Trig Identities

$tan(x) = \frac{sin(x)}{cos(x)}$, $csc(x) = \frac{1}{sin(x)}$, $sec(x) = \frac{1}{cos(x)}$
$sin^2(x) + cos^2(x) = 1$, $tan^2(x) + 1 = sec^2(x)$
$cot^2(x) + 1 = csc^2(x)$, $sin(2\theta) = 2sin(\theta)cos(\theta)$
$cos(2\theta) = cos^2(\theta) - sin^2(\theta)$
$tan(2\theta) = \frac{2tan(\theta)}{1-tan^2(\theta)}$, $sin^2(x) = \frac{1-cos(2x)}{2}$
$cos^2(x) = \frac{1+cos(2x)}{2}$

Derivatives

$d(sinx) = cosx$, $d(cosx) = -sinx$, $d(tanx) = sec^2x$
$d(secx) = secxtanx$, $d(cscx) = -cscxcotx$
$d(cotx) = -csc^2x$
$d(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$, $d(cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$
$d(tan^{-1}x) = \frac{1}{1+x^2}$, $d(sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$
$d(csc^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$, $d(cot^{-1}x) = \frac{-1}{1+x^2}$

Integrals

$\int sinx = -cosx + C$, $\int cosx = sinx + C$, $\int tanx = -ln|cosx| + C$, $\int secx = ln|secx + tanx| + C$, $\int cscx = -ln|cscx + cotx| + C$, $\int cotx = ln|sinx| + C$, $\int sec^2x = tanx + C$, $\int csc^2x = -cotx + C$, $\int secxtanx = secx + C$, $\int cscxcotx = -cscx + C$, $\int tan^2x = tanx - x + C$, $\int cot^2x = -cotx - x + C$, $\int tanx = -ln|cosx| + C$, $\int cotx = ln|sinx| + C$, $\int \frac{1}{a^2+x^2} = \frac{1}{a}tan^{-1}(\frac{x}{a}) + C$, $\int \frac{1}{\sqrt{a^2+x^2}} = ln|x + \sqrt{a^2+x^2}| + C$, $\int \frac{1}{\sqrt{a^2-x^2}} = sin^{-1}(\frac{x}{a}) + C$, $\int \frac{1}{x^2-a^2} = \frac{1}{2a}ln|\frac{x-a}{x+a}| + C$

Ch1: Basic Definitions

Ordinary: single independent variable. **Partial**: multiple independent variables. **Order**: highest derivative. **Linear diffeq**: additive combination of first powers: $a_n(x)\frac{d^ny}{dx^n} + a_{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_1(x)\frac{dy}{dx} + a_0(x)y = F(x)$.

Example of linear: $t^3\frac{dx}{dt} - x = t^3$. $\frac{dy}{dx} = sin(x)$ bc $0 * y$ **Nonlinear**: $\frac{d^2y}{dy^2} + y^3 = 0$ bc of y^3 term.

Another: $\frac{d^2y}{dx^2} - y\frac{dy}{dx} = cosx$ bc of ydy term.
Explicit Soln: function $\phi(x)$ when sub for y in eqn satisfies for $\forall x$ in interval. **Implicit Soln**: Verify by differentiate implicit soln wrt ind. var on both sides to check = orig.

Note: Diff of y wrt x becomes $\frac{dy}{dx}$
Existence & Uniqueness: 1. Find the form of the differential equation $\frac{dy}{dx} = f(x,y)$. 2. Identify the function $f(x,y)$ and the partial derivative $\frac{\partial y}{\partial x}$ such that both are continuous in the rectangle $(x, f(x))$. **Autonomous eqns**: $y' = f(y)$, rhs is fxn of dependent var only.
Method of Isoclines: $y' = f(x,y) = c$, solve for y to make isoclines (dotted lines), slope same along isocline.
Euler's Method: $y_{n+1} = y_n + hf(x_n, y_n)$, where h is step size ($\frac{dist}{numstep}$).

Ch2: Solving Linear 1st Order

Separable: $y' = f(x)g(y)$, sep. into $\frac{dy}{g(y)} = f(x)dx$, integrate both sides.
Linear $a_1(x)\frac{dy}{dx} + a_0(x)y = b(x)$. Special cases: $a_0(x) = 0$(no y term) then $y(x) = \int \frac{b(x)}{a_1(x)}dx + C$ $a_0(x) = a_1'(x)$ product rule $y(x) = \frac{1}{a_1(x)}(\int b(x)dx + C)$, **Standard Form**: $\frac{dy}{dx} + P(x)y = Q(x)$, **Integrating Factor**: $\mu(x) = exp(\int P(x)dx)$, Solve: $\mu(x)y(x) = \int \mu(x)Q(x)dx + C$, **Exact**: Form $M(x,y)dx + N(x,y)dy = 0$, check $M_y = N_x$ for exactness, Solving: 1. Text exactness, 2. $F(x,y) = \int M(x,y)dx + g(y)$, 3. $\frac{\partial F}{\partial y} = N(x,y)$ to get $g'(y)$, 4. $g(y) = \int g'(y)dy$, 5. sub $g(y)$ into $F(x,y) = C$, **Exactness Test Fail**: check $\frac{1}{N}(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x})$ only depends on x or $\frac{1}{M}(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y})$ only depends on y, then $\mu(\cdot) = exp(\int(\cdot)d(\cdot))$, plug in $\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0$, solve with exact. **Homogenous**: 1. Express dy/dx in terms of y/x, 2. Sub $v = y/x = f(x,v)$, 3. $x(dv/dx) + v = f(x,v)$ solve with separable, 4. Resub $v = y/x$. **Solving** $dy/dx = G(ax+by)$: 1. sub $z = ax + by$, 2. diff $dz/dx = a + b(dy/dx)$ then sub $dy/dx = (dz/dx - a)/b$, 3. solve separable 4. resub $ax + by = z$. **Bernoulli**: 1. $dy/dx + P(x)y = Q(x)y^n$, $v = y^{1-n}$, 2. $1/(1-n)dv/dx + P(x)v = Q(x)$, 3. solve linear w/ $\mu(x)$. **Linear Coeff**: Special cases: 1. $a_1b_2 = a_2b_1$ then $z = ax + by$, 2. $b_1 = a_2$ then exact, 3. $c_1 = c_2 = 0$ then homogenous. **Coeff Solve**: 1. solve system $a_1h + b_1k = -c$ and $a_2h + b_2k = -c$ for h, k, 2. sub in $x = u + h$ and $y = v + k$ for form $M(u,v)du + N(u,v)dv = 0$, 3. Rearr for dv/du, multiply by $(1/u)/(1/u)$ then sub $z = v/u$, 4. $u(dz/du) + z = step3$, 5. solve separable (partial frac) 6. resub z, u, v.

Ch3: 1st Order Applications

Mixing Problems: $dx/dt = r_{in} - r_{out}$, where r(t) can also be salt concentration, if so, then $dx/dt = r_{in}c_{in} - r_{out}x(t)/V$, where x(t) is [salt] **Malthusian/Exponential Model**: $dP/dt = kp, p(0) = p_0$, $p(t) = p_0 * exp(kt)$ **Logistic Model**: $dP/dt = k_1p - k_2\frac{p(p-1)}{2}$, $p(t) = \frac{p_0p_1}{p_0+(p_1-p_0)exp(-Ap_1t)}$ **Newton's Law of Cooling**: $T(t) = exp(-kt)\int exp(kt)(kM + H + U)dt + C * exp(-kt)$, where M is initial temp, H is heat (≥ 0), U is heating(+)/cooling rate(-). **Oscillating M Problem**: Given time constant (1/k) and M varies as sine wave w/ min/max time/temp given, find min/max time/temp of building. 1. Find period $\omega = 2\pi/24hr$, $T_{high} = M_0 + B, T_{low} = M_0 - B$, solve for $M_0 = B_0, B$ 2. $T(t) = B_0 - BF(t)$ $Cexp(-kt)$ dies off, $F(t) = \pm 1/\sqrt{1 + (k/\omega)^2}$, 3. Find T_{high} and T_{low} by plugging in F(t) w/ $/pm1$, 4. Find t_{high} and t_{low} by plugging in T_{high} and T_{low} into $T(t) = M_0 + (T_0 - M_0)exp(-kt)$.
Physics: $F(t,v) = m\frac{dv}{dt} = mg - bv, v(0) = v_0$
$v(t) = \frac{mg}{b} + (v_0 - \frac{mg}{b})exp(-bt/m)$
$x(t) = \frac{mg}{b}t + (v_0 - \frac{mg}{b})\frac{m}{b}(1 - exp(-bt/m))$
Parachute Problem: Find distance (d) fallen after given t_1 w/o parachute w/ $x_1(t_1), b_1$, calc $v(t)$ to use as v_0 in $v_2(t_2)$, distance left $d = h - x_1(t_1)$, Solve for t_2 in $x_2(t_2) = d$, total time $t_{total} = t_1 + t_2$. **RL Circuit**: $exp(Rt/L)I(t) = \int exp(Rt/L)(E(t)/L)dt + K$ to $I(t) = I_{max}(1 - exp(-Rt/L))$, Resistor Voltage: $E_R(t) = I(t)$. Inductor Voltage: $E_L(t) = L(dI/dt)$. Capacitor Voltage: $E_C(t) = q/C$. Capacitor Current: $R(dq/dt) + q/C = E(t)$. **RC Circuit**: $exp(t/RC)q(t) = \int exp(t/RC)(E(t)/R)dt + K$. If $E(t) = V = const$, then $q(t) = CV + (Q - CV)exp(-t/RC)$, Q is initial charge. If $Q = 0$, then $V(t) = V_{max}(1 - exp(-t/RC))$.

Ch4: Linear 2nd Order Applications

Homogenous 2nd-order const coeff: 1. turn $ay'' + by' + cy = 0$ into $ar^2 + br + c = 0$ and solve for r. 2. $y = e^{rt}$ are solns. 3. $y = c_1e^{r_1t} + c_2e^{r_2t}$ is the general soln. 4a. If $r_1 \neq r_2$, then $y = c_1e^{r_1t} + c_2e^{r_2t}$. 4b. If $r_1 = r_2$, then $y = c_1e^{r_1t} + c_2te^{r_2t}$.
Wronskian test for linear independence:
$W[y_1, y_2] = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$.
Homogenous Complex: 1. Find $r = \alpha \pm i\beta$ from $ar^2 + br + c = 0$. 2. $y = e^{\alpha t}(c_1\cos\beta t + c_2\sin\beta t)$ is the general soln.

Mass-spring system: $my'' + by' + ky = 0$ where m is mass, b is damping, k is spring const.