# General framework to analyze long-range degree correlations in complex networks

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We propose a general way to analyze degree correlations between nodes separated by more than one step in complex networks. While degree correlations between nearest neighbors are extensively studied, it has been pointed out recently that many properties and dynamics of complex networks are related strongly to long-range (i.e., beyond nearest-neighbor) degree correlations (LRDCs) in the sense of shortest path distance [1]. However, only limited quantities have been proposed to pick up some specific aspects of LRDCs up to now. For a global and multilateral analysis, a more general framework to describe the entire information of LRDCs is required.

Here, to completely describe LRDCs between degrees $k$ and $k'$ of two nodes separated by a shortest path distance $l$, we introduce one joint probability and four conditional probabilities as functions of $k$, $k'$, and $l$. Our framework based on these probability functions includes the conventional treatment describing nearest neighbor degree correlations, as a special case of $l = 1$. These five probabilities are not independent of each other. We present general relations among them using the Bayes' theorem and clarify the relevance to nearest-neighbor degree correlations [2].

Also, we analytically calculate these five probabilities for networks without any degree correlations at an arbitrary distance under the mean-field approximation and the local tree assumption [3]. The five probabilities for a long-range uncorrelated network are criteria to determine the existence or nonexistence of LRDCs in a given network. By comparing the probabilities for a given network with those for the corresponding long-range uncorrelated network, which are analytically calculated, one can capture the feature of LRDCs. We apply the present framework to synthetic models and real-world networks and demonstrate the validity of our argument.

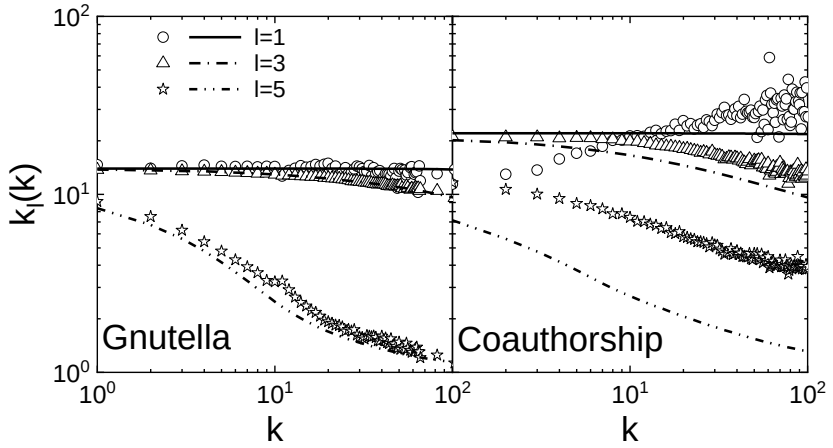


Figure 1. Average $l$th neighbor degree $k_l(k)$ is average degree of nodes separated by $l$ from a node of degree $k$. Symbols represent $k_l(k)$ of the real-world networks, the Gnutella network [4] and the coauthorship network [5], at fixed values of $l$, and curves indicate those of corresponding long-range uncorrelated networks.

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