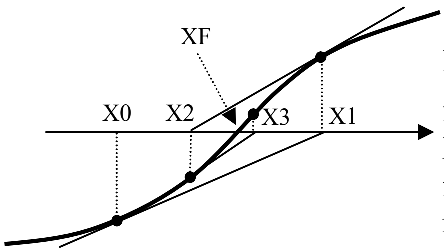
APS106 LAB # 6 - MONDAY, MARCH 3, 2:00 - 4:00

Unlike linear equations, it is hard to find closed-form (a kind of direct) solutions for non-linear equations. Some kind of iterative method is required for many non-linear equations. One of the most popular iterative methods is known as the Newton-Raphson method. In the Newton-Raphson method of finding the solution, one starts with a rough estimate of the solution. A new improved solution is then obtained using the following scheme:

$$x^{New} = x^{Old} - \frac{f(x^{Old})}{f'(x^{Old})}$$

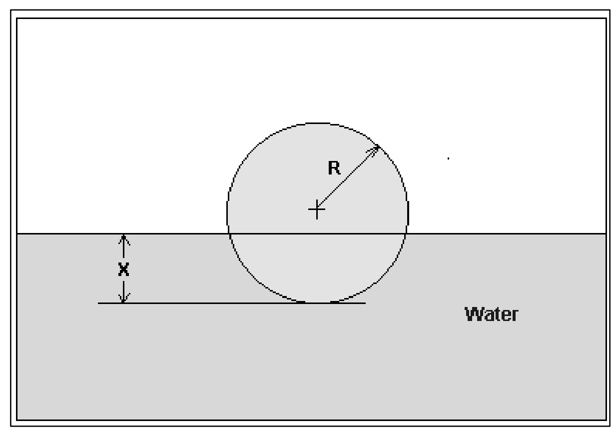


Here, $f(x) = 0$ is the non-linear equation, $f(x)$ is the function and $f'(x)$ is its derivative. In the diagram on the left, X0 is the starting point, X1, X2, X3… are the successive improved approximations. XF is the value of x when the required accuracy is attained.

Suppose, you are working for 'DOWN THE TOILET COMPANY' that makes floats for commodes. The ball has a specific gravity of 0.6 and has a radius of 5.5 cm. You are asked to find the distance to which the ball will get submerged when floating in water.

The equation that gives the depth 'x' (in metres) to which the ball is submerged under water is given by

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$



In this lab exercise,

a. Rearrange the equation into the form $f(x) = 0$.
b. Find the derivative, $f'(x)$ of the non-linear function, $f(x)$.
c. Write two C-functions that output the value of the function and the value of its derivative for any given value of x.
d. Write the calling program (main) that implements Newton-Raphson method to find the zero of the non-linear equation in the specified range. Analyze the problem to find a good initial guess.
e. Compute the value of x at equilibrium correct to five decimal places, and print it (to the console) with the required number of iterations. In other words, the difference between the solution XF and the value of x found on the iteration prior to the last one should not be greater than 0.000001.