# **Mechanics**

Spring 2017

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 $\verb|http://raw.githubusercontent.com/NanoScaleDesign/Mechanics/master/mechanics.pdf| \\$ 

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# Chapter 0

# Course information

#### 0.1 This course

This is the Spring 2017 Mechanics course studied by 2nd-year undegraduate international students at Kyushu University.

#### 0.1.1 What you need to do

- Borrow the book "Engineering Mechanics: Dynamics", 6th edition, by Meriam and Kraige from the Kikan-kyoiku office in the centre-zone. The course will be based on that book and you will need to refer to it in class.
- Prepare a challenge-log in the form of a workbook or folder where you can clearly write the calculations you perform to solve each challenge. This will be used in the final assessment and will be occasionally reviewed by the teacher.
- Submit a weekly feedback form by **9am on Monday** before class at https://goo.gl/forms/ 2PgFF0eqTOvbK0to2.
- Please bring a wifi-capable internet device to class, as well as headphones if you need to access online components of the course during class. If you let me know in advance, I can lend computers and provide power extension cables for those who require them (limited number).

#### 0.1.2 How this works

- This booklet forms part of an active-learning segment in the course. The learning is self-directed in contrast to the traditional lecture-style model.
- Learning is guided through solving a series of challenges combined with instant feedback about the correctness of your answer.
- Traditional lectures are replaced by discussion time. Here, you are encouraged to discuss any issues with your peers, teacher and any teaching assistants. You can also learn from explaining concepts to your peers.
- Discussion-time is from 13:00 to 14:30 on Mondays at room W4-766.
- Peer discussion is encouraged, however, if you have help to solve a challenge, always make sure you do understand the details yourself. You will need to be able to do this in an exam environment. The questions on the exam will be similar in nature to the challenges. If you can do all of the challenges, you can get 100% on the exam.
- Every challenge in the book typically contains a **Challenge** with suggested **Resources** which you are recommended to utilise in order to solve the challenge. **Solutions** will be given. Occasionally the teacher will provide extra **Comments** to help guide your thinking.
- For deep understanding, it is recommended to study the suggested resources beyond the minimum required to complete the challenge.
- The challenge document has many pages and is continuously being developed. Therefore it is advised to view the document on an electronic device rather than print it. The date on the front page denotes the version of the document. You will be notified when the document is updated.
- A target challenge will be set each week. This will set the pace of the course and define the examinable material. It's ok if you can't quite reach the target challenge for a given week, but you should be careful not to fall behind, since the date of the exam cannot be delayed.

#### 0.1.3 Assessment

In order to prove to outside parties that you have learned something from the course, we must perform summative assessments. You will receive a weighted score based on:

- $\bullet$  Challenge-log (10%) final state at the end of the course, showing your calculations for all the challenges in the course.
- Mid-term exam (30%)
- Coursework (20%)
- Final exam (40%)

 $Final\ score = MAX(Weighted\ score,\ Final\ exam)$ 

# 0.2 Timetable

|    | Discussion | Target | Note                              |
|----|------------|--------|-----------------------------------|
| 1  | 10 April   | -      |                                   |
| 2  | 17 April   | 1.4    |                                   |
| 3  | 24 April   | 1.7    |                                   |
| 4  | 8 May      | 1.12   |                                   |
| 5  | 15 May     | 1.17   |                                   |
| 6  | 22 May     | 1.22   |                                   |
| 7  | 29 May     | 1.27   |                                   |
| 8  | 5 June     |        |                                   |
| 9  | 12 June    | -      | Mid-term exam (covers up to 1.27) |
| 10 | 19 June    |        |                                   |
| 11 | 26 June    |        |                                   |
| 12 | 3 July     |        |                                   |
| 13 | 10 July    |        |                                   |
| 14 | 13 July    |        |                                   |
| -  | 24 July    | -      | Final exam                        |
| -  | 16 August  | -      | Retake exam (tentative)           |

### 0.3 Hash-generation

Some solutions to challenges are encrypted using MD5 hashes. In order to check your solution, you need to generate its MD5 hash and compare it to that provided. MD5 hashes can be generated at the following sites:

- Wolfram alpha: (For example: md5 hash of "q\_1.00") http://www.wolframalpha.com/input/?i= md5+hash+of+%22q\_1.00%22
- www.md5hashgenerator.com

Since MD5 hashes are very sensitive to even single-digit variation, you must enter the solution exactly. This means maintaining a sufficient level of accuracy when developing your solution, and then entering the solution according to the format below:

Unless specified otherwise, any number from 0.00 to  $\pm 9999.99$  should be represented as a normal number to two decimal places. All other numbers should be in scientific form. See the table below for examples.

| Solution   | Input                        |
|--|------------------------------|
| 1  | 1.00                         |
| -3   | -3.00                        |
| -3.5697  | -3.57                        |
| 0.05   | 0.05                         |
| 0.005  | 5.00e-3                      |
| 50   | 50.00                        |
| 500  | 500.00                       |
| 5000   | 5000.00                      |
| 50,000   | 5.00e4                       |
| $5 \times 10^{-476}$   | 5.00e-476                    |
| $5.0009 \times 10^{-476}$  | 5.00e-476                    |
| $-\infty$  | -infinity (never "infinite") |
| $2\pi$   | 6.28                         |
| i  | im(1.00)                     |
| 2i   | im(2.00)                     |
| 1+2i   | re(1.00)im(2.00)             |
| -0.0002548 i   | im(-2.55e-4)                 |
| 1/i = i/-1 = -i  | im(-1.00)                    |
| $e^{i2\pi} \left[ = \cos(2\pi) + i\sin(2\pi) = 1 + i0 = 1 \right]$     | 1.00                         |
| $e^{i\pi/3} \left[ = \cos(\pi/3) + i\sin(\pi/3) = 0.5 + i0.87 \right]$ | re(0.50)im(0.87)             |
| Choices in order A, B, C, D  | abcd                         |

Entry format is given with the problem. So "q\_X" means to enter "q\_X" replacing "X" with your solution. The first 6 digits of the MD5 sum should match the given solution  $(MD5(q_X) = ...)$ .

Note that although some answers can usually only be integers (eg, number of elephants), unless otherwise indicated you should always enter an integer to two decimal places (ie, with ".00" after it) to generate the correct hash.

# Chapter 1

Kinetics of systems of particles

# 1.1 System centre-of-mass position, mass and velocity: I

#### Resources

• Book sections 4/1 to 4/2

# Challenge

- 1.  $\overline{r}$ ,  $\dot{\overline{r}}$  and  $\ddot{\overline{r}}$  of Question 4/1.
- 2. Question 4/4

#### Solution

- 1. Given in book.
- 2. 316 N

# 1.2 System centre-of-mass position, mass and velocity: II

#### Resources

 $\bullet$  Book sections 4/1 to 4/2

# Challenge

Question 4/5. Determine the magnitude of the acceleration.

#### Solution

Given in book.

# 1.3 System centre-of-mass position, mass and velocity: III

#### Resources

 $\bullet$  Book sections 4/1 to 4/2

### Challenge

Question 4/13, but change the force to a  $10\,\mathrm{N}$  force and the mass of each bar to  $8\,\mathrm{kg}$ .

#### Solution

 $0.42\,\mathrm{m/s^2}$ 

# 1.4 Kinetic and potential energy

#### Resources

• Book section 4/3

### ${\bf Challenge}$

- 1. Calculate T in question 4/1
- 2. Question 4/10

#### Solution

- 1. Given in book.
- 2. To check your final answer, substitute b=2 metres into your final answer. You should obtain  $5.27\,\mathrm{m/s}$ .

# 1.5 Cross-product

#### Resources

• https://www.khanacademy.org/science/physics/magnetic-forces-and-magnetic-fields/electric-motors/v/calculating-dot-and-cross-products-with-unit-vector-notation

#### Challenge

- 1. Determine the angle between the two vectors  $\mathbf{a} = [3,0,0]$  and  $\mathbf{b} = [3,1,0]$  and use it to calculate  $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ . Which direction does the vector  $\mathbf{c}$  point?
- 2. Determine the cross product  $\mathbf{f} = \mathbf{d} \times \mathbf{e}$  where  $\mathbf{d} = 4\hat{i} + 2\hat{j} + 1\hat{k}$  and  $\mathbf{e} = -2\hat{i} 4\hat{j} + 8\hat{k}$  without calculating the angle between them.

#### Solution

Please compare your answer with your partner and discuss in class if answers differ.

# 1.6 Rotation I

#### Resources

 $\bullet$  Book section 4/4

### Challenges

Calculate the angular momentum and the rate of change of angular momentum with time for Question 4/1.

#### Solutions

Given in book.

# 1.7 Rotation II

#### Resources

• Book section 4/4

# Challenges

Question 4/16

If you have difficulty, consider doing question 4/15 first (optional).

#### Solutions

The required time should be  $2.72\,\mathrm{s}$ 

# 1.8 Rotation III

#### Resources

• Book section 4/4

#### Challenges

Question 4/2

#### Solutions

To check your answers substitute d=2 metres, m=7 kg, v=3 m/s and f=7 N into your final answers. You should obtain  ${\cal H}_G=432\hat{i}+144\hat{j}+168\hat{k}$  kgm²/s and  ${\dot {\cal H}}_G=-8\hat{i}-12\hat{j}+0\hat{k}$  Nm

# 1.9 Conservation of momentum

#### Resources

• Book section 4/5

### Challenges

- 1. In Question 4/17, at what point does the vehicle stop accelerating?
- 2. Solve Question 4/17
- 3. Question 4/18

#### Solutions

- 1. Please write your answer and compare with your partner in class
- 2. Given in book
- $3.~0.21~\mathrm{m/s}$

# 1.10 Conservation of momentum vs energy

#### Resources

• Book section 4/5

#### Challenges

- 1. Solve Question 4/19
- 2. Why is energy not conserved here? Where did the energy go? Under what conditions is momentum conserved, and under what conditions is energy conserved?

#### Solutions

- 1. Given in book
- $2.\ \,$  Please write your answers and compare with your partner in class.

# 1.11 Combined problems I

#### Resources

 $\bullet$  Book section 4/1 to 4/5

#### Challenge

Solve Question 4/22.

The question states that an impulse is imparted "over a negligibly short period of time" which is a little confusing since impulse is the integration of force over time which becomes zero as time goes to zero. Instead, here you can consider that whatever the time is, the final product of Force and Time is  $10\,\mathrm{N}\,\mathrm{s}$ .

#### Solution

 $4.7\,\mathrm{m/s}$ 

# 1.12 Combined problems II

#### Resources

 $\bullet$  Book section 4/1 to 4/5

#### Challenge

Solve Question 4/28

#### Solutions

You should obtain an algebraic expression for v and  $\dot{\theta}$ . To check your expression, you can substitute the following values into the expression:  $m_0=1\,\mathrm{kg},\,v_0=1000\,\mathrm{m\,s^{-1}},\,b=1.5\,\mathrm{m}$  and  $m=4\,\mathrm{kg}$ , whereby you should obtain  $v=111\,\mathrm{m\,s^{-1}}$  and  $\dot{\theta}=222\,\mathrm{rad\,s^{-1}}$ .

# 1.13 In-plane flow

### Resources

• Book section 4/6

# Challenge

Derive equation 4/19 in the book from equation 4/19a.

# 1.14 Force on vane

#### Resources

• Book section 4/6

# ${\bf Challenge}$

Show your working for sample problem 4/5 (a) and (b)

#### 1.15 Power and a vane

#### Resources

• Book section 4/6

#### Challenge

Considering sample problem 4/6,

- 1. Explain in words what is meant by "power by action of the fluid".
- 2. The power is defined here by measuring the force applied to move an object at a constant velocity. If force creates acceleration (F = ma), how can the velocity be constant?
- 3. Work through and solve the sample problem.

#### Solutions

Please compare your solutions with your partner. You may be asked to present your solutions to the class.

# 1.16 Balancing forces: Jet aeroplane example

#### Resources

• Book section 4/6

# Challenge

Work through sample problem 4/8 to obtain the equation of motion of the system as given in the book  $(m'_g u - m'_a v = mg \sin \theta + D)$ .

# 1.17 Balancing forces: Jet aeroplane

#### Resources

• Book section 4/6

# Challenge

Answer question 4/33.

#### Solution

Given in book.

# 1.18 Balancing forces: Fire tug

#### Resources

• Book section 4/6

# Challenge

Answer question 4/37.

#### Solution

Given in book.

# 1.19 Balancing ball on a water stream

#### Resources

• Book section 4/6

### ${\bf Challenge}$

Answer question 4/42. Take note about the conservation of energy in the jet stream, and the fact that the jet stream remains intact. You can assume that the water stream is fully deflected horizontally when it hits the ball.

#### Solution

 $4.8\,\mathrm{m}$ 

# 1.20 Pressure I

#### Resources

• Book section 4/6

# Challenge



A typical die has a side length of  $1.4\,\mathrm{cm}$  and weighs  $2.8\,\mathrm{g}$ . Consider the die at rest on a desk. Estimate the pressure on the bottom of the die due to the desk.

#### Solution

 $140\,\mathrm{Pa}$ 

# 1.21 Pressure II

#### Resources

• Book section 4/6

# ${\bf Challenge}$

Answer question 4/50

### Solution

 $1035\,\mathrm{Pa}$ 

# 1.22 Power and a Helicopter

#### Resources

• Book section 4/6

# Challenge

Answer question 4/59

#### Solutions

Given in book

#### Mass ejection 1.23

#### Resources

• Book section 4/7

#### Challenge

Consider rocket thrust where exhaust is emitted at a speed of  $220\,\mathrm{m\,s^{-1}}$ . The force on the rocket due to the thrust alone is  $400\,\mathrm{N}$ . Calculate (a) the mass flow rate m' and (b) the time-rate increase of the mass of the rocket  $\dot{m}$ .

#### Solutions

- $\begin{array}{l} {\rm (a)~MD5(a\_X)} = 300026. \ldots ~{\rm kg \, s^{-1}} \\ {\rm (b)~MD5(b\_X)} = {\rm a2fb8f...} ~{\rm kg \, s^{-1}} \end{array}$

# 1.24 Rocket sample problem

### Resources

• Book section 4/7

# Challenge

Complete the sample problem 4/11 using both solution I and II. Please be sure to follow the logic.

# 1.25 Rocket-style problem I

#### Resources

• Book section 4/7

# Challenge

Answer question 4/67

#### Solution

Given in book.

# 1.26 Rocket-style problem II

#### Resources

• Book section 4/7

# Challenge

Answer question 4/82

### Solution

 $4.8\,{\rm m\,s^{-1}}$ 

# 1.27 Mass intake and power

#### Resources

• Book section 4/7

# Challenge

Answer question 4/80

### Solution

 $1.6\,\mathrm{m\,s^{-2}}$  deceleration