PS2)

4.) VC(H) = Largest size S that H can shatter

Shatter = ~~A~~ Separate S (= Set of points) such that any labeling is possible

a) $H_1 \subseteq H_2$

True

Show: $VC(H_1) \leq VC(H_2)$

True. ~~As~~ $VC(H_1)$ can not be $> VC(H_2)$

because VC can only increase by adding more hypotheses to H. Can't decrease, as more elements in H means more labeling possibilities of S => VC can only increase.

But if H_1 is exactly H_2 then $VC(H_1) = VC(H_2)$

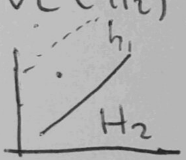
if H_1 is just one hypothesis then $VC(H_1) = 0$

But H_2 shatters at least as many as H_1 and $0 \leq VC(H_2)$

b) $H_1 = H_2 \cup \{h_1, \dots, h_k\}$

$\Rightarrow VC(H_1) \leq VC(H_2) + K$

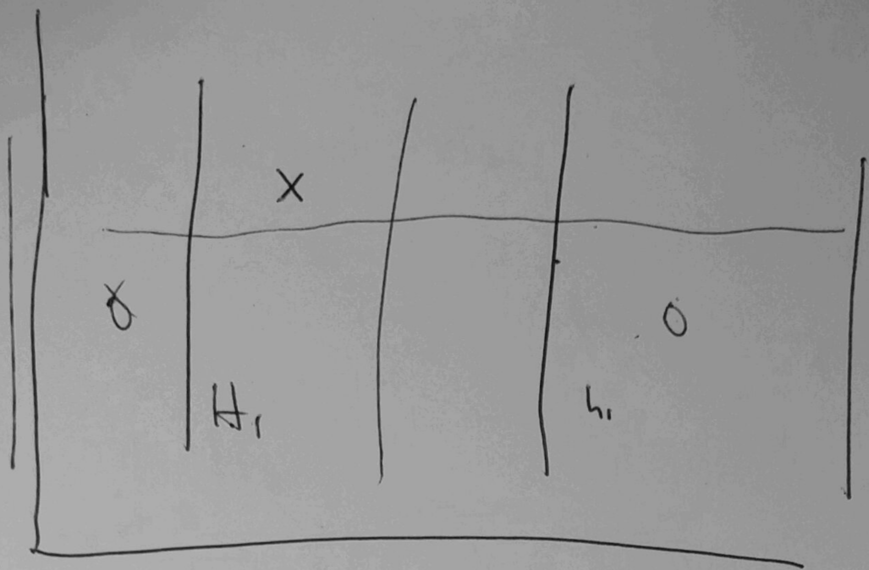
Assume S:



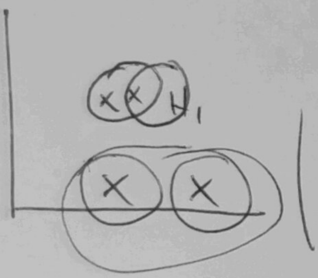
$\Rightarrow VC(H_2) = 0$ (can't relabel point p)

$\Rightarrow VC(H_2 \cup h_1) = 1 = 0+1$

b)



Labeling possibilities:



Num points	Poss	Min h
1	2	2
2	$2^2 = 4$	4

Can at most add one VC with each additional hypo, but not more than 1. As shown above it's possible that ~~d~~ 1 more hyp can increase VC by 1. Even if more hyps are added, (say 2) smaller or equal still holds as $1 \leq 2$.

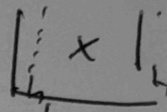
Show that cannot add more than 1 VC with 1 hyp:

1 additional hyp gives at best 1 more labeling ~~appear~~ set, but never multiple. But with every additional point, the number of possible labels increases by 2x.

PS 2)

a)

b). Conclusion: Can be shown that adding

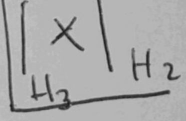
True 1 k can add 1 VC ().

But ~~k~~ 1 hyp can never increase VC by more than 1 because if set S is increased by 1 point, the number of labeling possibilities is 2x'd, while an ~~etc~~ additional hypo can only add 1 more labeling way at best.

$\Rightarrow VC(H_2) \leq VC(H_1) + k$

c) $H_1 = H_2 \cup H_3$

$VC(H_1) \leq VC(H_2) + VC(H_3)$.

False : ~~take~~ 

$\Rightarrow VC(H_2) = 0$

$VC(H_3) = 0$

$\Rightarrow VC(H_2) + VC(H_3) = 0$

But $VC(H_3 \cup H_2) = 1 > 0$