Ordinary Differential Equations

Autumn 2017

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http://www.jamescannon.net/teaching/ordinary-differential-equations

http://raw.githubusercontent.com/NanoScaleDesign/OrdinaryDifferentialEquations/master/ode.pdf

Contents

Chapter 0

Course information

0.1 This course

This is the Autumn 2017 Ordinary Differential Equations course studied by 2nd-year undegraduate international students at Kyushu University.

0.1.1 How this works

- In contrast to the traditional lecture-homework model, in this course the learning is self-directed and active via publicly-available resources.
- Learning is guided through solving a series of carefully-developed challenges contained in this book, coupled with suggested resources that can be used to solve the challenges with instant feedback about the correctness of your answer.
- There are no lectures. Instead, there is discussion time. Here, you are encouraged to discuss any issues with your peers, teacher and any teaching assistants. Furthermore, you are encouraged to help your peers who are having trouble understanding something that you have understood; by doing so you actually increase your own understanding too.
- Discussion-time is from 14:50 to 16:20 on Fridays at room Centre Zone 1409.
- Peer discussion is encouraged, however, if you have help to solve a challenge, always make sure you do understand the details yourself. You will need to be able to do this in an exam environment. If you need additional challenges to solidify your understanding, then ask the teacher. The questions on the exam will be similar in nature to the challenges. If you can do all of the challenges, you can get 100% on the exam.
- Every challenge in the book typically contains a **Challenge** with suggested **Resources** which you are recommended to utilise in order to solve the challenge. Occasionally the teacher will provide extra **Comments** to help guide your thinking. A **Solution** is also made available for you to check your answer. Sometimes this solution will be given in encrypted form. For more information about encryption, see section 0.3.
- For deep understanding, it is recommended to study the suggested resources beyond the minimum required to complete the challenge.
- The challenge document has many pages and is continuously being developed. Therefore it is advised to view the document on an electronic device rather than print it. The date on the front page denotes the version of the document. You will be notified by email when the document is updated. The content may differ from last-year's document.
- A target challenge will be set each week. This will set the pace of the course and defines the examinable material. It's ok if you can't quite reach the target challenge for a given week, but then you will be expected to make it up the next week.
- You may work ahead, even beyond the target challenge, if you so wish. This can build greater flexibility into your personal schedule, especially as you become busier towards the end of the semester.
- Your contributions to the course are strongly welcomed. If you come across resources that you found useful that were not listed by the teacher or points of friction that made solving a challenge difficult, please let the teacher know about it!

0.1.2 Assessment

In order to prove to outside parties that you have learned something from the course, we must perform summative assessments. This will be in the form of a mid-term exam (weighted 30%), coursework (weighted 20%), a satisfactory challenge-log (weighted 10%) and a final exam (weighted 40%).

Your final score is calculated as Max(final exam score, weighted score), however you must pass the final exam to pass the course.

0.1.3 What you need to do

- Prepare a challenge-log in the form of a workbook or folder where you can clearly write the calculations you perform to solve each challenge. This will be a log of your progress during the course and will be occasionally reviewed by the teacher.
- You need to submit a brief report at `https://goo.gl/forms/S69DuM4xCss0WtjH3` by 8am on the day of the class. Here you can let the teacher know about any difficulties you are having and if you would like to discuss anything in particular.
- Please bring a wifi-capable internet device to class, as well as headphones if you need to access online components of the course during class. If you let me know in advance, I can lend computers and provide power extension cables for those who require them (limited number).

0.2 Timetable

	Discussion	**Target**	**Note**
1	4 Oct	-	Wednesday class
2	13 Oct	3.2	
3	20 Oct	3.9	
4	27 Oct	4.7	
5	10 Nov		
6	17 Nov		
7	24 Nov		
8	1 Dec		Coursework instructions
9	8 Dec	Midterm exam	
10	15 Dec		
11	22 Dec		
12	15 Jan		Monday class
13	19 Jan		Submission of coursework
14	26 Jan		
15	2 Feb		
16	9 Feb	Final exam	

Example: To keep pace with the course, you should aim to complete challenge 2 of chapter 3 by the 13th of October.

0.3 Hash-generation

Some solutions to challenges are encrypted using MD5 hashes. In order to check your solution, you need to generate its MD5 hash and compare it to that provided. MD5 hashes can be generated at the following sites:

- Wolfram alpha: (For example: md5 hash of "q1.00") `http://www.wolframalpha.com/input/?i=md5+hash+of+%22q1.00%22`
- `www.md5hashgenerator.com`

Since MD5 hashes are very sensitive to even single-digit variation, you must enter the solution *exactly*. This means maintaining a sufficient level of accuracy when developing your solution, and then entering the solution according to the format suggested by the question. Some special input methods:

Solution	**Input**
5×10^{-476}	5.00e-476
5.0009×10^{-476}	5.00e-476
$-\infty$	-infinity (never "infinite")
2π	6.28
i	im(1.00)
2i	im(2.00)
1 + 2i	re(1.00)im(2.00)
-0.0002548 i	im(-2.55e-4)
1/i = i/-1 = -i	im(-1.00)
$e^{i2\pi}$ $[= cos(2\pi) + isin(2\pi) = 1 + i0 = 1]$	1.00
$e^{i\pi/3}$ $[= cos(\pi/3) + isin(\pi/3) = 0.5 + i0.87]$	re(0.50)im(0.87)
Choices in order A, B, C, D	abcd

The first 6 digits of the MD5 sum should match the first 6 digits of the given solution.

Chapter 1

Hash practise

1.1 Hash practise: Integer

X = 46.3847
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

hash of aX = e77fac

1.2 Hash practise: Decimal

X = 49
Form: Two decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

hash of bX = 82c9e7

1.3 Hash practise: String

X = abcdef
Form: String.
Place the indicated letter in front of the number.
Example: aX where $X = abc$ is entered as aabc

hash of cX = 990ba0

1.4 Hash practise: Scientific form

X = 500,765.99
Form: Scientific notation with the mantissa in standard form to 2 decimal place and the exponent in integer form.
Place the indicated letter in front of the number.
Example: aX where $X = 4 \times 10^{-3}$ is entered as a4.00e-3

hash of dX = be8a0d

Chapter 2

Definitions

2.1 Order of a differential equation

Resources

- Text: `http://tutorial.math.lamar.edu/Classes/DE/Definitions.aspx`

Challenge

What is the sum of the orders of the following equations?

$$\frac{dy}{dx}A = 5x^3 + 3 \tag{2.1}$$

$$cos(y)y'''(x) - y(x) = 25 \tag{2.2}$$

$$\frac{d}{dx}\frac{d^2y}{dx^2} = \frac{x^{-2}}{3} \tag{2.3}$$

Solution

X = Your solution
Form: Integer
Place the indicated letter in front of the number
Example: aX where $X = 46$ is entered as a46

hash of eX = 492585

2.2 Identifying linear and non-linear differential equations

Comment

Being able to identify linear and non-linear ODE's will help you understand how to approach different problems.

Generally speaking, the differential equation is linear if the functions and orders of the differentials are linear. For example,

$$y'' - 4yx = lnx - y$$

can be shown to be linear. Rearranging to collect all the y-terms together:

$$y'' - 4yx + y = lnx$$

the dependent variable y and its derivatives are each of the first degree and depend only on a constant or the independent variable.

An example of a non-linear equation however would be

$$5 + yy' = x - y$$

or

$$yy' + y = x - 5$$

The fact that y' is multiplied by y results in a non-linear equation in y.

Challenge

Sum the points corresponding to the equations that are linear. You may be able to judge some by eye, but you should prove mathematically that at least one of the equations are linear and at least one of the equations are non-linear.

1 point: $\dfrac{dy}{dt} = 5t^3 + 3$.

2 points: $cos(y)y'''(t) - y(t) = 25$.

4 points: $\dfrac{d}{dt}\dfrac{d^2y}{dt^2} = \dfrac{t^{-2}}{3}$.

8 points: $y'(t) - sin(y(t)) = 0$.

16 points: $y'(t) - y(t) = 0$.

32 points: $ty'(t) - y(t) = 0$.

Solution

X = Your solution
Form: Integer
Place the indicated letter in front of the number
Example: aX where $X = 46$ is entered as a46

hash of rX = f5d2c0

2.3 Linear differential equations vs non-linear differential equations

Resources

- Wikipedia: https://en.wikipedia.org/wiki/Nonlinear_system#Nonlinear_differential_equations
- Wikipedia: https://en.wikipedia.org/wiki/Linear_differential_equation

Challenge

Write no-more than 1 short paragraph describing in qualitative terms the difference between a linear and non-linear differential equation.

Solution

Please compare with your partner in class and discuss with the teacher if you are unsure.

2.4 Valid solutions

Resources

- Text: `http://tutorial.math.lamar.edu/Classes/DE/Definitions.aspx`

Challenge

Use substitution to prove that

$$y = \frac{5}{5 + x} \tag{2.4}$$

is a solution to the equation

$$xy' + y = y^2 \tag{2.5}$$

and state the value of x for which the solution is undefined.

Solution

Value of x for which solution is undefined:

X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of tX = 829f33

2.5 Range of valid solutions

Resources

- Text: `http://tutorial.math.lamar.edu/Classes/DE/Definitions.aspx`

Challenge

Use substitution to prove that

$$y = -\sqrt{100 - x^2} \tag{2.6}$$

is a solution to the equation

$$x + yy' = 0 \tag{2.7}$$

and state the range of x for which the solution is valid. Enter the value of the lower range as the solution below.

Solution

X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of yX = d96920

Chapter 3

1st-order differential equations

3.1 Determining a simple DE from a description

Resources

- Text: `http://tutorial.math.lamar.edu/Classes/DE/Definitions.aspx`

Challenge

Newton's law of cooling states that the rate of cooling of an object is proportional to the temperature difference with the ambient surroundings. (a) Write a differential equation describing this situation. (b) Assuming a proportionality constant of 0.2 /hour, what is the rate of temperature change when the object is at 30 °C and the ambient temperature is 20 °C?

Solution

(units: $^{\circ}\mathrm{C}\,\mathrm{h}^{-1}$)

X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of qX = 4aca8d

3.2 Direction (Slope) fields

Resources

- Text: `http://tutorial.math.lamar.edu/Classes/DE/DirectionFields.aspx`
- Video 1: `https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/differential-equations-intro/v/creating-a-slope-field`
- Video 2: `https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/differential-equations-intro/v/slope-field-to-visualize-solutions`

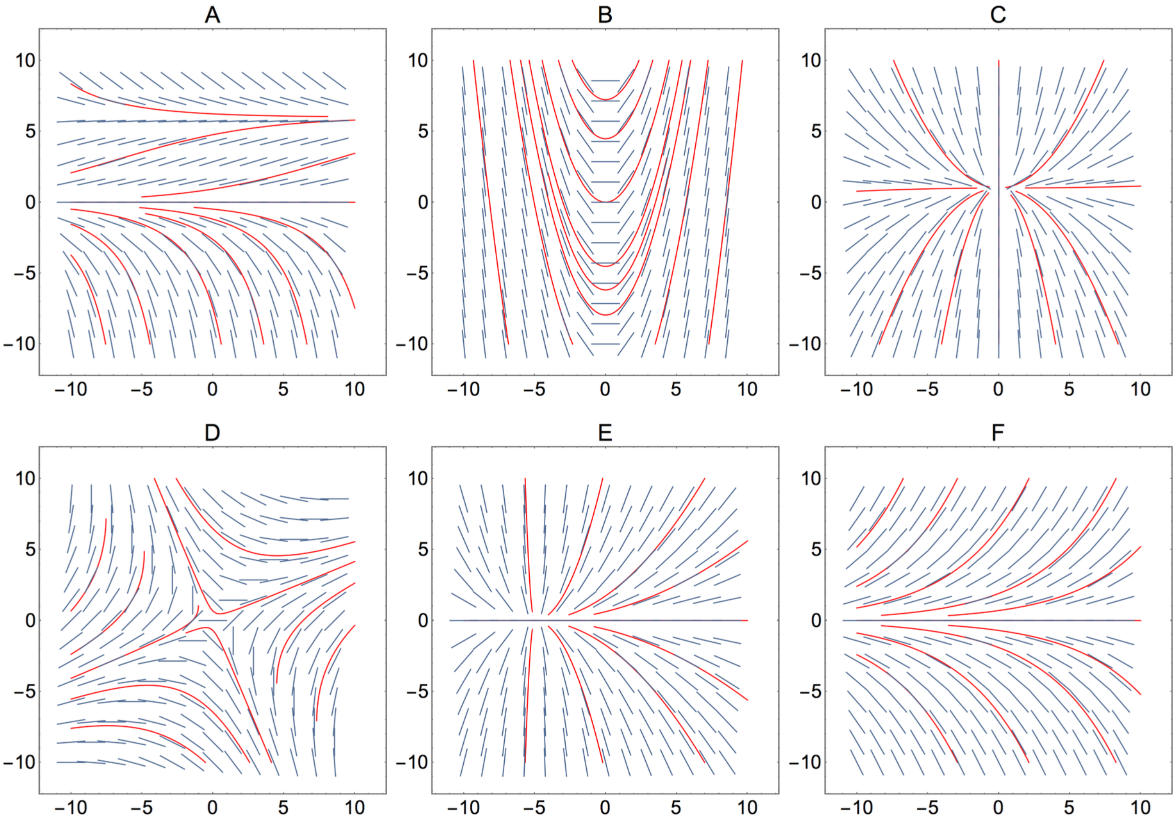
Comment

It is good practise to try drawing the below fields before looking at the next page. You need to be able to go in both directions (ie, drawing and recognising). You will not be given a glimps at the fields in the exam prior to being asked to draw them.

Question

Try drawing the slope field for at least 3 of the equations given below (your choice). Then, put the slope fields given on the next page in the same order as these equations.

1. $y' = x$
2. $y' = 0.2y$
3. $y' = 0.2y(1 - y/6)$
4. $y' = (x - y)/(x + y)$
5. $y' = 2(y - 1)/x$
6. $y' = 2y/(x + 5)$



Solution

X = Your solution
Form: String.
Place the indicated letter in front of the string.
Example: aX where $X = abcdef$ is entered as aabcdef

Hash of qX = e93bfe

3.3 Separable equations I

Resources

- Video I: https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/separable-equations/v/separable-differential-equations-introduction
- Video II: https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/separable-equations/v/particular-solution-to-differential-equation-example
- Text:http://tutorial.math.lamar.edu/Classes/DE/Separable.aspx

Comment

Let's start with a fundamental equation:

$$\frac{dy}{dt} = y \tag{3.1}$$

This is saying that the slope (the rate of change of y) linearly depends on y. That is, that as the value of y increases, the slope also increases; a positive feedback loop. In fact, you get an exponentially-increasing function.

So one aim of this course is to be able to solve such equations mathematically. But I also want you to understand the "physical" meaning of the relation between y and its slope, and how this leads to such a fundamental function such as an exponential.

Challenge

Considering the equation

$$\frac{dy}{dt} = y \tag{3.2}$$

solve for y.

Solution

To check your answer, solve for $y(5)$ given the initial condition $y(0) = 1$.

$y(5) = 148.413$

3.4 Separable equations II

Resources

- Video I: `https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/separable-equations/v/separable-differential-equations-introduction`
- Video II: `https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/separable-equations/v/particular-solution-to-differential-equation-example`
- Text:`http://tutorial.math.lamar.edu/Classes/DE/Separable.aspx`

Challenge

a) Now consider what is meant, physically speaking, by the relation:

$$\frac{dy}{dt} = -y \tag{3.3}$$

Why does it tend to zero for increasing t?

b) Solve for y.

Solution

a) Please compare your solution with your partner or discuss with the teacher.

b) To check your answer, solve for $y(5)$ given the initial condition $y(0) = 1$.

$y(5) = 0.00674$

3.5 Separable equations III

Resources

- Video I: `https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/separable-equations/v/separable-differential-equations-introduction`
- Video II: `https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/separable-equations/v/particular-solution-to-differential-equation-example`
- Text:`http://tutorial.math.lamar.edu/Classes/DE/Separable.aspx`

Challenge

a) Now consider when the slope of y not only depends on y but also on t:

$$\frac{dy}{dt} = ty \tag{3.4}$$

b) or on a constant a:

$$\frac{dy}{dt} = ay \tag{3.5}$$

See how the feedback is greater or lesser, depending on the constant or variable placed in front of y?

Solution

a) Solve for $y(5)$ under the initial condition $y(0) = 1$

268,337

b) Solve for $y(5)$ under the initial condition $y(0) = 1$ and with $a = 2$

22,026.5

3.6 Separable equations IV

Resources

- Video I: `https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/separable-equations/v/separable-differential-equations-introduction`
- Video II: `https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/separable-equations/v/particular-solution-to-differential-equation-example`
- Text:`http://tutorial.math.lamar.edu/Classes/DE/Separable.aspx`

Challenge

Determine $y(t)$ for

$$\frac{dy}{dt} = e^t \tag{3.6}$$

Again, think about what is happening here. Do you see the link with challenge 3.3? There we wrote in terms of y. Here we write in terms of e^t. Do you see they're the same thing?

Solution

To check your answer, solve for $y(3)$ given the initial condition $y(0) = 1$.

$y(3) = 20.09$

3.7 Rate of growth

Resources

- Video: `https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/logistic-differential-equation/v/modeling-population-with-differential-equations`

Comment

One interesting application of 1st-order differential equations is that of population growth.

Challenge

Assuming there is no-limit on growth, a given bacteria would be able to reproduce at such a rate that the amount of bacteria measured in mg increases by 20% every 25 hours. Derive an expression for the rate of growth.

Solution

To check your answer, calculate the rate of growth when there are 20 mg of bacteria.

0.146 mg/hour

3.8 Logistic equation

Resources

- Videos: The 4 remaining logistic differential equation videos starting at: `https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/logistic-differential-equation/v/logistic-differential-equation-intuition`

Comment

We considered exponential growth, but in real life there is often a limit to this. This is where the logistic equation is useful.

Challenge

Assuming there is no-limit on growth, a given bacteria would be able to reproduce at such a rate that the amount of bacteria measured in mg increases by 20% every 25 hours. However, due to environmental factors the limiting (maximum) amount of bacteria that can exist in the system at any one time is 400 mg. Assuming an initial amount of bacteria of 20 mg, how much time, rounded to the nearest integer hours, must one wait to reach 100 mg of bacteria?

Solution

253.1 hours

3.9 Autonomous differential equations

Resources

- Wikipedia: `https://en.wikipedia.org/wiki/Autonomous_system_(mathematics)`

Challenge

The logistic equation is an example of an autonomous differential equation. Add the points of the autonomous differential equations in the following list:

1 point: $y' = cos(y) - 5$

2 points: $y' = cos(y)/x - 5$

4 points: $y' = cos(y)/x - 5/x$

8 points: $y^2 = y'y + 5$

16 points: $xy' = 5y$

32 points: $y' = 1$

Solution

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of fX = 1227c7

3.10 The stability of solutions I

Resources

- Text: `http://tutorial.math.lamar.edu/Classes/DE/EquilibriumSolutions.aspx`
- Text: `http://www.math.psu.edu/tseng/class/Math251/Notes-1st%20order%20ODE%20pt2.pdf`

Challenge

Considering the logistic equation $N' = 0.2N(1 - N/6)$, make 3 separate lists containing any equilibrium, semi-stable and unstable y-values.

To check your answer, sum the value of each list. If there are no values in a list, enter -999 to check the result.

Solution

Stable

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of gX = 4a4314

Semi-stable

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of hX = 9df203

Unstable

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of jX = 17cb7f

3.11 The stability of solutions II

Resources

- Text: `http://tutorial.math.lamar.edu/Classes/DE/EquilibriumSolutions.aspx`
- Text: `http://www.math.psu.edu/tseng/class/Math251/Notes-1st%20order%20ODE%20pt2.pdf`

Challenge

Considering the differential equation $y' = (y^2 - 16)(y + 3)^2$, make 3 separate lists containing any equilibrium, semi-stable and unstable y-values.

To check your answer, sum the value of each list. If there are no values in a list, simply enter "none" to check the result.

Solution

Stable

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of kX = ffc446

Semi-stable

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of zX = f76cc4

Unstable

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

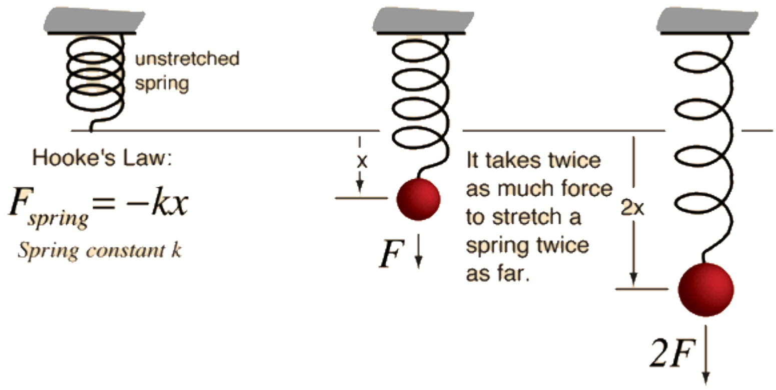
Hash of xX = bf947d

Chapter 4

2nd-order differential equations

4.1 Hooke's law

Comment



(Image from HyperPhysics by Rod Nave, Georgia State University)

Second-order differential equations deal with oscillations. Here we consider harmonic oscillation of a spring. The aim of this challenge is to give you the opportunity to think about how the terms of a 2nd-order ODE relate to force and stiffness in the context of a spring.

Equation 4.1 is a fundamental equation of mechanics describing oscillatory motion such as the spring here. Hooke's law states that the force leading to acceleration of the mass m is proportional to the stretching distance x. The proportionality constant is Hooke's constant, k.

$$mx'' + kx = 0 \tag{4.1}$$

or alternatively

$$mx'' = -kx \tag{4.2}$$

This leads to perfectly oscillating motion,

$$x(t) = \cos(\omega t) \tag{4.3}$$

which oscillates forever since there is no damping term.

Challenge

By considering the oscillatory motion (equation 4.3) as a solution of the 2nd-order differential equation given by Hooke's law (equations 4.1 and 4.2), determine the oscillation frequency ω in terms of the mass and spring constant.

Solution

To check your answer, calculate the oscillation frequency for a harmonic spring with a mass of 2 kg and spring-constant of 4 kg/s^2. Only enter numbers, without any units, in your answer.

X = Your solution
Form: Decimal to 2 decimal places.
Place the indicated letter in front of the number.
Example: aX where $X = 46.00$ is entered as a46.00

Hash of gX = 9553fe

4.2 Exponentials and trigonometry

Resources

- Text: `https://www.phy.duke.edu/~rgb/Class/phy51/phy51/node15.html`

Challenge

Write $\sin(x)$ and $\cos(x)$ in exponential form.

Solution

Please compare your solution with your partner or discuss with the teacher.

4.3 Characteristic equation: understanding

Resources

- Book (`http://tutorial.math.lamar.edu/getfile.aspx?file=B,1,N`) from page 106.

Comment

It is possible to add a damping term B to Hooke's law that is proportional to the velocity of the movement. You could imagine this as a friction term, with the force from friction becoming stronger as the velocity increases.

Challenge

$$A\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Cy = 0 \tag{4.4}$$

Show that, assuming that all solutions to a 2nd-order differential equation of the form above will have solutions $y(t) = e^{rt}$, the value of r can in principle be determined by solving the following a quadratic equation of the form

$$Ar^2 + Br + C = 0 \tag{4.5}$$

Solution

If you are unsure of your derivation, please ask someone.

4.4 Characteristic equation: roots

Resources

- Book (`http://tutorial.math.lamar.edu/getfile.aspx?file=B,1,N`) from page 106.

Challenge

Sum the points of the differential equations that have characteristic equations with

- Real, distinct roots
- Complex roots
- Equal roots

1 point: $-3y'' - 5y' + 2y = 0$

2 points: $3y'' - 4y' + 3y = 0$

4 points: $3y'' - 6y' + 3y = 0$

8 points: $3y'' - 5y' + 2y = 0$

16 points: $3y'' - 5y' + 4y = 0$

32 points: $3y'' + 5y' + 2y = 0$

Solution

Real, distinct roots

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of iX = dc6ada

Complex roots

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of jX = 7c030b

Equal roots

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of kX = c90b44

4.5 Characteristic equation: real roots with positive B

Resources

- Book (`http://tutorial.math.lamar.edu/getfile.aspx?file=B,1,N`) from page 108.

Challenge

1. Solve the following 2nd-order differential equation that has real roots:

$$y'' + 3y' + 2y = 0 \tag{4.6}$$

2. What is the effect of including a positive damping (friction) term?

Solution

1. $y(1) = 1.14$ given initial conditions $y(0) = 5$ and $y'(0) = -8$.
2. Please compare your answer with your partner or discuss with the teacher in class.

4.6 Characteristic equation: real roots with negative B

Resources

- Book (`http://tutorial.math.lamar.edu/getfile.aspx?file=B,1,N`) from page 108.

Comment

Here we include a damping term again, but this time it is positive.

Challenge

1. Solve the following 2nd-order differential equation that has real roots.

$$y'' - 3y' + 2y = 0 \tag{4.7}$$

2. What is the effect of changing the damping term from positive to negative?

Solution

1. $y(1) = -47.13$ given initial conditions $y(0) = 5$ and $y'(0) = -8$.
2. Please compare your answer with your partner or discuss with the teacher in class.

4.7 Characteristic equation: B in equations with real roots

Challenge

(Note that there are two parts to this challenge.)

Considering the equation

$$Ay'' + By' + Cy = 0 \tag{4.8}$$

1 point: Positive damping (positive B) leads to solutions with exponentials with positive exponents.

2 points: Positive damping (positive B) leads to solutions with exponentials with negative exponents.

4 points: Negative damping (negative B) leads to solutions with exponentials with positive exponents.

8 points: Negative damping (negative B) leads to solutions with exponentials with negative exponents.

16 points: Exponentials with positive exponents (eg, e^t) lead to exponential growth (instability).

32 points: Exponentials with negative exponents (eg, e^{-t}) lead to exponential growth (instability).

64 points: Exponentials with positive exponents (eg, e^t) lead to a damped signal (stability).

128 points: Exponentials with negative exponents (eg, e^{-t}) lead to a damped signal (stability).

Solution

X = Your solution
Form: Integer.
Place the indicated letter in front of the number.
Example: aX where $X = 46$ is entered as a46

Hash of oX = 8e808b