$\cancel{\nabla_u = \sum_i \sum_j -2 x_j^{(i)} x^{(i)} + 2 x^{(i)2} u_j - \lambda(u^T u - 1) =}$

PS 4 3 $\qquad \times \ 1/2$

$$f_u(x) = (x^T u) \cdot u = (u^T x) \cdot u$$

$$\Rightarrow \min \sum_{i=1}^{m} \left(x^{(i)} - (x^T u) u \right)^2$$

$$\text{s.t. } u^T u = 1 \Rightarrow u^T u - 1 = 0$$

$$\Rightarrow \min_u \sum_{i=1}^{m} \left(x^{(i)} - (x^T u) u \right)^T \left(x^{(i)} - (x^T u) u \right) - \lambda (u^T u - 1)$$

$$\Rightarrow \min \sum_{i=1}^{m} x^{(i)T} x^{(i)} - 2 \cdot x^{(i)T} (x^{(i)T} u) u + ((x^T u) u)^T (x^T u) u - \lambda (u^T u - 1)$$

$$\Rightarrow \min \sum_{i=1}^{m} x^T x - 2 \cdot (x^T u)^2 + (x^T u)^2 \cdot u^T u - \lambda (u^T u - 1)$$

$\Rightarrow$

$\cancel{\nabla_u \min \sum_{i=1}^{m} - 4.}$ $\qquad x^T u = \left(\sum_j (x_j u_j)\right)^2$

Since $u^T u = 1$ $\qquad (x^T u)' = x$

$\Rightarrow$

$$\min \sum_{i=1}^{m} -(x^T u)^2 + \lambda(1 - u^T u)$$

$$\nabla_u \Rightarrow \sum_{i=1}^{m} -\left(x(x^T u) + \cancel{u}\, x(x^T u)\right) - 2\lambda u$$

$$\Rightarrow \sum_{i=1}^{m} -2x(x^T u) \quad \cancel{*} \; -2\lambda u$$

$$\Rightarrow \left(\sum_{i=1}^{m} x x^T\right) u = -\lambda u$$

$$\Rightarrow \Sigma u = \lambda u \quad ✓$$

Since $\Sigma = \sum_{i=1}^{m} x^{(i)} x^{(i)T}$