

Mechanics

Spring 2017

Last updated:
29th May 2017 at 12:24

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<http://raw.githubusercontent.com/NanoScaleDesign/Mechanics/master/mechanics.pdf>

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Chapter 0

Course information

0.1 This course

This is the Spring 2017 Mechanics course studied by 2nd-year undergraduate international students at Kyushu University.

0.1.1 What you need to do

- Borrow the book “Engineering Mechanics: Dynamics”, 6th edition, by Meriam and Kraige from the Kikan-kyoiku office in the centre-zone. The course will be based on that book and you will need to refer to it in class.
- Prepare a challenge-log in the form of a workbook or folder where you can clearly write the calculations you perform to solve each challenge. This will be used in the final assessment and will be occasionally reviewed by the teacher.
- Submit a weekly feedback form by **9am on Monday** before class at <https://goo.gl/forms/2PgFF0eqT0vbK0to2>.
- Please bring a wifi-capable internet device to class, as well as headphones if you need to access online components of the course during class. If you let me know in advance, I can lend computers and provide power extension cables for those who require them (limited number).

0.1.2 How this works

- This booklet forms part of an active-learning segment in the course. The learning is self-directed in contrast to the traditional lecture-style model.
- Learning is guided through solving a series of challenges combined with instant feedback about the correctness of your answer.
- Traditional lectures are replaced by discussion time. Here, you are encouraged to discuss any issues with your peers, teacher and any teaching assistants. You can also learn from explaining concepts to your peers.
- Discussion-time is from 13:00 to 14:30 on Mondays at room W4-766.
- Peer discussion is encouraged, however, if you have help to solve a challenge, always make sure you do understand the details yourself. You will need to be able to do this in an exam environment. The questions on the exam will be similar in nature to the challenges. If you can do all of the challenges, you can get 100% on the exam.
- Every challenge in the book typically contains a **Challenge** with suggested **Resources** which you are recommended to utilise in order to solve the challenge. **Solutions** will be given. Occasionally the teacher will provide extra **Comments** to help guide your thinking.
- For deep understanding, it is recommended to study the suggested resources beyond the minimum required to complete the challenge.
- The challenge document has many pages and is continuously being developed. Therefore it is advised to view the document on an electronic device rather than print it. The date on the front page denotes the version of the document. You will be notified when the document is updated.
- A target challenge will be set each week. This will set the pace of the course and define the examinable material. It's ok if you can't quite reach the target challenge for a given week, but you should be careful not to fall behind, since the date of the exam cannot be delayed.

0.1.3 Assessment

In order to prove to outside parties that you have learned something from the course, we must perform summative assessments. You will receive a weighted score based on:

- Challenge-log (10%) - final state at the end of the course, showing your calculations for all the challenges in the course.
- Mid-term exam (30%)
- Coursework (20%)
- Final exam (40%)

Final score = MAX(Weighted score, Final exam)

0.2 Timetable

	Discussion	Target	Note
1	10 April	-	
2	17 April	1.4	
3	24 April	1.7	
4	8 May	1.12	
5	15 May	1.17	
6	22 May	1.22	
7	29 May	1.27	
8	5 June	1.34	
9	12 June	-	Mid-term exam (covers up to 1.27)
10	19 June		
11	26 June		
12	3 July		
13	10 July		
14	13 July		
-	24 July	-	Final exam
-	16 August	-	Retake exam (tentative)

0.3 Hash-generation

Some solutions to challenges are encrypted using MD5 hashes. In order to check your solution, you need to generate its MD5 hash and compare it to that provided. MD5 hashes can be generated at the following sites:

- Wolfram alpha: (For example: md5 hash of “q_1.00”) http://www.wolframalpha.com/input/?i=md5+hash+of+%22q_1.00%22
- www.md5hashgenerator.com

Since MD5 hashes are very sensitive to even single-digit variation, you must enter the solution exactly. This means maintaining a sufficient level of accuracy when developing your solution, and then entering the solution according to the format below:

Unless specified otherwise, any number from 0.00 to ± 9999.99 should be represented as a normal number to two decimal places. All other numbers should be in scientific form. See the table below for examples.

Solution	Input
1	1.00
-3	-3.00
-3.5697	-3.57
0.05	0.05
0.005	5.00e-3
50	50.00
500	500.00
5000	5000.00
50,000	5.00e4
5×10^{-476}	5.00e-476
5.0009×10^{-476}	5.00e-476
$-\infty$	-infinity (never “infinite”)
2π	6.28
i	im(1.00)
2i	im(2.00)
$1 + 2i$	re(1.00)im(2.00)
$-0.0002548 i$	im(-2.55e-4)
$1/i = i/-1 = -i$	im(-1.00)
$e^{i2\pi} [= \cos(2\pi) + i\sin(2\pi) = 1 + i0 = 1]$	1.00
$e^{i\pi/3} [= \cos(\pi/3) + i\sin(\pi/3) = 0.5 + i0.87]$	re(0.50)im(0.87)
Choices in order A, B, C, D	abcd

Entry format is given with the problem. So “q_X” means to enter “q_X” replacing “X” with your solution. The first 6 digits of the MD5 sum should match the given solution (MD5(q_X)= ...).

Note that although some answers can usually only be integers (eg, number of elephants), unless otherwise indicated you should always enter an integer to two decimal places (ie, with “.00” after it) to generate the correct hash.

Chapter 1

Kinetics of systems of particles

1.1 System centre-of-mass position, mass and velocity: I

Resources

- Book sections 4/1 to 4/2

Challenge

1. $\bar{\mathbf{r}}$, $\dot{\bar{\mathbf{r}}}$ and $\ddot{\bar{\mathbf{r}}}$ of Question 4/1.
2. Question 4/4

Solution

1. Given in book.
2. 316 N

1.2 System centre-of-mass position, mass and velocity: II

Resources

- Book sections 4/1 to 4/2

Challenge

Question 4/5. Determine the *magnitude* of the acceleration.

Solution

Given in book.

1.3 System centre-of-mass position, mass and velocity: III

Resources

- Book sections 4/1 to 4/2

Challenge

Question 4/13, but change the force to a 10 N force and the mass of each bar to 8 kg.

Solution

0.42 m/s^2

1.4 Kinetic and potential energy

Resources

- Book section 4/3

Challenge

1. Calculate T in question 4/1
2. Question 4/10

Solution

1. Given in book.
2. To check your final answer, substitute $b = 2$ metres into your final answer. You should obtain 5.27 m/s .

1.5 Cross-product

Resources

- <https://www.khanacademy.org/science/physics/magnetic-forces-and-magnetic-fields/electric-motors/v/calculating-dot-and-cross-products-with-unit-vector-notation>

Challenge

1. Determine the angle between the two vectors $\mathbf{a} = [3, 0, 0]$ and $\mathbf{b} = [3, 1, 0]$ and use it to calculate $\mathbf{c} = \mathbf{a} \times \mathbf{b}$. Which direction does the vector \mathbf{c} point?
2. Determine the cross product $\mathbf{f} = \mathbf{d} \times \mathbf{e}$ where $\mathbf{d} = 4\hat{i} + 2\hat{j} + 1\hat{k}$ and $\mathbf{e} = -2\hat{i} - 4\hat{j} + 8\hat{k}$ without calculating the angle between them.

Solution

Please compare your answer with your partner and discuss in class if answers differ.

1.6 Rotation I

Resources

- Book section 4/4

Challenges

Calculate the angular momentum and the rate of change of angular momentum with time for Question 4/1.

Solutions

Given in book.

1.7 Rotation II

Resources

- Book section 4/4

Challenges

Question 4/16

If you have difficulty, consider doing question 4/15 first (optional).

Solutions

The required time should be 2.72 s

1.8 Rotation III

Resources

- Book section 4/4

Challenges

Question 4/2

Solutions

To check your answers substitute $d = 2$ metres, $m = 7$ kg, $v = 3$ m/s and $f = 7$ N into your final answers. You should obtain $\mathbf{H}_G = 432\hat{i} + 144\hat{j} + 168\hat{k}$ kgm²/s and $\dot{\mathbf{H}}_G = -8\hat{i} - 12\hat{j} + 0\hat{k}$ Nm

1.9 Conservation of momentum

Resources

- Book section 4/5

Challenges

1. In Question 4/17, at what point does the vehicle stop accelerating?
2. Solve Question 4/17
3. Question 4/18

Solutions

1. Please write your answer and compare with your partner in class
2. Given in book
3. 0.21 m/s

1.10 Conservation of momentum vs energy

Resources

- Book section 4/5

Challenges

1. Solve Question 4/19
2. Why is energy not conserved here? Where did the energy go? Under what conditions is momentum conserved, and under what conditions is energy conserved?

Solutions

1. Given in book
2. Please write your answers and compare with your partner in class.

1.11 Combined problems I

Resources

- Book section 4/1 to 4/5

Challenge

Solve Question 4/22.

The question states that an impulse is imparted “over a negligibly short period of time” which is a little confusing since impulse is the integration of force over time which becomes zero as time goes to zero. Instead, here you can consider that whatever the time is, the final product of Force and Time is 10 N s .

Solution

4.7 m/s

1.12 Combined problems II

Resources

- Book section 4/1 to 4/5

Challenge

Solve Question 4/28

Solutions

You should obtain an algebraic expression for v and $\dot{\theta}$. To check your expression, you can substitute the following values into the expression: $m_0 = 1 \text{ kg}$, $v_0 = 1000 \text{ m s}^{-1}$, $b = 1.5 \text{ m}$ and $m = 4 \text{ kg}$, whereby you should obtain $v = 111 \text{ m s}^{-1}$ and $\dot{\theta} = 222 \text{ rad s}^{-1}$.

1.13 In-plane flow

Resources

- Book section 4/6

Challenge

Derive equation 4/19 in the book from equation 4/19a.

1.14 Force on vane

Resources

- Book section 4/6

Challenge

Show your working for sample problem 4/5 (a) and (b)

1.15 Power and a vane

Resources

- Book section 4/6

Challenge

Considering sample problem 4/6,

1. Explain in words what is meant by “power by action of the fluid”.
2. The power is defined here by measuring the force applied to move an object at a constant velocity. If force creates acceleration ($F = ma$), how can the velocity be constant?
3. Work through and solve the sample problem.

Solutions

Please compare your solutions with your partner. You may be asked to present your solutions to the class.

1.16 Balancing forces: Jet aeroplane example

Resources

- Book section 4/6

Challenge

Work through sample problem 4/8 to obtain the equation of motion of the system as given in the book ($m'_g u - m'_a v = mg \sin \theta + D$).

1.17 Balancing forces: Jet aeroplane

Resources

- Book section 4/6

Challenge

Answer question 4/33.

Solution

Given in book.

1.18 Balancing forces: Fire tug

Resources

- Book section 4/6

Challenge

Answer question 4/37.

Solution

Given in book.

1.19 Balancing ball on a water stream

Resources

- Book section 4/6

Challenge

Answer question 4/42. Take note about the conservation of energy in the jet stream, and the fact that the jet stream remains intact. You can assume that the water stream is fully deflected horizontally when it hits the ball.

Solution

4.8 m

1.20 Pressure I

Resources

- Book section 4/6

Challenge



A typical die has a side length of 1.4 cm and weighs 2.8 g. Consider the die at rest on a desk. Estimate the pressure on the bottom of the die due to the desk.

Solution

140 Pa

1.21 Pressure II

Resources

- Book section 4/6

Challenge

Answer question 4/50

Solution

1035 Pa

1.22 Power and a Helicopter

Resources

- Book section 4/6

Challenge

Answer question 4/59

Solutions

Given in book

1.23 Mass ejection

Resources

- Book section 4/7

Challenge

Consider rocket thrust where exhaust is emitted at a speed of 220 m s^{-1} . The force on the rocket due to the thrust alone is 400 N . Calculate (a) the mass flow rate m' and (b) the time-rate increase of the mass of the rocket \dot{m} .

Solutions

- (a) MD5(a_X) = 300026... kg s^{-1}
(b) MD5(b_X) = a2fb8f... kg s^{-1}

1.24 Rocket sample problem

Resources

- Book section 4/7

Challenge

Complete the sample problem 4/11 using both solution I and II. Please be sure to follow the logic.

1.25 Rocket-style problem I

Resources

- Book section 4/7

Challenge

Answer question 4/67

Solution

Given in book.

1.26 Rocket-style problem II

Resources

- Book section 4/7

Challenge

Answer question 4/82

Solution

4.8 m s^{-1}

1.27 Mass intake and power

Resources

- Book section 4/7

Challenge

Answer question 4/80

Solution

1.6 m s^{-2} deceleration

1.28 Chain style sample problem

Resources

- Book section 4/7

Challenge

Work through sample problem 4/9

1.29 Rope style sample problem

Resources

- Book section 4/7

Challenge

Work through sample problem 4/10

1.30 Chain vs Rope style sample problem difference

Resources

- Book section 4/7

Challenge

Considering the chain sample problem and the unconstrained rope problem, why was the kinetic energy different in these two cases? What assumptions were made in the chain problem compared to the unconstrained rope problem, and how did this impact the calculation of kinetic energy? How might the kinetic energy change if the rope in sample problem 4/10 was made of a combination of small loops like the chain in sample problem 4/9?

Please write a few sentences summarising your understanding.

Solution

Please compare your writing with your partner's writing and discuss any differences.

1.31 Constrained and unconstrained rope style sample problem

Resources

- Book section 4/7

Challenge

Considering the unconstrained and constrained rope sample problem, how is the approach different? How do the different values for “P” and “R” arise? What assumptions are different?

Please write a few sentences summarising your understanding.

Solution

Please compare your writing with your partner’s writing and discuss any differences.

1.32 Lifting a chain

Resources

- Book section 4/7

Challenge

An 18K gold chain has a mass of 1.12 g and a length of 40 cm. You pick up one end of the chain and lift it up vertically at a constant velocity. There will be two downward forces present: one due to the hanging weight of the chain due to earth's gravity (A), and another induced by the constant addition of mass to the hanging part of the chain (B). If the chain is lifted up at 5 cm s^{-1} , calculate B . State what simplifying assumptions you make. How does this compare to A ?

Solution

B : $7 \mu\text{N}$

1.33 Chain on a pully

Resources

- Book section 4/7

Challenge

Answer question 4/83

Solution

P is given in book, and R should match your understanding of the weight of the pile.

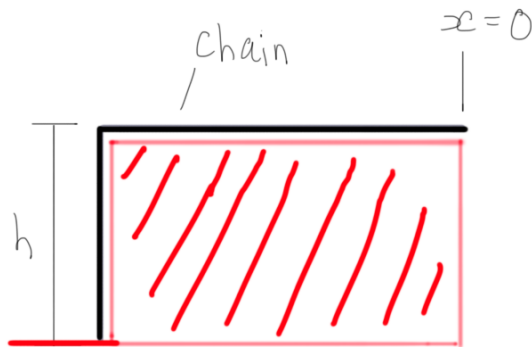
1.34 An accelerating chain

Resources

- Book section 4/7

Challenge

Consider a chain hanging over the edge of a block of height h . The chain has a total length L and mass per unit length of ρ . The left end of the chain is barely touching the ground. Ignoring friction at the corner and making other idealised assumptions, obtain an expression for the acceleration of the chain as a function of the position of the end of the chain (x) as it slides along the horizontal surface.



Solution

To check your answer, determine the acceleration for a height of 5 m and chain length of 10 m, when the chain has slid 2 m. You should obtain an acceleration of 6.13 m s^{-2}