Metrics Class notes

Class 4 and 5

October 2, 2020

The model

$$y_i = e(z_i, \epsilon_i, \theta)$$

where $\epsilon \sim f(\cdot|z, \theta)$ which is the unobserved heterogeneity *often multidimensional). We have many inconditional moments

Simulation-Based Methods

$$\mathbb{E}[y^h|x, \theta] = \int e^h(x, \epsilon, \theta) f(\epsilon|x, \theta) d\epsilon \equiv m_h(z, \theta)$$

The conditional moment $E[y^h - m_h(x, \theta_0)|x] = 0$ is intractable because of multidimensionality of ϵ (≥ 4) or $e(\cdot)$ is intractable. But the condition imply many unconditional moment conditions:

$$\mathbb{E}[\psi_h(z)[y^h - m_h(z; \theta_0)]] = 0$$

If the problem is the dimensionality of ϵ then we can do the following

$$E[y^h|x, \theta] 0 \frac{1}{S} \sum_{s=1}^{S} e^h(x_i, \epsilon_{is}; \theta) \frac{f()}{g()}$$

. . .

Back to Pakes and Pollard

$$\mathbb{E}[\psi_h(z)[y^h - m_h(z; \theta_0)]] = 0$$

Stack this H moments in one vector $G(\theta)$

$$G(\theta) = \int h(y, x, \theta) dP(y, x)$$

since we can not compute m^h, $h()$ is intractable. They assume that there exist a function H such that

$$h(y, x; \theta) = \int H(y, x, \epsilon) dP(\epsilon|y, x)$$

If the problem is the dimensionality of ϵ

$$H(y, x, \theta) = \psi_h(x) \left(y^h - e^h(x, \epsilon, \theta) \frac{f(\epsilon|x, \theta)}{g(\epsilon|x, \theta)} \right)$$

where we used the fact that ϵ is independent of y so $E[\epsilon|y, x, \theta] = E[\epsilon|x, \theta]$. So we can approximate

$$\hat{h}(y_i, x_i, \theta) = \frac{1}{S} \sum_{s=1}^{S} H(y_i, x_i, \epsilon)$$

where $\epsilon_{is} \sim P(\cdot|y_i, x_i)$ iid. In the example,

$$\hat{h}(y_i, x_i, \theta) = \psi_h(x_i) \left[y_i^h - \frac{1}{S} \sum_{s=1}^{S} e^h(x_i, \epsilon_{is}, \theta) \frac{f(\epsilon_{is}|x_i, \theta)}{g(\epsilon_{is}|x_i, \theta)} \right] \tag{1}$$

SMM

In GMM the idea is to $||\frac{1}{n} \sum^N h(y_i, x_i; \theta|| \simeq 0$. In SMM the idea es to replace h by $\hat{h}$.

Multinomial Choice Model (McFadden 89')

Individual has $m \geq 2$ alternatives. Vector of covariates $[z_1, ..., z_m]$ and a random vector of individual weights α_i. Utility from j-th alternative is $z_j' \alpha$ and chooses it if $z_j' \alpha_i \geq z_k' \alpha_i$ for all $k \neq j$.

Individual unobserved heterogeneity is $\alpha_i \sim h(\eta, \theta_0)$ with $\eta \sim g(\cdot)$, where h and g are known.

Multinomial probit is the case where $\alpha_i \sim N_h(\mu, \Sigma) = h(\eta, \mu, \Sigma)$.

[I got lost here..] he explains why α is generates as a $k \times 1$ vector function $h(\eta, \theta_0)$

of an r-dimensional random vector η with known distribution. [Some discussion about independence that I did not understand]

so... the utility from individual is

$$u_{ji} = z_j'\alpha_i = z_{1j}\alpha_{1i} + ... + z_h\alpha_{hi}$$

[He talks about limitations of multinomial logit]

if covariates are stacked in a $m \times k$ matrix Z, the choice is specified by the response vector

$$d = D[Zh(\eta, \theta_0)]$$

where D maps into $\{0,1\}^m$. Note that $d = y$ and $D = e(x, \epsilon, \theta)$. So let π

$$E[d|x, \theta] = \pi(z, \theta)$$

Why is π intractable?

$$\pi(z, \theta) = \int_{dimK} D[Zh(\eta, \theta_0)]dP(\eta)$$

so the problem is the dimensionality.

Class 5

Now we go back to example 4.2 of the paper.

Individual utility from j to agent i gives

$$U_{i,j} = z_j'\alpha_i$$

The Random Utility Model establishes that i choose alternative j iff

$$U_{i,j} \geq U_{i,j'}, \forall j' \neq j$$

The unobserved heterogeneity is on α

$$\alpha_i = h(\eta_i, \theta_0) \text{ where } \eta_i \sim g(\cdot)$$

with $h(\cdot)$ and $g(\cdot)$ are known. When $\alpha_i \sim \mathcal{N}(\mu_0, \Sigma_0)$ then we are in the Multinomial Probit model.

Stacking everything in matrices

$$\begin{bmatrix} u_{1i} \\ \dots \\ u_{ni} \end{bmatrix} = Z\beta_i = Zh(\eta_i, \theta_0)$$

and

$$d = \begin{bmatrix} d_{1i} \\ \dots \\ d_{ni} \end{bmatrix} = D[Zh(\eta_i, \theta_0)]$$

Last class we said that this is intractable because of the dimensionality. Now we are going to see how we simulate here. We want to look at

$$\pi(z, \theta) = E[d|z, \theta] = \int_{dimK+M} D[Zh(\eta, \theta_0)]g(\eta)d\eta$$

where the dimensionality problem comes from the integral being of dimension $K+M$ (which also kills the option to do MLE). We want to look for the θ that does

$$E[d|Z, \theta_0] - \pi(Z, \theta_0)] = 0$$

We know that this condition implies the unconditional moment

$$E[W(Z)[d - \pi(Z, \theta_0)]] = 0$$

but now d and Z are random, so we integrate over the measure $dP(d, Z)$

$$\underbrace{G(\theta)}_{k\times 1} = \int W(Z, \theta)[d - \pi(Z, \theta)]dP$$

where $G(\theta_0) = 0$. So now we are going to simulate π. We need to replace $\pi(Z_i, \theta)$ by a simulation estimator. For each individual generate s new random variables $\eta_{i1}, ..., \eta_{is}$ and replace

$$\pi(Z_i, \theta) \simeq \hat{\pi}_s(Z_i, \theta) = \frac{1}{s}\sum^{s} D[Z_i h(\eta, \theta)]$$

and we do

$$G_n(\theta) = n^{-1} \sum_{i=1}^{n} W(Z_i, \theta)[d_i - \hat{\pi}_s(Z_i, \theta)]$$

this works for $s = 1$ but the precision of the estimator increase with s.

[I was lost in the variance discussion]

Indirect inference - Gourieroux et al.

Consider the dynamic model

$$y_t = e(y_{t-1}, x_t, u_t, \theta)$$
$$u_t = \phi(u_{t-1}, \epsilon_t, \theta), \theta \in \Theta \in \mathbb{R}^{\mathbb{P}}$$