4b) $\boxed{\text{Show condition I:}}$

$$\log\left(\phi^{y}(1-\phi)^{1-y}\right) = y\cdot\log(\phi) + (1-y)\cdot\log(1-\phi)$$

$$\Rightarrow d\phi = \frac{y}{\phi} + (1-y)\cdot\frac{1}{1-\phi}\cdot -1$$

$$= \frac{y}{\phi} - \frac{1-y}{1-\phi}$$

$$\Rightarrow \frac{y}{\phi} = \frac{1-y}{1-\phi}$$

$$\Rightarrow \frac{\phi}{y} = \frac{1-\phi}{1-y}$$

$$\Rightarrow \frac{\phi}{1-\phi} = \frac{y}{1-y}$$

$$\Rightarrow \frac{1-\phi}{\phi} = \frac{1-y}{y}$$

$$\frac{1}{\phi} - \cancel{1} = \frac{1}{y} - \cancel{1}$$

$$\phi = y$$

$$\Rightarrow \sum_{i=1}^{m}\phi = \sum_{i=1}^{m} y^{i}$$

$$\Rightarrow m\cdot\phi = \sum_{i=1}^{m}\cdot y^{i}$$

$$\boxed{\Rightarrow \phi = \frac{1}{m}\cdot\sum_{i=1}^{m} 1\{y^{(i)}=1\}}$$

for μ_0:

At the end:

$$\sum_{i=1}^{m} (1-y) \cdot \Sigma \cdot (x - \mu_0) = 0$$

$\Rightarrow 0 = 0$ for i where $y^i = 1$

for all others: $\sum_{i=1}^{p} x^i = \cancel{\$}\, p \cdot \mu_0$

$$\Rightarrow \mu_0 = \frac{1}{p} \sum_{i=1}^{p} x^i$$

$\boxed{\text{Show } \Sigma:}$

$$\frac{dl}{d\Sigma} = \frac{dl}{dp} \quad (\text{assume } p = \Sigma)$$

also $z = 2\pi^{\frac{n}{2}}$

$$\Rightarrow \frac{dl}{dp} = d\Big(y \cdot \log\Big(\frac{1}{z \cdot p^{\frac{1}{2}}} \cdot \exp\Big(-\frac{1}{2} (x-\mu_1) p^{-1} (x-\mu_1) \Big)\Big)$$

$$+ (1-y) \log\Big(\frac{1}{z \cdot p^{\frac{1}{2}}} \cdot \exp\Big(-\frac{1}{2} (x-\mu_0)^2 \cdot p^{-1} \Big)\Big) \Big) \Big/ dp$$

$$= y \cdot \Big(-\log(z \cdot p^{\frac{1}{2}}) \cancel{+} - \frac{1}{2} (x-\mu_1)^2 \cdot p^{-1} \Big) + (1-y) \cdot \Big(-\log(z\, p^{\frac{1}{2}}) + -\frac{1}{2} (x-\mu_0)^2 p \Big)$$

$$\Rightarrow \frac{d}{dp} =$$

$$= -y \cancel{\log(z \cdot p^{\frac{1}{2}})} - y \cdot \frac{1}{2} \cdot (x-\mu_1)^2 \cdot p$$

$$- \log(z p^{\frac{1}{2}}) - \frac{1}{2}(x-\mu_0)^2 p + y \cdot \cancel{\log(z \cdot p^{\frac{1}{2}})} + y \cdot \frac{1}{2} \cdot (x-\mu_0)^2 \cdot p$$

$$\frac{d}{d_6} = -\cancel{\log(z} \frac{1}{z \cdot p^{\frac{1}{2}}} \cdot \cancel{z p^{\frac{1}{2}}} \frac{1}{2} \cdot z \cdot p^{-\frac{1}{2}}$$

$$= -\frac{\cancel{z} \cdot p^{-\frac{1}{2}}}{2 \cancel{z}} \cdot p^{-\frac{1}{2}} = -\frac{1}{2p}$$

$$\sum_{i=1}^{m} (x-\mu_1) = \sum_{i=1}^{m} p$$

if: $y=1$

if $y=0$:

$$\sum_{i=1}^{m} (x-\mu_0) = \sum_{i=1}^{m} p$$

$\Rightarrow$ taken together

$\#$

$$\Rightarrow \frac{d(-\log(z \cdot p^{\frac{1}{2}}))}{d_p} = -\frac{1}{2p}$$

$$\frac{d(-\frac{1}{2}(x-\mu_1)^2 \cdot p^{-1})}{d_p} = +\frac{\frac{1}{2}(x-\mu_1)^2}{p^2}$$

$$\cancel{\Rightarrow +\frac{1}{2p} = +\frac{1}{2}(x-\mu_1)^2}$$

$$\cancel{\Rightarrow \frac{1}{p} = (x-\mu_1)^2}$$

$$\Rightarrow -\frac{1}{2p} + \frac{(x-\mu_1)^2}{2p^2} = 0$$

$$\Rightarrow \frac{(x-\mu_1)^2}{2p^2} = \frac{1}{2p}$$

$$\Rightarrow (x-\mu_1)^2 = p$$

Taking it all together:

$$y\left(-\frac{1}{2p}+\frac{(x-\mu_1)^2}{2p^2}\right)+(1-y)\left(-\frac{1}{2p}+\frac{(x-\mu_0)^2}{2p^2}\right)=0$$

$$-\frac{\cancel{y}}{\cancel{2}p}+y\frac{(x-\mu_1)^2}{2p^2}-\frac{1}{2p}+\frac{(x-\mu_0)^2}{2p^2}+\cancel{\frac{y}{2p}}-y\frac{(x-\mu_0)^2}{2p^2}=0$$

$$y\frac{(x-\mu_1)^2}{\cancel{2}p^2}+(1-y)\frac{(x-\mu_0)^2}{\cancel{2}p^2}=\frac{1}{\cancel{2}p}$$

$$y(x-\mu_1)^2+(1-y)(x-\mu_0)^2=p$$

$$\Rightarrow\sum_{i=1}^{m}y^i(x-\mu_1)^2+(1-y^i)(x-\mu_0)^2=m\cdot p$$

$$\Rightarrow\boxed{p=\frac{1}{m}\cdot\sum_{i=1}^{m}(x-\mu_{y^i})^2}$$

4a) $p(y|x) = \dfrac{p(x|y)\ p(y)}{p(x)}$

$$p(y=1 \mid x; \phi, \Sigma, \mu_0, \mu_1) = \frac{p(x \mid y=1) \cdot p(y=1)}{p(x \mid y=1)\, p(y=1) + p(x \mid y=0)\, p(y=0)}$$

$p(y=1) = \phi \quad p(y=0) = 1-\phi$

$$\Rightarrow p(y=1 \mid x; \ldots) = \frac{\phi \cdot \cancel{A} \cdot \exp(K(\mu_1))}{\phi \cdot \cancel{A} \cdot \exp(K(\mu_1)) + (1-\phi) \cdot \cancel{A} \cdot \exp(K(\mu_0))}$$

now divide by $\phi \cdot \exp(K(\mu_1))$

~~by $\phi \cdot \exp($~~

$$\Rightarrow \frac{1}{1 + \dfrac{(1-\phi) \cdot \exp(K(\mu_0))}{\phi \cdot \exp(K(\mu_1))}}$$

4a continued)

$$\Rightarrow \exp(-\theta^T x) = \frac{\exp(K(\mu_0)) - \phi \cdot \exp(K(\mu_0))}{\phi \cdot \exp(K(\mu_1))}$$

or: $$\exp(\theta^T x) = \frac{\phi \exp(K(\mu_1))}{\exp(K(\mu_0)) \cdot (1-\phi)}$$

$$\Rightarrow \theta^T x = \log \phi + K(\mu_1) - K(\mu_0) - \log(1-\phi)$$

$$= \log \frac{\phi}{1-\phi} + \underbrace{K(\mu_1) - K(\mu_0)}$$

$$-\frac{1}{2}(x-\mu_1)^T \Sigma^{-1} (x-\mu_1) + \frac{1}{2}(x-\mu_0)^T \Sigma^{-1} (x-\mu_0)$$

$$= \frac{1}{2}\Big(\cancel{x^T \Sigma^{-1} x} - x^T \Sigma^{-1} \mu_0 - \mu_0^T \Sigma^{-1} x + \mu_0^T \Sigma^{-1} \mu_0$$

$$- \cancel{x^T \Sigma^{-1} x} + x^T \Sigma^{-1} \mu_1 + \mu_1^T \Sigma^{-1} x - \mu_1^T \Sigma^{-1} \mu_1$$

$$= \frac{1}{2}\Big(\big(-\mu_0^T \Sigma^{-1} - (\Sigma^{-1}\mu_0)^T + (\Sigma^{-1}\mu_1)^T + \mu_1^T \Sigma^{-1}\big) x$$

$$+ \mu_0^T \Sigma^{-1} \mu_0 - \mu_1^T \Sigma^{-1} \mu_1$$

$\rightarrow$ Const

$$= \frac{1}{2}\left(2\mu_1^T \Sigma^{-1} - 2\mu_0^T \Sigma^{-1} \right) x$$

$$= \left(\mu_1^T \Sigma^{-1} - \mu_0^T \Sigma^{-1} \right) \cdot x$$

$$\boxed{\Rightarrow \theta = \begin{pmatrix} \log \frac{\phi}{1-\phi} + \frac{1}{2}\left(\mu_0^T \Sigma^{-1} \mu_0 - \mu_1^T \Sigma^{-1} \mu_1 \right) \\ \mu_1^T \Sigma^{-1} - \mu_0^T \Sigma^{-1} \end{pmatrix}}$$