 y^3}\ (y \gg a)$$



$$E_y = -2\cdot\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2+a^2}\cdot\frac{a}{\sqrt{x^2+a^2}}$$

$$= \frac{-2Qa}{4\pi\varepsilon_0(x^2+a^2)^{\frac{3}{2}}} \approx \frac{-p}{4\pi\varepsilon_0 x^3}$$

无限长线电荷



$$\lambda = \frac{Q}{L}$$

$$dE = \frac{\lambda\, dx\cdot r}{4\pi\varepsilon_0 (x^2+r^2)^{\frac{3}{2}}}$$

$$E = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\lambda r}{4\pi\varepsilon_0}\,\frac{dx}{(x^2+r^2)^{\frac{3}{2}}}$$

$x = \frac{r}{\tan\theta}$ $dx = -\frac{r}{\sin^2\theta}\,d\theta$

$$E = \frac{\lambda r}{4\pi\varepsilon_0}\int_{\theta_1}^{\theta_2} -\frac{\sin\theta}{r^2}\,d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\cos(\theta_2-\theta_1) = \frac{\lambda}{2\pi\varepsilon_0 r}$$

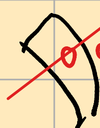
电偶极矩在电场中:

$\vec{p} = q\vec{d}$ $\tau = \vec{p}\times\vec{E}$

$$W = \int dw = \int_{\theta_0}^{\theta} \tau\cdot d\theta$$

高斯定理:

通量，以法向为正



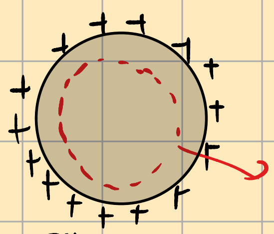
$d\Phi = E\cdot n\cdot dA$

$$\oiint \vec{E}\,d\vec{A} = \Phi_E = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\Downarrow$$

$$\nabla\cdot E = \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{y} + \frac{\partial E_z}{\partial z}\right) = \frac{1}{\varepsilon_0}\rho$$

应用：1. 导体内没有净电荷



$$\oiint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0} \qquad q=0$$

$E=0$

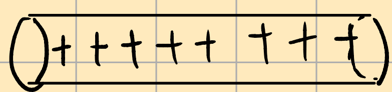
Gaussian surface

2. 异形：

$$E\Delta A = \frac{1}{\varepsilon_0}\sigma\Delta A$$

$$E = \frac{\sigma}{\varepsilon_0}$$

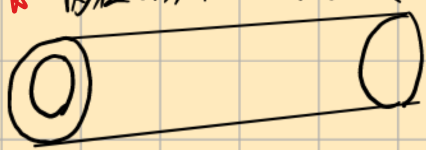
3. 无穷长导线



$$E\cdot 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi r\varepsilon_0}$$

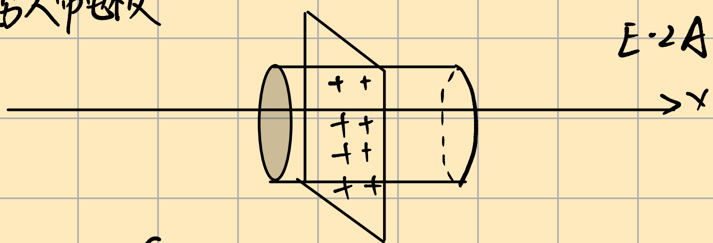
☆ 内径 a，外径 b，轴线上放一线电荷，线电荷密度为 λ，求外表面 σ.



$$E = \frac{\sigma}{\varepsilon_0} = \frac{\lambda}{2\pi b\varepsilon_0}$$

$$\sigma = \frac{\lambda}{2\pi b}$$

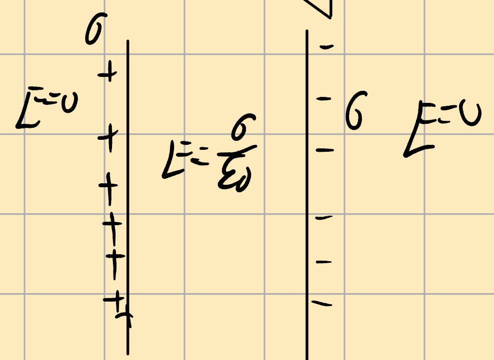
4. 无穷大带电板



$$E\cdot 2A = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

5.



$$-2\pi R\cdot h\cdot\sigma_{inner} + \lambda h = 0$$

$$\sigma_{inner} = \frac{\lambda}{2\pi R}$$

势能:

静电力是保守力.

$$\Delta U_{ab}=U_b-U_a=-\int_a^b F\cdot dl=-q\int_a^b E dl$$

对点电荷 $\Delta U_{ab}=-q\int_{r_a}^{r_b}\frac{Q}{4\pi\varepsilon_0 r^2}dr$

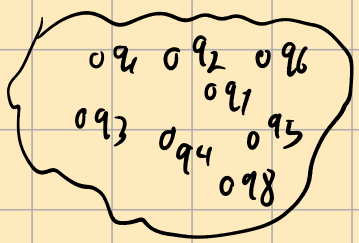
$$=\frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{r_b}-\frac{1}{r_a}\right)$$

$r_a=\infty$时 定义 $U_\infty=0$ 则 $U(r)=\frac{q_1q_2}{4\pi\varepsilon_0 r}=\int_r^\infty F\cdot dl$.

电势能是系统具有的

$$U(r)=\frac{q_1q_2}{4\pi\varepsilon_0 r}$$

多个点电荷系统



$$U=\sum_{i\neq j}\frac{1}{2}U_{ij}$$

静电场环路定律

$$\oint E dl=0 \Rightarrow \oint E dl=\iint \nabla\times E\, dS=0$$

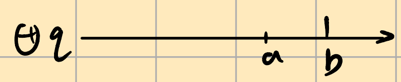
$$\nabla\times E=0$$

定义电势: $V=\frac{U}{q_0}=\frac{q}{4\pi\varepsilon_0 r}$ (对点电荷)

$$V_B-V_A=\frac{W_{AB}}{q}=-\int_A^B E\cdot dl$$

定义 $V_\infty=0$. $V_p=-\int_\infty^p E dl=\int_p^\infty E\cdot dl$

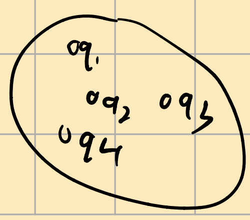
点电荷电势.



$$V_b-V_a=-\int_a^b E\cdot dl=-\int_{r_a}^{r_b}\frac{q}{4\pi\varepsilon_0 r^2}dr$$

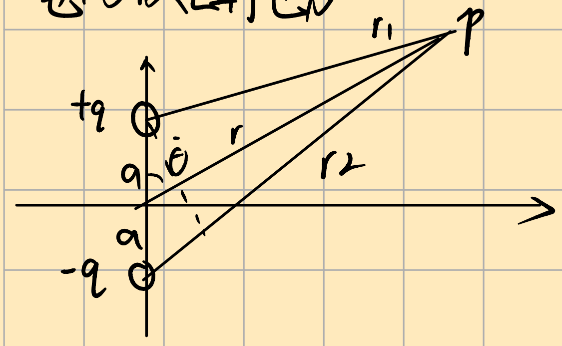
$$=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_b}-\frac{1}{r_a}\right)$$

$$V=\frac{q}{4\pi\varepsilon_0 r}\quad (r_a=\infty\ \ V_a=0)$$



point p $\quad V_p=\sum_{i=1}^{N}\frac{q_i}{4\pi\varepsilon_0 r_i}$

电偶极矩的电势.



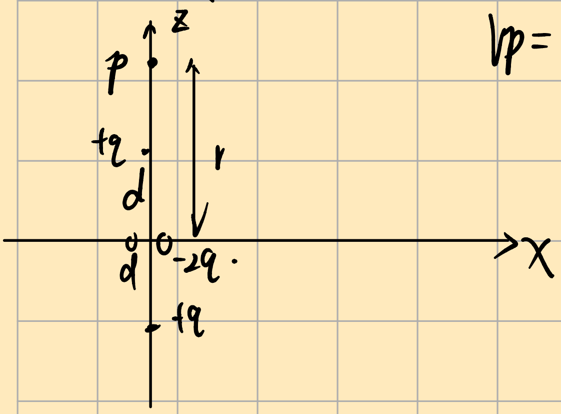
$$V_p = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

for $r \gg a$

$$= \frac{q}{4\pi\varepsilon_0} \frac{2a\cos\theta}{r^2}$$

$$= \frac{p\cdot\hat{r}}{4\pi\varepsilon_0 r^2}.$$

电四偶极矩.



$$V_p = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r-d} + \frac{1}{r+d} - \frac{2}{r}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{2r}{r^2-d^2} - \frac{2}{r}\right)$$

$$= \frac{2q}{4\pi\varepsilon_0} \frac{d^2}{r(r^2-d^2)}$$

$$\approx \frac{2qd^2}{4\pi\varepsilon_0 r^3} = \frac{Q}{4\pi\varepsilon_0 r^3}$$

$Q = 2qd^2$，称为电四偶极矩

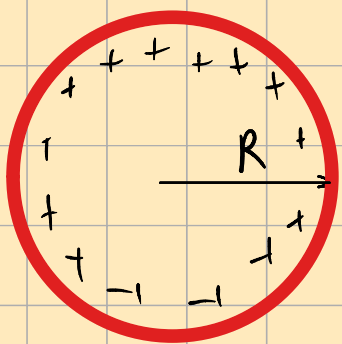
连续分布电荷体的电势能:

线电荷密度 $dq = \lambda dx$

面 $dq = \sigma dA$

体 $dq = \rho dV$

eg. 1. 球壳(充电)



$\cdot P$

$E = 0 \quad (r < R)$

$E = \frac{q}{4\pi\varepsilon_0 r^2} \quad (r > R)$

$r_p > R \quad V_p = \int_p^\infty E\,dr = \frac{q}{4\pi\varepsilon_0 r_p}$

$r_p < R \quad V_p = \int_R^\infty E\,dr = \frac{q}{4\pi\varepsilon_0 R}$

球壳自身的能量(电势能)

$$U = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}$$

$$= \frac{1}{2}\sum_{i=1}^{n} q_i V_i$$ (其它点在q_i产生的电势)

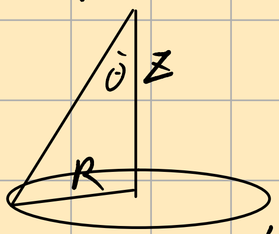
$$= \frac{1}{2}\int V\,dq = \frac{q^2}{8\pi\varepsilon_0 R}$$

→ 计算电子半径

$$W = mc^2 = \frac{e^2}{8\pi\varepsilon_0 R}$$

$$R = \frac{e^2}{8\pi\varepsilon_0 mc^2}$$

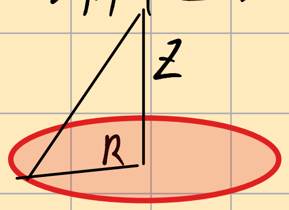
2. 均匀带电圆环



$$dV=\frac{dq}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$$

$$V=\frac{q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$$

3. 均匀带电圆盘



$$dV=\frac{dq}{4\pi\varepsilon_0\sqrt{w^2+z^2}}=\frac{\sigma\, 2\pi w\, dw}{4\pi\varepsilon_0\sqrt{w^2+z^2}}=\frac{\sigma}{4\varepsilon_0}\frac{d(w^2+z^2)}{\sqrt{w^2+z^2}}=\frac{\sigma}{2\varepsilon_0}d\sqrt{w^2+z^2}$$

$$V=\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2}-z\right)$$

$z\gg R$

$$V=\frac{\sigma z}{2\varepsilon_0}\left(\sqrt{1+\frac{R^2}{z^2}}-1\right)\approx\frac{\sigma z}{2\varepsilon_0}\left(\frac{R^2}{2z^2}\right)=\frac{q}{4\pi\varepsilon_0 z}$$

等势面. 电场与他们垂直. 导体表面电场一定是垂直的

从V计算E

$$E=-\left(\frac{dV}{dl}\right)_{max}\quad or:\ E\cdot\hat{l}=-\frac{dV}{dl}$$

$$E_x=-\frac{\partial V}{\partial x}\quad E_y=-\frac{\partial V}{\partial y}\quad E_z=-\frac{\partial V}{\partial z}$$

$$E=-\nabla V=-\left(\frac{\partial V}{\partial x}\hat{x}+\frac{\partial V}{\partial y}\hat{y}+\frac{\partial V}{\partial z}\hat{z}\right)$$

↳ 注意此处是对V梯度, 和Gauss中 $\nabla\cdot E$ 不同 $\left(\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}\right)$

球坐标系:

$$-\nabla V=-\left[\frac{\partial V}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial V}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\hat{\phi}\right]$$

柱坐标系:

$$\nabla V=\frac{\partial}{\partial r}(Vr)\hat{r}+\frac{1}{r}\frac{\partial V}{\partial\phi}\hat{\phi}+\frac{\partial V}{\partial z}\hat{z}$$

例: 1. 得到球坐标系下电偶极子的 $\vec{E}$.

$$V=\frac{2aq\cos\theta}{4\pi\varepsilon_0 r^2}$$

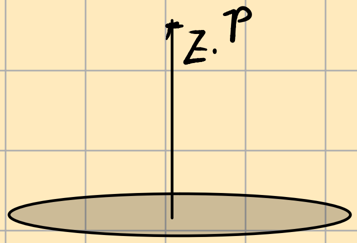
$$\nabla V=\frac{\partial V}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial V}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\hat{\phi}$$

$$E_r=\frac{aq\cos\theta}{\pi\varepsilon_0 r^3}\hat{r}$$

$$E_\theta=\frac{1}{r}\cdot\frac{2aq\sin\theta}{4\pi\varepsilon_0 r^2}\hat{\theta}$$

$E_\phi=0$

2. 均匀带电圆盘



$$V=\frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2}-z\right)$$

$$E_z=-\frac{\partial V}{\partial z}\hat{z}$$

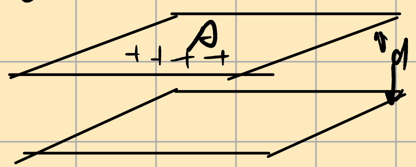
$$=-\frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{R^2+z^2}}-1\right)$$

$$=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{R^2+z^2}}\right)$$

电容器和电介质

1. $C=\frac{Q}{\Delta V}$. 求C: 设q. 求E, $\Rightarrow \Delta V$

e.g. 1. 平行板电容器

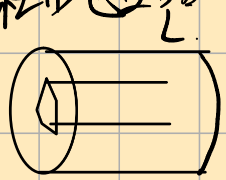


$$q=\sigma\cdot A \qquad C=\frac{\varepsilon_0 A}{d}$$

$$E=\frac{\sigma}{\varepsilon_0}$$

$$\Delta V=\frac{\sigma d}{\varepsilon_0}$$

2. 圆柱形电容器



内径 a −Q

外径 b +Q.

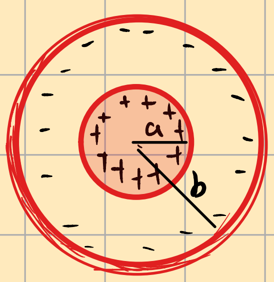
$$E\cdot 2\pi r\cdot l=\frac{Q}{\varepsilon_0}$$

$$E=\frac{Q}{\varepsilon_0 2\pi l r}$$

$$\Delta V=\left|\int E\,dl\right|=\frac{Q}{\varepsilon_0 2\pi l}\ln\frac{b}{a}$$

$$C=\frac{\varepsilon_0 2\pi l}{\ln\frac{b}{a}}$$

3. 球形电容器



$$E=\frac{Q}{4\pi\varepsilon_0 r^2}$$

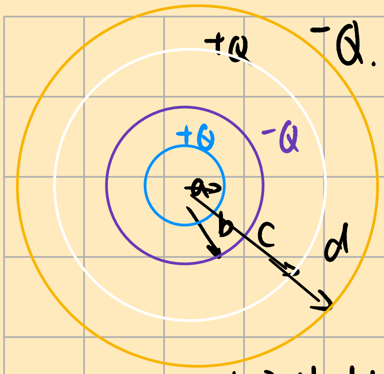
$$\Delta V=\left|-\int_a^b E\,dr\right|$$

$$=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$C=\frac{Q}{\Delta V}=\frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}}=\frac{4\pi\varepsilon_0 ab}{b-a}$$

电容并联：电阻串联

电容串联：电阻并联



ad间电容：
C_{ab}与C_{cd}串联.

电容器储存的能量：

$$dW = V(q)dq = \frac{q}{C}dq$$

$$W = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

$$C = \frac{\varepsilon_0 A}{d} \quad E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}. \quad V = E\cdot d \quad W = \frac{1}{2}\varepsilon_0 E^2 A d$$

单体体积能量，也即能量密度 $u = \frac{W}{Ad} = \frac{1}{2}\varepsilon_0 E^2$

这是普适的

电介质.

$$C = k_e C_0$$

极化强度矢量.

$$\vec{P} = \frac{\sum \vec{p_i}}{\Delta V} = nq\vec{l}. \quad dN = nd V = nl dA\cos\theta. \quad dq' = q dN = nql dA\cos\theta = p\cdot dA.$$

$$\oint p\cdot dA = \sum q'_{out} = -\sum q'_{in}$$

$$dq' = q'_n dV = p\cdot dA. \quad p dA\cos\theta = dq' \quad \vec{P_n} = \vec{P_n} = P\cos\theta = \sigma' \text{（束缚电荷面密度）}$$

极化规律：

$$\vec{P} \Rightarrow \sigma e' \Rightarrow \vec{E'} \Rightarrow E.$$

对各向同性材料 $\vec{P} = \chi_e \varepsilon_0 \vec{E}$

$$\chi_e = k_e - 1.$$

电位移矢量

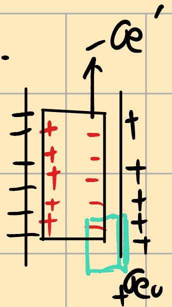
$$\vec{D} = (\varepsilon_0\vec{E} + \vec{P}),$$

$$\varepsilon_0 \oint E\cdot dA = \sum q = \sum_{in}(q_{自由} + q'_{in})$$
$$= \sum_{in} q_{自由} - \oint p\cdot dA$$

$$\oint(\varepsilon_0\vec{E} + \vec{P})\, dA = \sum_{in} q_{自由}.$$

对各向同性材料 $\vec{P} = \chi_e\varepsilon_0\vec{E} \quad \vec{D} = \varepsilon_0 k_e \vec{E}.$

e.g.



$$\oint D\cdot dA=\sum q_{in}$$

导体中无电场: $\Downarrow$

$$D_1\, A=\sigma A$$

$$\kappa_e\varepsilon_0 E=\sigma_{e0}$$

$$\vec{E}=\frac{\sigma_{e0}}{\kappa_e\varepsilon_0}=\frac{E_0}{\kappa_e}$$

稳恒电流:

电流密度 j. $\quad di=j\,dA$. $\quad i=\iint_A j\,dA=\iint_A j\cos\theta\, dA$

电流连续方程: $\oiint_A j\,dA=-\frac{dq}{dt}$. $\quad R=\int\rho\frac{dl}{A}$.

欧姆定律积分形式:

$$\begin{cases} i=\frac{\Delta V}{R}=\frac{\int E\,dl}{R} \\ i=\iint j\,dA \\ R=\int\rho\frac{dl}{A} \end{cases}$$

微分形式:

$$\Delta i=\frac{\Delta V}{R}.$$

$$j\Delta A=\frac{E\Delta l}{\rho\frac{\Delta l}{\Delta A}}$$

$$j=\frac{E}{\rho}=\sigma E.$$

微观解释: $j=neu$ $\quad u=\frac{u_1}{2}=\frac{1}{2}\frac{eE}{m}\cdot\underbrace{\tau}_{\text{平均自由时间}}=\frac{1}{2}\frac{eE}{m}\frac{\lambda}{\bar{v}_t}$

$\lambda\to$ 平均自由程

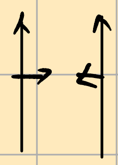
$\bar{v}_t\to$ 平均热运动速度.

$$j=\sigma E \qquad \sigma=\frac{j}{E}=\frac{neu}{E}=\frac{ne^2\lambda}{2m\bar{v}_t}\propto\frac{1}{\sqrt{T}}$$

磁学

1. 安培定律.

$$dF_{12} = \frac{\mu_0}{4\pi}\,\frac{i_2 ds_2 \times (i_1 ds_1 \times \hat{r}_{12})}{r_{12}^2}$$



两电流元平行，互相吸引.

2. $$dF_2 = \frac{\mu_0}{4\pi}\,\frac{i_2 ds_2 \times (i_1 ds_1 \times \hat{r}_{12})}{r_{12}^2}$$

$$= i_2 ds_2 \left(\frac{\mu_0}{4\pi}\,\frac{i_1 ds_1 \times \hat{r}_{12}}{r_{12}^2} \right)$$

定义 $B_1 = \frac{\mu_0}{4\pi} \oint_{L_1} \frac{i_1 ds_1 \times \hat{r}_{12}}{r_{12}^2}$.

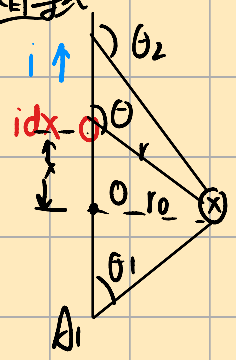
$$dF_2 = i_2 ds_2 \times B_1$$

3. Biot-Savart law.

$$B = \frac{\mu_0}{4\pi} \oint_L \frac{i ds \times \hat{r}}{r^2}.$$

常见磁场分布

1. 长直导线



$$dB = \frac{\mu_0}{4\pi}\,\frac{i dx \times \hat{r}}{r^2}.$$

$$r^2 = \frac{r_0^2}{\sin^2\theta}.$$

$$-\tan\theta = \frac{r_0}{x}$$

$$x = -\frac{r_0}{\tan\theta} \qquad \frac{\cos\theta}{\sin\theta}$$

$$dx = \frac{r_0}{\sin^2\theta}\, d\theta$$

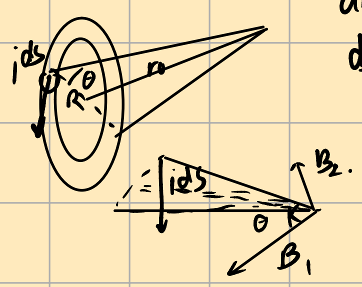
$$\frac{dx \times \hat{r}}{r^2} = \frac{\frac{r_0}{\sin\theta} d\theta}{r_0^2} \cdot \sin^2\theta = \frac{\sin\theta}{r_0}\, d\theta.$$

$$dB = \frac{\mu_0 i \sin\theta\, d\theta}{4\pi r_0}$$

$$B = \frac{\mu_0 i}{4\pi r_0} (\cos\theta_1 - \cos\theta_2)$$

无限长导线 $B = \frac{\mu_0 i}{2\pi r_0}$

2. 圆电流环.



$$dB = \frac{\mu_0}{4\pi}\,\frac{i ds \times r}{r^2}.$$

$$dB_{合} = 2 dB \cos\theta$$

$$r = \frac{r_0}{\sin\theta}.$$

$$dB_{合} = 2\cos\theta\, \frac{\mu_0}{4\pi}\, \frac{\sin^2\theta\, i ds}{r_0^2}$$

$$= \frac{\mu_0 i}{2\pi r_0^2} \sin^2\theta \cos\theta\, ds$$

$$B = \frac{\mu_0 i R^2}{2} \cdot \frac{1}{(r_0^2 + R^2)^{\frac{3}{2}}}.$$

$$\sin\theta = \frac{r_0}{\sqrt{r_0^2 + R^2}} \qquad \cos\theta = \frac{R}{\sqrt{r_0^2 + R^2}}.$$

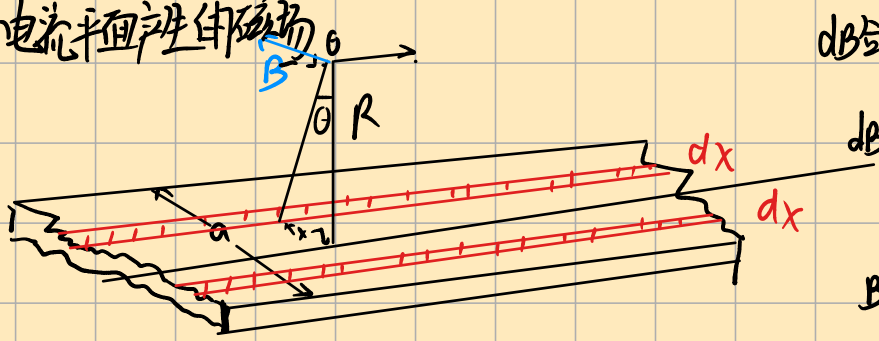
$r_0 = 0$: $B = \frac{\mu_0 i}{2R}.$

$r_0 \gg R$: $B = \frac{\mu_0 i R^2}{2 r_0^3}.$

定义 磁偶极矩：$\mu = iA = i\pi R^2$. $B=\frac{\mu_0 i R^2}{2 r^3} = \frac{\mu_0}{2\pi}\cdot\frac{\mu}{r^3}$.

N匝线圈要$\times N$.

3. 电流平面产生的磁场



$dB_{总} = 2dB\cos\theta$ $\quad dx = d(R\tan\theta) = R\frac{1}{\cos^2\theta}d\theta$

$dB = \frac{\mu_0\cos\theta\, i}{2\pi R\, a}dx$

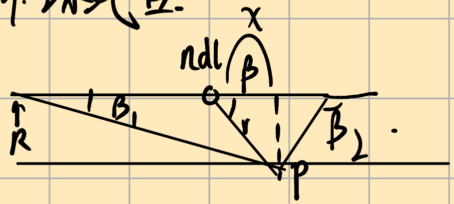
$= \frac{\mu_0\cos\theta\, i}{2\pi a R}\frac{R}{\cos^2\theta}d\theta$

$B_{总} = \frac{\mu_0 i}{\pi a}\tan^{-1}\frac{a}{2R}$

$R\gg a$时 $B\approx\frac{\mu_0 i}{\pi a}\cdot\frac{a}{2R} = \frac{\mu_0 i}{2\pi R}$

$R\to 0$ $\quad B_{总}\approx\frac{\mu_0 i}{2a}$

4. 螺线管.



$dx = \frac{-R}{\sin^2\beta}d\beta$

$dB = \frac{\mu_0 i n\,dl\,R^2}{2(R^2+x^2)^{\frac{3}{2}}}$ $\qquad dB = \frac{\mu_0 i n R^2}{2(R^2+\frac{R^2}{\tan^2\beta})^{\frac{3}{2}}}\cdot\frac{-R}{\sin^2\beta}d\beta$

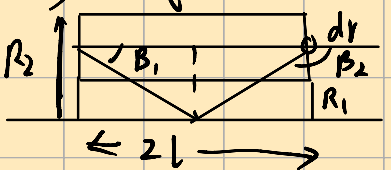
$\tan\beta = \frac{R}{x}$ $\quad x = \frac{R}{\tan\beta}$. $\qquad$ 分母 $R^3(1+\frac{\cos^2}{\sin^2})^{\frac{3}{2}} = R^3\frac{1}{\sin^3\beta}$

$dB = \frac{-\mu_0 i n\sin\beta}{2}d\beta$ $\qquad B = \frac{\mu_0 i n}{2}(\cos\beta_1 - \cos\beta_2)$

$= \frac{\mu_0 i n}{2}d\cos\beta$

$L\to\infty$ $\quad\beta_1 = 0$ $\quad\beta_2 = \pi$ $\quad B = \mu_0 n i$ $\quad$ 在两端 $B = \frac{1}{2}\mu_0 n i$

5. 多层螺线管.



中间：$B = \frac{1}{2}\mu_0 n i(\cos\beta_1 - \cos\beta_2)$

$n = \frac{N}{2l}$ →总数.

$ni = \frac{N}{2l}i = \frac{N}{2l}\cdot j$

$dB = \frac{1}{2}\mu_0\frac{Ni}{2l(R_2-R_1)}\cdot\frac{2l}{\sqrt{l^2+r^2}}dr$ $\qquad \gamma = \frac{l}{R_1}$

$= \mu_0 j\, l\,\frac{1}{\sqrt{l^2+r^2}}dr$ $\qquad \alpha = \frac{R_2}{R_1}$

$B = \mu_0 j l\int_{R_1}^{R_2}\frac{1}{\sqrt{l^2+r^2}}dr$ $\qquad B_0 = \mu_0 j\cdot R_1\gamma\ln\left(\frac{\alpha+\sqrt{\alpha^2+\gamma^2}}{1+\sqrt{1+\gamma^2}}\right)$

$= \mu_0 j l\ln\frac{R_2+\sqrt{l^2+R_2^2}}{R_1+\sqrt{l^2+R_1^2}}$.

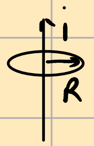
4. 磁场高斯定理：$\oint_S \vec{B}\cdot d\vec{s}=0$ $\nabla\cdot B=0$

回路定理：$\oint B\,dl=\mu_0\sum i$

适用于高对称体系.

e.g. 1. 无限长导线

线半径R. 在内外：



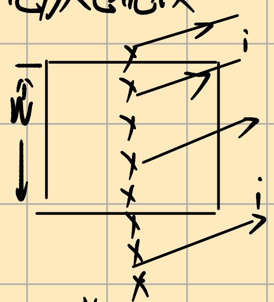
$r>R$ $B\cdot 2\pi r=\mu_0 i$

$B=\frac{\mu_0 i}{2\pi r}$

$r<R$ $B=\frac{\mu_0}{2\pi r}\frac{i r^2}{R^2}$

$=\frac{\mu_0 i r}{2\pi R^2}$.

2. 无穷大电流板

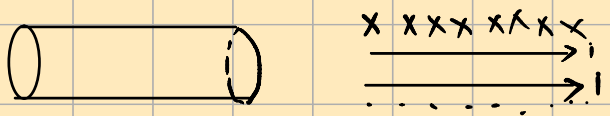


由对称性，B只有沿竖直方向的分量.

$\int B\cdot dl=2Bw=\mu_0 n w i$

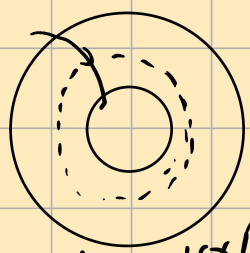
$B=\frac{1}{2}\mu_0 n i$

3. 螺线管.



$B=\mu_0 n i$

4.

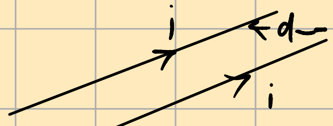


$B\cdot 2\pi r=\mu_0 N i$

$B=\frac{\mu_0 N i}{2\pi r}=\mu_0 n i$

5. 安培力：$dF=i\,ds\times B$

e.g. 1 两通相同电流的导线



单位长度受力：$f=\frac{\mu_0 i^2}{2\pi d}$

$i=\sqrt{\frac{2\pi f d}{\mu_0}}$ 定义 1A.

2. 力矩：

$\mu=iA\hat{n}$ ($\hat{n}$ 由右手定则确定)

$\tau=\mu\times B$

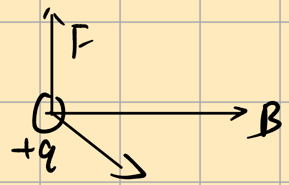
$U=-\int \tau\,d\theta=\vec{\mu}\cdot\vec{B}$.

$\theta=90°$, $U=0$ $U=-\vec{\mu}\cdot\vec{B}$

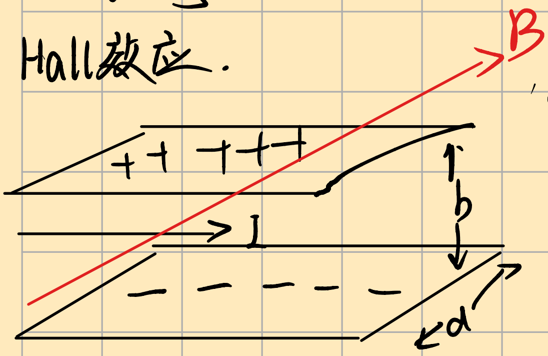
5. 带电粒子在磁场中运动

1. 洛伦兹力　$\vec F = q\vec v\times\vec B$　　安培力的微观解释.

$d\vec F = i\,d\vec s\times\vec B$.



Hall效应.



$qvB = qE$

$v = \frac{E}{B}$

AA'的电势 $\Delta V = Eb = vBb = \frac{j}{nq}Bb$

$= \frac{jbd}{nqd}B$

$= \frac{IB}{nqd}$.

Hall电阻: $R_H = \frac{\Delta V}{I} = \frac{B}{nqd}$.

6. 电磁感应

1. $\varepsilon = -\frac{d\Phi_B}{dt}$

负号是相对于右手螺旋确定的方向来的.

2. 动生电动势.

电子在B中受力 $f = -e(v\times B)$

非静电力: 单位正电荷受力 $\vec K = \vec v\times\vec B$.

$$\varepsilon = \int_{低电势}^{高电势} \vec K\cdot dl = \int_{-}^{+}(\vec v\times\vec B)\,dl.$$

注意高低电势点, 以及点积正负

3. 感生电动势.

$\varepsilon = \oint E\cdot dl$　　E方向由右手螺旋判断　　$\Phi_B = \iint B\cdot dA$.

$\oint E\cdot dl = \varepsilon = -\frac{d\Phi_B}{dt}$　　$\oint E\,dl = -\iint \frac{\partial B}{\partial t}\cdot dA$

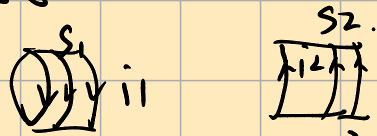
感生电场: $\oint E\,dl \neq 0 = \varepsilon$.　　$\nabla\times E = -\frac{\partial B}{\partial t}$

在空间中任何一点电场

$E = E_{sta} + E_{ind}$

$\oint E\cdot dl = \oint (E_{sta}+E_{ind})\,dl = 0 + \left(-\frac{d\Phi_B}{dt}\right) = -\frac{d\Phi_B}{dt}$

7. 电感和材料磁性质



i_1 变化：S_1 对 S_2 的磁通匝链数：$\Psi_{12} = (N_2 \Phi_{12})$ $\Psi_{12} = M_{12} i_1$

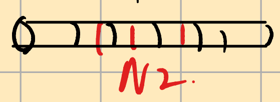
i_2 变化 ~ $\Psi_{21} = (N_1 \Phi_{21})$ $\Psi_{21} = M_{21} i_2$

由 i_1 变化，2 中 $\varepsilon_2 = -\frac{d\Psi_{12}}{dt} = -M_{12}\frac{di}{dt}$

i_2 变化，1 中 ~ $\varepsilon_1 = -\frac{d\Psi_{21}}{dt} = -M_{21}\frac{di_2}{dt}$

一般情况 $M_{12} = M_{21} = M$

e.g

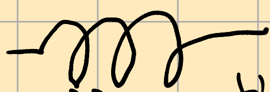


$B = \mu_0 n i_1$

计算 1 对 2 的互感系数

$$M_{12} = \frac{\Psi_{12}}{i_1} = \frac{N_2 B \cdot A}{i_1} = N_2 \mu_0 n A = N_2 \mu_0 \frac{N_1 A}{l}$$

自感



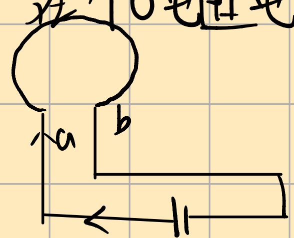
i 变化，B 变化，感应电动势

$$\Psi = NB \cdot A = Li$$

$$\varepsilon_L = -\frac{d\Psi}{dt} = -L\frac{di}{dt}$$

$$V_b - V_a = -L\frac{di}{dt}$$

存在 0 电阻电路中

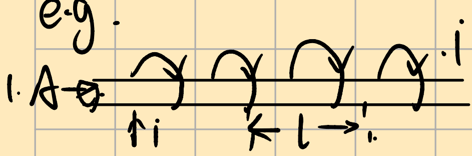


i 增大：B 增大，产生 ε 感抵消原电动势

$$\Delta V = -\frac{d\Phi_B}{dt} = -L\frac{di}{dt}$$

计算自感系数：① 假设 $i \longrightarrow B, \longrightarrow \Psi$

e.g.



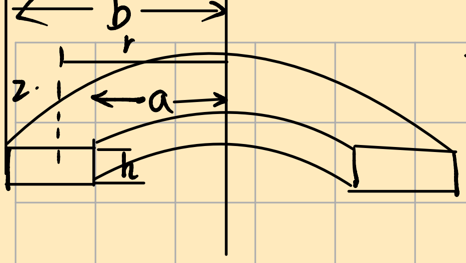
$B = \mu_0 n i$
$= \mu_0 n i$

$\Psi = \mu_0 n i A \cdot nl$

$L = \frac{\Psi}{i} = \mu_0 n^2 A l$
$= \mu_0 n^2 V$

单位体积电感 $L_V = \frac{L}{V} = \mu_0 n^2$

单位长度电感 $L_l = \frac{L}{l} = \mu_0 n^2 A$



假设 i

$B\cdot 2\pi r=\mu_0 i N$

$B=\frac{\mu_0 i N}{2\pi r}$.

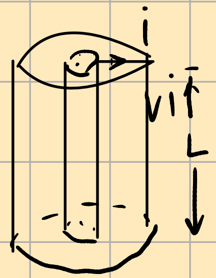
$\phi=\iint B\cdot dA=\int_a^b \frac{\mu_0 i N}{2\pi r}h\,dr$

$=\frac{\mu_0 i h N}{2\pi}\ln\frac{b}{a}$.

$\psi=N\phi$

$=\frac{\mu_0 i h}{2\pi}\ln\frac{b}{a}N^2$

$L=\frac{N\phi}{i}=\frac{\psi}{i}=\frac{\mu_0 h N^2}{2\pi}\ln\frac{b}{a}$



$B\cdot 2\pi r=\mu_0 i$

$B=\frac{\mu_0 i}{2\pi r}$

$\psi=\int_a^b\frac{\mu_0 i}{2\pi r}dr\,L$.

$=\frac{\mu_0 i L}{2\pi}\ln\frac{b}{a}$

$=\frac{\mu_0 L}{2\pi}\ln\frac{b}{a}$

互感和自感的关系

没有磁通量泄漏时：$M=\sqrt{L_1L_2}$ 串联(同方向) $L=(\sqrt{L_1}+\sqrt{L_2})^2$

反方向 $L=(\sqrt{L_1}-\sqrt{L_2})^2$

磁化材料.

$L=\kappa_m L_0$ (插入材料后).

微观上：磁性质是由分子中的磁偶极矩决定的

$\mu=iA=\frac{ev}{2\pi r}\cdot\pi r^2=\frac{1}{2}erv$.

角动量：$l=mvr$.

轨道磁矩 $\mu_l=\frac{el}{2m}$ $\vec{\mu}_l=\frac{-e}{2m}\vec{L}$. 对个电子是这样，对原子中(多个电子)：$\vec{\mu}_{tot}=\frac{-e}{2m}\vec{L}_{tot}$

$\vec{L}=\sqrt{L(L+1)}\cdot\hbar$ L为角量子数 (0,1,2,3,…)

$\vec{L}_z$ $(0,\pm1,\pm2,\cdots\pm L)\hbar$.

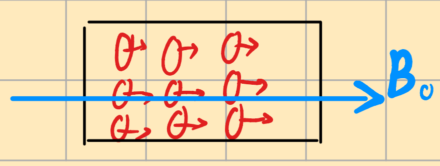
自旋磁矩. $\vec{\mu}_s=-\frac{e}{m}\vec{S}$ (自旋角动量) $S=\frac{1}{2}\hbar$ 对电子

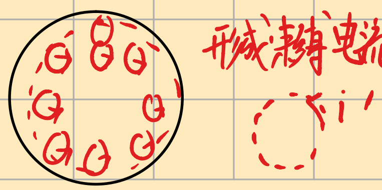
总自旋磁矩：$\vec{\mu}_S=-\frac{e}{m}\vec{S}$.

总磁矩 $\vec{\mu}_J=-\frac{e}{2m}\vec{J}=-\frac{e}{2m}(\underbrace{\vec{L}+2\vec{S}}_{\triangleq\vec{J}})=-\frac{e}{2m}\vec{J}$

原子核磁矩几乎可以忽略.

从宏观上：材料插入螺线管后，原本杂乱磁矩在 $\vec{B}$ 作用下取向一致.



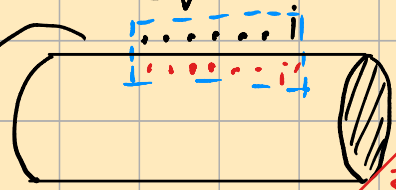


形成磁化电流

$$\vec{B}=\vec{B_0}+\vec{B_m}$$

磁化强度矢量 $\vec{M}$.

$\vec{M}=\frac{\sum \vec{m}}{\Delta V}$ (单位体积原子磁矩和)



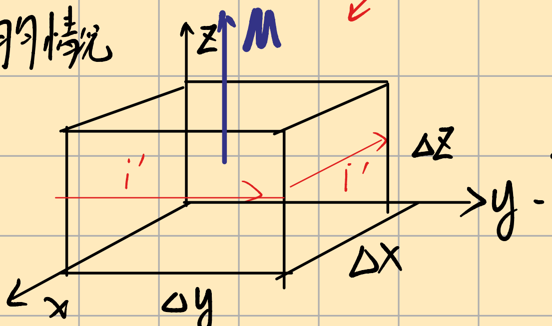
$\oint M\cdot dl=\sum i'$

$\vec{M}\times\vec{n}=\vec{j'}$ (单位长度电流, 表面电流密度)

和之前j不同.

推.

均匀情况



$j'=\frac{i'}{\Delta z}$

$\Delta m=i'\cdot\Delta A=j'\Delta x\Delta y\Delta z$

$M=\frac{\Delta m}{\Delta V}=j'$

$M\cdot\Delta z=i'$

非均匀情况: 分割为很小的均匀磁化之和

$\oint M dl=\sum i'$

$\vec{M}\times\vec{n}=\vec{j'}$

$$\oint_L \vec{B}\cdot d\vec{l}=\mu_0\sum_{in} i$$
$$=\mu_0\left(\sum_{in}(i_0+i')\right)$$
$$=\mu_0\sum_{in} i_0+\mu_0\oint M d\vec{l}$$

$$\oint_L\left(\frac{\vec{B}}{\mu_0}-\vec{M}\right)\cdot d\vec{l}=\sum_{in} i_0$$

△: $\vec{H}=\frac{\vec{B}}{\mu_0}-\vec{M}$ (磁场强度) 新安培定律 $\oint_l \vec{H}\cdot d\vec{l}=\sum_{in} i_0$

真空中 $\vec{M}=0$ $\vec{H}=\frac{\vec{B}}{\mu_0}$

软铁棒:

$\oint H\cdot dl=\sum i_0$

$H\cdot\Delta l=N i_0$

$H=n i_0$

$B_0=\mu_0 n i_0$

$=\mu_0 H$

$\vec{B}=\mu_0(\vec{H}+\vec{M})$

$=\vec{B_0}+\mu_0\vec{M}$

$B=k_m\mu_0 H$

$=k_m\mu_0 n i_0$

$=k_m B_0$

$\frac{\psi}{\psi_0}=k_m$ $\frac{L}{L_0}=k_m$

对磁性材料 $\vec{M}=\chi_m\vec{H}$. $\vec{B}=k_m\mu_0\vec{H}$

$=\mu_0\vec{H}+\mu_0\chi_m\vec{H}$

$k_m=1+\chi_m$

顺磁: $\chi_m>0$ $k_m>1$ ($k_m\approx 1$)

抗磁: $\chi_m<0$ $k_m<1$ $|\chi_m|\ll 1$. $k_m\approx 1$.

铁磁: $k_m\approx 10^2\sim 10^3$

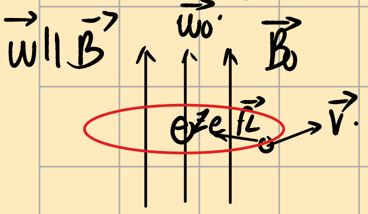
顺磁材料 $\mu_m \neq 0$

$\boxed{\vec{B} = \vec{B_0} + \mu_0 \vec{M}}$　　$\vec{M} = \chi_m \vec{H} = \frac{C}{T}\vec{H}$ (居里定律)

抗磁材料.

$\mu_m = 0$　$\vec{J} = 0$.　本身没有磁矩，处于B中，洛伦兹力影响向心力，产生$\Delta\vec{\mu}$.

$\vec{\omega} \parallel \vec{B}$



之前: $\frac{Ze^2}{4\pi\varepsilon_0 r^2} = m\omega_0^2 r$

$\omega_0 = \left(\frac{Ze^2}{4\pi m r^3}\right)^{\frac{1}{2}}$

现在 $\frac{Ze^2}{4\pi\varepsilon_0 r^2} + evB_0 = m(\omega_0 + \Delta\omega)^2 r$.

$evB_0 = 2m\omega_0\Delta\omega r$.

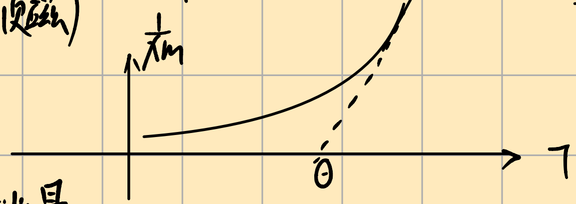
$\Delta\omega = \frac{eB_0 \cancel{v}}{2m\cancel{\omega_0 r}} = \frac{eB}{2m}$.

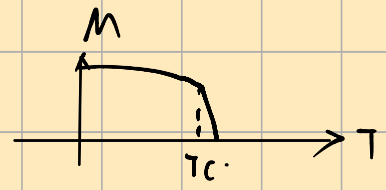
$\vec{\omega} \parallel -\vec{B}$　　$\omega = \omega_0 - \Delta\omega$　　$\Delta\omega = \frac{eB}{2m}$.

$\mu = iA = \frac{ev}{2\pi r}\pi r^2 = \frac{er^2}{2}\omega$.　　$\vec{\mu} = -\frac{er^2}{2}\vec{\omega}$　　$\Delta\vec{\mu} = -\frac{er^2}{2}\Delta\omega = -\frac{e^2B_0 r^2}{4m}$

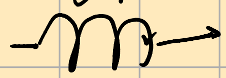
铁磁材料: 磁滞回线，磁畴.

(当$T \gg T_C$时表现顺磁)　$\chi_m = \frac{M}{H} = \frac{C}{T-\theta}$　$\theta > 0$　　T_C:





磁场能量



$dW = -\varepsilon_L dq = -\varepsilon_L i dt = L di\, i$

$W = \int_0^{i_{max}} L i\, di = \frac{1}{2}Li^2$.

做正功，E储存在磁场中

两线圈.

互感能: $W = W_1 + W_2 = -\int_0^{\infty} \varepsilon_{21} i_1 dt - \int_0^{\infty} \varepsilon_{12} i_2 dt$

$= -\int_0^{\infty}\left[-M_{21}\frac{di_2}{dt}i_1 - M_{12}\frac{di_1}{dt}i_2\right]dt$

$= \int_0^{I_1 I_2}(M_{21} i_1 di_2 + M_{12} i_2 di_1)$

$= M I_1 I_2$　$(M_{21} = M_{12} = M)$

$U = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$.

$= \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + \frac{1}{2}M_{12} I_1 I_2 + \frac{1}{2}M_{21} I_1 I_2$

$U_m = \frac{1}{2}\sum_{i=1}^{k} L_i I_i^2 + \frac{1}{2}\sum_{i \neq j=1}^{k} M_{ij} I_i I_j$

磁场能量密度　　对长直螺线管

$U=\frac{1}{2}LI^2$　　$L=\mu_0 n^2 lA$　　$B=\mu_0 nI$.

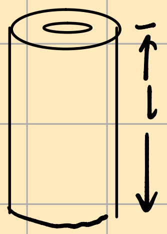
$$u_B=\frac{U}{V}=\frac{\frac{1}{2}\mu_0 lA\cdot n^2I^2}{lA}=\frac{1}{2}\mu_0 n^2 I^2$$

$$=\frac{(\mu_0 nI)^2}{2\mu_0}=\frac{B^2}{2\mu_0}.$$

$$u_B=\frac{B^2}{2\mu_0}=\frac{1}{2}\vec{B}\cdot\vec{H}.$$

$$u_E=\frac{1}{2}\varepsilon_0 E^2=\frac{1}{2}\vec{D}\cdot\vec{E}$$

e.g. 同轴电缆.



设: i, $B=\frac{\mu_0 i}{2\pi r}$　　$L=\frac{\psi}{i}=\frac{\mu_0 l}{2\pi}\ln\frac{b}{a}$.

$$d\psi=dr\cdot l\cdot\frac{\mu_0 i}{2\pi r}$$　　$$u_B=\frac{B^2}{2\mu_0}=\frac{\mu_0^2 i^2}{4\pi^2 r^2\cdot 2\mu_0}=\frac{\mu_0 i^2}{8\pi^2 r^2}.$$

$$\psi=\int_a^b\frac{\mu_0 i l}{2\pi r}dr$$　　$$E=\int u_B\, dV$$

$$=\frac{\mu_0 i l}{2\pi}\ln\frac{b}{a}.$$　　$$=\int_a^b\frac{\mu_0 i^2}{8\pi^2 r^2}\cdot 2\pi r l\, dr.$$

$$=\int_a^b\frac{\mu_0 i^2 l}{4\pi r}dr$$

$$=\frac{\mu_0 i^2 l}{4\pi}\ln\frac{b}{a}.=\frac{1}{2}Li^2$$

RL.RC电路, 电磁振荡 从略

MaxWell 方程.

在真空中:

$$\begin{cases} \oint E\cdot dA = \frac{q_0}{\varepsilon_0} \\ \oint B\cdot dA = 0 \\ \oint E\cdot dl = -\frac{d\Phi_B}{dt} = -\iint \frac{\partial B}{\partial t}\cdot dA \\ \oint \vec{B}\, d\vec{l} = \mu_0 i \end{cases}$$

在电介质和磁性材料中:

$$\begin{cases} \oint D\cdot d\vec{A} = q_0 \text{ 自由电荷} \\ \oint B\, d\vec{A} = 0 \\ \oint E\cdot dl = -\iint \frac{\partial B}{\partial t}\cdot dA \\ \oint H\cdot dl = \sum_{in} i_0 = \iint \vec{j_0}\cdot dA \end{cases}$$

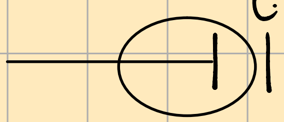
其中 $\vec{D} = \varepsilon_0\vec{E} + \vec{P} = K_e\varepsilon_0\vec{E}$.

$\vec{B} = \mu_0(\vec{H} + \vec{M}) = k_m\mu_0\vec{H}$ (各向同性的材料)

$\vec{j_0} = \sigma\vec{E}$ (欧姆定律微分形式)

$$\Rightarrow \begin{cases} \nabla\cdot D = \rho_{e0} \\ \nabla\cdot B = 0 \\ \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{H} = \vec{j_0} \end{cases}$$

电容器引起的安培定律失效问题, 被位移电流这个概念解决了.



用一个闭合S包裹电容器

$$\iint_S j_0\, dA \neq 0 = -\frac{dq_0}{dt}$$

引入位移电流 i_D $\oint H\cdot dl = i_0 + i_D$.

为了让这项为0: $\oiint_S D\cdot dA = q_0$

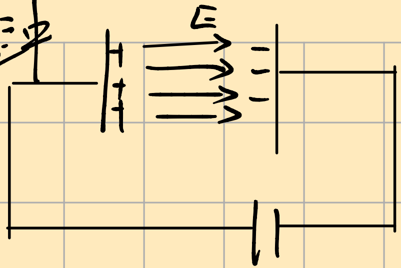
$$\iint_S j_0\, dA = -\oiint_S \frac{\partial D}{\partial t} dA.$$

则 $\iint_S \left(j_0 + \frac{\partial D}{\partial t}\right) dA = 0$

$$\begin{cases} \text{则位移电流}: i_D = \iint_S \frac{\partial D}{\partial t} dA \\ \text{电位移通量}: \Phi_D = \iint D\, dA. \\ \text{位移电流密度}: \vec{j_D} = \frac{\partial\vec{D}}{\partial t}. \end{cases}$$

新安培定律: $\oint H\cdot dl = i_0 + i_D = \oiint_S \left(\vec{j_0} + \frac{\partial D}{\partial t}\right)\cdot dA.$

真空中



$E=\frac{\sigma_e}{\varepsilon_0}=\frac{q}{\varepsilon_0 A}$.

$q=\varepsilon_0 AE=DA=\Phi_D$ $(\vec{P}=0)$

现在 在极板间存在位移电流 i_D.

$i_0=\frac{dq}{dt}=\varepsilon_0\frac{d\Phi_E}{dt}$

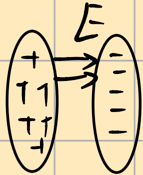
$=\frac{d\Phi_D}{dt}=i_D$.

此时极板间存在磁场.

$$\int H\cdot dl=i_0+i_D=\oiint_S \frac{\partial D}{\partial t}\,dA.$$

$$\oint \frac{B}{\mu_0}\cdot dl=\iint \varepsilon_0\frac{\partial E}{\partial t}\,dA.$$

$$\oint B\cdot dl=\mu_0\varepsilon_0\iint\frac{\partial E}{\partial t}\cdot dA$$



$r<R$: $B\cdot 2\pi r=\mu_0\varepsilon_0\pi r^2\frac{\partial E}{\partial t}$.

$$B=\frac{1}{2}\mu_0\varepsilon_0 r\frac{dE}{dt}.$$

$r>R$ $\quad B=\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}$

经过修正后，我们得到了麦克斯韦方程组

真空中

$$\begin{cases}\oiint E\cdot dA=\frac{q_0}{\varepsilon_0}\\ \oiint B\cdot dA=0\\ \oint E\cdot dl=-\iint\frac{\partial B}{\partial t}\cdot dA\\ \oint H\cdot dl=i_0+i_D=i_0+\varepsilon_0\iint\frac{\partial E}{\partial t}\cdot dA.\end{cases}$$

在电介质和磁性材料中.

$$\begin{cases}\oiint D\cdot dA=q_0.\\ \oiint B\cdot dA=0.\\ \oint E\cdot dl=-\iint\frac{\partial B}{\partial t}\cdot dA.\\ \oint H\cdot dl=i_0+\iint\frac{\partial D}{\partial t}\cdot dA\end{cases}$$

$$\begin{cases}\vec{D}=\varepsilon_0\vec{E}+\vec{P}=K_e\varepsilon_0\vec{E}.\\ \vec{B}=\mu_0(\vec{H}+\vec{M})=K_m\mu_0\vec{H}.\\ \vec{j_0}=\sigma\vec{E}.\end{cases}$$

$$\begin{cases}\nabla\cdot D=\rho_{e0}\\ \nabla\cdot B=0\\ \nabla\times E=-\frac{\partial B}{\partial t}\end{cases}\qquad \nabla\times H=j_0+\frac{\partial D}{\partial t}.$$

电磁波发射：

(1) 发射功率 $\frac{dW}{dt} \propto f^4$. $f > 10^5 Hz$.

$$f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

(2) 电磁波在真空中的5个性质：

① $\rho_{e0}=0$，$\vec{j}_0=0$.

② 电磁波是横波 $\vec{E}\perp\vec{k}$，$\vec{H}\perp\vec{k}$.

③ $\vec{E}\perp\vec{H}$.

④ $\vec{E}$，$\vec{H}$ 同相

⑤ $v=\frac{1}{\sqrt{k_e\varepsilon_0 k_m\mu_0}}$

在自由空间中 ($\rho_{e0}=0$，$\vec{j}_0=0$)

$$\begin{cases}\nabla\cdot E=0\\ \nabla\times E=-\frac{\partial B}{\partial t}=-k_m\mu_0\frac{\partial H}{\partial t}\\ \nabla\cdot H=0\\ \nabla\times H=\frac{\partial D}{\partial t}=k_e\varepsilon_0\frac{\partial E}{\partial t}.\end{cases} \Rightarrow \begin{cases}\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=0\\ |\quad|=-k_m\mu_0\left(\frac{\partial H_x}{\partial t}\hat{i}+\frac{\partial H_y}{\partial t}\hat{j}+\frac{\partial H_z}{\partial t}\hat{k}\right)\\ \frac{\partial H_x}{\partial x}+\frac{\partial H_y}{\partial y}+\frac{\partial H_z}{\partial z}=0\\ |\quad|=k_e\varepsilon_0(\quad)\end{cases}$$

0 0 ⟹ 0

注意，E、H与x,y无关

可得 $E_z(z,t)$，$H_z(z,t)$ 为常数.

那么它和EMW也无关，可令为0.

说明E、H是横波.

$E_z(z,t,x,y)=0$

$$\Rightarrow\begin{cases}\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}=-k_m\mu_0\frac{\partial H_x}{\partial t}\\ \frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}=-k_m\mu_0\frac{\partial H_y}{\partial t}.\\ \frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=-k_m\mu_0\frac{\partial H_z}{\partial t}\\ \frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}=k_e\varepsilon_0\frac{\partial E_x}{\partial t}.\\ \frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x}=k_e\varepsilon_0\frac{\partial E_y}{\partial t}\\ \frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}=k_e\varepsilon_0\frac{\partial E_z}{\partial t}.\end{cases}$$

0 0 0

$$\frac{\partial E_x}{\partial z}=-k_m\mu_0\frac{\partial H_y}{\partial t}$$

$$\frac{\partial H_y}{\partial z}=-k_e\varepsilon_0\frac{\partial E_x}{\partial t}.$$

$$\frac{\partial^2 E_x}{\partial z^2}=-k_m\mu_0\frac{\partial^2 H_y}{\partial t\partial z}.$$

场是连续的

$$=-k_m\mu_0\frac{\partial}{\partial t}\frac{\partial H_y}{\partial z}$$

$$=k_mk_e\mu_0\varepsilon_0\frac{\partial^2 E_x}{\partial t^2}$$

$$\Rightarrow\begin{cases}-\frac{\partial E_y}{\partial z}=-k_m\mu_0\frac{\partial H_x}{\partial t}\\ \frac{\partial E_x}{\partial z}=-k_m\mu_0\frac{\partial H_y}{\partial t}.\\ -\frac{\partial H_y}{\partial z}=k_e\varepsilon_0\frac{\partial E_x}{\partial t}\\ \frac{\partial H_x}{\partial z}=k_e\varepsilon_0\frac{\partial E_y}{\partial t}.\end{cases}$$

选取一个方向 令 $E_y=0$. $\frac{\partial H_x}{\partial t}=0$ $\frac{\partial H_x}{\partial z}=0$

$H_x=0$.

则只有 E_x 和 H_y 分量 $E\perp H$.

于是可以解出，$E_x = E_{x0} e^{i(\omega t - kz)}$　　$k = \sqrt{k_e\varepsilon_0 k_m\mu_0}\,\omega$.

$H_y = H_{y0} e^{i(\omega t - kz)}$，代入上述两个方程.

$\cancel{-i}kE_{x0}\cancel{e^{i(\omega t-kz)}} = -k_m\mu_0 \cdot \cancel{i}\omega H_{y0} \cancel{e^{i(\omega t - kz)}}$

$kE_{x0} = k_m\mu_0\omega H_{y0}$.

$E_{x0} = k_m\mu_0 v H_{y0} = \frac{\sqrt{k_m\mu_0}\, H_{y0}}{\sqrt{k_e\varepsilon_0}}$

$\sqrt{k_e\varepsilon_0}\,E_{x0} = \sqrt{k_m\mu_0}\,H_{y0}$.

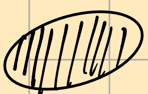
$\sqrt{k_e\varepsilon_0}\,E_0 e^{i\varphi_E} = \sqrt{k_m\mu_0}\,H_0 e^{i\varphi_H}$

$E_0 = \frac{B_0}{\sqrt{k_e k_m \mu_0 \varepsilon_0}} = cB_0$.

$$\Leftarrow \quad \begin{cases} \sqrt{k_e\varepsilon_0}\,E_0 = \sqrt{k_m\mu_0}\,H_0 \\ \varphi_E = \varphi_H \end{cases}$$

则H、E同相

电磁波的能流密度和动量.



体空间V

$U = \iiint \left(\frac12\varepsilon_0E^2 + \frac{B^2}{2\mu_0}\right)dv$

$= \iiint\left(\frac12\vec D\cdot\vec E + \frac12\vec B\cdot\vec H\right)dV$

若有电介质、磁性材料

$\frac{dU}{dt} = \frac{d}{dt}\iiint\left(\frac12\vec D\cdot\vec E + \frac12\vec B\cdot\vec H\right)dV$

$= \frac12\iiint\frac{\partial}{\partial t}(\vec D\cdot\vec E+\vec B\cdot\vec H)dV$.

$j_0 + \frac{\partial D}{\partial t} = \nabla\times\vec H$.

$\frac{\partial B}{\partial t} = -\nabla\times\vec E$.

$\frac{\partial}{\partial t}(\vec D\cdot\vec E+\vec B\cdot\vec H)$

$= k_e\varepsilon_0\frac{\partial}{\partial t}(E^2) + k_m\mu_0\frac{\partial}{\partial t}(H^2)$

$= 2k_e\varepsilon_0 E\frac{\partial E}{\partial t} + 2k_m\mu_0 H\frac{\partial H}{\partial t}$

$= 2E\cdot\frac{\partial D}{\partial t} + 2H\cdot\frac{\partial B}{\partial t}$

$= 2E(\nabla\times\vec H - j_0) - 2H\cdot\nabla\times\vec E$

$= -2\nabla(E\times H) - 2j_0\cdot\vec E$

$j_0 = \sigma(\vec E + \vec K)$　非静电场强　　$\vec E = \frac1\sigma \vec j_0 - \vec K = \rho\vec j_0 - \vec K$.

$j_0\cdot E = \rho(j_0)^2 - Kj_0$

$\vec E\cdot(\nabla\times\vec H) - \vec H\cdot(\nabla\times\vec E) = -\nabla\cdot(\vec E\times\vec H)$

$\frac{dU}{dt} = -\iiint\nabla\cdot(E\times H)dV - \iiint j_0\cdot E\, dV$

$= -\oint_S E\times H\, dA - \iiint j_0\cdot E\, dV$

$\iiint(j_0\cdot E)dV = \iiint j_0\cdot E\,\Delta A\Delta l$

$= \iiint j_0^2\rho\Delta A\Delta l - Kj_0\Delta A\Delta l$

$= \iiint j_0^2\frac{\rho\Delta l}{\Delta A}\cdot\Delta A^2 - (j_0\Delta A)(K\Delta l)$

$= i_0^2R - i_0\mathcal{E}$

$= Q - P$

$\frac{dU}{dt} = -\oint\vec E\times\vec H\, dA - \iiint j_0\cdot E\, dV$

$= -\oint\vec S\, dA - Q + P$

$\vec S = \vec E\times\vec H$：玻印廷矢量

$\vec S = \vec E\times\vec H = \frac{\vec E\times\vec B}{\mu_0}$　单位时间单位面积向外辐射能

$= \frac{EB}{\mu_0}$ (数值)

$= \frac{E^2}{\mu_0 c}$　　$Z_0 = \mu_0 c = 377\Omega$.

$S = \frac{E^2}{377\Omega}$.

光强：S平均值：

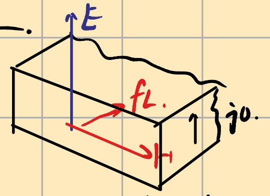
$$I=\langle S\rangle=\frac{\langle E^2\rangle}{Z_0}=\frac{E_{max}^2}{377}\langle\sin^2(kz-\omega t)\rangle=\frac{1}{2}\frac{E_{max}^2}{377\Omega}.$$

注意，电和磁能量密度是相等的！ $\frac{1}{2}\vec{D}\cdot\vec{E}=\frac{1}{2}\vec{B}\cdot\vec{H}$ $u=u_E+u_B=\varepsilon_0E^2$

电磁波能量密度：$\langle u\rangle=\varepsilon_0\langle E^2\rangle=\frac{\varepsilon_0E_{max}^2}{2}$

$$I=\langle u\rangle\cdot c=\frac{c\varepsilon_0E_{max}^2}{2}=\frac{E_{max}^2}{2\mu_0c}=\frac{E_{max}^2}{2\cdot 377\Omega}.$$

动量.



$f_L=-e\vec{v}\times\vec{B}$

$=-\mu_0e\vec{v}\times\vec{H}.$

$j_0=\sigma\vec{E}.$

力·位移/

$$\Delta F\cdot c\Delta t=(\vec{S}_{in}-\vec{S}_{ref})\Delta A\cdot\Delta t$$

$$\Delta F=\frac{1}{c}(\vec{S}_{in}-\vec{S}_{ref})\Delta A.$$

$$p=\frac{\Delta F}{\Delta A}=\frac{1}{c}(|\vec{S}_{in}|+|\vec{S}_{ref}|.)$$

电磁波动量：$\Delta G=\Delta G_p=-\Delta F\cdot\Delta t=\frac{1}{c}(\vec{S}_{out}-\vec{S}_{in})\Delta A\cdot\Delta t$

Δt 内 EMW 体积 $\Delta V=c\Delta t\cdot\Delta A$.

动量密度 $\Delta g=\frac{\Delta G}{\Delta V}=\frac{1}{c^2}(\vec{S}_{out}-\vec{S}_{in}).$

$\longrightarrow$ $g=\frac{1}{c^2}\vec{S}=\frac{1}{c^2}\vec{E}\times\vec{H}$

多普勒效应：

$$f=f_0\frac{\sqrt{1-u^2/c^2}}{1+\frac{u}{c}\cos\theta}$$

u S'相对S速度

θ 反射波方向与 u 的夹角

光学

几何光学三定律：折射，反射，直线传播

① 光在介质中传播时：$v=\frac{1}{\sqrt{k_e k_m \varepsilon_0 \mu_0}}$ （大多数情况 $k_m \simeq 1$）

$= \frac{c}{\sqrt{k_e}}$. $n \simeq \sqrt{k_e} > 1$.

n 与波长有关系

② 全反射 光密 → 光疏 （n大 → n小）

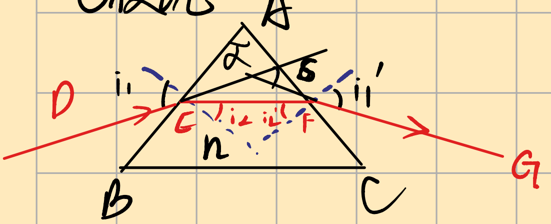
$\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2}$

$\theta_2 = 90^\circ$

$\sin\theta_1 = \frac{n_2}{n_1}$ 发生全反射.

③色散. 折射率随 ω 变化 $n(\omega) = 1 + \frac{A}{\omega_0^2 - \omega^2}$

④棱镜



$\frac{\sin i_2}{\sin i_1} = \frac{1}{n}$.

$\frac{\sin i_1'}{\sin i_2'} = n$.

求 δ_{min}： $\delta = i_1' - i_2' + i_1 - i_2$

$= i_1' + i_1 - \alpha$

$\frac{d\delta}{di_1} = 1 + \frac{di_1'}{di_1}$ $\frac{di_1'}{di_1} = -1$

$\frac{di_1'}{di_1} = -\frac{\cos i_1 \cos(\alpha - i_2)}{\cos i_1' \cos i_2}$.

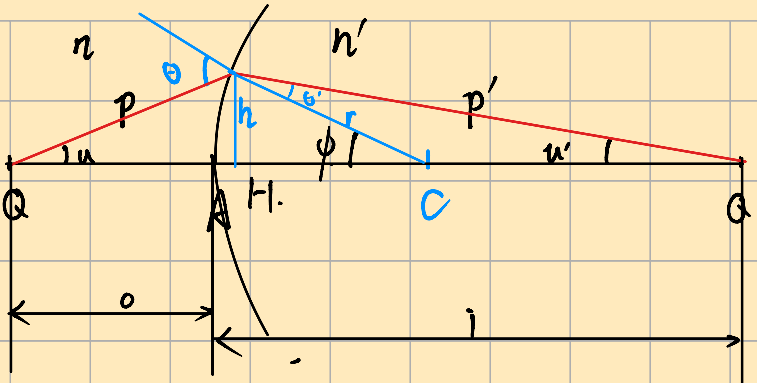
极值条件：$i_1' = i_1$ / $i_2' = i_2$

$n = \frac{\sin\frac{\alpha + \delta_{min}}{2}}{\sin\frac{\alpha}{2}}$

⑤ 惠更斯原理. /费马原理

成像

1. 球面镜成像



$$\begin{cases}\dfrac{p}{\sin\varphi}=\dfrac{r}{\sin u}=\dfrac{o+r}{\sin\theta} & ① \\ \dfrac{p'}{\sin\varphi}=\dfrac{r}{\sin u'}=\dfrac{i-r}{\sin\theta'} & ②\end{cases}$$

$$\begin{cases} n\sin\theta = n'\sin\theta' \\ \theta-u=\varphi=\theta'+u' \end{cases}$$

$\dfrac{①}{②}$: $\dfrac{p}{p'}=\dfrac{\sin u'}{\sin u}=\dfrac{o+r}{\sin\theta}\cdot\dfrac{\sin\theta'}{i-r}=\dfrac{o+r}{i-r}\cdot\dfrac{n}{n'}$

$$\frac{p}{n(o+r)}=\frac{p'}{n'(i-r)}$$

又
$$\begin{cases} p^2=(o+r)^2+r^2-2r(o+r)\cos\varphi \\ \quad = o^2+2r^2+2or-2r(o+r)\cos\varphi \\ p'^2 = 2r^2+i^2-2ir+2r(i-r)\cos\varphi \end{cases}$$

$\cos\varphi = 1-2\sin^2\frac{\varphi}{2}$

$$\Rightarrow \begin{cases} p^2 = o^2+4r(o+r)\sin^2\frac{\varphi}{2} \\ p'^2 = i^2-4r(i-r)\sin^2\frac{\varphi}{2}\end{cases}$$

平方代入

$$\frac{o^2+4r(o+r)\sin^2\frac{\varphi}{2}}{n^2(o+r)^2}=\frac{i^2-4r(i-r)\sin^2\frac{\varphi}{2}}{n'^2(i-r)^2}$$

$$\frac{o^2}{n^2(o+r)^2}-\frac{i^2}{n'^2(i-r)^2}=-4r\sin^2\frac{\varphi}{2}\left[\frac{1}{n^2(o+r)}+\frac{1}{n'^2(i-r)}\right]$$

观察式 φ 不同时，i 会不同，所以，一般球面镜是没办法成像的。

唯一组点可以成像：左右都为0.

$$\begin{cases}\dfrac{o^2}{n^2(o+r)^2}-\dfrac{i^2}{n'^2(i-r)^2}=0 \\ \dfrac{1}{n^2(o+r)}+\dfrac{1}{n'^2(i-r)}=0\end{cases}$$

实际中，我们会利用傍轴近似：$h^2 \ll o^2, i^2, r^2$，直接去掉等式右边的项。

$$\frac{o^2}{n^2(o+r)^2}=\frac{i^2}{n'^2(i-r)^2}$$

$$\Rightarrow \frac{n(o+r)}{o}=\frac{n'(i-r)}{i}$$

$$n+\frac{nr}{o}=n'-\frac{n'r}{i}$$

$$\frac{n}{o}+\frac{n'}{i}=\frac{n'-n}{r}$$

$$\boxed{\frac{n}{o}+\frac{n'}{i}=\frac{n'-n}{r}}$$

定义焦点：

第一焦点：$i\to\infty$ $o=f=\dfrac{n}{n'-n}r$.

第二焦点 $o\to\infty$ $i=f'=\dfrac{n'}{n'-n}r$.

$$\frac{f}{o}+\frac{f'}{i}=1$$

$$\boxed{\frac{f}{o}+\frac{f'}{i}=1}$$

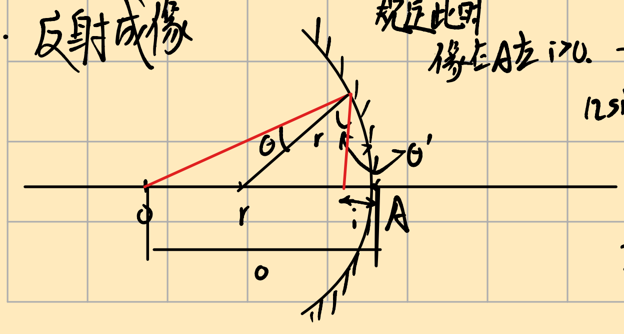
符号约定

(1) 物在A左边 $o>0$

(2) 像在B右边 $i>0$

(3) 球心在A右边 (凸) $r>0$

2. 反射成像



规定此时像在A左 $i>0$. $\dfrac{f}{o}+\dfrac{f'}{(-i)}=1$.

$n\sin\theta = n' -\sin\theta'$

$\theta=\theta'$

$n=-n'$

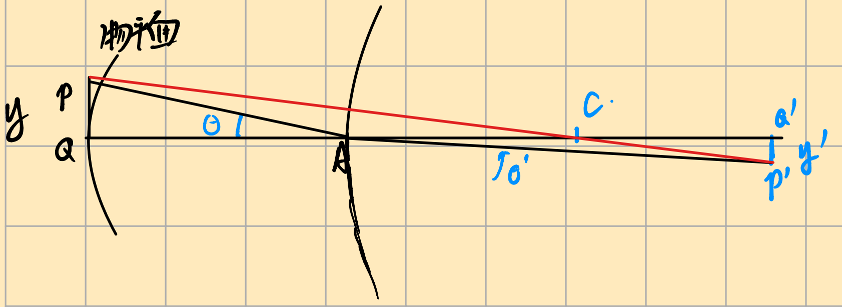
$f=\dfrac{n}{n'-n}r=-\dfrac{r}{2}$ $f'=\dfrac{n'}{n'-n}=\dfrac{r}{2}$. $-n=n'$

$$\frac{1}{o}+\frac{1}{i}=-\frac{2}{r}$$

$$\frac{1}{o}+\frac{1}{i}=-\frac{2}{r}$$

(符号与一般相反)

3. 傍轴物点成像和横向放大率.



傍轴近似:

$y^2, y'^2 \ll o^2, i^2, r^2$ $\quad \left\{ \begin{aligned} n\sin\theta &\simeq n\theta \\ &= n'\theta' \end{aligned} \right.$

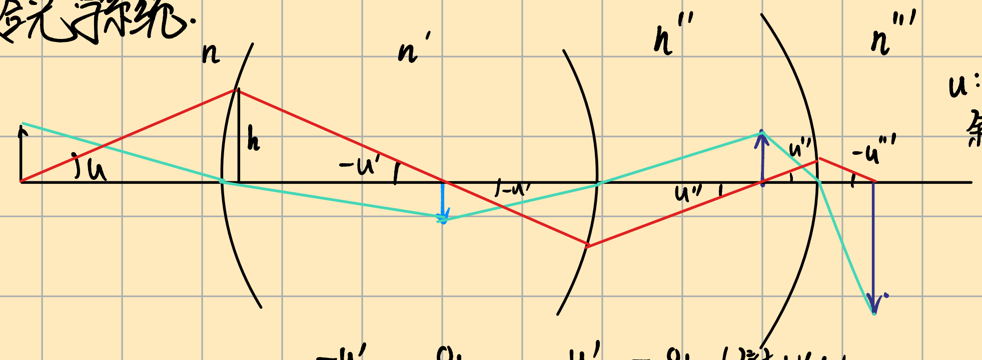
符号: 物/像在光轴上: $y>0$

放大倍数:

$$m = \frac{y'}{y} = \frac{-i\theta'}{o\theta} = -\frac{i n}{o n'}$$

对反射 $m = -\frac{i}{o}$

4. 复合光学系统.



u: 傍轴光线与光轴夹角, 向上倾斜为正向下为负

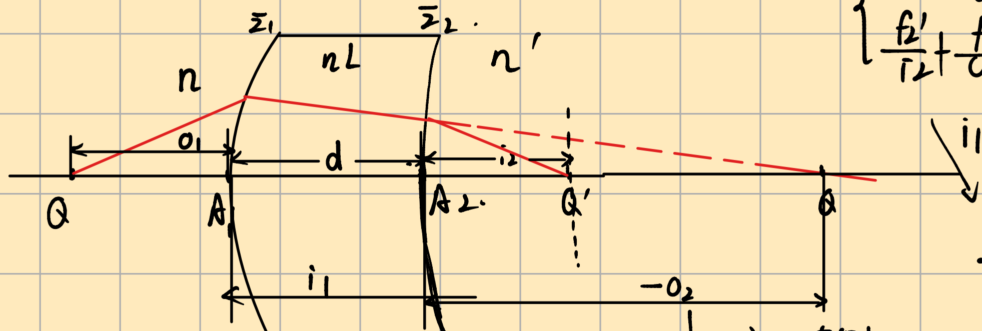
$$\frac{-u'}{u} = \frac{o_1}{i_1} \qquad \frac{u'}{u} = -\frac{o_1}{i_1} \text{ (傍轴近似)}$$

$$m = \frac{y'}{y} = -\frac{in}{on'} = \frac{un}{u'n'}$$

Lagrange-Helmholtz 定律: $ynu = y'n'u' = y''n''u'' = \cdots$

5. 薄透镜成像.

把第一次像看作第二次的物



物在Σ2右侧

$$\begin{cases} \dfrac{f_1'}{i_1}+\dfrac{f_1}{o_1}=1 & f_1=\dfrac{n}{n_L-n}r_1 \quad f_1'=\dfrac{n_L}{n_L-n}r_1 \\ \dfrac{f_2'}{i_2}+\dfrac{f_2}{o_2}=1 & f_2=\dfrac{n_L}{n'-n_L}r_2 \quad f_2'=\dfrac{n'}{n'-n_L}r_2 \end{cases}$$

$i_1 = -o_2 + d \approx -o_2$.

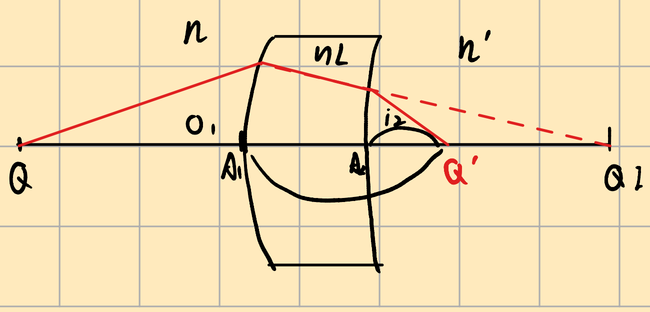
$$\begin{cases} \dfrac{f_2 f_1'}{i_1}+\dfrac{f_1 f_2}{o_1}=f_2 \\ \dfrac{f_1' f_2'}{i_2}-\dfrac{f_1' f_2}{i_1}=f_1' \end{cases}$$

相加 $$\frac{\frac{f_1 f_2}{f_2+f_1'}}{o_1}+\frac{\frac{f_1' f_2'}{f_1'+f_2}}{i_2}=1$$

定义 $f' = \dfrac{f_1' f_2'}{f_2+f_1'}$ $\quad f = \dfrac{f_1 f_2}{f_2+f_1'}$.

$$\frac{f}{o}+\frac{f'}{i}=1$$

6. 磨镜者公式 (len's maker's Equation)



$$\frac{f}{o}+\frac{f'}{i}=1$$

$$f = \frac{f_1 f_2}{f_2+f_1'} = \frac{\frac{n\,r_1}{n_L-n}\cdot\frac{n_L\,r_2}{n'-n_L}}{\frac{n_L\,r_2}{n'-n_L}+\frac{n_L}{n_L-n}r_1} = \frac{n}{\frac{n_L-n}{r_1}+\frac{n'-n_L}{r_2}}$$

$$f' = \frac{f_1' f_2'}{f_2+f_1'} = \frac{\frac{n_L}{n_L-n}r_1\cdot\frac{n'}{n'-n_L}r_2}{\frac{n_L}{n'-n_L}r_2+\frac{n_L}{n_L-n}r_1} = \frac{n'}{\frac{n_L-n}{r_1}+\frac{n'-n_L}{r_2}}$$

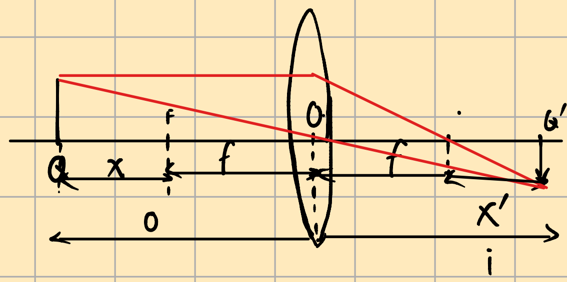
$$f=\frac{n}{\frac{n_L-n}{r_1}+\frac{n'-n_L}{r_2}} \qquad f'=\frac{n'}{\frac{n_L-n}{r_1}+\frac{n'-n_L}{r_2}}$$

$n=n'=1$ $\quad f=f'=\dfrac{1}{(n_L-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$ $\qquad f=f'=\dfrac{1}{(n_L-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$ $(n=n'=1)$

$f>0, f'>0$，凸透镜.

$f<0, f'<0$ 凹透镜.

7. $n=n'$时 $\quad \frac{f}{o}+\frac{f'}{i}=1 \Rightarrow \frac{1}{o}+\frac{1}{i}=\frac{1}{f}$. Gauss形式



$o=f+x$

$i=f'+x'$

$$\frac{1}{f+x}+\frac{1}{f+x'}=\frac{1}{f}$$

$$f^2+fx'+f^2+fx=f^2+fx'+fx+xx'.$$

$$xx'=f^2=ff'$$

放大倍数:

$m_1=\dfrac{-n\,i_1}{n_L\,o_1}$ $\qquad -o_2\approx i_1.$ $\qquad -o_2\approx i_1$

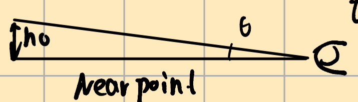
$m_2=-\dfrac{n_L\,i_2}{n'\,o_2}.$ $\quad m=m_1m_2=\dfrac{n\,i_2}{n'\,o_1}=\dfrac{-n\,i}{n'\,o}=\dfrac{-f\,i}{f'\,o}$ $\quad m=\dfrac{-n\,i}{n'\,o}=-\dfrac{f\,i}{f'\,o}$

牛顿形式: $ff'=x^2 \Rightarrow m=-\frac{i}{o}=-\frac{f+x'}{f+x}=-\frac{f+\frac{f^2}{x}}{f+x}=-\frac{f}{x}.$

屈光度 $P=\frac{1}{f(m)}$ 如 $f=-50cm=-0.5m$ $\quad P=\frac{1}{-0.5}=-2.0D.$ $\Rightarrow$ 200度

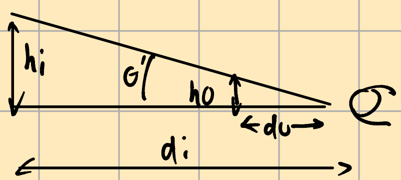
放大镜:

角放大率



$\tan\theta\approx\theta=\frac{h_0}{N}$

放大镜 协助



$$M=\frac{\theta'}{\theta}=\frac{h_0}{d_0}\cdot\frac{N}{h_0}=\frac{N}{d_0}$$

放大镜: $\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}.$

$$\frac{1}{d_0}=\frac{1}{f}-\frac{1}{d_i}$$

令 $d_i=-N$. 有Max放大倍数: $\frac{1}{d_0}=\frac{1}{f}+\frac{1}{N}$

$$M=\frac{N}{f}+1$$

$d_i=\infty$ $\qquad$ min: $\qquad M=\frac{N}{f}.$

人眼成像略: 近点; 远点,

25cm $\uparrow$ $\quad \infty$ $\uparrow$

近视: 透镜把像成在远点上: $f_{lens}=-d_{far}.$

$$\frac{1}{\infty}+\frac{1}{-d_{far}}=\frac{1}{f_{lens}}$$

远视: $\frac{1}{d_0}+\frac{1}{-d_{near}}=\frac{1}{f}$ $\quad$ 患者 $d_{near}=50cm$

光学

1. 定态波：　平面波　$k=\frac{2\pi}{\lambda}$　$A=constant$

$$\varphi(p)=\vec{k}\cdot\vec{r}+\varphi_0$$

球面波　$A(p)=\frac{a}{r}$

$$\varphi(p)=kr+\varphi_0$$

光：$\vec{E}(p,t)=\vec{E_0}(p)\cos[\omega t-\varphi(p)]$

复数描述　$\tilde{U}(p,t)=A(p)e^{-i(\omega t-\varphi(p))}$

光强：　$I=[A(p)]^2=\tilde{U}^*(p)\cdot\tilde{U}(p)$

这里 $\tilde{U}(p)$ 是复振幅：$A(p)e^{i\varphi(p)}$

波的叠加：

叠加后，波的强度 $I(p)=\tilde{U}^*(p)\,\tilde{U}(p)$

$$=[A_1e^{-i\varphi_1(p)}+A_2e^{-i\varphi_2(p)}][A_1e^{i\varphi_1(p)}+A_2e^{i\varphi_2(p)}]$$

$$=A_1^2+A_2^2+2A_1A_2\cos(\varphi_1-\varphi_2).$$

若 $I_1(p)=A_1^2(p)$　　$I(p)=I_1(p)+I_2(p)+2\sqrt{I_1I_2}\cos(\varphi_1-\varphi_2)$

$I_2(p)=A_2^2(p)$

$I=I_1+I_2+2\sqrt{I_1I_2}\cos(\varphi_1-\varphi_2)$

$\cos(\varphi_1-\varphi_2)>0$，$I>I_1+I_2$.

$\cos(\varphi_1-\varphi_2)<0$，$I<I_1+I_2$

发生以下情况时，不会发生干涉：　均值为0

① $\delta(p)=\varphi_1-\varphi_2$ 不是定值：　$\overline{\cos\delta(p)}=\overline{\cos(\varphi_1-\varphi_2)}=0$.

② 矢量波垂直：$\tilde{U}^2(p,t)=U_1^2+U_2^2$

即 $I=I_1+I_2$.

③ $\omega_1\neq\omega_2$ 时

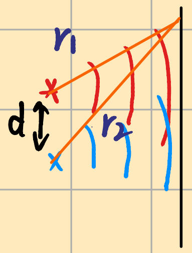
$$I(p,t)=I_1+I_2+2\sqrt{I_1I_2}\cos[(\varphi_1-\varphi_2)-(\omega_1-\omega_2)t]$$

$\omega_1-\omega_2\neq0$　$\overline{\cos(\quad)}=0$

$$I(p)=I_1(p)+I_2(p)$$

干涉条件：$\omega_1=\omega_2$　U_1 与 U_2 有平行分量　相差稳定.

两球面波干涉:



$r_1, r_2 \gg d$

$I_1(p) = (A_1(p))^2 \quad A_1(p) \propto \frac{1}{r_1}$

$I_2(p) = A_2^2(p) \quad A_2(p) \propto \frac{1}{r_2}$

$r_1, r_2 \gg d \quad A_1(p) \simeq A_2(p) = A$

$I = 2A^2(1+\cos\delta)$

$= 4A^2\cos^2\frac{\delta(p)}{2}$

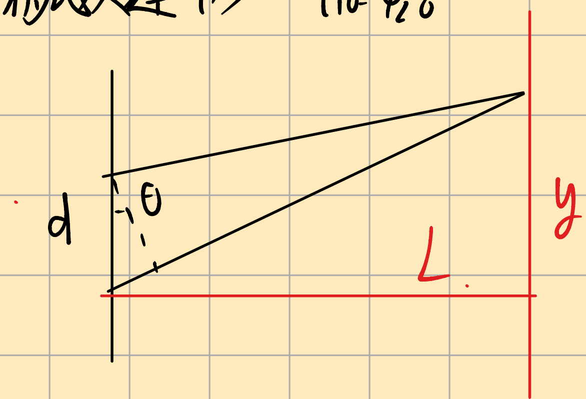
$\varphi_1(p) = kr_1 + \varphi_{10}$ $\quad \delta = \varphi_{10} - \varphi_{20} + k(r_1 - r_2)$

$\varphi_2(p) = kr_2 + \varphi_{20}.$ $\quad \varphi_{10} - \varphi_{20} = 0 \quad \delta = \frac{2\pi}{\lambda}(r_1 - r_2)$

$\Delta L = r_1 - r_2 = m\lambda$ I最大

$= r_1 - r_2 = (m+\frac{1}{2})\lambda$ I最小

杨氏双缝干涉 $\varphi_{10} - \varphi_{20}$



$d\sin\theta = m\lambda$ 相长干涉

$d\sin\theta = (m+\frac{1}{2})\lambda$ 相消干涉.

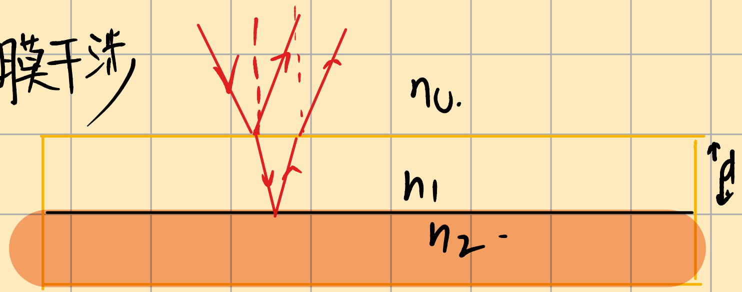
$\lambda < d$ 致 $\sin\theta$ 取不了值、

$\sin\theta \simeq \theta \simeq \tan\theta = \frac{y}{L}$

$y = \frac{m\lambda L}{d}$ 相长

$y = \frac{(m+\frac{1}{2})\lambda L}{d}$ 相消

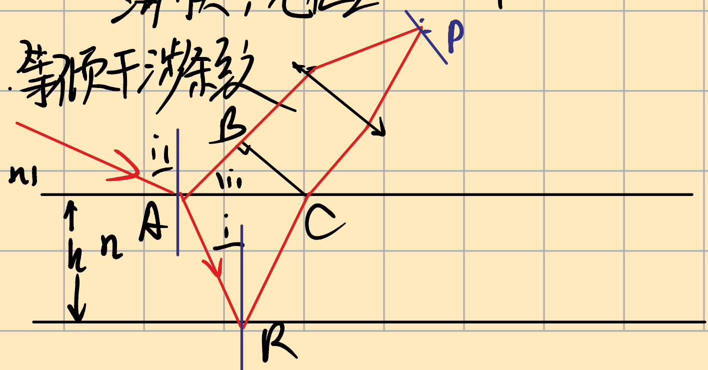
薄膜干涉



折射率小进大 发生半波损失, 光程差$+\frac{\lambda}{2}$.

薄膜, 光程差$\simeq 2d$

等倾干涉条纹



不考虑半波损失

$\Delta L = (ABC) - (AB)$

$= n\left(\frac{2h}{\cos i}\right) - n_1 AC\cdot\sin i_1$

$= \frac{n2h}{\cos i} - n_1\sin i_1 \cdot 2h\tan i$

$= 2h\left[\frac{n}{\cos i} - \frac{n_1\sin i_1\sin i}{\cos i}\right]$

$= 2h\left[\frac{n}{\cos i} - \frac{n\sin^2 i}{\cos i}\right]$

$= 2hn\cos i$

$\Delta L = 2nh\cos i = m\lambda$，最大.

$= (m+\frac{1}{2})\lambda$，最小

i 最小 $\cos i$ 最大 m 最大

m 级：$\Delta L_m = 2nh\cos i_m = m\lambda$　　$\cos i_m = \frac{m\lambda}{2nh}$

$(m+1)$ 级：$\Delta L_{m+1} = 2nh\cos i_{m+1} = (m+1)\lambda$　　$\cos i_{m+1} = \frac{(m+1)\lambda}{2nh}$.

$\cos i_{m+1} - \cos i_m = \frac{\lambda}{2nh}$.

$(\cos i_{m+1} - \cos i_m) = \left(\frac{\partial \cos i}{\partial i}\right)_{i=i_m}(i_{m+1} - i_m) = -\sin i_m (i_{m+1} - i_m) = \frac{\lambda}{2nh}$.

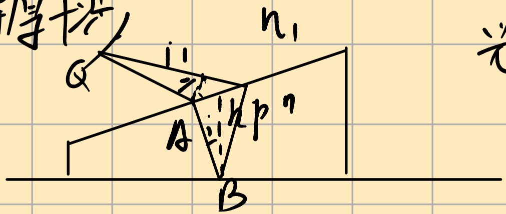
$\Delta r_m = r_{m+1} - r_m = f(i_{m+1} - i_m) = -\frac{f\lambda}{2nh\sin i_m}$

$r_{m+1} < r_m$　　$\Delta r \downarrow$

迈克尔逊干涉仪

$\Delta L_m = 2d = m\lambda$ （最大）

等厚干涉



光程差 $(QABP) - (QP)$

$= (QA) - (QP) + (ABP)$.

$(QA) - (QP) = -n_1 AP\sin i_1$

$= -APn\sin i$

$= -2hn\frac{\sin^2 i}{\cos i}$

$ABP = \frac{2n\,h}{\cos i}$.

光程差 $2nh\cos i$

$\Delta L = 2nh\cos i = m\lambda$　max值

$= (m+\frac{1}{2})\lambda$　极小值.

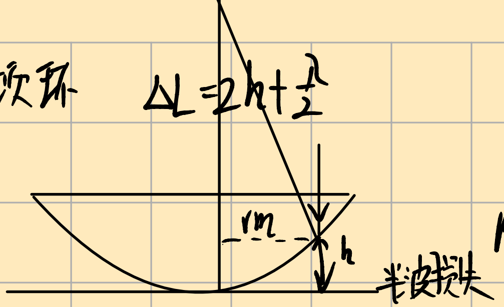
$i = 0$ 时　$\Delta L = 2nh$　h 相等则 m 相同

$2n\Delta h = \Delta\lambda$

$\Delta h = \frac{\Delta\lambda}{2n}$

条纹间距 $\Delta x = \frac{\Delta h}{\theta} = \frac{\Delta\lambda}{2n\theta}$　　$\theta = \frac{\Delta\lambda}{2n\Delta x}$　（对 air, $n=1$）

牛顿环 $\Delta L = 2h + \frac{\lambda}{2}$



对暗纹.

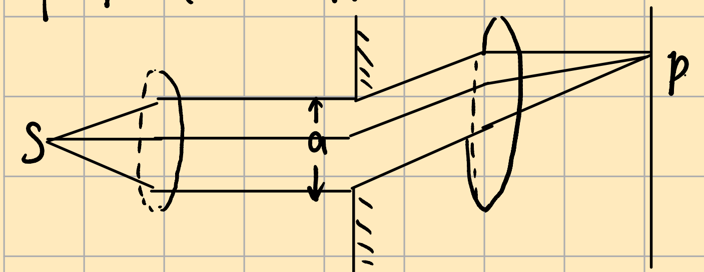
$$2h_m = m\lambda \quad (m=0,1,2\cdots)$$

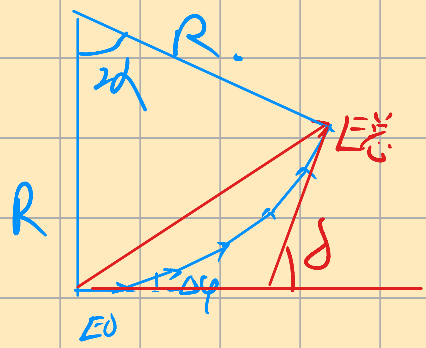
$$R-h_m=\sqrt{R^2-r_m^2}$$

$$h_m = R-\sqrt{R^2-r_m^2} = R\left(1-\sqrt{1-\frac{r_m^2}{R^2}}\right) = \frac{r_m^2}{2R} = \frac{1}{2}m\lambda.$$

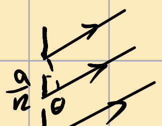
$$r_m=\sqrt{mR\lambda}.$$

弗朗和夫单缝衍射





把单缝看作n个 $\frac{a}{n}$ 小缝.



相邻光光程差 $\Delta x = \frac{a}{n}\sin\theta$.

相位差 $\Delta\varphi = \frac{2\pi}{\lambda}\frac{a}{n}\sin\theta$.

以第一支光相位为0: $E_1 = E_0 e^{i0}$

$E_2 = E_0 e^{i\Delta\varphi}$

$E_n = E_0 e^{i(n-1)\Delta\varphi}$

P点总光强: $\vec{E} = \vec{E_1}+\vec{E_2}+\cdots+\vec{E_n}$

$E = 2R\sin\alpha$ $\quad R=\frac{\overset{\frown}{MN}}{2\alpha}$ $\quad E=\overset{\frown}{MN}\frac{\sin\alpha}{\alpha}.$ $\quad 2\alpha=\delta=(n-1)\Delta\varphi \overset{n\to\infty}{\approx} \frac{2\pi}{\lambda}a\sin\theta$

$\alpha=\frac{\pi}{\lambda}a\sin\theta$

$\overset{\frown}{MN} = E_m$ ($\theta=0$时, 多矢量标量和, 即圆心·弧长)

$$E=E_m\frac{\sin\alpha}{\alpha}. \qquad I=I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \alpha=\frac{\pi a\sin\theta}{\lambda}.$$

$$\boxed{E=E_m\frac{\sin\alpha}{\alpha} \quad I=I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \alpha=\frac{\pi a\sin\theta}{\lambda}}$$

最小: 当 $\frac{\pi a\sin\theta}{\lambda}=m\pi$ I 暗纹. ($\sin\alpha=0$ $\alpha\neq0$)

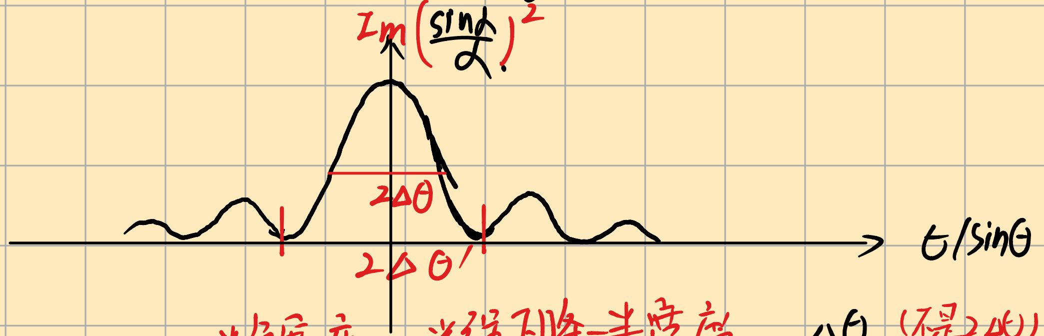
$(m=\pm1,\pm2)$.

最大: $\theta=0$时 $\quad I=I_m$ $\left(\frac{\sin\alpha}{\alpha}\to1\right)$

次极大: 近似 $\alpha=\frac{1}{2}\pi+m\pi.$ $\quad a\sin\theta=(m+\frac{1}{2})\lambda.$

($m=1$, 注意是从1开始)

实际上，应该令 $\frac{d\sin\alpha}{d\alpha}=0$ $\alpha=tg\alpha$ $\alpha=\pm1.43\pi,\pm2.46\pi,\pm3.47\pi$.



半角宽度，光强下降一半宽度 $\Delta\theta$ (不是 $2\Delta\theta$)

约等于到第一级最小的宽度 $\Delta\theta'$

$a\sin\theta=\lambda$ $\Delta\theta\simeq\sin\theta=\frac{\lambda}{a}$.

在光屏上：$\Delta y_m=f\Delta\theta=\frac{f\lambda}{a}$ (对 θ 很小时)

半波带．通过计算 N 数量 $N=\frac{a\sin\theta}{\frac{\lambda}{2}}$ N奇：亮

N偶：暗

$\theta=0$ 时，中央亮纹

夫朗和夫．
圆孔衍射 $I_\theta=I_0\left(\frac{2J_1(x)}{x}\right)^2$ $x=\frac{2\pi a\sin\theta}{\lambda}$

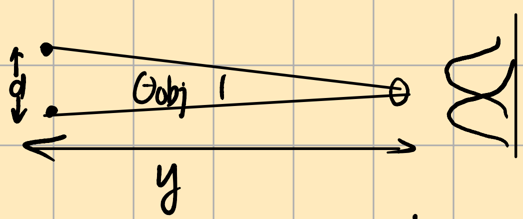
半角宽度 $\Delta\theta=0.61\frac{\lambda}{a}=1.22\frac{\lambda}{D}$

(a指圆孔半径).

分辨率 (对圆孔衍射)

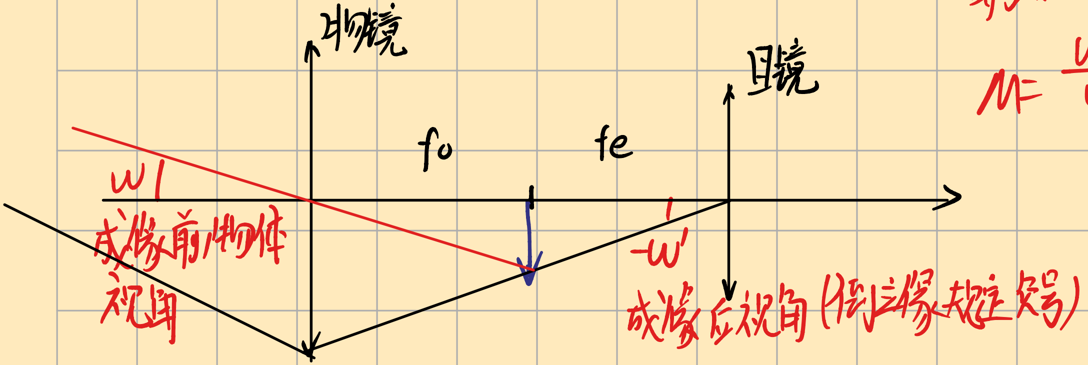
当一光源主极大位落在另一光的第一级暗纹时，刚好能分辨

$\theta_R=\theta_{min}=1.22\frac{\lambda}{D}$ R(分辨本领) $=\frac{1}{\theta_R}$



θ_{object} 可近似为 $\frac{d}{y}$. 为了分辨，$\theta_{object}>\theta_{min}=\frac{1.22\lambda}{D}$

望远镜分辨率



角放大率

$$M=\frac{w'}{w}=\frac{-f_o}{f_e}$$

$$\theta_R=1.22\frac{\lambda}{D}$$

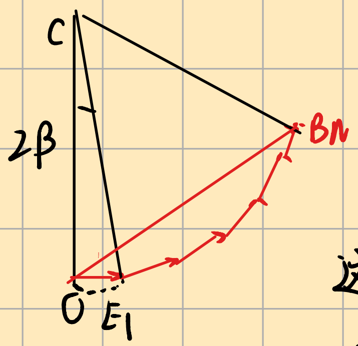
人眼与仪器 θ 比值，$M=\frac{\theta_e}{\theta_R}$，至少放大这么多倍，才能让人眼观测到望远镜可分辨的最小角

多缝光栅

类似单缝：　相邻缝 $\Delta L=d\cdot\sin\theta$　　$\delta=\frac{2\pi}{\lambda}d\sin\theta=2\beta$

$\beta=\frac{\pi d\sin\theta}{\lambda}$

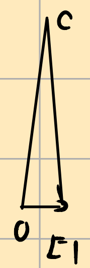
注意，在这里，缝的数是有限的，因此推导思路与之前略有不同



$$E=\vec{E_1}+\vec{E_2}+\cdots\vec{E_N}$$
$$=\overline{OB_N}$$
$$=2\overline{OC}\cdot\sin N\beta$$

这里 $N\not\to\infty$，不能用圆弧近似，

观察 OE_1C 扇形



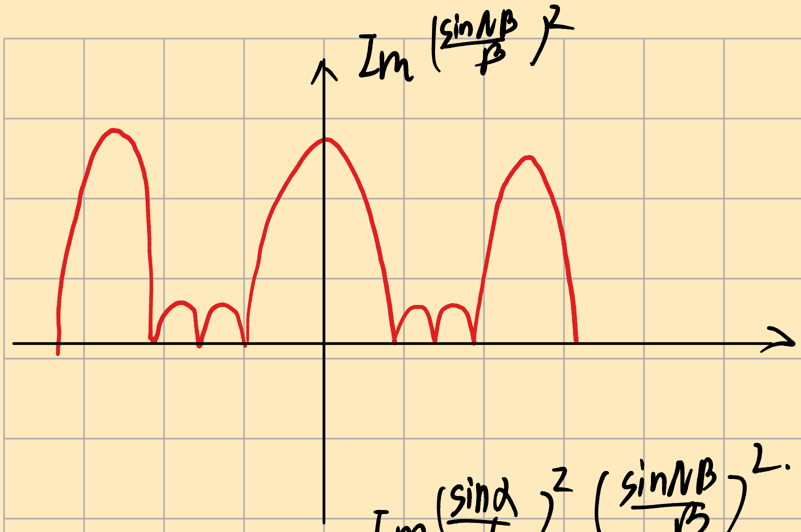
$2\overline{OC}\sin\beta=E_1$

$2\overline{OC}=\frac{E_1}{\sin\beta}$

$$E=E_1\frac{\sin N\beta}{\sin\beta}$$
$$=E_m\frac{\sin\alpha}{\alpha}\cdot\frac{\sin N\beta}{\sin\beta}$$

$$I_\theta=I_m\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\beta}\right)^2$$

$\alpha=\frac{\pi a\sin\theta}{\lambda}$　　$\beta=\frac{\pi d\sin\theta}{\lambda}$　　$(d=a+b)$ 不透光部分



$\beta = k\pi$ ($k \neq 0$, $k=\pm 1, \cdots$) 时

$(\frac{\sin N\beta}{\sin\beta})^2 \to N^2$.

产生主极大.

k级主极大与k+1级之间

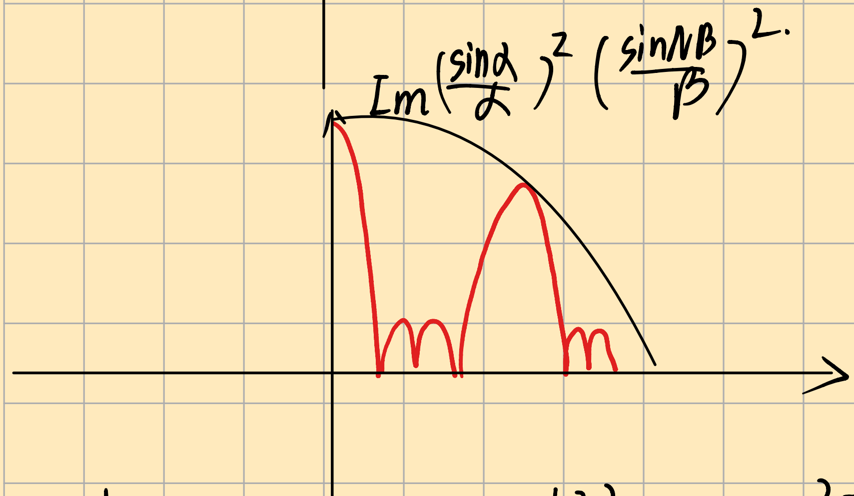
$N\beta$取π的整数倍时有极小.

$Nk\pi$ ______ $(Nk+N)\pi$

$(Nk+1)$, $(Nk+2)$ … $(Nk+N-1)\pi$

共有$N-1$个极小

$N-2$ 次极大



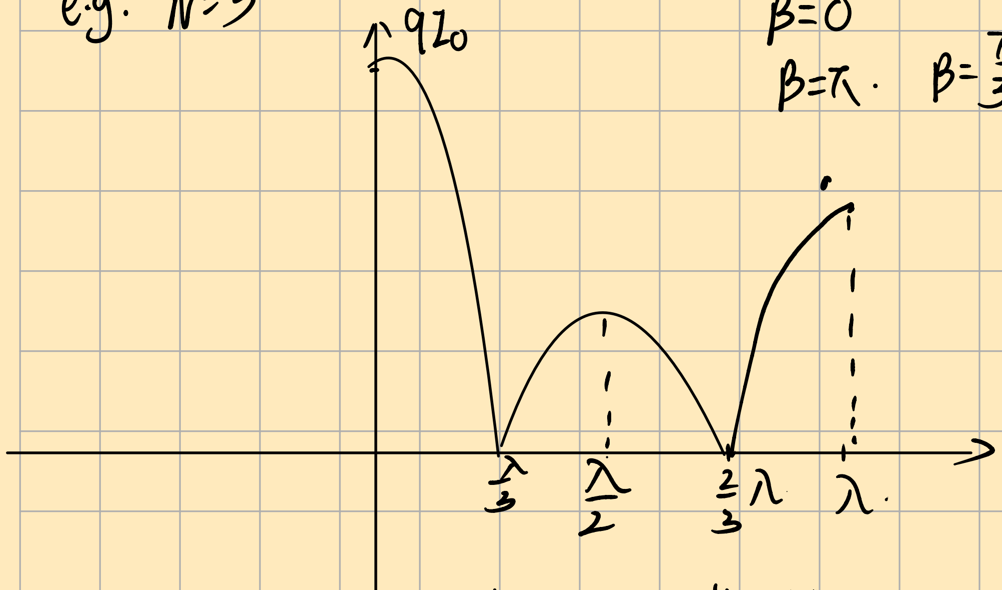
具体来说：$\beta = m\pi$. $\Rightarrow$ 主极大. $I_\theta = N^2 I_m$ ($\theta=0$时是这样,其它会乘上$(\frac{\sin\alpha}{\alpha})^2$).

$d\sin\theta = m\lambda$

$\sin\theta = m\frac{\lambda}{d}$ 要 $\lambda<d$, 否则 m只能取0

$m = \frac{d}{\lambda}\sin\theta \leq \frac{d}{\lambda}$.

e.g. $N=3$



$\beta=0$

$\beta=\pi$. $\beta = \frac{\pi}{3} = \frac{\pi d\sin\theta}{\lambda}$.

令$I=0$. $N\beta=\pi$ … $N-1\pi$. $\beta=\frac{\pi}{N}$ … $\frac{N-1}{N}\pi$.

$d\sin\theta = \frac{\lambda}{3}, \frac{2}{3}\lambda$.

令$3\beta = \frac{3}{2}\pi$.

(粗略地, 精确解: $N\tan\beta = \tan N\beta$).

$d\sin\theta$.

半角宽度. $d\sin\theta = \frac{1}{N}\lambda$

$\sin\theta \simeq \Delta\theta$

$\Delta\theta = \frac{\lambda}{Nd}$

若θ不能近似.

$d\cos\theta\,\Delta\theta = \frac{1}{N}\lambda$

$\Delta\theta = \frac{\lambda}{Nd\cos\theta}$

主极大缺失。当干涉项在衍射极大为0时，会缺失。

即 $\beta = m\pi$ 而 $\alpha = k\pi$。

此时 $d\sin\theta = m\lambda$

$a\sin\theta = k\pi$

$d = \frac{m}{k}a$.

typical width: 角宽 xf.

色散:

$d\sin\theta = m\lambda$ (主极大) 我们由主极大来区分不同颜色(λ不同)光

$d\cos\theta\Delta\theta = m\Delta\lambda$.

定义 $D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}$

$$\boxed{D = \frac{m}{d\cos\theta} \quad R = Nm}$$

光栅分辨本领，同样服从瑞利判据。

最小波长 对应最小角差，也即半角宽度 $\Delta\theta_w = \frac{\lambda}{Nd\cos\theta}$.

由 $D = \frac{\Delta\theta_w}{\Delta\lambda_{min}}$

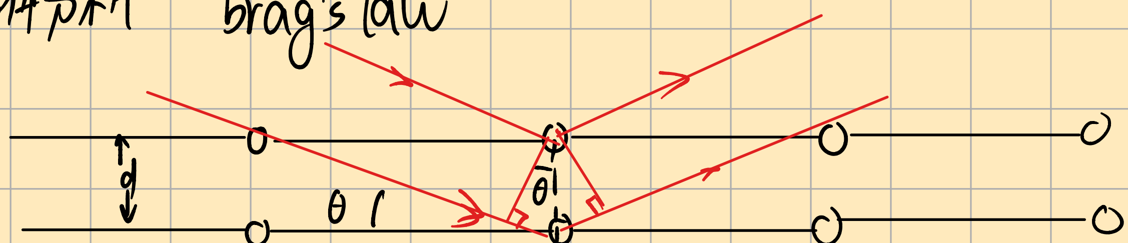
$$\Delta\lambda_{min} = \frac{\lambda}{Nd\cos\theta \cdot D_0} = \frac{\lambda}{Nd\cos\theta \frac{m}{d\cos\theta}} = \frac{\lambda}{Nm}.$$

$d\cos\theta\Delta\theta = m\Delta\lambda$.

定义: $R = \frac{\lambda}{\Delta\lambda} = Nm$

一般λ取要分辨对象的均值。

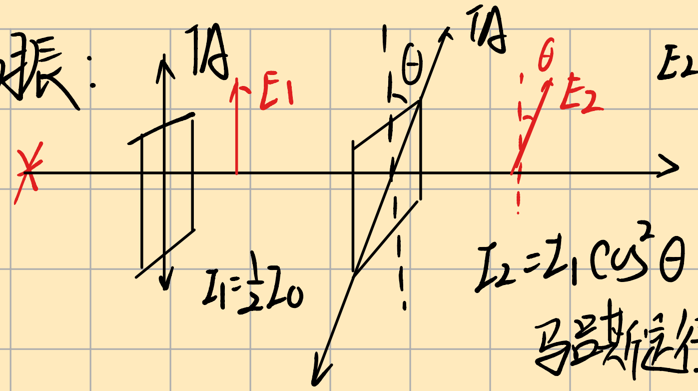
晶体分析: brag's law



$2d\sin\theta = m\lambda$ (形成主极大)

可测出 d.

偏振：



$E_2=\hat{E_1}\cdot\hat{n}$.

$I\propto E^2$.

$I_2=I_1\cos^2\theta$

马吕斯定律.

线偏振：

$E_x=E_{x0}\sin(kz-\omega t+\varphi_x)$

$E_y=E_{y0}\sin(kz-\omega t+\varphi_y)$.

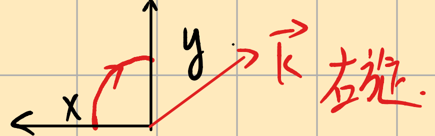
$\phi=\phi_x-\phi_y=0$ 时. $\frac{E_y}{E_x}=\frac{E_{y0}}{E_{x0}}$. 沿 $\arctan\left(\frac{E_{y0}}{E_{x0}}\right)$ 方向振动

圆偏振：

$E_{x0}=E_{y0}$. $\phi=\phi_x-\phi_y=\frac{\pi}{2}$.

$E_x=E\cos(kz-\omega t)$

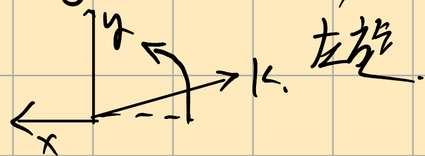
$E_y=E\sin(kz-\omega t)$



$-\frac{\pi}{2}$

$E_x=-E\cos(kz-\omega t)$

$E_y=E\sin(kz-\omega t)$



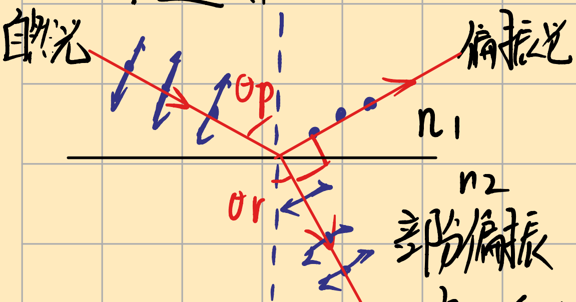
五种偏振光：

1. 无偏振光
2. 线～
3. 圆
4. 部分偏振（介于无偏与线偏间），不同方向I不一样.
5. 椭圆偏振（$\phi\neq\pm\frac{\pi}{2}$ 或 $E_{x0}\neq E_{y0}$）

用 $p=\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$ 描述偏振度：线偏 $p=1$

自然光 $p=0$.

反射起偏



多次反射也可得线偏振光

（如果是brewster角，反射光是部分偏振.）

条件：$\begin{cases}\theta_p+\theta_r=90^\circ\\ n_1\sin\theta_p=n_2\sin\theta_r.\end{cases}$

$n_1\sin\theta_p=n_2\cos\theta_p$

$\tan\theta_p=\frac{n_2}{n_1}$

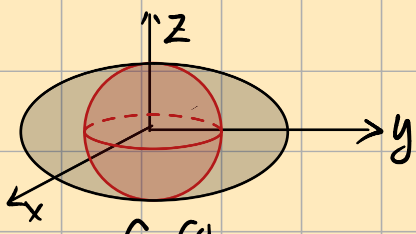
双折射

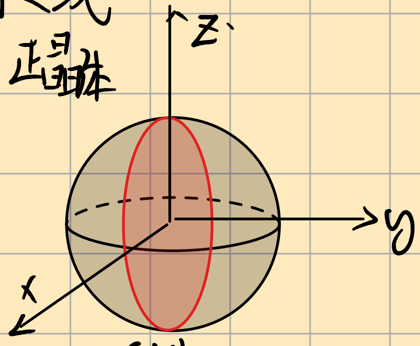
o光：任意方向速度一样．折射率n_o → 振动方向垂直传播方向与光轴平面平行.

e光：不同方向折射率不同 n_o → n_e 主折射率

$n_e<n_o$ 负晶体 $n_e>n_o$ 正．

光轴：o.e两光速度相同连线





¼入片，o光与e光光程差 $\Delta L=|(n_e-n_o)d|$ $\Delta\phi=\frac{2\pi}{\lambda}(n_o-n_e)d$.

$\Delta\phi=\pm\frac{\pi}{2}$ $(\Delta L=\pm\frac{1}{4}\lambda)$ 圆偏振 ⇔ 线偏振.

(当然要保证e光o光的振幅相同)

量子力学.

1. 总辐出度：黑体单位面积在T下的总辐射功率.（对λ从0至∞积分）

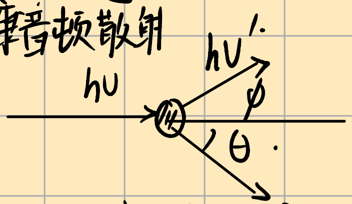
$$I=\int_0^\infty R(\lambda,T)\,d\lambda$$

$$R=\frac{dI}{d\lambda}.$$

2. 玻尔兹曼式 $I(T)=\sigma T^4$

3. 维恩公式 $T\lambda_{max}=b=2.898\times10^{-3}\,m\cdot K$

4. 康普顿散射



$p=\frac{h}{\lambda}$ $E=h\nu$.

$$\Delta\lambda=\lambda'-\lambda_0=\frac{h}{m_0c}(1-\cos\phi)$$

$$\begin{cases} h\nu+m_0c^2=h\nu'+mc^2 \\ \frac{h}{\lambda_0}=\frac{h}{\lambda}\cos\phi+\frac{m_0v}{\sqrt{1-(\frac{v}{c})^2}}\cos\theta \\ \frac{h}{\lambda_0}\sin\phi=\frac{m_0v}{\sqrt{1-(v/c)^2}}\sin\theta \end{cases}$$

$$\Delta\lambda=\frac{h}{m_0c}[1-\cos\phi]$$

德布罗意波：

$$\begin{cases} E = h\nu = \hbar\omega \\ \hbar k = \frac{h}{\lambda} = p \end{cases} \qquad \nu = \frac{E}{h}. \quad \lambda = \frac{h}{p} \qquad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

波函数

$$\psi = \psi_0 e^{i(kx-\omega t)} = \psi_0 e^{\frac{i}{\hbar}(px-Et)}$$

$$p(x) = \psi\psi^* \qquad \int_{x_1}^{x_2} p(x)\,dx = \int_{x_1}^{x_2} \psi\psi^* dx$$

$\psi\psi^* dx$表示：在 x 到 $x+dx$ 找到该粒子的概率

$$\int_{-\infty}^{+\infty} p(x)\,dx = 1$$

确定 $p(k)$，$\Delta p = 0$ $\Delta x = \infty$，粒子均匀分布.

推广：

$$\bar{x} = \frac{\int_{-\infty}^{+\infty} x\psi^*\psi\,dx}{\int_{-\infty}^{+\infty}\psi^*\psi\,dx} = \int_{-\infty}^{+\infty}\psi^* x\psi\,dx$$

$$= \langle\psi|x|\psi\rangle = \langle|x|\rangle.$$

$$\psi = \psi_0 e^{i(kx-\omega t)} \qquad p = \hbar k \quad E = \hbar\omega.$$

$$\frac{\partial\psi}{\partial x} = ik\psi_0 e^{i(kx-\omega t)}$$

$$-i\hbar\frac{\partial\psi}{\partial x} = \hbar k\psi_0 e^{i(kx-\omega t)}$$

$$-i\hbar\frac{\partial}{\partial x}\psi = p\psi.$$

$$p \leftrightarrow i\hbar\frac{\partial}{\partial x}. \qquad p \leftrightarrow -i\hbar\frac{\partial}{\partial x}$$

$$\frac{\partial\psi}{\partial t} = -i\psi_0\omega e^{i(kx-\omega t)}$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi. \qquad E \leftrightarrow i\hbar\frac{\partial}{\partial t}$$

薛定谔方程：

$$E = \frac{1}{2}mv^2 + U = \frac{p^2}{2m} + U.$$

$$E\psi = \frac{p^2}{2m}\psi + U\psi.$$

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U\psi.$$

$$i\hbar\frac{\partial}{\partial t}\psi = \left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + U(x,t)\right]\psi(x,t)$$ (一维含时).

一维： $\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + U(x,t)$

三维： $\hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + U(x,t)$

定态方程： U 不含 t，ψ 可分离为 $\psi(x,t) = \psi(x)e^{-i\omega t}$.

代入 $i\hbar\frac{\partial}{\partial t}\psi = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2}(\psi e^{-i\omega t}) + U\psi e^{-i\omega t}$

$$\hbar\omega\,\psi(x) = -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E-U]\psi(x) = 0$$

或 $\underbrace{\left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + U\right)}_{\triangleq \hat{H}}\psi(x) = E\psi(x)$

易忘/试 快速过

环 $\frac{1}{4\pi\varepsilon_0}\frac{Qz}{(R^2+z^2)^{\frac{3}{2}}}$

盘 $\frac{Q}{2\pi\varepsilon_0 R^2}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$

线：$\frac{\lambda}{2\pi\varepsilon_0 r}$

$\vec{p}=q\vec{d}$ $\tau=\vec{p}\times\vec{E}$

$\vec{E}=-\nabla V=-\left(\frac{\partial V}{\partial x}\hat{x}+\frac{\partial V}{\partial y}\hat{y}+\frac{\partial V}{\partial z}\hat{z}\right)$

球坐标 $-\left(\frac{\partial V}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\hat{\varphi}\right)$

柱 $-\left(\frac{\partial V}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta}+\frac{\partial V}{\partial z}\hat{z}\right)$

板：$C=\frac{\varepsilon_0 A}{d}$

柱 $C=\frac{\varepsilon_0 2\pi l}{\ln\frac{b}{a}}$

球：$C=\frac{4\pi\varepsilon_0 ab}{b-a}$

$u=\frac{1}{2}\varepsilon_0 E^2$

$\vec{P}=\frac{\sum\Delta p_m}{\Delta V}=nq\vec{l}$ $\quad P\cdot n=\sigma e$ (束缚)

$\oint P\cdot dA=\sum q_{束out}=-\sum q_{in}$

$\vec{P}=\chi_e\varepsilon_0\vec{E}$ $\quad k_e=1+\chi_e$

$\vec{D}=k_e\varepsilon_0\vec{E}=\varepsilon_0\vec{E}+\vec{P}$

$\oint D\cdot dA=\sum_{in} q_{自由}$

$\vec{j}=\sigma\vec{E}$

$R=\int\rho\frac{dl}{A}$

$dF_{12}=\frac{\mu_0}{4\pi}\frac{i_2 ds_2\times(i_1 ds_1\times\hat{r}_{12})}{r_{12}^2}$

$B_1=\frac{\mu_0}{4\pi}\oint_{L_1}\frac{i_1 ds_1\times\hat{r}_{12}}{r_{12}^2}$

$B=\frac{\mu_0}{4\pi}\oint_L\frac{i ds\times\hat{r}}{r^2}$

导线 $B=\frac{\mu_0 i}{4\pi r_0}(\cos\theta_1-\cos\theta_2)$ $\quad\left(\frac{\mu_0 i}{2r_0}\right)$

环 $B=\frac{\mu_0 i R^2}{2(R^2+z^2)^{\frac{3}{2}}}$

平面 $B=\frac{\mu_0 i}{\pi a}\tan^{-1}\left(\frac{a}{2R}\right)$ $\quad\left(\frac{\mu_0 i}{2a}\right)$

螺线管 $B=\frac{\mu_0 n i}{2}(\cos\beta_1-\cos\beta_2)$ $\quad(\mu_0 n i)$

多层 $\mu_0 j l\ln\frac{R_2+\sqrt{R_2^2+l^2}}{R_1+\sqrt{R_1^2+l^2}}$

$j=\frac{Ni}{2l(R_2-R_1)}$

Hall效应 $\Delta V=Eb=vBb=\frac{BI}{nqd}$

$R_H=\frac{\Delta V}{I}=\frac{B}{nqd}$

互感 $\Psi_{12}=M_{12}i_1=N_2B_1A$

自感 $\Psi=Li=NBA$

螺管 $L=\mu_0 n^2 Al$

同轴 $L=\frac{\mu_0 l}{2\pi}\ln\frac{b}{a}$

$\vec{M}=\frac{\sum\mu_m}{\Delta V}$ $\quad\oint M\cdot dl=\sum i_束$

$M\times\vec{n}=\vec{j}'$ (单位长度束缚)

$\oint H\cdot dl=\sum_{in} i_0$

$k_m=1+\chi_m$ $\quad B=k_m\mu_0 H=k_m B_0$

$u_{\vec{B}}=\frac{B^2}{2\mu_0}=\frac{1}{2}\vec{B}\cdot\vec{H}$

$u_{\vec{E}}=\frac{1}{2}\varepsilon_0 E^2=\frac{1}{2}\vec{D}\cdot\vec{E}$

Maxwell 方程组

位移电流 $I_D=\iint_S\frac{\partial D}{\partial t}\cdot dA$

电位移通量 $\Phi_D=\iint D\,dA$

位移电流密度 $i_D=\frac{\partial D}{\partial t}$

$$\begin{cases}\oint D\,dA=q_{自由}\\ \oint B\cdot dA=0\\ \oint E\cdot dl=-\iint\frac{\partial B}{\partial t}\cdot dA\\ \oint H\cdot dl=\sum i_0+i_D=\iint\left(j_0+\frac{\partial D}{\partial t}\right)\cdot dA\end{cases}$$

$\nabla\cdot D=\rho_{e0}$

$\nabla\cdot B=0$

$\nabla\times E=-\frac{\partial B}{\partial t}$

$\nabla\times H=j_0+\frac{\partial D}{\partial t}$

$$\nabla\cdot D=\begin{cases}\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}+\frac{\partial D_z}{\partial z}\\ 柱:\ \frac{1}{r}\frac{\partial rD_r}{\partial r}+\frac{1}{r}\frac{\partial D_\phi}{\partial \phi}+\frac{\partial D_z}{\partial z}\\ 球\ \frac{1}{r^2}\frac{\partial(r^2D_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial \sin\theta\cdot D_\theta}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}\end{cases}$$

$\frac{1}{r}\cdot r$ $\frac{1}{r}$ ϕ θ

$\frac{1}{r^2}\cdot r^2$ $\frac{1}{r\sin\theta}\sin\theta$ $\frac{1}{r\sin\theta}$

$$\sqrt{k_e\varepsilon_0}\,E_0=\sqrt{k_m\mu_0}\,H_0$$

$$\frac{dU}{dt}=-\oint S\,dA-Q+P \qquad \vec{S}=\vec{E}\times\vec{H}=\frac{\vec{E}\times\vec{B}}{\mu_0}=\frac{EB}{\mu_0}=\frac{E^2}{\mu_0 c}.$$

$B=\frac{E}{c}$ $\qquad S=\frac{E^2}{377}$

$$I=\langle S\rangle=\frac{\langle E^2\rangle}{Z_0}=\frac{E_{max}^2}{2\mu_0 c}.\ \ \frac{1}{2}\varepsilon_0(E_{max})^2\cdot c$$

$$p=\frac{1}{c}\left(|S_{in}|+|S_{ref}|\right) \qquad g=\frac{1}{c^2}\vec{S}.$$

$$\Delta g=\frac{1}{c^2}[\vec{S_{out}}-\vec{S_{in}}]$$

$$f=f_0\frac{\sqrt{1-u^2/c^2}}{1+\frac{u}{c}\cos\theta}$$

光

$$\frac{o^2}{n^2(o+r)^2}-\frac{i^2}{n'^2(i-r)^2}=-4r\sin^2\frac{\psi}{2}\left[\frac{1}{n^2(o+r)}+\frac{1}{n'^2(i-r)}\right].$$

$$\frac{f}{o}+\frac{f'}{i}=1 \qquad f=\frac{n}{n'-n}r \qquad f'=\frac{n'}{n'-n}r.$$

$$\frac{n}{o}+\frac{n'}{o}=\frac{n'-n}{r}.$$

$$m=\frac{-in}{on'}$$

$$\frac{f}{o}+\frac{f'}{i}=1 \qquad f=\frac{f_1f_2}{f_2+f_1'} \qquad f'=\frac{f_1'f_2'}{f_1'+f_2}.$$

磨镜者试:
$$f=\frac{n}{\frac{n_L-n}{r_1}+\frac{n'-n_L}{r_2}} \qquad f'=\frac{n'}{\frac{n_L-n}{r_1}+\frac{n'-n_L}{r_2}}.$$

$$(n=n'=1)\quad f=f'=\frac{n}{\frac{n_L-n}{r_1}+\frac{n'-n_L}{r_2}}=\frac{1}{(n_L-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

$$y=\frac{m\lambda L}{d}$$

$2nh\cos i$

$$E=E_m\frac{\sin\alpha}{\alpha}$$

$$I=I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \alpha=\frac{\pi a\sin\theta}{\lambda}.$$

$$\Delta\theta=\frac{\lambda}{a}.$$

$$\Delta\theta=0.66\frac{\lambda}{a}=1.22\frac{\lambda}{D}$$

$$R=\frac{1}{\theta_R}$$

$$E=E_m\frac{\sin\alpha}{\alpha}\ \frac{\sin N\beta}{\beta}$$

$$I=I_m\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\beta}\right)^2$$

$$\alpha=\frac{\pi a\sin\theta}{\lambda} \qquad \beta=\frac{\pi d\sin\theta}{\lambda}$$

$$\Delta\theta=\frac{\lambda}{Nd}=\frac{\lambda}{Nd\cos\theta}$$

$$D=\frac{m}{d\cos\theta} \quad R=Nm=\frac{\lambda}{\Delta\lambda}.$$

$$I=\sigma T^4.$$

$$\lambda_{max}T=b=2.898\times10^{-3}\,m\cdot K$$

$$\Delta\lambda=\frac{h}{m_0c}(1-\cos\phi)$$

$$E=h\nu=\hbar\omega$$

$$p=\frac{h}{\lambda}=\hbar K.$$

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mE}}$$

$$\vec{p}\leftrightarrow-i\hbar\frac{\partial}{\partial x}$$

$$\vec{E}\leftrightarrow i\hbar\frac{\partial}{\partial t}.$$

$$E=\frac{p^2}{2m}+U$$

$$E\psi=\frac{p^2}{2m}\psi+U\psi$$

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{-\hbar^2\partial^2}{2m\partial x^2}\psi+U\psi.$$

$$\psi=\phi(x)e^{-i\omega t}$$

$$E\phi(x)=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\phi^2(x)+U\phi(x)$$

$$\frac{\partial^2}{\partial x^2}\phi(x)+\frac{2m}{\hbar^2}(E-U)\phi(x)=0$$

$$\frac{o^2}{n^2(o+r)^2}-\frac{i^2}{n'^2(i-r)^2}=-4r\sin^2\left(\frac{1}{n^2(o+r)}+\frac{1}{n'^2(i-r)}\right)$$

$$-\frac{n}{o}+\frac{n'}{i}=\frac{n'-n}{r}.$$

$$\frac{1}{o}+\frac{1}{i}=-\frac{2}{r}.$$

$$\frac{f}{o}+\frac{f'}{i}=1 \qquad f=\frac{f_1f_2}{f_1'+f_2} \qquad f'=\frac{f_1'f_2'}{f_1'+f_2}$$

$$=\frac{n}{\frac{n_L-n}{r_1}+\frac{n'-n_L}{r_2}} \qquad =\frac{n'}{\frac{n_L-n}{r_1}+\frac{n'-n_L}{r_2}}$$