Mass Transfer

Spring 2018

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http://www.jamescannon.net/teaching/mass-transfer

http://raw.githubusercontent.com/NanoScaleDesign/MassTransfer/master/mass_transfer.pdf

Contents

Chapter 0

Course information

0.1 This course

This is the Spring 2018 Mass Transfer graduate course at Kyushu University.

0.1.1 What you need to do

- Borrow the book "Principles of Heat and Mass Transfer", 7th Edition, by Incropera et. al. from the Mechanical Engineering Office on the 4th floor of West 4. The course will be based on that book and you will need to refer to it in class.
- Prepare a challenge-log in the form of a workbook or folder where you can clearly write the calculations you perform to solve each challenge. This will be used in the final assessment and will be occasionally reviewed by the teacher.
- Submit a weekly feedback form by **8am on Monday** before class at `https://goo.gl/forms/slmT8LNxM10vtlSs2`.
- Please bring a wifi-capable internet device to class, as well as headphones if you need to access online components of the course during class. If you let me know in advance, I can lend computers and provide power extension cables for those who require them (limited number).

0.1.2 How this works

- This booklet forms part of an active-learning segment in the course. The learning is self-directed in contrast to the traditional lecture-style model.
- Learning is guided through solving a series of challenges combined with instant feedback about the correctness of your answer.
- Traditional lectures are replaced by discussion time. Here, you are encouraged to discuss any issues with your peers, teacher and any teaching assistants. You can also learn from explaining concepts to your peers.
- Discussion-time is from 10:30 to 12:00 on Mondays at room Engineering-2.
- Peer discussion is encouraged, however, if you have help to solve a challenge, always make sure you do understand the details yourself. You will need to be able to do this in an exam environment. The questions on the exam will be similar in nature to the challenges.
- Every challenge in the book typically contains a **Challenge** with suggested **Resources** which you are recommended to utilise in order to solve the challenge. **Solutions** will be given. Occasionally the teacher will provide extra **Comments** to help guide your thinking.
- For deep understanding, it is recommended to study the suggested resources beyond the minimum required to complete the challenge.
- The challenge document has many pages and is continuously being developed. Therefore it is advised to view the document on an electronic device rather than print it. The date on the front page denotes the version of the document. You will be notified when the document is updated.
- A target challenge will be set each week. This will set the pace of the course and define the examinable material. It's ok if you can't quite reach the target challenge for a given week, but you should be careful not to fall behind, since the date of the exam cannot be delayed.

0.1.3 Assessment

In order to prove to outside parties that you have learned something from the course, we must perform summative assessments. You will receive a weighted score based on:

- Challenge-log (10%) - final state at the end of the course, showing your calculations for all the challenges in the course.
- Coursework (20%)
- Final exam (70%)

Final score = MAX(Weighted score, Final exam)

0.2 Timetable

	Discussion	**Target**	**Note**
1	28 May	-	
2	4 June	2.7	
3	11 June	2.13	
4	18 June	2.21	
5	25 June		
6	2 July		
-	23 July	-	Final exam

0.3 Hash-generation

Some solutions to challenges are encrypted using MD5 hashes. In order to check your solution, you need to generate its MD5 hash and compare it to that provided. MD5 hashes can be generated at the following sites:

- Wolfram alpha: (For example: md5 hash of "q1.00") `http://www.wolframalpha.com/input/?i=md5+hash+of+%22q1.00%22`
- `www.md5hashgenerator.com`

Since MD5 hashes are very sensitive to even single-digit variation, you must enter the solution *exactly*. This means maintaining a sufficient level of accuracy when developing your solution, and then entering the solution according to the format suggested by the question. Some special input methods:

Solution	**Input**
5×10^{-476}	5.00e-476
5.0009×10^{-476}	5.00e-476
$-\infty$	-infinity (never "infinite")
2π	6.28
i	im(1)
2i	im(2)
1 + 2i	re(1)im(2)
-0.0002548 i	im(-2.55e-4)
1/i = i/-1 = -i	im(-1)
$e^{i2\pi}$ $[= cos(2\pi) + isin(2\pi) = 1 + i0 = 1]$	1.00
$e^{i\pi/3}$ $[= cos(\pi/3) + isin(\pi/3) = 0.5 + i0.87]$	re(0.50)im(0.87)
Choices in order A, B, C, D	abcd

The first 6 digits of the MD5 sum should match the first 6 digits of the given solution.

Chapter 1

Hash practise

1.1 Hash practise: Integer

X = 46.3847
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

hash of aX = e77fac

1.2 Hash practise: Decimal

X = 49
Form: Two decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

hash of bX = 82c9e7

1.3 Hash practise: String

X = abcdef
Form: String.
Place the indicated letter in front of the number.
Example: aX where $X = abc$ is entered as aabc

hash of cX = 990ba0

1.4 Hash practise: Scientific form

X = 500,765.99
Form: Scientific notation with the mantissa in standard form to 2 decimal place and the exponent in integer form.
Place the indicated letter in front of the number.
Example: aX where $X = 4 \times 10^{-3}$ is entered as a4.00e-3

hash of dX = be8a0d

Chapter 2

Diffusion

2.1 Definition of Mass Transfer

Resources

- Chapter 14, introduction

Challenge

Add the points of the following conditions which constitute diffusive mass-transfer

1 point: Evaporation of water vapour into the air

2 points: Water being pumped through a pipe

4 points: Dissolving of sugar into tea

8 points: Aeration of waste-water

16 points: Motion of air around a room due to the presence of a fan

Solution

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of aX = d16142

2.2 Diffusion in the long time limit

Resources

- Book: 14.1.1 to 14.1.2
- Video: `https://www.youtube.com/watch?v=-FLv0uxLrDI`

Challenge

Consider a box of volume $1\,\mathrm{m}^3$. The box contains 1 mole of gas. At time $t = 0$, all the gas molecules in the left 1/4 of the box are labeled A. As time goes to $t = \infty$, what will the density of the molecules labeled A be in the right half of the box? Note that there is only 1 species of gas in the box.

Solution

(Units: Moles / m^3)

X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of bX = e1f90a

2.3 Definitions of quantities I

Resources

- Book: 14.1.1 to 14.1.2
- Video: `https://www.youtube.com/watch?v=-FLv0uxLrDI`

Challenge

Assuming air is made up exclusively of oxygen and nitrogen with their partial pressures in the ratio 0.21:0.79, what are their mass-fractions?

Solutions

Oxygen: 0.233
Nitrogen: 0.767

2.4 Definitions of quantities II

Resources

- Book: 14.1.1 to 14.1.2
- Video: `https://www.youtube.com/watch?v=-FLv0uxLrDI`

Challenge

Japan imports substantial amounts of LNG which is a mixture of the following gases:

Liquid	**Mol %**
Methane	93.5
Ethane	4.6
Propane	1.2
Carbon dioxide	0.7

The masses of Methane, Ethane, Propane and Carbon Dioxide are 16, 30, 44 and 44 g/mol respectively.

Assuming ideal gases, calculate the following:

1. The mole-fraction of ethane
2. The mass-fraction of ethane
3. The average molecular mass of the mixture
4. The mass-density of the gas when heated to 207 K under a total pressure of 1.4×10^5 Pa
5. The partial pressure of the methane when the total pressure is 1.4×10^5 Pa

Solutions

1.
X = Your solution
Form: Decimal to 3 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.000$ is entered as a46.000

Hash of cX = bf4ce8

2.
X = Your solution
Form: Decimal to 3 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.000$ is entered as a46.000

Hash of dX = d34e49

3. (Units: g/mol)
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of eX = 6b58f1

4.
$1397\,\mathrm{kg\,m^{-3}}$

5. (Units: kPa)
X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of fX = 4d1d94

2.5 Mass diffusivity

Resources

- Book: 14.1.3 - 14.1.4, Table A-8

Challenge

Estimate the mass diffusivity of the following gases in air at 350 K and 1 atm pressure:

1. Ammonia
2. Hydrogen

Solutions

1. $3.6 \times 10^{-5}\,\mathrm{m}^2\,\mathrm{s}^{-1}$
2. $5.2 \times 10^{-5}\,\mathrm{m}^2\,\mathrm{s}^{-1}$

2.6 Cases of diffusion

Resources

- Book: 14.1.3 - 14.1.4

Challenge

Considering air as a mixture of two gases (O_2 and N_2), situated in a closed, cylindrical container with its axis vertical and with opposite ends maintained at different temperatures. Assume the total pressure of the air is uniform throughout the container.

Consider each of the following conditions:

A) The bottom surface is colder than the top surface
B) The top surface is colder than the bottom surface

1. Add the points for the following *true* statements:

1 point There is motion of the air in case (A)

2 points There is no motion of the air in case (A)

4 points There is motion of the air in case (B)

8 points There is no motion of the air in case (B)

16 points There is diffusive mass transfer inside the cylinder in case (A)

32 points There is no diffusive mass transfer inside the cylinder in case (A)

64 points There is diffusive mass transfer inside the cylinder in case (B)

128 points There is no diffusive mass transfer inside the cylinder in case (B)

2. Explain your reasoning

Solutions

1.
X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of gX = 389975

2.
Please compare your answer with your partner in class.

2.7 Diffusion coefficient equivalency

Resources

- Video: `https://www.youtube.com/watch?v=NTlR18NyqAE`

Challenge

Prove that in a binary mixture, the diffusion coefficient of gas "A" in "B" is the same as the diffusion coefficient of gas "B" in "A" (ie, $D_{AB} = D_{BA}$).

2.8 Column evaporation derivation

Resources

- Book 14.2

Challenge

Starting from basic knowledge of absolute flux in terms of both diffusion and advection:

$$N''_A = -CD_{AB}\Delta x_A + x_A N''_A \tag{2.1}$$

show that evaporation in a stationary column (figure 14.2 in the book) can be given by

$$N''_A = \frac{CD_{AB}}{L} ln\left(\frac{1 - x_{A,L}}{1 - x_{A,0}}\right) \tag{2.2}$$

where $x_{A,L}$ and $x_{A,0}$ are the molar fractions at the top and bottom the column, respectively.

Solution

Compare your answer with your partner

2.9 Saturated water vapour pressure

Resources

- http://www.chemguide.co.uk/physical/phaseeqia/vapourpress.html
- https://sciencing.com/water-vapor-pressure-vs-humidity-19402.html

Challenge

1) Write a few sentences briefly explaining what is meant by "Saturated Vapour Pressure" and how it is related to "Relative humidity".

2) Considering an air-water interface at equilibration, if the relative humidity is 50% and the saturated water density is $0.02\,\mathrm{kmol\,m^{-3}}$, what is the density of water vapour near the interface? In addition to a numerical answer, explain in a few words the reasoning behind your answer.

3) Under the same conditions as part (2), what is the mole-fraction of water vapour above the surface? In addition to a numerical answer, explain in a few words the reasoning behind your answer.

4) Considering the saturation pressure of water is 0.03 atm, what is the mole-fraction of water vapour in the air at 1 atm?

Solution

1)
Compare your answer with your partner.

2)
(Units: $\mathrm{kmol\,m^{-3}}$)
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of aX = 9bb791

Compare your reasoning with your partner.

3)
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of bX = 06ac46

Compare your reasoning with your partner.

4)
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of cX = f2a860

2.10 Evaporation through a pore

Resources

- Book 14.2

Challenge

In your challenge log, work through example 14.2 (you do not need to include the "comments" part in your challenge log).

2.11 Evaporation pan

Resources

- Book 14.2, Table A-8

Challenge

Evaporation pans like the one shown are used to measure the rate of evaporation of water in a local area.

(image: Bidgee, Wikipedia)

An evaporation pan is placed in a location with an ambient air temperature of 298 K and relative humidity of 25%. The pan contains water at the same temperature as the surrounding air. The pan has a diameter of 20 cm and height of 160 mm, and it starts half-full of water.

1. Assuming only diffusive mass transport, what is the initial evaporation rate?
2. Including the effects of advection, what is the initial evaporation rate?

(The saturation pressure of water at 298 K is 0.035 31 bar, and its specific volume at an atmospheric pressure of 1 atm is 39.13 $m^3\,kg^{-1}$.)

Solution

1. 1.087×10^{-8} kmol s^{-1}
2. 1.116×10^{-8} kmol s^{-1}

2.12 Stationary Medium

Resources

- Video: `https://www.youtube.com/watch?v=F0deXOH_YEM`

Challenge

1. Briefly explain what is meant by a stationary medium.
2. Considering a stationary medium of 3 species "A", "B" and "C" with equal molar concentration, if the flux of species "A" is $2\,\mathrm{kmol\,m^{-2}\,s^{-1}}$ and "B" is $-8\,\mathrm{kmol\,m^{-2}\,s^{-1}}$, what is the flux of species "C"?
3. Considering a binary system of atoms "A" and "B" with concentration $5\,\mathrm{kmol\,m^{-3}}$ and $10\,\mathrm{kmol\,m^{-3}}$ respectively, if the molar velocity of species "A" is $2\,\mathrm{m\,s^{-1}}$, what is the molar velocity of species "B"?

Solutions

1.
Please compare your answer with your partner.

2.
(Units: $\mathrm{kmol\,m^{-2}}$)
X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of gX = 4a4314

3.
(Units: $\mathrm{m\,s^{-1}}$)
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of hX = 5ba818

2.13 Stationary Medium Approximation

Resources

- Book 14.3

Challenge

In a few sentences, describe what is meant by *the stationary medium approximation*. Give at least one real-world example each of case where the stationary medium approximation would and would-not be appropriate.

Solution

Compare your answer with your partner

2.14 Steady-state definition

Resources

- Web: `http://www.virginia.edu/bohr/mse209/chapter5.htm`

Challenge

1. Write a sentence or two explaining what is meant by steady-state conditions in this context.
2. What is $\delta J(x)/\delta t$ at any given position x under a steady-state condition?

Solutions

1.
Please compare your answer with your partner

2.
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of aX = 690969

2.15 Steady-state diffusion planer example I

Resources

- Book: 14.4.1 to 14.4.3
- Video: `https://youtu.be/4KACai1gYzc`

Challenge

Work through the calculation from equations 14.51 to 14.54, showing your reasoning along the way. Don't worry about the concept of diffusion resistance for now.

2.16 Steady-state diffusion planer example II

Resources

- Book: 14.4.1 to 14.4.3
- Video: `https://youtu.be/4KACai1gYzc`

Challenge

Work through example 14.3 in section 14.4.3.

2.17 Steady-state diffusion through flat surface

Resources

- Book: 14.4.1 to 14.4.3

Challenge

A thin plastic membrane is used to maintain separation between helium and an outer chamber. Under steady-state conditions, the concentration of helium is $0.02\,\mathrm{kmol\,m^{-3}}$ and $0.005\,\mathrm{kmol\,m^{-3}}$ at the inner and outer boundaries respectively. If the membrane is $1\,\mathrm{mm}$ thick and the binary diffusion coefficient of helium in the plastic membrane is $10^{-9}\,\mathrm{m^2\,s^{-1}}$, what is the diffusive flux through the membrane in terms of $\mathrm{kmol\,s^{-1}\,m^{-2}}$ and $\mathrm{kg\,s^{-1}\,m^{-2}}$?

Solution

$1.5 \times 10^{-8}\,\mathrm{kmol\,s^{-1}\,m^{-2}}$
$6.0 \times 10^{-8}\,\mathrm{kg\,s^{-1}\,m^{-2}}$

2.18 Steady-state diffusion in pipe walls

Resources

- Book: 14.4.1 to 14.4.3

Challenge

A pipe with inner-radius 10 cm and outer-radius 13 cm is carrying hydrogen. The concentration of hydrogen inside the pipe is $0.07\,\mathrm{kmol\,m^{-3}}$ and outside the pipe is $0.04\,\mathrm{kmol\,m^{-3}}$. If the hydrogen has a diffusion coefficient of $10^{-8}\,\mathrm{m^2\,s^{-1}}$ in the walls of the pipe, at what rate is hydrogen lost from the pipe, per metre length of pipe?

Solution

$7.18 \times 10^{-9}\,\mathrm{kmol\,s^{-1}}$

2.19 Raoult's law

Resources

- Book: 14.5.1

Challenge

Given that the saturated vapour pressure of water in the atmosphere at 17 °C is 0.019 17 bar, considering a puddle of water exposed to the atmosphere at 17 °C, calculate the following. You may assume that the average mass of air is 29 g mol^{-1}.

1. The molar fraction of water vapour on the air-side of the water-air interface.
2. The mass fraction of water vapour on the air-side of the water-air interface.
3. The molar fraction of water vapour on the water-side of the water-air interface.
4. The mass fraction of water vapour on the water-side of the water-air interface.

Solution

1. 0.019

2. 0.012

3.
X = Your solution
Form: Decimal, to 3 decimal places
Place the indicated letter in front of the number
Example: aX where $X = 42.544$ is entered as a42.544

hash of bX = 4b5ab9

4.
X = Your solution
Form: Decimal, to 3 decimal places
Place the indicated letter in front of the number
Example: aX where $X = 42.544$ is entered as a42.544

hash of cX = d69d92

2.20 Boundary conditions

Resources

- Book: First page of section 14.5

Challenge

State two different types of boundary conditions in terms of molar density, x_i.

Solution

Please discuss with your partner and the teacher.

2.21 Henry's law

Resources

- Book: 14.5.2

Challenge

Consider a gas stream flowing above a liquid. Species A has a mole fraction of 0.5 in the liquid while species B has a mole fraction of 0.005 in the liquid. Calculate Henry's constant for the gas where Henry's law is most applicable, if the partial pressure of both species in the gas stream is 0.5 bar.

Solution

X = Your solution
Form: Decimal, to 1 decimal place
Place the indicated letter in front of the number
Example: aX where $X = 42.5$ is entered as a42.5

hash of fX = 4e930b