Exact coherent state solutions in channel flow

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1 Introduction

Shear flows describe any flow in which a mainstream flow is parallel to a boundary surface of wall. The requirement of no-slip at such boundaries induces a shearing of the flow with movement away from the boundary, as the flow speed adjusts from zero relative to the boundary up to the mainstream value. Such flows find a large number of applications, including flow through pipes and ducts, or the flow over an aircraft wing for example. Studies in this area focus on a number of canonical flows, including pipe flow, channel flow, plane Couette flow and so on, with the expectation that all the important mechanisms and flow phenomena can be understood by understanding these canonical flows. Such understanding is as yet incomplete. In particular, in most cases transition appears abruptly, bypassing the linear stage (i.e. transition that can be explained by linear stability theory), and so alternative approaches are required.

The present study in particular seeks to add to understanding of transition and turbulence in channel flow, flow driven between 2 parallel plates by a constant imposed pressure gradient as shown in Fig.1. Linear stability theory predicts transition for Reynolds number $R > 5772$, but in practice transition can be observed for $R > 1000$ or so. We approach the problem from a dynamical systems viewpoint, and seek simple exact solutions to the governing Navier-Stokes equations. In particular we start from recently obtained travelling-wave solutions for channel flow subject to a spanwise system rotation[1], and attempt to continue these solutions by homotopy to the non-rotating case.

2 Mathematical Formulation

The governing dimensionless Navier-Stokes equations are expressed in a rotating frame of

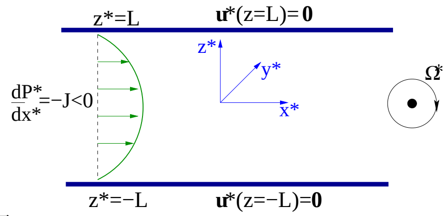

図 1. Channel configuration in dimensional coordinates.

reference by

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nabla^2\mathbf{u} - [0,\Omega,0]\times\mathbf{u}, \quad (1)$$

where velocity $\mathbf{u}$ additionally satisfies the incompressibility condition, p and t denote pressure and time respectively, and x, y, z denote distances in the streamwise, spanwise and wall-normal directions respectively. All lengths have been scaled by half the channel width (L), and the velocity by $V = \nu/L$, where ν denotes the kinematic viscosity. Two physical parameters control the flow: the rotation number $\Omega = 2\Omega^* L^2/\nu$, and Reynolds number $R = L^3 J/2\rho\nu^2$, where J is a constant imposed streamwise pressure gradient. We find solutions to (1) other than the laminar basic flow given by $\mathbf{u} = (u_0(z) = R(1-z^2), 0, 0)$, by seeking solutions for an imposed disturbance $\hat{\mathbf{u}} = \mathbf{u} - \mathbf{u_0}$, $\hat{p} = p - p_0$ in a travelling wave form, with solutions obtained by Newton iteration[1].

3 Results

Of the 15 distinct flows found[1]for the rotating channel flow case, only two, which we label 'TW1' and 'TW2' could be continued to the (non-rotating) channel flow case. Both these flows appear in saddle-node bifurcations with increasing R. Neither of these flows has symmetry about the channel centreplane, but the lower-branch of the TW1 flow approaches a symmetric flow structure as this solution branch approaches a termination point at $R = 1404$. Using this flow as an initial guess we have identified a third flow, 'TW3', which the