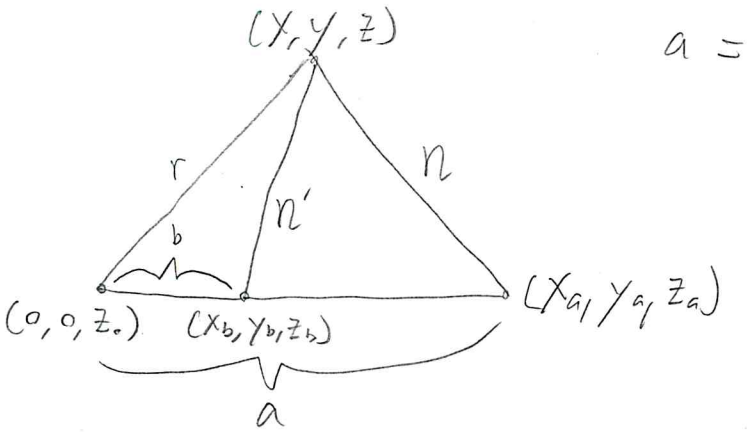
$$b = \sqrt{x_b^2 + y_b^2 + (z_b - z_0)^2}\,, \qquad n = \sqrt{(x-x_a)^2 + (y-y_a)^2 + (z-z_a)^2}$$

$$a = \sqrt{x_a^2 + y_a^2 + (z_a - z_0)^2}\,, \qquad n' = \sqrt{(x-x_b)^2 + (y-y_b)^2 + (z-z_b)^2}$$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ z_0 \end{pmatrix} + t \begin{pmatrix} x_a \\ y_a \\ z_a - z_0 \end{pmatrix}$$

$$\frac{x_b}{x_a} = \frac{y_b}{y_a} = \frac{z_b - z_0}{z_a - z_0}$$

$$z_b - z_0 = (z_a - z_0)\frac{x_b}{x_a}$$

$$y_b = x_b \frac{y_a}{x_a}$$

$$b = \frac{R^2}{a} = \frac{R^2}{\sqrt{x_a^2 + y_a^2 + (z_a - z_0)^2}} = \sqrt{x_b^2 + y_b^2 + (z_b - z_0)^2} \quad \Rightarrow \quad \frac{R^4}{x_a^2 + y_a^2 + (z_a - z_0)^2} = x_b^2 + y_b^2 + (z_b - z_0)^2$$

$$x_b^2 + x_b^2 \frac{y_a^2}{x_a^2} + (z_a - z_0)^2 \frac{x_b^2}{x_a^2} = \frac{R^4}{x_a^2 + y_a^2 + (z_a - z_0)^2} \quad \Rightarrow \quad x_b^2 \left(1 + \frac{y_a^2}{x_a^2} + \frac{(z_a - z_0)^2}{x_a^2}\right) = \frac{R^4}{x_a^2 + y_a^2 + (z_a - z_0)^2}$$

$$x_b = \frac{R^2}{\sqrt{(x_a^2 + y_a^2 + (z_a - z_0)^2)\left(1 + \frac{y_a^2}{x_a^2} + \frac{(z_a - z_0)^2}{x_a^2}\right)}}$$

$$V(x, y, z) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{(x-x_a)^2 + (y-y_a)^2 + (z-z_a)^2}} - \frac{R}{\sqrt{x_a^2 + y_a^2 + (z_a - z_0)^2}\sqrt{(x-x_b)^2 + (y-y_b)^2 + (z-z_b)^2}}\right)$$

$$\frac{\partial V}{\partial x} = \frac{q}{4\pi\epsilon_0}\left(\frac{x_a - x}{((x-x_a)^2+(y-y_a)^2+(z-z_a)^2)^{3/2}} - \frac{R(x_b-x)}{\sqrt{x_a^2+y_a^2+(z_a-z_0)^2}\left((x-x_b)^2+(y-y_b)^2+(z-z_b)^2\right)^{3/2}}\right)$$

$$\frac{\partial V}{\partial y} = \dots$$

$$\frac{\partial V}{\partial z} = \dots$$

$$x_b = (z_b - z_0)\frac{x_a}{z_a - z_0}, \quad y_b = (z_b - z_0)\frac{y_a}{z_a - z_0}$$

$$\frac{R^4}{x_a^2+y_a^2+(z_a-z_0)^2} = (z_b-z_0)^2\frac{x_a^2}{(z_a-z_0)^2} + (z_b-z_0)^2\frac{y_a^2}{(z_a-z_0)^2} + (z_b-z_0)^2$$

$$= (z_b-z_0)^2\left(1 + \frac{x_a^2}{(z_a-z_0)^2} + \frac{y_a^2}{(z_a-z_0)^2}\right)$$

$$z_b - z_0 = \frac{R^2}{\sqrt{1 + \frac{x_a^2}{(z_a-z_0)^2} + \frac{y_a^2}{(z_a-z_0)^2}}\sqrt{x_a^2+y_a^2+(z_a-z_0)^2}}$$