This is an example of inverse design of slow light waveguides based on supercell band structure calculations. For the h_z polarization, the band structure calculation is governed by

$$\boldsymbol{\nabla} \cdot \left(\frac{1}{\varepsilon_r} \boldsymbol{\nabla} h \right) + \left(\frac{\omega}{c} \right)^2 h = 0 \tag{1}$$

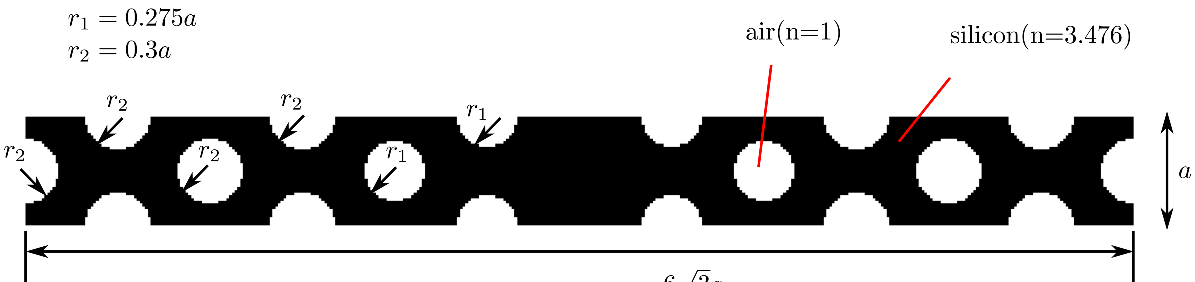
under the Floquet-Bloch wave boundary conditions as

$$h\left(x, a\right) = \exp\left(ika\right) h\left(x, 0\right) \qquad h\left(0, y\right) = h\left(b, y\right). \tag{2}$$

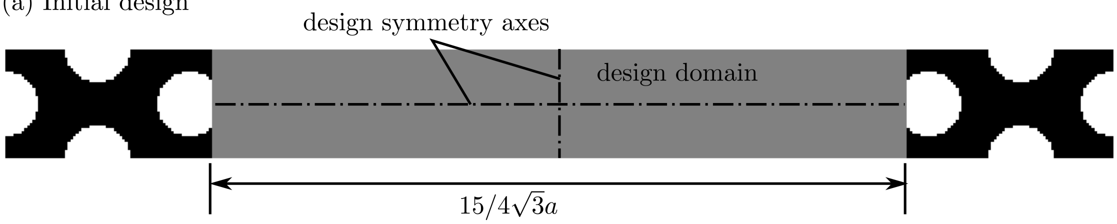
The discrete expression obtained using the finite element method is stated as

$$\left(\mathbf{K}_k - \omega^2 \mathbf{M}\right) \mathbf{h} = 0. \tag{3}$$

The considerd supercell, initial design and corresponding design domain are illustrated in the figure below.



(a) Initial design



(b) Design domain

The design problem is stated as

$$\begin{aligned}
\min_{\rho_j} \quad \max_{\eta} \quad \max_{k_i} \qquad & f\left(\overline{\boldsymbol{\rho}}_\eta\right) = \left(\frac{c\left(k_i - k_{i-1}\right)}{\omega_n^\eta\left(k_{i-1}\right) - \omega_n^\eta\left(k_i\right)} - n_g^* \right)^2 \\
s.t. \qquad & \left[\mathbf{K}_k^\eta - \left(\omega^\eta\right)^2 \mathbf{M}^\eta\right] \mathbf{h}^\eta = 0 \\
& \max_{k_{ii}} \ \ \omega_{n-1}^\eta\left(k_{ii}\right) \ \leq \ a_1 \ \min_{k_i} \ \ \omega_n^\eta\left(k_i\right) \\
& \omega_n^\eta\left(0\right) \ \geq \ a_2 \ \max_{k_i} \ \ \omega_n^\eta\left(k_i\right) \\
& \min_{k_{ii}} \ \ \omega_{n+1}^\eta\left(k_{ii}\right) \ \geq \ a_2 \ \max_{k_i} \ \ \omega_n^\eta\left(k_i\right) \\
& f_v = \frac{\sum_j \overline{\rho}_j^\eta v_j}{\sum_j v_j} \leq 1 \\
& 0 \leq \rho_j \leq 1 \\
& j = 1, \ldots, N, \quad i = 2, \ldots, m, \quad a_1 < 1, \quad a_2 > 1.
\end{aligned} \tag{4}$$

The relevant parameters are defined below:

- Discretization:

  408x40 bilinear quadrilateral elements

- Regularization:

  density filter (filter radius: 1/8a) + projection

- Continuation scheme in the projection

  For every 40th iteration or if ($\{ \Delta\rho < 1e-3$ or $\Delta f < 1e-3 \}$ and $\beta < 50$, set $\beta = 1.3\beta$.
  If $\Delta\rho < 1e-4$ or $\Delta f < 1e-4$, terminate.

- Interpolation of the relative permittivity of element e:

  $\frac{1}{\varepsilon_e^\eta} = (1-\overline{\rho}_e^\eta)\frac{1}{\varepsilon_{\text{air}}} + \overline{\rho}_e^\eta \frac{1}{\varepsilon_{\text{Si}}}$ where $\varepsilon_{\text{Si}} = (3.476)^2$.

- Robust formulation: $\eta \in [0.35, 0.5, 0.65]$.

- $n_g^* = 25$

- Target k points: 7 equidistant points $k \in [0.3875, 0.4625]2\pi/a$

- $a_1 = 0.9$ and $a_2 = 1.1$

The blue print design with $\eta = 0.5$ obtained using the robust optimization formulation considering the parameters above and corresponding performance are shown in the figure below.