e.g. $N = 4$ w: the twiddle factor

$$w^{nk} = \left(e^{-i2\pi/N}\right)^{nk}$$

$$w = e^{-i2\pi/N} = e^{-i\pi/2}$$

$$w^{nk} = \left(e^{-i\pi/2}\right)^{nk} = e^{-ink\pi/2}$$

$$= \cos nk\pi/2 - i \sin nk\pi/2$$

$$\left.\begin{array}{l} n = 0, 1, 2, 3 \\ \\ k = 0, 1, 2, 3 \end{array}\right\} \quad nk = 0, 1, 2, 3, 4, 6, 9$$

$$w^0 = \cos 0 - i \sin 0 = 1$$

$$w^1 = \cos \pi/2 - i \sin \pi/2 = -i$$

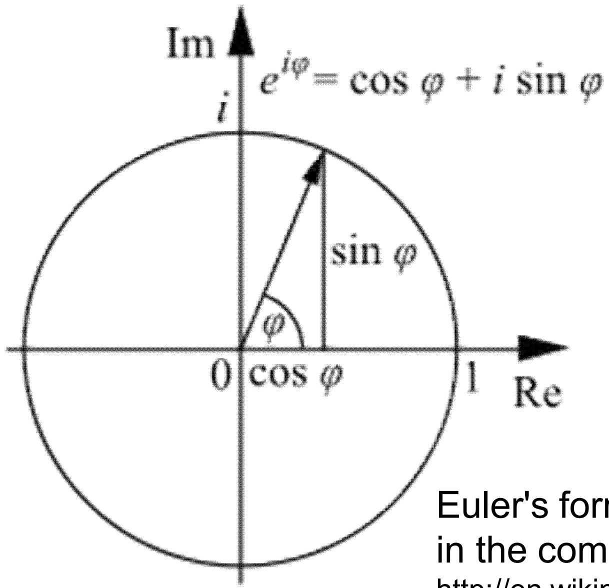
$$w^2 = \cos \pi - i \sin \pi = -1$$

$$w^3 = \cos 3\pi/2 - i \sin 3\pi/2 = i$$

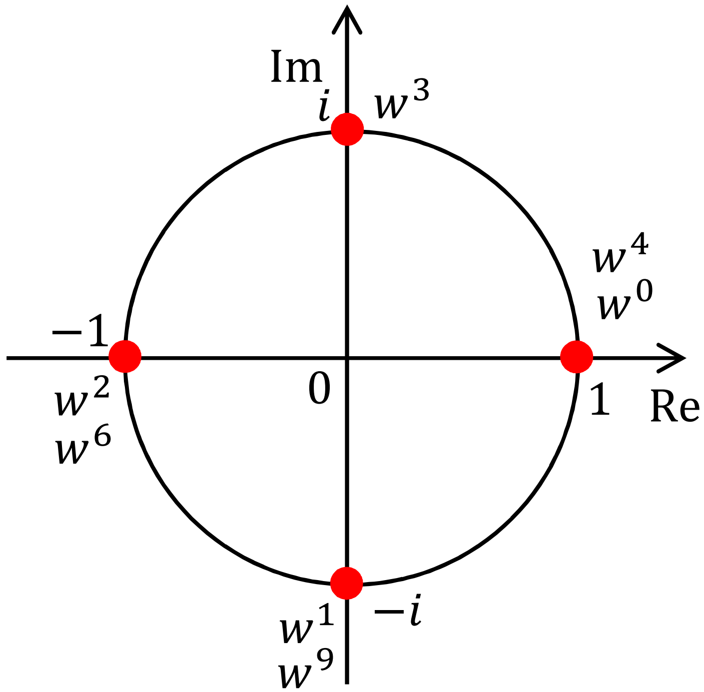
$$w^4 = \cos 2\pi - i \sin 2\pi = 1$$

$$w^6 = \cos 3\pi - i \sin 3\pi = -1$$

$$w^9 = \cos 9\pi/2 - i \sin 9\pi/2 = -i$$



Euler's formula illustrated in the complex plane
http://en.wikipedia.org/wiki/Euler%27s_formula



e.g. $N = 8$

$$\left.\begin{array}{l} n = 0, 1, 2, \cdots, 7 \\ k = 0, 1, 2, \cdots, 7 \end{array}\right\} \quad nk = 0, 1, 2,3,4,5,6,7,8,9, 10,12,14,15,16,18, 20,21,24,25,28,30, 35,36,42,49$$

$$w^{nk} = \left(e^{-i2\pi/N}\right)^{nk} = e^{-ink\pi/4}$$
$$= \cos nk\pi/4 - i \sin nk\pi/4$$

$$w^0 = \cos 0 - i \sin 0 = 1$$

$$w^1 = \cos \pi/4 - i \sin \pi/4 = \frac{1-i}{\sqrt{2}}$$

$$w^2 = \cos \pi/2 - i \sin \pi/2 = -i$$

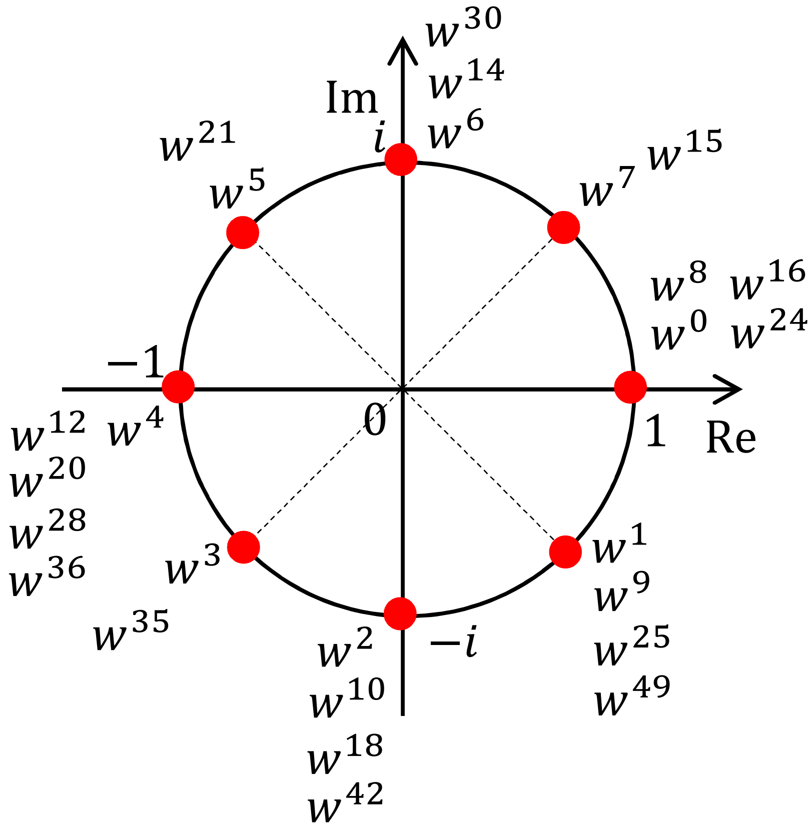
$$w^3 = \cos 3\pi/4 - i \sin 3\pi/4 = \frac{-1-i}{\sqrt{2}}$$

$$w^4 = \cos \pi - i \sin \pi = -1$$

$$w^5 = \cos 5\pi/4 - i \sin 5\pi/4 = \frac{-1+i}{\sqrt{2}}$$

$$w^6 = \cos 3\pi/2 - i \sin 3\pi/2 = i$$

$$w^7 = \cos 7\pi/4 - i \sin 7\pi/4 = \frac{1+i}{\sqrt{2}}$$



$$w^8 = w^{16} = w^{24} = w^0 \qquad w^{35} = w^3$$

$$w^9 = w^{25} = w^{49} = w^1 \qquad w^{12} = w^{20} = w^{28} = w^{36} = w^4$$

$$w^{10} = w^{18} = w^{42} = w^2 \qquad w^{21} = w^5 \qquad w^{14} = w^{30} = w^6 \qquad w^{15} = w^7$$