Mechanics

Spring 2017

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3rd July 2017 at 13:37

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http://www.jamescannon.net/teaching/mechanics

http://raw.githubusercontent.com/NanoScaleDesign/Mechanics/master/mechanics.pdf

Contents

Chapter 0

Course information

0.1 This course

This is the Spring 2017 Mechanics course studied by 2nd-year undegraduate international students at Kyushu University.

0.1.1 What you need to do

- Borrow the book "Engineering Mechanics: Dynamics", 6th edition, by Meriam and Kraige from the Kikan-kyoiku office in the centre-zone. The course will be based on that book and you will need to refer to it in class.
- Prepare a challenge-log in the form of a workbook or folder where you can clearly write the calculations you perform to solve each challenge. This will be used in the final assessment and will be occasionally reviewed by the teacher.
- Submit a weekly feedback form by **9am on Monday** before class at `https://goo.gl/forms/2PgFF0eqTOvbK0to2`.
- Please bring a wifi-capable internet device to class, as well as headphones if you need to access online components of the course during class. If you let me know in advance, I can lend computers and provide power extension cables for those who require them (limited number).

0.1.2 How this works

- This booklet forms part of an active-learning segment in the course. The learning is self-directed in contrast to the traditional lecture-style model.
- Learning is guided through solving a series of challenges combined with instant feedback about the correctness of your answer.
- Traditional lectures are replaced by discussion time. Here, you are encouraged to discuss any issues with your peers, teacher and any teaching assistants. You can also learn from explaining concepts to your peers.
- Discussion-time is from 13:00 to 14:30 on Mondays at room W4-766.
- Peer discussion is encouraged, however, if you have help to solve a challenge, always make sure you do understand the details yourself. You will need to be able to do this in an exam environment. The questions on the exam will be similar in nature to the challenges. If you can do all of the challenges, you can get 100% on the exam.
- Every challenge in the book typically contains a **Challenge** with suggested **Resources** which you are recommended to utilise in order to solve the challenge. **Solutions** will be given. Occasionally the teacher will provide extra **Comments** to help guide your thinking.
- For deep understanding, it is recommended to study the suggested resources beyond the minimum required to complete the challenge.
- The challenge document has many pages and is continuously being developed. Therefore it is advised to view the document on an electronic device rather than print it. The date on the front page denotes the version of the document. You will be notified when the document is updated.
- A target challenge will be set each week. This will set the pace of the course and define the examinable material. It's ok if you can't quite reach the target challenge for a given week, but you should be careful not to fall behind, since the date of the exam cannot be delayed.

0.1.3 Assessment

In order to prove to outside parties that you have learned something from the course, we must perform summative assessments. You will receive a weighted score based on:

- Challenge-log (10%) - final state at the end of the course, showing your calculations for all the challenges in the course.
- Mid-term exam (30%)
- Coursework (20%)
- Final exam (40%)

Final score = MAX(Weighted score, Final exam)

0.2 Coursework

Mechanics is a large subject with a wide-range of applications. This coursework is designed to give you the opportunity to follow your personal interest and investigate in depth an area of Mechanics of your choice.

The task is as follows:

1) Create a document, explaining about any application of Mechanics that interests you. The document should be **between 1 and 4 pages**, including any necessary figures, mathematics and references.

2) Create **1 to 2 challenge(s)** to accompany your report, so someone reading your document can test their knowledge.

3) Include **fully worked** solutions to challenges you make (ie, not only the final answer, but clearly show the steps involved in order to achieve the final answer).

0.2.1 Submission

Submission is electronic, and the file may be in any format, including PDF, LibreOffice, MS Word, Google docs, Latex, etc... If you submit a PDF, please also submit the source-files used to generate the PDF.

Submit the materials by **email** to the teacher **before the class on 10 July 2017** with the subject "[Mechanics] Coursework". I will confirm in the class that I received your coursework. If you cannot attend the class, you must request confirmation of receipt when you send the email.

Late submission:
By 23:59 on 11 July 2017: 90% of the final mark.
By 23:59 on 17 July 2017: 50% of the final mark.
Later submissions cannot be considered.

0.2.2 Marking

Marks will be assigned based on the degree to the report fulfills the following criteria: For maximum marks you should do the following:

- Clearly demonstrate your understanding of what you write about. You can do this by, for example, mathematically solving for a relevant case or explaining with words how it applies in different situations.
- Ensure your subject has some relation to Mechanics and is of Engineering relevance.
- Choose a subject that goes beyond the boundaries of the examples covered in the textbook.
- Ensure the work is your own and all references, images and text taken from other sources are properly cited.
- Pitch the description at a level appropriate level so that others in the class can follow your discussion.
- Explain in reasonable depth.
- Explain accurately and clearly.

Note: The application that you describe can does not have to be originally invented by you (although you are welcome to propose an application like this if you wish). The application may already exist, but you will need to demonstrate understanding about the application and calculations involved in the use of Mechanics with this application.

0.3 Timetable

	Discussion	Target	Note
1	10 April	-	
2	17 April	1.4	
3	24 April	1.7	
4	8 May	1.12	
5	15 May	1.17	
6	22 May	1.22	
7	29 May	1.27	
8	5 June	1.34	
9	12 June	-	Mid-term exam (covers up to 1.27)
10	19 June	1.34	
11	26 June	2.6	
12	3 July	2.12	
13	10 July	2.17	Coursework submission
14	13 July		
-	24 July	-	Final exam
-	16 August	-	Retake exam (tentative)

0.4 Hash-generation

Some solutions to challenges are encrypted using MD5 hashes. In order to check your solution, you need to generate its MD5 hash and compare it to that provided. MD5 hashes can be generated at the following sites:

- Wolfram alpha: (For example: md5 hash of "q_1.00") `http://www.wolframalpha.com/input/?i=md5+hash+of+%22q_1.00%22`
- `www.md5hashgenerator.com`

Since MD5 hashes are very sensitive to even single-digit variation, you must enter the solution exactly. This means maintaining a sufficient level of accuracy when developing your solution, and then entering the solution according to the format below:

Unless specified otherwise, any number from 0.00 to ±9999.99 should be represented as a normal number to two decimal places. All other numbers should be in scientific form. See the table below for examples.

Solution	**Input**
1	1.00
-3	-3.00
-3.5697	-3.57
0.05	0.05
0.005	5.00e-3
50	50.00
500	500.00
5000	5000.00
50,000	5.00e4
5×10^{-476}	5.00e-476
5.0009×10^{-476}	5.00e-476
$-\infty$	-infinity (never "infinite")
2π	6.28
i	im(1.00)
2i	im(2.00)
1 + 2i	re(1.00)im(2.00)
-0.0002548 i	im(-2.55e-4)
1/i = i/-1 = -i	im(-1.00)
$e^{i2\pi}$ $[= cos(2\pi) + isin(2\pi) = 1 + i0 = 1]$	1.00
$e^{i\pi/3}$ $[= cos(\pi/3) + isin(\pi/3) = 0.5 + i0.87]$	re(0.50)im(0.87)
Choices in order A, B, C, D	abcd

Entry format is given with the problem. So "q_X" means to enter "q_X" replacing "X" with your solution. The first 6 digits of the MD5 sum should match the given solution (MD5(q_X)= ...).

Note that although some answers can usually only be integers (eg, number of elephants), unless otherwise indicated you should always enter an integer to two decimal places (ie, with ".00" after it) to generate the correct hash.

Chapter 1

Kinetics of systems of particles

1.1 System centre-of-mass position, mass and velocity: I

Resources

- Book sections 4/1 to 4/2

Challenge

1. $\overline{\boldsymbol{r}}$, $\dot{\overline{\boldsymbol{r}}}$ and $\ddot{\overline{\boldsymbol{r}}}$ of Question 4/1.
2. Question 4/4

Solution

1. Given in book.
2. 316 N

1.2 System centre-of-mass position, mass and velocity: II

Resources

- Book sections 4/1 to 4/2

Challenge

Question 4/5. Determine the *magnitude* of the acceleration.

Solution

Given in book.

1.3 System centre-of-mass position, mass and velocity: III

Resources

- Book sections 4/1 to 4/2

Challenge

Question 4/13, but change the force to a 10 N force and the mass of each bar to 8 kg.

Solution

$0.42\,\mathrm{m/s^2}$

1.4 Kinetic and potential energy

Resources

- Book section 4/3

Challenge

1. Calculate T in question 4/1
2. Question 4/10

Solution

1. Given in book.
2. To check your final answer, substitute b = 2 metres into your final answer. You should obtain 5.27 m/s.

1.5 Cross-product

Resources

- https://www.khanacademy.org/science/physics/magnetic-forces-and-magnetic-fields/electric-motors/v/calculating-dot-and-cross-products-with-unit-vector-notation

Challenge

1. Determine the angle between the two vectors $\boldsymbol{a} = [3, 0, 0]$ and $\boldsymbol{b} = [3, 1, 0]$ and use it to calculate $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$. Which direction does the vector $\boldsymbol{c}$ point?

2. Determine the cross product $\boldsymbol{f} = \boldsymbol{d} \times \boldsymbol{e}$ where $\boldsymbol{d} = 4\hat{i} + 2\hat{j} + 1\hat{k}$ and $\boldsymbol{e} = -2\hat{i} - 4\hat{j} + 8\hat{k}$ without calculating the angle between them.

Solution

Please compare your answer with your partner and discuss in class if answers differ.

1.6 Rotation I

Resources

- Book section 4/4

Challenges

Calculate the angular momentum and the rate of change of angular momentum with time for Question 4/1.

Solutions

Given in book.

1.7 Rotation II

Resources

- Book section 4/4

Challenges

Question 4/16

If you have difficulty, consider doing question 4/15 first (optional).

Solutions

The required time should be 2.72 s

1.8 Rotation III

Resources

- Book section 4/4

Challenges

Question 4/2

Solutions

To check your answers substitute $d = 2$ metres, $m = 7$ kg, $v = 3$ m/s and $f = 7$ N into your final answers. You should obtain $\boldsymbol{H}_G = 432\hat{\imath} + 144\hat{\jmath} + 168\hat{k}$ kgm^2/s and $\dot{\boldsymbol{H}}_G = -8\hat{\imath} - 12\hat{\jmath} + 0\hat{k}$ Nm

1.9 Conservation of momentum

Resources

- Book section 4/5

Challenges

1. In Question 4/17, at what point does the vehicle stop accelerating?
2. Solve Question 4/17
3. Question 4/18

Solutions

1. Please write your answer and compare with your partner in class
2. Given in book
3. 0.21 m/s

1.10 Conservation of momentum vs energy

Resources

- Book section 4/5

Challenges

1. Solve Question 4/19

2. Why is energy not conserved here? Where did the energy go? Under what conditions is momentum conserved, and under what conditions is energy conserved?

Solutions

1. Given in book

2. Please write your answers and compare with your partner in class.

1.11 Combined problems I

Resources

- Book section 4/1 to 4/5

Challenge

Solve Question 4/22.

The question states that an impulse is imparted "over a negligibly short period of time" which is a little confusing since impulse is the integration of force over time which becomes zero as time goes to zero. Instead, here you can consider that whatever the time is, the final product of Force and Time is $10\,\mathrm{N\,s}$.

Solution

$4.7\,\mathrm{m/s}$

1.12 Combined problems II

Resources

- Book section 4/1 to 4/5

Challenge

Solve Question 4/28

Solutions

You should obtain an algebraic expression for v and $\dot{\theta}$. To check your expression, you can substitute the following values into the expression: $m_0 = 1\,\mathrm{kg}$, $v_0 = 1000\,\mathrm{m\,s^{-1}}$, $b = 1.5\,\mathrm{m}$ and $m = 4\,\mathrm{kg}$, whereby you should obtain $v = 111\,\mathrm{m\,s^{-1}}$ and $\dot{\theta} = 222\,\mathrm{rad\,s^{-1}}$.

1.13 In-plane flow

Resources

- Book section 4/6

Challenge

Derive equation 4/19 in the book from equation 4/19a.

1.14 Force on vane

Resources

- Book section 4/6

Challenge

Show your working for sample problem 4/5 (a) and (b)

1.15 Power and a vane

Resources

- Book section 4/6

Challenge

Considering sample problem 4/6,

1. Explain in words what is meant by "power by action of the fluid".

2. The power is defined here by measuring the force applied to move an object at a constant velocity. If force creates acceleration ($F = ma$), how can the velocity be constant?

3. Work through and solve the sample problem.

Solutions

Please compare your solutions with your partner. You may be asked to present your solutions to the class.

1.16 Balancing forces: Jet aeroplane example

Resources

- Book section 4/6

Challenge

Work through sample problem 4/8 to obtain the equation of motion of the system as given in the book ($m'_g u - m'_a v = mg \sin\theta + D$).

1.17 Balancing forces: Jet aeroplane

Resources

- Book section 4/6

Challenge

Answer question 4/33.

Solution

Given in book.

1.18 Balancing forces: Fire tug

Resources

- Book section 4/6

Challenge

Answer question 4/37.

Solution

Given in book.

1.19 Balancing ball on a water stream

Resources

- Book section 4/6

Challenge

Answer question 4/42. Take note about the conservation of energy in the jet stream, and the fact that the jet stream remains intact. You can assume that the water stream is fully deflected horizontally when it hits the ball.

Solution

4.8 m

1.20 Pressure I

Resources

- Book section 4/6

Challenge

A typical die has a side length of 1.4 cm and weighs 2.8 g. Consider the die at rest on a desk. Estimate the pressure on the bottom of the die due to the desk.

Solution

140 Pa

1.21 Pressure II

Resources

- Book section 4/6

Challenge

Answer question 4/50

Solution

1035 Pa

1.22 Power and a Helicopter

Resources

- Book section 4/6

Challenge

Answer question 4/59

Solutions

Given in book

1.23 Mass ejection

Resources

- Book section 4/7

Challenge

Consider rocket thrust where exhaust is emitted at a speed of $220\,\mathrm{m\,s^{-1}}$. The force on the rocket due to the thrust alone is $400\,\mathrm{N}$. Calculate (a) the mass flow rate m' and (b) the time-rate increase of the mass of the rocket $\dot{m}$.

Solutions

(a) MD5(a_X) = 300026… $\mathrm{kg\,s^{-1}}$
(b) MD5(b_X) = a2fb8f… $\mathrm{kg\,s^{-1}}$

1.24 Rocket sample problem

Resources

- Book section 4/7

Challenge

Complete the sample problem 4/11 using both solution I and II. Please be sure to follow the logic.

1.25 Rocket-style problem I

Resources

- Book section 4/7

Challenge

Answer question 4/67

Solution

Given in book.

1.26 Rocket-style problem II

Resources

- Book section 4/7

Challenge

Answer question 4/82

Solution

$4.8\,\mathrm{m\,s^{-1}}$

1.27 Mass intake and power

Resources

- Book section 4/7

Challenge

Answer question 4/80

Solution

$1.6\,\mathrm{m\,s^{-2}}$ deceleration

1.28 Chain style sample problem

Resources

- Book section 4/7

Challenge

Work through sample problem 4/9

1.29 Rope style sample problem

Resources

- Book section 4/7

Challenge

Work through sample problem 4/10

1.30 Chain vs Rope style sample problem difference

Resources

- Book section 4/7

Challenge

Considering the chain sample problem and the unconstrained rope problem, why was the kinetic energy different in these two cases? What assumptions were made in the chain problem compared to the unconstrained rope problem, and how did this impact the calculation of kinetic energy? How might the kinetic energy change if the rope in sample problem 4/10 was made of a combination of small loops like the chain in sample problem 4/9?

Please write a few sentences summarising your understanding.

Solution

Please compare your writing with your partner's writing and discuss any differences.

1.31 Constrained and unconstrained rope style sample problem

Resources

- Book section 4/7

Challenge

Considering the unconstrained and constrained rope sample problem, how is the approach different? How do the different values for "P" and "R" arise? What assumptions are different?

Please write a few sentences summarising your understanding.

Solution

Please compare your writing with your partner's writing and discuss any differences.

1.32 Lifting a chain

Resources

- Book section 4/7

Challenge

An 18K gold chain has a mass of 1.12 g and a length of 40 cm. You pick up one end of the chain and lift it up vertically at a constant velocity. There will be two downward forces present: one due to the hanging weight of the chain due to earth's gravity (A), and another induced by the constant addition of mass to the hanging part of the chain (B). If the chain is lifted up at $5\,\mathrm{cm\,s^{-1}}$, calculate B. State what simplifying assumptions you make. How does this compare to A?

Solution

B: $7\,\mu\mathrm{N}$

1.33 Chain on a pully

Resources

- Book section 4/7

Challenge

Answer question 4/83

Solution

P is given in book, and R should match your understanding of the weight of the pile.

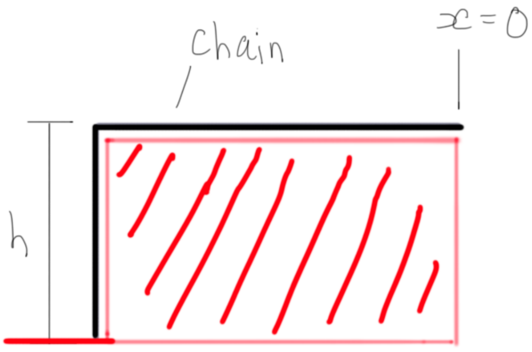
1.34 An accelerating chain

Resources

- Book section 4/7

Challenge

Consider a chain hanging over the edge of a block of height h. The chain has a total length L and mass per unit length of ρ. The left end of the chain is barely touching the ground. Ignoring friction at the corner and making other idealised assumptions, obtain an expression for the acceleration of the chain as a function of the position of the end of the chain (x) as it slides along the horizontal surface.



Solution

To check your answer, determine the acceraleration for a height of 5 m and chain length of 10 m, when the chain has slid 2 m. You should obtain an acceleration of $6.13\,\mathrm{m\,s^{-2}}$

Chapter 2

3D dynamics of rigid bodies

2.1 Radial velocity with horizontal connection

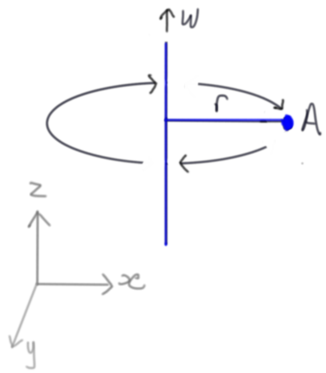
Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

A weight "A" is tethered to a pole by a stiff rod of length r. If the angular velocity is $5.5\,\mathrm{rad\,s^{-1}}\ \hat{k}$ and the length of the rod is $47\,\mathrm{m}$ along the x-axis, what is the linear velocity of the weight "A"?



Solution

X = Your solution
Units: $\mathrm{m\,s^{-1}}$
Form: Decimal, to 1 decimal place
Place the indicated letter in front of the number
Example: aX where $X = 42.5\ \mathrm{m\,s^{-1}}$ is entered as a42.5

$\hat{\imath}$ = hash of aX = 9497cd $\mathrm{m\,s^{-1}}$
$\hat{\jmath}$ = hash of bX = d17e5c $\mathrm{m\,s^{-1}}$
$\hat{k}$ = hash of cX = 347133 $\mathrm{m\,s^{-1}}$

2.2 Radial velocity with non-horizontal connection

Resources

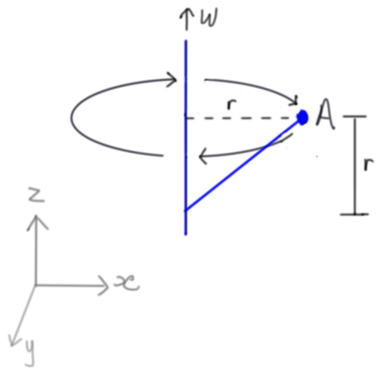
- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

1. The position of "A" and the pole are unchanged (the radial distance is the same) and the angular velocity remains the same, but "A" is now hinged to the pole from below instead of horizontally, as shown in the picture. Calculate the linear velocity of "A" (calculate mathematically, not just by comparison with the previous challenge).

2. Write a sentence or two comparing your answer with that obtained from the previous challenge, including reasoning why.



Solution

1.
X = Your solution
Units: $\mathrm{m\,s^{-1}}$
Form: Decimal, to 1 decimal place
Place the indicated letter in front of the number
Example: aX where $X = 42.5\ \mathrm{m\,s^{-1}}$ is entered as a42.5

$\hat{\imath}$ = hash of dX = c6e675 $\mathrm{m\,s^{-1}}$
$\hat{\jmath}$ = hash of eX = bcff19 $\mathrm{m\,s^{-1}}$
$\hat{k}$ = hash of fX = 979ed8 $\mathrm{m\,s^{-1}}$

2. Please discuss in class if you are unsure about your answer.

2.3 Linear acceleration

Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

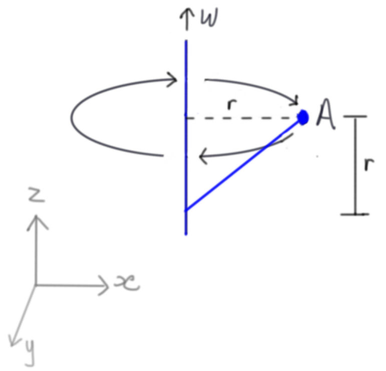
Challenge

Using information from previous challenges, determine the:

1. Linear acceleration towards the centre of pole.

2. The tangential linear acceleration

3. Is there linear acceleration towards the centre of the pole? Is there tangential linear acceleration? Write a sentence or two to explain why for both cases.



Solution

X = Your solution
Units: $\mathrm{m\,s^{-2}}$
Form: Decimal, to 2 decimal place
Place the indicated letter in front of the number
Example: aX where $X = 42.57\ \mathrm{m\,s^{-1}}$ is entered as a42.57

1.
$\hat{i}$ = hash of gX = e1993f $\mathrm{m\,s^{-2}}$
$\hat{j}$ = hash of hX = 5a16a7 $\mathrm{m\,s^{-2}}$
$\hat{k}$ = hash of iX = 2ebd7c $\mathrm{m\,s^{-2}}$

2.
$\hat{i}$ = hash of jX = 4b3090 $\mathrm{m\,s^{-2}}$
$\hat{j}$ = hash of kX = 28435f $\mathrm{m\,s^{-2}}$
$\hat{k}$ = hash of lX = 060ec3 $\mathrm{m\,s^{-2}}$

3. Please compare your answer with your partner.

2.4 Radial acceleration - only magnitude

Resources

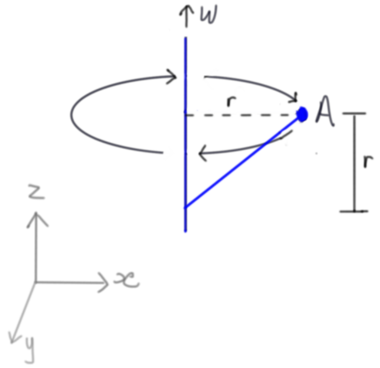
- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Now consider that the radial velocity is not constant, but is undergoing an acceleration so that the magnitude of the angular velocity $\boldsymbol{w}$ increases while it continues to point in the same direction.

If the acceleration is $2\,\mathrm{rad\,s^{-2}}$, what is the tangential acceleration of "A"?



Solution

X = Your solution
Units: $\mathrm{m\,s^{-1}}$
Form: Decimal, to 1 decimal place
Place the indicated letter in front of the number
Example: aX where $X = 42.5\ \mathrm{m\,s^{-1}}$ is entered as a42.5

$\hat{\imath}$ = hash of mX = b9f8f5 $\mathrm{m\,s^{-1}}$
$\hat{\jmath}$ = hash of nX = 57e394 $\mathrm{m\,s^{-1}}$
$\hat{k}$ = hash of oX = 0c8b72 $\mathrm{m\,s^{-1}}$

2.5 Radial acceleration - only direction (precession)

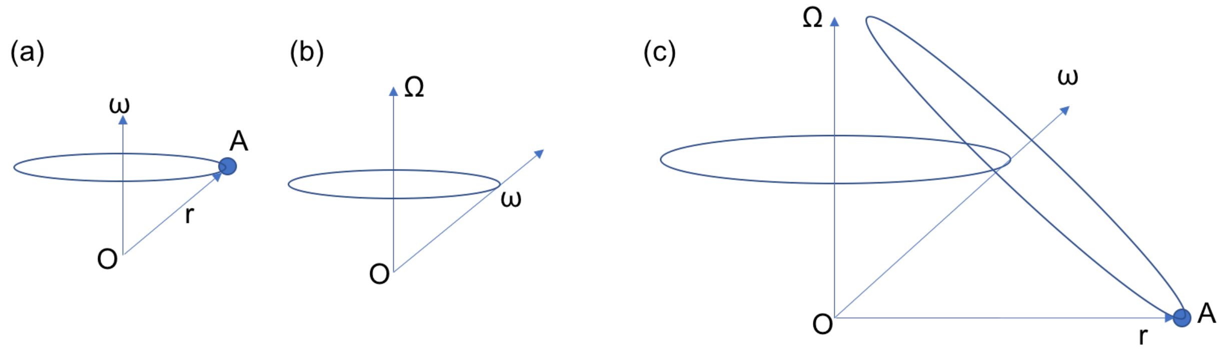
Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

The previous challenges considered the case (a) below, where the direction of the angular velocity vector ω was unchanging. Next consider that the angular velocity vector is precessing around an axis of symmetry, and this precession has an angular velocity of Ω, as shown in (b). Combining (a) and (b) we have (c).



1. Consider the point "A" rotating about the vector ω with angular velocity magnitude $5.5\,\text{rad}\,\text{s}^{-1}$, but now tilt the ω vector and allow the rotation to precess around a vector of symmetry Ω. Assume that only the direction (not the magnitude) of the angular velocity vector ω is changing with time. If $\Omega = 3\hat{k}$ $\text{rad}\,\text{s}^{-1}$ and angular velocity vector ω is inclined at 45° in the positive x-direction so that the vector $\boldsymbol{r} = 20\hat{\imath}$, calculate the linear acceleration of point "A". You may consider the origin to be at position "A" where the $\boldsymbol{\Omega}$, $\boldsymbol{\omega}$ and $\boldsymbol{r}$ vectors meet.

2. What is the direction of the acceleration of the angular velocity vector ω? Write 1 or 2 sentences to explain why.

3. What is the direction of the linear acceleration of "A"? Write one or two sentences (possibly with a diagram) to explain when the sign will be opposite but with same magnitude.

Solution

1. $-302.5\hat{\imath} + 69.15\hat{k}$

2. and 3. Please discuss in class.

2.6 Radial acceleration II

Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Question 7/4

Solution

$1285\,\mathrm{m\,s^{-1}}$

2.7 Unit vector of a rotation axis

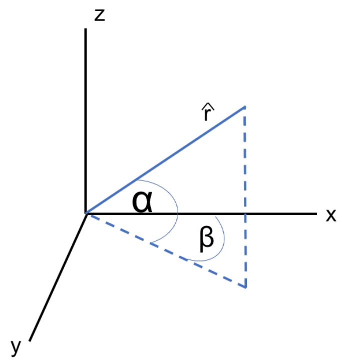
Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Considering vector $\boldsymbol{r}$ in the figure below, if the angles $\alpha = 45$ degrees and $\beta = 30$ degrees, write the unit vector $\hat{\boldsymbol{r}}$ in terms of the Cartesian unit vectors.



Solution

$\boldsymbol{r} = \frac{3}{2\sqrt{2}}\hat{\imath} + \frac{1}{2\sqrt{2}}\hat{\jmath} + \frac{1}{\sqrt{2}}\hat{k}$

2.8 Simultaneous rotation I

Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Work through sample problem 7/2.

2.9 Simultaneous rotation II

Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Work through sample problem 7/1, parts (a) and (b) only.

2.10 Simultaneous rotation III

Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Complete question 7/12

Solution

Velocity: $\frac{\pi}{8}(-4\hat{\imath} - 6\hat{\jmath} + 3\hat{k})\ \mathrm{m\,s^{-1}}$

Acceleration: $\frac{-\pi^2}{8}(25\hat{\jmath} + 18\hat{k})\ \mathrm{m\,s^{-2}}$

2.11 Time-dependent rotation of vectors I

Resources

- Book section 5/7

Comment

A vector $\boldsymbol{r}$ may be split into its magnitude and scaler components like $\boldsymbol{r} = r_i\hat{\boldsymbol{\imath}} + r_j\hat{\boldsymbol{\jmath}} + r_k\hat{\boldsymbol{k}}$. For fixed co-ordinate systems, the direction of $(\hat{\boldsymbol{\imath}},\ \hat{\boldsymbol{\jmath}},\ \hat{\boldsymbol{k}})$ does not vary with time, so the derivatives of the unit vectors are zero. For a rotating co-ordinate system however, the derivative chain rule must be used in order to take account of the changing directions of the unit vectors.

Challenge

Consider a disk that is initially flat in the x-y plane. The z-axis can be considered to point perpendicularly up from the centre of the disk. The disk then starts spinning with angular velocity depending on time t:

$$\boldsymbol{\omega} = \omega_i Sin(2\pi t)\hat{\boldsymbol{\imath}} + \omega_j Cos(2\pi t)\hat{\boldsymbol{\jmath}} + \omega_k t\hat{\boldsymbol{k}} \tag{2.1}$$

You can consider the $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ axes to be rotating with the disk and the $\hat{\boldsymbol{k}}$ vector to always point perpendicular to the plane of the disk.

Derive an expression for the angular acceleration $\dot{\boldsymbol{\omega}}$ of the disk as a function of time.

Solution

To check your answer, substitute the following values into your final equation: $t = 1$
$\omega_i = 10$ $\omega_j = 3$ $\omega_k = 2$

and you should obtain the final result

$\dot{\boldsymbol{\omega}} = (20\pi - 6)\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{k}}$

2.12 Time-dependent rotation of vectors II

Resources

- Book section 5/7

Challenge

Answer question 7/27 in the book.

Solution

Given in book.

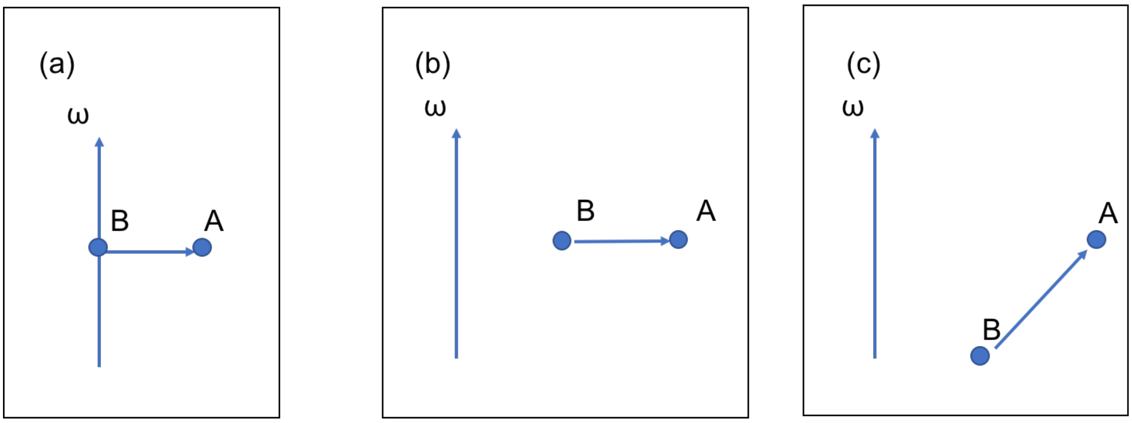
2.13 Relative velocity

Resources

- Book section 7/6

Challenge

Consider a rotating rigid body with angular velocity $\omega = \hat{k}$. Two points, "A" and "B" are chosen on the rigid body. The book describes how it is possible to calculate the velocity of point "A" given the velocity of point "B". By calculating the velocity of point "A" in the 3 cases below, prove that the velocity of "A" can be calculated accurately irrespective of choice of the location of "B".



a) $\boldsymbol{B} = \hat{k}$, $\boldsymbol{A} = \hat{\imath} + \hat{k}$
b) $\boldsymbol{B} = \hat{\imath} + \hat{k}$, $\boldsymbol{A} = 2\hat{\imath} + \hat{k}$
c) $\boldsymbol{B} = \hat{\imath}$, $\boldsymbol{A} = 2\hat{\imath} + \hat{k}$

Solution

You should be able to show that all 3 cases will result in the same value of v_A. If this is not the case, please discuss with your partner or the teacher.

2.14 Crank-style problem

Resources

- Book section 7/6

Comment

The angular velocity of the link $\bar{AB}$ is, by definition, perpendicular to the axis of the link. In the second part of this challenge, you use this fact along with other obtained equations to obtain the 3 Cartesian directions of the angular velocity vector. Note that although the concept of the angular momentum of the link $\bar{AB}$ is used to calculate w_2 in the first part of the problem, you don't actually need to calculate the value of the $\boldsymbol{w_n}$ vector in order to determine the value of w_2.

Challenge

Work through sample problem 7/3

2.15 Perpendicular position, velocity and rotation vectors

Challenge

Prove that $\boldsymbol{r}_{AB}$, $\boldsymbol{v}_{AB}$ and $\boldsymbol{\omega}_n$ in sample problem 7/3 are all perpendicular to each other.

2.16 Perpendicular double cross-product

Challenge

Considering a vector $\boldsymbol{a} = \hat{k}$ and a vector $\boldsymbol{b} = \hat{\imath}$, show that $\boldsymbol{a} \times (\boldsymbol{a} \times \boldsymbol{b}) = -a^2\boldsymbol{b}$.

2.17 3D acceleration

Comment

In this sample problem the concept that $\dot{\boldsymbol{\omega}}_n$ is normal to $\boldsymbol{r}_{AB}$, is included. “A” and “B” are part of a rigid body and therefore the separation between them is constant, even while the ijk components of $\boldsymbol{r}_{AB}$ vary with orientation of $\boldsymbol{r}_{AB}$. Despite this constant variation in orientation, angular velocity is always normal to $\boldsymbol{r}_{AB}$, by definition, because rotation parallel to $\boldsymbol{r}_{AB}$ has no influence on the motion of $\bar{AB}$. Any angular acceleration parallel to $\boldsymbol{r}_{AB}$ has no effect on the motion of $\hat{AB}$, and since the direction of $\boldsymbol{\omega}_n$ is always defined to be normal to $\boldsymbol{r}_{AB}$, the angular acceleration $\dot{\boldsymbol{\omega}}_n$ is also always normal to $\boldsymbol{r}_{AB}$.

Challenge

Work through sample problem 7/4.