

# Fourier Analysis

Autumn 2018

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<http://www.jamescannon.net/teaching/fourier-analysis>

[http://raw.githubusercontent.com/NanoScaleDesign/FourierAnalysis/master/fourier\\_analysis.pdf](http://raw.githubusercontent.com/NanoScaleDesign/FourierAnalysis/master/fourier_analysis.pdf)

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## Chapter 0

# Course information

## 0.1 This course

This is the Autumn 2018 Fourier Analysis course studied by 3rd-year undergraduate international students at Kyushu University.

### 0.1.1 How this works

- In contrast to the traditional lecture-homework model, in this course the learning is self-directed and active via publicly-available resources.
- Learning is guided through solving a series of carefully-developed challenges contained in this book, coupled with suggested resources that can be used to solve the challenges with instant feedback about the correctness of your answer.
- There are no lectures. Instead, there is discussion time. Here, you are encouraged to discuss any issues with your peers, teacher and any teaching assistants. Furthermore, you are encouraged to help your peers who are having trouble understanding something that you have understood; by doing so you actually increase your own understanding too.
- Discussion-time is from 10:30 to 12:00 on Wednesdays at room W4-529.
- Peer discussion is encouraged, however, if you have help to solve a challenge, always make sure you do understand the details yourself. You will need to be able to do this in an exam environment. If you need additional challenges to solidify your understanding, then ask the teacher. The questions on the exam will be similar in nature to the challenges. If you can do all of the challenges, you can get 100% on the exam.
- Every challenge in the book typically contains a **Challenge** with suggested **Resources** which you are recommended to utilise in order to solve the challenge. Occasionally the teacher will provide extra **Comments** to help guide your thinking. A **Solution** is also made available for you to check your answer. Sometimes this solution will be given in encrypted form. For more information about encryption, see section 0.3.
- For deep understanding, it is recommended to study the suggested resources beyond the minimum required to complete the challenge.
- The challenge document has many pages and is continuously being developed. Therefore it is advised to view the document on an electronic device rather than print it. The date on the front page denotes the version of the document. You will be notified by email when the document is updated. The content may differ from last-year's document.
- A target challenge will be set each week. This will set the pace of the course and defines the examinable material. It's ok if you can't quite reach the target challenge for a given week, but then you will be expected to make it up the next week.
- You may work ahead, even beyond the target challenge, if you so wish. This can build greater flexibility into your personal schedule, especially as you become busier towards the end of the semester.
- Your contributions to the course are strongly welcomed. If you come across resources that you found useful that were not listed by the teacher or points of friction that made solving a challenge difficult, please let the teacher know about it!

### 0.1.2 Assessment

In order to prove to outside parties that you have learned something from the course, we must perform summative assessments. This will be in the form of a satisfactory challenge-log (weighted 5%), coursework (weighted 15%), a mid-term exam (weighted 30%) and a final exam (weighted 50%).

Your final score is calculated as  $\text{Max}(\text{final exam score}, \text{weighted score})$ , however you must pass the final exam to pass the course.

### 0.1.3 What you need to do

- Prepare a challenge-log in the form of a workbook or folder where you can clearly write the calculations you perform to solve each challenge. This will be a log of your progress during the course and will be occasionally reviewed by the teacher.
- You need to submit a brief report at <https://goo.gl/forms/vPPCY0Z6nW5ns1mm1> by 8am on the day of the class. Here you can let the teacher know about any difficulties you are having and if you would like to discuss anything in particular.
- Please bring a wifi-capable internet device to class, as well as headphones if you need to access online components of the course during class. If you let me know in advance, I can lend computers and provide power extension cables for those who require them (limited number).

## 0.2 Timetable

	Discussion	Target	Note
<b>1</b>	3 Oct	-	
<b>2</b>	17 Oct	2.11	
<b>3</b>	24 Oct		
<b>4</b>	31 Oct		
<b>5</b>	7 Nov		
<b>6</b>	14 Nov		
<b>7</b>	21 Nov		
<b>8</b>	28 Nov		
<b>9</b>	5 Dec		
<b>10</b>	12 Dec		
<b>11</b>	19 Dec		
<b>12</b>	9 Jan		
<b>13</b>	16 Jan		
<b>14</b>	23 Jan		
<b>15</b>	6 Feb	Final exam	Tentative



### 0.3 Hash-generation

Some solutions to challenges are encrypted using MD5 hashes. In order to check your solution, you need to generate its MD5 hash and compare it to that provided. MD5 hashes can be generated at the following sites:

- Wolfram alpha: (For example: md5 hash of “q1.00”) <http://www.wolframalpha.com/input/?i=md5+hash+of+%22q1.00%22>
- [www.md5hashgenerator.com](http://www.md5hashgenerator.com)

Since MD5 hashes are very sensitive to even single-digit variation, you must enter the solution *exactly*. This means maintaining a sufficient level of accuracy when developing your solution, and then entering the solution according to the format suggested by the question. Some special input methods:

Solution	Input
$5 \times 10^{-476}$	5.00e-476
$5.0009 \times 10^{-476}$	5.00e-476
$-\infty$	-infinity (never “infinite”)
$2\pi$	6.28
i	im(1)
2i	im(2)
$1 + 2i$	re(1)im(2)
$-0.0002548 i$	im(-2.55e-4)
$1/i = i/-1 = -i$	im(-1)
$e^{i2\pi} [= \cos(2\pi) + i\sin(2\pi) = 1 + i0 = 1]$	1.00
$e^{i\pi/3} [= \cos(\pi/3) + i\sin(\pi/3) = 0.5 + i0.87]$	re(0.50)im(0.87)
Choices in order A, B, C, D	abcd

The first 6 digits of the MD5 sum should match the first 6 digits of the given solution.



## Chapter 1

# Hash practise

## 1.1 Hash practise: Integer

$X = 46.3847$

Form: Integer.

Place the indicated letter in front of the number.

Example: aX where  $X = 46$  is entered as a46

hash of aX = e77fac

## 1.2 Hash practise: Decimal

$X = 49$

Form: Two decimal places.

Place the indicated letter in front of the number.

Example: aX where  $X = 46.00$  is entered as a46.00

hash of bX = 82c9e7

## 1.3 Hash practise: String

$X = abcdef$

Form: String.

Place the indicated letter in front of the number.

Example: aX where  $X = abc$  is entered as aabc

hash of cX = 990ba0

## 1.4 Hash practise: Scientific form

$X = 500,765.99$

Form: Scientific notation with the mantissa in standard form to 2 decimal place and the exponent in integer form.

Place the indicated letter in front of the number.

Example: aX where  $X = 4 \times 10^{-3}$  is entered as a4.00e-3

hash of dX = be8a0d

## 1.5 Hash practise: Numbers with real and imaginary parts

$X = 1 + 2i$

Form: Integer (*for imaginary numbers, “integer” means to write both the real and imaginary parts of the number as integers. If you were instructed to enter to two decimal places, then you would need to enter each of the real and imaginary parts to two decimal places. Refer to section 0.3 to see an example of how to handle input of imaginary numbers.*)

Place the indicated letter in front of the number.

Example: aX where  $X = 46$  is entered as a46

hash of eX = 4aa75a

## 1.6 $\pi$ and imaginary numbers

$$X = -2\pi i$$

Form: Two decimal places. Place the indicated letter in front of the number.

Example: aX where  $X = 46.00$  is entered as a46.00

hash of fX = ad3e8b

## 1.7 Imaginary exponentials

Note that you will need to understand how to expand exponentials in terms of their sines and cosines in order to do this. If you do not understand how to do this yet, skip this challenge and come back to it later.

$$X = 4e^{i3\pi/4}$$

Form: Two decimal places. Place the indicated letter in front of the number.

Example: aX where  $X = 46.00$  is entered as a46.00

hash of gX = 59a753



## Chapter 2

# Periods and frequencies

## 2.1 Period at 50 THz

### Resources

- Video: <https://www.youtube.com/watch?v=v3CvAW8BDHI>

### Challenge

A signal is oscillating at a frequency of 50 THz. What is the period?

### Solution

X = Your solution

Form: Scientific notation with the mantissa in standard form to 2 decimal places and the exponent in integer form.

Place the indicated letter in front of the number.

Example: aX where  $X = 4.0543 \times 10^{-3}$  is entered as a4.05e-3

Hash of qX = 3faf81



## 2.2 Frequency with $k=1$

### Comment

Note that we're working in radians here. From now on a factor of  $2\pi$  will be included in the oscillations so that  $\sin(2\pi t)$  will complete 1 cycle in 1 second. (If your calculator defaults to degrees, be sure to change it to radians for this course.)

### Challenge

What is the frequency of  $\sin(2\pi kt)$ , where  $t$  is time in seconds and  $k = 1$ ?

### Solution

(Hz)

X = Your solution

Form: Decimal to 2 decimal places.

Place the indicated letter in front of the number.

Example: aX where  $X = 46.00$  is entered as a46.00

Hash of eX = 720149

## 2.3 Frequency with k=2

### Challenge

What is the frequency of  $\sin(2\pi kt)$ , where  $t$  is time in seconds and  $k = 2$ ?

### Solution

(Hz)

X = Your solution

Form: Decimal to 2 decimal places.

Place the indicated letter in front of the number.

Example: aX where  $X = 46.00$  is entered as a46.00

Hash of rX = 96ba66

## 2.4 The meaning of $k$

### Challenge

Considering the previous two challenges, what does  $k$  physically represent in those challenges?

### Solution

Please compare your answer with your partner in class or discuss with the teacher.

## 2.5 Smallest period with $k=1$

### Resources

- Book: 1.3 (<https://see.stanford.edu/materials/lsoftaee261/book-fall-07.pdf>)
- Video (14m10s to 17m00s): <https://youtu.be/1rqJl7Rs6ps?t=14m10s>

### Challenge

What is the smallest period of  $\sin(2\pi kt)$ , where  $t$  is time in seconds and  $k = 1$ ?

### Solution

(s)

X = Your solution

Form: Decimal to 2 decimal places.

Place the indicated letter in front of the number.

Example: aX where  $X = 46.00$  is entered as a46.00

Hash of wX = 25c4fb

## 2.6 Smallest period with $k=2$

### Resources

- Book: 1.3 (<https://see.stanford.edu/materials/lsoftaee261/book-fall-07.pdf>)
- Video (14m10s to 17m00s): <https://youtu.be/1rqJl7Rs6ps?t=14m10s>

### Challenge

What is the smallest period of  $\sin(2\pi kt)$ , where  $t$  is time in seconds and  $k = 2$ ?

### Solution

(s)

X = Your solution

Form: Decimal to 2 decimal places.

Place the indicated letter in front of the number.

Example: aX where  $X = 46.00$  is entered as a46.00

Hash of tX = bb995f

## 2.7 Phase

### Comments

Another important concept is phase. For a simple sine signal  $\theta(t) = \sin(2\pi t)$ , at  $t = 0$  the angle  $\theta$  is zero, but one can shift the phase (starting point) of the signal by effectively making the sine-curve non-zero at  $t = 0$ . Another way to think about it is to say the sine curve doesn't reach zero until a time  $t - \phi$  where  $\phi$  is the phase-shift added.

### Challenge

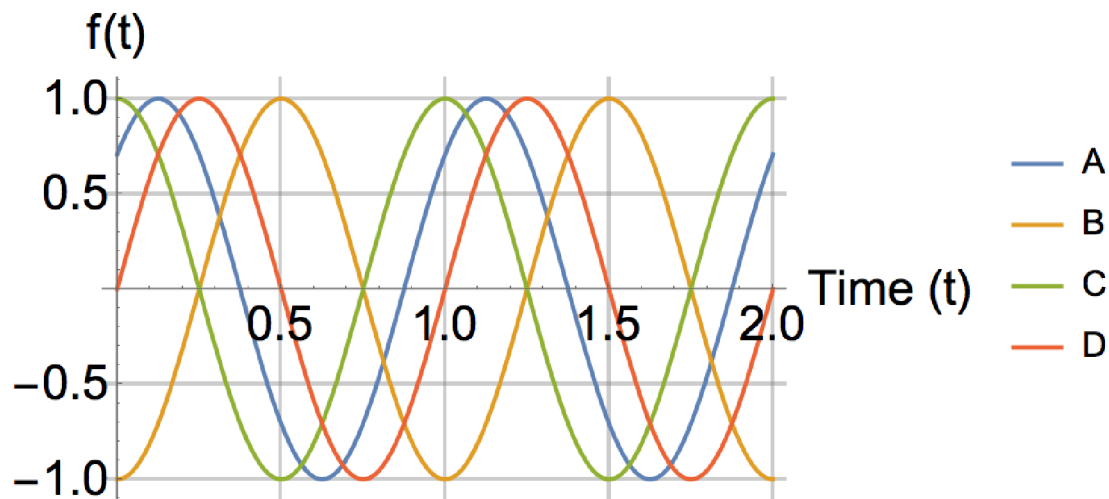
Place the following four graphs in the following order:

$$\sin(2\pi t + \pi/2)$$

$$\sin(2\pi t - \pi/2)$$

$$\sin(2\pi t + \pi/4)$$

$$\sin(2\pi t + 2\pi)$$



### Solution

X = Your solution

Form: String.

Place the indicated letter in front of the string.

Example: aX where X=abcd is entered as aabcd

Hash of iX = 5c0e8b

## 2.8 Amplitude

### Comments

Another important concept is amplitude.  $\sin(2\pi t)$  has an amplitude of 1, but this can be easily modified to go between  $\pm A$  by multiplication with  $A$ .

### Challenge

The following 4 graphs correspond to the equation  $A \sin(2\pi kt)$  with variation in the values of  $A$  and  $k$ . What is the sum of the values of  $A$  for the following graphs?



### Solution

X = Your solution

Form: Integer.

Place the indicated letter in front of the number.

Example: aX where  $X = 46$  is entered as a46

Hash of uX = 6bce05

## 2.9 Periodic and non-periodic signals

### Resources

- Video: [https://www.youtube.com/watch?v=F\\_pdpbu8bgA](https://www.youtube.com/watch?v=F_pdpbu8bgA)
- Book: 1.3 (<https://see.stanford.edu/materials/lsoftaee261/book-fall-07.pdf>)

### Challenge

The list below contains periodic and non-periodic signals. Sum the points of the signals below that are *periodic*.

1 point:  $x(t) = t^2$

2 points:  $x(t) = \sin(t)$

4 points:  $x(t) = \sin(2\pi t)$

8 points:  $x(t) = \sin(2\pi t + t)$

16 points:  $x(t) = \sin(2\pi t) + \sin(t)$

32 points:  $x(t) = \sin(5\pi t) + \sin(2\pi t)$

64 points:  $x(t) = \sin(5\pi t) + \sin(37\pi t)$

128 points:  $x(t) = \sin(5\pi t) + \sin(37.01\pi t)$

256 points:  $x(t) = \sin(5\pi t) + \sin(\sqrt{2}\pi t)$

512 points:  $x(t) = \sin(5\sqrt{2}\pi t) + \sin(\sqrt{2}\pi t)$

### Solution

X = Your solution

Form: Integer.

Place the indicated letter in front of the number.

Example: aX where  $X = 46$  is entered as a46

Hash of uX = 5d906b



## 2.10 Making non-periodic signals from periodic signals

### Challenge

It is not immediately intuitive that it is possible to make a non-periodic signal by simply adding two periodic signals. Referring to the previous challenge, in no-more than 1 paragraph, explain how this is possible.

### Solution

Please compare your answer with your partner or ask the teacher in class.

## 2.11 Fundamental frequency

### Challenge

Considering the periodic signals in challenge 2.9 in order of increasing point-score, calculate the fundamental frequency and period of the last periodic signal in the list.

### Solution

Frequency (Hz) (be careful about rounding up or down to 2 decimal places):

X = Your solution

Form: Decimal to 2 decimal places.

Place the indicated letter in front of the number.

Example: aX where  $X = 46.00$  is entered as a46.00

Hash of aX = ac6698

Period (s):

X = Your solution

Form: Decimal to 2 decimal places.

Place the indicated letter in front of the number.

Example: aX where  $X = 46.00$  is entered as a46.00

Hash of bX = 6fdff9