PS3)

1a) $\psi_i = P(z_i = 1)$

$$z_i = 1\{|\hat{\phi}_i - \phi_i| > \gamma\}$$

$$\Rightarrow \boxed{P(|\hat{\phi}_i - \phi_i| > \gamma) \leq 2\exp(-2\gamma^2 m)}$$

Bernoulli PDF: PS3 1b)

Every V_i and W_i is either 0 or 1

$\cancel{P(V_i \geq 1) = E(V_i)}$ ~~thus~~ i same as $P(V_i = 1)$

$\cancel{P(V_i < 0) = 0}$

For any Bernoulli var X:

~~PRO~~ For $t \geq 1$:

$P(X > t) = 0$ $\quad (= P(X > 1) = 0)$

$0 \leq t < 1$:

$P(X > t) = E(X)$ (because 0 is not part of it)

$\Rightarrow$ same as $P(X = 1)$

$t < 0$:

$P(X > t) = 1$ (i.e. ~~for~~ $= P(X \geq 0) = 1$)

$\Rightarrow P(V_i > t) \leq P(W_i > t)$ for all t, as $0 = 0$

$E(V_i) \leq E(W_i)$

$1 = 1$

1b continued

$$P\left(\sum_{i=1}^{2} V_i > t\right) = P(V_1 + V_2 > t)$$

given that for every i: $P(V_i > t) \le P(W_i > t)$ needs to hold

More explicitly:

For $0 \le t < 1$

$$P(V_1 + \dots + V_k > t)$$

= any V_i needs to be 1

$\Rightarrow 1 - P(\text{No } V_i \text{ is } 1)$

$$= 1 - \underbrace{\prod \underbrace{(1 - \underbrace{E(V_i)}_{\le W_i})}_{\ge W_i}}_{\le V_i} \Rightarrow \text{qed}$$

c) $\psi_i = P(z_i = 1) \leq 2\exp(-2\gamma^2 m)$

$\Rightarrow E(z_i) \leq 2\exp(-2\gamma^2 m)$

$P\left(\frac{1}{n}\sum_{i=1}^{n} z_i > \tau\right)$

$P(\bar{Z} = 1)$ = All are inaccurate = $\prod_i P(z_i = 1) \leq 2^n \cdot \exp(-2\gamma^2 mn)$

= Highest upper bound

$P(\bar{Z} = 0)$ = None are inaccurate $\geq 1 - 2^n \exp(-2\gamma^2 mn)$

Markov's inequality:

$\cancel{P\left(\sum_{i=1}\right)}$ $\leq \dfrac{\frac{n}{n} 2\exp(-2\gamma^2 m)}{\tau}$