Mechanics

Spring semester 2018

Last updated:
7th May 2018 at 12:28

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http://www.jamescannon.net/teaching/mechanics

http://raw.githubusercontent.com/NanoScaleDesign/Mechanics/master/mechanics.pdf

Contents

Chapter 0

Course information

0.1 This course

This is the Spring 2018 Mechanics course studied by 2nd-year undergraduate international students at Kyushu University.

0.1.1 What you need to do

- Borrow the book "Engineering Mechanics: Dynamics", 6th edition, by Meriam and Kraige from the Mechanical Engineering office on the 4th floor of West 4. The course will be based on that book and you will need to refer to it in class.
- Prepare a challenge-log in the form of a workbook or folder where you can clearly write the calculations you perform to solve each challenge. This will be used in the final assessment and will be occasionally reviewed by the teacher.
- Submit a weekly feedback form by **9am on Monday** before class at `https://goo.gl/forms/ULjtgmL0Q2fZbG292`.
- Please bring a wifi-capable internet device to class, as well as headphones if you need to access online components of the course during class. If you let me know in advance, I can lend computers and provide power extension cables for those who require them (limited number).

0.1.2 How this works

- This booklet forms part of an active-learning segment in the course. The learning is self-directed in contrast to the traditional lecture-style model.
- Learning is guided through solving a series of challenges combined with instant feedback about the correctness of your answer.
- There are no lectures. Instead, there is discussion time. Here, you are encouraged to discuss any issues with your peers, teacher and any teaching assistants. Furthermore, you are encouraged to help your peers who are having trouble understanding something that you have understood; by doing so you actually increase your own understanding too.
- Discussion-time is from 13:00 to 14:30 on Mondays at room W4-766.
- Peer discussion is encouraged, however, if you have help to solve a challenge, always make sure you do understand the details yourself. You will need to be able to do this in an exam environment. The questions on the exam will be similar in nature to the challenges.
- Every challenge in the book typically contains a **Challenge** with suggested **Resources** which you are recommended to utilise in order to solve the challenge. Occasionally the teacher will provide extra **Comments** to help guide your thinking. A **Solution** is also made available for you to check your answer. Sometimes this solution will be given in encrypted form. For more information about encryption, see section 0.4.
- For deep understanding, it is recommended to study the suggested resources beyond the minimum required to complete the challenge.
- The challenge document has many pages and is continuously being developed. Therefore it is advised to view the document on an electronic device rather than print it. The date on the front page denotes the version of the document. You will be notified by email when the document is updated. The content may differ from last-year's document.
- A target challenge will be set each week. This will set the pace of the course and defines the examinable material. It's ok if you can't quite reach the target challenge for a given week, but you should be careful not to fall behind, since the exam cannot be delayed.

- You may work ahead, even beyond the target challenge, if you so wish. This can build greater flexibility into your personal schedule, especially as you become busier towards the end of the semester.
- Your contributions to the course are strongly welcomed. If you come across resources that you found useful that were not listed by the teacher or points of friction that made solving a challenge difficult, please let the teacher know about it!

0.1.3 Assessment

In order to prove to outside parties that you have learned something from the course, we must perform summative assessments. You will receive a weighted score based on:

- Challenge-log (10%) - final state at the end of the course, showing your calculations for all the challenges in the course.
- Coursework (20%)
- Mid-term exam (30%)
- Final exam (40%)

Final score = MAX(Weighted score, Final exam score)

0.2 Peerwise

Part of this course will involve your creation of questions using the PeerWise platform. To register, go to `http://peerwise.cs.auckland.ac.nz/at/?kyushu_u_jp` and register. Access the course "Advanced Mechanics (Spring 2018)" and enter the following information:

- Course ID = **17143**
- Identifier = **Your student ID**

Once you have logged in, select the "Advanced Mechanics (Spring 2018)" course, then under "Your questions" click "view". You can then click "Create new question" or view other questions by clicking "view" under "Unanswered questions".

Note: There is a leaderboard system with points, etc. I encorage you to vote-up the questions that you learn most from, so as to help other students identify the best questions. However, the marking of the questions by the teacher is not influenced by these votes.

0.3 Timetable

	Discussion	**Target**	**Note**
1	9 April	-	
2	16 April	2.4	
3	23 April	2.10	
4	7 May	2.14	
5	14 May	2.21	
6	21 May		
7	28 May		
8	11 June	-	Mid-term exam
9	18 June	-	
10	25 June		
11	2 July		
12	9 July		Coursework information
13	12 July		
14	23 July	-	Coursework submission
-	August	-	Final exam
-	September	-	Retake exam

0.4 Hash-generation

Some solutions to challenges are encrypted using MD5 hashes. In order to check your solution, you need to generate its MD5 hash and compare it to that provided. MD5 hashes can be generated at the following sites:

- Wolfram alpha: (For example: md5 hash of "q1.00") `http://www.wolframalpha.com/input/?i=md5+hash+of+%22q1.00%22`
- `www.md5hashgenerator.com`

Since MD5 hashes are very sensitive to even single-digit variation, you must enter the solution *exactly*. This means maintaining a sufficient level of accuracy when developing your solution, and then entering the solution according to the format suggested by the question. Some special input methods:

Solution	**Input**
5×10^{-476}	5.00e-476
5.0009×10^{-476}	5.00e-476
$-\infty$	-infinity (never "infinite")
2π	6.28
i	im(1)
2i	im(2)
1 + 2i	re(1)im(2)
-0.0002548 i	im(-2.55e-4)
1/i = i/-1 = -i	im(-1)
$e^{i2\pi}$ [$= cos(2\pi) + isin(2\pi) = 1 + i0 = 1$]	1.00
$e^{i\pi/3}$ [$= cos(\pi/3) + isin(\pi/3) = 0.5 + i0.87$]	re(0.50)im(0.87)
Choices in order A, B, C, D	abcd

The first 6 digits of the MD5 sum should match the first 6 digits of the given solution.

Chapter 1

Hash practise

1.1 Hash practise: Integer

X = 46.3847
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

hash of aX = e77fac

1.2 Hash practise: Decimal

X = 49
Form: Two decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

hash of bX = 82c9e7

1.3 Hash practise: String

X = abcdef
Form: String.
Place the indicated letter in front of the number.
Example: aX where $X = abc$ is entered as aabc

hash of cX = 990ba0

1.4 Hash practise: Scientific form

X = 500,765.99
Form: Scientific notation with the mantissa in standard form to 2 decimal place and the exponent in integer form.
Place the indicated letter in front of the number.
Example: aX where $X = 4 \times 10^{-3}$ is entered as a4.00e-3

hash of dX = be8a0d

Chapter 2

Kinetics of systems of particles

2.1 System centre-of-mass position, mass and velocity: I

Resources

- Book sections 4/1 to 4/2

Comment

You may use the following text as an alternative explanation to that in the book.

Introduction

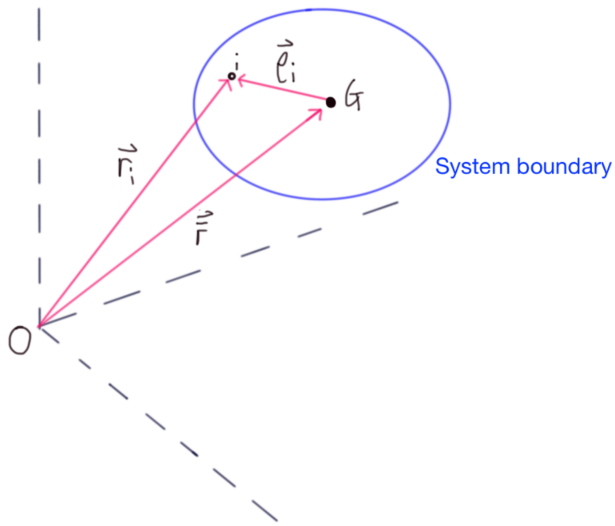
You are already largely familiar with the kinetics of a single particle. The idea of this short course is to extend these principles to the motion of a general system of particles. This will enable you to describe the motion of both rigid and non-rigid bodies and systems.

A rigid body is defined as a solid system of particles where the distance between the particles does not change with respect to time. Examples of rigid-bodies are numerous, including many machines operating in the air, on land or under the sea, including rockets and spacecraft. A non-rigid body can come in two forms:

- A single solid body that changes shape over time, perhaps due to elastic or non-elastic deformations.
- A defined mass of liquid or gaseous particles flowing at a given rate. Important examples include air and fuel flowing through the turbine of an aircraft engine, the exhaust from the nozzle of a rocket, or water passing through a pump.

Generalising Newton's 2nd law

Here we extend Newton's second law of motion to cover a general mass system, modelled by considering n particles each with mass m_i bounded by a closed surface in space:



The centre-of-mass of the system is denoted by the point G and the distance of particle i from the centre-of-mass is $\boldsymbol{\rho_i}$. The distance of particle i from an arbitrary non-accelerating origin O of a Newtonian set of reference axes is $\boldsymbol{r_i}$ and the distance of the centre-of-mass from this origin is $\bar{\boldsymbol{r}}$. Note that point G does not have to correspond to the position of any one particle, and may just represent the position of the centre-of-mass of the particles at that particular instant.

The total mass of the system m is of course just sum of all the particle masses m_i:

$$m = \sum m_i \tag{2.1}$$

so that the centre-of-mass G of the system can be determined from the position of all particles:

$$m\bar{\boldsymbol{r}} = \sum m_i \boldsymbol{r}_i \tag{2.2}$$

Each particular in the system is subject to forces. It is useful to group these forces into two contributions:

- Sources internal to the system boundary (ie, other particles). For example, the force on particle i by particle j.
- Sources external to the system boundary (eg, electric fields, gravity, etc).

One can consider the total force acting on the system as a whole. Note that the force on on particle i by particle j is equal and opposite to the force on particle j by particle i. Thus the sum of forces from internal forces must all cancel out to be zero. If there are multiple external forces $\boldsymbol{F}_i$ on the system, then the total force on the system must be $\sum \boldsymbol{F}$. Thus the equation of motion for the system can be written as

$$\sum \boldsymbol{F} = m\ddot{\bar{\boldsymbol{r}}} = m\bar{\boldsymbol{a}} \tag{2.3}$$

where $\bar{\boldsymbol{a}}$ corresponds to the acceleration of the centre-of-mass G.

Equation 2.3 is the generalised form of Newton's second law of motion. Each component in a Cartesian system may be treated independently (eg, $\sum F_x = m\bar{a}_x$ etc). The resultant external force $\sum \boldsymbol{F}$ will have the same direction as the acceleration of the system $\bar{\boldsymbol{a}}$ at that instant of time, however the resultant external force does not necessarily pass through the centre-of-mass (usually it does not).

Challenge

1. $\bar{\boldsymbol{r}}$, $\dot{\bar{\boldsymbol{r}}}$ and $\ddot{\bar{\boldsymbol{r}}}$ of Question 4/1.
2. Question 4/4.

Solution

1. Given in book.
2. 534 N

2.2 System centre-of-mass position, mass and velocity: II

Resources

- Book sections 4/1 to 4/2

Challenge

Question 4/5. Determine the *magnitude* of the acceleration.

Solution

$4\,\mathrm{m/s^2}$

2.3 System centre-of-mass position, mass and velocity: III

Resources

- Book sections 4/1 to 4/2

Challenge

Question 4/13

Solution

$4.2\,\mathrm{m/s^2}$

2.4 Kinetic and potential energy

Resources

- Book section 4/3

Challenge

1. Calculate T in question 4/1
2. Question 4/10

Solution

1. Given in book.
2. To check your final answer, substitute b = 2 metres into your final answer. You should obtain 5.27 m/s.

2.5 Cross-product

Resources

- `https://www.khanacademy.org/science/physics/magnetic-forces-and-magnetic-fields/electric-motors/v/calculating-dot-and-cross-products-with-unit-vector-notation`

Challenge

1. Determine the angle between the two vectors $\boldsymbol{a} = [3, 0, 0]$ and $\boldsymbol{b} = [3, 1, 0]$ and use it to calculate $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$. Which direction does the vector $\boldsymbol{c}$ point?

2. Determine the cross product $\boldsymbol{f} = \boldsymbol{d} \times \boldsymbol{e}$ where $\boldsymbol{d} = 4\hat{i} + 2\hat{j} + 1\hat{k}$ and $\boldsymbol{e} = -2\hat{i} - 4\hat{j} + 8\hat{k}$ without calculating the angle between them.

Solution

Please compare your answer with your partner and discuss in class if answers differ.

2.6 Rotation of particles I

Resources

- Book section 4/4

Challenges

Calculate the angular momentum and the rate of change of angular momentum with time for Question 4/1.

Solutions

Given in book.

2.7 Rotation of particles II

Resources

- Book section 4/4

Challenges

Question 4/15

Solutions

Given in book.

2.8 Rotation of particles III

Resources

- Book section 4/4

Challenges

Question 4/16

Solutions

The required time should be $2.72\,\mathrm{s}$

2.9 Rotation of particles IV

Resources

- Book section 4/4

Challenges

Question 4/2

Solutions

To check your answers substitute $d = 2$ metres, $m = 7$ kg, $v = 3$ m/s and $f = 7$ N into your final answers. You should obtain $\boldsymbol{H}_G = 432\hat{\imath} + 144\hat{\jmath} + 168\hat{k}$ kgm^2/s and $\dot{\boldsymbol{H}}_G = -8\hat{\imath} - 12\hat{\jmath} + 0\hat{k}$ Nm

2.10 Energy and Rotation I

Resources

- Book section 4/1 to 4/5

Challenge

Solve Question 4/22.

Solution

4.7 m/s

2.11 Energy and Rotation II

Resources

- Book section 4/1 to 4/5

Challenge

Solve Question 4/28

Solutions

You should obtain an algebraic expression for v and $\dot{\theta}$. To check your expression, you can substitute the following values into the expression: $m_0 = 1\,\mathrm{kg}$, $v_0 = 1000\,\mathrm{m\,s^{-1}}$, $b = 1.5\,\mathrm{m}$ and $m = 4\,\mathrm{kg}$, whereby you should obtain $v = 111\,\mathrm{m\,s^{-1}}$ and $\dot{\theta} = 222\,\mathrm{rad\,s^{-1}}$.

2.12 Mass ejection

Resources

- Book section 4/7

Challenge

Consider rocket thrust where exhaust is emitted at a speed of $220\,\mathrm{m\,s^{-1}}$. The force on the rocket due to the thrust alone is $400\,\mathrm{N}$. Calculate (a) the mass flow rate m' and (b) the time-rate increase of the mass of the rocket $\dot{m}$.

Solutions

(a)
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of aX = b54e89 ($\mathrm{kg\,s^{-1}}$)

(b)
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of bX = 6a83d8 ($\mathrm{kg\,s^{-1}}$)

2.13 Rocket sample problem

Resources

- Book section 4/7

Challenge

Complete the sample problem 4/11 using both solution I and II. Please be sure to follow the logic and understand the link between the two methods.

2.14 Rocket-style problem I

Resources

- Book section 4/7

Challenge

Answer question 4/67

Solution

Given in book.

2.15 Rocket-style problem II

Resources

- Book section 4/7

Challenge

Answer question 4/82

Solution

$4.8\,\mathrm{m\,s^{-1}}$

2.16 Mass intake I

Resources

- Book section 4/7

Challenge

Answer question 4/80

Solution

$1.6\,\mathrm{m\,s^{-2}}$ deceleration

2.17 Mass intake II

Resources

- Book section 4/7

Challenge

Answer question 4/76

Solution

$0.152\,\mathrm{m\,s^{-2}}$

2.18 Chain style sample problem

Resources

- Book section 4/7

Challenge

Work through sample problem 4/9

2.19 Rope style sample problem

Resources

- Book section 4/7

Challenge

Work through sample problem 4/10

2.20 Chain vs Rope style sample problem difference

Resources

- Book section 4/7

Challenge

Considering the chain sample problem and the unconstrained rope problem, why was the kinetic energy different in these two cases? What assumptions were made in the chain problem compared to the unconstrained rope problem, and how did this impact the calculation of kinetic energy?

Please write a few sentences summarising your understanding.

Solution

Please compare your writing with your partner's writing and discuss any differences.

2.21 Constrained and unconstrained rope style sample problem

Resources

- Book section 4/7

Challenge

Considering the unconstrained and constrained rope sample problem, how are the approaches different? How do the different values for “P” and “R” arise? What assumptions are different?

Please write a few sentences summarising your understanding.

Solution

Please compare your writing with your partner’s writing and discuss any differences.