

This is an example of inverse design of slow light waveguides based on supercell band structure calculations. For the h_z polarization, the band structure calculation is governed by

$$\nabla \cdot \left(\frac{1}{\varepsilon_r} \nabla h \right) + \left(\frac{\omega}{c} \right)^2 h = 0 \quad (1)$$

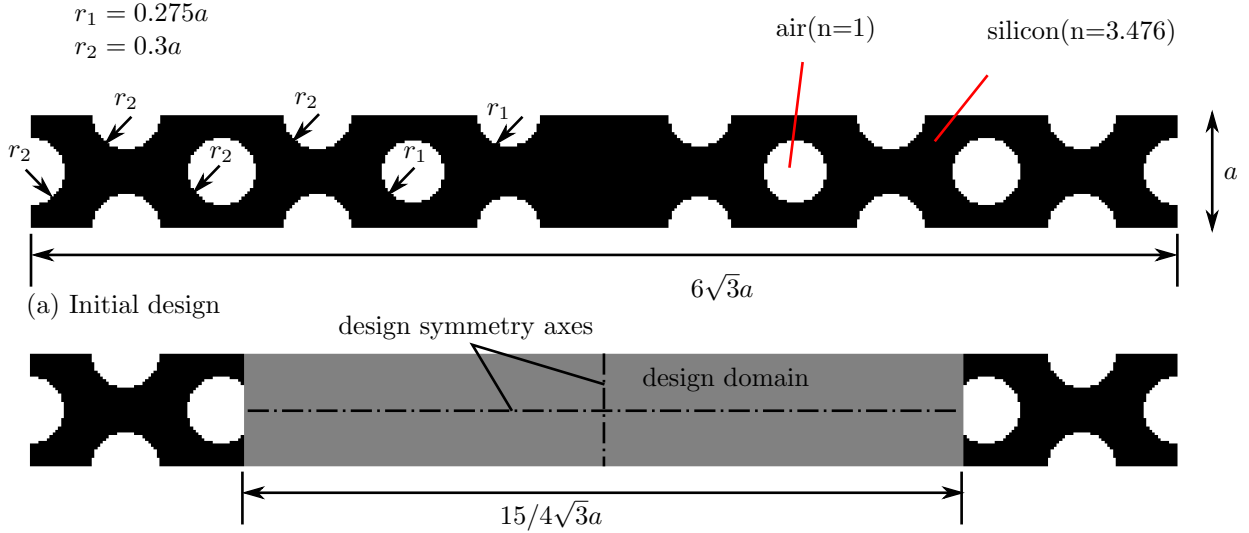
under the Floquet-Bloch wave boundary conditions as

$$h(x, a) = \exp(ika) h(x, 0) \quad h(0, y) = h(b, y). \quad (2)$$

The discrete expression obtained using the finite element method is stated as

$$(\mathbf{K}_k - \omega^2 \mathbf{M}) \mathbf{h} = 0. \quad (3)$$

The considered supercell, initial design and corresponding design domain are illustrated in the figure below.



(b) Design domain

The design problem is stated as

$$\begin{aligned} \min_{\rho_j} \quad & \max_{\eta} \quad \max_{k_i} \quad f(\bar{\rho}_{\eta}) = \left(\frac{c(k_i - k_{i-1})}{\omega_n^{\eta}(k_{i-1}) - \omega_n^{\eta}(k_i)} - n_g^* \right)^2 \\ s.t. \quad & \left[\mathbf{K}_k^{\eta} - (\omega^{\eta})^2 \mathbf{M}^{\eta} \right] \mathbf{h}^{\eta} = 0 \\ & \max_{k_{ii}} \omega_{n-1}^{\eta}(k_{ii}) \leq a_1 \min_{k_i} \omega_n^{\eta}(k_i) \\ & \omega_n^{\eta}(0) \geq a_2 \max_{k_i} \omega_n^{\eta}(k_i) \\ & \min_{k_{ii}} \omega_{n+1}^{\eta}(k_{ii}) \geq a_2 \max_{k_i} \omega_n^{\eta}(k_i) \\ & f_v = \frac{\sum_j \bar{\rho}_j^{\eta} v_j}{\sum_j v_j} \leq 1 \\ & 0 \leq \rho_j \leq 1 \\ & j = 1, \dots, N, \quad i = 2, \dots, m, \quad a_1 < 1, \quad a_2 > 1. \end{aligned} \quad (4)$$

The relevant parameters are defined below:

- Discretization:

408x40 bilinear quadrilateral elements

- Regularization:

density filter (filter radius: $1/8a$) + projection

- Continuation scheme in the projection

For every 40th iteration or if ($\{ \Delta\rho < 1e-3$ or $\Delta f < 1e-3$ } and $\beta < 50$, set $\beta = 1.3\beta$.
If $\Delta\rho < 1e-4$ or $\Delta f < 1e-4$, terminate.

- Interpolation of the relative permittivity of element e :

$$\frac{1}{\varepsilon_e^\eta} = (1 - \bar{\rho}_e^\eta) \frac{1}{\varepsilon_{\text{air}}} + \bar{\rho}_e^\eta \frac{1}{\varepsilon_{\text{Si}}}$$

- Robust formulation: $\eta \in [0.35, 0.5, 0.65]$.

- $n_g^* = 25$

- Target k points: 7 equidistant points $k \in [0.3875, 0.4625]2\pi/a$

- $a_1 = 0.9$ and $a_2 = 1.1$

The blue print design with $\eta = 0.5$ obtained using the robust optimization formulation considering the parameters above and corresponding performance are shown in the figure below.

