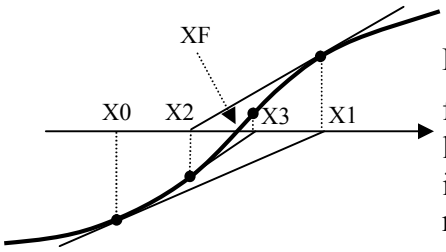
# APS106 LAB # 6 - THURSDAY, MARCH 6, 12:00 - 2:00

Unlike linear equations, it is hard to find closed-form (direct) solutions for non-linear equations. Instead, some kind of iterative method is required for many non-linear equations. One of the most popular iterative methods is known as the Newton-Raphson method. In the Newton-Raphson method of finding the solution, one starts with a rough estimate of the solution. A new improved solution is then obtained using the following scheme:

$$x^{New} = x^{Old} - \frac{f(x^{Old})}{f'(x^{Old})}$$




Here, $f(x) = 0$ is the non-linear equation, $f(x)$ is the function and $f'(x)$ is its derivative. In the diagram on the left, X0 is the starting point, X1, X2, X3… are the successive improved approximations. XF is the value of *x* when the required accuracy is attained.

An artillery gun fires a shell at an initial angle $\alpha$ to the horizontal with an initial speed $V_0$. The shell undergoes significant air resistance proportional to its velocity. Under this condition the horizontal range is given by the equation:

$$x = B\left(1 - e^{-x/A}\right) \quad \text{where } A = \frac{V_0 g \cos\alpha}{\gamma\left(V_0 \gamma \sin\alpha + g\right)} \quad \text{and} \quad B = \frac{V_0 \cos\alpha}{\gamma}$$

You may assume the following numerical values:

initial speed $V_0 = 750 m/s$,

firing angle $\alpha = 50^o$,

gravitational acceleration $g = 9.8 m/s^2$, and

drag coefficient $\gamma = 0.1/s$

**Evaluate the horizontal range:**

a. Rearrange the projectile equation in the form $f(x) = 0$.
b. Find the derivative, $f'(x)$ of the non-linear function, $f(x)$.
c. Write two functions that output the value of the function and the value of its derivative for any given value of *x*.
d. Write the calling program (main) that implements the Newton-Raphson method to find the zero of the non-linear equation. (**Hint:** Artillery shells could reach 3 –15 km)
e. Compute the range of the artillery accurate to 2 decimal points and print it to the console with the required number of iterations. In other words, the difference between the

solution XF and the value of $x$ found on the iteration prior to the last one should not be greater than 0.001.