

Mechanics

Spring semester 2018

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<http://raw.githubusercontent.com/NanoScaleDesign/Mechanics/master/mechanics.pdf>

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Contents

0	Course information	5
0.1	This course	6
0.1.1	What you need to do	6
0.1.2	How this works	6
0.1.3	Assessment	7
0.2	Peerwise	8
0.3	Timetable	9
0.4	Hash-generation	10
1	Hash practise	11
1.1	Hash practise: Integer	12
1.2	Hash practise: Decimal	12
1.3	Hash practise: String	12
1.4	Hash practise: Scientific form	12
2	Kinetics of systems of particles	13
2.1	System centre-of-mass position, mass and velocity: I	14
2.2	System centre-of-mass position, mass and velocity: II	16
2.3	System centre-of-mass position, mass and velocity: III	17
2.4	Kinetic and potential energy	18
2.5	Cross-product	19
2.6	Rotation of particles I	20
2.7	Rotation of particles II	21
2.8	Rotation of particles III	22
2.9	Rotation of particles IV	23
2.10	Energy and Rotation I	24
2.11	Energy and Rotation II	25
2.12	Mass ejection	26
2.13	Rocket sample problem	27
2.14	Rocket-style problem I	28
2.15	Rocket-style problem II	29
2.16	Mass intake I	30
2.17	Mass intake II	31
2.18	Chain style sample problem	32
2.19	Rope style sample problem	33
2.20	Chain vs Rope style sample problem difference	34
2.21	Constrained and unconstrained rope style sample problem	35
2.22	Lifting a chain	36
2.23	Chain on a pulley	37
2.24	An accelerating chain	38
2.25	Making a question	39
2.26	Answering Peerwise question(s)	40
3	3D dynamics of rigid bodies	41

3.1	Radial velocity with horizontal connection	42
3.2	Radial velocity with non-horizontal connection	43
3.3	Linear acceleration	45
3.4	Radial acceleration - increase in magnitude of radial velocity	47
3.5	Radial acceleration - only direction (precession)	49
3.6	Radial acceleration II	50
3.7	Unit vector of a rotation axis	51
3.8	Simultaneous rotation I	52
3.9	Simultaneous rotation II	53
3.10	Simultaneous rotation III	54
3.11	Time-dependent rotation of vectors I	55
3.12	Time-dependent rotation of vectors II	56
3.13	Relative velocity	57
3.14	Crank-style problem	58
3.15	Perpendicular position, velocity and rotation vectors	59
3.16	Perpendicular double cross-product	60
3.17	3D acceleration	61
3.18	3D velocity calculation	62
3.19	3D velocity and acceleration calculation	63

Chapter 0

Course information

0.1 This course

This is the Spring 2018 Mechanics course studied by 2nd-year undergraduate international students at Kyushu University.

0.1.1 What you need to do

- Borrow the book “Engineering Mechanics: Dynamics”, 6th edition, by Meriam and Kraige from the Mechanical Engineering office on the 4th floor of West 4. The course will be based on that book and you will need to refer to it in class.
- Prepare a challenge-log in the form of a workbook or folder where you can clearly write the calculations you perform to solve each challenge. This will be used in the final assessment and will be occasionally reviewed by the teacher.
- Submit a weekly feedback form by **9am on Monday** before class at <https://goo.gl/forms/ULjtgmL0Q2fZbG292>.
- Please bring a wifi-capable internet device to class, as well as headphones if you need to access online components of the course during class. If you let me know in advance, I can lend computers and provide power extension cables for those who require them (limited number).

0.1.2 How this works

- This booklet forms part of an active-learning segment in the course. The learning is self-directed in contrast to the traditional lecture-style model.
- Learning is guided through solving a series of challenges combined with instant feedback about the correctness of your answer.
- There are no lectures. Instead, there is discussion time. Here, you are encouraged to discuss any issues with your peers, teacher and any teaching assistants. Furthermore, you are encouraged to help your peers who are having trouble understanding something that you have understood; by doing so you actually increase your own understanding too.
- Discussion-time is from 13:00 to 14:30 on Mondays at room W4-766.
- Peer discussion is encouraged, however, if you have help to solve a challenge, always make sure you do understand the details yourself. You will need to be able to do this in an exam environment. The questions on the exam will be similar in nature to the challenges.
- Every challenge in the book typically contains a **Challenge** with suggested **Resources** which you are recommended to utilise in order to solve the challenge. Occasionally the teacher will provide extra **Comments** to help guide your thinking. A **Solution** is also made available for you to check your answer. Sometimes this solution will be given in encrypted form. For more information about encryption, see section 0.4.
- For deep understanding, it is recommended to study the suggested resources beyond the minimum required to complete the challenge.
- The challenge document has many pages and is continuously being developed. Therefore it is advised to view the document on an electronic device rather than print it. The date on the front page denotes the version of the document. You will be notified by email when the document is updated. The content may differ from last-year’s document.
- A target challenge will be set each week. This will set the pace of the course and defines the examinable material. It’s ok if you can’t quite reach the target challenge for a given week, but you should be careful not to fall behind, since the exam cannot be delayed.

- You may work ahead, even beyond the target challenge, if you so wish. This can build greater flexibility into your personal schedule, especially as you become busier towards the end of the semester.
- Your contributions to the course are strongly welcomed. If you come across resources that you found useful that were not listed by the teacher or points of friction that made solving a challenge difficult, please let the teacher know about it!

0.1.3 Assessment

In order to prove to outside parties that you have learned something from the course, we must perform summative assessments. You will receive a weighted score based on:

- Challenge-log (10%) - final state at the end of the course, showing your calculations for all the challenges in the course.
- Coursework (20%)
- Mid-term exam (30%)
- Final exam (40%)

Final score = MAX(Weighted score, Final exam score)

0.2 Peerwise

Part of this course will involve your creation of questions using the PeerWise platform. To register, go to http://peerwise.cs.auckland.ac.nz/at/?kyushu_u_jp and register. Access the course “Advanced Mechanics (Spring 2018)” and enter the following information:

- Course ID = **17143**
- Identifier = **Your student ID**

Once you have logged in, select the “Advanced Mechanics (Spring 2018)” course, then under “Your questions” click “view”. You can then click “Create new question” or view other questions by clicking “view” under “Unanswered questions”.

There is more detailed information about usage here: <https://peerwise.cs.auckland.ac.nz/docs/students/>.

0.3 Timetable

	Discussion	Target	Note
1	9 April	-	
2	16 April	2.4	
3	23 April	2.10	
4	7 May	2.14	
5	14 May	2.21	
6	21 May	2.26	
7	28 May		
8	11 June	-	Mid-term exam
9	18 June	-	
10	25 June		
11	2 July		
12	9 July		Coursework information
13	12 July		
14	23 July	-	Coursework submission
-	August	-	Final exam
-	September	-	Retake exam

0.4 Hash-generation

Some solutions to challenges are encrypted using MD5 hashes. In order to check your solution, you need to generate its MD5 hash and compare it to that provided. MD5 hashes can be generated at the following sites:

- Wolfram alpha: (For example: md5 hash of “q1.00”) <http://www.wolframalpha.com/input/?i=md5+hash+of+%22q1.00%22>
- www.md5hashgenerator.com

Since MD5 hashes are very sensitive to even single-digit variation, you must enter the solution *exactly*. This means maintaining a sufficient level of accuracy when developing your solution, and then entering the solution according to the format suggested by the question. Some special input methods:

Solution	Input
5×10^{-476}	5.00e-476
5.0009×10^{-476}	5.00e-476
$-\infty$	-infinity (never “infinite”)
2π	6.28
i	im(1)
2i	im(2)
$1 + 2i$	re(1)im(2)
-0.0002548 i	im(-2.55e-4)
$1/i = i/-1 = -i$	im(-1)
$e^{i2\pi} [= \cos(2\pi) + i\sin(2\pi) = 1 + i0 = 1]$	1.00
$e^{i\pi/3} [= \cos(\pi/3) + i\sin(\pi/3) = 0.5 + i0.87]$	re(0.50)im(0.87)
Choices in order A, B, C, D	abcd

The first 6 digits of the MD5 sum should match the first 6 digits of the given solution.

Chapter 1

Hash practise

1.1 Hash practise: Integer

$X = 46.3847$

Form: Integer.

Place the indicated letter in front of the number.

Example: aX where $X = 46$ is entered as a46

hash of aX = e77fac

1.2 Hash practise: Decimal

$X = 49$

Form: Two decimal places.

Place the indicated letter in front of the number.

Example: aX where $X = 46.00$ is entered as a46.00

hash of bX = 82c9e7

1.3 Hash practise: String

$X = abcdef$

Form: String.

Place the indicated letter in front of the number.

Example: aX where $X = abc$ is entered as aabc

hash of cX = 990ba0

1.4 Hash practise: Scientific form

$X = 500,765.99$

Form: Scientific notation with the mantissa in standard form to 2 decimal place and the exponent in integer form.

Place the indicated letter in front of the number.

Example: aX where $X = 4 \times 10^{-3}$ is entered as a4.00e-3

hash of dX = be8a0d

Chapter 2

Kinetics of systems of particles

2.1 System centre-of-mass position, mass and velocity: I

Resources

- Book sections 4/1 to 4/2

Comment

You may use the following text as an alternative explanation to that in the book.

Introduction

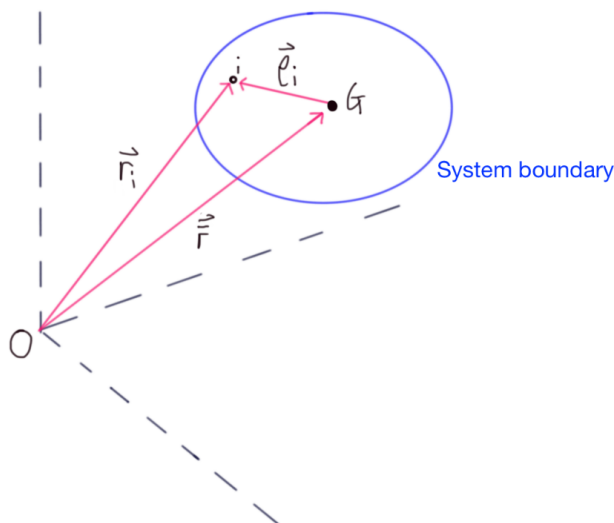
You are already largely familiar with the kinetics of a single particle. The idea of this short course is to extend these principles to the motion of a general system of particles. This will enable you to describe the motion of both rigid and non-rigid bodies and systems.

A rigid body is defined as a solid system of particles where the distance between the particles does not change with respect to time. Examples of rigid-bodies are numerous, including many machines operating in the air, on land or under the sea, including rockets and spacecraft. A non-rigid body can come in two forms:

- A single solid body that changes shape over time, perhaps due to elastic or non-elastic deformations.
- A defined mass of liquid or gaseous particles flowing at a given rate. Important examples include air and fuel flowing through the turbine of an aircraft engine, the exhaust from the nozzle of a rocket, or water passing through a pump.

Generalising Newton's 2nd law

Here we extend Newton's second law of motion to cover a general mass system, modelled by considering n particles each with mass m_i bounded by a closed surface in space:



The centre-of-mass of the system is denoted by the point G and the distance of particle i from the centre-of-mass is ρ_i . The distance of particle i from an arbitrary non-accelerating origin O of a Newtonian set of reference axes is r_i and the distance of the centre-of-mass from this origin is \bar{r} . Note that point G does not have to correspond to the position of any one particle, and may just represent the position of the centre-of-mass of the particles at that particular instant.

The total mass of the system m is of course just sum of all the particle masses m_i :

$$m = \sum m_i \quad (2.1)$$

so that the centre-of-mass G of the system can be determined from the position of all particles:

$$m\bar{\mathbf{r}} = \sum m_i \mathbf{r}_i \quad (2.2)$$

Each particular in the system is subject to forces. It is useful to group these forces into two contributions:

- Sources internal to the system boundary (ie, other particles). For example, the force on particle i by particle j .
- Sources external to the system boundary (eg, electric fields, gravity, etc).

One can consider the total force acting on the system as a whole. Note that the force on on particle i by particle j is equal and opposite to the force on particle j by particle i . Thus the sum of forces from internal forces must all cancel out to be zero. If there are multiple external forces \mathbf{F}_i on the system, then the total force on the system must be $\sum \mathbf{F}$. Thus the equation of motion for the system can be written as

$$\sum \mathbf{F} = m\ddot{\bar{\mathbf{r}}} = m\bar{\mathbf{a}} \quad (2.3)$$

where $\bar{\mathbf{a}}$ corresponds to the acceleration of the centre-of-mass G .

Equation 2.3 is the generalised form of Newton's second law of motion. Each component in a Cartesian system may be treated independently (eg, $\sum F_x = m\bar{a}_x$ etc). The resultant external force $\sum \mathbf{F}$ will have the same direction as the acceleration of the system $\bar{\mathbf{a}}$ at that instant of time, however the resultant external force does not necessarily pass through the centre-of-mass (usually it does not).

Challenge

1. $\bar{\mathbf{r}}$, $\dot{\bar{\mathbf{r}}}$ and $\ddot{\bar{\mathbf{r}}}$ of Question 4/1.
2. Question 4/4.

Solution

1. Given in book.
2. 534 N

2.2 System centre-of-mass position, mass and velocity: II

Resources

- Book sections 4/1 to 4/2

Challenge

Question 4/5. Determine the *magnitude* of the acceleration.

Solution

4 m/s^2

2.3 System centre-of-mass position, mass and velocity: III

Resources

- Book sections 4/1 to 4/2

Challenge

Question 4/13

Solution

4.2 m/s²

2.4 Kinetic and potential energy

Resources

- Book section 4/3

Challenge

1. Calculate T in question 4/1
2. Question 4/10

Solution

1. Given in book.
2. To check your final answer, substitute $b = 2$ metres into your final answer. You should obtain 5.27 m/s .

2.5 Cross-product

Resources

- <https://www.khanacademy.org/science/physics/magnetic-forces-and-magnetic-fields/electric-motors/v/calculating-dot-and-cross-products-with-unit-vector-notation>

Challenge

1. Determine the angle between the two vectors $\mathbf{a} = [3, 0, 0]$ and $\mathbf{b} = [3, 1, 0]$ and use it to calculate $\mathbf{c} = \mathbf{a} \times \mathbf{b}$. Which direction does the vector \mathbf{c} point?
2. Determine the cross product $\mathbf{f} = \mathbf{d} \times \mathbf{e}$ where $\mathbf{d} = 4\hat{i} + 2\hat{j} + 1\hat{k}$ and $\mathbf{e} = -2\hat{i} - 4\hat{j} + 8\hat{k}$ without calculating the angle between them.

Solution

Please compare your answer with your partner and discuss in class if answers differ.

2.6 Rotation of particles I

Resources

- Book section 4/4

Challenges

Calculate the angular momentum and the rate of change of angular momentum with time for Question 4/1.

Solutions

Given in book.

2.7 Rotation of particles II

Resources

- Book section 4/4

Challenges

Question 4/15

Solutions

Given in book.

2.8 Rotation of particles III

Resources

- Book section 4/4

Challenges

Question 4/16

Solutions

The required time should be 2.72 s

2.9 Rotation of particles IV

Resources

- Book section 4/4

Challenges

Question 4/2

Solutions

To check your answers substitute $d = 2$ metres, $m = 7$ kg, $v = 3$ m/s and $f = 7$ N into your final answers. You should obtain $\mathbf{H}_G = 432\hat{i} + 144\hat{j} + 168\hat{k}$ kgm²/s and $\dot{\mathbf{H}}_G = -8\hat{i} - 12\hat{j} + 0\hat{k}$ Nm

2.10 Energy and Rotation I

Resources

- Book section 4/1 to 4/5

Challenge

Solve Question 4/22.

Solution

4.7 m/s

2.11 Energy and Rotation II

Resources

- Book section 4/1 to 4/5

Challenge

Solve Question 4/28

Solutions

You should obtain an algebraic expression for v and $\dot{\theta}$. To check your expression, you can substitute the following values into the expression: $m_0 = 1 \text{ kg}$, $v_0 = 1000 \text{ m s}^{-1}$, $b = 1.5 \text{ m}$ and $m = 4 \text{ kg}$, whereby you should obtain $v = 111 \text{ m s}^{-1}$ and $\dot{\theta} = 222 \text{ rad s}^{-1}$.

2.12 Mass ejection

Resources

- Book section 4/7

Challenge

Consider rocket thrust where exhaust is emitted at a speed of 220 ms^{-1} . The force on the rocket due to the thrust alone is 400 N . Calculate (a) the mass flow rate m' and (b) the time-rate increase of the mass of the rocket \dot{m} .

Solutions

(a)

X = Your solution

Form: Decimal to 2 decimal places.

Place the indicated letter in front of the number.

Example: aX where $X = 46.00$ is entered as a46.00

Hash of aX = b54e89 (kg s^{-1})

(b)

X = Your solution

Form: Decimal to 2 decimal places.

Place the indicated letter in front of the number.

Example: aX where $X = 46.00$ is entered as a46.00

Hash of bX = 6a83d8 (kg s^{-1})

2.13 Rocket sample problem

Resources

- Book section 4/7

Challenge

Complete the sample problem 4/11 using both solution I and II. Please be sure to follow the logic and understand the link between the two methods.

2.14 Rocket-style problem I

Resources

- Book section 4/7

Challenge

Answer question 4/67

Solution

Given in book.

2.15 Rocket-style problem II

Resources

- Book section 4/7

Challenge

Answer question 4/82

Solution

4.8 m s^{-1}

2.16 Mass intake I

Resources

- Book section 4/7

Challenge

Answer question 4/80

Solution

1.6 m s^{-2} deceleration

2.17 Mass intake II

Resources

- Book section 4/7

Challenge

Answer question 4/76

Solution

0.152 m s^{-2}

2.18 Chain style sample problem

Resources

- Book section 4/7

Challenge

Work through sample problem 4/9

2.19 Rope style sample problem

Resources

- Book section 4/7

Challenge

Work through sample problem 4/10

2.20 Chain vs Rope style sample problem difference

Resources

- Book section 4/7

Challenge

Considering the chain sample problem and the unconstrained rope problem, why was the kinetic energy different in these two cases? What assumptions were made in the chain problem compared to the unconstrained rope problem, and how did this impact the calculation of kinetic energy?

Please write a few sentences summarising your understanding.

Solution

Please compare your writing with your partner's writing and discuss any differences.

2.21 Constrained and unconstrained rope style sample problem

Resources

- Book section 4/7

Challenge

Considering the unconstrained and constrained rope sample problem, how are the approaches different? How do the different values for “P” and “R” arise? What assumptions are different?

Please write a few sentences summarising your understanding.

Solution

Please compare your writing with your partner’s writing and discuss any differences.

2.22 Lifting a chain

Resources

- Book section 4/7

Challenge

An 18K gold chain has a mass of 1.12 g and a length of 40 cm. You pick up one end of the chain and lift it up vertically at a constant velocity. There will be two downward forces present: one due to the hanging weight of the chain due to earth's gravity (A), and another induced by the constant addition of mass to the hanging part of the chain (B). If the chain is lifted up at 5 cm s^{-1} , calculate B . State what simplifying assumptions you make. How does this compare to A ?

Solution

B : $7 \mu\text{N}$

2.23 Chain on a pully

Resources

- Book section 4/7

Challenge

Answer question 4/83

Solution

P is given in book, and R should match your understanding of the weight of the pile.

2.24 An accelerating chain

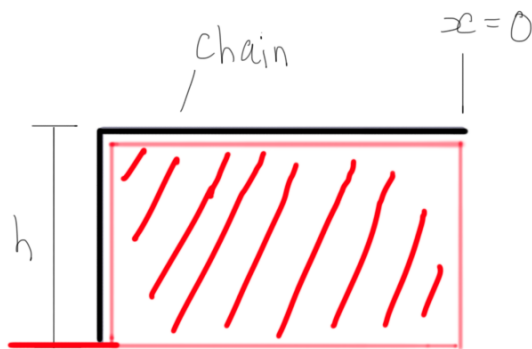
Resources

- Book section 4/7

Challenge

a) Consider a chain hanging over the edge of a block of height h . The chain has a total length L and mass per unit length of ρ . The left end of the chain is barely touching the ground. Ignoring friction at the corner and making other idealised assumptions, obtain an expression for the acceleration of the chain as a function of the position of the end of the chain (x) as it slides along the horizontal surface.

b) Did you need to consider the force generated by the stopping of the chain as it landed on the ground? If so, how did it come into the equation? If not, why not?



Solution

a) To check your answer, determine the acceleration for a height of 5 m and chain length of 10 m, when the chain has slid 2 m. You should obtain an acceleration of 6.13 m s^{-2} .

b) Please discuss your answer with your partner

2.25 Making a question

Coursework challenge

Resources

- Book section 4

Challenge

Create an original question concerning anything covered in this chapter using the Peerwise platform (see section 0.2 for details). If you create more than one question, the question awarded highest marks will be counted.

Your question must contain the following:

1. A diagram to support your question.
2. 5 possible answer options (A-E) that can be used by the student to check if their answer is correct. These may be numerical, mathematical, word-based or hashes.
3. It is important that it is not possible to derive the method from the answer-options that you give.
4. All the answer options should be plausible.
5. For at least one of your wrong-answer options, ensure that the answer corresponds to a typical mistake that a student might make when answering your question.
6. Provide a detailed solution to your question, including explanation and mathematics. *Demonstrate your own thinking and understanding.*
7. Related to item 5, explain why the student might have chosen that wrong answer, and explain where they may have gone wrong in their thinking.

2.26 Answering Peerwise question(s)

Resources

- Book section 4

Challenge

Choose at least one question from Peerwise and attempt to answer it. Explain your reasoning (don't just write mathematics). Clearly write an alternative explanation or possible improvement to the question you answered.

Solution

Please discuss in class.