Mass Transfer

Spring 2017

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http://www.jamescannon.net/teaching/mass-transfer

http://raw.githubusercontent.com/NanoScaleDesign/MassTransfer/master/mass_transfer.pdf

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Chapter 0

Course information

0.1 This course

This is the Spring 2017 Mass Transfer graduate course at Kyushu University.

0.1.1 What you need to do

- Borrow the book "Principles of Heat and Mass Transfer", 7th Edition, by Incropera et. al. from the Mechanical Engineering Office on the 4th floor of West 4. The course will be based on that book and you will need to refer to it in class.
- Prepare a challenge-log in the form of a workbook or folder where you can clearly write the calculations you perform to solve each challenge. This will be used in the final assessment and will be occasionally reviewed by the teacher.
- Submit a weekly feedback form by **8am on Monday** before class at `https://goo.gl/forms/bFAcVvwXstWgwXbG3`.
- Please bring a wifi-capable internet device to class, as well as headphones if you need to access online components of the course during class. If you let me know in advance, I can lend computers and provide power extension cables for those who require them (limited number).

0.1.2 How this works

- This booklet forms part of an active-learning segment in the course. The learning is self-directed in contrast to the traditional lecture-style model.
- Learning is guided through solving a series of challenges combined with instant feedback about the correctness of your answer.
- Traditional lectures are replaced by discussion time. Here, you are encouraged to discuss any issues with your peers, teacher and any teaching assistants. You can also learn from explaining concepts to your peers.
- Discussion-time is from 10:30 to 12:00 on Mondays at room Engineering-2.
- Peer discussion is encouraged, however, if you have help to solve a challenge, always make sure you do understand the details yourself. You will need to be able to do this in an exam environment. The questions on the exam will be similar in nature to the challenges. If you can do all of the challenges, you can get 100% on the exam.
- Every challenge in the book typically contains a **Challenge** with suggested **Resources** which you are recommended to utilise in order to solve the challenge. **Solutions** will be given. Occasionally the teacher will provide extra **Comments** to help guide your thinking.
- For deep understanding, it is recommended to study the suggested resources beyond the minimum required to complete the challenge.
- The challenge document has many pages and is continuously being developed. Therefore it is advised to view the document on an electronic device rather than print it. The date on the front page denotes the version of the document. You will be notified when the document is updated.
- A target challenge will be set each week. This will set the pace of the course and define the examinable material. It's ok if you can't quite reach the target challenge for a given week, but you should be careful not to fall behind, since the date of the exam cannot be delayed.

0.1.3 Assessment

In order to prove to outside parties that you have learned something from the course, we must perform summative assessments. You will receive a weighted score based on:

- Challenge-log (20%) - final state at the end of the course, showing your calculations for all the challenges in the course.
- Presentation (30%)
- Final exam (50%)

Final score = MAX(Weighted score, Final exam)

0.2 Timetable

	Discussion	**Target**	**Note**
1	5 June	-	
2	12 June	1.7	
3	19 June	1.12	
4	26 June	1.17	
5	3 July	1.23	
6	13 July	1.26	Presentations
-	24 July	-	Final exam

So for example, you should aim to complete challenge 1.7 by the 12th of June.

0.3 Presentations

Due to limited time, classes will mostly focus on diffusive mass transfer, however convective mass transfer is also an important mode of mass transport. Your task is to undertake a research project to learn about convective mass transfer, and then present to the class about any application of convective mass transfer that you find interesting. Presentation time will be 8 minutes. Chapter 9 of the book has a good summary of the subject.

For maximum marks you should do the following:

- Ensure that your presentation is 7:00-8:00 minutes long. Timing will be stricly kept.
- Include a basic description of equations related to your chosen subject.
- Include at least 1 graph.
- Clearly demonstrate understanding (showing calculation examples is a good way to do this).
- Demonstrate a novel application of convective heat mass transfer.
- Ensure the work is your own and you cite all references, as well as images and text taken from other sources.
- Pitch the presentation at a level whereby your classmates can follow your discussion.
- Explain accuratly and clearly.
- Talk in either English or Japanese.
- Write any text on the slides mostly (90%+) in English.

Note: The application that you describe can does not have to be originally invented by you (although you are welcome to propose an application like this if you wish). The application may already exist, but you will need to demonstrate understanding about the application and calculations involved in the use of convective mass transfer.

0.4 Hash-generation

Some solutions to challenges are encrypted using MD5 hashes. In order to check your solution, you need to generate its MD5 hash and compare it to that provided. MD5 hashes can be generated at the following sites:

- Wolfram alpha: (For example: md5 hash of "q_1.00") `http://www.wolframalpha.com/input/?i=md5+hash+of+%22q_1.00%22`
- `www.md5hashgenerator.com`

Since MD5 hashes are very sensitive to even single-digit variation, you must enter the solution exactly. This means maintaining a sufficient level of accuracy when developing your solution, and then entering the solution according to the format below:

Unless specified otherwise, any number from 0.00 to ±9999.99 should be represented as a normal number to two decimal places. All other numbers should be in scientific form. See the table below for examples.

Solution	**Input**
1	1.00
-3	-3.00
-3.5697	-3.57
0.05	0.05
0.005	5.00e-3
50	50.00
500	500.00
5000	5000.00
50,000	5.00e4
5×10^{-476}	5.00e-476
5.0009×10^{-476}	5.00e-476
$-\infty$	-infinity (never "infinite")
2π	6.28
i	im(1.00)
2i	im(2.00)
1 + 2i	re(1.00)im(2.00)
-0.0002548 i	im(-2.55e-4)
1/i = i/-1 = -i	im(-1.00)
$e^{i2\pi}$ $[= cos(2\pi) + isin(2\pi) = 1 + i0 = 1]$	1.00
$e^{i\pi/3}$ $[= cos(\pi/3) + isin(\pi/3) = 0.5 + i0.87]$	re(0.50)im(0.87)
Choices in order A, B, C, D	abcd

Entry format is given with the problem. So "q_X" means to enter "q_X" replacing "X" with your solution. The first 6 digits of the MD5 sum should match the given solution (MD5(q_X)= ...).

Note that although some answers can usually only be integers (eg, number of elephants), unless otherwise indicated you should always enter an integer to two decimal places (ie, with ".00" after it) to generate the correct hash.

Chapter 1

Diffusion

1.1 Definition of Mass Transfer

Resources

- Chapter 14, introduction

Challenge

Add the points of the following conditions which constitute diffusive mass-transfer

1 point: Evaporation of water vapour into the air

2 points: Water being pumped through a pipe

4 points: Dissolving of sugar into tea

8 points: Aeration of waste-water

16 points: Motion of air around a room due to the presence of a fan

Solution

(Enter as an integer)

MD5(a_X) = b786dd...

1.2 Diffusion in the long time limit

Resources

- Book: 14.1.1 to 14.1.2
- Video: `https://www.youtube.com/watch?v=-FLv0uxLrDI`

Challenge

Consider a box of volume $1\,\mathrm{m}^3$. The box contains 1 mole of gas. At time $t = 0$, all the gas molecules in the left 1/4 of the box are labeled A. As time goes to $t = \infty$, what will the density of the molecules labeled A be in the right half of the box? Note that there is only 1 species of gas in the box.

Solution

Units: Moles / m^3

(enter to two decimal places)

MD5(b_X) = 13c60a...

1.3 Definitions of quantities I

Resources

- Book: 14.1.1 to 14.1.2
- Video: `https://www.youtube.com/watch?v=-FLv0uxLrDI`

Challenge

Assuming air is made up exclusively of oxygen and nitrogen with their partial pressures in the ratio 0.21:0.79, what are their mass-fractions?

Solutions

Oxygen: 0.233
Nitrogen: 0.767

1.4 Definitions of quantities II

Resources

- Book: 14.1.1 to 14.1.2
- Video: `https://www.youtube.com/watch?v=-FLv0uxLrDI`

Challenge

Japan imports substantial amounts of LNG which is a mixture of the following gases:

Liquid	**Mol %**
Methane	93.5
Ethane	4.6
Propane	1.2
Carbon dioxide	0.7

The masses of Methane, Ethane, Propane and Carbon Dioxide are 16, 30, 44 and 44 g/mol respectively.

Assuming ideal gases, calculate the following:

1. The mole-fraction of ethane
2. The mass-fraction of ethane
3. The average molecular mass of the mixture
4. The mass-density of the gas when heated to 207 K under a total pressure of 1.4×10^5 Pa
5. The partial pressure of the methane when the total pressure is 1.4×10^5 Pa

Solutions

1. (enter as a decimal to 3 decimal places) MD5(c_X) = 117398...
2. (enter as a decimal to 3 decimal places) MD5(d_X) = 6801da...
3. (enter as a decimal to 3 decimal places in units of g/mol) MD5(e_X) = e3a65e...
4. $1397\,\mathrm{kg\,m^{-3}}$
5. (enter as an integer in units of kPa) MD5(f_X) = f54c28...

1.5 Mass diffusivity

Resources

- Book: 14.1.3 - 14.1.4, Table A-8

Challenge

Estimate the mass diffusivity of the following gases in air at 350 K and 1 atm pressure:

1. Ammonia
2. Hydrogen

Solutions

1. $0.36 \times 10^{-4}\,\mathrm{m}^2\,\mathrm{s}^{-1}$
2. $0.52 \times 10^{-4}\,\mathrm{m}^2\,\mathrm{s}^{-1}$

1.6 Cases of diffusion

Resources

- Book: 14.1.3 - 14.1.4

Challenge

Considering air in a closed, cylindrical container with its axis vertical and with opposite ends maintained at different temperatures. Assume the total pressure of the air is uniform throughout the container.

Consider each of the following conditions:

1. The bottom surface is colder than the top surface
2. The top surface is colder than the bottom surface

For each condition, write a few sentences explaining a) if there is any motion of the air and b) if mass transfer occurs.

Solutions

Please compare your answer with your partner.

1.7 Diffusion coefficient equivalency

Resources

- Video: `https://www.youtube.com/watch?v=NTlR18NyqAE`

Challenge

Prove that in a binary mixture, the diffusion coefficient of gas "A" in "B" is the same as the diffusion coefficient of gas "B" in "A" (ie, $D_{AB} = D_{BA}$).

1.8 Saturated water vapour pressure

Resources

- http://www.chemguide.co.uk/physical/phaseeqia/vapourpress.html

Challenge

Write a few sentences briefly explaining what is meant by "Saturated Vapour Pressure".

Solution

Compare your answer with your partner

1.9 Evaporation through a pore

Resources

- Book 14.2 (be sure to follow the derivation of column evaporation)

Challenge

In your challenge log, work through example 14.2 (you do not need to include the "comments" part in your challenge log).

1.10 Evaporation pan

Resources

- Book 14.2, Tables A-6 and A-8
- Challenges in Appendix A

Comments

You have studied that the molar fraction of species i (x_i) can be calculated directly from the ratio of molar densities (C_i/C) or equivalently through Dalton's law (p_i/p). In the case where p is the saturation partial pressure of water vapour (ie, $p_{\mathrm{wv}}/p_{\mathrm{wv,sat}}$), the molar fraction x_{wv} is equivalent to the relative humidity.

Challenge

Evaporation pans like the one shown are used to measure the rate of evaporation of water in a local area.

(image: Bidgee, Wikipedia)

An evaporation pan is placed in a location with an ambient air temperature of 300 K and relative humidity of 25%. The pan contains water at the same temperature as the surrounding air. The pan has a diameter of 20 cm and height of 160 mm, and it starts half-full of water.

1. Assuming only diffusive mass transport, what is the initial evaporation rate?
2. Including the effects of advection, what is the initial evaporation rate?

Solution

1. $1.087 \times 10^{-8}\,\mathrm{kmol\,s^{-1}}$
2. $1.108 \times 10^{-8}\,\mathrm{kmol\,s^{-1}}$

1.11 Stationary Medium

Resources

- Video: `https://www.youtube.com/watch?v=F0deXOH_YEM`

Challenge

1. Briefly explain what is meant by a stationary medium.
2. Considering a stationary medium of 3 species "A", "B" and "C", if the flux of species "A" is $2\,\mathrm{kmol\,m^{-2}\,s^{-1}}$ and "B" is $-8\,\mathrm{kmol\,m^{-2}\,s^{-1}}$, what is the flux of species "C"?
3. Considering a binary system of atoms "A" and "B" with concentration $5\,\mathrm{kmol\,m^{-3}}$ and $10\,\mathrm{kmol\,m^{-3}}$ respectively, if the molar velocity of species "A" is $2\,\mathrm{m\,s^{-1}}$, what is the molar velocity of species "B"?

Solutions

1. Please compare your answer with your partner
2. (enter to two decimal places in units of $\mathrm{kmol\,m^{-2}\,s^{-1}}$) MD5(g_X) = 2c32d8...
3. (enter to two decimal places in units of $\mathrm{m\,s^{-1}}$) MD5(h_X) = 3ccbb4...

1.12 Stationary Medium Approximation

Resources

- Book 14.3

Challenge

In a few sentences, describe what is meant by *the stationary medium approximation.* Give at least one real-world example each of case where the stationary medium approximation would and would-not be appropriate.

Solution

Compare your answer with your partner

1.13 Steady-state definition

Resources

- Web: `http://www.virginia.edu/bohr/mse209/chapter5.htm`

Challenge

The example in section 14.4.3 talks of steady state conditions.

1. Write a sentence or two explaining what is meant by steady-state conditions in this context.
2. What is $\delta J(x)/\delta t$ at any given position x under a steady-state condition?

Solutions

1. Please compare your answer with your partner

2.
X = Your solution
Form: Decimal, to 1 decimal place
Place the indicated letter in front of the number
Example: aX where $X = 42.5$ is entered as a42.5

hash of aX = 9497cd

1.14 Steady-state diffusion planer example I

Resources

- Book: 14.4.1 to 14.4.3
- Video: `https://youtu.be/4KACai1gYzc`

Challenge

Work through the calculation from equations 14.51 to 14.54, showing your reasoning along the way. Don't worry about the concept of diffusion resistance for now.

1.15 Steady-state diffusion planer example II

Resources

- Book: 14.4.1 to 14.4.3
- Video: `https://youtu.be/4KACai1gYzc`

Challenge

Work through example 14.3 in section 14.4.3.

1.16 Steady-state diffusion through flat surface

Resources

- Book: 14.4.1 to 14.4.3

Challenge

A thin plastic membrane is used to maintain separation between helium and an outer chamber. Under steady-state conditions, the concentration of helium is $0.02\,\mathrm{kmol\,m^{-3}}$ and $0.005\,\mathrm{kmol\,m^{-3}}$ at the inner and outer boundaries respectively. If the membrane is 1 mm thick and the binary diffusion coefficient of helium in the plastic membrane is $10^{-9}\,\mathrm{m^2\,s^{-1}}$, what is the diffusive flux through the membrane in terms of $\mathrm{kmol\,s^{-1}\,m^{-2}}$ and $\mathrm{kg\,s^{-1}\,m^{-2}}$?

Solution

$1.5 \times 10^{-8}\,\mathrm{kmol\,s^{-1}\,m^{-2}}$
$6.0 \times 10^{-8}\,\mathrm{kg\,s^{-1}\,m^{-2}}$

1.17 Steady-state diffusion in pipe walls

Resources

- Book: 14.4.1 to 14.4.3

Challenge

A pipe with inner-radius 10 cm and outer-radius 13 cm is carrying hydrogen. The concentration of hydrogen inside the pipe is $0.07\,\mathrm{kmol\,m^{-3}}$ and outside the pipe is $0.04\,\mathrm{kmol\,m^{-3}}$. If the hydrogen has a diffusion coefficient of $10^{-8}\,\mathrm{m^2\,s^{-1}}$ in the walls of the pipe, at what rate is hydrogen lost from the pipe, per metre length of pipe?

Solution

$7.18 \times 10^{-9}\,\mathrm{kmol\,s^{-1}}$

1.18 Raoult's law

Resources

- Book: 14.5.1

Challenge

Given that the saturated vapour pressure of water in the atmosphere at 17 °C is 0.019 17 bar, considering a puddle of water exposed to the atmosphere at 17 °C, calculate the following. You may assume that the average mass of air is $29\,\mathrm{g\,mol^{-1}}$.

1. The molar fraction of water vapour on the air-side of the water-air interface.
2. The mass fraction of water vapour on the air-side of the water-air interface.
3. The molar fraction of water vapour on the water-side of the water-air interface.
4. The mass fraction of water vapour on the water-side of the water-air interface.

Solution

1. 0.019

2. 0.012

3.
X = Your solution
Form: Decimal, to 3 decimal places
Place the indicated letter in front of the number
Example: aX where $X = 42.544$ is entered as a42.544

hash of bX = 4b5ab9

4.
X = Your solution
Form: Decimal, to 3 decimal places
Place the indicated letter in front of the number
Example: aX where $X = 42.544$ is entered as a42.544

hash of cX = d69d92

1.19 Boundary conditions

Resources

- Book: First page of section 14.5

Challenge

State two different types of boundary conditions in terms of molar density, x_i.

Solution

Please discuss with your partner and the teacher.

1.20 Henry's law

Resources

- Book: 14.5.2

Challenge

Consider a gas stream flowing above a liquid. Species A has a mole fraction of 0.5 in the liquid while species B has a mole fraction of 0.005 in the liquid. Calculate Henry's constant for the gas where Henry's law is most applicable, if the partial pressure of both species in the gas stream is 0.5 bar.

Solution

X = Your solution
Form: Decimal, to 1 decimal place
Place the indicated letter in front of the number
Example: aX where $X = 42.5$ is entered as a42.5

hash of fX = 4e930b

1.21 Solubility

Resources

- Book: 14.5.2

Challenge

1. Considering a solid-gas interface, calculate the solubility of species A if the density of A is $10^{-3}\,\mathrm{kmol\,m^{-3}}$ in the solid given a partial pressure of $0.5\,\mathrm{bar}$ in the gas.

2. What volume will species A occupy in the gas-phase at STP if it occupies $20\,\mathrm{m^3}$ in the solid phase?

Solution

1.
X = Your solution
Units: kmol / m^3 (solid) / bar (partial pressure in gas phase)
Form: Scientific notation with the mantissa in standard form to 1 decimal place and the exponent in integer form
Place the indicated letter in front of the number
Example: aX where $X = 4.0 \times 10^{-3}$ is entered as a4.0e-3

hash of gX = 0174a0

2. $0.908\,\mathrm{m^3}$

1.22 Flux through membrane with solubility

Resources

- Book: 14.5.2

Challenge

Work through example 14.4

1.23 Flux through spherical membrane

Resources

- Book: 14.5.2

Challenge

Consider helium gas stored in a silicon spherical container at 25 °C and 4 bar pressure, with an inner radius of 100 mm and outer radius of 110 mm. Assume a diffusion coefficient of helium in the walls of the container of $0.4 \times 10^{-13}\,\mathrm{m^2\,s^{-1}}$ and a solubility of Helium in Silicon of $4.5 \times 10^{-4}\,\mathrm{kmol\,m^{-3}\,bar^{-1}}$. Since the thickness of the walls is similar to the radius, you cannot approximate the surface as being flat, as was done in the previous challenge. In particular, the surface area is now a function of radius.

Solution

$4 \times 10^{-15}\,\mathrm{kg\,s^{-1}}$

1.24 Chemical reaction types

Resources

- Book: 14.5.3
- Wikipedia: `https://en.wikipedia.org/wiki/Catalysis#Types`

Challenge

Draw a digram such as figure 14.7 in the book and mark the location where Heterogeneous and Homogeneous reactions would each take place.

Solution

Please discuss with your partner and the teacher if you are unsure.

1.25 Reaction orders

Resources

- Wikipedia: https://en.wikipedia.org/wiki/Order_of_reaction#First_order

Challenge

Write a sentence describing what is meant by a zero-order and first-order reaction.

Solution

Please compare with your partner and ask the teacher if you are unsure.

1.26 Reaction limitations

Resources

- Book: 14.5.2

Challenge

Consider species "A" that is consumed at the surface shown in figure 14.7 through a chemical reaction. The height of the system is 20 cm and the mole-fraction at this height ($x = L$) is 0.5. The total concentration of the mixture can be considered to be uniform at $4\,\mathrm{kmol\,m^{-3}}$. The diffusion coefficient of "A" in the mixture is $10 \times 10^{-3}\,\mathrm{m^2\,s^{-1}}$.

1. What is the concentration of "A" at $x = 0$ in the following cases?
a) Reaction-limited case
b) Diffusion-limited case

2. What is the flux of "A" at $x = 0$ in the following case?
a) Diffusion-limited case

Solution

1.a)
X = Your solution
Units: $\mathrm{kmol\,m^{-3}}$
Form: Decimal to 1 decimal place
Place the indicated letter in front of the number
Example: aX where $X = 46$ is entered as a46.0

hash of hX = 3fec6b

1.b) X = Your solution
Units: $\mathrm{kmol\,m^{-3}}$
Form: Decimal to 1 decimal place
Place the indicated letter in front of the number
Example: aX where $X = 46$ is entered as a46.0

hash of iX = 46d0a7

2.a) X = Your solution
Units: $\mathrm{kmol\,m^{-3}}$
Form: Decimal to 2 decimal places
Place the indicated letter in front of the number
Example: aX where $X = 46$ is entered as a46.00

hash of jX = 3f0fa2

Appendix A

Supporting challenges

A.1 Partial pressure and humidity

Challenge

If the saturation pressure of water in air at a given temperature is 0.02 bar and the relative humidity of the air is 30%, calculate the partial pressure of the water vapour in the air.

Solution

X = Your solution
Form: Decimal, to 3 decimal places
Place the indicated letter in front of the number
Example: aX where $X = 42.544$ is entered as a42.544

hash of dX = f8dcaa

A.2 Pressure and molar density

Challenge

Given that the saturation pressure of water in air at a given temperature is 0.05 bar with a saturated molar density of $5\,\mathrm{kmol\,m^{-3}}$, if the partial pressure of water in the air is 0.01 bar, calculate the molar density of the water in the air.

Solution

X = Your solution
Form: Decimal, to 1 decimal place
Place the indicated letter in front of the number
Example: aX where $X = 42.5$ is entered as a42.5

hash of eX = 1c8d37

A.3 Specific volume

Challenge

Referring to the specific volume of saturated water at 290 K in table A-6 (v_g), what is the saturated molar concentration? You cannot assume this is an ideal gas.

Solution

$7.97 \times 10^{-4}\,\mathrm{kmol\,m^{-3}}$

A.4 Molar density of air

Challenge

Assuming air to be an ideal gas, calculate the molar density of air, C_{air} at 300 K and atmospheric pressure.

Solution

$4.1 \times 10^{-2}\,\mathrm{kmol\,m^{-3}}$