Mechanics

Spring semester 2018

Last updated:
23rd July 2018 at 12:38

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http://www.jamescannon.net/teaching/mechanics

http://raw.githubusercontent.com/NanoScaleDesign/Mechanics/master/mechanics.pdf

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Chapter 0

Course information

0.1 This course

This is the Spring 2018 Mechanics course studied by 2nd-year undergraduate international students at Kyushu University.

0.1.1 What you need to do

- Borrow the book "Engineering Mechanics: Dynamics", 6th edition, by Meriam and Kraige from the Mechanical Engineering office on the 4th floor of West 4. The course will be based on that book and you will need to refer to it in class.
- Prepare a challenge-log in the form of a workbook or folder where you can clearly write the calculations you perform to solve each challenge. This will be used in the final assessment and will be occasionally reviewed by the teacher.
- Submit a weekly feedback form by **9am on Monday** before class at `https://goo.gl/forms/ULjtgmL0Q2fZbG292`.
- Please bring a wifi-capable internet device to class, as well as headphones if you need to access online components of the course during class. If you let me know in advance, I can lend computers and provide power extension cables for those who require them (limited number).

0.1.2 How this works

- This booklet forms part of an active-learning segment in the course. The learning is self-directed in contrast to the traditional lecture-style model.
- Learning is guided through solving a series of challenges combined with instant feedback about the correctness of your answer.
- There are no lectures. Instead, there is discussion time. Here, you are encouraged to discuss any issues with your peers, teacher and any teaching assistants. Furthermore, you are encouraged to help your peers who are having trouble understanding something that you have understood; by doing so you actually increase your own understanding too.
- Discussion-time is from 13:00 to 14:30 on Mondays at room W4-766.
- Peer discussion is encouraged, however, if you have help to solve a challenge, always make sure you do understand the details yourself. You will need to be able to do this in an exam environment. The questions on the exam will be similar in nature to the challenges.
- Every challenge in the book typically contains a **Challenge** with suggested **Resources** which you are recommended to utilise in order to solve the challenge. Occasionally the teacher will provide extra **Comments** to help guide your thinking. A **Solution** is also made available for you to check your answer. Sometimes this solution will be given in encrypted form. For more information about encryption, see section 0.4.
- For deep understanding, it is recommended to study the suggested resources beyond the minimum required to complete the challenge.
- The challenge document has many pages and is continuously being developed. Therefore it is advised to view the document on an electronic device rather than print it. The date on the front page denotes the version of the document. You will be notified by email when the document is updated. The content may differ from last-year's document.
- A target challenge will be set each week. This will set the pace of the course and defines the examinable material. It's ok if you can't quite reach the target challenge for a given week, but you should be careful not to fall behind, since the exam cannot be delayed.

- You may work ahead, even beyond the target challenge, if you so wish. This can build greater flexibility into your personal schedule, especially as you become busier towards the end of the semester.
- Your contributions to the course are strongly welcomed. If you come across resources that you found useful that were not listed by the teacher or points of friction that made solving a challenge difficult, please let the teacher know about it!

0.1.3 Assessment

In order to prove to outside parties that you have learned something from the course, we must perform summative assessments. You will receive a weighted score based on:

- Challenge-log (10%) - final state at the end of the course, showing your calculations for all the challenges in the course.
- Coursework (20%)
- Mid-term exam (30%)
- Final exam (40%)

Final score = MAX(Weighted score, Final exam score)

Course evaluation (unofficial): `https://goo.gl/forms/YycJxFHOuuoBYCru1`

0.2 Peerwise

Part of this course will involve your creation of questions using the PeerWise platform. To register, go to `http://peerwise.cs.auckland.ac.nz/at/?kyushu_u_jp` and register. Access the course "Advanced Mechanics (Spring 2018)" and enter the following information:

- Course ID = **17143**
- Identifier = **Your student ID**

Once you have logged in, select the "Advanced Mechanics (Spring 2018)" course, then under "Your questions" click "view". You can then click "Create new question" or view other questions by clicking "view" under "Unanswered questions".

There is more detailed information about usage here: `https://peerwise.cs.auckland.ac.nz/docs/students/`.

0.3 Timetable

	Discussion	Target	Note
1	9 April	-	
2	16 April	2.4	
3	23 April	2.10	
4	7 May	2.14	
5	14 May	2.21	
6	21 May	2.26	
7	28 May	3.6	
8	11 June	-	Mid-term exam
9	18 June	-	First coursework due
10	25 June	3.12	
11	2 July	3.18	
12	9 July	3.28	
13	12 July	3.32	
14	23 July	3.34	Final coursework due
-	30 July	-	Final exam
-	September	-	Retake exam

0.4 Hash-generation

Some solutions to challenges are encrypted using MD5 hashes. In order to check your solution, you need to generate its MD5 hash and compare it to that provided. MD5 hashes can be generated at the following sites:

- Wolfram alpha: (For example: md5 hash of "q1.00") `http://www.wolframalpha.com/input/?i=md5+hash+of+%22q1.00%22`
- `www.md5hashgenerator.com`

Since MD5 hashes are very sensitive to even single-digit variation, you must enter the solution *exactly*. This means maintaining a sufficient level of accuracy when developing your solution, and then entering the solution according to the format suggested by the question. Some special input methods:

Solution	**Input**
5×10^{-476}	5.00e-476
5.0009×10^{-476}	5.00e-476
$-\infty$	-infinity (never "infinite")
2π	6.28
i	im(1)
2i	im(2)
1 + 2i	re(1)im(2)
-0.0002548 i	im(-2.55e-4)
1/i = i/-1 = -i	im(-1)
$e^{i2\pi}$ $[= cos(2\pi) + isin(2\pi) = 1 + i0 = 1]$	1.00
$e^{i\pi/3}$ $[= cos(\pi/3) + isin(\pi/3) = 0.5 + i0.87]$	re(0.50)im(0.87)
Choices in order A, B, C, D	abcd

The first 6 digits of the MD5 sum should match the first 6 digits of the given solution.

Chapter 1

Hash practise

1.1 Hash practise: Integer

X = 46.3847
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

hash of aX = e77fac

1.2 Hash practise: Decimal

X = 49
Form: Two decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

hash of bX = 82c9e7

1.3 Hash practise: String

X = abcdef
Form: String.
Place the indicated letter in front of the number.
Example: aX where $X = abc$ is entered as aabc

hash of cX = 990ba0

1.4 Hash practise: Scientific form

X = 500,765.99
Form: Scientific notation with the mantissa in standard form to 2 decimal place and the exponent in integer form.
Place the indicated letter in front of the number.
Example: aX where $X = 4 \times 10^{-3}$ is entered as a4.00e-3

hash of dX = be8a0d

Chapter 2

Kinetics of systems of particles

2.1 System centre-of-mass position, mass and velocity: I

Resources

- Book sections 4/1 to 4/2

Comment

You may use the following text as an alternative explanation to that in the book.

Introduction

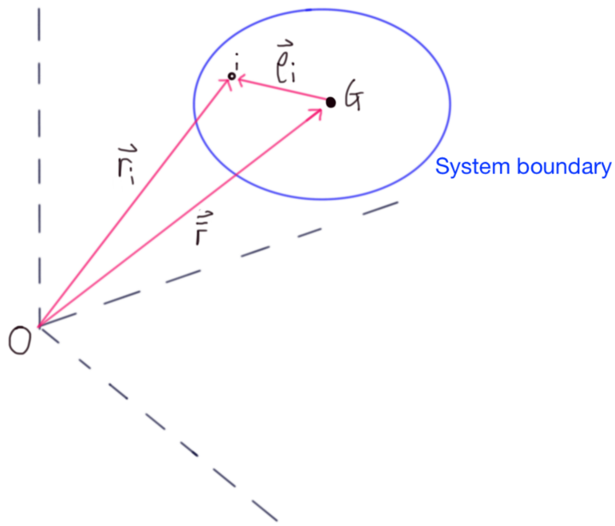
You are already largely familiar with the kinetics of a single particle. The idea of this short course is to extend these principles to the motion of a general system of particles. This will enable you to describe the motion of both rigid and non-rigid bodies and systems.

A rigid body is defined as a solid system of particles where the distance between the particles does not change with respect to time. Examples of rigid-bodies are numerous, including many machines operating in the air, on land or under the sea, including rockets and spacecraft. A non-rigid body can come in two forms:

- A single solid body that changes shape over time, perhaps due to elastic or non-elastic deformations.
- A defined mass of liquid or gaseous particles flowing at a given rate. Important examples include air and fuel flowing through the turbine of an aircraft engine, the exhaust from the nozzle of a rocket, or water passing through a pump.

Generalising Newton's 2nd law

Here we extend Newton's second law of motion to cover a general mass system, modelled by considering n particles each with mass m_i bounded by a closed surface in space:



The centre-of-mass of the system is denoted by the point G and the distance of particle i from the centre-of-mass is $\boldsymbol{\rho_i}$. The distance of particle i from an arbitrary non-accelerating origin O of a Newtonian set of reference axes is $\boldsymbol{r_i}$ and the distance of the centre-of-mass from this origin is $\bar{\boldsymbol{r}}$. Note that point G does not have to correspond to the position of any one particle, and may just represent the position of the centre-of-mass of the particles at that particular instant.

The total mass of the system m is of course just sum of all the particle masses m_i:

$$m = \sum m_i \tag{2.1}$$

so that the centre-of-mass G of the system can be determined from the position of all particles:

$$m\bar{\boldsymbol{r}} = \sum m_i \boldsymbol{r}_i \tag{2.2}$$

Each particular in the system is subject to forces. It is useful to group these forces into two contributions:

- Sources internal to the system boundary (ie, other particles). For example, the force on particle i by particle j.
- Sources external to the system boundary (eg, electric fields, gravity, etc).

One can consider the total force acting on the system as a whole. Note that the force on on particle i by particle j is equal and opposite to the force on particle j by particle i. Thus the sum of forces from internal forces must all cancel out to be zero. If there are multiple external forces $\boldsymbol{F}_i$ on the system, then the total force on the system must be $\sum \boldsymbol{F}$. Thus the equation of motion for the system can be written as

$$\sum \boldsymbol{F} = m\ddot{\bar{\boldsymbol{r}}} = m\bar{\boldsymbol{a}} \tag{2.3}$$

where $\bar{\boldsymbol{a}}$ corresponds to the acceleration of the centre-of-mass G.

Equation 2.3 is the generalised form of Newton's second law of motion. Each component in a Cartesian system may be treated independently (eg, $\sum F_x = m\bar{a}_x$ etc). The resultant external force $\sum \boldsymbol{F}$ will have the same direction as the acceleration of the system $\bar{\boldsymbol{a}}$ at that instant of time, however the resultant external force does not necessarily pass through the centre-of-mass (usually it does not).

Challenge

1. $\bar{\boldsymbol{r}}$, $\dot{\bar{\boldsymbol{r}}}$ and $\ddot{\bar{\boldsymbol{r}}}$ of Question 4/1.
2. Question 4/4.

Solution

1. Given in book.
2. 534 N

2.2 System centre-of-mass position, mass and velocity: II

Resources

- Book sections 4/1 to 4/2

Challenge

Question 4/5. Determine the *magnitude* of the acceleration.

Solution

$4\,\mathrm{m/s^2}$

2.3 System centre-of-mass position, mass and velocity: III

Resources

- Book sections 4/1 to 4/2

Challenge

Question 4/13

Solution

$4.2\,\mathrm{m/s^2}$

2.4 Kinetic and potential energy

Resources

- Book section 4/3

Challenge

1. Calculate T in question 4/1
2. Question 4/10

Solution

1. Given in book.
2. To check your final answer, substitute b = 2 metres into your final answer. You should obtain 5.27 m/s.

2.5 Cross-product

Resources

- https://www.khanacademy.org/science/physics/magnetic-forces-and-magnetic-fields/electric-motors/v/calculating-dot-and-cross-products-with-unit-vector-notation

Challenge

1. Determine the angle between the two vectors $\boldsymbol{a} = [3, 0, 0]$ and $\boldsymbol{b} = [3, 1, 0]$ and use it to calculate $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$. Which direction does the vector $\boldsymbol{c}$ point?

2. Determine the cross product $\boldsymbol{f} = \boldsymbol{d} \times \boldsymbol{e}$ where $\boldsymbol{d} = 4\hat{i} + 2\hat{j} + 1\hat{k}$ and $\boldsymbol{e} = -2\hat{i} - 4\hat{j} + 8\hat{k}$ without calculating the angle between them.

Solution

Please compare your answer with your partner and discuss in class if answers differ.

2.6 Rotation of particles I

Resources

- Book section 4/4

Challenges

Calculate the angular momentum and the rate of change of angular momentum with time for Question 4/1.

Solutions

Given in book.

2.7 Rotation of particles II

Resources

- Book section 4/4

Challenges

Question 4/15

Solutions

Given in book.

2.8 Rotation of particles III

Resources

- Book section 4/4

Challenges

Question 4/16

Solutions

The required time should be $2.72\,\mathrm{s}$

2.9 Rotation of particles IV

Resources

- Book section 4/4

Challenges

Question 4/2

Solutions

To check your answers substitute $d = 2$ metres, $m = 7$ kg, $v = 3$ m/s and $f = 7$ N into your final answers. You should obtain $\boldsymbol{H}_G = 432\hat{\imath} + 144\hat{\jmath} + 168\hat{k}$ kgm^2/s and $\dot{\boldsymbol{H}}_G = -8\hat{\imath} - 12\hat{\jmath} + 0\hat{k}$ Nm

2.10 Energy and Rotation I

Resources

- Book section 4/1 to 4/5

Challenge

Solve Question 4/22.

Solution

4.7 m/s

2.11 Energy and Rotation II

Resources

- Book section 4/1 to 4/5

Challenge

Solve Question 4/28

Solutions

You should obtain an algebraic expression for v and $\dot{\theta}$. To check your expression, you can substitute the following values into the expression: $m_0 = 1\,\mathrm{kg}$, $v_0 = 1000\,\mathrm{m\,s^{-1}}$, $b = 1.5\,\mathrm{m}$ and $m = 4\,\mathrm{kg}$, whereby you should obtain $v = 111\,\mathrm{m\,s^{-1}}$ and $\dot{\theta} = 222\,\mathrm{rad\,s^{-1}}$.

2.12 Mass ejection

Resources

- Book section 4/7

Challenge

Consider rocket thrust where exhaust is emitted at a speed of $220\,\mathrm{m\,s^{-1}}$. The force on the rocket due to the thrust alone is $400\,\mathrm{N}$. Calculate (a) the mass flow rate m' and (b) the time-rate increase of the mass of the rocket $\dot{m}$.

Solutions

(a)
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of aX = b54e89 ($\mathrm{kg\,s^{-1}}$)

(b)
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of bX = 6a83d8 ($\mathrm{kg\,s^{-1}}$)

2.13 Rocket sample problem

Resources

- Book section 4/7

Challenge

Complete the sample problem 4/11 using both solution I and II. Please be sure to follow the logic and understand the link between the two methods.

2.14 Rocket-style problem I

Resources

- Book section 4/7

Challenge

Answer question 4/67

Solution

Given in book.

2.15 Rocket-style problem II

Resources

- Book section 4/7

Challenge

Answer question 4/82

Solution

$4.8\,\mathrm{m\,s^{-1}}$

2.16 Mass intake I

Resources

- Book section 4/7

Challenge

Answer question 4/80

Solution

$1.6\,\mathrm{m\,s^{-2}}$ deceleration

2.17 Mass intake II

Resources

- Book section 4/7

Challenge

Answer question 4/76

Solution

$0.152\,\mathrm{m\,s^{-2}}$

2.18 Chain style sample problem

Resources

- Book section 4/7

Challenge

Work through sample problem 4/9

2.19 Rope style sample problem

Resources

- Book section 4/7

Challenge

Work through sample problem 4/10

2.20 Chain vs Rope style sample problem difference

Resources

- Book section 4/7

Challenge

Considering the chain sample problem and the unconstrained rope problem, why was the kinetic energy different in these two cases? What assumptions were made in the chain problem compared to the unconstrained rope problem, and how did this impact the calculation of kinetic energy?

Please write a few sentences summarising your understanding.

Solution

Please compare your writing with your partner's writing and discuss any differences.

2.21 Constrained and unconstrained rope style sample problem

Resources

- Book section 4/7

Challenge

Considering the unconstrained and constrained rope sample problem, how are the approaches different? How do the different values for "P" and "R" arise? What assumptions are different?

Please write a few sentences summarising your understanding.

Solution

Please compare your writing with your partner's writing and discuss any differences.

2.22 Lifting a chain

Resources

- Book section 4/7

Challenge

An 18K gold chain has a mass of 1.12 g and a length of 40 cm. You pick up one end of the chain and lift it up vertically at a constant velocity. There will be two downward forces present: one due to the hanging weight of the chain due to earth's gravity (A), and another induced by the constant addition of mass to the hanging part of the chain (B). If the chain is lifted up at $5\,\mathrm{cm\,s^{-1}}$, calculate B. State what simplifying assumptions you make. How does this compare to A?

Solution

B: $7\,\mu\mathrm{N}$

2.23 Chain on a pully

Resources

- Book section 4/7

Challenge

Answer question 4/83

Solution

P is given in book, and R should match your understanding of the weight of the pile.

2.24 An accelerating chain

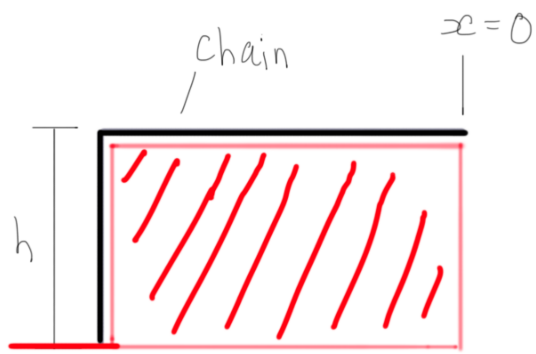
Resources

- Book section 4/7

Challenge

a) Consider a chain hanging over the edge of a block of height h. The chain has a total length L and mass per unit length of ρ. At time t=0, the right end of the chain lays on top of the block at its right side at position x=0, while the left end of the chain is barely touching the ground. Initially the chain is stationary, but then the chain is released, causing the right side to accelerate horizontally in the left direction and the chain to pile up to the left of the block. Ignoring friction at the corner and making other idealised assumptions, obtain an expression for the acceleration of the chain as a function of the position of the end of the chain (x) as it slides along the horizontal surface, before it reaches the end of the block.

b) Did you need to consider the force generated by the stopping of the chain as it landed on the ground? If so, how did it come into the equation? If not, why not?



Solution

a) To check your answer, determine the acceleration for a height of 5 m and chain length of 10 m, when the chain has slid 2 m. You should obtain an acceleration of 6.13 m s^{-2}.

b) Please discuss your answer with your partner

2.25 Making a question (particles)

Coursework challenge

Resources

- Book section 4

Challenge

Create an original question concerning anything covered in this chapter using the Peerwise platform (see section 0.2 for details). If you create more than one question, the question awarded highest marks will be counted.

Your question must contain the following:

1. A diagram to support your question.
2. 5 possible answer options (A-E) that can be used by the student to check if their answer is correct. These may be numerical, mathematical, word-based or hashes.
3. It is important that it is not possible to derive the method from the answer-options that you give.
4. All the answer options should be plausible.
5. For at least one of your wrong-answer options, ensure that the answer corresponds to a typical mistake that a student might make when answering your question.
6. Provide a detailed solution to your question, including explanation and mathematics. *Demonstrate your own thinking and understanding.*
7. Related to item 5, explain why the student might have chosen that wrong answer, and explain where they may have gone wrong in their thinking.

2.26 Answering Peerwise question(s)

Resources

- Book section 4

Challenge

Choose at least one question from Peerwise and attempt to answer it. Explain your reasoning (don't just write mathematics). Clearly write an alternative explanation or possible improvement to the question you answered.

Solution

Please discuss in class.

Chapter 3

3D dynamics of rigid bodies

3.1 Radial velocity with horizontal connection

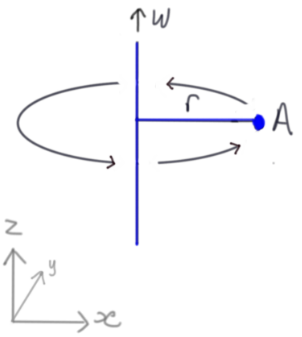
Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

A weight "A" is tethered to a pole by a stiff rod of length r. If the angular velocity is $5.5\,\mathrm{rad\,s^{-1}}\ \hat{k}$ and the length of the rod is $47\,\mathrm{m}$ along the x-axis, what is the linear velocity of the weight "A"?



Solution

(Units: $\mathrm{m\,s^{-1}}$)

$\hat{\boldsymbol{i}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of iX = 2ebd7c

$\hat{\boldsymbol{j}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of jX = bbc682

$\hat{\boldsymbol{k}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of kX = 28435f

3.2 Radial velocity with non-horizontal connection

Resources

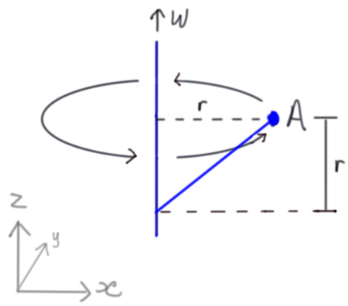
- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

1. The position of "A" and the pole are unchanged (the radial distance is the same) and the angular velocity remains the same, but "A" is now hinged to the pole from below instead of horizontally, as shown in the picture. Calculate the linear velocity of "A" (calculate mathematically, not just by comparison with the previous challenge).

2. Write a sentence or two explaining the effect of the choice of tethering position on the calculation and the final answer.



Solution

(Units: $\mathrm{m\,s^{-1}}$)

$\hat{\boldsymbol{i}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of aX = 690969

$\hat{\boldsymbol{j}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of bX = e73aa7

$\hat{\boldsymbol{k}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.

Example: aX where $X = 46.00$ is entered as a46.00

Hash of cX = 11e1c5

2. Please discuss in class if you are unsure about your answer.

3.3 Linear acceleration

Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

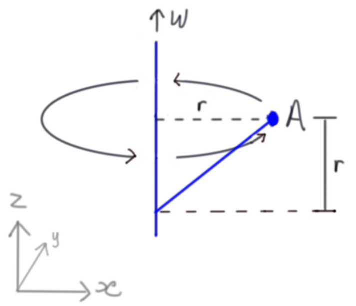
Challenge

Using information from previous challenges, determine the:

1. Linear acceleration towards the centre of pole.

2. The tangential linear acceleration

3. Is there linear acceleration towards the centre of the pole? Is there tangential linear acceleration? Write a sentence or two to explain why for both cases.



Solution

(Units: $\mathrm{m\,s^{-2}}$)

1.
$\hat{\boldsymbol{\imath}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of dX = 3df1ec

$\hat{\boldsymbol{\jmath}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of eX = ea8d91

$\hat{\boldsymbol{k}}$
X = Your solution
Form: Decimal to 2 decimal places.

Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of fX = 645427

2.
$\hat{\boldsymbol{\imath}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of gX = a120f7

$\hat{\boldsymbol{\jmath}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of hX = 5a16a7

$\hat{\boldsymbol{k}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of iX = 2ebd7c

3. Please compare your answer with your partner.

3.4 Radial acceleration - increase in magnitude of radial velocity

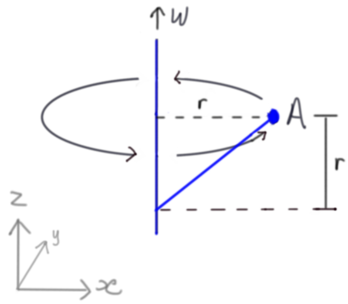
Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Now consider that the radial velocity is not constant, but is undergoing an angular acceleration of $2\,\mathrm{rad\,s^{-2}}$, so that the magnitude of the angular velocity $\boldsymbol{w}$ increases while it continues to point in the same direction. What is the tangential acceleration of "A"?



Solution

(Units: $\mathrm{m\,s^{-2}}$)

$\hat{\boldsymbol{\imath}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of mX = 6d0eb9

$\hat{\boldsymbol{\jmath}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of nX = 3c7c40

$\hat{\boldsymbol{k}}$
X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of oX = 29f8f8

3.5 Radial acceleration - only direction (precession)

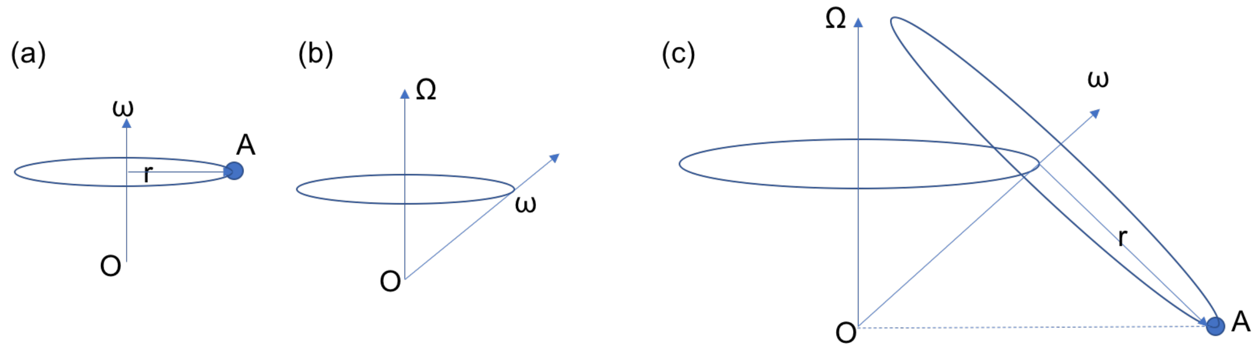
Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

The previous challenges considered the case (a) below, where the direction of the angular velocity vector ω was unchanging. Next consider that the angular velocity vector is precessing around an axis of symmetry, and this precession has an angular velocity of Ω, as shown in (b). Combining (a) and (b) we have (c).



1. Consider the point "A" rotating about the vector ω with angular velocity magnitude $5.5\,\mathrm{rad\,s^{-1}}$, but now tilt the ω vector and allow the rotation to precess around a vector of symmetry Ω. Assume that only the direction (not the magnitude) of the angular velocity vector ω is changing with time. If $\Omega = 3\hat{k}$ $\mathrm{rad\,s^{-1}}$ and angular velocity vector ω is inclined at 45° in the x-z plane so that the point "A" lays on the x-axis.

1. Calculate the linear acceleration of point "A".

2. Write one or two sentences to explain when the signs of the linear acceleration will be opposite but with same magnitude.

Solution

1. $-3155\hat{i} + 1005\hat{k}$

2. Please discuss in class.

3.6 Radial acceleration II

Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Question 7/4

Solution

$1285\,\mathrm{m\,s^{-1}}$

3.7 Unit vector of a rotation axis

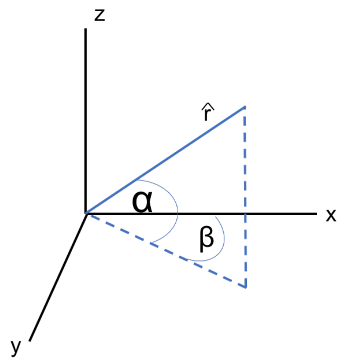
Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Considering vector $\boldsymbol{r}$ in the figure below, if the angles $\alpha = 45$ degrees and $\beta = 30$ degrees, write the unit vector $\hat{\boldsymbol{r}}$ in terms of the Cartesian unit vectors.



Solution

$\boldsymbol{r} = \frac{\sqrt{3}}{2\sqrt{2}}\hat{i} + \frac{1}{2\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$

3.8 Simultaneous rotation I

Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Work through sample problem 7/2.

3.9 Simultaneous rotation II

Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Work through sample problem 7/1

3.10 Simultaneous rotation III

Resources

- Book sections 7/1 to 7/5

Correction to book: Figure 7/9 should read $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$ *(not* $\boldsymbol{\Omega} \times \boldsymbol{r}$*)*

Challenge

Complete question 7/12

Solution

Velocity: $\frac{\pi}{8}(-4\hat{\imath} + 6\hat{\jmath} - 3\hat{k})\ \mathrm{m\,s^{-1}}$

Acceleration: $\frac{\pi^2}{8}(-25\hat{\jmath} - 18\hat{k})\ \mathrm{m\,s^{-2}}$

3.11 Time-dependent rotation of vectors

Resources

- Book section 5/7

Challenge

Answer question 7/27 in the book by:

a) Considering the axes fixed and not rotating with the shaft.

b) Considering the axes rotating with the shaft. In this case, you will need to consider the velocity components of the axes themselves as well as the velocity component induced by the swinging of the pendulum, using the derivative chain rule.

Solution

The solution is given in the book and you should obtain the same answer in both cases.

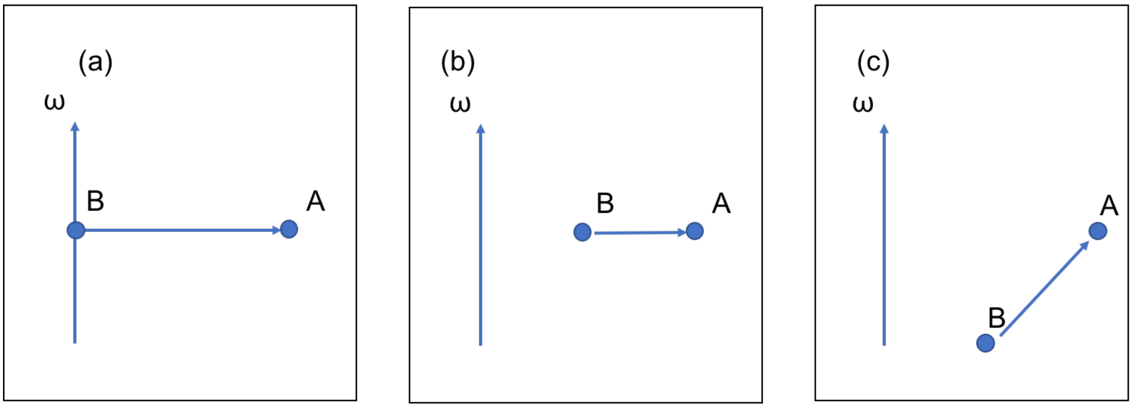
3.12 Relative velocity

Resources

- Book section 7/6

Challenge

Consider a rotating rigid body with angular velocity $\omega = \hat{k}$. Two points, "A" and "B" are chosen on the rigid body. The book describes how it is possible to calculate the velocity of point "A" given a set of axes (xyz) with the origin at point "B". Thus from the perspective of an observer on xyz, point "A" is rotating about point "B". Show that the velocity of point "A" relative to point "B" is always the same, irrespective of the choice of location of point "B". You may show this by demonstrating consistency for the 3 cases below.



a) $\boldsymbol{B} = \hat{k}$, $\boldsymbol{A} = 2\hat{\imath} + \hat{k}$
b) $\boldsymbol{B} = \hat{\imath} + \hat{k}$, $\boldsymbol{A} = 2\hat{\imath} + \hat{k}$
c) $\boldsymbol{B} = \hat{\imath}$, $\boldsymbol{A} = 2\hat{\imath} + \hat{k}$

Solution

You should be able to show that all 3 cases will result in the same value of v_A. If this is not the case, please discuss with your partner or the teacher.

3.13 Crank-style problem

Resources

- Book section 7/6

Comment

The angular velocity of the link $\bar{AB}$ is, by definition, perpendicular to the axis of the link. In the second part of this challenge, you use this fact along with other obtained equations to obtain the 3 Cartesian directions of the angular velocity vector. Note that although the concept of the angular momentum of the link $\bar{AB}$ is used to calculate w_2 in the first part of the problem, you don't actually need to calculate the value of the $\boldsymbol{w_n}$ vector in order to determine the value of w_2.

Challenge

Work through sample problem 7/3

3.14 Perpendicular position, velocity and rotation vectors

Challenge

Prove that $\boldsymbol{r}_{AB}$, $\boldsymbol{v}_{AB}$ and $\boldsymbol{\omega}_n$ in sample problem 7/3 are all perpendicular to each other. Draw a diagram to show the 3 vectors.

Solution

Please compare with your partner in class.

3.15 Perpendicular double cross-product

Challenge

Considering a vector $\boldsymbol{a} = \hat{k}$ and a vector $\boldsymbol{b} = \hat{\imath}$, show that $\boldsymbol{a} \times (\boldsymbol{a} \times \boldsymbol{b}) = -a^2 \boldsymbol{b}$.

3.16 3D linear acceleration

Resources

- Book section 7/6

Challenge

Calculate the linear acceleration of points A and B in sample problem 7/4.

Solution

$\boldsymbol{a}_A = 1800\hat{\boldsymbol{\imath}} + 50\dot{\omega}_2\hat{\boldsymbol{\jmath}}$ mm s^{-2}

$\boldsymbol{a}_B = 3600\hat{\boldsymbol{k}}$ mm s^{-2}

3.17 3D angular acceleration

Resources

- Book section 7/6

Comment

The last few challenges have all been building up to this challenge. This is not easy but shows how powerful the tools are that you have gained.

Challenge

Work through sample problem 7/4.

3.18 3D velocity calculation

Resources

- Book section 7/6

Challenge

Answer question 7/38.

Solution

$-0.64\hat{\imath} - 4.87\hat{\jmath} + 1.27\hat{k}\ \mathrm{m\,s^{-1}}$

3.19 3D velocity and acceleration calculation

Resources

- Book section 7/6

Challenge

Answer question 7/43.

Solution

Given in book.

3.20 Inertia

Resources

- Book appendix B

Challenge

Write a paragraph describing what inertia I is, explain its mathematical derivation.

3.21 Inertia: A simple cylinder

Resources

- Book appendix B
- Video: `https://www.youtube.com/watch?v=Sx-5YJbszuI`

Challenge

Work through sample problem B/1

3.22 Inertia: A simple sphere

Resources

- Book appendix B

Challenge

Work through sample problem B/2

3.23 Inertia: A 2D rectangle

Resources

- Book appendix A
- Video: `https://www.youtube.com/watch?v=zhpxO0zm_E4`

Challenge

Determine the area moment of inertia about the x-axis I_x of a rectangle which is centred on the x-y axes with x and y lengths "X" and "Y" respectively.

Solution

You should find that your answer is consistent with $I_x = 2$ for $X = 3$ and $Y = 2$.

3.24 Inertia: A simple parallelepiped

Resources

- Book appendix B

Challenge

Work through sample problem B/3

3.25 Inertia: A cone

Resources

- Book appendix B

Challenge

Answer question B/4

Solution

Your solution should be consistent with a total mass of 8 kg and a radius of 3.5 m having an intertia I_{zz} of 49 kgm^2.

3.26 Inertia: A rod

Resources

- Book appendix B

Challenge

Answer question B/3

Solution

Given in book.

3.27 Inertia: A rod and a disk

Resources

- Book appendix B

Challenge

Answer question B/18. Please fully derive your answer, except you may use the fact that I_{xx} for each rod is $mL^2/6$.

Solution

Your answer should be consistent with a length of 1.73 m for a disk radius of 2 m.

3.28 Inertia: A bent cylinder

Resources

- Book appendix B
- Book appendix D

Challenge

Answer question B/27. You do not need to derive solutions from first principles, and instead may refer to appendix D to build your answer.

Solution

Given in book.

3.29 Off-diagonal inertia

Resources

- https://www.colorado.edu/physics/phys3210/phys3210_fa15/lecnotes.2015-10-14.The_Inertia_Tensor.html

Challenge

Consider a cube. The x, y an z axes pass through the centre of the cube perpendicular to the faces of the cube (ie, each axis passes through the centre of a face). The mass of the cube is evenly distributed so the centre-of-mass of the cube is in the centre, where the 3 axes cross.

The off-diagonal values of inertia I_{xy}, I_{xz} and I_{yz} are zero. Show this mathematically.

Solution

Please compare your answer with your partner.

3.30 Angular momentum and kinetic energy of two plates

Resources

- Book section 7/7 and 7/8

Challenge

Work through sample problem 7/6.

3.31 Angular momentum and kinetic energy of a cone

Resources

- Book section 7/7 and 7/8

Challenge

Answer question 7/65. Note that the question only concerns the cone, and not the cone-rod system.

Solution

Given in book, however there is a mistake in the book and the angular momentum H should read:
$\frac{3}{10}mr^2\left[\left(\frac{1}{2}+2\frac{h^2}{r^2}\right)\Omega\hat{i}+p\hat{k}\right]$

3.32 Angular momentum and kinetic energy of a rolling disk

Resources

- Book section 7/7 and 7/8

Challenge

Answer question 7/71.

Solution

Given in book.

3.33 Making a question (3D dynamics)

Coursework challenge

Resources

- Book section 7 and appendix B

Challenge

Create an original question concerning anything covered in this chapter using the Peerwise platform (see section 0.2 for details). If you create more than one question, the question awarded highest marks will be counted.

Your question must contain the following:

1. A diagram to support your question.
2. 5 possible answer options (A-E) that can be used by the student to check if their answer is correct. These may be numerical, mathematical, word-based or hashes.
3. It is important that it is not possible to derive the method from the answer-options that you give.
4. All the answer options should be plausible.
5. For at least one of your wrong-answer options, ensure that the answer corresponds to a typical mistake that a student might make when answering your question.
6. Provide a detailed solution to your question, including explanation and mathematics. *Demonstrate your own thinking and understanding.*
7. Related to item 5, explain why the student might have chosen that wrong answer, and explain where they may have gone wrong in their thinking.

3.34 Answering Peerwise question(s)

Resources

- Book section 7 and appendix B

Challenge

Choose at least one question from Peerwise and attempt to answer it. Explain your reasoning (don't just write mathematics). Clearly write an alternative explanation or possible improvement to the question you answered.

Solution

Please discuss in class.