

**GOVERNMENT OF TAMILNADU  
DIRECTORATE OF TECHNICAL EDUCATION  
CHENNAI – 600 025**

**STATE PROJECT COORDINATION UNIT**

**Diploma in Electronics and Communication Engineering**

**Course Code: 1040**

**M – Scheme**

**e-TEXTBOOK**

**on**

**ELECTRICAL CIRCUITS AND INSTRUMENTATION**  
**for**  
**III Semester DECE**

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## 34032 ELECTRICAL CIRCUITS AND INSTRUMENTATION DETAILED SYLLABUS

### UNIT I

D.C. CIRCUITS AND THEOREMS Definition and unit for voltage, current, power, resistance, conductance, resistivity- Ohm's law – only simple problems in ohm's law- Kirchoff's current law and voltage law. Series circuits –parallel circuits, series parallel circuits. Mesh Method (simple problems) Thevenin's - Norton's theorems, Super position and Maximum power transfer theorem – Statement and Explanation (simple problems)

### UNIT II

A.C. CIRCUITS AND RESONANCE A.C. CIRCUITS AC through single pure resistance, pure inductance, pure capacitance - voltage and current relationship and (to mention only) the equation for power and power factor in each case ( only simple problems). Definition for impedance, reactance, admittance, conductance, impedance, Phase angle, power factor and power. AC circuits – Derivation for impedance and admittance, power and power factor in Series and Parallel R-L ,R-C ,R-L-C circuits. Analysis of Parallel R-L circuit, R-C circuit, R-LC circuit (qualitative treatment only). RESONANCE Resonance- series resonance – parallel resonance - condition for resonance-resonant frequency-Q factor - resonance curve-bandwidth (only simple problems).

### UNIT III

#### TRANSFORMERS AND MACHINES TRANSFORMERS

Transformer – Ideal transformer – construction - working principle –EMF equation Losses in transformer- core loss, copper loss- Efficiency- Regulation OC, SC test on transformer -List of applications (qualitative treatment only) MACHINES D.C. Machines - DC–Generator –Working principle - Types- Applications DC motor- working principle - types- applications (qualitative treatment only ) Single phase induction motor- types- construction and principle of operation of capacitor start induction motor- Applications - stepper motor-working principle-uses (qualitative treatment only)- Universal Motor (qualitative treatment only) Difference between single phase and three phase supply.

### UNIT IV

MEASURING INSTRUMENTS AND CRO MEASURING INSTRUMENTS      Definition for Measurement, Instrument- Errors in Measurement - Calibration- Indicating instruments – Basic forces for indicating instruments - construction and operation of permanent magnet moving coil Instrument -Advantages – Disadvantages of PMMC - Shunts and Multipliers - DC ammeter-DC voltmeter-voltmeter sensitivity. Bridges- Types - Wheat stone bridge - applications -Universal impedance bridge arrangements to measure R, L,C CRO CRO- Block diagram and principle of operation of CRO- operation of CRT Electrostatic focusing- Electrostatic deflection (no derivation ) - Block diagram of vertical deflection system- Applications of CRO - Types of CRO- Block diagram and operation of dual trace CRO- Dual beam CRO - Comparison between dual trace and dual beam CRO – Digital storage Oscilloscope - Block diagram- advantage. Block diagram-working principle of Function Generator

## UNIT V

**TRANSDUCERS , SENSORS & TEST INSTRUMENTS**      **TRANSDUCERS** Transducers – Classification of transducers Strain gauge - Types-uses. Construction, operation and applications of photo electric transducer, LVDT, RVDT and Load cell. Principle of working of Thermocouple- Temperature measurement using thermocouple - list of applications- Principle of working of Thermistor –Temperature measurement using thermistors - Types (NTC, PTC) – List of applications. **SENSORS** IR range sensor – IR proximity sensor- Ultrasonic range sensor- Touch Sensor. **TEST INSTRUMENTS** Digital voltmeter –Types ( to list only ) - Basic block diagram of DVM - Block diagram of Digital multimeter- Advantages over analog instruments - Block diagram of Digital frequency counter– Simple PC based Data Acquisition system – Block

## CONTENT

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## UNIT – I

### D.C CIRCUITS AND THEOREMS

#### INTRODUCTION

In the study of electric circuits we are interesting in the flow of electricity from one device to another. A simplest of electric devices will have a pair of terminals. Electricity enters at one terminals and leaves from the another.

The energy associated with an electric system is measured by quantities like electric charge, potential and current. Historically electric charges were first observed by rubbing certain dry substances together. Atomic physicists now picture the electric charges as one of the building blocks of the universe. The negatively charged electron is one of the constituents of the atom. Its mass is  $9.107 \times 10^{-28}$  gm. As the electron is very small unit, the practical unit of charge is the coulomb.

$$1 \text{ coulomb} = 9.107 \times 10^{-28} \times Q_e$$

Where  $Q_e$  is the electronic charge.

#### BASIC CONCEPT

All matter consists of minute particles called molecules. Molecules are made up of more minute particles called atoms. Inside the atom there a central nucleus and a number of electrons around the nucleus. The nucleus contains protons and neutrons. Protons are positively charged particles and neutrons do not carry any charge. The electron carries a negative charge.

In a normal atom, the positive charge on the nucleus is exactly equal to the total negative charge due to all the electrons surrounding the nucleus.

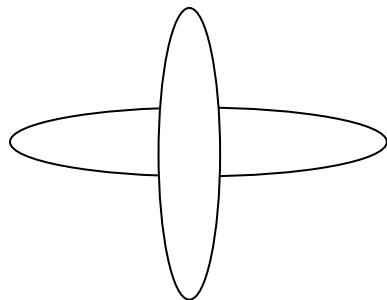


Fig 1.1

Every atom in which the positive and negative charges are equal has no net positive and negative charge.

Electrons revolve around the nucleus in elliptical orbits. The first orbit has a maximum of 2 electrons, second 8 electrons, third 18 electrons and so on. Number of electrons present in the orbit is  $2n^2$ , where 'n' is the orbit number.

## IMPORTANT DEFINITIONS

### 1.1.1. Current

Flow of electrons in any conductor is called current. It is represented by the letter 'I'

The unit of the current is **ampere**

$$\text{Current, } I = \frac{\text{Charge}}{\text{Time}}$$
$$I = \frac{Q}{T}$$

Where 'Q' is in coulomb and 't' is in second.

### 1.1.2. One Ampere

One ampere can be defined as one coulomb of charge is transferred in one second.

### 1.1.3 Voltage

The difference of potentials between two points is called voltage or potential difference.

The voltage represented by the letter 'V'

The unit of the voltage is **volt**.

### 1.1.4. One volt

One volt is defined as the potential difference across a resistance of one ohm carrying a current of one ampere

### 1.1.5. Resistance

The opposition offered by a substance to the flow of current is called resistance.

The resistance represented by the letter 'R'

The unit of the resistance is **Ohm ( $\Omega$ )**

### Laws of Resistance

The resistance (R) of a conductor

- is directly proportional to its length (l)
- is inversely proportional to its area of cross section (a)

- depends up on the nature of material
- depends on its temperature

$$R \propto \frac{l}{a}$$

$$R = \frac{\rho l}{a}$$

Where  $\rho$  (Rho) is a constant for the material called resistivity (or) specific resistance

### Conductance , G

Conductance is the reciprocal of resistance.

The conductance represented by the letter 'G'

The unit of the conductance is mho ( )

$$G = \frac{1}{R} = \frac{a}{\rho l}$$

$$= \frac{\sigma a}{l}$$

Where  $\sigma = \frac{1}{\rho}$

where  $\sigma$  (sigma) is called the conductivity.

### Conductivity

The reciprocal of resistivity is called conductivity.

### Problems

#### Example 1.1

Find the resistance of a copper wire of 0.75 km long and having a cross sectional area of 0.01cm<sup>2</sup>. (Take  $\rho = 1.72 \times 10^{-8}$  ohm-m)

#### Given data

Length of wire, $l$	=	0.75km	=	$0.75 \times 10^3$ m
cross sectional area , $a$	=	0.01cm <sup>2</sup>	=	$0.01 \times 10^{-4}$ m <sup>2</sup>
specific resistance, $\rho$	=	$1.72 \times 10^{-8}$ ohm-m		

#### Solution

$$\begin{aligned}
 \text{Resistance, } R &= \frac{\rho l}{a} \\
 &= \frac{1.72 \times 10^{-8} \times 0.75 \times 10^3}{0.01 \times 10^{-4}} \\
 &= \frac{1.29 \times 10^{-8}}{0.01 \times 10^{-4}} \\
 R &= 2.9 \Omega
 \end{aligned}$$

### Example 1.2

Find the cross sectional area of a aluminum wire of 700m long and having a resistance of 0.24 ohm (Take=  $\rho = 2.83 \times 10^{-8}$  ohm-m)

#### Given data

$$\begin{aligned}
 \text{Length of wire, } l &= 700 \text{ m} \\
 \text{Resistance, } R &= 0.24 \text{ ohm} \\
 \text{Specific resistance, } \rho &= 2.83 \times 10^{-8} \text{ ohm-m}
 \end{aligned}$$

#### Solution

$$\begin{aligned}
 \text{Resistance, } R &= \frac{\rho l}{a} \\
 \text{Cross sectional area , } a &= \frac{\rho l}{R} \\
 &= \frac{2.83 \times 10^{-8} \times 700}{0.24} \\
 &= \frac{1.81 \times 10^{-5}}{0.24} \\
 a &= 8.254 \times 10^{-5} \text{ m}^2
 \end{aligned}$$

### Example 1.3

Find the resistance of a copper wire 1 km long and 0.5 cm dia given the specific resistance of copper as  $1.7 \times 10^{-8} \Omega\text{-m}$

#### Given data

$$\text{Length of wire in meter, } l = 1\text{Km} = 1000 \text{ m}$$

$$\frac{\pi}{8} \left( \frac{0.5}{2} \right)^2$$

$$\text{Area , } a = \frac{\text{-----} \times \text{-----}}{4 \quad 100}$$

$$\text{Specific resistance, } \rho = 1.7 \times 10^{-8} \text{ ohm-m}$$

### Solution

$$\begin{aligned}\text{Resistance, } R &= \frac{\rho l}{a} \\ &= \frac{1.7 \times 10^{-8} \times 1000}{0.785 \times 2.5 \times 10^{-5}} \\ &= \frac{1700 \times 10^{-8}}{1.962 \times 10^{-5}} \\ R &= 0.866 \text{ ohm}\end{aligned}$$

### 1.1.6.Ohm's Law

The current flowing in an electric circuit depends on two quantities, the applied emf and the resistance in the circuit. The connecting the three factors was established first by G.S. Ohm in 1826 and is popularly known as the Ohm's law.

#### Statement of Ohm's Law

At constant temperature, the current flowing through the conductor is directly proportional to the potential difference (voltage) between the two ends of the conductor

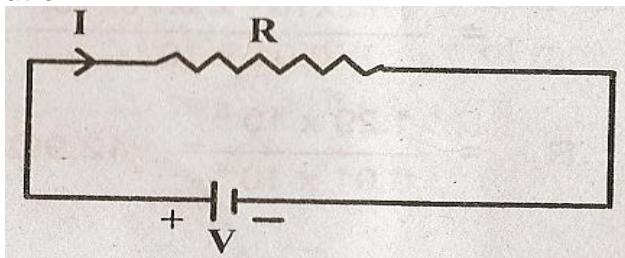
$$\text{i.e., } V \propto I$$

$$V = RI$$

$$V = IR$$

Where 'R' is the resistance of the conductor in ohms.

#### Explanation



**Fig. 1.2**

Consider a simple closed circuit as shown in the Fig.1.2.

Let, 'V' be the supply voltage (d.c)

'I' be the current flowing through the circuit (amp).

'R' be the resistance of the circuit (ohm).

According to Ohm's law,  $V \propto I$

$$\frac{V}{I} = R$$

i.e.,  $\frac{V}{I} = \text{Resistance (constant)}$

The unit of the resistance is Ohm

### 1.1.7. Power

Power is the rate of doing work.

$$\text{Power} = \frac{\text{Work done}}{\text{Time}}$$

The power represented by the letter 'P'

The unit of the power is **watt** or joules / second

Power is the product of voltage and current

$$\text{Power, } P = \text{voltage} \times \text{current}$$

$$P = VI \text{ watts}$$

(or)

$$P = (IR) I$$

$$P = I^2 R \text{ watts}$$

(or)

$$P = V \left( \frac{V}{R} \right)$$

$$P = \frac{V^2}{R} \text{ watts}$$

The unit of power can also be expressed in kilowatts (Kw) ,  $1 \text{ Kw} = 1000 \text{ watts}$

### 1.1.8. Energy

Energy is the capacity to do work. Energy is the product of the power ant time

$$\text{Energy} = \text{Power} \times \text{time}$$

$$= P \times t$$

$$= VI \times t$$

(or)

$$\text{Energy} = I^2R \times t$$

(or)

$$V^2 \times t$$

$$\text{Energy} = \frac{V^2}{R} \times t$$

The unit of energy is watt-sec. Watt-sec. is the smallest unit of energy. So kilo watt hour (Khr) is generally used.

## Problem

### Example 1.4

An electric iron is rated 1000 W and 240 V. Find the current drawn and resistance of the heating element.

#### Given data

$$\text{Power}, P = 1000 \text{ watts}$$

$$\text{Voltage}, V = 240 \text{ volts}$$

#### Solution

$$\text{Power}, P = \frac{V^2}{R}$$

$$\text{Resistance}, R = \frac{V^2}{P} = \frac{(240)^2}{1000}$$

$$= 57.6 \text{ Ohms}$$

$$\text{Current}, I = \frac{V}{R} = \frac{240}{57.6}$$

$$I = \frac{240}{57.6}$$

$$I = 4.17 \text{ amps.}$$

### Example 1.5

Calculate the current and resistance of a 50 w , 100 v electric bulb.

#### Given data

$$\text{Power , P} = 50 \text{ watts}$$

$$\text{Voltage. V} = 100 \text{ volts}$$

#### Solution

$$\text{Power , P} = VI \text{ watts}$$

$$\begin{aligned} \text{Current, I} &= \frac{P}{V} \\ &= \frac{50}{100} \end{aligned}$$

$$I = \frac{0.5 \text{ amps}}{V}$$

$$\text{Resistance ,R} = \frac{V}{I}$$

$$\begin{aligned} &= \frac{100}{0.5} \\ R &= 200 \text{ Ohms} \end{aligned}$$

### Example 1.6

Calculate the power rating of a heater coil when used on 200V supply taking 4 amps.

#### Given data

$$\text{Current , I} = 4 \text{ amps}$$

$$\text{Voltage. V} = 200 \text{ volts}$$

#### Solution

$$\text{Power , P} = VI \text{ watts}$$

$$= 200 \times 4$$

$$P = 800 \text{ watts}$$

### Example 1.7

A filament lamp connected to a 230V DC supply draws 300mA. Find the power absorbed by the lamp.

**Given data**

$$\text{Current, } I = 300 \text{ mA} = 300 \times 10^{-3} \text{ amps}$$

$$\text{Voltage, } V = 230 \text{ volts}$$

**Solution**

$$\begin{aligned}\text{Power, } P &= VI \text{ watts} \\ &= 230 \times 300 \times 10^{-3}\end{aligned}$$

$$P = 69 \text{ watts}$$

**Example 1.8**

A lamp having a resistance of  $500 \Omega$  works on 220 supply system. Determine the energy consumed by operating 30 days at the rate of 4 hours a day.

**Given data**

$$\text{Resistance, } R = 500 \Omega$$

$$\text{Voltage, } V = 220 \text{ volts}$$

**Solution**

$$\text{Energy} = \text{Power} \times \text{Energy} \text{ watt-sec.}$$

$$\text{Power, } P = \frac{V^2}{R}$$

$$= \frac{220^2}{500}$$

$$P = 96.8 \text{ watts}$$

$$\text{Total hours} = 30 \times 4 = 120$$

$$\begin{aligned}\text{Energy} &= 96.8 \times 120 = 11616 \text{ hr} \\ &= 11.616 \text{ Kwhr}\end{aligned}$$

**Example 1.9**

A building has following loads : twenty 100 watt lamps operated 4 hours daily and thirty 60 watt lamps operated 3 hours daily, all connected to a 230 volt source. Calculate (a) the total power (b) the total current (c) the monthly consumption electrical energy (d) the monthly electrical energy charges at 35 paise per unit

**Given data**

$$\text{Voltage, } V = 230 \text{ volts}$$

**Solution**

$$(a) \text{ Total power, } P = 20 \times 100 + 30 \times 60$$

$$P = 3800 \text{ watts} = 3.8 \text{ Kw}$$

$$\begin{aligned} (b) \text{ Total current, } I &= \frac{P}{V} \\ &= \frac{3800}{230} \\ &= 16.5 \text{ amps} \end{aligned}$$

$$\begin{aligned} (c) \text{ Energy consumption daily} &= 20 \times 100 \times 4 + 30 \times 60 \times 3 \\ &= 13,400 \text{ watt-hours} \end{aligned}$$

$$\begin{aligned} \text{Monthly consumption} &= 13400 \times 30 \\ &= 13400 \text{ Whr} = 13.4 \text{ Kwh} \end{aligned}$$

$$\begin{aligned} (d) \text{ the monthly electrical energy charges at } 35 \text{ paise per unit} &= 13.4 \times 30 \times 0.35 = \\ &= \text{Rs. 140.70} \end{aligned}$$

### 1.1.9. Resistors in series

A circuit is said to be a series circuit if the current flowing through all the elements is the same.

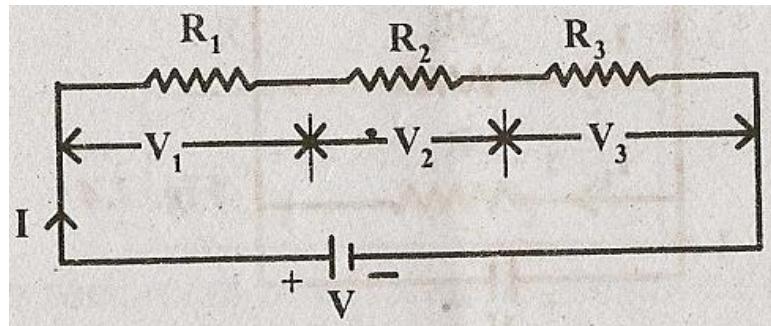


Fig.1.3

In fig.1.3 three resistors \$R\_1\$, \$R\_2\$, \$R\_3\$ are connected in series. So the current (\$I\$) flowing through each resistor is same. But the voltage drop across each resistor is different due to its different resistance value. The total voltage in this circuit is '\$V\$'

**According to Ohm's Law , \$V = IR\$**

Voltage drop across \$R\_1\$ is \$V\_1 = IR\_1\$

Voltage drop across \$R\_2\$ is \$V\_2 = IR\_2\$

Voltage drop across  $R_3$  is  $V_3 = IR_3$

$$\begin{aligned}\text{Total voltage } V &= V_1 + V_2 + V_3 \\ &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \\ IR &= I(R_1 + R_2 + R_3)\end{aligned}$$

$$R = (R_1 + R_2 + R_3)$$

The total (or) equivalent resistance is equal to the sum of all the resistances which are connected in series

#### 1.1.10. Resistors in Parallel

A circuit is said to be a parallel circuit if the potential difference ( voltage ) across all the elements is the same.

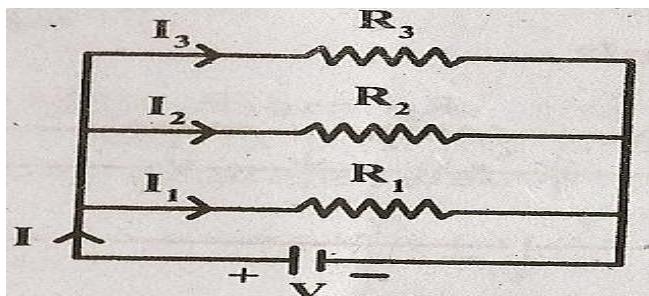


Fig.1.4

In fig.1.4 three resistors  $R_1$ ,  $R_2$ ,  $R_3$  are connected in parallel . So the potential drop across each resistor is same. But the current flowing through each resistor is different due its different resistance value. The total current  $I$  in the circuit divides to pass through the individual resistances.

$$\text{According to Ohm's Law , } I = \frac{V}{R}$$

$$\text{The current flowing through resistor } R_1 \text{ is } I_1 = \frac{V}{R_1}$$

$$\text{The current flowing through resistor } R_2 \text{ is } I_2 = \frac{V}{R_2}$$

The current flowing through resistor  $R_3$  is  $I_3 = \frac{V}{R_3}$

Total current  $I = I_1 + I_2 + I_3$

$$\begin{aligned} &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ &= V \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \\ \frac{V}{R} &= V \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \end{aligned}$$

$$\frac{1}{R} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

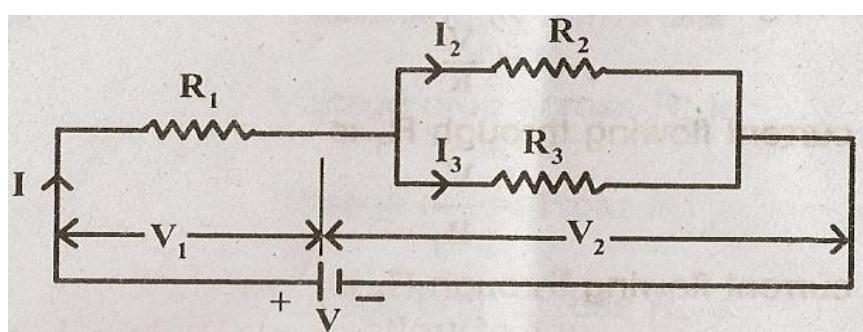
The reciprocal of equivalent resistance is equal to the sum of resistances which are connected in parallel.

When two resistor  $R_1$  and  $R_2$  are connected , then equivalent resistance is

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

#### 1.1.11.Resistors in Series - Parallel



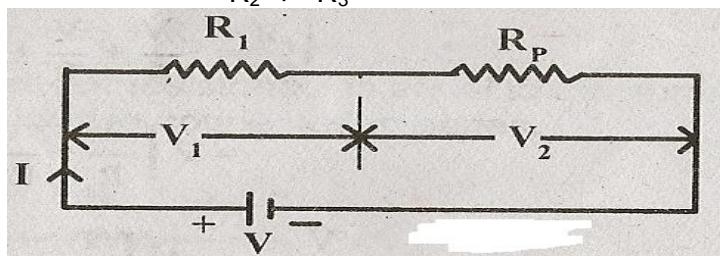
**Fig.1. 5**

A series parallel circuit is shown in fig.1.5. Here the resistors  $R_2$  and  $R_3$  are connected in parallel. But the resistor  $R_1$  is connected in series with this parallel combination.

Equivalent resistance of the resistors  $R_2$  and  $R_3$  (parallel combination) is

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_p = \frac{R_2 R_3}{R_2 + R_3}$$



**Fig .1.6**

$$\text{Total voltage } V = V_1 + V_2$$

$$= IR_1 + IR_p$$

$$= I (R_1 + R_p)$$

$$IR = I (R_1 + R_p)$$

$$R = R_1 + R_p$$

$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

The current flowing through resistor  $R_2$  is  $I_2 = \frac{IR_3}{R_2 + R_3}$

The current flowing through resistor  $R_3$  is  $I_3 = \frac{IR_2}{R_2 + R_3}$

## Problem

### Example 1.10

In the circuit shown in fig. 1.7 , Find the current , the voltage across each resistor and the power dissipated in each resistor.

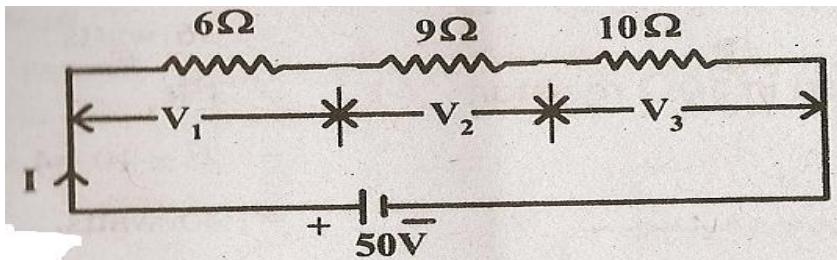


Fig. 1.7.

#### Given data

$$\text{Voltage } V = 50 \text{ volts}$$

$$\text{Resistors, } R_1 = 6\Omega, R_2 = 9\Omega, R_3 = 10\Omega$$

#### Solution

$$\text{Total resistance, } R = R_1 + R_2 + R_3$$

$$= 6 + 9 + 10$$

$$R = 25\Omega$$

$$I = \frac{V}{R}$$

$$I = \frac{50}{25}$$

$$I = 2 \text{ amps.}$$

$$\begin{aligned} \text{Voltage drop across } 6\Omega \text{ resistor, } V_1 &= IR_1 \\ &= 2 \times 6 \\ &= 12 \text{ volts} \end{aligned}$$

$$\begin{aligned} \text{Voltage drop across } 9\Omega \text{ resistor, } V_2 &= IR_1 \\ &= 2 \times 9 \\ &= 18 \text{ volts} \end{aligned}$$

$$\begin{aligned} \text{Voltage drop across } 10\Omega \text{ resistor, } V_3 &= IR_1 \\ &= 2 \times 10 \\ &= 20 \text{ volts} \end{aligned}$$

$$\begin{aligned} \text{Power dissipated in } 6\Omega \text{ resistor, } P_1 &= I^2 R_1 \\ &= 2^2 \times 6 = 4 \times 6 \\ &= 24 \text{ watts} \end{aligned}$$

$$\begin{aligned}\text{Power dissipated in } 6 \Omega \text{ resistor, } P_1 &= I^2 R_1 \\ &= 2^2 \times 9 = 4 \times 9 \\ &= 36 \text{ watts}\end{aligned}$$

$$\begin{aligned}\text{Power dissipated in } 6 \Omega \text{ resistor, } P_1 &= I^2 R_3 \\ &= 2^2 \times 10 = 4 \times 10 \\ &= 40 \text{ watts}\end{aligned}$$

### Example 1.11

Three resistors A, B and C are connected in series to a 240 volts source, as shown in fig. 1.8. If  $R_A = 60 \Omega$  and  $E_B = 40$  volts when the current is 0.5 amp. Calculate the resistance  $R_B$  and  $R_C$ .

**Fig. 1.8.**

#### Given data

$$\begin{aligned}\text{Total voltage, } V &= 240 \\ \text{Current, } I &= 0.5 \text{ amp.} \\ \text{Voltage, } E_B &= 40 \text{ volts} \\ \text{Resistor, } R_A &= 60 \Omega\end{aligned}$$

#### Solution

$$\text{Voltage } E_B = 40 \text{ V} = IR_B$$

$$R_B = \frac{40}{0.5} = 80 \Omega$$

$$\text{According to Ohm's Law, } I = \frac{V}{R} = \frac{V}{R_A + R_B + R_C}$$

$$0.5 = \frac{240}{60 + 80 + R_C}$$

$$240$$

$$60 + 80 + R_C = \frac{\dots}{0.5}$$

$$60 + 80 + R_C = 480$$

**Resistance  $R_C = 340 \Omega$**

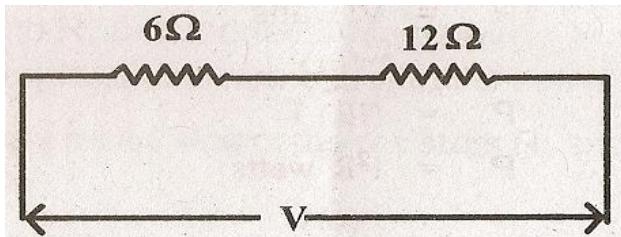
### Example 1.12

Two resistors resistance  $6 \Omega$  and  $12 \Omega$  are connected in series . Find the equivalent Also find the equivalent resistance when they are connected in parallel.

**Given data**

**Case 1:** Series combination of  $6 \Omega$  and  $12 \Omega$  resistors.

$$R_1 = 6 \Omega, R_2 = 12 \Omega$$



**Fig.1.9**

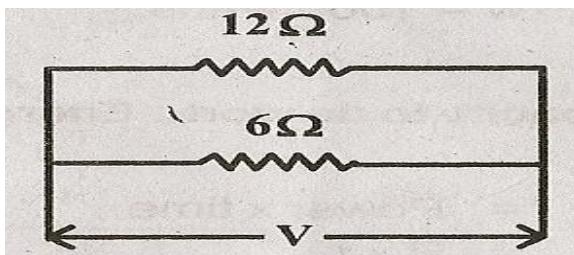
**Solution**

$$\begin{aligned} \text{Equivalent resistance, } R &= R_1 + R_2 \\ &= 6 + 12 \end{aligned}$$

$$R = 18 \Omega$$

**Case 2:** Parallel combination of  $6 \Omega$  and  $12 \Omega$  resistors.

$$R_1 = 6 \Omega, R_2 = 12 \Omega$$



**Fig.1.10**

$$\text{Equivalent resistance, } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$$R = \frac{6 \times 12}{6 + 12} = \frac{72}{18}$$

$$R = 4 \Omega$$

### Example 1.13

In the circuit shown in fig.1.11 , find the total resistance and current flowing through each branch.

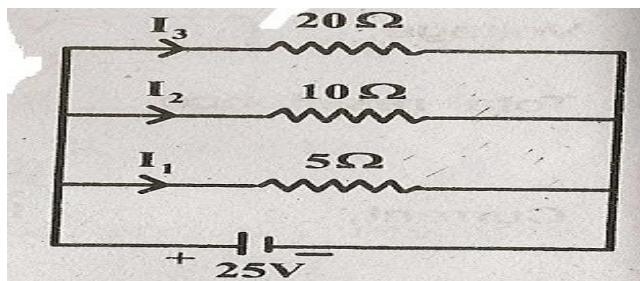


Fig.1.11

#### Given data

$$R_1 = 5 \Omega, R_2 = 10 \Omega, R_3 = 20 \Omega$$

#### Solution

$$\text{Equivalent resistance, } \frac{1}{R} = \frac{1}{5} + \frac{1}{10} + \frac{1}{20}$$

$$= 0.2 + 0.1 + 0.05$$

$$\frac{1}{R} = 0.35$$

$$\frac{1}{0.35} = R$$

$$\text{Total resistance, } R = 2.86 \Omega$$

$$\text{The current flowing through } 5\Omega \text{ resistor } I_1 = \frac{V}{R_1} = \frac{25}{5}$$

$$I_1 = 5 \text{ amps.}$$

The current flowing through  $10\Omega$  resistor  $I_2 = \frac{V}{R_2} = \frac{25}{10}$

$$I_2 = 2.5 \text{ amps.}$$

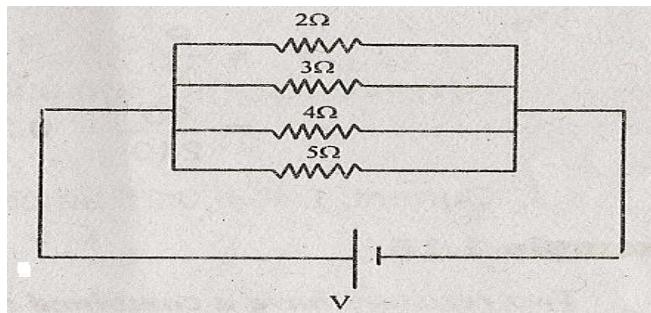
The current flowing through  $20\Omega$  resistor  $I_3 = \frac{V}{R_3} = \frac{25}{20}$

$$I_3 = 1.25 \text{ amps.}$$

### Example 1.14

Resistors of values 2, 3, 4, and 5 ohm are connected in parallel. If the total power absorbed by all the resistors is 200w, find the voltage applied to the circuit.

**Given data**



**Fig.1.12**

$$\text{Power, } P = 200 \text{ watts}$$

$$R_1 = 2 \Omega, R_2 = 3 \Omega, R_3 = 4 \Omega, R_4 = 5 \Omega$$

**Solution**

$$\text{Total resistance } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\begin{aligned} \text{Total resistance } \frac{1}{R} &= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \\ &= 0.5 + 0.33 + 0.25 + 0.2 \end{aligned}$$

$$\frac{1}{R} = 1.28$$

$$R = \frac{V^2}{P} = \frac{V^2}{1.28}$$

**Total resistance, R = 0.78 ohm**

$$\text{Power, } P = \frac{V^2}{R}$$

$$V^2 = P \times R$$

$$V = \sqrt{P \times R}$$

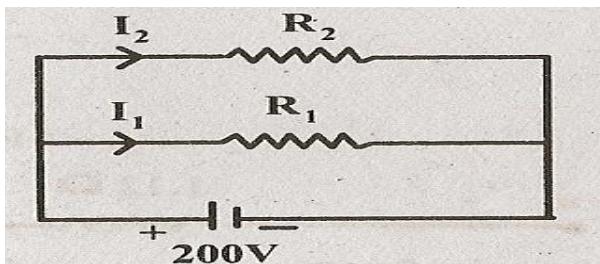
$$V = \sqrt{200 \times 0.78}$$

$$\text{Voltage, } V = 12.5 \text{ volts}$$

### Example 1.15

Two resistors are connected in parallel across a 200V mains take a total current of 10amps. The power dissipated in one of the resistors is 1200. watts. Find the value of each resistor.

**Given data**



**Fig.1.13**

$$\text{Voltage, } V = 200 \text{ volts}$$

$$\text{Current, } I = 10 \text{ amps.}$$

$$\text{Power dissipated in } R_1 \text{ of the resistors } P_1 = 1200 \text{ watts}$$

**Solution**

$$I_1 = \frac{P_1}{V} = \frac{1200}{200} \\ I_1 = 6 \text{ amps.}$$

$$\text{Resistors, } R_1 = \frac{V}{I_1}$$

$$R_1 = \frac{200}{6} = 33.33 \Omega$$

$$\begin{aligned} \text{Total current, } I &= I_1 + I_2 \\ 10 &= 6 + I_2 \\ I_2 &= 10 - 6 \\ &= 4 \text{ amps.} \end{aligned}$$

$$\text{Resistors, } R_2 = \frac{V}{I_2}$$

$$R_2 = \frac{200}{4} = 50 \Omega$$

### Example 1.16

A resistance is connected in parallel with a resistor  $R$  whose ohmic value is unknown. If the total current taken is 10.5 amps. When the circuit voltage is 210 V , calculate  $R$ , shown in fig.1.14.

#### Given data

**Fig.1.14**

$$\text{Voltage, } V = 210 \text{ volts}$$

$$\text{Current, } I = 10.5 \text{ amps.}$$

$$\text{Resistance, } R_1 = 30 \text{ Ohms} =$$

$$\begin{aligned} \text{Solution} \quad I_1 &= \frac{V}{R_1} = \frac{210}{30} \\ I_1 &= 7 \text{ amps.} \end{aligned}$$

**Total current,**       $I_1 = I_1 + I_2$   
 $I_2 = I - I_1$   
 $= 10.5 - 7$   
 $I_2 = 3.5 \text{ amps.}$

210

**Resistance, R**       $= \frac{210}{3.5} = 60 \text{ Ohms.}$

### Example 1.17

Three resistors of  $2\Omega$ ,  $7\Omega$ , and  $4\Omega$  are connected in parallel. This combination is put in series with  $3.5\Omega$  resistor. Determine the equivalent resistance of the combination.

#### Given data

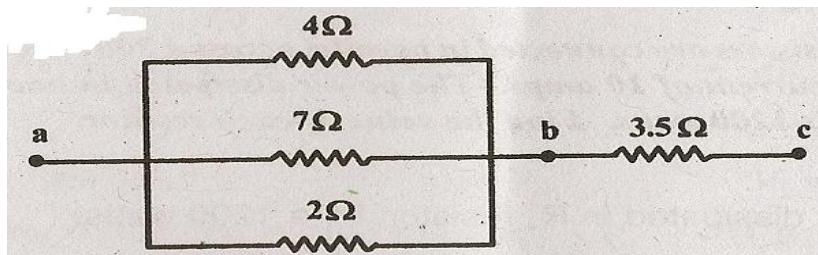


Fig.1.15

Resistors       $R_1 = 2\Omega$ ,  $R_2 = 4\Omega$ ,  $R_3 = 7\Omega$

#### Solution

$$\frac{1}{R_{AB}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{AB}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{7}$$

$$= 0.893$$

$R_{AB} = 1.12\Omega$

Total resistance,  $R_{AC}$        $= R_{AB} + R_{BC}$   
 $= 1.12 + 3.5$

$$R_{AC} = 4.62 \Omega$$

### Example 1.18

Two resistors of  $10 \Omega$  and  $20\Omega$  are connected in parallel. This combination is connected in series with  $4 \Omega$  resistor. Determine the current and voltage across  $15V$  resistor in the given circuit.

**Given data**

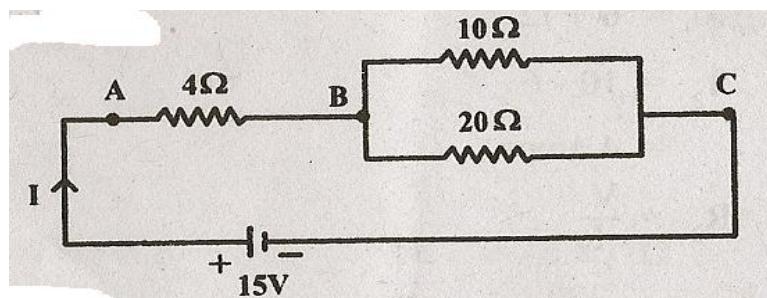


Fig.1.16

$$\text{Voltage, } V = 15V$$

$$\text{Resistors } R_1 = 4 \Omega, R_2 = 10 \Omega, R_3 = 20 \Omega$$

**Solution**

$$\text{Total resistance, } R_{AC} = R_{AB} + R_{BC}$$

$$R_{AB} = 4 \Omega$$

$$R_{BC} = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC}}$$

$$= \frac{10 \times 20}{10 + 20} = 6.67 \Omega$$

$$R_{AC} = 4 + 6.67$$

$$\text{Total resistance, } R_{AC} = \frac{10.67 \Omega}{V}$$

$$\text{Current, } I = \frac{V}{R}$$

$$I = \frac{15}{10.67} = 1.4 \text{ amps.}$$

Current through 20 ohm resistor

$$= I \times \frac{R_1}{R_1 + R_2}$$

$$= 1.4 \times \frac{10}{10 + 20} = 0.467 \Omega$$

Voltage drop across 4 Ω resistor,  $V_{AB}$

$$= I \times R \quad [ V=IR ]$$

$$= 1.4 \times 4$$

$$V_{AB} = 5.6 \text{ omhs}$$

Applied voltage , V

$$V_{BC} = V_{AB} + V_{BC}$$

$$= 15 - 5.6$$

$$= 9.6 \text{ volts.}$$

**Voltage drop across 20 Ω resistor,  $V_{BC}$  = 9.4 volts**

### Example 1.19

Two 50 Ω resistors are connected in series. When a resistor R is connected across one of them , the total circuit resistance is 60 Ω. Calculate the value of R , shown in fig.1.16

#### Given data

**Fig.1.17**

Total resistance = 60 Ω

#### Solution

Total resistance ,  $R_{AC} = R_{AB} + R_{BC}$

$$60 = R_{AB} + R_{BC}$$

$$R_{AB} = 50 \Omega$$

$$R_{BC} = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC}}$$

$$= \frac{50 \times R}{50 + R}$$

$$60 = 50 + \frac{50 \times R}{50 + R}$$

$$60 = \frac{2500 + 50R + 50R}{50 + R}$$

$$R = 12.5 \Omega$$

### Example 1.20

When two resistors have a combined resistance of 10 ohm when connected in series and 2.4 ohm when connected in parallel. Find the resistance of each resistor.

#### Solution

Let the two resistors be  $R_1$  and  $R_2$ .

When connected in series,

$$R_1 + R_2 = 10 \Omega \quad \dots \quad (1)$$

When connected in parallel,

$$\frac{R_1 R_2}{R_1 + R_2} = 2.4 \Omega \quad \dots \quad (2)$$

Substitute the equation (1) in equation in (2)

$$\frac{R_1 R_2}{10} = 2.4$$

$$\frac{R_1 R_2}{R_1 + R_2} = 24$$

$$R_1 = \frac{24}{R_2} \quad \dots \quad (3)$$

Substitute the equation (3) in equation in (1)

$$\frac{24}{R_2} + R_2 = 10$$

$$R_2^2 + 24 = 10R_2$$

$$R_2^2 - 10R_2 + 24 = 0$$

$$(R-6)(R-4) = 0$$

$$R_2 = 6 \Omega \text{ (or) } R_2 = 4 \Omega$$

Substitute the value of  $R_2$  in equation (2)

$$R_2 = 6 \Omega,$$

$$R_1 + 6 = 10$$

$$R_1 = 10 - 6 = 4$$

$$R_1 = 4 \Omega$$

$$R_2 = 4 \Omega$$

$$R_1 + 4 = 10$$

$$R_1 + 4 = 10$$

$$R_1 = 10 - 4 = 6$$

$$R_1 = 6 \Omega$$

$$R_1 = 4 \Omega \text{ (or) } R_1 = 6 \Omega$$

$R_1 = 4 \Omega$
$R_2 = 6 \Omega$

(or)

$R_1 = 6 \Omega$
$R_2 = 4 \Omega$

### 1.1.12. Kirchoff's Laws

#### (a) First Law (or) current law (KCL)

The sum of currents flowing towards a junction is equal to the sum of current flowing away from the junction.

(or)

The algebraic sum of currents at a junction (node) of a network is zero.

#### Explanation

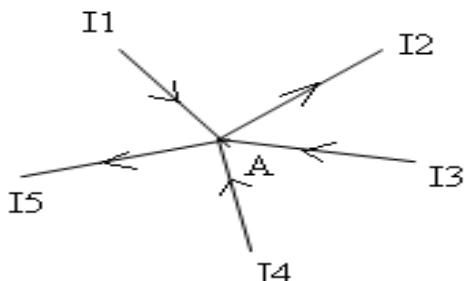


Fig. 1.18

Consider five conductors carrying currents  $I_1, I_2, I_3, I_4$  and  $I_5$  meeting at point 'o' as shown in fig.1.17. The arrow marks indicate the direction of the current flowing through the conductor.

Assuming signs of currents flowing towards point 'o' as positive and the signs of currents flowing away from point 'o' as negative.

### According to Kirchoff's current law

$$(I_1) + (-I_2) + (I_3) + (I_4) + (-I_5) = 0$$

$$\begin{aligned} I_1 - I_2 + I_3 + I_4 - I_5 &= 0 \\ I_1 + I_3 + I_4 &= I_2 + I_5 \end{aligned}$$

Sum of incoming currents = Sum out going currents

### (a) Second Law (or) voltage law (KVL)

In a closed circuit the sum of the potential drops is equal to the sum of the potential rises.

(or)

The algebraic sum voltage in a closed circuit (mesh) is equal to zero.

### Explanation

#### (i) For e.m.f source (battery)

- (a) If we go from the positive terminal of the battery to the negative terminal., there is a fall (drop) in potential. Hence the sign is negative.
- (b) If we go from the negative terminal of the battery to the positive terminal., there is a rise in potential. Hence the sign is positive.

#### (ii) For voltage drop (IR drop)

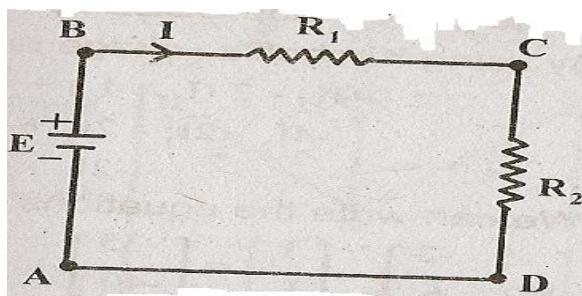


Fig. 1.19

- (a) If we go through the resistor in the same direction as the current there is fall (drop) in potential (because current flows from higher potential to lower potential). Hence the sign is negative.
- (b) If we go through the resistor opposite to the current direction, there is a rise (drop) in potential. Hence the sign is positive.

In fig.1.19 ABCD is a closed loop. According to Kirchoff's voltage law,

$$+ E - IR_1 - IR_2 = 0$$

$$\begin{aligned} & \text{(or)} \\ E &= IR_1 + IR_2 \end{aligned}$$

Therefore , E.m.f's in the circuit = Voltage drop in the circuit

### PROBLEM

#### Example 1.21

Calculate the currents in different branches of the circuit shown in the fig.1.20

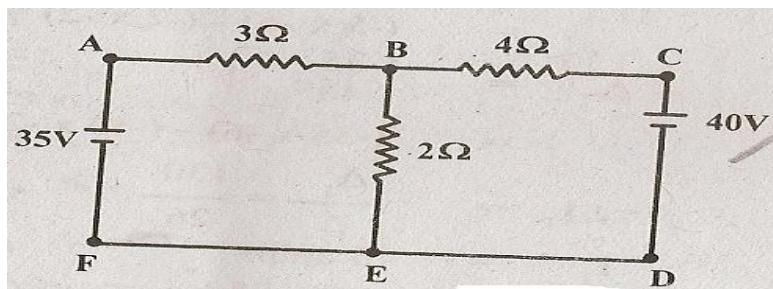


Fig.1.20

### Solution

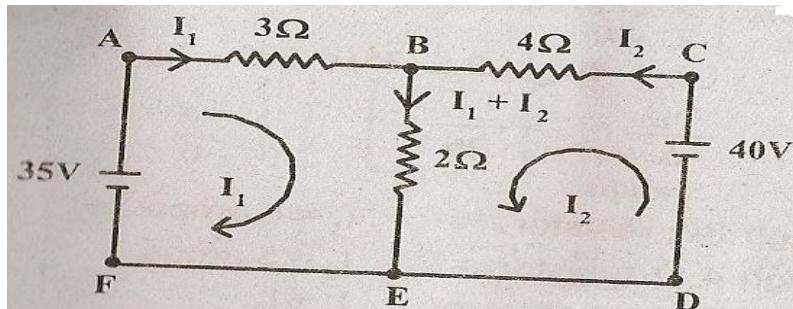


Fig.1.21

**In the closed loop ABEFA ,**

By using Kirchoff's voltage law,

$$\begin{aligned} -3I_1 - 2(I_1 + I_2) + 35 &= 0 \\ -3I_1 - 2I_1 - 2I_2 + 35 &= 0 \\ -5I_1 - 2I_2 &= -35 \quad \text{--- (1)} \end{aligned}$$

In the closed loop ABEFA ,

By using Kirchoff's voltage law,

$$-4I_2 - 2(I_1 + I_2) + 40 = 0$$

$$-4I_2 - 2I_1 - 2I_2 + 40 = 0$$

$$-2I_1 - 6I_2 = -40 \quad \dots \quad (2)$$

We can write the equations (1) and (2) in matrix form

$$\begin{pmatrix} -5 & -2 \\ -2 & -6 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -35 \\ -40 \end{pmatrix}$$

$$\begin{aligned} \Delta &= \begin{pmatrix} -5 & -2 \\ -2 & -6 \end{pmatrix} \\ &= (-5 \times -6) - (-2 \times -2) = 30 - 4 = 26 \end{aligned}$$

$$\begin{aligned} \Delta_1 &= \begin{pmatrix} -35 & -2 \\ -40 & -6 \end{pmatrix} \\ &= (-35 \times -6) - (-40 \times -2) = 210 - 80 = 130 \end{aligned}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{130}{26}$$

$$I_1 = 5 \text{ amps.}$$

$$\begin{aligned} \Delta_2 &= \begin{pmatrix} -5 & -35 \\ -2 & -40 \end{pmatrix} \\ &= (-5 \times -40) - (-2 \times -35) = 200 - 70 = 130 \end{aligned}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{130}{26}$$

$$I_2 = 5 \text{ amps.}$$

$$\text{Current through } 3 \Omega \text{ resistor} = I_1 = 5 \text{ amps.}$$

$$\text{Current through } 4 \Omega \text{ resistor} = I_2 = 5 \text{ amps.}$$

$$\begin{aligned}\text{Current } I \text{ through } 2\Omega \text{ resistor} &= I_1 + I_2 \\ &= 5 + 5 = 10 \text{ amps.}\end{aligned}$$

### Example 1.22

A wheat stone bridge consists of  $AB = 10\Omega$ ,  $BC = 10\Omega$ ,  $CD = 4\Omega$  and  $DA = 5\Omega$ . A galvanometer of  $20\Omega$  resistance is connected across BD. Calculate currents through the galvanometer when a p.d of 10V is maintained across AC.

### Solution

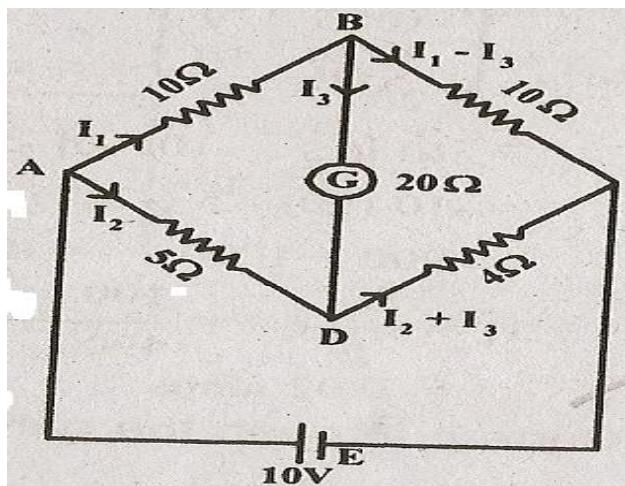


Fig.1.22

In the closed loop ABDA ,

By using Kirchoff's voltage law,

$$\begin{aligned}-10I_1 - 20I_3 + 5I_2 &= 0 \\ -10I_1 + 5I_2 - 20I_3 &= 0 \quad \dots \dots \dots (1)\end{aligned}$$

In the closed loop ADCEA ,

By using Kirchoff's voltage law,

$$\begin{aligned}-5I_2 - 4(I_2 + I_3) + 10 &= 0 \\ -5I_1 - 4I_2 - 4I_3 + 10 &= 0 \\ -9I_2 - 4I_3 &= -10 \quad \dots \dots \dots (2)\end{aligned}$$

In the closed loop ABCEA ,

By using Kirchoff's voltage law,

$$-10I_1 - 10(I_1 + I_3) + 10 = 0$$

$$\begin{aligned} -10I_1 - 10I_2 - 10I_3 + 10 &= 0 \\ -20I_2 - 10I_3 &= -10 \end{aligned} \quad \text{----- (3)}$$

We can write the equations (1), (2) and (3) in matrix form

$$\begin{pmatrix} -10 & 5 & -20 \\ 0 & -9 & -4 \\ -20 & 0 & 10 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -10 \\ -10 \end{pmatrix}$$

$$\begin{aligned} \Delta &= \begin{pmatrix} -10 & 5 & -20 \\ 0 & -9 & -4 \\ -20 & 0 & 10 \end{pmatrix} \\ &= -10 [(-9 \times 10) - 0] - 5 [(0 \times 10) - (-20 \times -4)] - 20 [0 - (-20 \times -9)] \\ &= -10 [(-90)] - 5 [(-80)] - 20(-180) \\ &= 900 + 400 + 3600 = 4900 \end{aligned}$$

$$\begin{aligned} \Delta_3 &= \begin{pmatrix} -10 & 5 & 0 \\ 0 & -9 & -10 \\ -20 & 0 & -10 \end{pmatrix} \\ &= -10 [(-9 \times -10) - 0] - 5 [(0 \times -10) - (-20 \times -10)] - 0 \\ &= -10 [(90)] - 5 [(-200)] \\ &= -900 + 1000 = 100 \end{aligned}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{100}{4900} = 0.02$$

$$I_3 = 5 \text{ amps.}$$

**Current through the galvanometer,  $I_3 = 0.02$  amps.**

### 1.2.1.NETWORK THEOREMS

#### 1.2.1 Super Position Theorems

In a network containing two or more e.m.f sources , the current in any resistor is equal to the algebraic sum of the separate currents in the resistor when each source acts separately.

##### Step by step procedure

1. Keep one source at one time
2. Short circuit the other voltage sources by their internal resistances and open circuit the current sources
3. Find the current in the resistor due to first source.
4. Repeat step 1 to 3 for other sources.
5. Add the currents algebraically.

##### Explanation

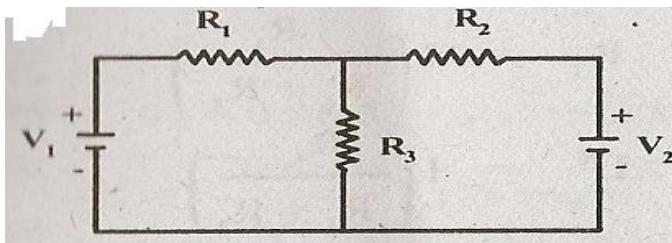


Fig.1.23

**Step -1 :** Keep  $V_1$  voltage source and short circuit the voltage source  $V_2$

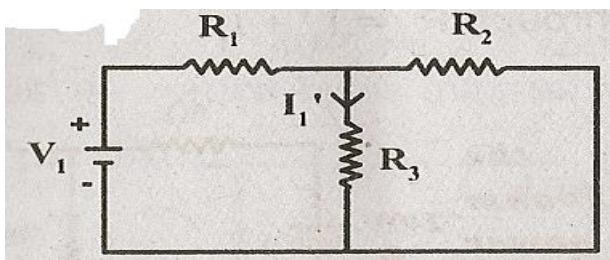


Fig.1.24

$$\text{Total resistance, } R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$\text{Total current, } I = \frac{V_1}{R}$$

I [ ]

$$\text{Current through } R_3 = I \frac{R_2}{R_2 + R_3}$$

$$I_1' = I \left( \frac{R_2}{R_2 + R_3} \right)$$

**Step -1 :** Keep  $V_2$  voltage source and short circuit the voltage source  $V_1$

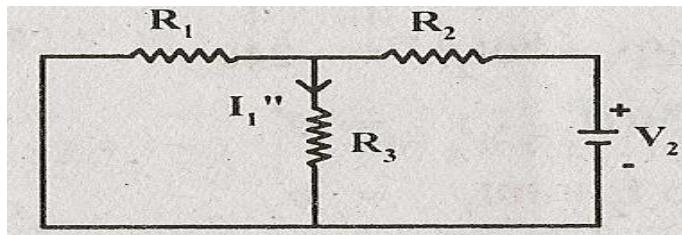


Fig.1.25

$$\text{Total resistance, } R = R_2 + \frac{R_1 R_3}{R_2 + R_3}$$

$$\text{Total current, } I = \frac{V_1}{R}$$

$$\text{Current through } R_3 = I \left( \frac{R_1}{R_1 + R_3} \right)$$

$$I_1'' = I \left( \frac{R_1}{R_1 + R_3} \right)$$

Add the currents algebraically.

$$\text{i.e., Current through } R_3 = I_1' + I_1''$$

### Example 1.23

Determine the current in the 150 ohm resistor and the power consumed by using superposition theorem.

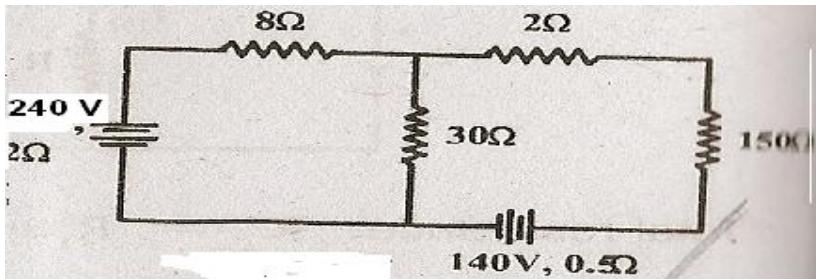


Fig.1.26

**Step -1 :** Keep 240 V voltage source and short circuit the 140 voltage source by its internal resistance.

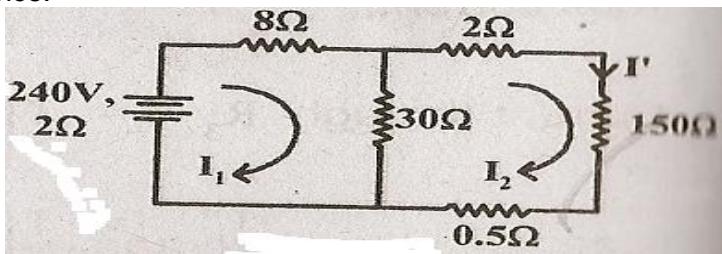


Fig.1.27

By using Kirchoff's voltage law,

$$240 - 2I_1 - 8I_1 - 30(I_1 - I_2) = 0 \\ -40I_1 + 30I_2 = -240 \quad \dots \dots \dots (1)$$

$$-2I_2 - 150I_2 - 0.5I_2 - 30(I_2 - I_1) = 0 \\ -182.5I_2 + 30I_1 = 0 \quad \dots \dots \dots (2)$$

$$\begin{pmatrix} 40 & 30 \\ 30 & -182.5 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -240 \\ 0 \end{pmatrix}$$

$$\Delta_2 = \begin{pmatrix} -40 & 30 \\ 30 & -182.5 \end{pmatrix} = (-40 \times -182.5) - (-30 \times 30) = 6400$$

$$\Delta_2 = \begin{pmatrix} -40 & -240 \\ 30 & 0 \end{pmatrix} = 0 - (30 \times -240) = 7200$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{7200}{6400} = 1.125 \text{ amps}$$

Current through 150 ohm resistor,  $I'' = I_2 = 1.125$  amps

**Step -2 :** Keep 140 V voltage source and short circuit the 240 voltage source by its internal resistance.

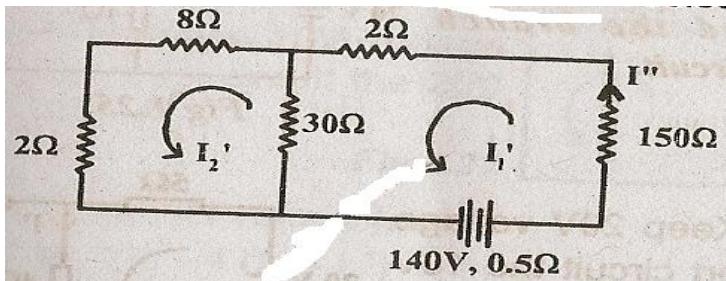


Fig.1.28

By using Kirchoff's voltage law,

$$\begin{aligned} -2I'_1 - 8I'_1 - 30(I'_2 - I'_1) &= 0 \\ -40I'_1 + 30I'_2 &= 0 \quad \dots \dots \dots (1) \end{aligned}$$

$$\begin{aligned} -150I'_1 - 2I'_1 - 30(I'_1 - I'_2) + 140 - 0.5I'_1 &= 0 \\ -182.5I'_1 + 30I'_2 &= -140 \quad \dots \dots \dots (2) \end{aligned}$$

$$\begin{pmatrix} 30 & -40 \\ -182.5 & 30 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -40 \end{pmatrix}$$

$$\Delta_1 = \begin{pmatrix} 30 & -40 \\ -182.5 & 30 \end{pmatrix} = (30 \times 30) - (-40 \times -182.5) = 6400$$

$$= \begin{pmatrix} 0 & -40 \\ -140 & 30 \end{pmatrix} = 0 - (40 \times -140) = 5600$$

$$I'_1 = \frac{\Delta_1}{\Delta} = \frac{-5600}{-6400} = 0.875 \text{ amps}$$

Current through 150 ohm resistor,  $I'' = I' = 0.875$  amps

**Step -3**

Current through 150 ohm resistor  $= I' - I''$

$$= 1.125 - 0.875 = 0.25 \text{ amps}$$

Power consumed by 150 ohm resistor

$$\begin{aligned} &= I^2 R \\ &= 0.25^2 \times 150 \\ &= \mathbf{9.375 \text{ watts}} \end{aligned}$$

### Example 1.24

Apply superposition theorem to find the branch currents in the circuit.

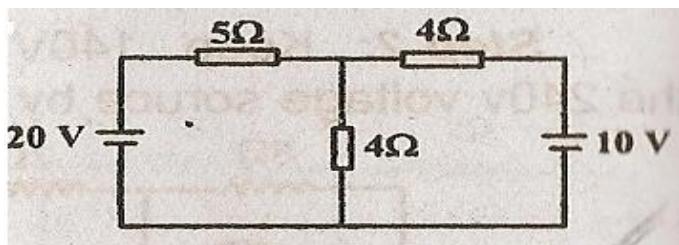


Fig.1.29

**Step -1 :** Keep **20 V** voltage source and short circuit the **10** voltage source by its internal resistance.

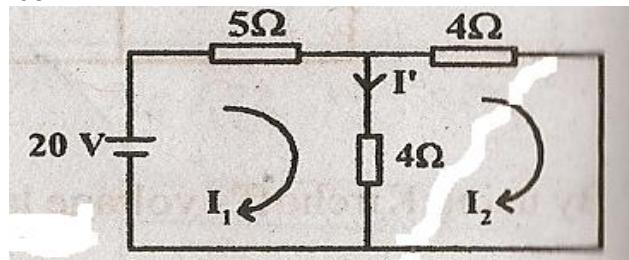


Fig.1.30

By using Kirchoff's voltage law,

$$\begin{aligned} 20 - 5I_1 - 8I_1 - 4(I_1 - I_2) &= 0 \\ -9I_1 + 4I_2 &= -20 \quad \dots \quad (1) \\ -4I_2 - 4(I_2 - I_1) &= 0 \\ -8I_2 + 4I_1 &= 0 \quad \dots \quad (2) \end{aligned}$$

$$\begin{pmatrix} -9 & 4 \\ 4 & -8 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -20 \\ 0 \end{pmatrix}$$

$$\Delta = \begin{pmatrix} -9 & 4 \\ 4 & -8 \end{pmatrix} = (-9 \times -8) - (4 \times 4) = 56$$

$$\Delta_1 = \begin{pmatrix} -20 & 4 \\ 0 & -8 \end{pmatrix} = (-20 \times -8) - 0 = 160$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{160}{56} = 2.857 \text{ amps}$$

$$\Delta_2 = \begin{pmatrix} -9 & -20 \\ 4 & 0 \end{pmatrix} = 0 - (-20 \times 4) - 0 = 80$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{80}{56} = 1.428 \text{ amps}$$

Current through 5 ohm resistor ,  $I_1 = 2.857$  amps

Current through 4 ohm resistor ,  $I_2 = 1.428$  amps

Current through 4 ohm (middle) resistor ,  $I' = I_1 - I_2$

$$= 2.857 - 1.428$$

$$= 1.429 \text{ amps}$$

**Step -2 :** Keep 10 V voltage source and short circuit the 20 voltage source by its internal resistance.

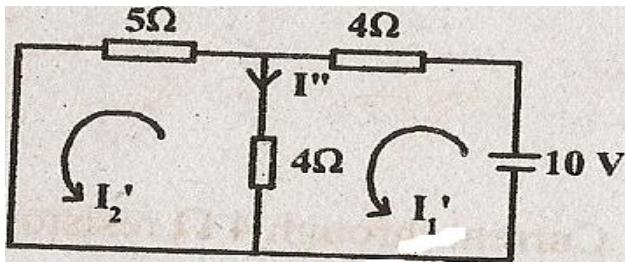


Fig.1.31

By using Kirchoff's voltage law,

$$-5I'_2 - 4(I'_2 - I'_1) = 0$$

$$-9I'_2 + 4I'_1 = 0 \quad \dots \quad (1)$$

$$-4I'_1 - 4(I'_1 - I'_2) = 0$$

$$-8I'_1 + 4I'_2 = -10 \quad \dots \quad (2)$$

$$\begin{pmatrix} 4 & -9 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} I'_1 \\ I'_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$$

$$\begin{array}{ccc} -8 & 4 & I'_2 \\ & & = \\ & & -10 \end{array}$$

$$\Delta = \begin{pmatrix} 4 & -9 \\ -8 & 4 \\ 0 & -9 \\ -10 & 4 \end{pmatrix} = (4 \times 4) - (-9 \times -8) = -56$$

$$\Delta_1 = \begin{pmatrix} 4 & -9 \\ -8 & 4 \\ 0 & -9 \\ -10 & 4 \end{pmatrix} = 0 - (-9 \times -10) = -90$$

$$I'_1 = \frac{\Delta_1}{\Delta} = \frac{-90}{-56} = 1.6 \text{ amps}$$

$$\Delta_2 = \begin{pmatrix} 4 & 0 \\ -8 & -10 \end{pmatrix} = (4 \times -10) = -40$$

$$I'_2 = \frac{\Delta_2}{\Delta} = \frac{-40}{-56} = 0.714 \text{ amps}$$

$$\text{Current through } 5 \text{ ohm resistor} = I'_2 = 0.714 \text{ amps}$$

$$\text{Current through } 4 \text{ ohm resistor} = I'_1 = 1.6 \text{ amps}$$

$$\text{Current through } 4 \text{ ohm (middle) resistor, } I'' = I'_1 - I'_2$$

$$= 1.6 - 0.714$$

$$= 0.886 \text{ amps}$$

### Step -3

$$\text{Current through } 5 \text{ ohm resistor} = I_1 - I'_2$$

$$= 2.857 - 0.714$$

$$= 2.143 \text{ amps}$$

$$\text{Current through } 4 \text{ ohm resistor (left to right)} = I'_1 - I'_2$$

$$= 1.428 - 1.6$$

$$= -0.172 \text{ amps}$$

$$\text{Current through } 4 \text{ ohm (middle) resistor} = I'_1 + I''_1$$

$$= 1.429 + 0.886$$

$$= 2.315 \text{ amps}$$

### Example 1.25

Find the through the  $6 \Omega$  resistor by using superposition theorem.

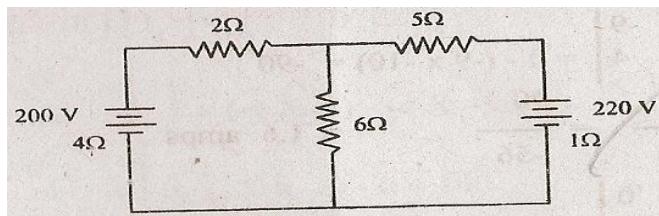


Fig.1.32

**Solution**

**Step -1 :** Keep 200 V voltage source and short circuit the 220 voltage source by its internal resistance.

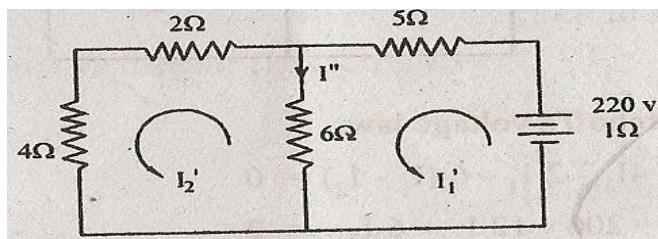


Fig.1.33

By using Kirchoff's voltage law,

$$200 - 4I_1 - 2I_1 - 6(I_1 - I_2) = 0$$

$$200 - 12I_1 + 6I_2 = 0$$

$$-12I_1 + 6I_2 = -200 \quad \dots \quad (1)$$

$$-5I_2 - I_2 - 6(I_2 - I_1) = 0$$

$$-6I_1 - 12I_2 = 0 \quad \dots \quad (2)$$

$$\begin{pmatrix} -12 & 6 \\ 6 & -12 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -200 \\ 0 \end{pmatrix}$$

$$\Delta = \begin{pmatrix} -12 & 6 \\ 6 & -12 \end{pmatrix} = (-12 \times -12) - (6 \times 6) = 108$$

$$\Delta_1 = \begin{pmatrix} -200 & 6 \\ 0 & -12 \end{pmatrix} = (-200 \times -12) - (0 \times 6) = 2400$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{2400}{108} = 22.22 \text{ amps}$$

$$\Delta_2 = \begin{pmatrix} -12 & -200 \\ 6 & 0 \end{pmatrix} = (-12 \times 0) - (-200 \times 6) = 1200$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{1200}{108} = 11.11 \text{ amps}$$

Current through 6 ohm resistor,  $I' = I_1 - I_2$   
 $= 22.22 - 11.11$   
 $I' = 11.11 \text{ amps}$

**Step -2 :** Keep 200 V voltage source and short circuit the 220 voltage source by its internal resistance.

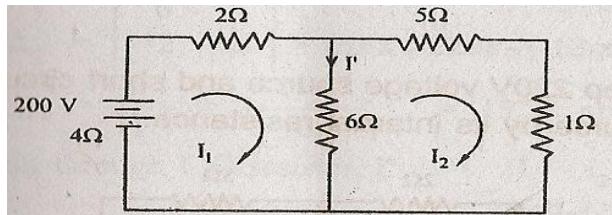


Fig.1.34

By using Kirchoff's voltage law,

$$\begin{aligned} -4I'_2 - 2I'_2 - 6(I'_2 - I'_1) &= 0 \\ -6I'_2 - 6I'_2 + 6I'_1 &= 0 \\ -6I'_1 - 12I'_2 &= 0 \quad \dots \dots \dots (1) \\ 220 - I'_1 - 5I'_1 - 6(I'_1 - I'_2) &= 0 \\ 220 - 12I'_1 - 6I'_2 &= 0 \\ -12I'_1 + 6I'_2 &= -220 \quad \dots \dots \dots (2) \end{aligned}$$

$$\begin{pmatrix} -12 & 6 \\ 6 & -12 \end{pmatrix} \begin{pmatrix} I'_1 \\ I'_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -220 \end{pmatrix}$$

$$\Delta = \begin{pmatrix} -12 & 6 \\ -6 & -12 \end{pmatrix} = (6 \times 6) - (-12 \times -12) = -108$$

$$\begin{pmatrix} \quad \\ \quad \end{pmatrix}$$

$$\Delta_1 = \begin{vmatrix} 0 & -12 \\ -220 & 6 \end{vmatrix} = (0 \times 6) - (-12 \times -220) = -2640$$

$$I'_1 = \frac{\Delta_1}{\Delta} = \frac{-2640}{-108} = 24.44 \text{ amps}$$

$$\Delta_2 = \begin{vmatrix} 6 & 0 \\ 12 & -220 \end{vmatrix} = (6 \times -220) - (-12 \times 0) = -1320$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-1320}{-108} = 12.22 \text{ amps}$$

Current through 6 ohm resistor,  $I'' = (I'_1 - I_2)$   
 $= 24.44 - 12.22$   
 $I'' = 12.22 \text{ amps}$

### Step -3

Current through 6 ohm resistor,  $I'' = I' + I''$   
 $= 24.44 + 12.22$   
 $= 36.66 \text{ amps}$

### THEVENIN'S THEOREM

Any two terminal network can be replaced by a voltage source  $V_{Th}$  in series with resistance  $R_{Th}$ .

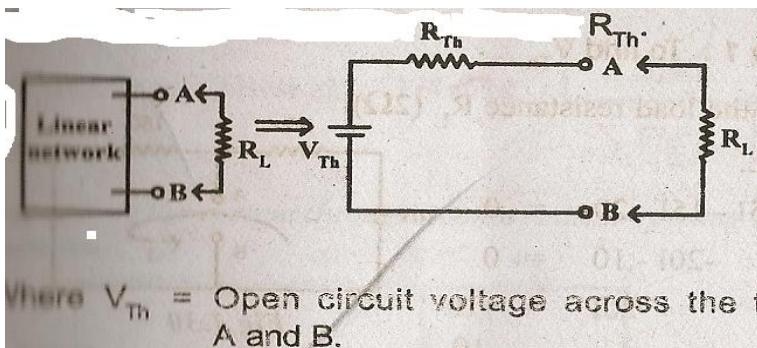


Fig. 1.35

Where  $V_{Th}$  = Open circuit voltage across the terminals A and B

$R_{Th}$  = Resistance of the network as looking into the circuit from terminals A and B

### STEP BY STEP PROCEDURE

1. Remove the load resistance  $R_L$ .

2. Find the open circuit voltage  $V_{Th}$ , which is the across the two terminals where the load is removed.
3. Replace all voltage sources by their internal resistances.
4. Calculate the Thevenin's looking back resistance  $R_{Th}$  from the load terminals.
5. Replace the entire network by the open circuit voltage  $V_{Th}$  in series with equivalent resistance  $R_{Th}$ .
6. Connect the load resistance  $R_L$  where it was already removed.
7. Find the current in the load resistance  $R_L$ .

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

### Example 1.26

Find the current through the  $20\Omega$  resistor by using Thevenin's theorem.

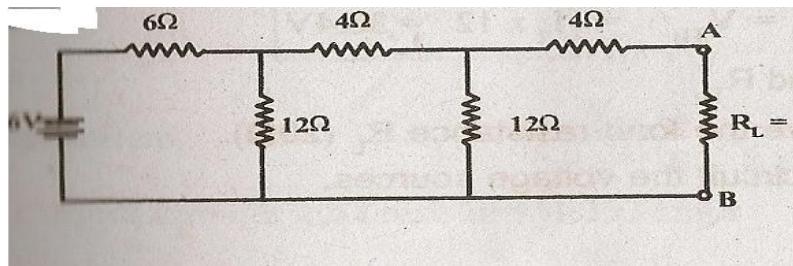


Fig.1.36

### Solution

#### Step -1 : To find $V_{Th}$

1. Remove the load resistance  $R_L(20\Omega)$

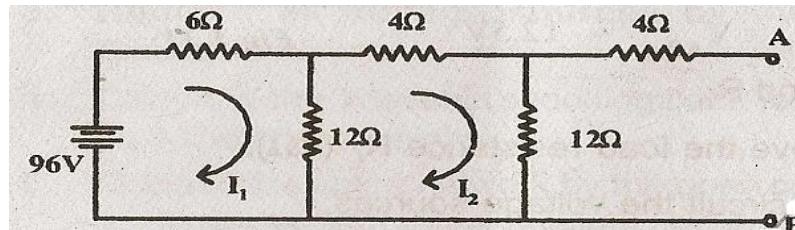


Fig.1.37

By using Kirchoff's voltage law,

$$96 - 4I_1 - 12(I_2 - I_1) = 0$$

$$-18I_1 + 6I_2 = 96 \quad \dots \quad (1)$$

$$-14I_2 - 12I_1 - 12(I_2 - I_1) = 0$$

$$12I_1 + 28I_2 = 0 \quad \dots \quad (2)$$

$$\Delta = \begin{pmatrix} -18 & 12 \\ 12 & -28 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -96 \\ 0 \end{pmatrix}$$

$$\Delta = \begin{pmatrix} -18 & 12 \\ 12 & -28 \end{pmatrix} = (-18 \times -28) - (12 \times 12) = 360$$

$$\Delta_2 = \begin{pmatrix} -18 & 12 \\ -12 & 0 \end{pmatrix} = 0 - (-96 \times 12) = 1152$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{1152}{360} = 3.2 \text{ amps}$$

$$V_{AB} = V_{Th} = I_2 \times 12 \\ = 3.2 \times 12 = 38.4 \text{ volts}$$

### Step -2 : To find $R_{Th}$

1. Remove the load resistance  $R_L(20\Omega)$
2. Short circuit the voltage sources.

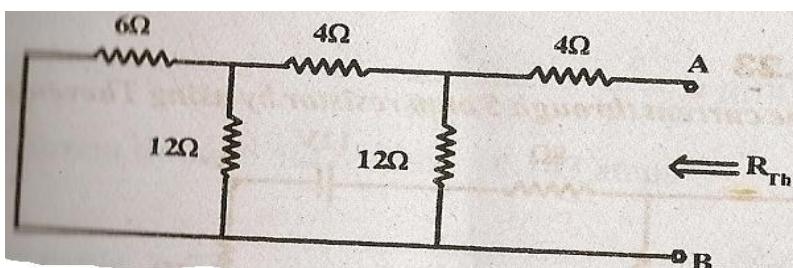


Fig.1.38  
6Ω and 12Ω are in parallel

$$\frac{6 \times 12}{6 + 12} = 4\Omega$$

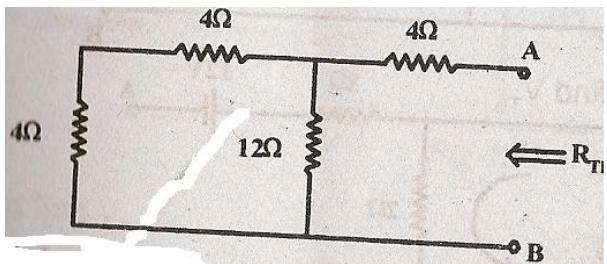


Fig.1.39

$4\Omega$  and  $4\Omega$  are in series

$$4 + 4 = 8\Omega$$

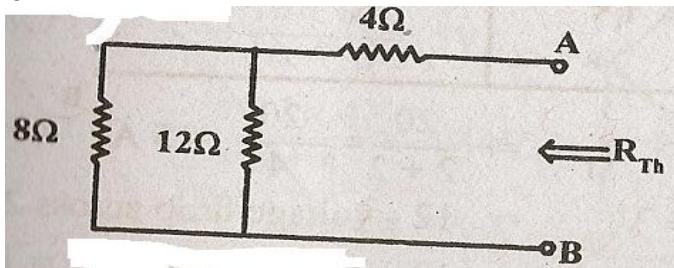


Fig.31.40

$8\Omega$  and  $12\Omega$  are in parallel with series  $4\Omega$  resistor

$$R_{Th} = \frac{8 \times 12}{8 + 12} + 4 = 4.8\Omega + 4 = 8.8\Omega$$

**Step -3 :** Thevenin's equivalent circuit

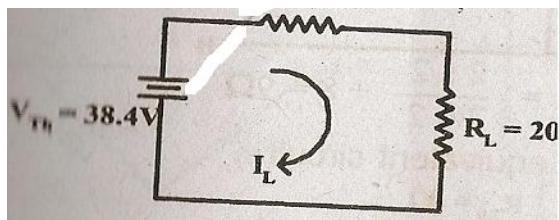


Fig. 1.41

$$\text{Load current, } I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{38.4}{8.8 + 20} = 1.33$$

**Current through  $20\Omega$  resistor,  $I_L$**  = 1.33 amps

**Example 1.27**

Find the through the  $5\Omega$  resistor by using Thevenin's theorem.

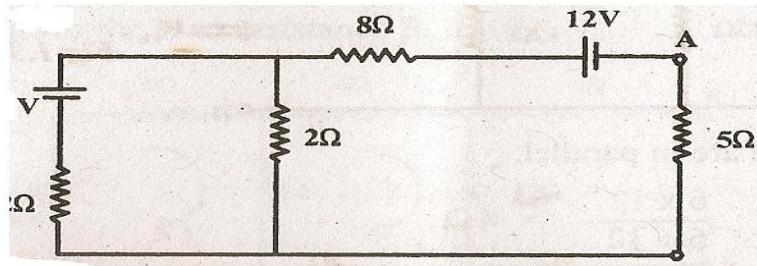


Fig.1.42

### Solution

#### Step -1 : To find $V_{Th}$

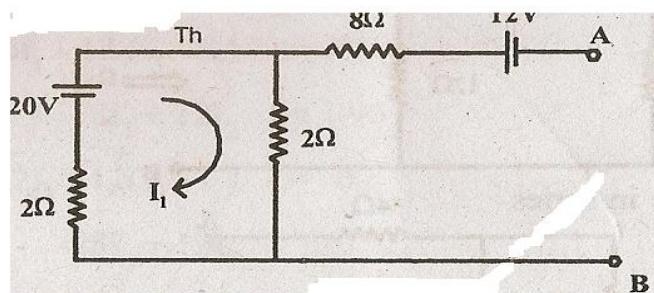


Fig.1.43

$$I_1 = \frac{20}{2 + 2} = \frac{20}{4} = 5A$$

$$\begin{aligned} V_{AB} &= V_{Th} = 12 - \text{voltage drop across } 2\Omega \text{ resistor} \\ &= 12 - 2 \times I_1 = 12 - 2 \times 5 = 2V \end{aligned}$$

#### Step -2 : To find $R_{Th}$

1. Remove the load resistance  $R_L(20\Omega)$
2. Short circuit the voltage sources.

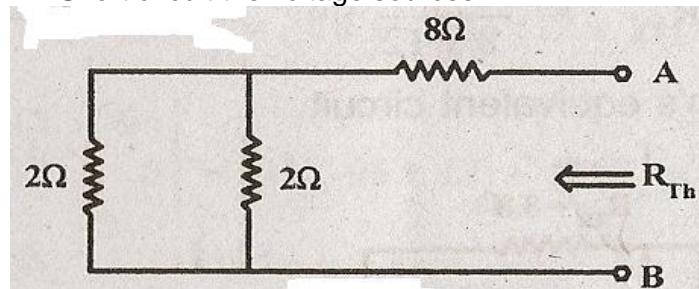


Fig.1.44

$2\Omega$  and  $2\Omega$  are in parallel with series  $8\Omega$  resistor  
 $2 \times 2$

$$R_{Th} = \frac{2}{2+2} = 1 \Omega$$

**Step -3 :** Thevenin's equivalent circuit

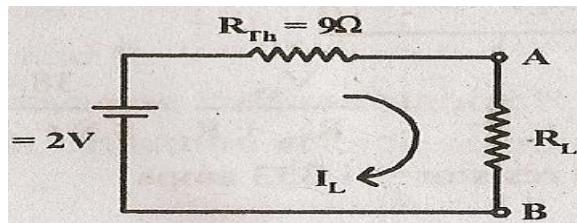


Fig. 1.45

$$\text{Load current, } I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{2}{9 + 5} = 0.143$$

**Current through 20Ω resistor,  $I_L = 0.143$  amps**

### Example 1.28

Find the through the 2Ω resistor by using Thevenin,s theorem.

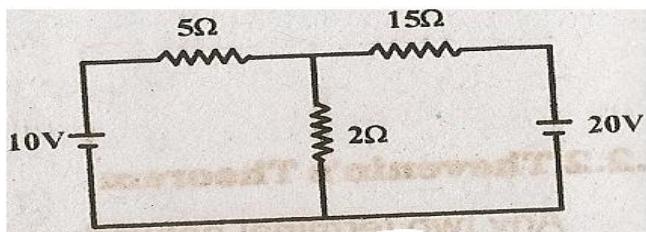


Fig.1.46

**Solution**

**Step -1 : To find  $V_{Th}$**

1.Remove the load resistance  $R_L$  (2Ω)

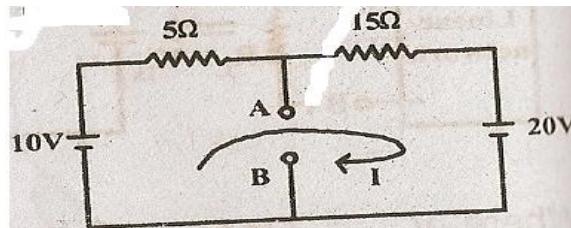


Fig.1.47

Apply Kirchhoff's Voltage Law,

$$\begin{aligned} 10 - 5I - 15I - 20 &= 0 \\ -20I - 10 &= 0 \\ -20 &= 10 \\ -10 & \end{aligned}$$

$$I = \frac{20}{20} = -0.5 \text{ amps}$$

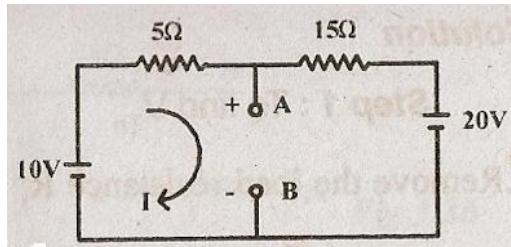


Fig.1.48

From fig. 1.42

$$10 - 5I - V_{AB} = 0$$

$$10 - 5 \times (-0.5) - V_{AB} = 0$$

$$10 + 2.5 - V_{AB} = 0$$

$$V_{AB} = 12.5 \text{ V}$$

$$V_{Th} = V_{AB} = 12.5 \text{ V}$$

### Step -2 : To find $R_{Th}$

1. Remove the load resistance  $R_L(2\Omega)$
2. Short circuit the voltage sources.
- 3.

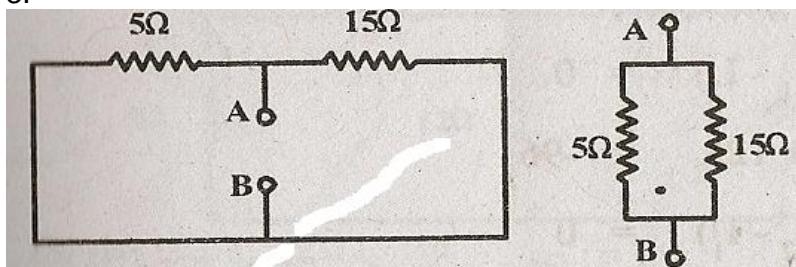


Fig.1.49

$$R_{Th} = \frac{5 \times 15}{5 + 15} = 3.75\Omega$$

### Step -3 : Thevenin's equivalent circuit

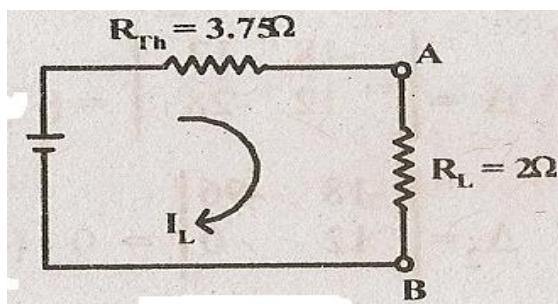


Fig. 1.50

$$\text{Load current, } I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{12.5}{3.75 + 2} = 2.174$$

**Current through  $20\Omega$  resistor,  $I_L = 2.174$  amps**

### NORTON'S THEOREM

Any two terminal network can be replaced by a current source  $I_N$  in parallel with resistance  $R_N$ .

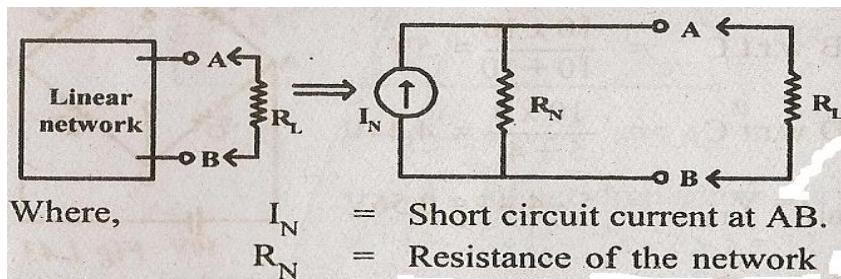


Fig. 1.51

Where  $I_N$  = Short circuit current at AB.

$R_N$  = Resistance of the network as looking into the circuit from terminals A and B

### STEP BY STEP PROCEDURE

1. Remove the load resistance  $R_L$  and put a short circuit
2. Find the short circuit current  $I_N$
3. Replace all voltage sources by their internal resistances.
4. Calculate the Norton's looking back resistance  $R_N$  from the load terminals.
5. Replace the entire network by the short circuit current  $I_N$  in parallel with equivalent resistance  $R_N$ .
6. Connect the load resistance  $R_L$  where it was already removed.
7. Find the current in the load resistance  $R_L$ .

$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

**Example 1.29**

Find the through  $R_L$  resistor by using Nortorn's theorem.

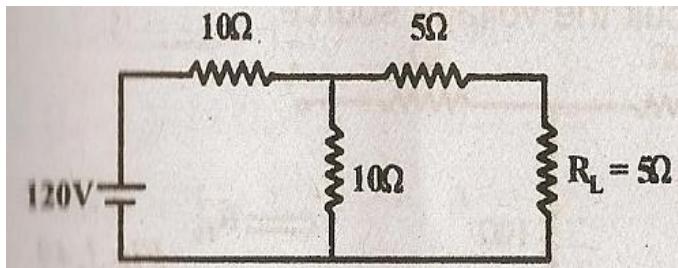


Fig.1.52

**Solution****Step -1 : To find  $I_N$** 

1. Short circuit the load resistance  $R_L (5\Omega)$

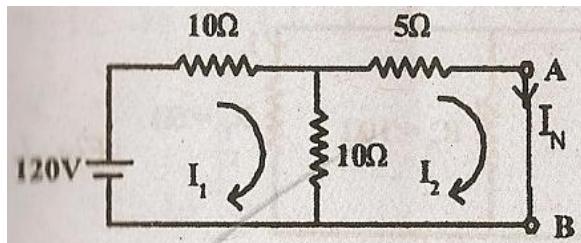


Fig.1.53

Apply Kirchhoff's Voltage Law,

$$\begin{aligned} 120 - 10I_1 - 10(I_1 - I_2) &= 0 \\ -20I_1 + 10I_2 &= -120 \quad \dots \dots \dots (1) \\ -5I_2 - 10(I_2 - I_1) &= 0 \\ 10I_1 - 15I_2 &= 0 \quad \dots \dots \dots (2) \end{aligned}$$

$$\begin{pmatrix} -20 & 10 \\ 10 & -15 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -120 \\ 0 \end{pmatrix}$$

$$\Delta = \begin{pmatrix} -20 & 10 \\ 10 & -15 \end{pmatrix} = (-20 \times -15) - (10 \times 10) = 200$$

$$\begin{pmatrix} & \\ & \end{pmatrix}$$

$$-20 \quad -120$$

$$\Delta_2 = \begin{vmatrix} -20 & 0 \\ 10 & 0 \end{vmatrix} = (-20 \times 0) \times (-120 \times 10) = 1200$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{1200}{200} = 6 \text{ amps}$$

$$I_N = 6 \text{ amps}$$

### Step -2 : To find $R_N$

1. Remove the load resistance  $R_L(20\Omega)$
2. Short circuit the voltage sources.

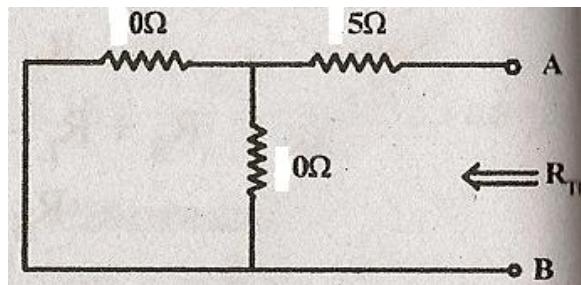


Fig.1.54

$$R_N = \frac{10 \times 10}{10 + 10} + 5 = 5 + 5 = 10\Omega$$

### Step -3 : Norton's equivalent circuit

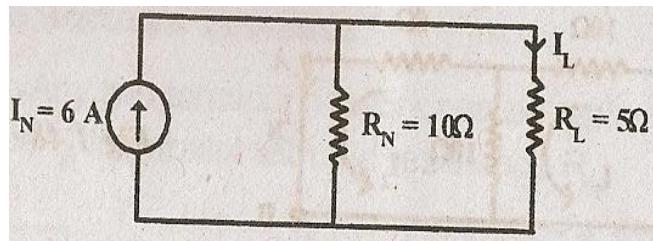


Fig. 1.55

$$\text{Current through } R_L, I_L = I_N \times \frac{R_N}{R_N + R_L} = 6 \times \frac{10}{10 + 5} = 4$$

$$\text{Current through } R_L, I_L = 4 \text{ amps}$$

### Example 1.30

Find the through  $20\Omega$  resistor by using Norton's theorem.

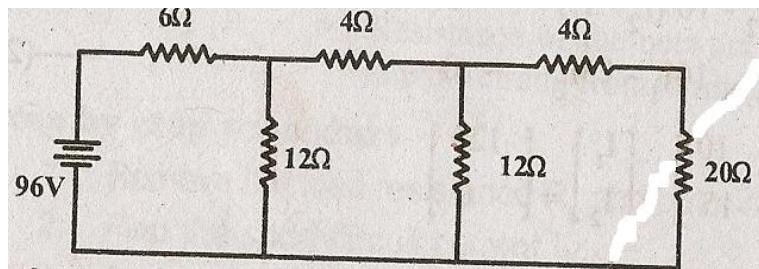


Fig.1.57

### Solution

#### Step -1 : To find $I_N$

1. Short circuit the load resistance  $R_L (5\Omega)$

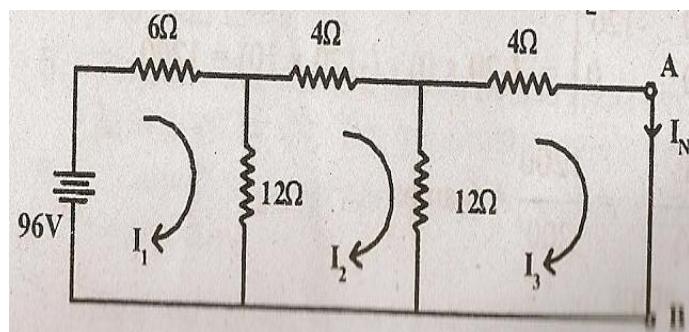


Fig.1.58

Apply Kirchhoff's Voltage Law,

$$\begin{aligned} 96 - 6I_1 - 12(I_1 - I_2) &= 0 \\ -18I_1 + 12I_2 &= -96 \quad \dots \quad (1) \end{aligned}$$

$$\begin{aligned} -4I_2 - 12(I_2 - I_3) - 12(I_2 - I_1) &= 0 \\ -28I_2 + 12I_3 + 12I_1 &= 0 \\ -16I_3 + 12I_2 &= 0 \quad \dots \quad (2) \end{aligned}$$

$$\left( \begin{array}{ccc} -18 & 12 & 0 \\ 10 & -28 & 12 \\ 0 & 12 & -16 \end{array} \right) \left( \begin{array}{c} I_1 \\ I_2 \\ I_3 \end{array} \right) = \left( \begin{array}{c} -96 \\ 0 \\ 0 \end{array} \right)$$

$$\Delta = \begin{pmatrix} -18 & 12 & 0 \\ 10 & -28 & 12 \\ 0 & 12 & -16 \end{pmatrix}$$

$$= -18 [(-28 \times -16) - (12 \times 12)] - 12 [(-12 \times -16) - (0 \times 12)] + 0 [(12 \times 12) - (0 \times -28)] \\ = -18(304) - 12(-192) = -3168$$

$$\Delta = \begin{pmatrix} -18 & 12 & 0 \\ 10 & -28 & 12 \\ 0 & 12 & -16 \end{pmatrix}$$

$$= -18 [(-28 \times 0) - (12 \times 0)] - 12 [(12 \times 0) - (0 \times 0)] - 96 (12 \times 12) - (0 \times -28)] \\ = -96(144) = 13824$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-13824}{-3168} = 4.36 \text{ amps}$$

$$I_3 = I_N = 6 \text{ amps}$$

### Step -2 : To find $R_N$

1. Remove the load resistance  $R_L$  ( $20\Omega$ )
2. Short circuit the voltage sources.

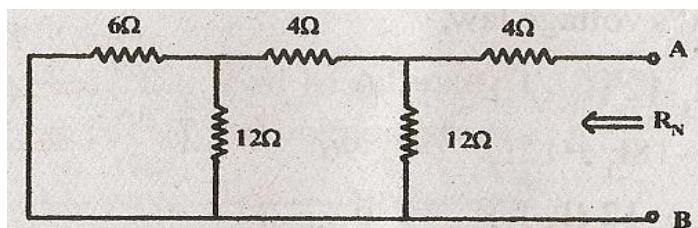


Fig.1.59

$6 \Omega$  and  $12 \Omega$  are in parallel.

$$\frac{6 \times 12}{6 + 12} = 4 \Omega$$

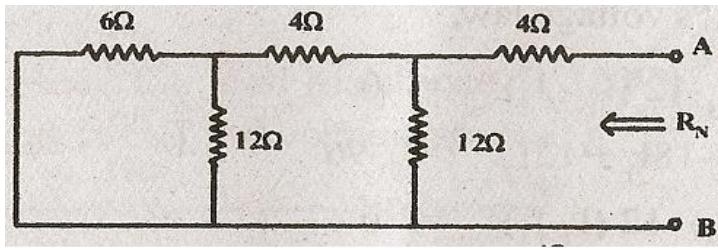


Fig.1.60

$4\ \Omega$  and  $4\ \Omega$  are in series.

$$4 + 4 = 8\ \Omega$$

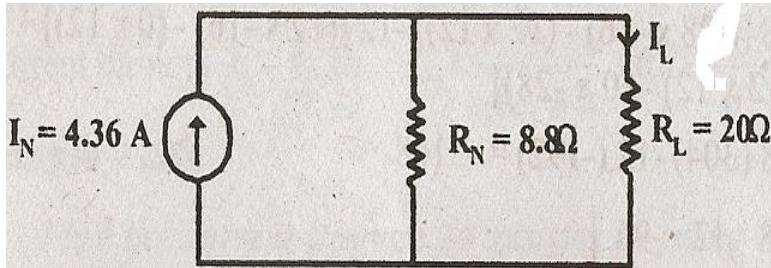


Fig.1.62

$$\text{Current through } R_L = I_L = I_N \times \frac{R_N}{R_N + R_L} = 4.36 \times \frac{10}{10 + 5} = 4$$

### MAXIMUM POWER TRANSFER THEOREM

Maximum power is transferred from source to load when the load resistance is made equal to the internal resistance of the source.

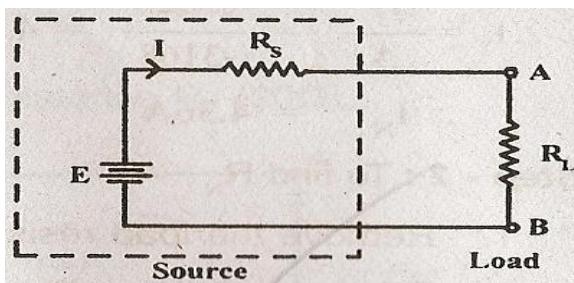


Fig.1.60

An electric circuit can be replaced by Thevenin's equivalent circuit consisting of Thevenin's voltage  $V_{Th}$  ( $= E$ ) in series with Thevenin's looking back resistance  $R_{Th}$  as shown in fig. 1.52

$$\text{Circuit Current , } I = \frac{E}{R_s + R_L}$$

$$\text{Power consumed by the load, } P_L = I^2 R$$

$$P_L = \left( \frac{E}{R_S + R_L} \right)^2 \times R_L = \frac{E^2 R_L}{(R_S + R_L)^2}$$

$$\frac{d P_L}{d R_L} = \frac{(R_S + R_L)^2 \cdot E^2 \cdot 1 - E^2 R_L [2(R_S + R_L) \cdot 1]}{[ (R_S + R_L)^2]^2}$$

$$= \frac{E^2 (R_S + R_L)^2 - 2 E^2 R_L (R_S + R_L)}{(R_S + R_L)^4}$$

$$d P_L$$

For maximum power ,  $\frac{d P_L}{d R_L} = 0$

$$\frac{E^2 (R_S + R_L)^2 - 2 E^2 R_L (R_S + R_L)}{(R_S + R_L)^4} = 0$$

$$E^2 (R_S + R_L)^2 - 2 E^2 R_L (R_S + R_L) = 0$$

$$E^2 (R_S + R_L)^2 = 2 E^2 R_L (R_S + R_L)$$

$$I = \frac{E^2 (R_S + R_L)^2}{2 E^2 R_L (R_S + R_L)}$$

$$R_S + R_L = 2 R_L$$

$$R_S + R_L - 2 R_L = 0$$

$$R_S - R_L = 0$$

$$R_S = R_L$$

$$\text{For maximum power , } P_L(\max) = \frac{E^2 R_L}{(R_L + R_L)^2}$$

$$\text{For maximum power , } P_L(\max) = \frac{E^2 R_L}{(2R_L)^2}$$

$$\text{For maximum power , } P_L(\max) = \frac{E^2}{4R_L} \text{ watts}$$

**Example 1.31**

Calculate the value of load resistance for maximum power transferred from the circuit shown in fig. Also find the value maximum power.

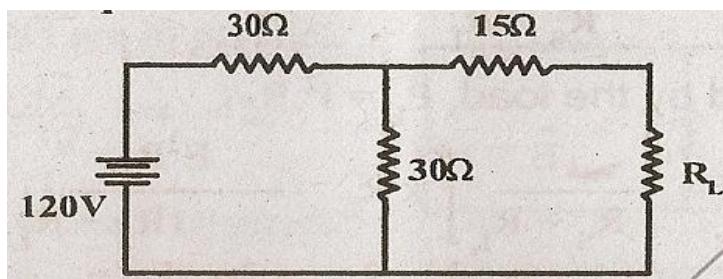


Fig.1.53

### Solution

#### Step -1 : To find $V_{Th}$ ( $E$ )

1. Remove the load resistance  $R_L$

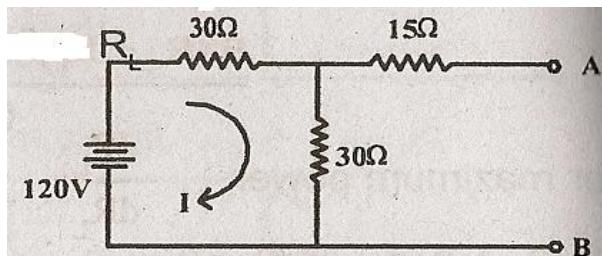


Fig.1.54

$$120 - 30I - 30I_1 = 0$$

$$120 - 60I = 0$$

$$\text{Current, } I = \frac{120}{60} = 2 \text{ amps}$$

$V_{Th} = E$  = voltage drop across  $30\Omega$  resistor

$$= 30 \times I = 30 \times 2 = 60 \text{ volts}$$

#### Step -2 : To find $R_{Th}$

1. Remove the load resistance  $R_L$
2. Short circuit the voltage sources.

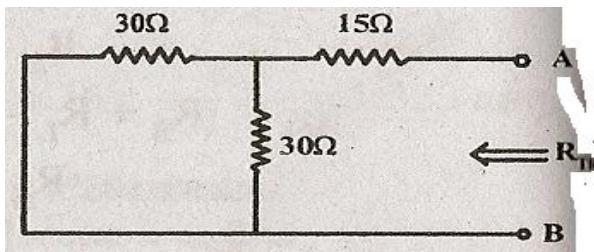


Fig.1.55

$$R_{Th} = \frac{30 \times 30}{30 + 30} + 15 = 15 + 15$$

$$R_S = R_{Th} = 30\Omega$$

**Step -3 :** Thevenin's equivalent circuit

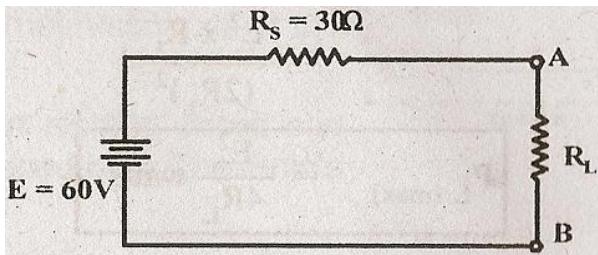


Fig. 1.56

For maximum power transfer  $R_S = R_L$

$$R_L = 30\Omega$$

$$\frac{E^2}{4R_L} = \frac{60^2}{4 \times 30}$$

$$\text{maximum power} = \frac{E^2}{4R_L} = \frac{60^2}{4 \times 30} = 30$$

Maximum power,  $P_{L_{max}} = 30 \text{ watts}$

## UNIT II

### A.C. CIRCUITS AND RESONANCE

#### Introduction

At constant temperature, the current flowing the conductor is directly proportional to the potential difference (voltage) between the two ends of the conductor. This is ohm's law

The relation may be written as,

$$V = IR$$

Where V and I now vary with time. When a sinusoidal if  $I = I_m \sin \omega t$  is applied to a resistor the potential across it.

$$\begin{aligned} E &= I_m R \sin \omega t \\ &= V_m \sin \omega t \end{aligned}$$

#### 2.1 VOLTAGE AND CURRENT RELATIONSHIP [A.C.CIRCUIT]

##### 2.1.1 A.C through pure resister only

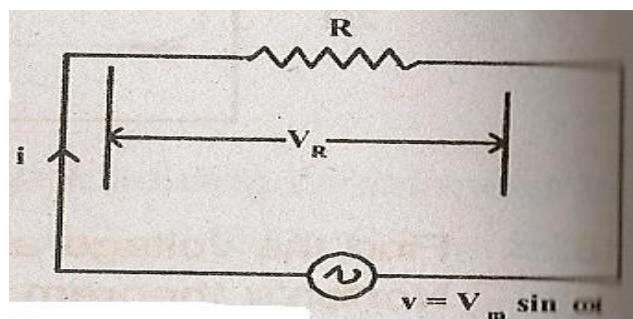


Fig.2.1

A pure resistive circuit is shown in fig.2.1 the alternating voltage applied across the resistor is

$$V = V_m \sin \omega t$$

Let 'I' be the alternating current through the circuit

$V_R$  = voltage drop across the resistor.

$$V = I \times R$$

$$I = \frac{V}{R}$$

$$I = \frac{V_m \sin \omega t}{R}$$

$$I = I_m \sin \omega t \quad \longrightarrow \quad 2$$

$$I_m \sin \omega t = \frac{V_m \sin \omega t}{R}$$

$$I_m = \frac{V_m \sin \omega t}{R \sin \omega t}, \quad I_m = \frac{V_m}{R}$$

From the equ. 1 and 2, the phase angle voltage and current is zero.

$$\Theta = 0$$

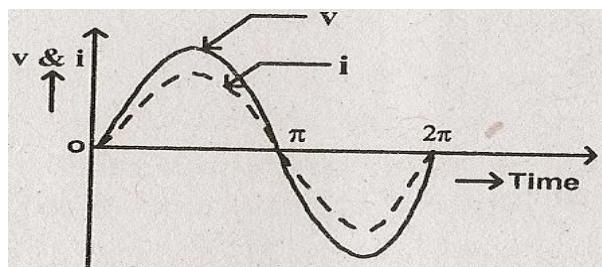
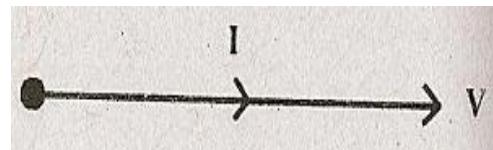


Fig.2.2 (a)



(b)

In pure resistive circuit, the circuit is in phase with the voltage .the waveform and vector diagram are shown in the fig.2.2 (a) and (b)

### Power

Instantaneous power,  $P = V \times I$

$$= V_m \sin \omega t \times I_m \sin \omega t$$

$$\begin{aligned}
&= V_m I_m \sin^2 \omega t \\
\text{average power, } P &= \int_0^{2\pi} \frac{\mathbf{v} \times \mathbf{i}}{2\pi} d(\omega t) \\
&= \int_0^{2\pi} \frac{V_m \sin \omega t \times I_m \sin \omega t}{2\pi} d(\omega t) \\
&= \frac{V_m I_m}{2\pi} \int_0^{2\pi} \sin^2 \omega t d(\omega t) \\
&= \frac{V_m I_m}{2\pi} \int_0^{2\pi} \left[ \frac{1 - \cos 2\omega t}{2} \right] d(\omega t) \\
&= \frac{V_m I_m}{4\pi} \left[ \omega t - \frac{\sin 2\omega t}{2} \right]_0^{2\pi} \\
&= \frac{V_m I_m}{4\pi} \left[ 2\pi - \frac{\sin 2 \times 2\pi}{2} - 0 - 0 \right] \\
&= \frac{V_m I_m}{2} [2\pi] \\
&= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}
\end{aligned}$$

### P=VI watts

Where V and I are R.M.S values

### Phase Angle

Phase angle is an angle between the voltage and current.

In a pure resistive current, the voltage and the current inphase with each other hence the phase angle is zero.

### Power factor

$$\begin{aligned}
\text{Power factor} &= \cos \theta \\
&= \cos \theta \quad [\theta=0] \\
&= 1 \text{ (Unity)}
\end{aligned}$$

Power factor is also defined as the ratio of resistance to impedance

$$\cos \theta \frac{R}{Z}$$

Its value cannot be more than one

### 2.1.2 A.C through pure inductor only

Pure inductive current is shown in fig.2.3

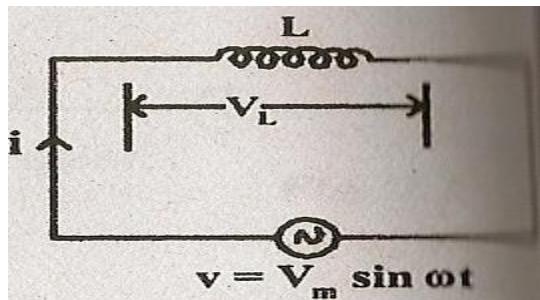


Fig.2.3

When an alternating current flows through a pure inductive coil a back e.m.f is induced due to the inductance of coil. This e.m.f opposes the applied voltage at every instant since there is no resistance the induced e.m.f will be equal and opposite to the applied voltage.

i.e., Applied voltage = back e.m.f

$$\text{Applied voltage, } V = V_m \sin \omega t \quad \longrightarrow 1$$

$$\text{Back e.m.f induced in the coil} = L \frac{di}{dt}$$

From equ(1)

$$di \frac{V_m \sin \omega t}{L} dt$$

$$\text{Integrating on both sides} = \int di = \int \frac{V_m}{L} \sin \omega t dt$$

$$i = \frac{V_m}{L} \left[ \frac{-\cos \omega t}{\omega} \right]$$

$$I = I_m \sin (\omega t - 90^\circ) \quad \longrightarrow 3$$

$$\text{Where } I_m = \frac{V_m}{\omega L}$$

$$I_m \sin (\omega t - 90^\circ) = \frac{V_m}{\omega L} \sin (\omega t - 90^\circ)$$

$$I_m = \frac{V_m}{\omega L} \times \frac{\sin (\omega t - 90^\circ)}{\sin (\omega t - 90^\circ)}$$

$$I_m = \frac{V_m}{\omega L}$$

from equ.2 and 3, we find that the current lags behind the applied voltage by  $90^\circ$ . the waveform and vector diagram are shown in fig.2.4 (a) and (b)

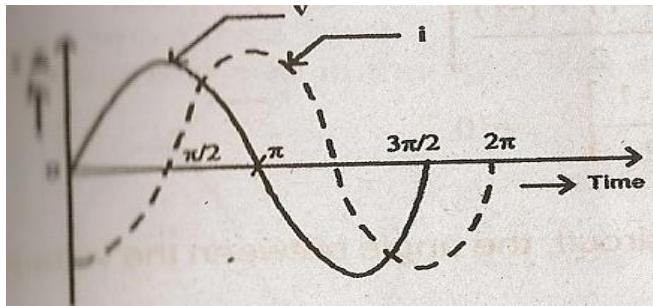
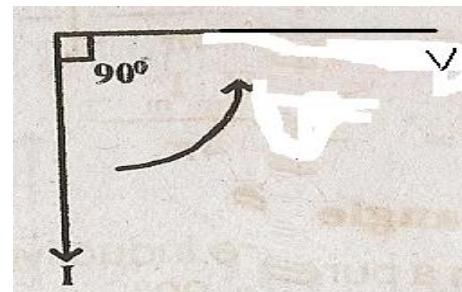


Fig.2.4 (a)



(b)

### Power (P)

$$V = V_m \sin \omega t, i = I_m \sin (\omega t - 90^\circ)$$

$$\text{Instantaneous power} = V \times i = V_m \sin \omega t \times I_m \sin (\omega t - 90^\circ)$$

$$\text{Average power} = \int_0^{2\pi} \frac{V_m \sin \omega t \times I_m \sin (\omega t - 90^\circ)}{2\pi} d(\omega t)$$

$$\frac{V_m I_m}{2\pi} \int_0^{2\pi} \sin \omega t \sin (\omega t - 90^\circ) d(\omega t)$$

$$\begin{aligned} &= \frac{V_m I_m}{2\pi} \int_0^{2\pi} \frac{\cos(\omega t - \omega t + 90^\circ) - \cos(\omega t + \omega t - 90^\circ)}{2} d(\omega t) \\ &= \frac{V_m I_m}{4\pi} \int_0^{2\pi} \cos 90^\circ - \cos(2\omega t - 90^\circ) d(\omega t) \\ &= \frac{V_m I_m}{4\pi} \int_0^{2\pi} [0 - \cos(2\omega t - 90^\circ)] d(\omega t) \\ &= \frac{V_m I_m}{4\pi} \left[ \frac{-\sin 2\omega t - 90^\circ}{2} \right]_0^{2\pi} \\ &= \frac{V_m I_m}{4\pi} \left[ \frac{-\sin (2 \times 2\pi - 90^\circ) + \sin (0 - 90^\circ)}{2} \right] \\ &= \frac{V_m I_m}{4\pi} \left[ \frac{(-(-1)) + (-1)}{2} \right] \\ &= \frac{V_m I_m}{4\pi} [0] = 0 \end{aligned}$$

$$\text{Power} = 0$$

### Phase angle

Phas angle is an angle between the voltage and current

In inductive circuit, the angle between the voltage and current. is  $90^\circ$

i.e.  $\theta = 90^\circ$       The current is always lagging the voltage by  $90^\circ$

### Power factor

$$\begin{aligned}\text{Power factor} &= \cos\theta \\ &= \cos(90^\circ) \\ &= 0 \quad (\text{lag}) \quad (\theta = 90^\circ)\end{aligned}$$

### 2.1.3 Ac through pure capacitor only

The circuit shown in the fig.2.5 is a pure capacitive circuit.

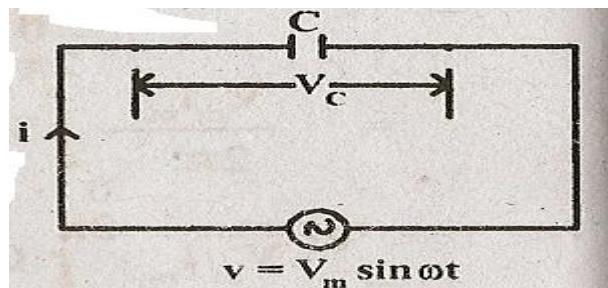


Fig.2.5

When an alternating voltage is applied to a capacitor the capacitor is charged first in one direction and then in the opposite direction.

$$V = V_m \sin \omega t \longrightarrow 1$$

$$Q = CV = C V_m \sin \omega t$$

$$\begin{aligned}\text{Current } i &= \frac{dQ}{dt} \\ &= \frac{d(C V_m \sin \omega t)}{dt} \\ &= C V_m \sin \omega t \cdot \omega = \omega C V_m \sin \omega t\end{aligned}$$

$$= \omega C V_m \sin \omega t (\omega t - 90^\circ)$$

$$I = I_m \sin(\omega t - 90^\circ) \longrightarrow 2$$

$$I_m \sin(\omega t - 90^\circ) = \omega C V_m \sin(\omega t)$$

$$I_m = \omega C V_m \text{ (or)}$$

$$I_m = \frac{V_m}{1/\omega C}$$

From the equ, 1 and 2, we find that the current leads the voltage by  $90^\circ$  the waveform and vector diagram. Are shown in fig.2.6 (a) and (b)

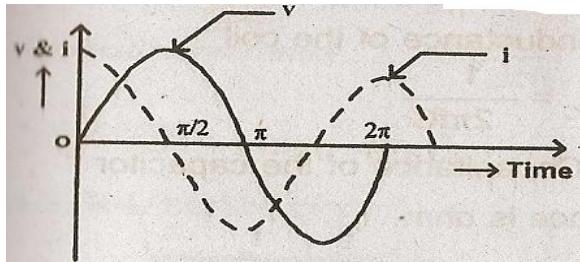
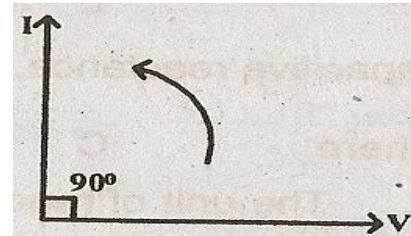


Fig. 2.6(a)



(b)

### Power (p)

$$V = V_m \sin \omega t, i = I_m \sin(\omega t + 90^\circ)$$

$$\text{Instantaneous power} = V \times i = V_m \sin \omega t \times I_m \sin(\omega t + 90^\circ)$$

$$\text{average power} = \int_0^{2\pi} V_i d(\omega t) = 0$$

$$\text{Average power} = \int_0^{2\pi} \frac{V_m \sin \omega t \times I_m \sin(\omega t + 90^\circ)}{2\pi} d(\omega t)$$

$$\frac{V_m I_m}{2\pi} \int_0^{2\pi} \sin \omega t \sin(\omega t + 90^\circ) d(\omega t)$$

$$= \frac{V_m I_m}{2\pi} \int_0^{2\pi} \frac{\cos(\omega t - \omega t - 90^\circ) - \cos(\omega t + \omega t + 90^\circ)}{2} d(\omega t)$$

$$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} \cos(-90^\circ) - \cos(2\omega t + 90^\circ) d(\omega t)$$

$$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} [0 - \cos(2\omega t + 90^\circ)] d(\omega t)$$

$$= \frac{V_m I_m}{4\pi} \left[ \frac{-\sin 2\omega t + 90^\circ}{2} \right]_0^{2\pi}$$

$$= \frac{V_m I_m}{4\pi} \left[ \frac{-\sin (2 \times 2\pi - 90^\circ) + \sin(0 - 90^\circ)}{2} \right]$$

$$\begin{aligned}
 &= \frac{V_m I_m}{4\pi} \left[ \frac{-(1) + 1}{2} \right] \\
 &= \frac{V_m I_m}{4\pi} \left[ \frac{-1 + 1}{2} \right] \\
 &= \frac{V_m I_m}{4\pi} [0] = 0
 \end{aligned}$$

### Phase angle

Phase angle is an angle between the voltage and current

In inductive circuit, the angle between the voltage and current is  $90^\circ$

$$\text{i.e. } \theta = 90^\circ$$

The current is always lagging the voltage by  $90^\circ$

### Power factor

$$\text{Power factor} = \cos\theta$$

$$= \cos(90^\circ)$$

$$= 0 \text{ (lag)}$$

## 2.1.4 IMPORTANT TERMS

### 1. Impedance

It is ratio of the applied voltage to the resulting current.

It is represented by the letter 'Z'.

Its unit is Ohm. ( $\Omega$ )

$$\text{i.e. } Z = \frac{V}{I} \text{ ohm.}$$

### 2. Admittance

It is defined as the reciprocal of impedance.

It is represented by the letter 'Y'.

Its unit is mho.

$$\text{i.e. } Y = \frac{1}{Z} = \frac{I}{V} \text{ mho}$$

### **3. Reactance**

Inductive reactance,  $X_L = 2\pi f L$

Where  $f$  = freq,

$L$ =inductance of the coil

Capacitive reactance,  $X_C = 1 / 2\pi f C$

Where

$C$ = capacitance of the capacitor

Unit of the reactance is Ohm

### **4. Susceptance**

It is defined as the reciprocal of reactance.

It is represented by the letter 'b' Its unit is mho

$$\therefore B = 1 / X \text{ mho}$$

### **5. Conductance**

It is defined as the reciprocal of resistance.

It is represented by the letter 'g'. Its unit is 'mho'.

$$\therefore G = 1/R \text{ mho}$$

## **PROBLEM**

### **Example 2.1**

A voltage of 100V at 50 Hz is applied across a pure inductor of 0.04 henry. Find the current solution

Given Data

Voltage,  $V = 100$  Volts

Frequency,  $f = 50$ Hz

Inductance,  $L = 0.04$  henry

$$\text{current, } I = \frac{V}{X_L}$$

$$= \frac{V}{2\pi fL}$$

$$= \frac{100}{\pi \times 50 \times 0.04}$$

Current ,  $I = 7.96$  amps.

### Example 2.2

An a.c circuit with a pure capacitor has a reactance of 6 ohms at 50 Hz. a voltage of 100V,50Hz is applied to the terminal of the capacitor. find (a) current (b) capacitance

**Solution**

**Given Data**

Voltage,  $V = 100$  Volts

Frequency,  $f = 50$ Hz

Capacitive reactance,  $X_C = 6 \Omega$

$$\text{Current } I = \frac{V}{X_C} = \frac{100}{6}$$

$$I = 16.66 \text{ amps.}$$

$$\text{Capacitive reactance, } X_C = \frac{1}{2\pi f C}$$

$$\text{Capacitance, } C = \frac{1}{2\pi \times 50 \times 6}$$

$$\text{Capacitance, } C = 5.3 \times 10^{-4} \text{ farad}$$

### Example 2.3

A voltage  $V = 200 \sin \omega t$  is applied to a 25 ohm resistor. Find the current  $I$  , the instantaneous power,  $p$  and the average power.

**Solution**

**Given Data**

Voltage, $V$	$= 200 \sin \omega t$
Resistance, $R$	$= 25 \Omega$
Current, $i$	$= V / R$

$$= \frac{200 \sin \omega t}{25}$$

$$I = 8 \sin \omega t$$

instantaneous Power,  $P = v \cdot i$   
 $= (200 \sin \omega t) (8 \sin \omega t)$   
 $= 1600 \sin^2 \omega t$

Average power  $p = \frac{1}{2\pi} \int_0^{2\pi} 1600 \sin^2 \omega t$

$$P = E \cdot I$$

$$p = \frac{200}{\sqrt{2}} \times \frac{8}{\sqrt{2}} = 800 \text{w}$$

**Power ,  $P= 800 \text{ watts}$**

### Example 2.4

A pure inductance  $L = 0.1 \text{ H}$  has applied voltage of  $V = 200 \sin 314t$ . Find the current the instantaneous and average power. Find also the inductive reactance and the RMS current.

#### Solution

##### Given Data

Inductance,  $L = 0.1 \text{ H}$   
Applied voltage ,  $V = 200 \sin 314t$

$$I = l \frac{di}{dt} \quad (\text{or}) \quad I = \frac{1}{L} \int e dt$$

$$= \frac{1}{0.1} \int 200 \sin 314t dt$$

$$= 10 \left[ 200 \frac{(-\cos 314t)}{314} \right]$$

$$= \frac{-2000}{314} \cos 314t$$

$$I = -6.37 \cos 314t$$

##### Power, $P$

$$p = e \times i \Rightarrow (200 \sin 314t) \times (-6.37 \cos 314t)$$

$$= -1274 \sin 314t \cos 314t$$

$$= -1274 (1/2 \sin 2(314t))$$

$$= \frac{-1274}{2} \sin 628 t$$

$$\text{Power, } P = -637 \sin 628 t$$

$$\begin{aligned} E \\ I_{\text{RMS}} &= \frac{\text{_____}}{XL} \\ &= \frac{200\sqrt{2}}{XL} \end{aligned}$$

$$\text{inductive reactance, } XL = 2\pi fL = 2 \times 50 \times 0.1$$

$$XL = 31.4 \text{ ohm}$$

$$\frac{E}{XL} = \frac{200\sqrt{2}}{XL} = 4.5 \text{ amps}$$

**R.M.S. current,, I = 4.5 amps.**

### 2.1.5 R-L SERIES CIRCUIT

In this Fig. 2.7 a pure resistance of 'R' ohms is connected in series with a pure inductance of L Henry

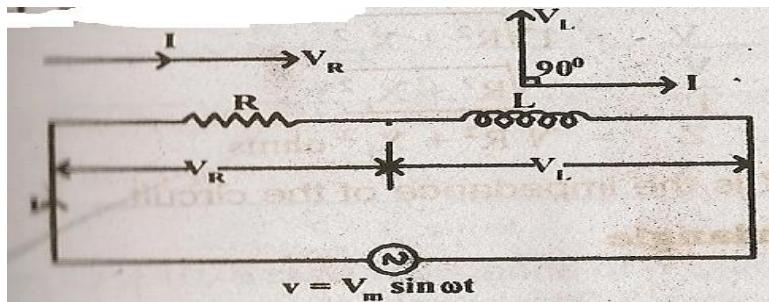


Fig. 2.7

Let

V = applied voltage (RMS)

I = current inn the circuit (RMS)

$V_R$  = voltage drop across the resistor ( $IR$ )

$V_L$  = voltage drop across inductor ( $IXL$ )

The phasor diagram of the circuit is shown Fig. 2.8 (a) and (b) taking of as the reference phasor.

OA → represents the voltage drop  $V_R$  and it is in phase with the current I

AB → represents the voltage  $V_L$  which leads the current by  $90^\circ$ .

OB → represents the applied voltage V, the phasor sum of the two drops.

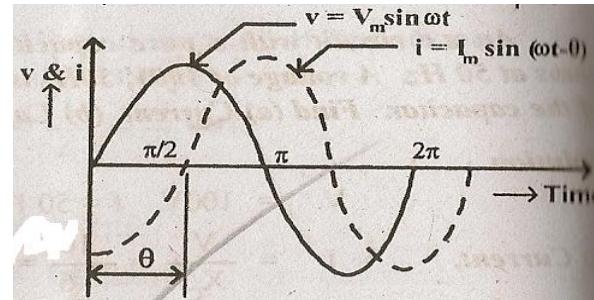
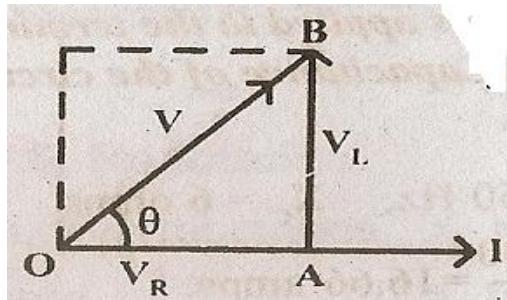


fig. 2.8 (a)

$$I.e., \quad OB = \sqrt{OA^2 + AB^2}$$

(b)

$$\begin{aligned} V &= \sqrt{VR^2 + VL^2} \\ V &= \sqrt{IR^2 + (IXL)^2} \\ V &= I\sqrt{R^2 + XL^2} \\ \frac{V}{I} &= \sqrt{R^2 + XL^2} \\ I.e., Z &= \sqrt{R^2 + XL^2} \text{ ohms.} \end{aligned}$$

Where Z is the impedance of the circuit.

### Impedance triangle

#### Phase angle

From the phasor diagrams that current lags the applied voltage by an angle  $\theta$ .

$$\tan \theta = \frac{AB}{OA} = \frac{VL}{VR} = \frac{IXL}{IR} = \frac{XL}{R}$$

$$\therefore \theta = \tan^{-1} \frac{XL}{R}$$

#### Power factor

$$\text{Power factor} = \cos \theta$$

$$\begin{aligned} &= \frac{OA}{OB} = \frac{VR}{V} = \frac{IR}{IZ} \\ \cos \theta &= \frac{R}{Z} \end{aligned}$$

## Power (P)

$$\begin{aligned} \text{Voltage } V &= V_m \sin \omega t, \\ \text{Current , i} &= I_m \sin (\omega t - \theta) \end{aligned}$$

$$\text{Instantaneous power, p} = vxi$$

$$\text{average power} = \int_0^{2\pi} \frac{V_m \sin \omega t \times I_m \sin(\omega t - 0)}{2\pi} d(\omega t)$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \theta$$

$$\text{Power , P} = V I \cos \theta \text{ watts}$$

## 2.1.6 R-C SERIES CIRCUIT

In Fig.2.9 a pure resistance of 'R' ohms is connected in series with a pure capacitance of C farads.

Let,  $V$  = applied voltage

$I$  = current inn the circuit

$V_R$  = voltage drop across the resistor ( $IR$ )

$V_C$  = voltage drop across capacitor ( $IXC$ )

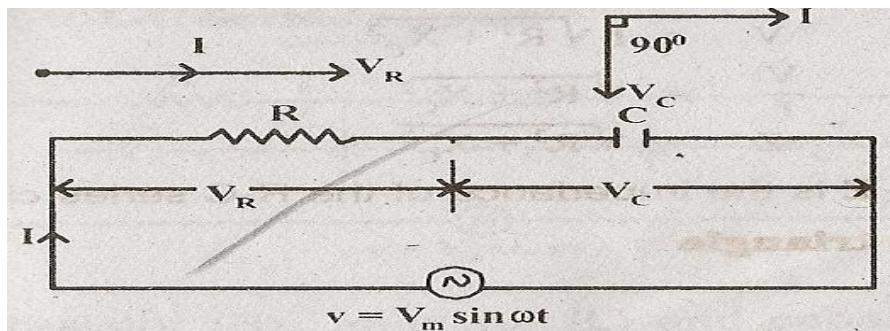


Fig. 2.9

The phasor diagram of the circuit is shown Fig.2.10 (a) and (b) taking of as the reference phasor.

OA → represents the voltage drop  $V_R$  and it is in phase with the current  $I$

AB → represents the voltage  $V_C$  which lags the current by  $90^\circ$ .

OB → represents the applied voltage  $V$ , the phaser sum of the two drops.

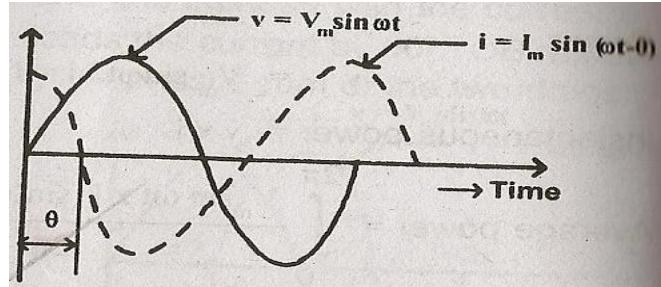
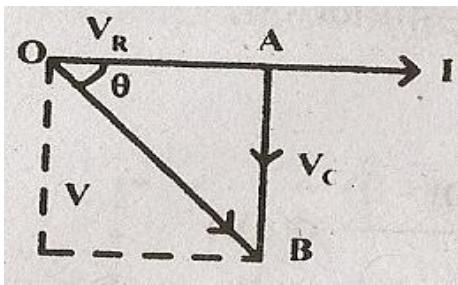


Fig. 2.10 (a)

$$I.e., \quad OB = \sqrt{OA^2 + AB^2}$$

$$V = \sqrt{VR^2 + VC^2}$$

$$V = \sqrt{IR^2 + (IXC)^2}$$

$$V = I\sqrt{R^2 + C}$$

$$\frac{V}{I} = \sqrt{R^2 + XC^2}$$

$$i.e., Z = \sqrt{R^2 + XC^2} \text{ ohms.}$$

Where  $Z$  is the impedance of the circuit. R - C series circuit.

### Impedance triangle

### Phase angle

From the phasor diagrams that current lags the applied voltage by an angle  $\theta$ .

$$\tan \theta = \frac{AB}{OA} = \frac{VC}{VR} = \frac{IXC}{IR} = \frac{XC}{R}$$

$$\therefore \theta = \tan^{-1} \frac{XC}{R}$$

### Power factor:

$$\text{Power factor} = \cos \theta$$

$$\begin{aligned} &= \frac{OA}{OB} = \frac{VR}{V} = \frac{IR}{IZ} \\ \cos \theta &= \frac{R}{Z} \end{aligned}$$

### Power (P)

$$V = V_m \sin \omega t, \quad i = I_m \sin(\omega t + \theta)$$

Instantaneous power,  $p = v \times i$

$$\text{average power} = \int_0^{2\pi} \frac{V_m \sin \omega t \times I_m \sin(\omega t + 0)}{2\pi} d(\omega t)$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \theta$$

$$\text{Power, } P = V I \cos \theta \text{ watts}$$

### 2.1.7 R-L-C series circuit

R-L-C series circuit is shown in the Fig.2.11 resister; inductor and capacitor are connected in series across 'V' volts. The current in the circuit is 'I' amps.

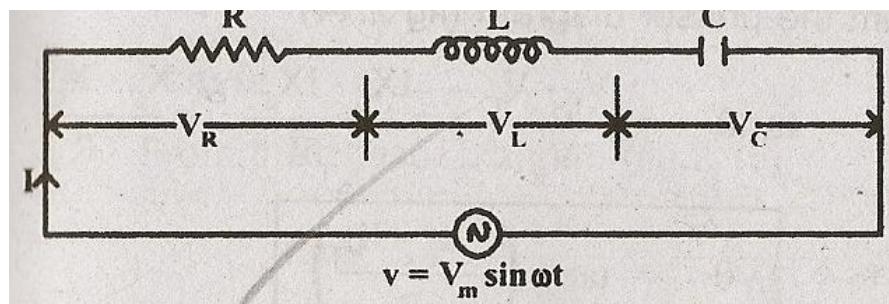


Fig. 2.11

$V_R$  = voltage drop across the resistor R ( $IR$ )

$V_C$  = voltage drop across capacitor C ( $IX_C$ )

$V_L$  = voltage drop across capacitor L ( $IX_L$ )

**Case (1)**  $X_L > X_C$

Fig. 2.12

If  $X_L$  is greater than  $X_C$ , then the result reactance is  $X_L - X_C$

$$OD = \sqrt{OA^2 + AD^2}$$

$$\begin{aligned} V &= \sqrt{VR^2 + (Vl - VC)^2} \\ &= \sqrt{(IR)^2 + (IXL - IXC)^2} \\ &= I\sqrt{R^2 + (XL - XC)^2} \\ \frac{V}{I} &= \sqrt{R^2 + (XL - XC)^2} \\ \therefore Z &= \sqrt{R^2 + (XL - XC)^2} \quad \text{Ohms.} \end{aligned}$$

### Phase angle

From the phasor diagram

$$\begin{aligned} \tan \theta &= \frac{VL - VC}{VR} = IXL - IXC \\ &= \frac{IXL - IXC}{IR} \\ &= \frac{I(XL - XC)}{IR} = \frac{XL - XC}{R} \\ \therefore \theta &= \tanh^{-1} \left[ \frac{XL - XC}{R} \right] \end{aligned}$$

### Power factor

$$\begin{aligned} \text{Power factor} &= \cos \theta \\ \cos \theta &= \frac{OA}{OD} = \frac{VR}{V} = \frac{IR}{IZ} = \frac{R}{Z} \\ \cos \theta &= \frac{R}{\sqrt{R^2 + (RL - Xl)^2}} \end{aligned}$$

**Power=VI cosθ watts.**

**Case (2)**  $X_C > X_L$

Fig. 2.13

If  $X_L$  is greater than  $X_C$ , then the result reactance is  $X_C - X_L$

$$OD = \sqrt{OA^2 + AD^2}$$

$$V = \sqrt{VR^2 + (VC - VL)^2}$$

$$= \sqrt{(IR)^2 + (IXC - IXL)^2}$$

$$= I\sqrt{R^2 + (XC - XL)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (XC - XL)^2}$$

$$\therefore Z = \sqrt{R^2 + (XC - XL)^2} \text{ Ohms.}$$

### Phase angle

From the phasor diagram

$$\begin{aligned}\tan \theta &= \frac{VC - VL}{VR} = IXC - IXL \\ &= \frac{IXC - IXL}{IR} \\ &= \frac{I(XC - XL)}{IR} = \frac{XC - XL}{R} \\ \therefore \theta &= \tanh^{-1} \left[ \frac{XC - XL}{R} \right]\end{aligned}$$

### Power factor

**Power factor** =  $\cos\theta$

$$\begin{aligned}\cos\theta &= \frac{\text{Adjacent Side}}{\text{Hyp..side}} \\ &= \left[ \cos\theta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (XC - XL)^2}} \right]\end{aligned}$$

$$= \frac{VR}{V} = \frac{IR}{IZ} = \frac{R}{\sqrt{R^2 + (XC - XL)^2}}$$

## Power, P

**Power = VI cosθ watts.**

- If the net reactance is inductive ( $XL - XC$ ) then the power factor is lagging.
- If net reactance is capacitive ( $XC - XL$ ), then the power factor is leading

## Example 2.5

A current of 10A flows in a circuit with a  $30^\circ$  angle of lag when the applied voltage is 100V. Find (a) resistance, reactance and impedance (b) the conductance, susceptance and admittance

### Given data

Current, I = 10 A

Applied voltage = 100 v

Phase angle =  $30^\circ$

### Solution

#### (a) (i) Impedance

$$Z = \frac{V}{I} = \frac{100}{10}$$

$$Z = 10\Omega$$

#### (ii) Resistance , R

$$\cos \theta = \frac{R}{Z} \Rightarrow R = Z \cos \theta$$

$$\begin{aligned} R &= 10 \times \cos 30^\circ \\ &= 10 \times 0.866 = 8.66\Omega \end{aligned}$$

$$R = 8.66\Omega$$

#### (iii) Reactance, XL

$$Z = \sqrt{R^2 + XL^2}$$

$$Z^2 = R^2 + XL^2$$

$$XL^2 = Z^2 - R^2$$

$$XL = \sqrt{Z^2 - R^2}$$

$$= \sqrt{(10)^2 - (8.66)^2}$$

$$XL = 5\Omega$$

**(b) (i) Conductance, G**

$$G = (1/R)$$

$$\begin{aligned} G &= 1/R \\ &= 1/8.66 \end{aligned}$$

$$G = 0.115 \text{ S}$$

**(ii) Admittance (1/Z)**

$$\frac{1}{Z} = \frac{1}{10} = 0.1 \text{ S}$$

**(iii) Susceptance (1/XL)**

$$\frac{1}{XL} = \frac{1}{5} = 0.2 \text{ S}$$

**Example 2.6**

A circuit consists of a resistance R ohm and inductance L Henry connected in series. If it is connected to a 240V, 50Hz supply mains it consumes 300Watts and the voltage drop across 'R' is 100V. Calculate the value of inductance 'L'

**Given Data**

Voltage drop across,  $V_R = 100V$

Power consumed,  $P = 300 \text{ watts.}$

Frequency,  $f = 50\text{Hz}$

**Solution**

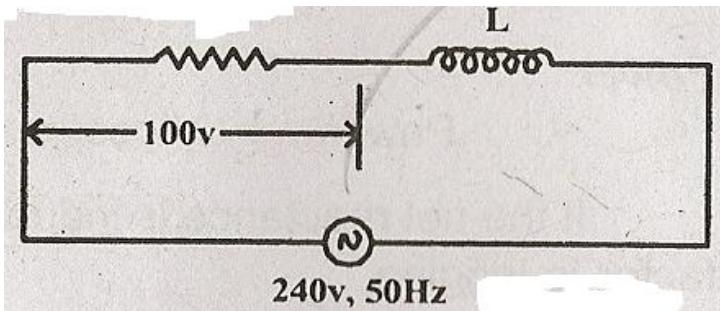


Fig. 2.14

$$\text{Inductance ,} XL = 2\pi f L$$

$$L = \frac{XL}{2\pi f}$$

$$XL = \sqrt{Z^2 + R^2}$$

$$I = \frac{P}{VR} = \frac{300}{100} = 3 \text{ amps}$$

$$\text{Current , } I = 3 \text{ amps}$$

### Impedance, Z

$$Z = \frac{V}{I} = \frac{240}{3} = 80$$

$$Z = 80\Omega$$

$$\text{Power consumed, } I^2R = 300$$

$$R = \frac{300}{I^2} = \frac{300}{(3)^2}$$

$$R = 33.33\Omega$$

$$XL = \sqrt{Z^2 - R^2} \Rightarrow XL = \sqrt{80^2 - (33.33)^2}$$

$$XL = 72.73\Omega$$

$$\therefore XL = 2\pi f L \Rightarrow \frac{XL}{2\pi f}$$

$$72.73$$

$$\text{Inductance , } L = \frac{XL}{2\pi f}$$

$$L = 0.232 \text{ Henry}$$

### Example 2.7

Calculate (a) the admittance (b) the conductance, G and (c) susceptance, B of a circuit consisting of a resistor of  $10\Omega$  in series with an inductor of  $0.1H$  when the freq. is  $50HZ$  and  $100HZ$ .

Resistance, R =  $10\Omega$   
Inductance ,L =  $0.1H$   
Frequency, f =  $50HZ$

### Solution

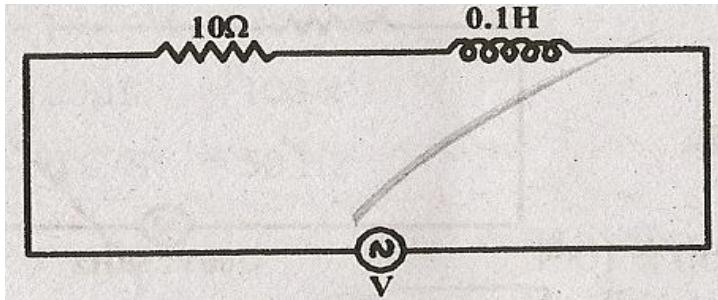


Fig.2.15

#### Case (I)

(a) Admittance ,Y =

$$Z = \sqrt{R^2 + XL^2}$$

$$XL = 2\pi fl = 2 \times \pi \times 50 \times 0.1$$

$$XL = 31.42\Omega$$

$$Z = \sqrt{(10)^2 + (31.42)^2}$$

$$Z = 32.97\Omega$$

$$Y = \frac{1}{Z} = \frac{1}{32.97}\Omega$$

Admittance, Y =  $0.03 mho$

(b) conductance, G =  $1/R$

$$G = \frac{1}{R} = \frac{1}{10} = 0.1 mho$$

$$G = 0.1 mho$$

(c) susceptance, B

$$B = \frac{1}{XL} = \frac{1}{31.42} = 0.032 mho$$

$$B = 0.032 mho$$

### Case (ii)

When freq. f=100HZ

(a) admittance, Y

$$Y = \frac{1}{Z} \quad Z = \sqrt{R^2 + XL^2}$$

$$XL = 2\pi fl \Rightarrow 2\pi \times 100 \times 0.1$$

$$XL = 62.83\Omega$$
$$Z = \sqrt{(10)^2 + (62.83)^2}$$

$$Z = 63.62\Omega$$
$$Y = \frac{1}{Z} = \frac{1}{63.62} \Omega$$

Admittance, Y= 0.0157mho

(b) conductance, G =1/R

$$G = \frac{1}{R} = \frac{1}{10} = 0.1 \text{ mho}$$

G= 0.1 mho

(c) susceptance, B

$$B = \frac{1}{XL} = \frac{1}{62.83} = 0.0159 \text{ mho}$$

$$B = 0.0159 \text{ U}$$

### Example 2.8

A resistor of  $100\Omega$  is connected in series with a  $50\mu\text{f}$  capacitor to a supply of 200V,50HZ  
find(a) the impedance (b)the current (c) the P.F(d) phase angle (e) the voltage across the resistor and across the capacitor.

#### Given data

Resistor, R	= $100\Omega$
Capacitor, C	= $50\mu\text{f} = 50 \times 10^{-6} \text{ F}$
Supply voltage, V	= 200V
Frequency, f	= 50HZ.

**Solution**

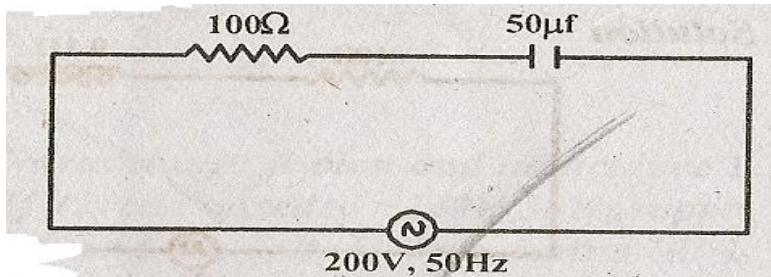


Fig.2.16

(a) Impedance

$$Z = \sqrt{R^2 + XC^2}$$

$$XC = \frac{1}{2\pi f c} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$$

$$X_C = 63.69\Omega$$

$$Z = \sqrt{(100)^2 + (63.69)^2}$$

$$Z = 118.56\Omega$$

(b) Current. I

$$I = \frac{V}{Z} = \frac{200}{118.56} = 1.69$$

$$I = 1.69 \text{ amps.}$$

(c) Power factor

$$\cos \theta = \frac{R}{Z} = \frac{100}{118.56} = 0.843$$

$$\cos \theta = 0.843$$

(d) Phase angle, θ

$$\theta = \tan^{-1} \left( \frac{XC}{R} \right)$$

$$\theta = \tan^{-1} \left[ \frac{63.69}{100} \right] = 32.49^\circ$$

$$\theta = 32.49^\circ$$

(e) Voltage drop across the resistor

$$V_R = IR = 1.69 \times 100$$

$$V_R = 169 \text{ volts.}$$

(f) Voltage drop across the capacitor

$$V_C = I \times C = 1.69 \times 63.69$$

$$V_C = 107.64 \text{ volts.}$$

### Example 2.9

A series circuit has  $R=10\Omega$ ,  $L=50\text{mH}$  and  $C= 100\mu\text{f}$  and supplied with  $200\text{V}, 50 \text{c/s}$ . Find  
 (a) impedance (b) current (c) power (d) Power factor (e) voltage drop across each element.

#### Given data

Resistor, R	= $10\Omega$
Capacitor, C	= $100\mu\text{f} = 100 \times 10^{-6} \text{ F}$
Inductance, L	= $50\text{mH}$
Supply voltage, V	= $200\text{V}$
Frequency, f	= $50\text{Hz}$ .

#### Solution

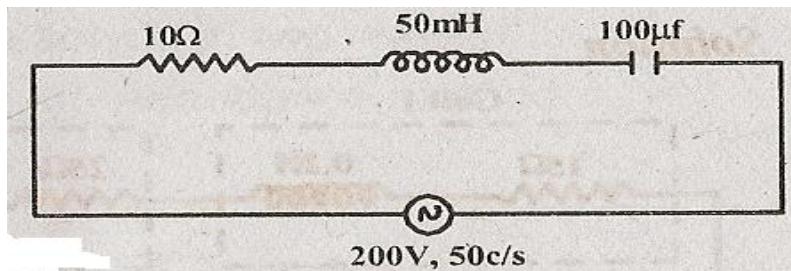


Fig.2.17

#### (a) Impedance, Z

[since  $XC > XL$ ]

$$Z = \sqrt{R^2 + (XC - XL)^2}$$

$$\text{Inductive reactance, } XL = 2\pi fl = 2 \times \pi \times 50 \times 50 \times 10^{-3}$$

$$XL = 15.7$$

#### (b) Inductive reactance, $X_C$

$$XC = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$XC = 31.83\Omega$$

$$Z = \sqrt{(10)^2 + (31.83 - 15.7)^2}$$

$$Z = \sqrt{100 + 260.18}$$

$$Z = 18.98\Omega$$

**(c) Current:**

$$I = \frac{V}{Z} = \frac{200}{18.98}$$

$$I = 10.54 \text{ amps.}$$

**(c) Power, p**

$$P = I^2 R = (10.54)^2 \times 10$$

$$P = 1110.9 \text{ Watts}$$

**(d) Power factor = Cosθ**

$$P.f = \cos \theta = \frac{R}{Z} = \frac{10}{18.98}$$

$$P.f = \cos \theta = 0.527$$

**(e) (i) Voltage drop across R , V<sub>R</sub>**

$$V_R = I_R = 10.54 \times 10$$

$$V_R = 105.4 \text{ volts.}$$

**(ii) Voltage drop across L , V<sub>L</sub>**

$$V_L = I \times X_L = 10.54 \times 15.7$$

$$= 165.48 \text{ volts}$$

**(iii) Voltage drop across C , V<sub>C</sub>**

$$V_C = I \times X_C$$

$$= 10.54 \times 31.83$$

$$C = 335.49 \text{ volts}$$

### Example 2.10

A coil having a resistance of  $15\Omega$  and an inductance of  $0.2\text{H}$  is connected in series with another coil having a resistance of  $25\Omega$  and an inductance of  $0.04\text{ H}$  to a  $230\text{V}$ ,  $50\text{HZ}$  supply. Determine the voltage across each coil, the power dissipated in each coil, the p.f of the circuit. as whole.

#### Given data

$$\text{Resistors } R_1 = 15\Omega \quad R_2 = 25\Omega$$

$$\text{Inductance }, L_1 = 0.2\text{H} \quad L_2 = 0.04\text{H}$$

$$\text{Supply voltage, } V = 230\text{V}$$

$$\text{Frequency, } f = 50\text{HZ.}$$

#### Solution

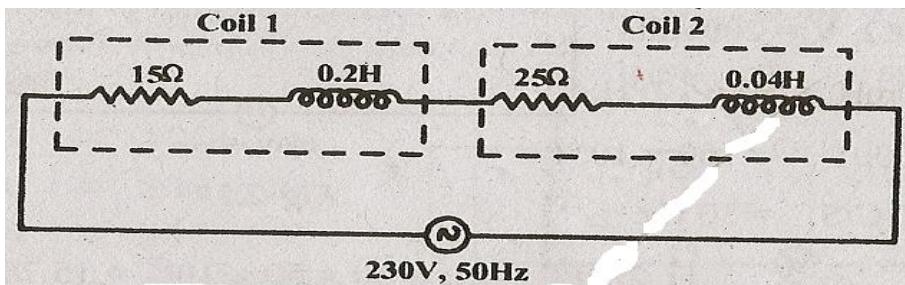


Fig.2.18

**(a) Voltage across each coil**

$$X_{L1} = 2\pi f L_1 = 2\pi \times 50 \times 0.2 = 62.83\Omega$$

$$X_{L1} = 62.83\Omega$$

$$X_{L2} = 2\pi f L_2 = 2\pi \times 50 \times 0.04 = 12.57$$

$$X_{L2} = 12.57\Omega$$

$$Z_1 = \sqrt{R_1^2 + X_{L1}^2} = \sqrt{(15)^2 + (62.83)^2}$$

$$Z_1 = 64.6\Omega$$

$$Z_2 = \sqrt{R_2^2 + X_{L2}^2} = \sqrt{(25)^2 + (12.57)^2}$$

$$Z_2 = 27.98\Omega$$

$$\therefore R = R_1 + R_2 = 15 + 25 = 40\Omega$$

$$X_L = X_{L1} + X_{L2} = 62.83 + 12.57$$

$$X_L = 75.4\Omega$$

$$Total Z = \sqrt{R^2 + X_L^2} = \sqrt{(40)^2 + (75.4)^2}$$

Total Impedance,  $Z = 85.35\Omega$

$$current, I = \frac{V}{Z} = \frac{230}{85.35} = 2.69$$

$$I = 2.69 \text{ amps}$$

Voltage drop across coil 1,  $V_1 = IZ_1$

$$= 2.69 \times 64.6$$

$$V_1 = 173.77 \text{ volts}$$

Voltage drop across coil 2,  $V_2 = IZ_2$

$$= 2.69 \times 27.98$$

$$V_2 = 75.27 \text{ volts}$$

**Power of dissipated in each coil**

$$\text{Power of coil } L_1, P_1 = I^2 R_1$$

$$= (2.69)^2 \times 15$$

$$P_1 = 108.5$$

Power of coil L2,  $P_2 = I^2 R_2$

$$= (2.69)^2 \times 25$$

$$P_2 = 180.9 \text{ watts}$$

### Power factor

$$\text{Power factor} = \cos\theta$$

$$\cos\theta = R / Z$$

$$= \frac{40}{85.35} = 0.469$$

$$\text{Power factor} = 0.469$$

### Example 2.11 (April , 2004)

A resistor of 3 ohms and an inductive reactor of 4 ohm are connected in series. This circuit connected across a single phase supply of 100V. find the line current and power consumed.

#### Given data

$$\text{Resistors } R = 3\Omega$$

$$\text{Inductive reactor, } X_L = 0.2H$$

$$\text{Supply voltage, } V = 100V$$

#### Solution

(1) Line current,  $I$

$$I = V/Z$$
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(3)^2 + (4)^2}$$

$$Z = \sqrt{9 + 16}$$

$$Z = \sqrt{25}$$

$$Z = 5\Omega$$

$$\text{current, } I = \frac{V}{Z} = \frac{100}{5} = 20$$

$$I = 20A$$

(2) Power consumed,  $P$

$$P = I^2 R = (20)^2 \times 3$$

$$P = 1200 \text{ Watts}$$

### Example 2.12

A resistance of  $20\Omega$  an inductance of  $0.2 \text{ H}$  and a capacitance  $100 \text{ micro farad}$  connected in series across a  $220V$   $50\text{Hz}$  supply. Determine (a) impedance (b) current (c) voltage across resistance, inductance and capacitance (d) power factor (e) phase angle (f) power in watts.

#### Given data

Resistors R	= $20\Omega$
Inductance, L	= $0.2\text{H}$
Supply voltage, V	= $100\text{V}$
Capacitance, C	= $100\mu\text{f} = 100 \times 10^{-6} \text{ f}$
Supply voltage, V	= $220\text{V}$
Frequency, f	= $50\text{HZ.}$

#### Solution

##### (a) Impedance

$$Z = \sqrt{R^2 + (XC - XL)^2}$$

$$\text{Inductive reactance, } X_L = 2\pi fL = 2\pi \times 50 \times 0.2 = 62.83\Omega$$

$$X_{L1} = 62.83\Omega$$

$$\text{Capacitive reactance, } XC$$

$$XC = \frac{1}{2\pi f c} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$XC = 31.83\Omega$$

$$Z = \sqrt{R^2 + (XC - XL)^2}$$

$$Z = \sqrt{(20)^2 + (62.83 - 31.83)^2}$$

$$Z = 36.89\Omega$$

##### (b) Current , I

$$\text{current, } I = \frac{V}{Z} = \frac{220}{36.89}$$

$$I = 5.96 \text{amps}$$

##### (c) Voltage drop across each element

$$\begin{aligned} \text{Voltage drop across resistor , } V_R &= I R \\ &= 5.96 \times 20 \\ V_R &= 119.2\text{V} \end{aligned}$$

$$\begin{aligned}\text{Voltage drop across inductor } V_L &= I \times L \\ &= 5.96 \times 62.83 \\ V_L &= 374.46V\end{aligned}$$

$$\begin{aligned}\text{Voltage drop across capacitor } V_C &= I \times C \\ V_C &= 5.96 \times 31.83 \\ V_C &= 189.7V\end{aligned}$$

**(d) Power factor**

$$\text{Power factor, } = \cos\theta$$

$$\cos\theta = R/Z$$

$$= \frac{20}{36.89}$$

$$\cos\theta = 0.542 \text{ lag}$$

**(e) Phase angle,  $\theta$**

$$\theta = \cos^{-1}(0.542)$$

$$\theta = 57.17$$

**(f) Power , P**

$$\begin{aligned}\text{Power, } &= VI \cos\theta \text{ watts} \\ &= 220 \times 5.96 \times 0.542 \\ \mathbf{P} &= 710.87 \text{ watts}\end{aligned}$$

### 2.1.8 PARALLEL CIRCUIT

Parallel circuit may be solved in two ways (a) Impedance method (b) admittance

**(a) Impedance method**

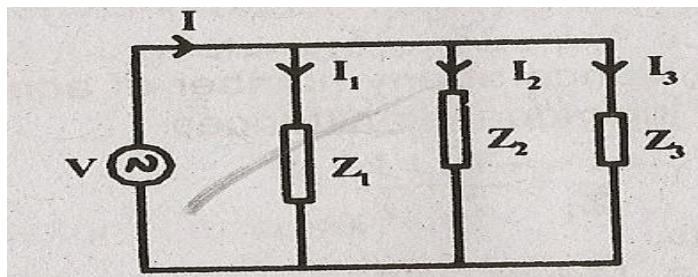


Fig.2.19

Let,

$I_1$  be the current following through the impedance  $Z_1$

$I_2$  be the current following through the impedance  $Z_2$

$I_3$  be the current following through the impedance  $Z_1$

'V' be the applied voltage

'I' is the vector sum of  $I_1, I_2, I_3$

$$I_1 = \frac{V}{Z_1}, I_2 = \frac{V}{Z_2}, I_3 = \frac{V}{Z_3}$$

$$\therefore I = I_1 + I_2 + I_3$$

$$= \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3}$$

$$= V \left( \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right)$$

$$\begin{array}{cccccc} I & & 1 & & 1 & \\ \hline V & = & Z_1 & + & Z_2 & + \\ 1 & & 1 & & 1 & \\ \hline Z & = & Z_1 & + & Z_2 & + \\ & & & & & Z_3 \end{array}$$

### (b) Admittance method

Admittance method is the reciprocal of impedance. Its unit is

$$\therefore Y = \frac{1}{Z}$$

$$Y = \frac{1}{V}$$

$$I = \frac{V}{Z}$$

$$I = V \cdot Y$$

$$I = I_1 + I_2 + I_3$$

$$I = V \cdot Y_1 + V \cdot Y_2 + V \cdot Y_3$$

$$VY = V(Y_1 + Y_2 + Y_3)$$

$$Y = Y_1 + Y_2 + Y_3$$

Equation admittance, Y

$$Y = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

Y is also expressed as

$$Y = G + jB$$

Where,

G= conductance

B= susceptance

$$\text{conductance, } G = \frac{R}{(R - jXC)} \text{ (capacitive)}$$

$$\text{susceptance, } B_1 = \frac{XL}{(R + jXL)} \text{ (inductive)}$$

$$\text{susceptance, } B_2 = \frac{XC}{(R + jXC)} \text{ (capacitive)}$$

### 2.1.9 Resistance – inductance parallel circuit

A resistance of 'R' ohms ad an inductance of L henry are connected in parallel as shown in fig.2.20

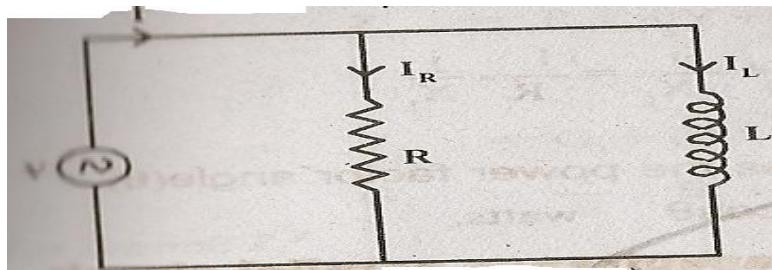


Fig.2.20

#### Impedance method

Let,

IR be the current flowing through 'R'

IL be the current flowing through 'L'

IR is in phase with the voltage V

IL circuit lags the voltage by 90°

Total of I= vertical addition of IR & IL

$$P.f = \cos \theta = \frac{IR}{I} \text{ (lag)} I = \sqrt{IR^2 + IL^2}$$

**Power = VIcosθ watts**

### Admittance method

For branch 1,

$$Z_1 = R + j0$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R + j0} = \frac{1}{R}$$

$$Y_1 = G_1 + jB_1$$

$$\therefore G_1 = \frac{1}{R}, B_1 = 0$$

For branch,2

$$Z_2 = 0 + jX_L$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{0 + jX_L} = \frac{-j}{X_L}$$

$$Y_2 = G_2 + jB_2$$

$$= -j$$

$$G_2 = 0, B_2 = \frac{-j}{X_L}$$

$$\text{Total } Y = Y_1 + Y_2$$

$$= \frac{1}{R} + \frac{-j}{X_L}$$

$$= \frac{1}{R} - \frac{j}{X_L}$$

$$\text{Current, } I = V \cdot Y$$

Angle between  $V$  &  $I$  give the power factor angle ( $\theta$ )

$$\text{Power} = VI \cos\theta \text{ watts}$$

### 2.1.10 Resistance-capacitance parallel circuit

A resistance of 'R' ohms and a capacitive of 'C' farad are connected in parallel as shown in the fig.2.21

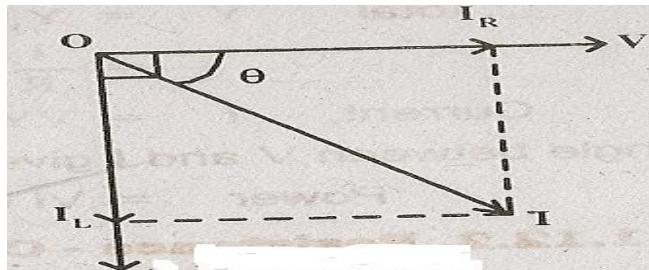


Fig.2.21

### Impedance method

Let,

$I_R$  be the current flowing through 'R'

$I_L$  be the current flowing through 'L'

$I_R$  is in phase with the voltage V

$I_L$  circuit lags the voltage by  $90^\circ$

Total current  $I = \text{vertical addition of } I_R \text{ & } I_c$

Power =  $VI\cos\theta$  watts

### Admittance method

For branch 1,

$$Z_1 = R + j0$$

$$\therefore G_1 = \frac{1}{R}, B_1 = 0$$

For branch 2

$$Z_2 = 0 - jXC$$

$$\frac{1}{Z_2} = \frac{1}{0 - jXC} = \frac{1}{jXC} = \frac{j}{XC}$$

$$\frac{1}{2} = G_2 + jB_2$$

$$\therefore B_2 = \frac{1}{XC}, G_2 = 0$$

Total  $Y = Y_1 + Y_2$

$$= \frac{1}{R} + \left( \frac{j}{RC} \right) = \frac{1}{R} + \frac{j}{XC}$$

Current,  $I = V \cdot Y$

$$\text{Power} = VI \cos\theta \text{ watts}$$

### 2.1.11 RL AND RC IN PARALLEL

Consider a parallel circuit consisting of two branches and connected to an a.c voltage of 'V' volts as shown in the fig.2.22

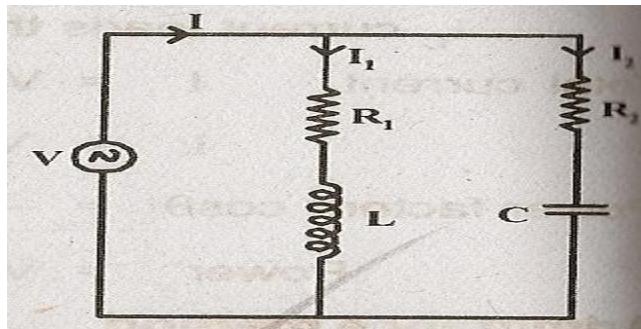


Fig.2.22

#### Impedance method

For branch:1

$$\begin{aligned} Z_1 &= \sqrt{R_1^2 + X_L^2} \\ Z_1 &= \sqrt{R_1^2 + (2\pi f L)^2} \\ I_1 &= V/Z_1 \\ \theta_1 &= \tan^{-1} \left( \frac{X_L}{R_1} \right) \end{aligned}$$

The lags the voltage by  $\theta_1^\circ$

For branch 2

$$\begin{aligned} Z_1 &= \sqrt{R_1^2 + X_C^2} \\ Z_1 &= \sqrt{R_1^2 + (2\pi f C)^2} \\ I_1 &= V/Z_2 \\ \theta_1 &= \tan^{-1} \left( \frac{X_C}{R_1} \right) \end{aligned}$$

The lags the voltage by  $\theta_2^\circ$

#### Admittance method

For branch, 1

Impedance triangle

Admittance triangle:

$$Z1 = R1 + jXl$$

$$Y1 = \frac{1}{Z1} = \frac{1}{R1 + jXl}$$

$$\frac{1}{R1 + jXl} \times \frac{R1 - jXL}{R1 - jXL}$$

Fig.2.23 (a)

(b)

$$\frac{R1 - jXL}{(R^2 + XL^2)}$$

$$\frac{R1}{(R1^2 + XL^2)} - \frac{jXL}{R1^2 + XL^2}$$

$$\frac{R1}{Z1^2} - \frac{jXL}{Z1^2}$$

$$Y1 = G1 - jB1$$

For branch 2,

$$Z2 = R2 - jXc$$

$$Y2 = \frac{1}{Z2} = \frac{1}{R2 - jXc}$$

Fig.2.24 (a)

(b)

$$\frac{1}{R2 - jXc} \times \frac{R2 + jXc}{R2 + jXc}$$

$$\frac{R2 + jXc}{(R2^2 + Xc^2)}$$

$$\frac{R2}{(R2^2 + Xc^2)} + \frac{jXc}{R2^2 + Xc^2}$$

$$\therefore Y2 = \frac{R2}{Z2^2} + \frac{jXc}{Z2^2}$$

$$Y2 = G2 + jB2$$

$$\text{Total } Y = Y1 + Y2 = G1 - jB1 + G2 + jB2$$

$$= (G1 + G2) + j(-B1 + B2)$$

$$= \sqrt{(G1 + G2)^2 + (-B1 + B2)^2}$$

$$\text{Phase angle, } \theta = \tan^{-1} \left[ \frac{(-B + B)}{G1 + G2} \right]$$

$$\text{Power} = VI \cos\theta \text{ watts}$$

### 2.1.12 R-L-C Parallel Circuit

R-L-C parallel circuit is shown in the fig.2.25( a) (b)

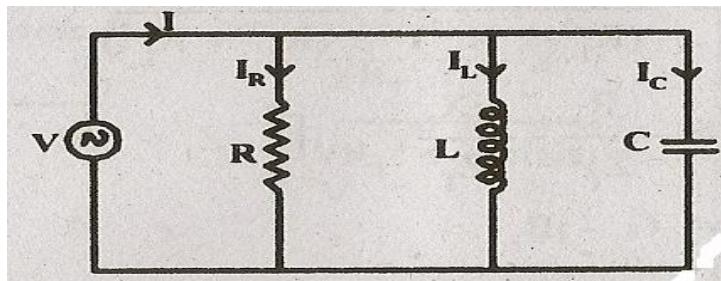
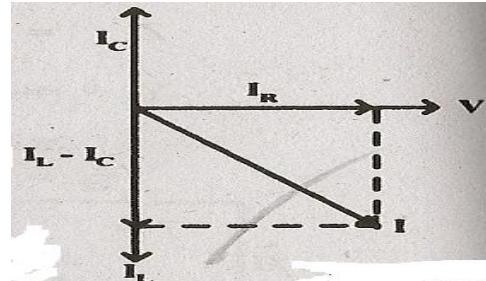


Fig.2.25(a)



(b)

#### Impedance method

$$IR = \frac{V}{R}, IL = \frac{V}{XL}, IC = \frac{V}{Xc}$$

$$= \frac{V}{2\pi fL}, IC = \frac{V}{Xc}$$

(1) If  $I_C$  is more than  $I_L$ , then the circuit is capacitive and power factor is leading.

(2) If  $I_L$  is more than  $I_C$ , then the circuit is inductive and the power factor is lagging.

The vector diagram for the resultant circuit is shown in the fig.

Component,  $IX = (IC - IL)$  or  $(IL - IC)$

Current,  $I = IR + IX$

Power factor,  $\cos\theta = IR/I$

Power =  $VI \cos\theta$  watts

#### Admittance method

For branch 1,

$$Z_1 = R + j0$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R + j0} = \frac{1}{R}$$

$$\therefore G_1 = \frac{1}{R}, B_1 = 0$$

For branch 2,

$$Z_2 = 0 + jXL$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{0 + jXL} = \frac{1}{jXL} = \frac{-j}{XL}$$

$$\frac{1}{2} = G_1 + jB_2$$

$$\therefore B_2 = \frac{-j}{XL}, G_2 = 0$$

For branch, 3

$$Z_3 = 0 - jXC$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{0 - jXC} = \frac{j}{XC}$$

$$Y_3 = G_3 + jB_3$$

$$\therefore B_3 = \frac{j}{XC}, G_3 = 0$$

Total  $Y = Y_1 + Y_2 + Y_3$

$$= \frac{1}{R} + \left( \frac{-j}{XL} \right) + \frac{j}{XC}$$

$$Y = \frac{1}{R} - \frac{j}{XL} + \frac{1}{XC}$$

Current,  $I = V \cdot Y$

**Power =  $VI \cos\theta$  watts.**

### 2.1.13 Series - Parallel Circuit

Series parallel circuit is shown in fig. 2.26

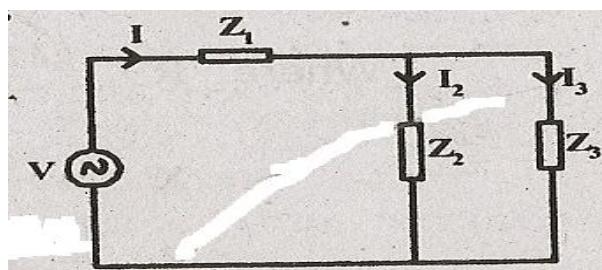


Fig. 2.26

Let  $Z_1$  be the series impedance.

$Z_2$  and  $Z_3$  are connected in parallel.

$Z_1$  is in series with parallel combination of  $Z_2$  and  $Z_3$ .

$$\therefore Z = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

current,  $I = \frac{V}{Z}$

$$I_2 = I \left[ \frac{Z_3}{Z_2 + Z_3} \right], I_3 = I \left[ \frac{Z_2}{Z_2 + Z_3} \right]$$

Angle between V and I is the phase angle ( $\theta$ ).

Power factor =  $\cos\theta$

### Admittance Method

$Z_2$  and  $Z_3$  are connected in parallel.

$$Y_{23} = Y_2 + Y_3$$

$$Y_2 = G_2 + jB_2$$

$$Y_3 = G_3 + jB_3$$

$$G_2 = \frac{R_2}{Z_2^2}, B_2 = \frac{-X_2}{Z_2^2}$$

$$G_3 = \frac{R_3}{Z_3^2}$$

$$B_3 = \frac{-X_3}{Z_3^2}$$

$$Y_{23} = (G_2 + G_3) + j(B_2 + B_3)$$

$$Z_{23} = \frac{1}{Y_{23}} = R + jX$$

$$\text{Where } R = \frac{G}{Y_2}$$

$$X = \frac{-B}{Y_2}$$

$$\therefore Z = Z_1 + Z_{23}$$

$$\text{Current, } I = \frac{V}{Z}$$

$$\text{Current, } I = \frac{V}{Z}$$

$$I_2 = I = \left( \frac{Z_3}{Z_2 + Z_3} \right), I_3 = I \left( \frac{Z_2}{Z_2 + Z_3} \right)$$

### Example 2.9

A coil of resistance 10 ohms and inductance 0.03H is connected in parallel with a non inductive resistor of 15 ohm. Determine (1) the current in each branch circuit (2) the total current (3) the phase angle of the combination when a voltage of 200v at 50 Hz is applied.

#### Given data

$$\text{Inductance, } L = 0.03 \text{ H}$$

$$\text{Resistance, } R_1 = 10 \Omega$$

$$\text{Resistance, } R_2 = 15 \Omega$$

$$\text{Voltage, } V = 200V$$

$$\text{Frequency, } F = 50\text{Hz}$$

#### Solution

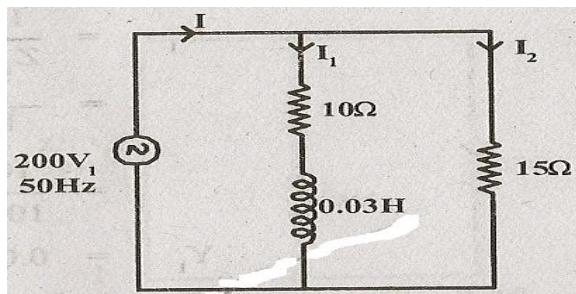


Fig.2.27

#### Impedance Method

$$I_1 = \frac{V}{Z_1}$$

$$Z_1 = R_1 + jX_1$$

$$= 10 + j2\pi fl = 10 + j2\pi \times 50 \times 0.03$$

$$= 10 + j9.43$$

$$\therefore I_1 = \frac{200}{10 + j9.43}$$

$$= \frac{200}{13.74 \angle 43.32} = 14.56 \angle -43.32 \text{ amps.}$$

$$I_2 = \frac{V}{Z_2}, Z_2 = 15\Omega$$

$$\therefore I_2 = \frac{200}{15} = 13.33 \text{ amps.}$$

$$\begin{aligned}
 I &= I_1 + I_2 \\
 &= 23.92 - j9.99 = 25.92 \angle -22.67^\circ \\
 &= 14.56 \angle -43.32^\circ + 13.33 = 10.59 - j9.99 + 13.33
 \end{aligned}$$

Current = 25.92 amps.

Phase angle,  $\theta = -22.67^\circ - 0^\circ = -22.67^\circ$

(Angle between V and I)

$$\therefore \text{power factor} = \cos \theta = \cos(-22.67^\circ) = 0.9226$$

(or)

Admittance method

$$Z_1 = 10 + jXl = 10 + j2\pi fl$$

$$\therefore Z_1 = 10 + j9.43, 2 = 15 + j0$$

$$\begin{aligned}
 \therefore Y_1 &= \frac{1}{Z_1} = \frac{1}{10 + j9.43} \\
 &= \frac{1}{10 + j9.43} \times \frac{10 - j9.43}{10 - j9.43} \\
 &= \frac{10 - j9.43}{10 + j9.43} \times \frac{10 - j9.43}{188.92}
 \end{aligned}$$

$$Y_1 = 0.053 - j0.0499$$

$$Y_2 = \frac{1}{15 + j0} = 0.066 + j0$$

Current in branch  $I_1 = V \cdot Y_1$

$$= 200(0.53 - j0.0499) = 10.6 - j9.98 A$$

Current in branch 2,  $I_2 = V \cdot Y_2$

$$= 200(0.066 + j0) = 13.2 A$$

2. Total current,  $I = I_1 + I_2 = 10.6 - j9.98 + 13.2$

$$= 23.8 - j9.98 = 25.807 \angle -22.749^\circ$$

Current = 25.807 amps

3. Phase angle,  $\theta = 22.749^\circ - 0^\circ = 22.749^\circ$

Power factor =  $\cos \theta = \cos (-22.749^\circ) = 0.9422$

## 2.12 Example

A resistor of 80 ohms, an inductance of 0.15 Hz and the capacitor of 100 $\mu$ f are connected parallel across 200V, 50Hz supply. Determine (1) the current in each branch (2) the total current (3) phase angle of the circuit.

### Given data

Resistance , R	= 80 $\Omega$ , L=0.15H
Capacitance	= 100 $\mu$ f = $100 \times 10^{-6}$ F
Voltage , V	= 200V, f=50Hz.

### Solution

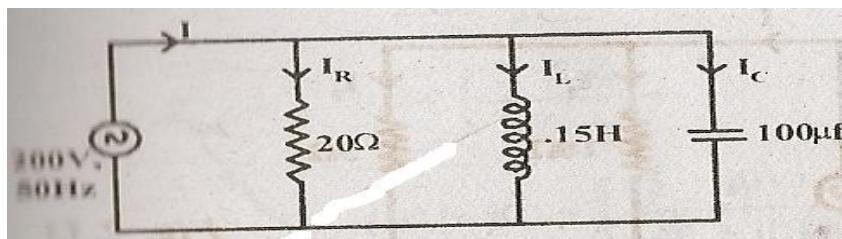


Fig.2.30

$$\text{Impedance , } Z_1 = R+j0$$

$$\therefore I_R = \frac{V}{Z_1} = \frac{200}{80} = 2.5 \text{ amps.}$$

$$Z_2 = 0+jX_L$$

$$X = 2\pi f l = 2\pi \times 50 \times 0.15 = 47.12$$

$$\therefore I_L = \frac{V}{Z_2} = \frac{200}{j47.12} = \frac{200}{47.12 \angle 90^\circ}$$

$$= 4.24 \angle -90^\circ \text{ amps}$$

$$Z_3 = 0-jX_C$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83$$

$$\therefore I_C = \frac{V}{Z_3} = \frac{200}{0 - j31.83} = \frac{200}{-j31.83}$$

$$= \frac{200}{31.83 \angle -90^\circ} = 6.28 \angle 90^\circ \text{ amps.}$$

$$2. \text{ Total current, } I = I_1 + I_2 + I_3$$

$$= 2.5 + 4.24 \angle -90^\circ + 6.28 \angle 90^\circ$$

$$= 2.5 + 0 - j4.24 + 0 + j6.28$$

$$= 2.5 + j 2.04 = 3.23 \angle 39.2^\circ \text{ amps.}$$

3. Phase angle,  $\theta = 39.2^\circ$

## RESONANCE

### Resonance

An a.c circuit is said to be in resonance when the circuit power factor is unity. i.e., At  $X_L = X_C$  resonance occurs. In a resonant circuit, resistors, Inductors and capacitors are connected either in series or in parallel combination. This circuit acts as a pure resistive circuit.

The frequency response of a resonant circuit is shown in the fig.2.37.

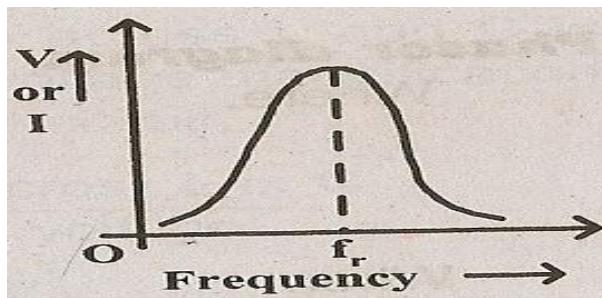


Fig.2.37

The frequency at which resonance occurs is called the resonant frequency ( $f_r$ ). From fig 2.37, the frequency response decreases sharply on either side of the resonant frequency. Hence a resonant circuit selects a range of frequencies for which the response is nearly equal to the maximum.

Two types of response are,

1. Series resonance
2. Parallel resonance

### SERIES RESONANCE

In a series resonance circuit, resistor (R), inductor (L) and capacitor(C) are connected in series across a sinusoidal voltage of variable frequency source. An R-L-C series resonance circuit is shown in the fig.2.38 (a).

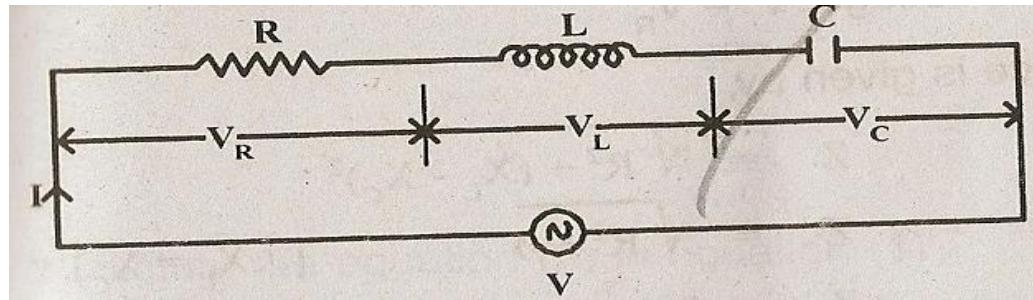


Fig. 2.38(a)

## Resonant frequency

At resonance  $X_L = X_C$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$4\pi^2 f_r^2 LC = 1$$

$$\therefore f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}$$

Phasor diagram

$$\text{Where, } V_L = IX_L$$

## Phasor Diagram

$$V_C = IX_C, \quad V_R = IR$$

$$\text{When } X_L = X_C$$

$$IX_L = IX_C$$

$$\therefore V_L = V_C$$

Fig.2.38(b)

Therefore,  $V_L$  and  $V_C$  are equal in magnitude but opposite in direction, hence they cancel each other.

Applied voltage,  $V = V_R$

Impedance is given by,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + 0} \quad [:\because X_L = X_C]$$

$$\therefore Z = R$$

$$\text{current, } I = \frac{V}{Z} = \frac{V}{R}$$

At resonance, the circuit current is maximum and the impedance is minimum. This is shown in the fig.2.39.

Fig.2.39

### Power factor

$$\cos \theta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}} = 1 \text{ (unity)}$$

### Quality factor of a series resonant circuit

At series resonance, the voltage across L and C is greater than the applied voltage. This voltage magnification is called Q-factor of the series resonant circuit.

$$Q - \text{factor} = \frac{\text{Voltage across } L \text{ or } C}{\text{Applied voltage}}$$

$$\begin{aligned} &= \frac{V_L}{V} = \frac{IX_L}{IR} = \frac{X_L}{R} \\ &= \frac{\omega_r L}{R} \rightarrow (1) \end{aligned}$$

$$\text{Resonant frequency, } f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \rightarrow (2)$$

$$\therefore Q \text{ factor} = \frac{1}{\sqrt{LC}} \times \frac{L}{R}$$

$$= \frac{1}{R} \sqrt{\frac{L}{C}}$$

### Resonance curve of a series resonant circuit

The curve drawn between current and frequency is known as resonance curve. Fig.2.40 shows the resonance curve of a typical R-L-C series circuit.

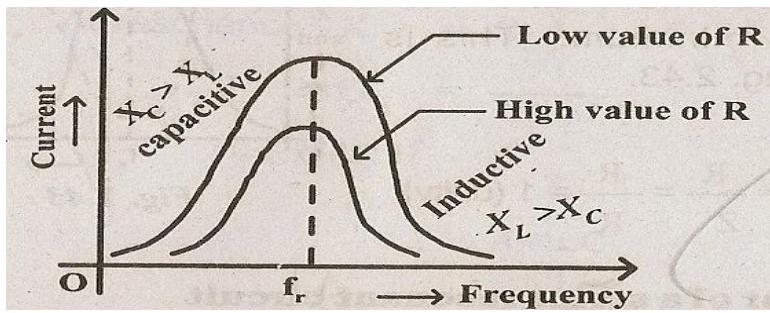


Fig.2.40

At resonant frequency ( $f_r$ ), the inductive reactance  $X_L$  is equal to the capacitive reactance  $X_C$ . The impedance of the circuit is only resistive and equal to  $R$ . hence the current at resonance is maximum and the power factor is unity.

If the frequency of the circuit is less than resonant frequency, then  $X_C > X_L$  and the circuit behaves as a R-C circuit. The current is decreased due to increased impedance, and the power factor is leading.

If the frequency of the circuit is greater than resonant frequency, then  $X_L > X_C$  and the circuit behaves as a R-L circuit. The current is also decreased and the power factor is lagging.

### Bandwidth of a series resonant circuit

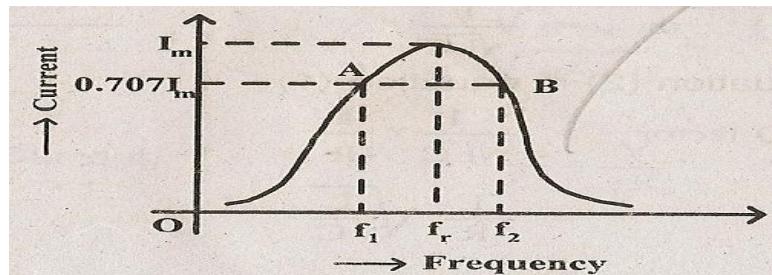


Fig.2.41

The current versus frequency curve for a series resonant circuit is shown in the fig 2.41. At resonance, the current is maximum.

Bandwidth of a circuit is defined as the frequencies which lie between two points on either side of the resonant frequency where current is 0.707 of its maximum value.

As shown in the fig.2.45,

$$\text{Bandwidth, } BW = f_2 - f_1$$

Where,  $f_1$  =Lower cut-off frequency.

$f_2$  =upper cut-off frequency.

The current at points A and B = $0.707 I_m$

$$= \frac{I_m}{\sqrt{2}}$$

Where,  $I_M = \frac{V}{R}$  (maximum current at resonance)

power at points A and B =  $I^2 R$

$$= \left( \frac{I_M}{\sqrt{2}} \right)^2 \times R = \frac{I_M^2}{2} \times R$$

$$= \frac{1}{2} I_M^2 R$$

Hence these two points A and B on the resonance curve are known as half power frequency points.

Bandwidth may also be expressed as

$$BW = \frac{f_r}{Q}$$

Where,  $f_r$  = Resonant frequency

$Q$  = Quality factor

## PARALLEL RESONANCE

Resistor (R), inductor (L) and capacitor (C) are connected in parallel across a sinusoidal voltage of variable frequency source. An R-L-C parallel resonance circuit is shown in the fig 2.43(a) and its vector diagram is shown in the fig 2.42(b).

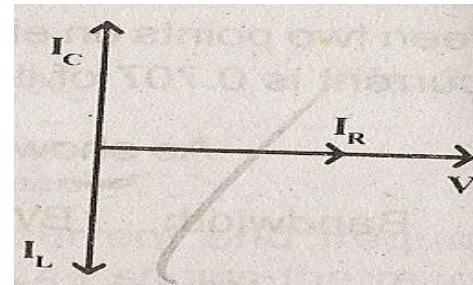
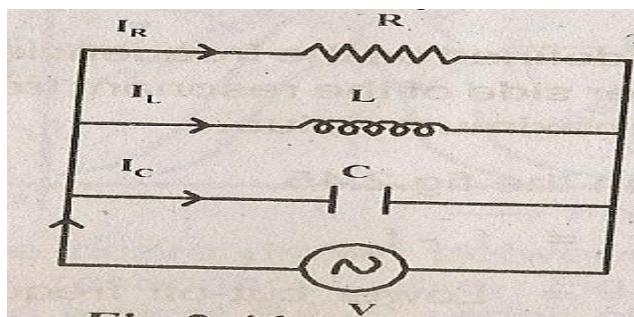


Fig.2.42(a)

(b)

$$I_R = \frac{V}{R}, I_L = \frac{V}{X_L}, I_C = \frac{V}{X_C}$$

At resonance,

$$X_L = X_C$$

$$\frac{V}{X_L} = \frac{V}{X_C}$$

$$\therefore I_L = I_C$$

Therefore,  $I_L$  and  $I_C$  are equal in magnitude but opposite in direction.

## Resonant frequency

At resonance,

$$X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}$$

## PARALLEL RESONANCE (TWO BRANCH CIRCUIT)

A practical parallel circuit consisting of a coil of resistor (R) and inductor (L) is connected in parallel with a capacitor (C) as shown in the fig.2.43 (a). the vector diagram is shown in the fig.2.43(b).

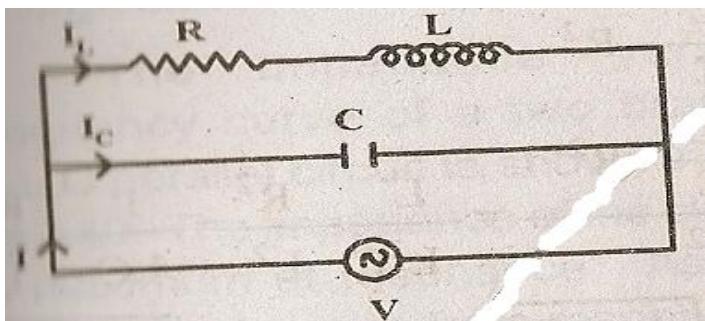


Fig.2.43(a)

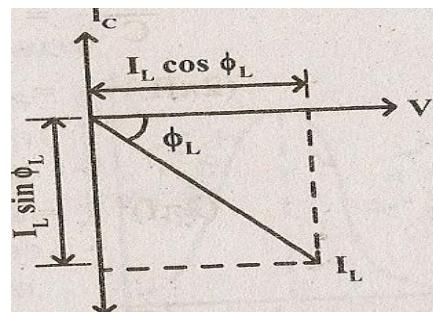


Fig.2.43(b)

The current in the coil ( $I_L$ ) is lagging behind the applied voltage  $V$  by an angle  $\Phi_L$ .

The current in the capacitor ( $I_C$ ) leads the voltage by  $90^\circ$ .

For  $I$  is in phase with the applied voltage, the current  $I_L \sin \Phi_L$  must be equal to  $I_C$ .

At resonance,  $I_C = I_L \sin \Phi_L$

$$\text{Where } I_C = \frac{V}{X_C}, I_L = \frac{V}{Z_L}, \sin \Phi_L = \frac{X_L}{Z_L}$$

$$\therefore I_C = I_L \sin \Phi_L$$

$$\frac{V}{X_C} = \frac{V}{Z_L} \cdot \frac{X_L}{Z_L}$$

$$X_L X_C = Z_L^2$$

$$2\pi f L \times \frac{1}{2\pi f C} = Z_L^2$$

$$\frac{L}{C} = R^2 + X_L^2 \quad [\therefore Z_L = R + jX_L]$$

$$(2\pi fL)^2 = \frac{L}{C} - R^2$$

$$(2\pi f)^2 = \frac{\frac{L}{C} - R^2}{L^2} = \frac{L}{L^2 C} - \frac{R^2}{L^2} = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$\text{if } R \text{ is negligible, then } f = \frac{1}{2\pi\sqrt{LC}}$$

### Resultant current at resonance

At resonance, the current in the circuit is in phase with the applied voltage.  
From the vector diagram (fig 2.44),

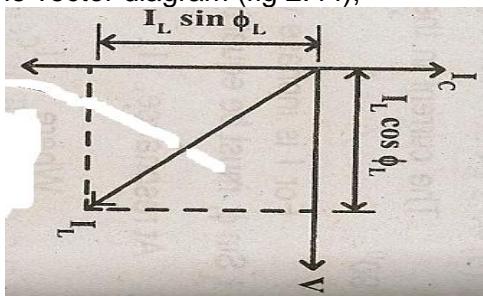


Fig.2.44

$$I = I_L \cos \phi_L$$

$$I_L = \frac{V}{Z_L} \text{ and } \cos \phi_L = \frac{R}{Z_L} \therefore I = \frac{V}{Z_L} \times \frac{R}{Z_L} = \frac{VR}{Z_L^2}$$

$$\text{But } X_L X_C = Z_L^2 \text{ or } \frac{L}{C} = Z_L^2$$

$$\therefore I = \frac{VR}{L/C} = \frac{V}{L/CR}$$

$L/CR$  is the equivalent impedance and is much larger than actual resistance. So parallel resonant circuit offers maximum impedance and the current is minimum.

### Resonance curve

#### Impedance versus frequency curve

The impedance versus frequency curve of a two branch R.L.C parallel circuit is shown in the fig 2.45. the impedance of the circuit is maximum. At resonance. As the frequency changes from resonance, the circuit impedance decreases rapidly.

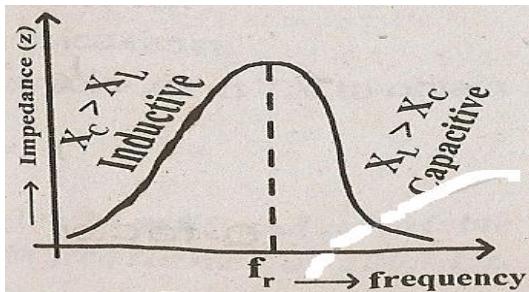


Fig.2.45

For frequencies below resonance, the capacitive reactance  $X_C$  is higher. Therefore, more current will flow through the coil. Thus the circuit behaves as inductive and the current lags behind the applied voltage.

For frequencies above resonance,  $X_L$  is higher. Therefore, more current will flow through the capacitor. Thus the circuit behaves as capacitive and the current leads behind the applied voltage.

#### Current versus frequency curve

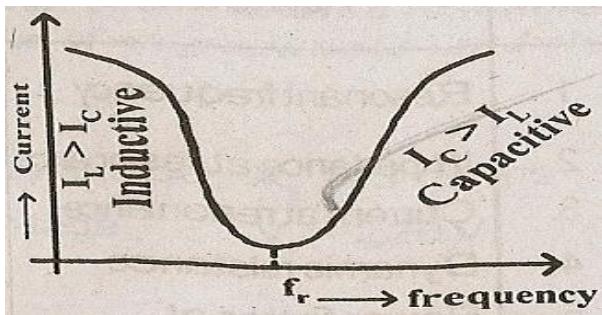


fig.2.46

Fig 2.46 shows the current versus frequency curve of the parallel circuit. The current at resonance is minimum. As the frequency changes from resonance, the circuit current increases rapidly.

#### Q-factor of a parallel resonant circuit

The ratio of circulating current between L and C to the supply current is known as current magnification or Q-factor of the parallel resonant circuit.

$$Q - \text{factor} = \frac{\text{circulating current between } L \text{ and } C}{\text{supply current}}$$

$$= \frac{I_C}{I}$$

$$I_C = \frac{V}{X_C} = \frac{V}{I/\omega C} = V\omega C$$

$$I = \frac{V}{L/CR} = VCR/L$$

$$\therefore Q - \text{factor} = \frac{V\omega C}{VCR/L}$$

$$= \frac{\omega L}{R} = \frac{X_L}{R} = \frac{2\pi f L}{R}$$

Resonant frequency,  $f_r = \frac{1}{2\pi\sqrt{LC}}$  (neglecting resistance R)

$$\therefore Q - \text{factor} = \frac{2\pi L}{R} \times \frac{1}{2\pi\sqrt{LC}} = \frac{L}{R\sqrt{LC}}$$

$$= \frac{1}{R} \sqrt{\frac{L}{C}}$$

### Comparison of series and parallel resonance

S. No.	Particulars	Series resonance circuit	Parallel resonance circuit
1.	Resonant frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$	$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$
2.	Impedance at resonance	Minimum	Maximum
3.	Current at resonance	Maximum	Minimum
4.	Dynamic resistance	R	$\frac{L}{CR}$
5.	Power factor at resonance	Unity (1)	Unity (1)
6.	Q factor	$\frac{X_L}{R}$ or $\frac{1}{R} \sqrt{\frac{L}{C}}$	$\frac{X_L}{R}$ or $\frac{1}{R} \sqrt{\frac{L}{C}}$

### Applications of resonance

- It is used in tank circuit of an oscillator.
- It is used in micro wave communication.
- It is used in the tuning circuit of ratio and T.V to obtain the required station.

### Example 2.18

A series circuit with R=5 ohms L=20 mH and a variable capacitor C has an applied voltage with frequency of 1000 Hz. Find the value of C for series resonance.

#### Given Data

Resistance , R = 5 Ω  
 Inductance , L = 20 H  
 Frequency = 100 Hz

### Solution

$$\text{At resonance } X_L = X_C, 2\pi f_L = 1/2\pi f_C$$

$$\therefore C = \frac{1}{4\pi^2 f^2 L}$$

$$= \frac{1}{4\pi^2 \times 1000^2 \times 20 \times 10^{-3}} = 1.27 \times 10^{-6} f.$$

$$\text{Capacitance , C} = 1.27 \times 10^{-6} f$$

### Example 2.19

A coil having a resistance of 6 ohms and reactance of 9 ohms in series with a capacitor such that the combination resonates at a frequency of 50Gz. Find the value of capacitance and the current if the supply voltage is 220V.

#### Given Data

Resistance , R = 6 Ω  
 Inductive reactance X L = 9 Ω  
 Frequency , f = 50 Hz  
 Voltage , V = 220 volts

### Solution

$$\text{At resonance } X_L = X_C$$

$$9 = 1/2\pi f_C$$

$$\therefore C = \frac{1}{2\pi \times 50 \times 9} = 353.67 \times 10^{-6} f$$

$$\therefore C = 353.67 \mu f$$

$$\text{Current, } I = \frac{V}{R} = \frac{220}{6} = 36.66 \text{ amps}$$

$$\text{Current , I} = 36.66 \text{ amps}$$

### Example 2.20

A series circuit contains a resistance of 5 ohms and inductance of 0.4H and a variable capacitor across 100 V , 50 hertz supply. Find (a) the capacitance for getting resonance (b) the p.d across inductance and capacitance (c) the Q factor for the series circuit.

### Given Data

Resistance , R	= 5 Ω
Inductance , L	= 0.4 H
Frequency , f	= 50 Hz
Voltage , V	= 100 volts

### Solution

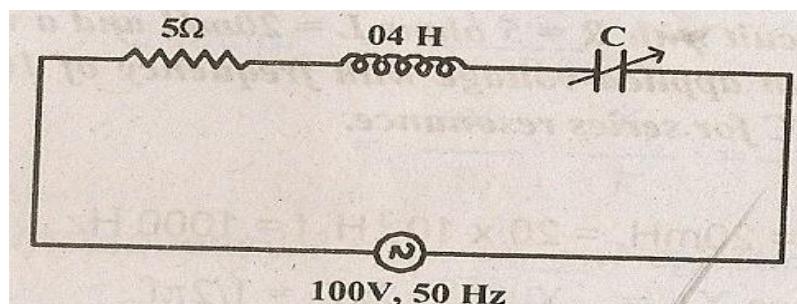


Fig. 2.47

$$\text{At resonance } X_L = X_C$$

#### (a) the capacitance for getting resonance

$$C = \frac{1}{\frac{2\pi f L}{2\pi f C}} = \frac{1}{\frac{1}{4\pi^2 f^2 L}} = \frac{4\pi^2 (50)^2 \times 0.4}{1}$$

$$C = \frac{25.333 \times 10^{-6}}{\frac{V}{R}} = \frac{25.33 \mu F}{5} = 5.066 \mu F$$

$$\text{Circuit Current , I} = \frac{V}{R} = \frac{100}{5} = 20 \text{ A}$$

$$I = 20 \text{ amps}$$

$$(b) \text{ P.d across inductance , } V_L = IX_L = 20 \times 2\pi f L$$

$$= 20 \times 2\pi \times 50 \times 0.4$$

$$= 2513.27 \text{ volts}$$

$$\text{P.d across capacitance, } V_C = I X_C = \frac{I}{\frac{1}{2\pi f C}} = \frac{20}{2\pi \times 50 \times 25.33 \times 10^{-6}}$$

$$= 2513.27 \text{ v}$$

(c) Q factor for the series circuit

$$\begin{aligned} \text{Q factor} &= \frac{X_L}{R} = \frac{2\pi f L}{R} \\ \text{Q factor} &= \frac{2\pi \times 50 \times 0.4}{5} \\ \text{Q factor} &= 25.133 \end{aligned}$$

### Example 2.21

For the circuit shown in figure determine the frequency at which the circuit resonates. Also find the voltage across inductor at resonance and Q factor of the circuit.

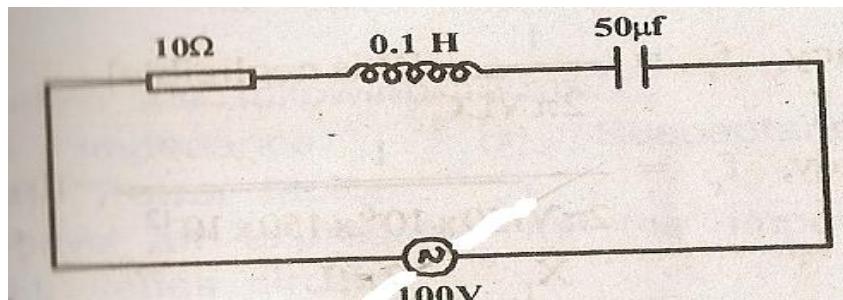


Fig.2.48

#### Given Data

Resistance , R	= 10 Ω
Inductance , L	= 0.1 H
Frequency , f	= 50 Hz
Voltage , V	= 100 volts

#### Solution

$$\text{At resonance } X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{50 \times 0.1 \times 10^{-6}}}$$

**Resonant frequency**  $f_r = 71.176 \text{ Hz}$

$$\text{Circuit Current, } I = \frac{V}{R} = \frac{100}{10}$$

**I = 10 amps**

$$\begin{aligned} (\text{b}) \text{ P.d across inductance, } V_L &= IX_L = 10 \times 2\pi f L \\ &= 20 \times 2\pi 50 \times 71.176 \times 0.1 \\ &= 447.21 \text{ volts} \end{aligned}$$

(c) Q factor for the series circuit

$$\text{Q factor} = \frac{X_L}{R} = \frac{2\pi f L}{R}$$

$$\text{Q factor} = \frac{2\pi \times 71.176 \times 0.1}{10}$$

**Q factor = 4.4721**

### Example 2.22

For the circuit shown in fig. determine the frequency at which the circuit resonates. Also find Q factor, bandwidth, impedance of the circuit and current drawn by the circuit.

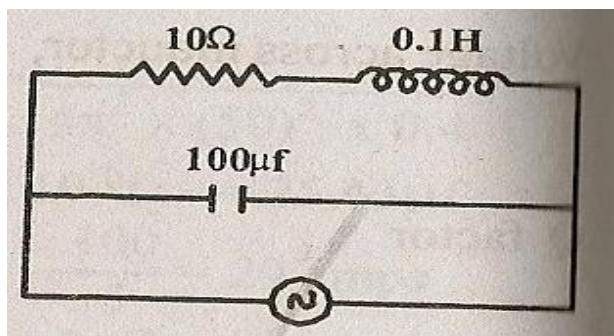


Fig.2.49

#### Given Data

Resistance, R	= 5 Ω
Inductance, L	= 150 μH
Capacitance, C	= 150 Pf = $150 \times 10^{-12} \text{ F}$

$$\text{Frequency, } f = 50 \text{ Hz}$$

$$\text{Voltage, } V = 0.2 \text{ volt}$$

### Solution

$$\text{At resonance } X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{150 \times 10^{-6} \times 150 \times 10^{-12}}}$$

$$\text{Resonant frequency, } f_r = 1061 \text{ KHz}$$

### Q factor

$$\text{Q factor} = \frac{X_L}{R} = \frac{2\pi f L}{R}$$

$$\text{Q factor} = \frac{2\pi \times 1061 \times 10^3 \times 150 \times 10^{-6}}{5} = 199.99$$

$$\text{Q factor} = 199.99$$

$$\text{Bandwidth} = \frac{f_r}{Q} = \frac{1061 \times 10^3}{199.99}$$

$$= 5.3 \text{ KHz}$$

$$\text{Impedance, } Z = \frac{L}{CR} = \frac{150 \times 10^3}{150 \times 10^{-12} \times 5}$$

$$Z = 200 \text{ K}\Omega$$

$$\text{Circuit Current, } I = \frac{V}{Z_{\max}} = \frac{0.2}{200 \times 10^3}$$

$$I = 1 \mu\text{A}$$

## UNIT - III

### TRANSFORMER AND MACHINES

#### TRANSFORMER

##### Introduction

A transformer is a static electrical machine which transfers electrical energy from one circuit to another circuit without change in its frequency. Due to electromagnetic induction principle the transfer of energy takes place.

(or)

A transformer is an electrical device that transfers electrical energy between two or more circuits through electromagnetic induction. Electromagnetic induction produces an electromotive force within a conductor

#### Transformer Construction

The physical basics of a transformer is mutual induction between two circuits linked by a common magnetic flux. In its simple form, it consists of two inductive coils which are electrically separated but magnetically linked through a path of low reluctance as shown in fig. 3.1

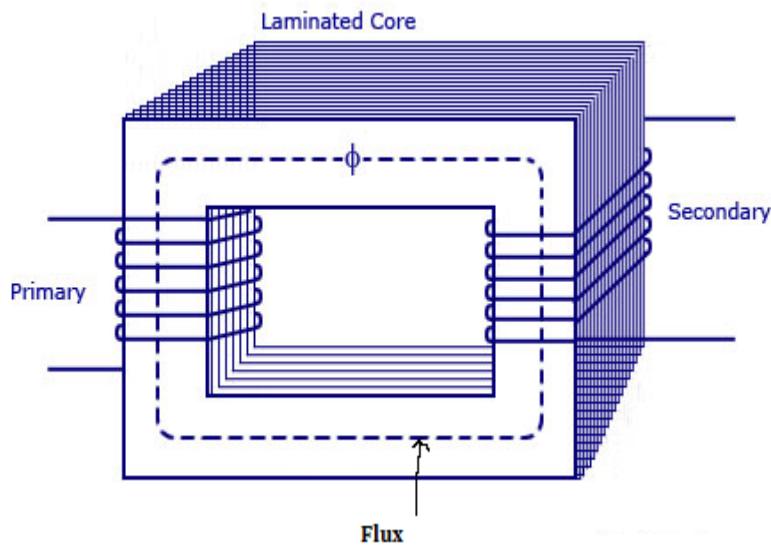


Fig.3.1

It consists of three essential parts , they are

- 1.Primary winding
- 2.Secondary winding

### 3.Laminated iron core

I/p ————— primary

O/p ————— secondary

#### Principle of operation

Transformer works on the principle of mutual induction. When an A.C supply is given to primary winding, an alternating flux is set up in the core. This flux cuts both primary and secondary windings. An e.m.f is induced in the primary winding according to self induction principle. According to Faraday's mutual induction principle, an e.m.f is induced in the secondary winding. If we connect a load to the secondary winding, current will flow through the load. In this way, electrical energy is transferred from the primary to secondary circuit.

#### Types of transformer

1.Step - down

2.Step - up

##### 1.Step - down

If the number of turns in the secondary winding is less than that of the primary winding, the e.m.f induced in the secondary winding will be less than the e.m.f induced in the primary winding. This type of transformers is called step down transformers.

##### 2.Step-up

If the number of turns in the secondary winding is more than that of the primary winding, the e.m.f induced in the secondary winding will be more than the e.m.f induced in the primary winding. This type of transformers is called step up transformers.

#### Ideal Transformer

An ideal transformer is a static electric machine which transfers electrical energy from one circuit to another circuit without any losses. It consists of two purely inductive coils of has loss-free core

#### Characteristics

1. no winding resistance therefore, there is no  $I^2R$  losses
2. no leakage flux
3. no core losses (or) iron losses in the core.

According to construction, transformer divided into two types

1.core type transformer

## 2.shell type transformer

### CORE TYPE TRANSFORMER

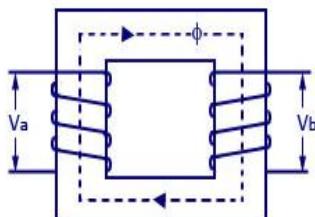


Fig.3.2 (a)

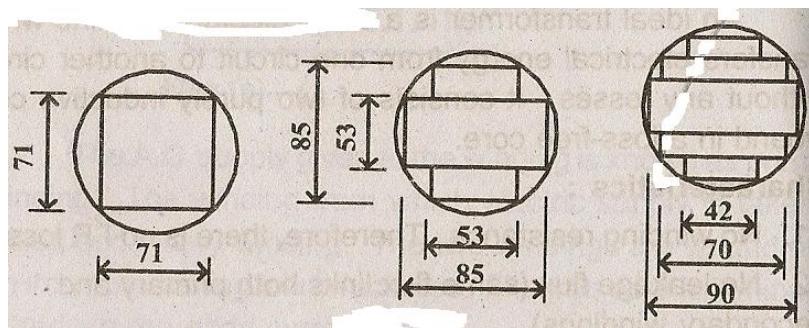


Fig.3.2(b)

In core type transformers the winding surround the core as shown in fig.3.2 (a) & (b)

- 1 The core is made up of thin laminated silicon steel.
- 2 The laminated steel cores are insulated from each other by means of varnish.
- 3 The laminated core minimizes the eddy current loss.
- 4 The thickness of lamination varies from 0.35 mm to 0.55 mm since the core
- 5 Since the core is made up of silicon steel, the hysteresis loss is reduced
- 6 The cross section of the core may be rectangular for small transformers
- 7 For large, the cross section of the core should be either square (or) stepped as shown in the fig.3.2(b)
- 8 The primary and secondary winding of core type transformer are wound helically.
- 9 Coil is insulated from one another using mica.

## SHELL TYPE OF TRANSFORMER:

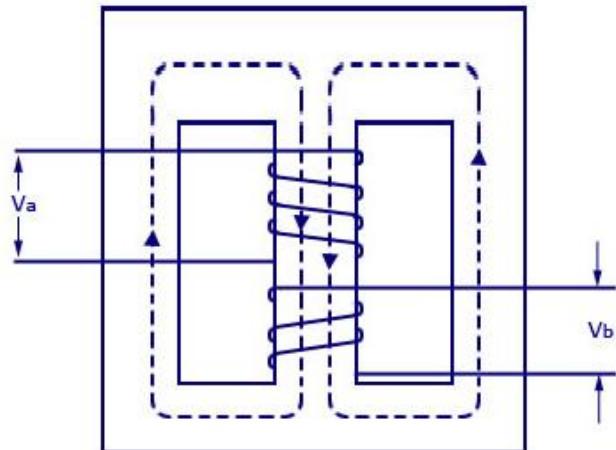


Fig.3.3

In shell type transformers the iron core surrounds the windings.

1. The core is made up of thin laminated silicon steel.
2. The laminated steel cores are insulated from each other by means of varnish.
3. The laminated core minimizes the eddy current loss.
4. The thickness of lamination varies from 0.35 mm to 0.
5. Since the core is made up of silicon steel, the hysteresis loss is reduced.
6. The cross section of the core may be rectangular for small transformers
7. For large, the cross section of the core should be either square (or) stepped as shown in the fig.3.3
8. Coil is insulated from one another using mica.

In this transformer, the iron core surrounds the windings the entire flux passes through the central part of iron core. But outside the central core it divides into two parts going in each direction. In this type low voltage winding is placed next to the core and the high voltage winding is placed around the low voltage winding.

## E.M.F EQUATION OF A TRANSFORMER

Consider a transformer having

Let,

$N_1$  = primary turns

$N_2$  = secondary turns

$\Phi$  = maximum flux in the core

$f$ =freq of the a.c voltage applied

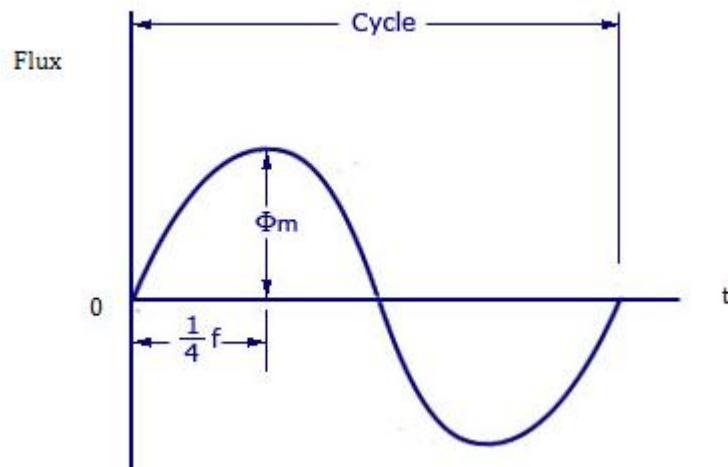


Fig. 3.4

Flux in the core will vary sinusoidal as shown in the waveform

The flux increases from zero value to max value ' $\Phi_m$ ' is  $\frac{1}{4} f$  second.

The change of flux is  $\frac{1}{4} f$  second =  $(\Phi_m - 0)$  Weber's.

$$\begin{aligned}\text{Rate of change of flux is the second} &= \frac{\Phi_m}{(1/4f)} \\ &= 4f\Phi_m\end{aligned}$$

Since the flux is varying sinusoidally, the r.m.s value of induced e.m.f is obtained by multiplying the average value with the form factor,

Form factor of sine wave =R.M.S value/average value = 1.11

$$\begin{aligned}\text{R.m.s value of e.m.f induced is one turn} &= 4f\Phi_m * 1.11 \\ &= 4.44 f\Phi_m\end{aligned}$$

R.m.s value of e.m.f induced in primary winding  $E_1 = 4.44 f \Phi_m N_1$

R.m.s value of e.m.f induced in secondary winding  $E_2 = 4.44 f \Phi_m N_2$

In an ideal transformer, e.m.f induced in any winding is equal to the voltage across its terminals on no load.

Applied voltage,  $V_1 = E_1$

Secondary terminal voltage,  $V_2 = E_2$

Voltage transformation ratio

$$\text{Voltage transformation ratio} = \frac{E_1}{E_2} = \frac{4.44 f \Phi_m N_1}{4.44 f \Phi_m N_2} = \frac{N_2}{N_1} = K$$

If  $N_2 > N_1$   $k > 1$ , then the transformer is called step up transformer

If  $N_1 > N_2$   $k < 1$ , then the transformer is called step down transformer

Current transformation ratio

In an ideal transformer,

Apparent input power = Apparent output power

$$V_1 I_1 = V_2 I_2$$

$$I_1 / I_2 = V_2 / V_1 = W_2 / W_1 = K$$

## Problems

### Example 3.1

A 200 KVA, 3300 / 240 volts, 50Hz 1Φ transformers has 80 turns on the secondary winding. Assuming an ideal transformer calculate (1) the primary and secondary current on full load (2) the maximum value of flux (3) the number of primary turns.

### Given data

Rating of the transformer =  $200\text{KVA} = 200 \times 10^3 \text{ VA}$

Primary voltage ,  $E_1 = 3300 \text{ volts}$

Secondary voltage ,  $E_2 = 240 \text{ volts}$

Secondary turns  $N_2 = 80 \text{ nos}$

Frequency,  $f = 50 \text{ Hz}$

**Solution**

Voltage transformation ratio  $\frac{E_2}{E_1} = \frac{N_2}{N_1}$

$$\therefore N_1 = \frac{E_2}{E_1} \times N_2 = \frac{3300}{240} \times 80 = 1110 \text{ turns}$$

Secondary output (rating) =  $200 \times 103 \text{ VA}$

$$\therefore \text{Secondary current, } I_2 = \frac{\text{Rating}}{E_2} = \frac{200 \times 103 \text{ VA}}{3300} = 833.33 \text{ A}$$

$$I_2 = 833.33 \text{ amps}$$

$$\text{Primary current, } I_1 = \frac{\text{Rating}}{E_1} = \frac{200 \times 103 \text{ VA}}{240} = 60.606 \text{ A}$$

$$E_1 = 4.44 \Phi_{mf} N_1$$

$$I_1 = 60.606 \text{ amps}$$

$$\text{Maximum flux, } \Phi_m = \frac{E_1}{4.44 f N_1} = \frac{3300}{4.44 \times 50 \times 1100} = 0.0135$$

$$\Phi_m = 0.0135 \text{ webers}$$

**Example 3.2**

A 6000/600 volt, 50 Hz, single phase transformer has a maximum flux density of 1.4 web/m<sup>2</sup> in its core. If the net cross sectional area of the iron core is 0.02 m<sup>2</sup>, calculate the number of turns in the primary and the secondary of the transformer.

**Given data**

Primary voltage,  $E_1 = 6000 \text{ volts}$

Secondary voltage,  $E_2 = 600 \text{ volts}$

Maximum flux density,  $B_m = 1.4 \text{ web/m}^2$

Net cross sectional area of the iron core =  $0.02 \text{ m}^2$

Frequency,  $f = 50 \text{ Hz}$

**Solution**

$E_1$

$$\text{Primary turns, } N_1 = \frac{\text{_____}}{4.44 \Phi_m f}$$

$$\text{Maximum flux } \Phi_m = B_m \times A = 1.4 \times 0.02$$

$$\Phi_m = 0.028 \text{ wb}$$

$$\text{Primary turns, } N_1 = \frac{E_1}{4.44 \Phi_m f} = \frac{6000}{4.44 \times 0.028 \times 50} = 965$$

$$N_1 = 965 \text{ turns}$$

$$\Phi_m = B_m \times A = 1.4 \times 0.02 = 0.028 \text{ wb}$$

$$\text{Secondary turns, } N_2 = \frac{E_2}{4.44 \Phi_m f} = \frac{600}{4.44 \times 0.028 \times 50} = 96.5 \text{ turns}$$

$$N_2 = 96.5 \text{ turns}$$

## Transformer test

The performance of transformer can be calculated on the basis of its equivalent circuit which contains four main parameters the equivalent resistance  $R_0$  referred to primary (or secondary  $R_0$ ), the equivalent leakage reactance  $X_0$  referred to primary (or secondary  $X_0$ ), the core loss conductance  $G_0$  and the magnetizing susceptance  $B_0$ . These constants are determined by two tests. open circuit test (O.C) short circuit test (S.C)

### Open Circuit (O.C) Test on Transformer

Open circuit test is used

- To determine no load constants  $R_0, X_0$
- To determine core loss

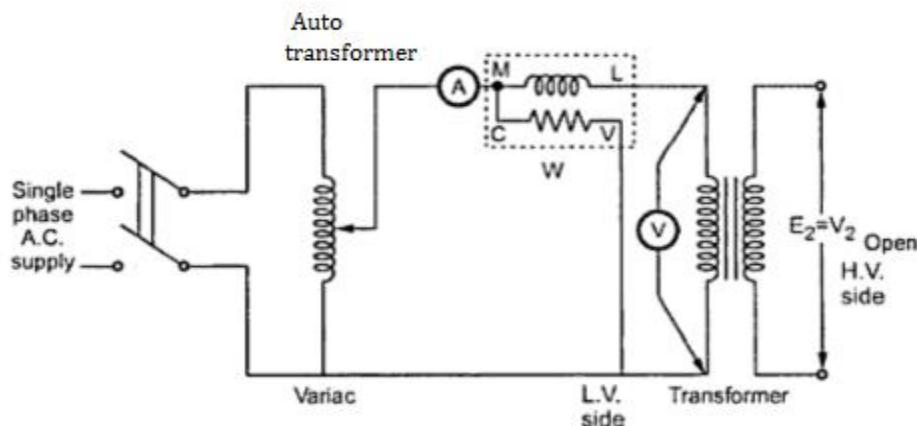


Fig.3.5

The connection diagram of O.C. test is shown in the fig.3.5. The rated voltage is applied to the primary winding , keeping the secondary is open. The readings of voltmeter , Ammeter and wattmeter are recorded. Let the respect readings are given below.

$$\text{Ammeter reading} = I_0$$

$$\text{Voltmeter reading} = V_0$$

$$\text{Wattmeter reading} = W_0$$

$W_0$  represents the power input to the transformer on no load. It is equal to sum of core loss and copper loss. Since the no load current is small , the copper loss, $(I_{20} R_{01})$  very small. Hence the copper loss at no load is neglected. Therefore  $W_0$  reading shows the core loss.

From this reading  $R_0$ ,  $X_0$  are calculated as followed,

$$V_0 I_0 \cos \theta_0 = W_0$$

$$\text{Power factor, } \cos \theta_0 = W_0 / V_0 I_0$$

$$\text{Core loss component of no load current } I_C = I_0 \cos \theta_0$$

$$\text{Magnetising component of no load current } I_m = I_0 \sin \theta_0$$

No load circuit constants

$$R_0 = V_0 / I_0$$

$$X_0 = V_0 / I_m$$

### Short Circuit (S.C ) Test on Transformer:

Short circuit test is used to

- 1 determine load constants  $R_{01}$  (or)  $R_{02}$ ,  $X_{01}$ (or)  $X_{02}$
- 2 determine copper loss at full load

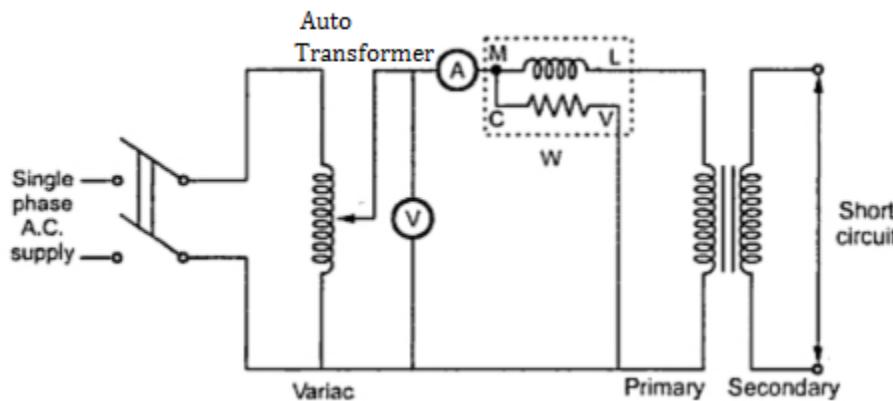


Fig.3.6

The connection diagram S.C. test is shown in the fig.3.6. A reduced voltage is applied to the primary winding and keeping the secondary short-circuited. The applied voltage is gradually increased until full load current flows through the transformer. The reading of ammeter, voltmeter and wattmeter are recorded. Let the respecting readings are  $I_s$ ,  $V_s$ ,  $W_s$ .

Here the core loss is negligibly small since the applied voltage is very small. So the wattmeter reading ( $W_s$ ) will give the full load copper loss of the transformer.

$$V_s I_s \cos\theta_s = W_s$$

$$\text{Power factor, } \cos\theta_s = W_s / V_s I_s$$

$$\text{Impedance, } Z_{01} = V_s / I_s$$

$$\text{Full load copper loss, } W_s = I_{s2} R_{01}$$

$$\text{Total resistance, } R_{01} = W_s / I_{s2} \quad (\text{R}_{01} \text{ is total resistance referred to primary side})$$

$$\text{Total reactance is primary side, } X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

## LOSSES IN A TRANSFORMER

Losses divided into two types,

1 core (or) iron losses

2 copper losses

### 1.Core (or) iron losses

These losses consist of hysteresis and eddy current losses and occur in the transformer core due to the alternating flux. These losses can be determined from open circuit.

$$\text{Hysteresis loss} \quad \alpha f B_m 1.6$$

$$\text{Eddy current loss} \quad \alpha f^2 B_m^2 t^2$$

Both hysteresis and eddy current losses depend upon maximum flux density ( $B_m$ ) in the core supply frequency( $f$ ). Since transformers are connected to constant frequency, constant supply voltage, both  $f$  and  $B_m$  are constant. Hence core (or) iron losses are practically same at all loads.

$$\text{Core (or) iron loss, } P_i = \text{hysteresis loss + eddy current loss}$$

$$= \text{constant losses}$$

### 2.Copper losses

These losses occur in both the primary and secondary windings due to ohmic resistance. These losses can be determined from short circuit test.

$$\begin{aligned}
 \text{Primary winding copper loss} &= I_1^2 R_1 \\
 \text{Secondary winding copper loss} &= I_2^2 R_2 \\
 \text{Total copper losses, } P_c &= I_1^2 R_1 + I_2^2 R_2 \\
 &= I_1^2 R_{01} \text{ (or) } I_2^2 R_{02}
 \end{aligned}$$

Copper loss occurs when the current flows through the winding. It is equal to  $I^2R$ . The loss varies as square of the load current. Since the copper losses vary with load current it is called variable loss.

$$\begin{aligned}
 \text{Total losses in a transformer} &= \text{constant loss} + \text{variable loss} \\
 &= P_i + P_c
 \end{aligned}$$

### Eddy current

When the armature core rotates, it also cuts the magnetic flux. Hence, an e.m.f is induced in the body of the core according to the law of electromagnetic induction. This e.m.f though small, set up large current in the body of the core due its small resistance. This current is known as eddy current.

### Efficiency of a transformer

Efficiency of a transformer is defined as the ratio of output power to input power

$$\% \text{ efficiency, } \eta = \frac{\text{Out power}}{\text{Input power}} \times 100$$

Efficiency of a transformer can be calculate, after determine the losses

$$\begin{aligned}
 &= \frac{\text{Output power}}{\text{Output power} + \text{losses}} \\
 &= \frac{\text{Output power}}{\text{Output power} + \text{copper losses} + \text{iron losses}}
 \end{aligned}$$

### All day efficiency

The all day efficiency is defined as the ratio of output is kWh to the input is kWh of a transformer during the whole day.

Output is kWh

$$\text{All day efficiency} = \frac{\text{Output power}}{\text{Input power}} \text{ for 24 hrs}$$

### Condition for maximum efficiency

The condition for maximum efficiency is copper loss should be equal to iron loss.

Condition of maximum efficiency is,

$$\begin{aligned}\text{Copper loss} &= \text{iron loss} \\ \text{Efficiency, } \eta &= \frac{\text{Output power}}{\text{Input power}} \\ \eta &= \frac{\text{Output power}}{\text{Output power} + \text{losses}}\end{aligned}$$

$$\text{Losses} = \text{iron loss} + \text{copper loss}$$

$$\begin{aligned}\text{Output power} &= V_2 I_2 \cos\theta_2 \\ \text{Core loss, } P_i &= \text{Hysteresis loss} + \text{Eddy current loss} \\ \text{Copper loss, } P_c &= I_{12} R_1 + I_2^2 R_2 = I_2^2 R_{02} \\ \text{Efficiency, } \eta &= \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_i + I_2^2 R_{02}}\end{aligned}$$

Divided both numerator and denominator by  $I_2$

$$\begin{aligned}&\frac{V_2 I_2 \cos\theta_2}{I_2} \\ &= \frac{V_2 I_2 \cos\theta_2}{I_2} + \frac{P_i}{I_2} + \frac{I_2^2 R_{02}}{I_2} \\ \eta &= \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_i + I_2^2 R_{02}}\end{aligned}$$

For maximum efficiency, denominator should be minimum  $I_2$

$$\frac{d}{d} I_2 ( V_2 \cos\theta_2 + P_i / I_2 + I_2 R_{02} ) = 0$$

$$0 - P_i / I_2^2 + R_{02} = 0$$

$$R_{02} = P_i / I_2^2$$

$$P_i = I_2 R_{02}$$

Core (or) iron loss=copper loss

This is the condition for maximum efficiency.

### Regulation of a Transformer

When the transformer secondary is connected to the load, the current flow through the secondary winding resistance and reactance causing a voltage drop. This voltage drop increases with increase in current. Therefore terminal voltage across at the secondary winding changes. The change in voltage from no load to full load is known as the regulation of the transformer.

No load voltage - full load voltage

$$\% \text{ regulation} = \frac{\text{No load voltage} - \text{full load voltage}}{\text{No load voltage}} \times 100$$

E-V

$$\% \text{ regulation} = \frac{E - V}{E} \times 100$$

Where, E = no load voltage

V = full load voltage

### Application of transformer

1. The transformer is used to either step up or step down the voltage
2. It is used in power supply for electronic circuit
3. In generating station the transformer step up the voltage for transmission.
4. It is used as an auto transformer starter for starting the induction motor.
5. It is used as an instrument transformer for increasing the range of meters.

### Example 3.3

Find the efficiency of a 150 KVA transformer at 75% of full load at u.p.f and 100% of full load at 0.8 p.f lag. If full load copper losses are 1.6 kw and the core losses are 1.4 kw.

### Given data

$$\begin{aligned} \text{Rating of the transformer} &= 150\text{KVA} = 150 \times 103 \text{ VA} \\ \text{core loss (or) iron loss, PI} &= 1.4 \text{ kw} \\ \text{copper loss, } P_c &= 1.6 \text{ kw} \end{aligned}$$

### Solution

Efficiency at 100% of full load (at 0.8 p.f lag)

$$\% \text{ of Efficiency, } \eta = \frac{\text{Output power}}{\text{Input power}}$$

Output power at any load = load x rating x power factor

$$\text{Output power at 0.8 p.f} = 1 \times 150 \times 0.8 = 120 \text{ Kw}$$

$$\text{Input power at 0.8 p.f} = \text{Output power} + \text{total losses}$$

$$\begin{aligned} (\text{copper loss at any load}) &= [(\text{load})]^2 \times \text{full load copper loss} \\ \text{copper loss at full load} &= [1]^2 \times \text{full load copper loss} \end{aligned}$$

$$\begin{aligned} &= 1 \times 1.6 \\ &= 1.6 \text{ Kw} \end{aligned}$$

$$\begin{aligned} \text{Total losses} &= \text{Iron loss} + \text{copper loss} \\ &= 1.4 + 1.6 = 3 \text{ Kw} \end{aligned}$$

$$\text{Input power at 0.8 p.f} = 120 + 3 = 123 \text{ Kw}$$

$$\begin{aligned} \eta &= \frac{\text{Output power}}{\text{Input power}} \\ &= \frac{120}{123} \times 100 \\ &= 97.56 \% \end{aligned}$$

Efficiency at 75% of full load (at u.p.f)

$$\text{copper loss at any load} = [\text{load}]^2 \times \text{full load copper loss}$$

$$\begin{aligned}\text{copper loss at 75\% of full load} &= \left( \frac{3}{4} \right)^2 \times 1.6 \\ &= 0.9 \text{ Kw}\end{aligned}$$

$$\text{Total losses} = 0.9 + 1.4 = 2.3 \text{ K}$$

$$\begin{aligned}\text{Output power at 75\% of full load} &= 0.75 \times 150 \times 1 \\ &= 112.5 \text{ Kw}\end{aligned}$$

$$\begin{aligned}\text{Input power at 75\% of full load} &= 112.5 + 2.3 \\ &= 114.8 \text{ Kw}\end{aligned}$$

Output power

$$\begin{aligned}\% \text{ of Efficiency at 0.8 p.f, } \dot{\eta} &= \frac{\text{Output power}}{\text{Input power}} \\ &= \frac{112.5}{114.8} \times 100\end{aligned}$$

$$\dot{\eta} = 97.99 \%$$

#### Example 3.4

A 60 KVA single phase transformer gave the following test results.

Open circuit test : 3000V applied to primary, power taken 430w

Short circuit test : primary output power is 525w. Calculate the efficiency (1) at full load  
(2) half load at 0.7 power factor.

#### Given data

$$\text{Rating of the transformer} = 60 \text{ KVA} = 60 \times 103 \text{ VA}$$

core loss (or) iron loss, PI = 430 w = 0.43 Kw

Full load copper loss, Pc = 525w = 0.525 kw

## Solution

Efficiency at full load (at 0.7 p.f lag)

$$\% \text{ of Efficiency}, \eta = \frac{\text{Output power}}{\text{Input power}}$$

Output power at any load = load x rating x power factor

Full load output power at 0.7 p.f = 60 x 0.7 = 42 Kw

Input power at 0.7 p.f = Output power + total losses

$$\begin{aligned} \text{Total losses} &= \text{Iron loss} + \text{copper loss} \\ &= 0.43 + 0.525 = 0.955 \text{Kw} \end{aligned}$$

$$\text{Input power at 0.7 p.f} = 42 + 0.955 = 42.955 \text{Kw}$$

$$\begin{aligned} \% \text{ of Efficiency at 0.8 p.f}, \eta &= \frac{\text{Output power}}{\text{Input power}} \\ &= \frac{42}{42.955} \times 100 \\ &= 97.77 \% \end{aligned}$$

Efficiency at half load (at 0.7 p.f)

$$\text{copper loss at any load} = (\text{load})^2 \times \text{full load copper loss}$$

$$\text{copper loss at 75\% of full load} = \left( \frac{1}{2} \right)^2 \times 0.525 = 0.131 \text{Kw}$$

$$\text{Total losses} = 0.131 + 0.43 = 0.561 \text{Kw}$$

$$\begin{aligned}\text{Output power at half load} &= 0.5 \times 60 \times 0.7 \\ &= 21 \text{ Kw}\end{aligned}$$

$$\begin{aligned}\text{Input power at half load} &= 21 + 0.561 \\ &= 21.561 \text{ Kw}\end{aligned}$$

$$\text{Output power}$$

% of Efficiency at 0.8 p.f,  $\eta$  =  $\frac{\text{Output power}}{\text{Input power}}$

$$= \frac{21}{21.561} \times 100$$

$$\eta = 97.39 \%$$

## DC MACHINES

### Introduction

D.C. generator is a electrical machine which converts mechanical energy (or power )in to electrical energy (or power ). The energy conversion is based on the principle of the production of dynamically or motionally. Induced e.m.f .

Whenever a conductor cuts magnetic flux and e.m.f is produced in it according to Faraday's laws of electromagnetic induction. This e.m.f causes a current to flow if the conductor circuit is closed. Mechanical energy is given o the generator by means of rotation of armature it is known as prime mover.

e.g. prime mover-diesel engine, turbine wised mill.

### Construction of dc machines

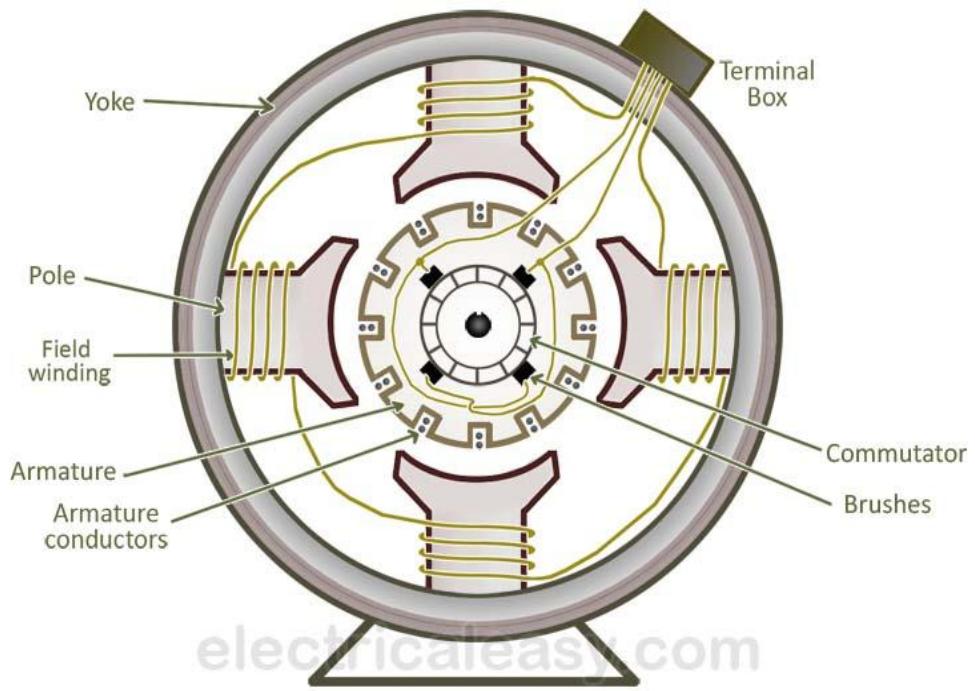


Fig.3.7

The construction diagram of DC machines is shown in fig. 3.7. It consists of following main parts are

1. Yoke (or) magnetic frame
2. Magnetic poles
3. Field coils
4. Inter poles (or) Commutation poles
5. Commutator
6. Brushes
7. Bearing and end cover

### **Yoke (or) magnetic frame**

1. It provides mechanical support for the machine and act as the magnetic flux produced the pole.
2. It forms the portion of magnetic circuit. it carries the magnetic flux produced by the poles.
3. It is made up of cast iron for smaller machine for larger machines it is made up of case steel.

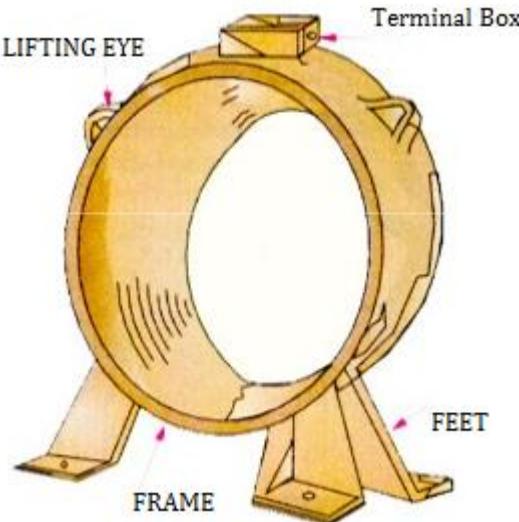


Fig.3.8

### Magnetic poles

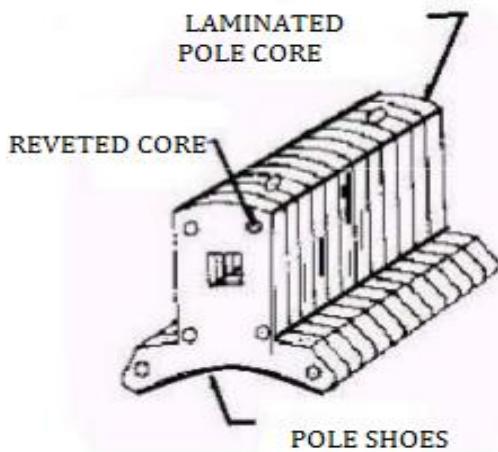


Fig.3.9

The field magnet consists of pole cores and pole shoes.

- (1) They spread out the flux in the air gap
- (2) They support the field coils.

Pole cores and pole shoes are built with these laminations of steel. These laminations are held together using rivets. The cores are laminated to reduce the eddy current loss. The magnetic poles are fitted inside the yoke by means of screws.

## Field coils

Field coils are usually wound with enameled copper wire. The magnetic field strength depends upon the current flowing through the coil. The north and South Pole depend upon the direction of the current flow through the field coil.

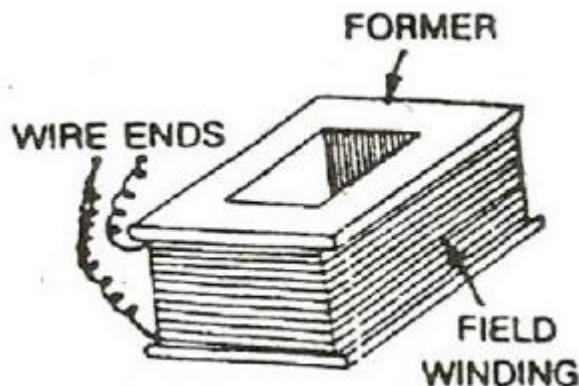


Fig.3.10

## Inter poles (or) Commutation poles

1. The function of inter pole is to improve the commutation and to reduce the armature reaction.
- 2 . The exciting coils on the interpoles are connected in series with the armature.

## Armature

The armature core is keyed to the machine shaft and it rotates between the field poles. It consists of slotted steel laminations. These laminations are stacked to form a cylindrical core.

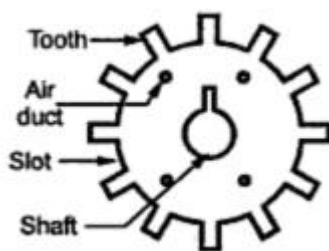


Fig.3.11



Fig.3.12

The laminations are insulated from each other by thin coating of varnish. The purpose of the lamination core is reduce the eddy current loss. Armature winding divided into two,

- (1) Lap winding: for low voltage, high current machine
- (2) Wave winding: for high voltage, low current machine

### Commutator

The commutator is made up of copper segments insulated from one another by mica sheets. The number of segments is equal to the number of armature coils. The segment is connected to armature conductor. Armature conductors are soldered to the commutator segment in a suitable manner to give rise to the armature winding.

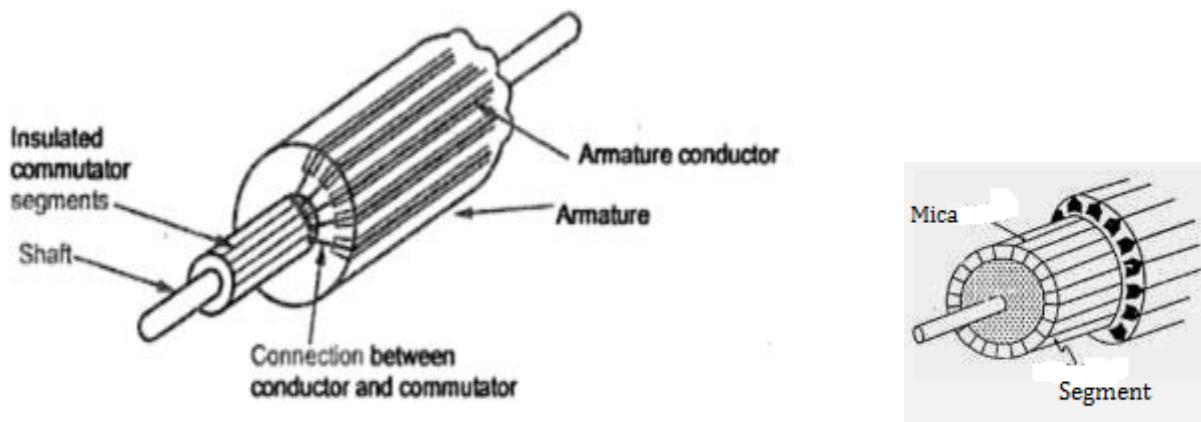


Fig.3.13

Function of commutator

1. Collection of current
2. Current from the armature conductor.
3. It converts alternating current induced in the armature conductors to unidirectional current.

## Brushes

Brushes are made up of carbon and rest on the commutator. The function of the brushes to collect current from the commutator to the external stationary load. The brushes are put inside the brush holders. The brush holders are kept pressed against the commutator by a spring as shown in fig. 3.14

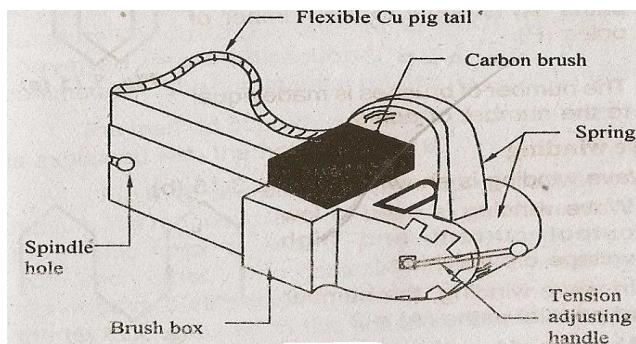


Fig.3.14

## Bearing and end cover

### Bearing

Ball bearings (or) rollers bearings are fitted inside the end cover. Armature shaft is mounted over these brings.

### End over

End over are made up of cast iron fabricated steel. They are fitted to both ends of yoke.

### Armature winding

## Type of winding

- (1) Lab winding
- (2) Wave winding

### Lab winding

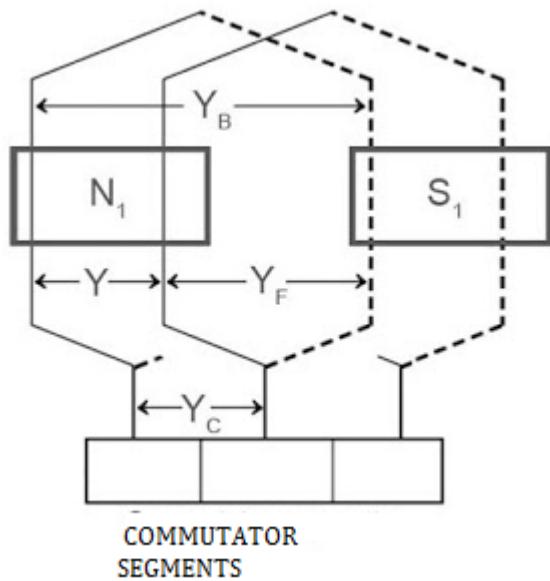


Fig.3.15 (a)

Lab winding is shown in fig. 3.15 (a)

1. Lab winding is used in large output current and low voltage dock machine.
2. In lab winding, the number of parallel paths (A) is equal to the number of poles (P).

$$\text{i.e., } A = P$$

3. The number of brushes is made equal to the number of poles.

### Wave winding

Wave winding is shown in fig. 3.15 (b)

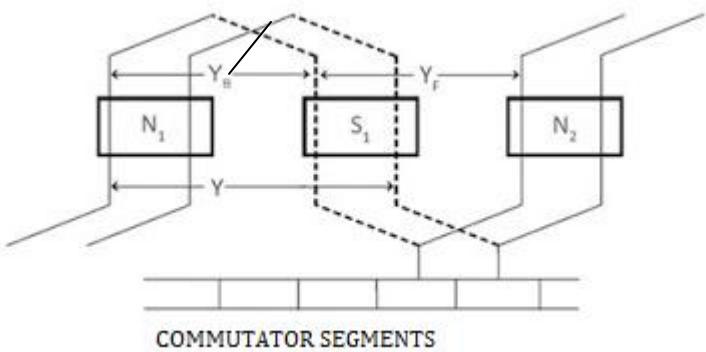


Fig.3.15 (b)

1. Wave winding is used in low output current and high voltage d.c machine.
2. In wave winding, the number of parallel paths ( $A$ ) =2
3. Number of brushes =2

## **D.C GENERATOR**

### **Introduction**

An electrical generator is a machine which converts mechanical energy into electrical energy.

### **D.C Generator Principle of Operation**

Electrical generator is based on the principle that whenever a conductor cuts magnetic flux an e.m.f is induced in the conductor. This e.m.f causes a current to flow if the circuit is closed. The direction of induced e.m.f is given by "Fleming's right hand rule."

The important components of d.c generator are,

1. A magnetic field
2. Conductor (or) group of conductors
3. Motion of conductor with respect to magnetic field.

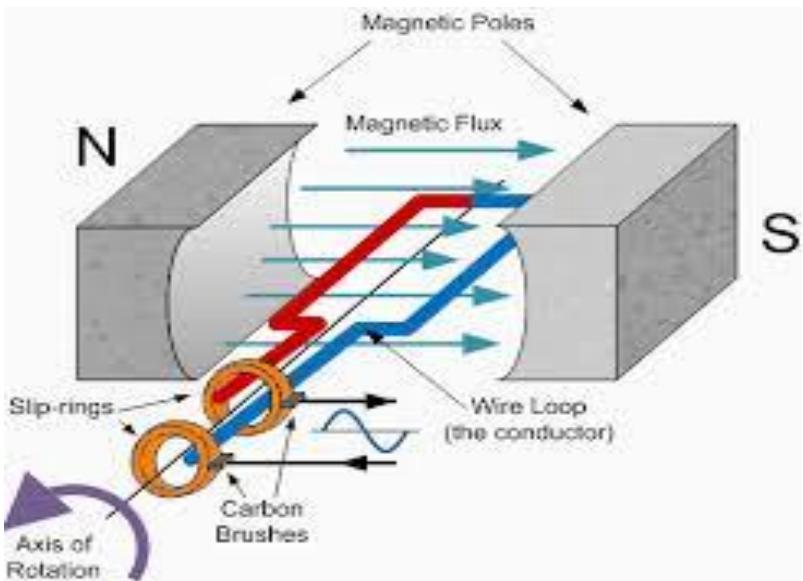


Fig.3.16 (a)

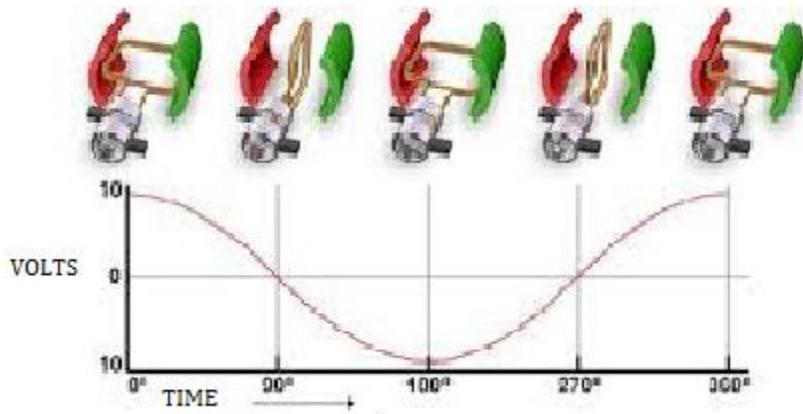


Fig.3.16 (b)

In D.C generators , a stationary magnetic field is produced by field magnets. The armature consisting of conductor is rotated inside this magnetic field by a prime mover. The prime mover may be a turbine or diesel engine petrol engine. The nature of e.m.f induce in the armature conductors is alternating (A.C). The A.C e.m.f converted into unidirectional e.m.f (D.C) by means commutator. The commutator rotates along with the armature.

The method of producing emf in a single loop generator is explained with the help of fig.

Fig.3.16 (a) & (b) shows a single loop of copper coil ABCD moving in a magnetic field. The two ends of coil are joined to two slip rings 'a' and 'b'. These slip rings are insulated from each

other. Two collecting brushes press against the slip rings a, b as shown in the fig.3.16 The brushes collect the current induced in the coil and supply it to the external load.

When the coil rotates inside the magnetic field the flux linked with the coil changes and hence e.m.f is induced in the coil, which is proportional to the rate of change of flux linkages. Imagine the coil to be rotating in the clockwise direction.

When the plane of the coil is right angles to flux lines i.e., in position -1 (where  $\theta = 0$ ) the flux linked with the coil is maximum, but rate of change of flux linkages is minimum. Because in this position the coil sides AB and CD do not cut or share the flux. Hence no e.m.f is induced in the coil at position-1.

Now the coil moves to at position-3 from position-1. When the coil reached the position-3 (where  $\theta = 90^\circ$ ), the coil plane horizontal to the flux lines the flux linked with coil is minimum, but the rate of change of flux linkages is maximum. Therefore maximum e.m.f is induced in the coil position 3.

In the next quarter revolution from position 3 position 5 ( $90^\circ$  to  $180^\circ$ ) the flux linked with coil is gradually increases but the rate of change of flux linkages is decreases. Therefore maximum e.m.f is induced is zero position 5.

Now the coil moves from position 5 to position 7 ( $180^\circ$  to  $270^\circ$ ) the e.m. induced in the coil is in the reverse direction. Therefore at position 7, the e.m.f induced is negative maximum. Then the coil moves from position 7 to position 1, the flux linked with the coil gradually increases, but the rate of change of flux linkages decreases. The e.m.f induced is zero at position 1. Thus the emf induced in the coil is an alternating e.m.f as shown in fig.3.17

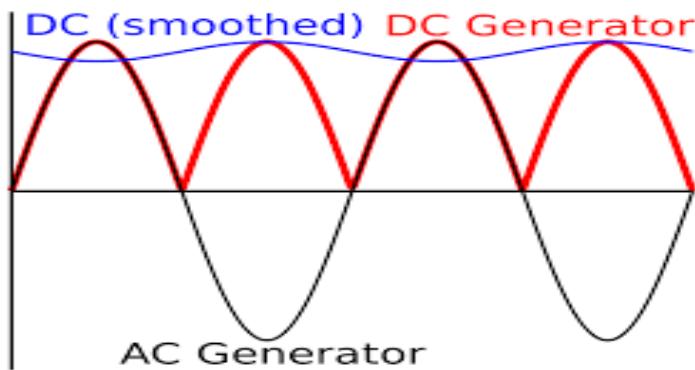


Fig.3.17

If the slip rings are replaced by split rings, the alternating e.m.f will become unidirectional current (D.C.). The split rings are made out of a conducting cylinder which is cut into two segments insulated from each other by thin sheet of mica. The coil ends are joined to this segment. Carbon brushes rest on the segments.

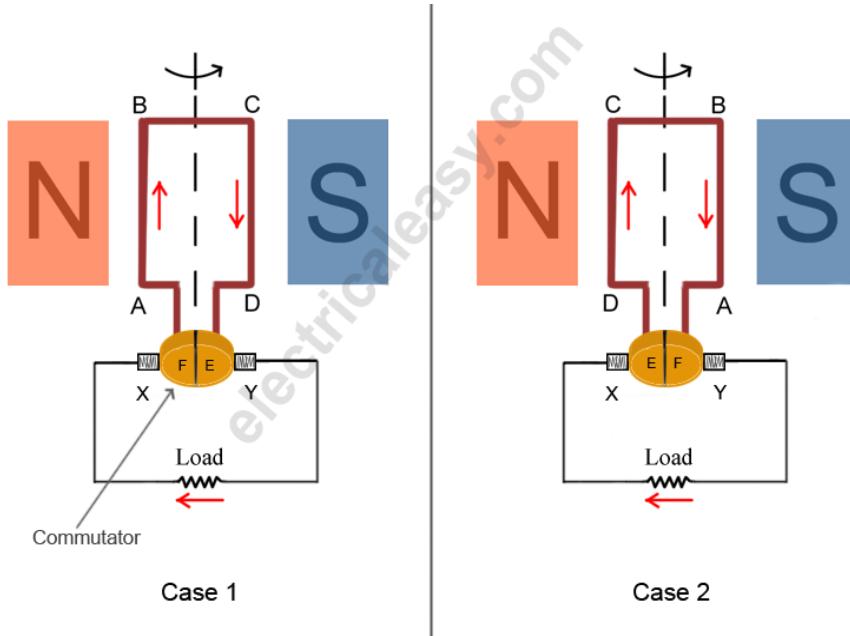


Fig.3.18(a)

Fig.3.18 (b)

Fig.3.18(a) shows the connection of coil ends with split rings 'a' and b. in the first half revolution current flows along ABLMCD in the brush No: 1 is contact with segment 'a' acts the positive end of e.m.f and 'b' act as the negative end.

In the next half revolution the direction of current in the coil has reversed as shown in fig.3.18 (b). But at the same time the positions of segment a and b have also been reversed. The segment 'a' is coming in contact with brush no: 2 and becomes negative end of induced e.m.f. Again the current in the load resistance flows in the same direction is from L to M. The current is unidirectional current due to rectifying action of split rings (also called as commutator)

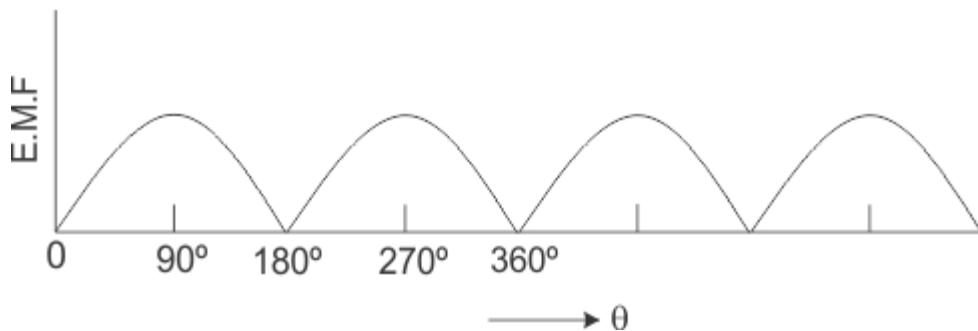


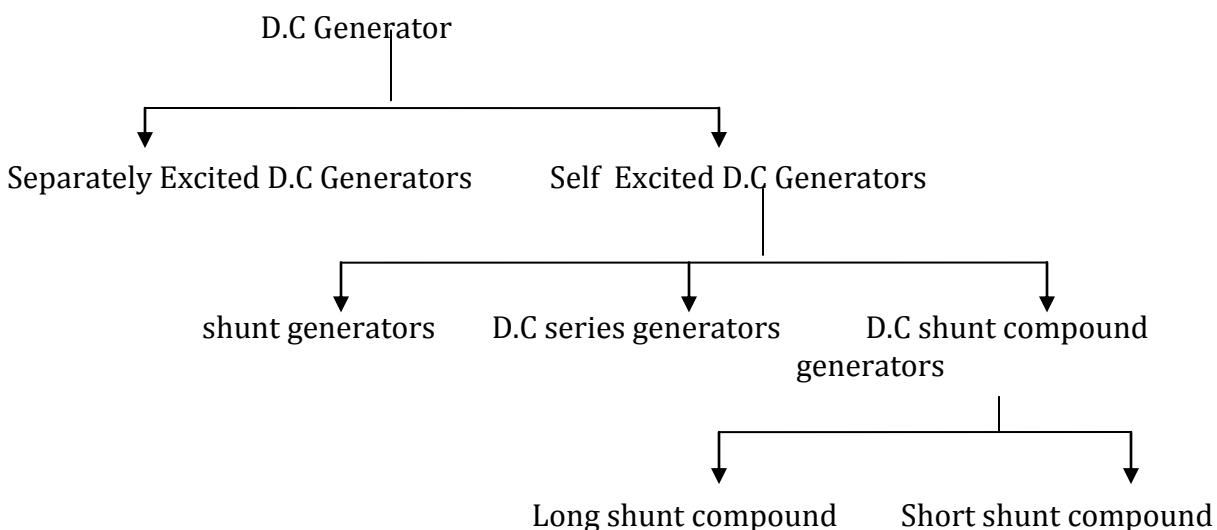
Fig.3.18 (c)

The unidirectional current is shown in the fig. To minimize the ripple in D.C current , the number of coil in the armature should be increased.

### Fleming's Right Hand rule

Keep the thumb, fore finger and middle finger of the right hand mutually perpendicular to each other. If the fore finger points the direction of magnetic flux lines and the thumb points the direction of motion of conductor , then the middle finger points the direction of induced e.m.f (or) current.

### Types of D.C Generator



### What is excitation?

The current flows through the field winding for production of magnetic flux are called excitation.

Depending upon the method of excitation D.C generators are classified as,

- (1).Separately excited D.C Generator
- (2).Self excited D.C Generator

#### **(1).Separately excited D.C Generator**

In separately excited D.C generator, the exciting field current supplied by a separate source as shown in fig.3.20. When current flows through the field winding, the main pole becomes magnet and the flux lines are produced.

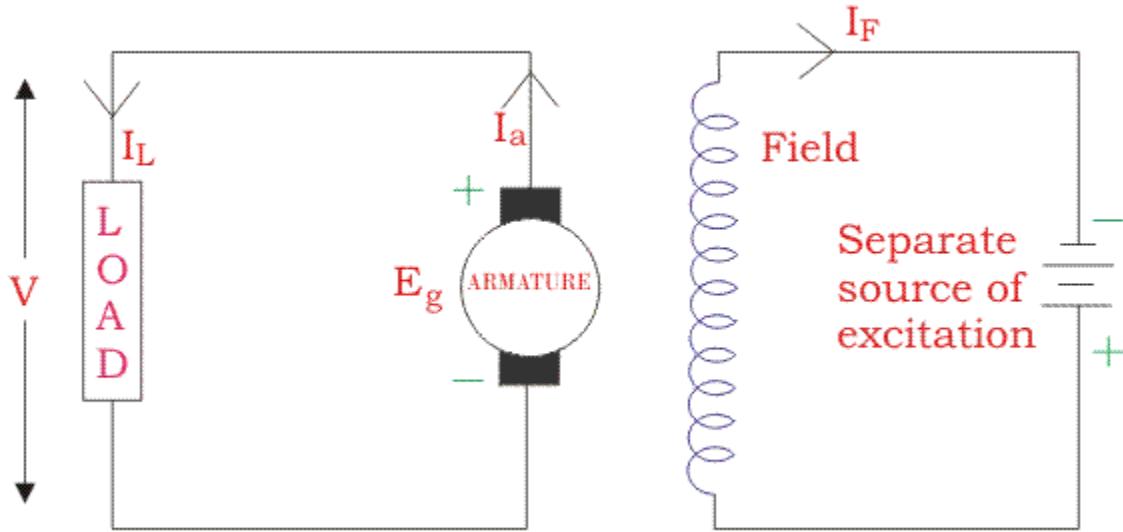


Fig.3.19

### Applications

If it has constant d.c output voltage , these generators are used for

1. Battery charging
2. Supply source to d.c motor

### (2). Self Excited D.C Generator

In self excited d.c generator, the emf induced in the armature supplies the exciting field current. Therefore , need not separate d.c supply for exciting field poles.

Depending upon the way in which the field winding is connected with armature , self excited d.c generators are classified as,

1. D.C shunt generator
- 2.D.C series generator
- 3.D.C compound generator

### 1.D.C Shunt Generator

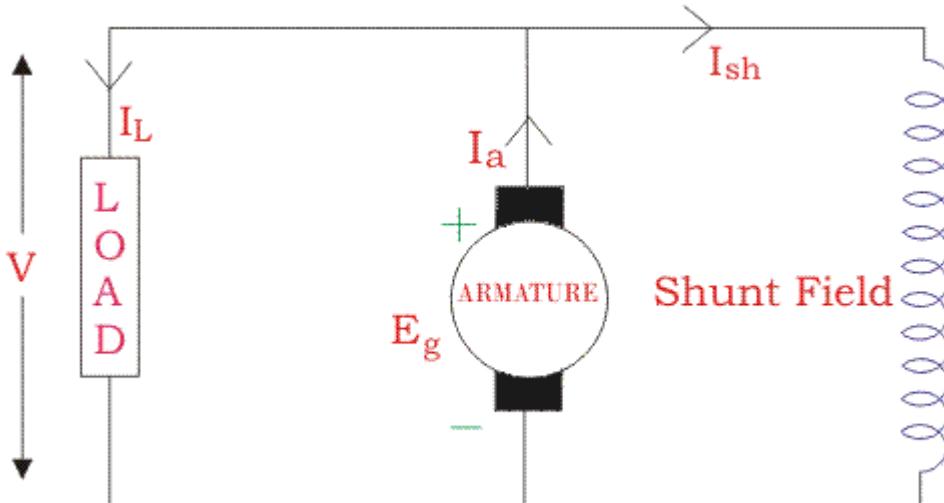


Fig.3.21

In D.C shunt generator, field winding is connected in parallel with the armature winding as shown in fig.3.21. Its field winding has many turns of thin wire having high resistance. Then armature rotates emf is induced due to the residual magnetism. Due to induced e.m.f, field current ( $I_{sh}$ ) flows and hence flux and voltage increases.

$$\text{D.c shunt generator, } I_a = I_L + I_{sh}$$

Where,       $I_a$  = armature current  
 $I_L$  = load current

$I_{sh}$ = shunt field current

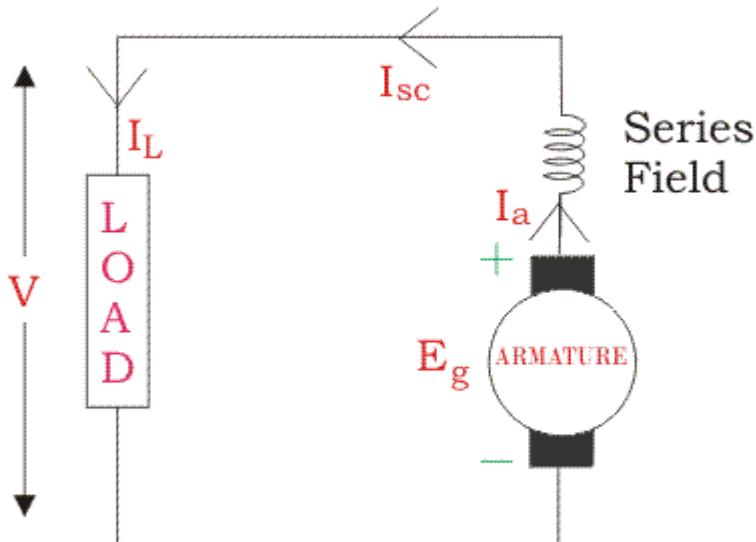
Application

1. It is used in battery charging
2. It is used in electroplating.
3. It is used as exciter for alternator.

### D.C Series Generator

In d.c series generator , the field winding is connected in series with the armature winding as shown in fig.3.22. It carries a large current. Its field winding has few turns of thick wire having low resistance.

$$I_a = I_L = I_{se}$$



## Series Wound Generator

Fig.3.22

Where,

$I_a$  = armature current

$I_L$  = load current

$I_{se}$  = shunt field current

For series generator no load characteristics is absent because the circuit is closed only when load is connected. The load current flows through the field winding and produces flux. In series generator, as the terminal voltage increases with the load current . Booster to compensate for load voltage drops.

## Application

1. It is used booster.
2. It is used for supply to arc lamps.

## D.C Compound Generator

Compound generator is a machine which consists of series field winding and shunt field winding. In d.c compound generator both series and shunt field winding are connected with the armature winding as shown in fig.3.23 & 3.24. The flux of the pole depends on the total flux of both field winding. Depends upon the connection field winding d.c compound are classified into two types.

1. Long shunt d.c compound generator
2. Short shunt d.c compound generator

### **1.Long Shunt Compound Generator**

In long shunt compound generator , the series field winding is connected in series with the armature winding and shunt field winding is connected in parallel with this arrangement as shown in fig.3.23

In long shunt compound generator,  $I_a = I_{se}$   
 $I_a = I_{sh} + I_L$

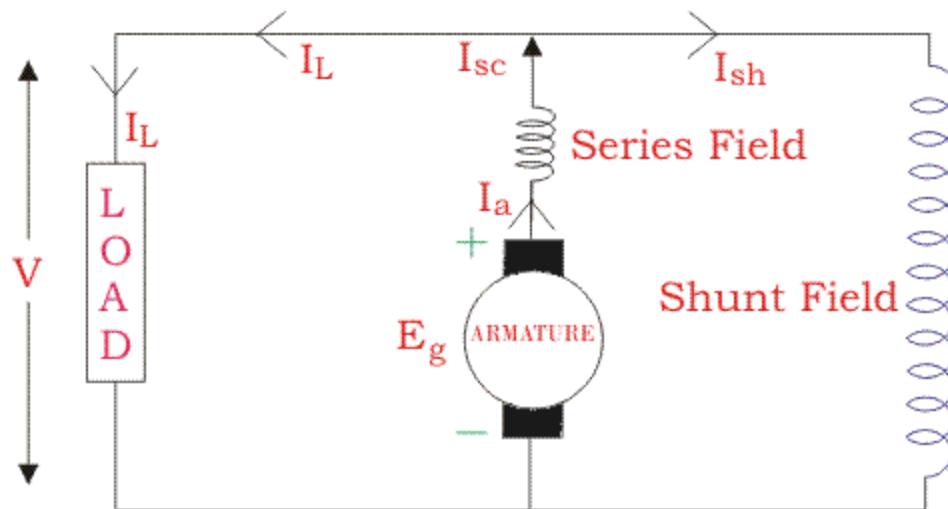


Fig.3.23

### **Short Shunt Compound Generator**

In short shunt compound generator , the series field winding is connected in series with the armature winding and shunt field winding is connected in parallel with this arrangement as shown in fig. 3.24.

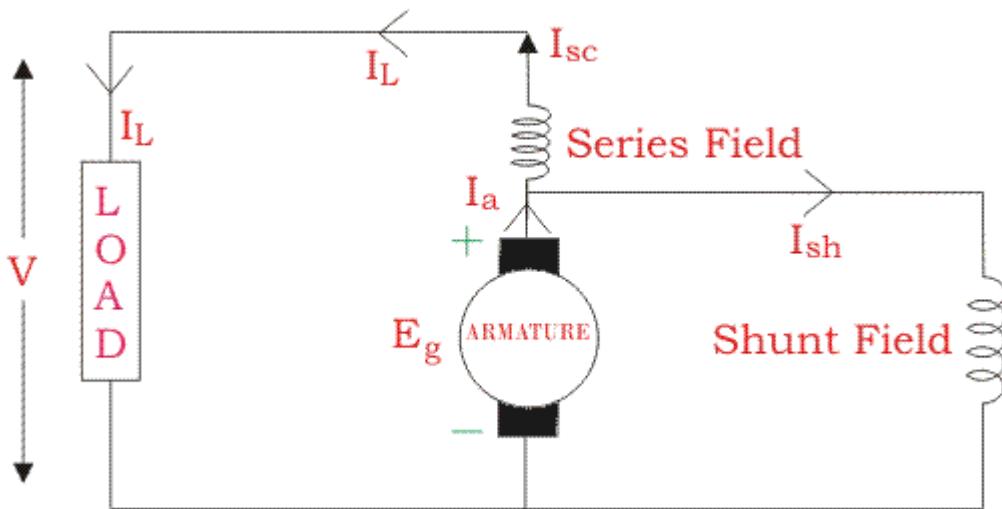


Fig.3.24.

$$I_{se} = I_c$$

$$I_a = I_{sh} + I_L$$

Depends upon the connection of field winding the d.c compound generator are further classified into two types.

1. Cumulative compound generator:
2. Differentially compound generator

### **1. Cumulative compound generator**

If the series field flux adds with shunt field flux it is called as cumulative compound generator.

In cumulative compound generator , the flux produced by the shunt field winding and flux produced by the series field winding acts in the same direction. Hence the total field flux acting in this type of generator is sum of these two fluxes.

$$\text{Total flux, } \Phi = \Phi_{sh} + \Phi_{se}$$

### **Differentially Compound Generator**

If the series field flux opposes with the shunt field flux, it is called as differential compound d.c generator.

In differentially compound generator , the flux produced by shunt field winding and the flux produced by the series field winding acts in the opposite direction. Hence the total field flux acting in this type of generator is difference of these two fluxes.

## Applications

1. Differentially compound generators are used for D.C welding purpose.
2. It is used in power offices and hotels.
3. It is used where power is to be transmitted to a long distance.

## D.C MOTORS

### Introduction

An electric motor is a machine which converts electrical energy into mechanical energy. The parts and construction of D.C motor are as same as that of D.C generator.

### Principle of Operation (Or) Motor Action

Whenever a current carrying conductor is placed in a magnetic field, a mechanical force is produced on the conductor. The direction of force is given by "fleming's left hand rule",

The magnitude of the force,  $F = B I L$  Newton

Where,  $F$  = force in Newton

$B$  = magnetic flux density in web/m<sup>2</sup>

$I$  = current in ampere.

$L$  = length of the conductor in meter.

To understand the principle of operation of d.c motor. Let us consider a two pole motor.

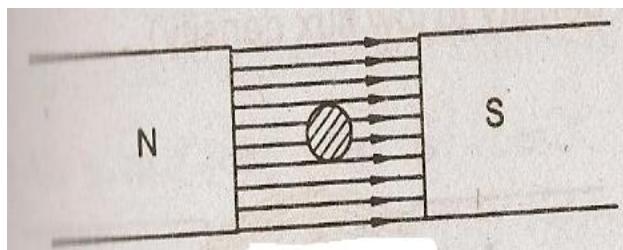


Fig.3.25 (a)

Fig.3.23(a) shows a uniform magnetic field in which no current carrying conductor is placed. The direction of magnetic flux is from north to south pole. There is no movement of the conductor.

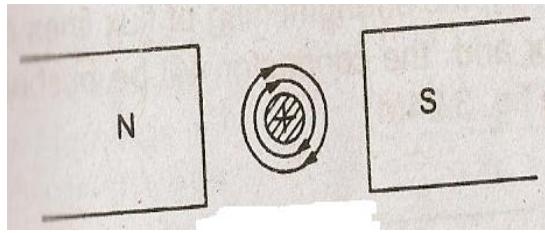


Fig.3.25 (b)

In fig. Fig.3.23 (b) there is no exciting current flow through the field winding and d.c current is sent through the conductor. Let the conductor carry the current away from the observer. It produces a magnetic flux lines around that in clockwise direction. There is no movement of the conductor.

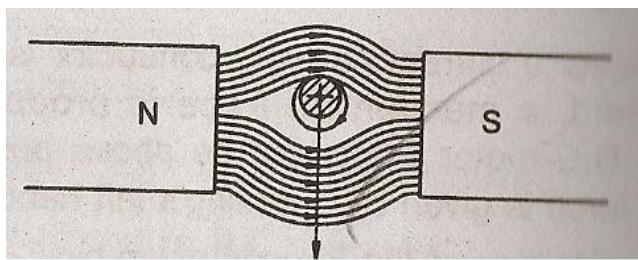


Fig.3.25 (c)

In fig. Fig.3.23 (c) the current carrying conductor is placed in magnetic field. The field due to the current in the conductor aids the main field above the conductor (flux strengthens) but opposes the main field below the conductor. (flux weakens). It is found that a force acts on the conductor, trying to push the conductor downwards as shown by the arrow (the conductor is pushed from high flux density to low flux density from high flux to low flux density).

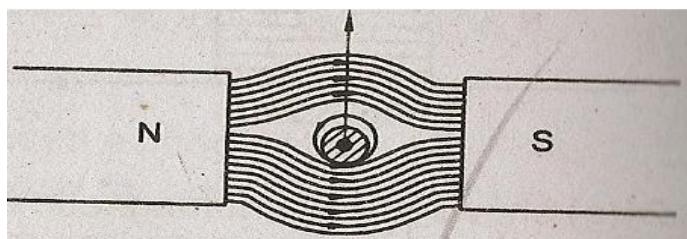


Fig.3.25 (d)

If the current conductor is reversed (current towards the observer ), the strengthening of flux lines occurs below the conductor will be pushed upward as shown in the fig. Fig.3.23 (d)

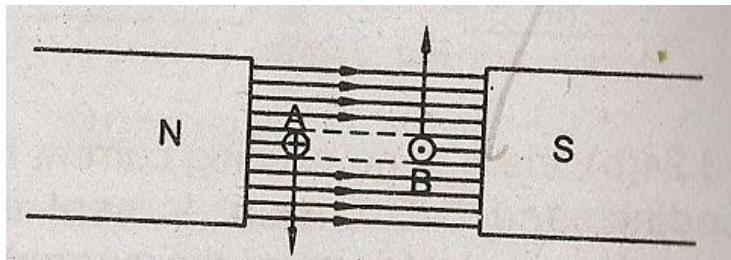


Fig.3.25 (e)

Now consider a single turn coil carrying a current as shown in the fig. Fig.3.23 (e). The coil side 'A' will be forced to move downwards, whereas the coil side 'B' will be forced to move upwards. The force acting on the coil sides 'A' and 'B' will be same magnitude but their direction is opposite to one another. As the coil is wound on the armature coil , which is supported by the bearings armature will rotate. The direction of rotation is found out by "Fleming's right hand rule".

### Fleming's Left Hand rule

Keep the thumb, fore finger and middle finger of the left hand mutually perpendicular to each other. If the fore finger points the direction of magnetic flux lines and the middle finger points the direction of current flow in the conductor, then the thumb points the direction of rotation of conductors.

### Back e.m.f (or) Counter e.m.f

When D.C supply is given to d.c motor its armature rotates. The armature rotates and cuts the magnetic flux lines. Therefore e.m.f is induced in the armature conductors (By faraday's law of electromagnetic induction). This induced e.m.f opposes the supply voltage (by Fleming's Left Hand rule). Hence the e.m.f induced in the armature is called back e.m.f. (or) counter e.m.f.

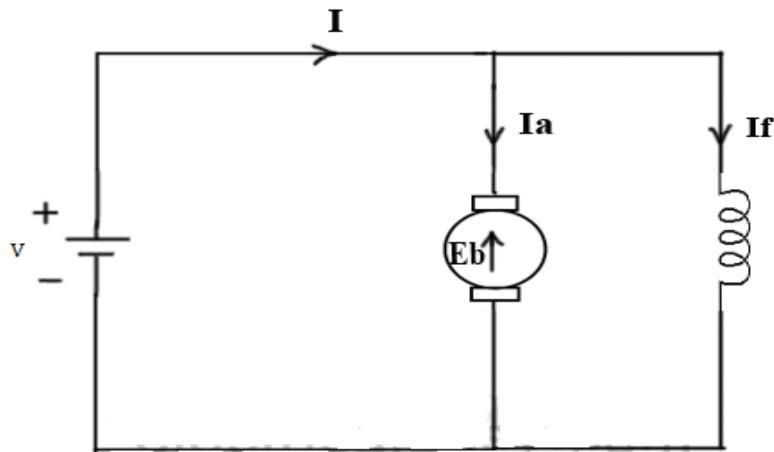


Fig.3.26

$$\text{Back e.m.f, } E_b = \frac{\Phi Z N P}{60A}$$

$$\text{Also, } E_b = V - I_a R_a$$

Where,

$\Phi$  = flux per pole is Weber's

Z = number of conductors in the armature

P = number of poles

A = number of parallel paths

V = supply voltage to the motor

$I_a$  = armature current

$R_a$  = armature resistance.

### Types of D.C Motor

Depending upon the way in which the field winding is connected with armature winding the D.C motors are classified as ,

1. D.C shunt motor
2. D.C series motor
3. D.C compound motor

### 1.D.C. Shunt Motor

In d.c shunt motor the field winding is connected in parallel with armature winding as shown in fig. 3.27 The shunt field winding has a large number of turns and smaller cross section area and high resistance.

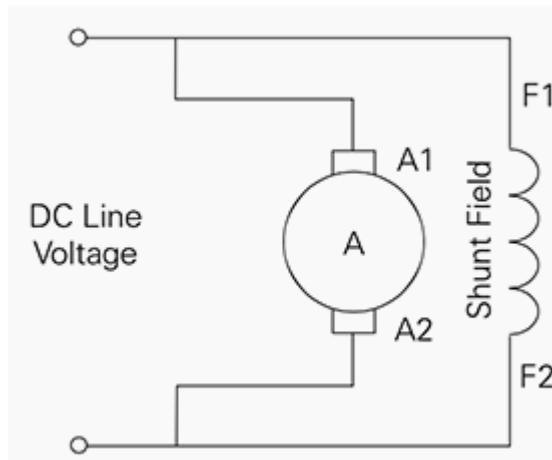


Fig. 3.27

$$\text{Applied voltage, } V = E_b + I_a R_a$$

$$\text{Armature current, } I_a = I_L - I_{sh}$$

Where,

$E_b$  = Back e.m.f

$I_a$  = Armature current

$I_L$  = Line current

$I_{sh}$  = Shunt field current

$R_a$  = Armature resistance.

### 2.D.C Series Motor

In D.C series motor, the field winding is connected in series with the armature as shown in the fig. 3.28. The series field winding has large cross sectional area , few number of turns and low resistance.

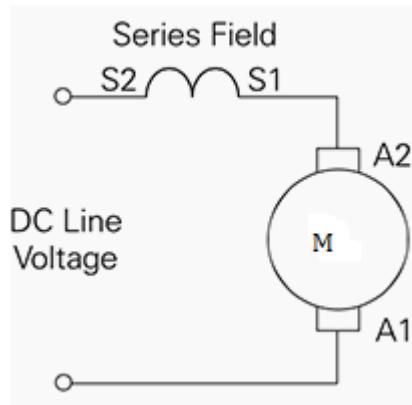


Fig. 3.28

In d.c series motor,  $I_L = I_a = I_{sh}$

$$\begin{aligned}\text{Applied voltage, } V &= E_b + I_a R_a + I_{se} R_{se} \\ &= E_b + I_a R_a + I_a R_{se} \\ &= E_b + I_a (R_a + R_{se}) \quad (I_a = I_{se})\end{aligned}$$

Where,  
 $E_b$  = Back e.m.f  
 $I_a$  = Armature current  
 $I_L$  = Line current  
 $I_{se}$  = Series field current  
 $R_{se}$  = Series field resistance  
 $R_a$  = Armature resistance

### 3.D.C Compound Motor

In d.c compound motor both series field and shunt field windings are connected with the armature winding. Depending upon the connection of field winding, d.c compound motors are classified into two types,

- (i) Long shunt d.c compound m
- (ii) Short shunt d.c compound motor

#### **(i) Long Shunt D.C Compound Motor**

In long shunt D.C compound motor the series field winding is connected in series with the armature winding and the shunt field winding is connected in parallel with this arrangement. This is shown in fig. 3.29

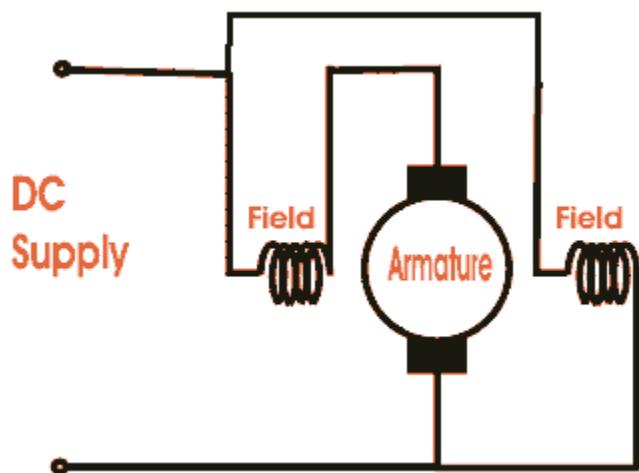


Fig. 3.29

In long shunt compound motor,

$$I_a = I_{se}$$

$$I_L = I_a + I_{sh}$$

### (ii) Short- Shunt D.C Compound Motor

In long shunt D.C compound motor the shunt field winding is connected in parallel with the armature winding and the series field winding is connected in series with this arrangement. This is shown in fig. 3.30

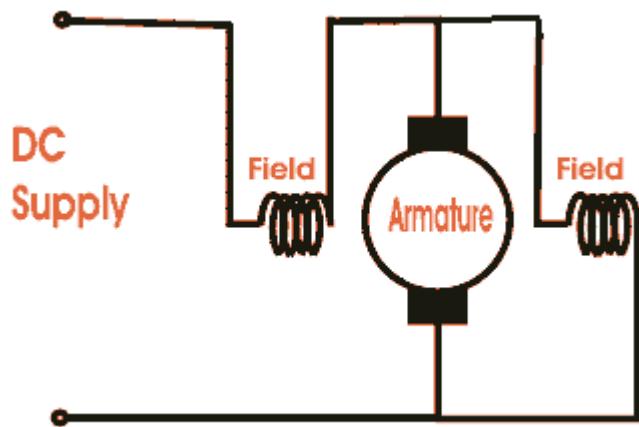


Fig. 3.30

In short shunt compound motor,

$$I_L = I_{se}$$

$$I_L = I_a + I_{sh}$$

Depending upon the way in which the shunt field flux and series field flux act with respect to each other, d.c compound motors are classified in to ,

- (a) Cumulative compound motor
- (b) Differentially compound motor

### **Applications of d.c motors**

Shunt motors are used where constant speed is required at low starting torque

#### **D.C. shunt motor**

1. It is used to drive centrifugal pumps
2. It is used to drive highs machine tool
3. It is used to drive wood working machine
4. It is used to drive paper mills
5. It is used to drive drilling machines etc.

#### **D.C series motor**

D.c series motors are used where high starting torque is required.

It is used to drive

1. Electric trains
2. Cranes
3. Lifts and Conveyors etc.

#### **D.c compound motor**

D.C compound motors are used where intermittent high starting torque is required.

It is used to drive

1. Rolling mills
2. Printing machines
3. Shears
4. Punches etc.

## **INDUCTION MOTOR**

### **Introduction**

As a general rule the conversion of electric power in to mechanical power takes place in the rotating part of an electric motor. In d.c motors the electric power is conducted directly to the armature (rotating part) through brushes and coommutator. Hence in this sense, a d.c motor can be called a conduction motor. However in a.c motors the rotor does not receive electric power by conduction by induction in exactly the same way as the secondary of a two winding transformer receive its power from the primary. That is why such motor are known as induction motor

### **Induction motor**

Induction motor is a machine which converts electrical energy into mechanical energy. There is no electrical connection between the supply voltage and the motor winding. E .m. f is induced in the rotor by electromagnetic induction. So it is called induction motor.  
Induction motor are classified into two types. They are,

- (1). Single phase Induction Motor
- (2). Three phase induction motor

### **SINGLE PHASE (1Φ) INDUCTION MOTOR**

It is motor with only one set of winding wound on the stator. It has a squirrel cage rotor. There is uniform air gap between stator and rotor. Stator and rotor are not electrically connected.

#### **Why single phase induction motor is not self starting?**

In single phase (1Φ) induction motor, the rotor is a squirrel cage rotor. In stator, only one winding (main winding) is provided. When an a.c supply (single phase) is given to the stator winding, an alternating (pulsating) magnetic field is produced. Therefore the rotor rotates in both directions alternatively. But the rotor does not rotate continuously. So the single phase (1Φ) induction motor is not self starting.

The following method is used to self start a single phase (1Φ) induction motor

1. Phase splitting method
  - (a) By using starting winding
  - (b) By using a capacitor in the winding
  - (c) By shaded poles.

2. Series motor method.
3. Repulsion method.

### Types of single phase ( $1\Phi$ ) induction motor

1. Split phase induction motor
2. Capacitor start induction motor
3. Shaded pole induction motor
4. Repulsion motor

### CAPACITOR START INDUCTION MOTOR

#### Construction

The construction of a capacitor start induction motor is shown in the fig. Fig. 3.31. two winding s are provided in the stator. They are,

1. Starting winding (or) auxiliary winding
2. Main winding (or) running winding

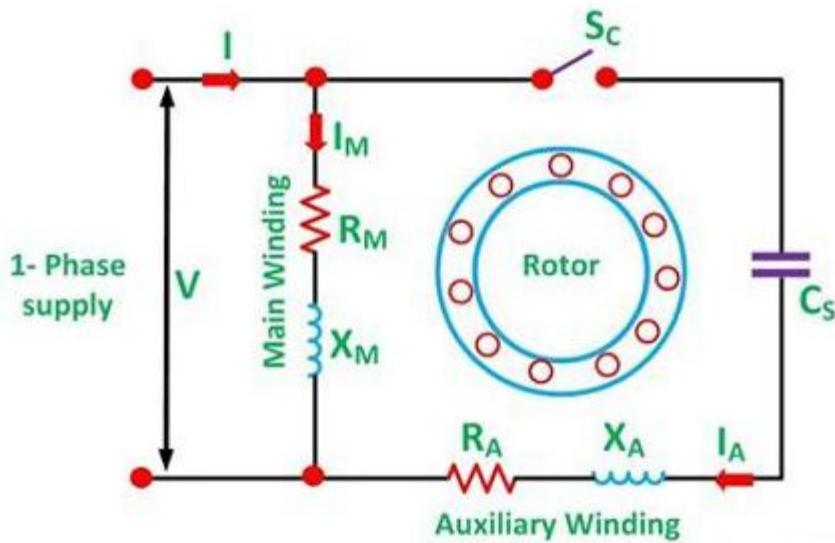


Fig. 3.31

A capacitor and a centrifugal switch are connected in series with the starting winding. The Main winding has low resistance and high reactance whereas starting winding has high resistance and low reactance. The main winding directly connected across the supply. The capacitor improves the power factor and leading current flows in the starting winding. Hence the torque of the motor will be increased and will be able to start at load. The capacitor is generally of electrolytic type and it is usually mounted on the outside of the motor as separate unit. Squirrel cage motor is used.

### Working

The current in starting winding Is always leading the current in main winding,  $I_M$  by about  $90^\circ$  by properly selecting the capacitance value. When the current flows through the stator winding a rotating magnetic field is produced. The rotating magnetic field cuts the rotor conductors. This induces an e.m.f in the rotor conductors. Hence a magnetic field is set up in the rotor. Due to the interaction of the two magnetic fields, (stator and rotor) the rotor rotates. The centrifugal switch disconnects the capacitor and starting winding from the supply when motor reaches about 75% of the rated speed.

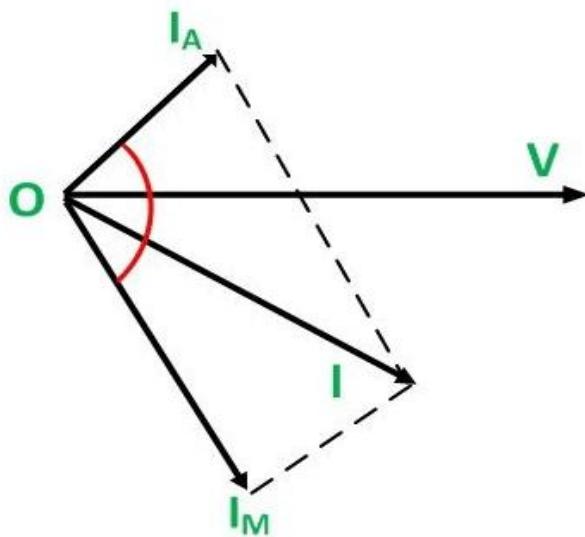


Fig. 3.32 (a)

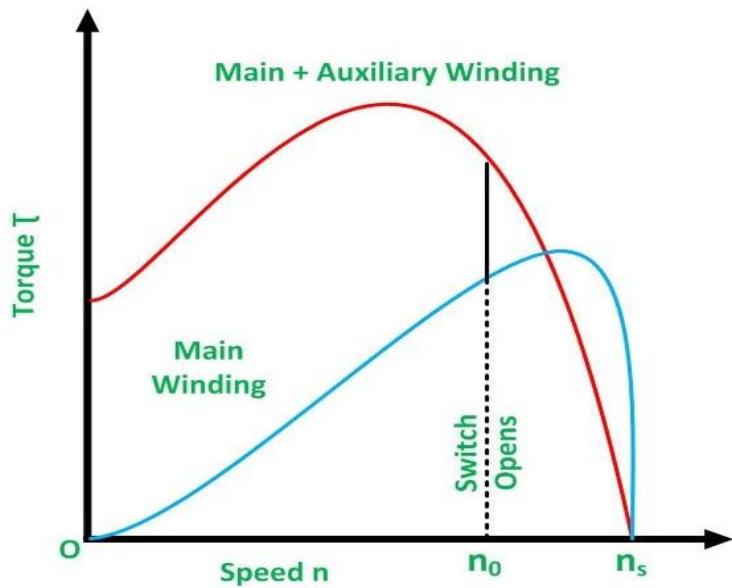


Fig. 3.32 (b)

The vector diagram is shown in the fig. 3.32(a) The current  $I_m$  lags the supply voltage  $V$  by a large angle and current  $I_s$  leads the supply voltage by a certain angle. The two currents are out of phase with each other by about  $90^\circ$ . Since the torque developed is proportional to the sine of angle between  $I_s$  &  $I_m$ . The speed -torque characteristics of capacitor start induction motor is shown in fig.3.30 (b)

## Applications

1. It is used in lathes
2. It is used in drilling machines.
3. It is used in fan and blowers.
4. It is used in wet grinders.
5. It is used in washing machines.
6. It is used in pumps.

## **STEPPER MOTOR**

### **Introduction**

Stepper motors are called stepping motors or step motors. The name stepper is used because this motor rotates through a fixed angular step in response to each input current pulse received by its controller. In recent years, there has been wide spread demand of stepping motors because of the explosive growth of the computer industry.

Stepper motor is a device which transforms electrical pulses into equal increments of rotary shaft motion called steps.

### **Step angle**

The angle through which the motor shaft rotates for each command pulse is called the step angle.

The stepper motors are classified into three types. They are,

1. Permanent magnet stepper motor
2. Variable reluctance stepper motor
3. Hybrid stepper motor

### **PERMANENT MAGNET STEPPER MOTOR**

The rotor is made of a permanent - magnet material like magnetically 'hard' ferrite. The stator has projecting poles but the rotor is cylindrical.

When a particular stator phase is energized, the rotor magnetic poles move into alignment with the excited stator poles. Consider a stepper motor having 4- pole stator with two phase winding as shown in fig. 3.33. Let the rotor made up of permanent magnet with 2-poles. The stator poles are marked as P,Q,R and T. They are excited with pulses supplied by power transistors. The power transistors are switched by digital controllers or computers.

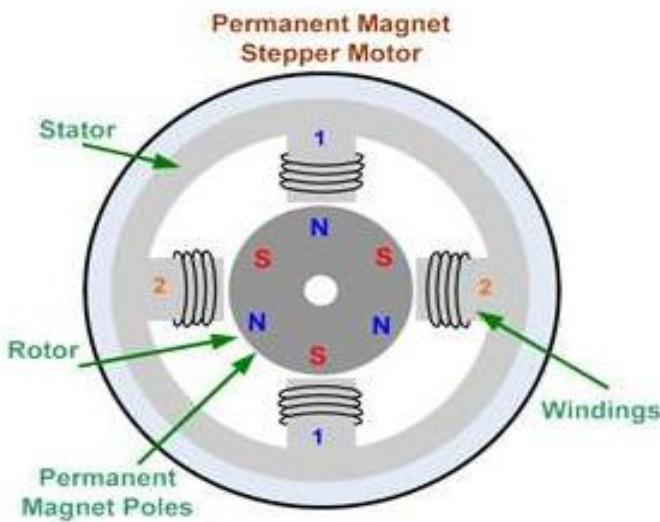


Fig. 3.33

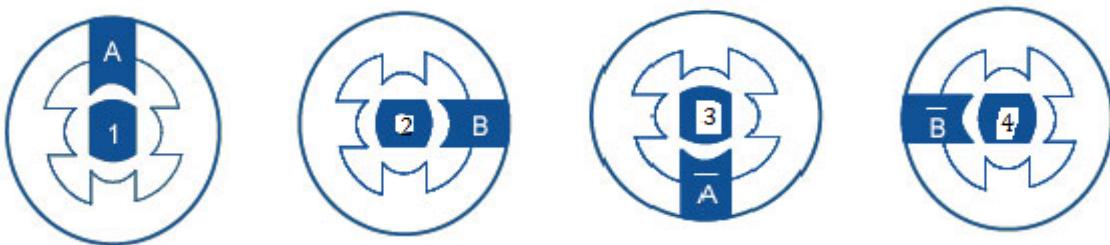


Fig. 3.34

If first excitation is applied to P and R , they develop the magnetic polarities indicated in step-1 as shown in fig. 3.34. Then the rotor sets itself vertically. If P and R are switched OFF and Q and T excited as in step-2 an alignment torque is developed on the rotor to turn its axis to 90°.This will move the rotor in horizontal position.

If Q and T are switched off and P and R re energized with reverse polarity, the rotor turns a further 90°. This will move the rotor to vertical position (step-3). If P and R are switched OFF and Q and T re-energized with reverse polarity the rotor turns a further 90°.This will move the rotor to horizontal position (step-4), Here the direction of rotation is anticlockwise. The direction of rotation of the stepper motor can be reversed by changing the current directions suitably.

## **Applications**

1. It is used in computer peripherals (floppy disk driver, line printer etc)
2. It is used in robots
3. It is used in X-Y plotters
4. It is used in machine tools.

## **Universal motor**

A **universal motor** is a special type of motor which is designed to run on either DC or single phase AC supply. These motors are generally series wound (armature and field winding are in series), and hence produce high starting torque. That is why, **universal motors** generally comes built into the device they are meant to drive. Most of the universal motors are designed to operate at higher speeds, exceeding 3500 RPM. They run at lower speed on AC supply than they run on DC supply of same voltage, due to the reactance voltage drop which is present in AC and not in DC.

There are two **basic types of universal motor**

- (i) Compensated type
- (ii) Uncompensated type

## **Construction of Universal motor**

Construction of a universal motor is very similar to the construction of a DC machine. It consists of a stator on which field poles are mounted. Field coils are wound on the field poles. However, the whole magnetic path (stator field circuit and also armature) is laminated. Lamination is necessary to minimize the eddy currents which induce while operating on AC.

The rotary armature is of wound type having straight or skewed slots and commutator with brushes resting on it. The commutation on AC is poorer than that for DC because of the current induced in the armature coils. For that reason brushes used are having high resistance. Working of universal motor

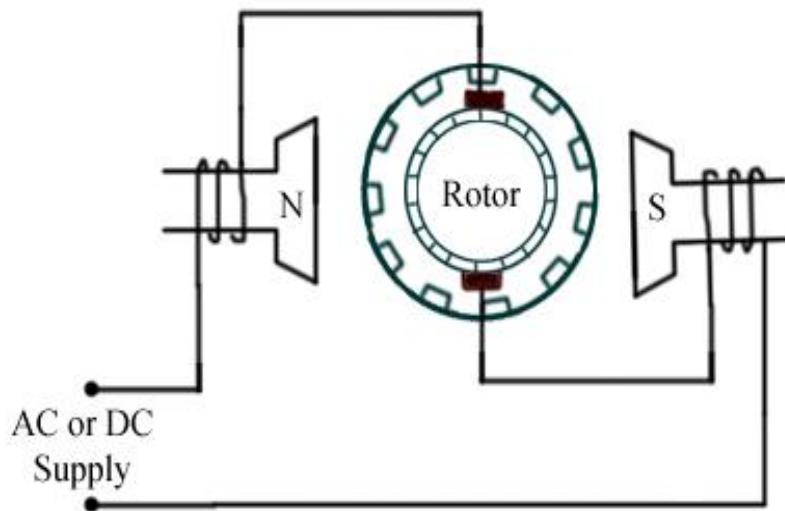


Fig. 3.35

A universal motor works on either DC or single phase AC supply. When the universal motor is fed with a DC supply, it works as a DC series motor. When current flows in the field winding, it produces an electromagnetic field. The same current also flows from the armature conductors. When a current carrying conductor is placed in an electromagnetic field, it experiences a mechanical force. Due to this mechanical force, or torque, the rotor starts to rotate. The direction of this force is given by Fleming's left hand rule.

When fed with AC supply, it still produces unidirectional torque. Because, armature winding and field winding are connected in series, they are in same phase. Hence, as polarity of AC changes periodically, the direction of current in armature and field winding reverses at the same time.

Thus, direction of magnetic field and the direction of armature current reverses in such a way that the direction of force experienced by armature conductors remains same. Thus, regardless of AC or DC supply, universal motor works on the same principle that DC series motor works.

## SPEED / LOAD CHARACTERISTICS

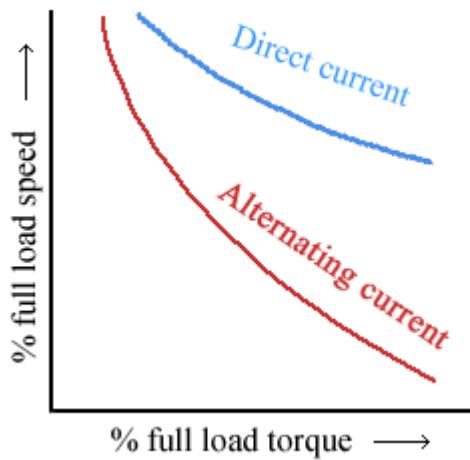


Fig. 3.36

Speed - load characteristics of a universal motor is similar to that of DC series motor. The speed of a universal motor is low at full load and very high at no load. Usually, gears trains are used to get the required speed on required load. The speed/load characteristics are shown in the figure. 3.36

### Applications of universal motor

- Universal motors find their use in various home appliances like vacuum cleaners, drink and food mixers, domestic sewing machine etc.
- The higher rating universal motors are used in portable drills, blenders etc.

## **UNIT IV**

### **MEASURING INSTRUMENTS AND CRO**

#### **1.1.INTRODUCTION**

These days most of the scientific and industrial measurements required very fast responses. The mechanical and electrical instruments and systems cannot cope up with this requirement. The necessity to step up response time and also the detection of dynamic changes in certain parameters, which require the monitoring time of the order of ms and many times  $\mu$ s have led to the design of today's electronic instruments and their associated circuitry.

##### **1.1.(a) Measurements**

The 'Measurement' of a given quantity is essentially an act or the result of comparison between the quantity and a predefined standard.

##### **1.1. (b) Instruments**

Measurement involves using an 'instrument' as a physical means of determining quantities or variables. Instrument is a device used for determining the value or magnitude of a quantity or variable.

##### **1.1.(c) Indicating Instruments**

Indicating instruments are those instruments which indicate the magnitude of the quantity being measured. The indications are given by a pointer moving over a calibrated scale.

Example: Ammeter, Voltmeter and Wattmeter.

#### **Uses of Instruments and Measurement System**

- (i) Monitoring of process and operation
- (ii) Control of process and operation
- (iii) Experimental engineering analysis

#### **1.2.BASIC FORCES FOR INDICATING INSTRUMENTS**

There are three types of basic forces for the proper operation of any indicating instruments, they are,

- (a) Deflecting force
- (b) Controlling force and
- (c) Damping force

### **(a) Deflecting Force**

Deflecting force that is used to move the point from its zero position in the deflecting system or moving system. Deflecting system of an instrument converts electric current or potential into a mechanical force. (i.e., deflecting Force)

The system which produced the deflecting force is called deflecting system. or moving systems.

The deflecting force can be produced by any one of the following effects.

- |                      |                           |
|----------------------|---------------------------|
| (i) Magnetic effect  | (ii) Electrostatic effect |
| (iii) Thermal effect | (iv) Induction effect     |
| (v) Chemical effect  |                           |

### **(b) Controlling Force**

Controlling force is required to keep the pointer at the final position steadily. It is also used to bring the pointer back to zero position. It is used to produce a force equal and opposite to the deflecting force of the final steady position. If this controlling force is absent the pointer will swing beyond the limit and deflection will be indefinite.

There are two types to provide controlling force, they are

- (i) Spring Control Method
- (ii) Gravity Control Method

#### **(i) Spring Control Method**

In spring control, the controlling force is provided by springs. The springs made up of phosphor bronze.

The essential requirements for instrument springs are

- (i) Non-magnetic
- (ii) Low resistivity
- (iii) Low temperature coefficient

The spring A and B is shown in fig. 4.1 which are coiled in the opposite directions. The pointer is attached to spindle. Under the influence of deflecting torque the pointer moves. During the pointer movement one of the springs unwinds itself and the other spring gets wounded. This wounding produces controlling torque. This is directly proportion to the angle of the deflection of the moving system. The pointer comes to reset, when deflecting torque ( $T_d$ ) is equal to the controlling torque

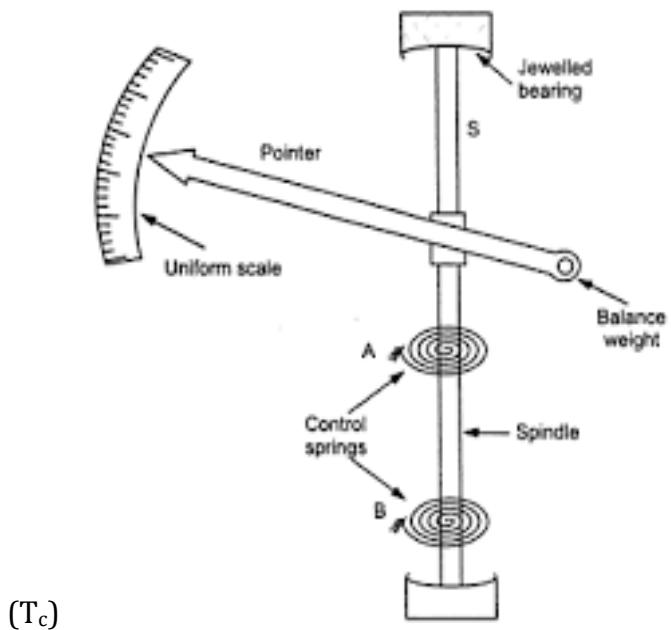


Fig.4.1

$$T_d = T_c$$

Controlling torque is directly proportional to the angle of twist ( $\theta$ )

$$T \propto \theta$$

Deflecting torque is directly proportional to the current (I) following through. i.e.

$$T_d \propto I$$

The spring controlled instruments have uniform scale.

## (ii) Gravity Control Method

In gravity control method, a small weight is placed on an arm of the moving system which is adjustable. This weight produces a controlling force during the deflection position.

Another arm has attached the spindle at an angle of  $90^{\circ}$  from the control weight arm. The controlling force is proportional of deflection moving system. In this arm a balance weight is attached for balance the weight of the pointer.

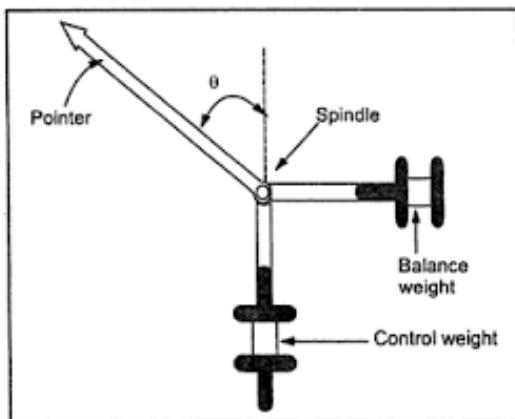


Fig.4.2

In zero position of the pointer control weight is vertical. When the pointer is deflected thro, an angle  $\theta$  from its zero position, the control weight will be in dotted position as shown in fig. 4.2

$$T_c = w \sin\theta \times l$$

Where,

$$wl = \text{constant}$$

$w$  = control weight

$l$  = distance from the axis of rotation

$\theta$  = angle of deflection.

$$T_c \propto \sin\theta$$

Final deflected position  $T_d = T_c$

$$\therefore I \propto \sin\theta$$

Current to be measured is proportional to since of the angle of deflection of the pointer. It has not uniform scale.

### (c) Damping Force

Damping force is used bring the pointer to rest rapidly and smoothly with out any oscillations.

They are four types to provide damping force , they are

- (i) Air friction damping
- (ii) Fluid friction damping
- (iii) Eddy current damping

### **(I) Air Friction Damping**

Air friction damping is one of the methods for producing damping force (damping torque).

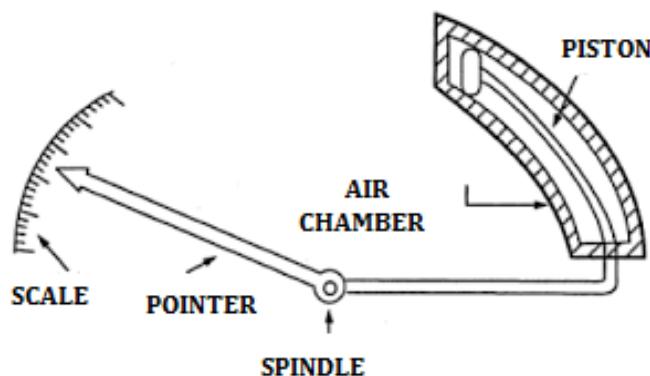


Fig.4.3

The fig.4.3 show the arrangement with an light aluminum piston attached to the moving system. The arrangement with a light aluminum piston attached to the spindle that carries the pointer and moves with in a air chamber. The piston moves in a fixed air chamber is closed at one end. Due to the oscillation, the piston will moves into and out of the air chamber.

If the piston moves rapidly into the chamber, the air inside the chamber is compressed and then the pressure inside the chamber opposes the motion of the piston. If the piston is moving outside, the pressure inside the chamber falls and therefore the outside pressure becomes grater than that of inside the chamber. Thus the motion of the piston is again opposed. Thus the movement of the piston in either direction is opposed and thus the oscillations are reduced.

### **(ii) Fluid Friction Damping**

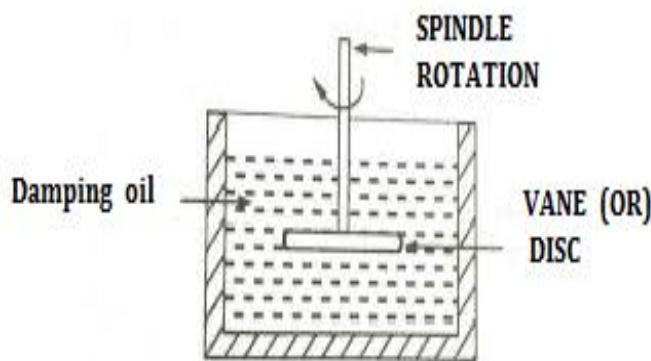


Fig.4.4

In the fluid friction damping circular disc (or) light vanes are attached to the spindle to the moving systems as shown in fig.4.4. The discs are dipped into a pot of damping oil are completely submerged into the oil. When the spindle moves by deflecting torque, the disc also moves in a circular motion and produces friction force. The frictional force always opposes the motion. The damping oil must be a good insulator non evaporating non-corrosive upon the metal of the discs.

### (iii) Eddy Current Damping

It is also known as magnetic or electromagnetic damping system. When conductor moves in a magnetic field an e.m.f is induced in it. If a closed path is provided for this e.m.f a current will flow and set up its own flux. This flux interacts with the main magnetic field and produces a damping torque.

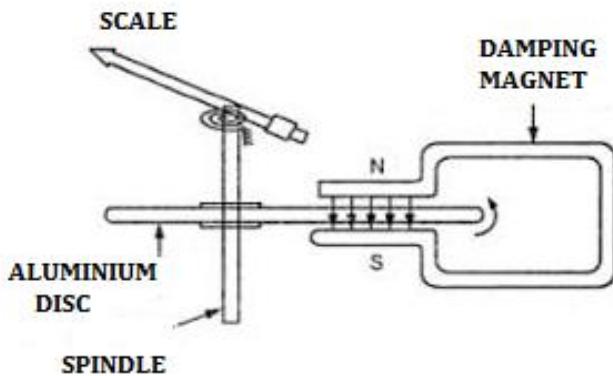


Fig.4.5

In the eddy current damping, the disc aluminum is mounted on the spindle as shown in the fig. 4.5. At its one end a permanent magnet is mounted in such a way that is enclosed a part of the disc within its pole faces. When the disc rotates along with the pointer some e.m.f produces eddy current. This eddy current in the disc, produces damping force and the pointer to

rest. The opposition to motion and torque produced is proportional to the strength of the magnetic field and the current produced.

## PERMANENT MAGNET MOVING COIL INSTRUMENT (PMMC)

The permanent magnet moving coil instrument is the most accurate type of direct current measurements.

### Principle

The working of this PMMC instruments based on the principle that when a current carrying conductor is placed in a uniform magnetic field, a mechanical force acts on the conductor.

### Construction

The fig.4.7 shows a permanent magnet moving coil instrument. The various parts of the instrument are

- |                               |                           |
|-------------------------------|---------------------------|
| (a) Powerful permanent magnet | (b) Soft iron pole pieces |
| (c) Moving coil               | (d) Pointer               |
| (e) Scale                     | (f) Spindle               |

**(a) Powerful Permanent Magnet :** The powerful permanent magnet is the heart of the instrument. It is made up of ALNICO.

**(b) Soft Iron Pole Pieces:** A soft iron core is mounted between the poles of the permanent magnet to reduce the air gap and give uniform radial field.

**(c)** Due to pole pieces arrangement more magnetic flux is present in between pole faces. U-shaped magnet is mostly used.

**(C) Moving Coil :** A coil of many turns of enameled or silk covered copper wire is wound on a light aluminum frame. This coil can move freely between the poles and iron core. The coil provides electro magnetic damping. The moving coil assembly is mounted on a pivoted spindle as shown in the fig.4.6.

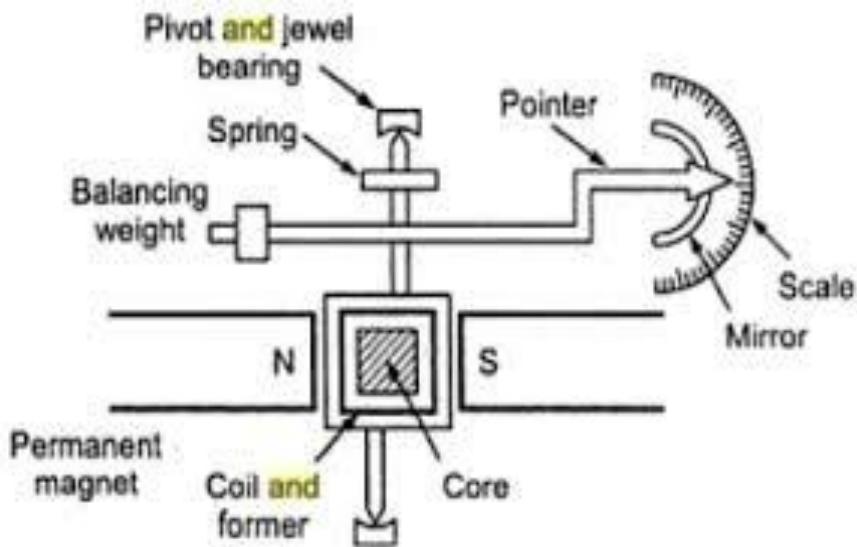


Fig.4.6

**(d) Pointer :** The pointer is carried by the spindle and moves over a graduated scale. It is light weight construction. It is indicated the reading.

**(e) Scale :** The scale has graduations marked in uniform. The scale marking either for  $90^\circ$  or  $120^\circ$

**(f) Spindle :** The pointer is carried by the spindle. The controlling torque is produced by two phosphor bronze hair springs. It attached at the top and bottom of the spindle. Eddy current damping is produced by the movement of the aluminum frame moving in the magnetic field of the permanent magnet.

### Working Principle

- When the current to be measured is passed through the coil, a magnetic field is produced.
- Due to the interaction between permanent magnetic field and the coil magnetic field a deflecting torque is produced.
- Hence the pointer moves over the graduated scale. At the same time the control spring produced controlling torque.
- When the two torques are equal the pointer comes to steady state and indicates the reading.

The control springs are attached to the moving coil in a such a manner that when the pointer moves the spring will move in rewind direction and so the control spring provides a mechanical torque which is opposite to the electromagnetic torque developed by the coil.

Deflecting torque in the moving coil ( $T$ ) =  $BAIN$  N-m

Where,  $T$  is the torque in N-m

$B$  = flux density in the air gap in web/m<sup>2</sup>

$A$  = area of the coil in m<sup>2</sup>

$I$  = current in amps

$N$  = number of turns of the coil

### **Advantages**

1. Consumes very small power.
2. Very high torque weighty ratio.
3. Uniform scale
4. No hysteresis loss.
5. No effect of stray magnetic field.
6. Very effective and efficient eddy current depending is provided.

### **Disadvantage**

1. High cost.
2. Errors due to ageing of permanent magnet and spring.
3. This instrument can not be used for A.C measurements.
4. Limited to D.C measurements

### **Error in Moving Coil Instrument**

1. Weakling of permanent magnets due to ageing at temperature effects.
2. Weakling a spring due to ageing at temperature effects

### **Shunts and Multipliers**

#### **Definition**

Shunts and multipliers are the common devices. Which are employed for extending the range of instrument. It is used to calibrate the instruments.

### Explanation

When heavy currents are to be measured the major part of the current is bypassed through a low resistance called a "shunt". The low resistance is connected in parallel with the low range ammeter as shown in fig.4.7.

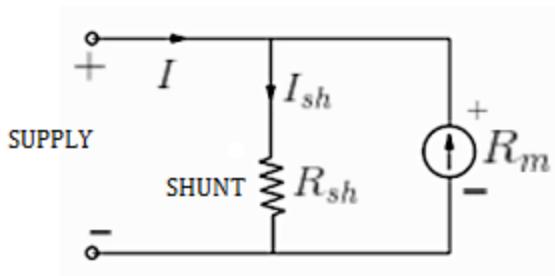


Fig.4.7

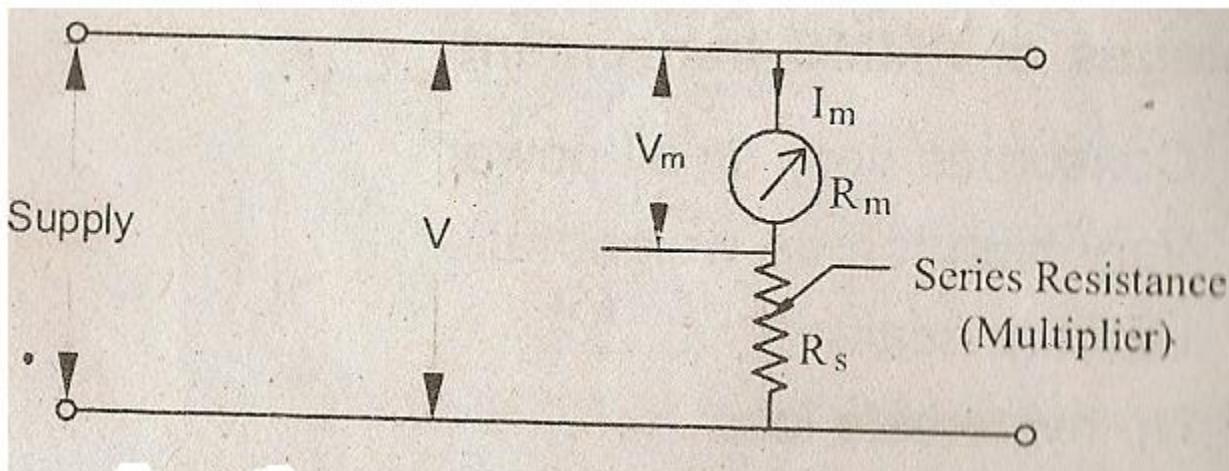


Fig.4.8

For measuring high voltages the moving coil meter is converted into a D.C voltmeter by connecting a series of resistance called "multiplier" as shown in fig.4.8.

### Requirements of a Shunt

1. Shunt must have a very low temperature coefficient of resistance.
2. Multiplying power of the shunt must be independent of temperature.
3. shunt resistance should not vary with time of usage.
4. Thermoelectric e.m.f of the shunt should be small.
5. Shunt should carry current without excessive temperature rise.

### Material Used for Shunt

1. 'Manganin' is a useful material for D.C measurement
2. 'Constantan' is a useful material for A.C measurement

### D.C AMMETER CIRCUIT

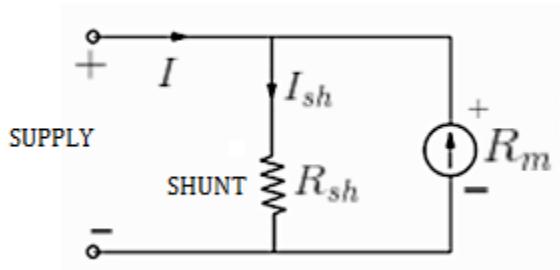


Fig.4.9

The D.C ammeter circuit is shown in fig.4.9. A basic PMMC instrument can be converted into an ammeter by the use of shunt to carry the major of the line current. The shunt resistance is in parallel with the meter resistance. So the voltage drop across the shunt is equal to the meter resistance voltage drop.

Let

$I$  = main circuit current

$I_m$  = meter current

$I_{sh}$  = shunt current

$R_m$  = meter resistance

$R_{sh}$  = shunt resistance

The main circuit current 'I' will be split into two parts. 'I<sub>m</sub>' is the low current flowing through the ammeter and I<sub>sh</sub> is the heavy current flow through the shunt.

$$\text{Thus } I = I_m + I_{sh} \longrightarrow 1$$

In the parallel circuit the voltage drop across the shunt and the meter must be the same.  
 Voltage drop across meter = I<sub>m</sub> R<sub>m</sub>  $\longrightarrow 2$

$$\text{Voltage drop across shunt} = I_{sh} R_{sh} \longrightarrow 3$$

Equating the equations 2 and 3

$$I_m R_m = I_{sh} R_{sh}$$

$$R_{sh} = \frac{I_m R_m}{I_{sh}} \longrightarrow 4$$

From the equation 1

$$\begin{aligned} I &= I_m + I_{sh} \\ I_{sh} &= I - I_m \end{aligned} \longrightarrow 5$$

Substituting the equation 5 in equation 4

$$R_{sh} = \frac{I_m R_m}{I - I_m}$$

## D.C VOLTMETER CIRCUIT

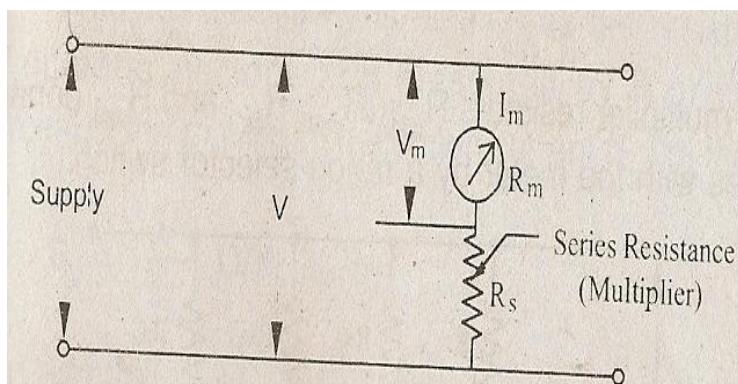


Fig.4.10

A basic PMMC meter movement is converted into a voltmeter by connecting a series resistance with it. This series resistance is known as multiplier. The combination is connected across the circuit whose voltage is to be measured as shown in fig.4.10. The multiplier limits the current through the movement. The current should not exceed the value of full scale deflection current, so multiplier is used.

The voltage range can be extended by means of different values of multiplier resistance.

Where

$I_m$  = meter current

$R_m$  = meter resistance

$R_{se}$  = multiplier resistance

$V_m$  = voltage across the meter

$V$  = voltage to be measured

$V_m = I_m R_m$

$V = \text{voltage across the meter} + \text{voltage across the multiplier}$

$V = I_m R_m + I_m R_{se}$

$I_m R_{se} = V - I_m R_m$

$$\therefore R_{se} = \frac{V - I_m R_m}{I_m}$$

### Requirements of a Multiplier

1. The resistance should remain constant with time.
2. Multiplier should be non inductive.
3. It should have a low temperature co-efficient.
4. Low thermoelectric e.m.f with copper

### Material Used for Multiplier

1. Manganin
2. Constantan

## **Voltmeter Sensitivity ( $S_v$ )**

Voltmeter sensitivity is a parameter that indicates the ability of the meter to indicate the input voltage accurately specified in term of  $\Omega/V$ . If the sensitivity is high, then the meter can indicate even very small input voltages accurately.

Voltmeter sensitivity is defined as the resistance of the voltmeter per one volt measured by it or it is the reciprocal of the maximum deflection current ( $I_m$ ) of the meter

$$sv = \frac{1}{Im} \Omega/V$$

## **BRIDGES**

Bridge circuits are used for measuring component values and other circuit parameters such as resistance, inductance and capacitance. In bridge circuit the unknown values of the components are compared to that of an accurately known component.

Basically there are two types of bridges, they are

1. D.C bridge
2. A.C bridge

### **1. D.C bridge**

D.C bridges are mainly used for measuring D.C resistance. "Wheatstone bridge" is used for measuring medium D.C resistances and "Kelvin Double bridge" is used for measuring low D.C resistances

### **2. A.C bridge**

A.C bridge consist of four arms , source of excitation and null detector. The A.C bridges are used to measure the values of inductance and capacitance. 'Maxwell's bridge' and 'Hay's bridge' are used for measurement of self inductance. 'Wein bridge' and 'universal impedance bridge' are used for measurement of frequency.

## **WHEAT STONE BRIDGE**

Wheatstone bridge is the simplest form of bridge used for measuring low and medium value of resistances. The fig.4.11 show the Wheatstone bridge circuit.

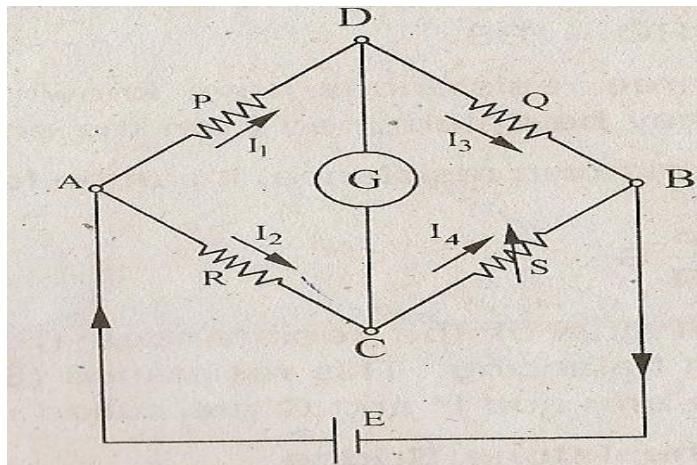


Fig.4.11

It consists of four resistive arms (P,Q,R,S), EMF source (E) and null deflector. Usually galvanometer (G) is used for null deflector. The source (E) is connected to 'A' and 'B', galvanometer connected to 'C' and 'D'. The galvanometer has zero center scale. When there is no current, the galvanometer reads zero. i.e., bridge is balanced.

When the bridge is balanced, the potential difference between the point C and D is zero. Hence no current flows through the galvanometer. Under this condition voltage drop between A and C is equal to the voltage drop between A and D. Similarly the voltage drop between B and C is equal to the voltage drop between B and D.

Hence under balanced condition

$$I_1 P = I_2 R \longrightarrow 1$$

Since the galvanometer current is zero.

$$I_1 = I_3 = \frac{E}{P+Q} \longrightarrow 2$$

$$I_2 = I_4 = \frac{E}{R+S} \longrightarrow 3$$

Where  $E$  is the e.m.f of the battery

Substitute the equation (2) and (3) on equation (1)

$$\frac{E}{P+Q}P = \frac{E}{R+S}R$$

$$\frac{P}{P+Q} = \frac{R}{R+S}$$

$$P(R+S) = R(P+Q)$$

$$PR+PS = PR+QR$$

$$PS = QR \quad \longrightarrow \star$$

Hence if four resistance have known value, the four resistance may be determined from the equation(4).

$$R = \frac{PS}{Q}$$

Where, 'R'- unknown resistance.

Resistance 'S' is made adjustable to permit balancing. This resistance is called standard resistance arm and P & Q are called ratio arm.

## Applications

1. To find the resistance value of unknown resistors of the range  $0.1\Omega$  to  $100000\Omega$
2. To measure the inductance, capacitance and frequency by suitable arranging the components of different arms.
3. Forms the basic four different bridge.
4. To measure Q-factor, dissipation factors of coils and capacitors.

## Effects of Connecting Leads

When measuring the low value of unknown resistance using wheat stone bridge, the error will occur due to the connecting leads of unknown resistance and contact resistance. This error is overcome in Kelvin's double bridge circuit.

## UNIVERSAL IMPEDANCE BRIDGE ARRANGEMENTS TO MEASURE R, L, C

The Universal Impedance Bridge is one of the most useful laboratory bridges. It combines several bridge configurations into a single portable instrument. It is capable of measuring both D.C and A.C resistance, inductance and a Q-factor of an inductor, capacitance and dissipation factor D of a capacitor. The universal impedance bridge consists of four basic bridge circuits. It has suitable A.C and D.C sources, A.C and D.C null detectors and impedance standards.

### Resistance Measuring Bridge

Wheatstone bridge is used for both D.C and A.C resistance measurements as shown in fig.4.24(a). For D.C resistance measurements a suspension type galvanometer having a sensitivity of  $0.5\mu A$  per scale division is used. The null indicator for A.C resistance measurements is usually a electron ray tube.

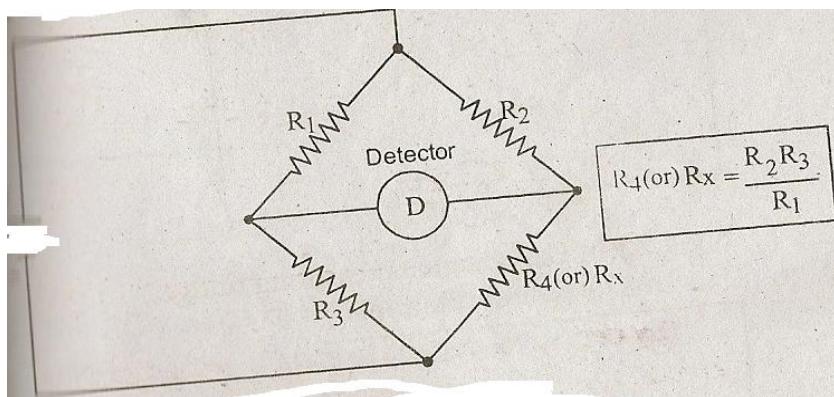


Fig.4.12 (a)

### Capacitance Measuring Bridge

Capacitance and dissipation factors are determined by modified Desaulty's bridge as shown in fig.4.12 (b)

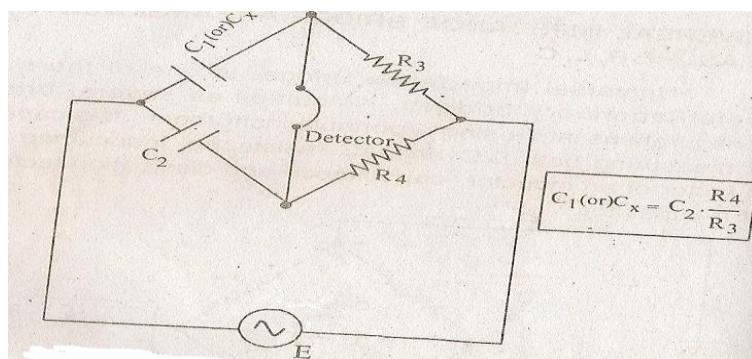


Fig.4.12 (b)

## Inductance Measuring Bridge

The Fig.4.12 (c) shows the arrangement of inductance measuring bridge. It is used for measuring inductance of medium of coil using Maxwell's Bridge.

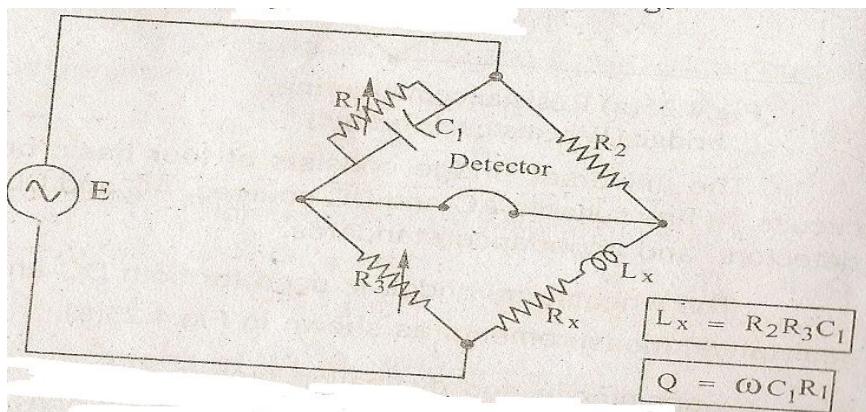


Fig.4.12 (c)

The Fig.4.12 (d) shows the arrangement of inductance measuring bridge. It is used for measuring inductance using Hay's Bridge .

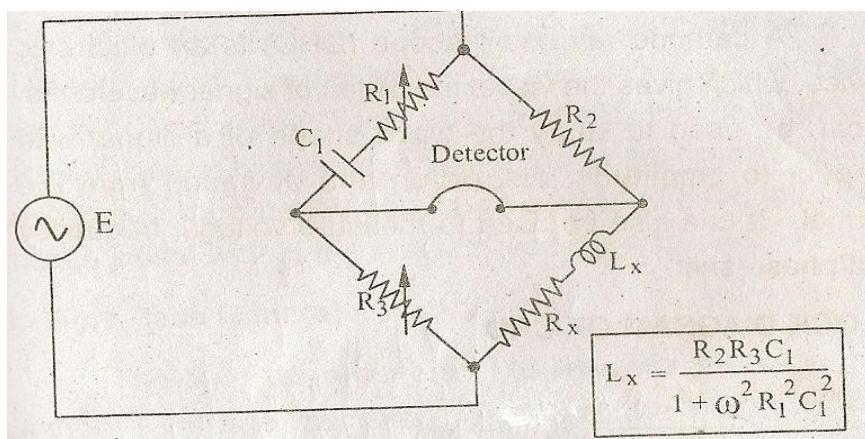


Fig.4.24(d)

## CATHODE RAY OSCILLOSCOPE ( CRO )

### Introduction

The **Cathode Ray Oscilloscope** (CRO) is an electronics device, which is laboratory instrument used for studying wave shapes of alternating currents and voltages as well as for measurement of voltage, current, power and frequency and phase shift with respect to amplitude distortion and deviation from the normal.

It allows the user to see the amplitude of electrical signals as a function of time on the screen. It is widely used for trouble shooting radio and TV receivers as well as laboratory work involving research and design. In true sense the cathode ray oscilloscope has been one of the most important tools in the design and development of modern electronic circuits.

### Block Diagram of a CRO

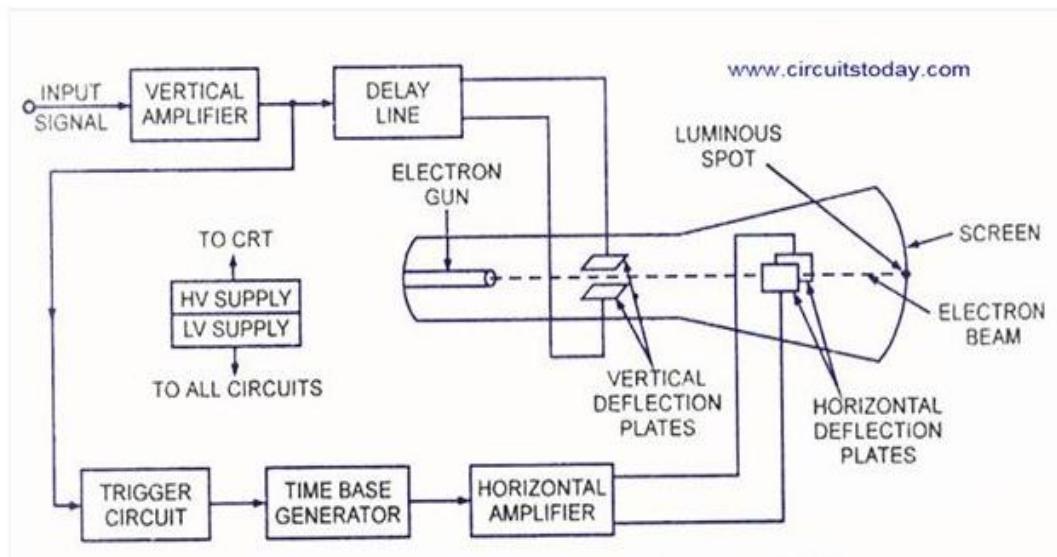


Fig.4.13 Block Diagram

The block diagram of CRO is shown in fig.4.13. the varies block of simple CRO are as follows.

1. Cathode Ray Tube ( CRT )
2. Vertical Amplifier
3. Horizontal Amplifier
4. Time Base Generator
5. Trigger Circuit

6. Delay Line

7. Power Supply

## **1.Cathode Ray Tube ( CRT )**

The instrument employs a **cathode ray tube** (CRT), which is the heart of the oscilloscope. It generates the electron beam, accelerates the beam to a high velocity, deflects the beam to create the image, and contains a phosphor screen where the electron beam eventually becomes visible. For accomplishing these tasks various electrical signals and voltages are required, which are provided by the power supply circuit of the oscilloscope. Low voltage supply is required for the heater of the electron gun for generation of electron beam and high voltage, of the order of few thousand volts, is required for cathode ray tube to accelerate the beam. Normal voltage supply, say a few hundred volts is required for other control circuits of the oscilloscope.

## **2. Vertical Amplifier**

Vertical Amplifier is also called as Y-amplifier. It is used to amplify the input signals. Vertical deflection plates are fitted between electron gun and screen to deflect the beam according to input signal. Electron beam strikes the screen and creates a visible spot. The signal to be viewed is supplied to the vertical deflection plates through the vertical amplifier, which raises the potential of the input signal to a level that will provide usable deflection of the electron beam.

## **3. Horizontal Amplifier**

Vertical Amplifier is also called as X-amplifier. The sawtooth voltage produced by the time base generator may not be sufficient. External signal is applied to horizontal deflection plates through the horizontal amplifier at the sweep selector switch as shown in figure. The horizontal amplifier, similar to the vertical amplifier, increases the amplitude of the input signal to the level required by the horizontal deflection plates of CRT.

## **4.Time Base Generator**

The linear deflection or sweep of the beam horizontally is accomplished by use of a sweep generator that is incorporated in the oscilloscope circuitry. The time base generator used to generate sawtooth voltage, require to deflect the beam in the horizontal section.

## **5.Trigger Circuit**

The trigger circuit is used to produces trigger pulse to start saw tooth signal. This circuit also employed to synchronize the horizontal deflection with the vertical input. It links the vertical input and horizontal input. The trigger pulse start the time base generator at the starting time of the input signal.

## **6. Delay Line**

The delay line is mainly used to delay the transmission of signal voltage to deflection plates.

## **7. Power Supply**

Power supply provides the voltages required by the CRT to generate and accelerate the electron beam. it also provides the voltages for various stages. High voltage is given to the accelerating and focusing anodes of the CRT and low voltages to all other circuits.

## **Principle of Operation of CRO**

A basic block diagram of a general purpose oscilloscope is shown in figure. Cathode ray tube and its various components will be discussed. The cathode ray is a beam of electrons which are emitted by the heated cathode (negative electrode) and accelerated toward the fluorescent screen. The assembly of the cathode, intensity grid, focus grid, and accelerating anode (positive electrode) is called an electron gun. Its purpose is to generate the electron beam and control its intensity and focus. Between the electron gun and the fluorescent screen are two pair of metal plates. One pair is provided horizontal deflection of the beam and another one pair to give vertical deflection to the beam.

The input voltage is amplified by vertical amplifier and passes through a delay line and applied to vertical deflection plates. In the most common use of the oscilloscope the signal to be studied is first amplified and then applied to the vertical (deflection) plates to deflect the beam vertically and at the same time a voltage that increases linearly with time is applied to the horizontal (deflection) plates thus causing the beam to be deflected horizontally at a uniform (constant) rate. The signal applied to the vertical plates is thus displayed on the screen as a function of time. The horizontal axis serves as a uniform time scale.

The horizontal voltage on the luminous spot moves from left to right over the display area of the screen. The spot moves from left to right on the screen with the uniform speed. The vertical input voltage moves the spot up and down on the screen in accordance with the instantaneous value of voltage. Due to effect of both voltages , the luminous spot traces the waveform of the input signal with respect to time and can be observed on the screen.

## CATHODE RAY TUBE ( CRT )

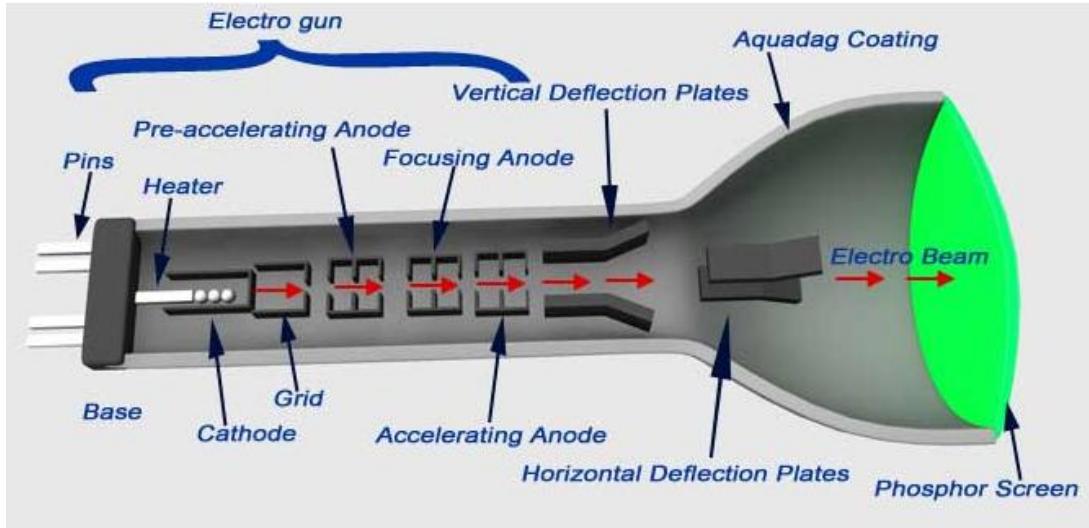


Fig.4.14

Cathode Ray Tube ( CRT ) is a heart of CRO. The internal structure of CRT is shown in fig. 4.14. it consists of varies parts. They are

1. Electron Gun Assembly
2. Deflection plate assembly
3. Fluorescent screen
4. Glass envelope

### 1. Electron Gun Assembly

The electron gun assembly consists of an indirectly heated cathode, a control grid surrounding the cathode, a focusing anode and an accelerating anode. The sole function of the electron gun assembly is to provide a focused electron beam which is accelerated towards the phosphor screen. The cathode is a nickel cylinder coated with an oxide coating and emits plenty of electrons, when heated. The emitting surface of the cathode should be as small as possible, theoretically a point. Rate of emission of electrons or say the intensity of electron beam depends on the cathode current, which can be controlled by the control grid. The control grid is a metal cylinder covered at one end but with a small hole in the cover.

The grid is kept at negative potential (variable) with respect to cathode and its function is to vary the electron emission and so the brilliancy of the spot on the phosphor screen.

### 2. Deflection plate assembly

Electron beam, after leaving the electron gun, passes through the two pairs of deflection plates. One pair of deflection plates is mounted vertically and deflects the beam in horizontal or X-direction and so called the horizontal or l-plates and the other pair is mounted horizontally and deflects the beam in vertical or Y-direction and called the vertical or Y-plates.

These plates are to deflect the beam according to the voltage applied across them. For example if a constant pd is applied to the set of Y-plates, the electron beam will be deflected upward if the upper plate is positive. In case the lower plate is positive then the beam will be deflected downward. Similarly if a constant pd is applied to the set of X-plates, the electron beam will be deflected to the left or right of the tube axis according to the condition whether the left or right plate is positive. When a sinusoidal voltage is applied to Y-plates, the beam will be moved up and down according to the variation of plate potential.

If the frequency of variation is more than 16 Hz the deflection will be a vertical line in the centre of the screen. In case the sinusoidal voltage is applied to X-plates and frequency of variation is more than 16 Hz the deflection will be a horizontal line. If potentials are applied to both sets of plates simultaneously, the deflection will be an oblique line. The amount of deflection is in proportion to the voltage applied to the pair of plates.

### **3. Fluorescent screen**

The inner face of the tube is coated with some fluorescent material such as zinc oxide etc. When high velocity of electron beam strikes the screen a spot light is produced at the point of impact. The colour of the spot depends upon the nature of fluorescent material

### **4. Glass envelope (Glass Body And Base).**

The whole assembly is protected in a conical highly evacuated glass housing through suitable supports. The inner walls of CRT between neck and screen are usually coated with a conducting material known as aquadag and this coating is electrically connected to the accelerating anode. The coating is provided in order to accelerate the electron beam after passing between the deflecting plates and to collect the electrons produced by secondary emission when electron beam strikes the screen. Thus the coating prevents the formation of –ve charge on the screen and state of equilibrium of screen is maintained. Horizontal and vertical marks are marked on the screen of the CRT to provide user a correct measurement.

### **Operation of CRT**

The sole function of the electron gun assembly is to provide a focused electron beam which is accelerated towards the phosphor screen. The hole in the grid is provided to allow passage for electrons through it and concentrate the beam of electrons along the axis of tube. Electron beam comes out from the control grid through a small hole in it and enters a pre-accelerating anode.

This electron beam is focused on the screen by an electrostatic lens consisting of two more cylindrical anodes called the focussing anode and accelerating anode apart from the pre-accelerating anode. The function of these anodes is to concentrate and focus the beam on the screen and also to accelerate the speed of electrons.

When the beam of electrons leaves the accelerating anode it comes under the influence of vertical and horizontal deflection plates. When no voltage is applied on the deflection plates, the beam of electron will produce a light spot at the centre of screen. The spot is made movable by applying appropriate signals on both the deflection plates.

## **FOCUSSING**

A potential difference is kept between these two electrodes so that an electric field is generated between them. Spreading of electric field is caused because of repulsion between electric lines.

In the electron gun of a CRT, middle anode is kept at lower potential with respect to other two anodes and it acts like an electrostatic lens and focal length of this lens can be varied by varying the potential of the middle anode with respect to other two anodes. So focusing of an electron beam is done by varying the potential of middle anode with the help of a potentiometer, as shown in figure. By increasing the positive potential applied to the focusing anode the electron beam can be narrowed and the spot on the screen can be made a pin point.

### **Electrostatic Focusing**

An electrostatic focusing system is shown in fig.4.15. Electrostatic lens consists of three anodes, with the middle anode at a lower potential than the other two electrodes.

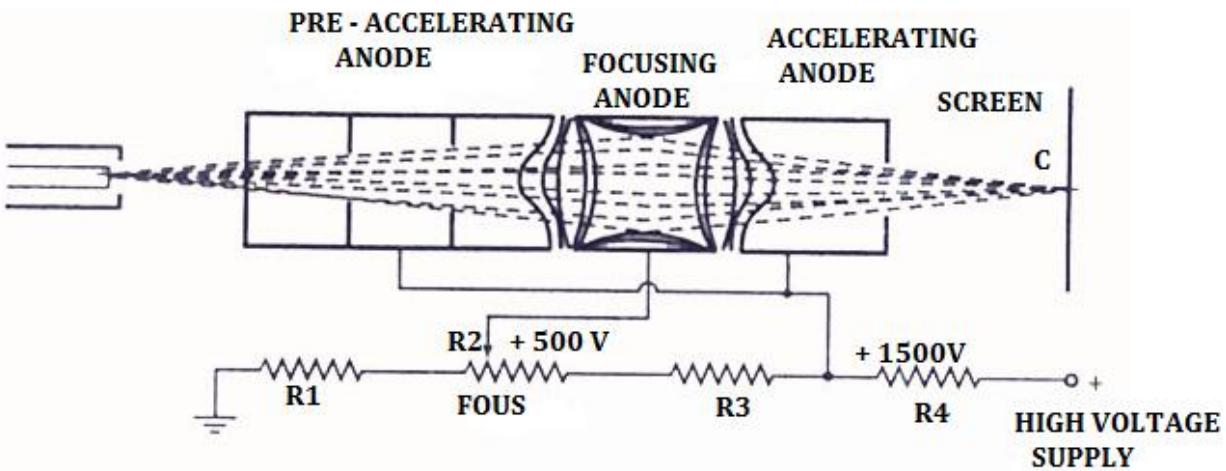


Fig.4.15

In CRO electrostatic focusing is used to coverage the electron beam into a single luminous spot on the screen. This method is called "Electron Lens System". in this system the pre-accelerating anode is the first electrode. The second electrode is the focusing anode. The third electrode is the accelerating electrode. The accelerating anode and pre-accelerating anodes are connected to high positive potential ( +1500v ). And the focusing anode is connected to low positive potential ( +500v ). The potential different between the focusing anode and accelerating anodes produces a non-uniform magnetic field on each side of the focusing anode. So it forms a "double concave lens"

The electrons enter the electric field between the pre-accelerating and focusing anode makes an angle with the CRT axis. They refracted towards the CRT axis. This approximated parallel beam then enters the electric field between the pre-accelerating and focusing anode. Now the beam is once again refracted to become convergence and focused on the screen at the centre of the CRT axis.

## DEFLECTION SYSTEM

Deflection system is used to deflect the electron beam vertically or horizontally by proper voltage applying across the deflection plates. There are two types of deflection system called electrostatic deflection and electromagnetic deflection. In CRO electrostatic deflection is used.

## Electrostatic Deflection

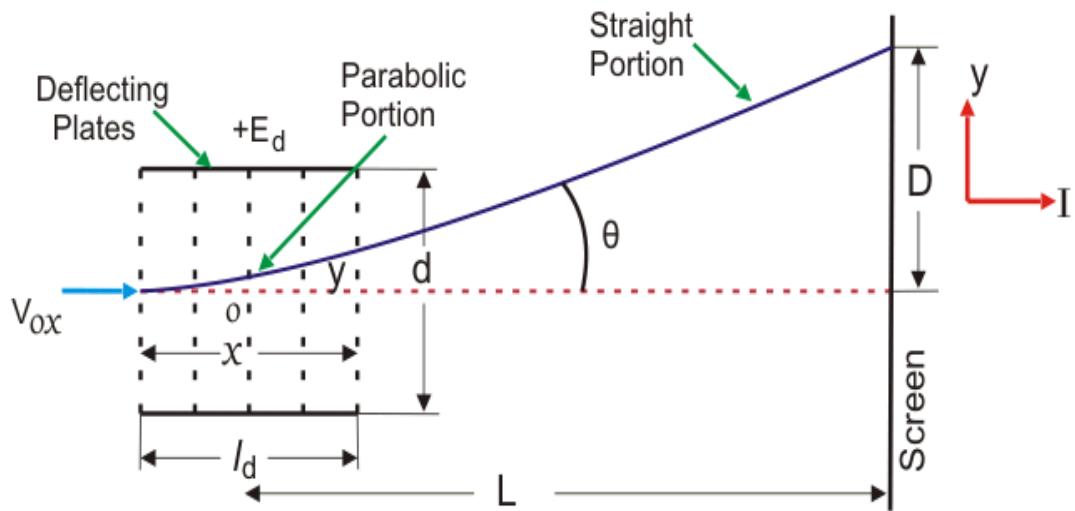


Fig.4.16

In electrostatic deflection system consists of two pair of deflection plates which are placed at right angles to each other. Electron beam after leaving the electron gun passes through the two pairs of deflection plates. One pair of deflection plates is mounted vertically and deflects the beam in horizontal or X-direction and so called the horizontal or l-plates and the other pair is mounted horizontally and deflects the beam in vertical or Y-direction and called the vertical or Y-plates. These plates are to deflect the beam according to the voltage applied across them. When no voltage is applied on the deflection plates, the beam of electron will produce a light spot at the centre of screen C. When voltage is applied to the vertical deflection plates such that upper plate potential is positive and lower plate potential is negative, a uniform electrostatic field is produced between the plates.

When electron beam passes between the plates it is attracted by the positive plate and repelled by the negative plate. When electron beam leaves from the deflection plates, it travels a straight line and touches the screen at 'A'. The deflection of the electron beam on the screen is proportional to the applied voltage of the deflection plates.

If horizontal deflection is required ( i.e, to move the beam on the screen to the right or left) a voltage is applied to the horizontal deflection plates.

## VERTICAL DEFLECTION SYSTEM

The vertical deflection system is used to providing faithful reproduction of the input voltage waveform.

### Block Diagram of Vertical Deflection System

The general block diagram of vertical deflection system is shown in fig.4.17. it consists of various parts. They are,

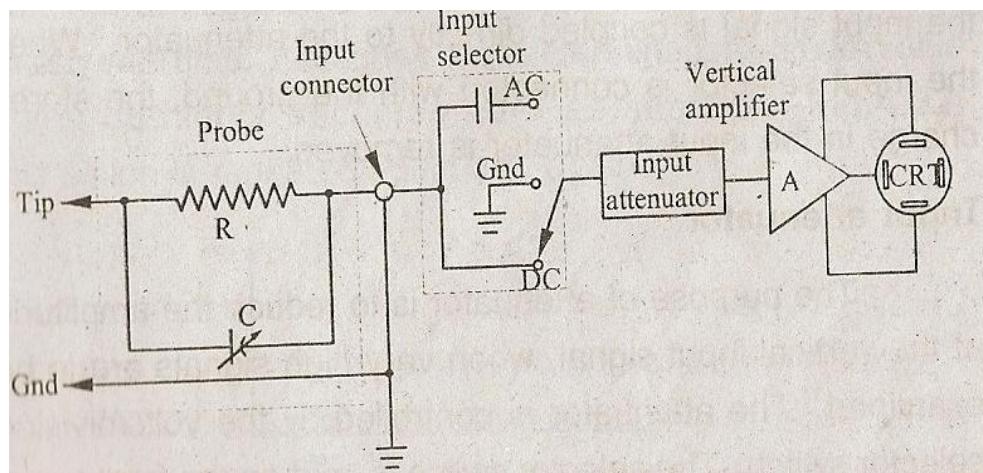


Fig.4.17

- a) CRO probe
- b) input selector
- c) input attenuator
- d) vertical amplifier

#### a) CRO Probe

The CRO probe is used to connect the vertical amplifier with the circuit under measurement. The general purpose probe is called passive probe. It consists of a series resistor and variable shunt capacitor. They are kept in probe body with a probe tip and ground connector.

#### b) Input Selector

It is three position switch such as A.C , D.C and ground. When the input selector switch is placed on the A.C position the signal is coupled to the attenuator through a capacitor. When the input selector switch is placed on the D.C position the signal is coupled directly to the attenuator. When the input selector switch is connected with the ground, the stored charge in the input attenuator is removed.

### c) Input Attenuator

The input attenuator is used to attenuate the signal to the required level. The input attenuator consists of a number of RC voltage dividers. The attenuator is controlled by the volts / division selector switch. The selector switch is kept on the front panel of the CRO. The selector switch is calibrated in term of the deflection factor (v / div). Usually the range of attenuator setting is 0.1, 0.2 ,0.5,1.2, 5,10,20 and 50 volts per division.

### D) Vertical Amplifier

The vertical amplifier consists of preamplifier and main vertical amplifier sections. The preamplifier consists of input amplifier and phase inverter. The main vertical amplifier consists of driver amplifier and class C push pull output amplifier. The input impedance of the vertical amplifier is represented by  $R_1$  in parallel with  $C_1$ . The output voltage is proportional to the ratio of amplifier input impedance and total circuit impedance.

The main vertical amplifier output stage delivers equal signal voltage of the opposite polarity to the vertical deflection plates of the CRT. It is expressed in terms of deflection factor (V / div)

### Need for Time Base Voltage

When A.C voltage is applied to the vertical deflecting plates, they become positive and negative alternatively. Hence according to the given frequency of voltage the luminous spot moves up and down on the screen. Hence only a vertical line appeared on the screen. But we can not see the actual waveform of the input A.C signal. To display a actual waveform it is required to apply a saw tooth voltage or a sweep voltage is applied to the X-X plates. This time base voltage is periodic voltage of periodic time. A saw tooth generator is used to generate a saw tooth waveform. This time base voltage increases linearly with time. It reaches a predetermined maximum value V and suddenly becomes zero as shown in fig.4.18

Fig.4.18

When a sweep voltage is applied to the X-X plates of the CRO , the spot moves slowly and at uniform speed from left to right. The spot reaches the position corresponding to the

amplitude  $V$  of the sweep voltage and the suddenly flies back to the original position. This period is fly back period.

## Sweep Generator

The circuit which develops ramp voltage is called as time base generators or sweep generators. The fig.4.31(a) and (b) shows a sweep generator circuit using UJT and fig.4.31(b) shows sweep waveform.

Sweep generator circuit using UJT, is noting but a relaxation oscillator. UJT acts as switch. When the supply is first applied, capacitor( $C_T$ ) charges exponentially through the resistor( $R_T$ ). The UJT emitter voltage  $V_E$  rises to  $V_{BB}$  when  $V_{BB}$  reaches the peak voltage  $V_P$ . The emitter to base diode become forward biased. Hence UJT triggers ON. This provides low resistance path between E and  $B_1$ . So, the capacitor discharges rapidly through the UJT. Emitter voltage  $V_E$  decreased until it can no longer sustain the minimum bias required for UJT conduction. Now low resistance path is broken and the capacitor recharges. This cycle of charging and discharging repeats and produces a saw tooth waveform or sweep waveform as shown if fig.4.31(b). the frequency of the oscillation depends on the values of  $R$  and  $C$ .

## Applications of CRO

1. Measurement of frequency
2. Measurement of phase of the waveform
3. Measurement of voltage
4. Measurement of current
5. Study of the waveform

### 1. Frequency Measurement using CRO

The unknown frequency can be accurately determined with the help of CRO. A known frequency is applied to horizontal input and unknown frequency to the vertical. A pattern with loop is obtained on the screen by adjusting the various controls. This pattern is called lissajous pattern. The unknown frequency is determined by using number of loops cut by the horizontal line and number of loops cut by the vertical line.

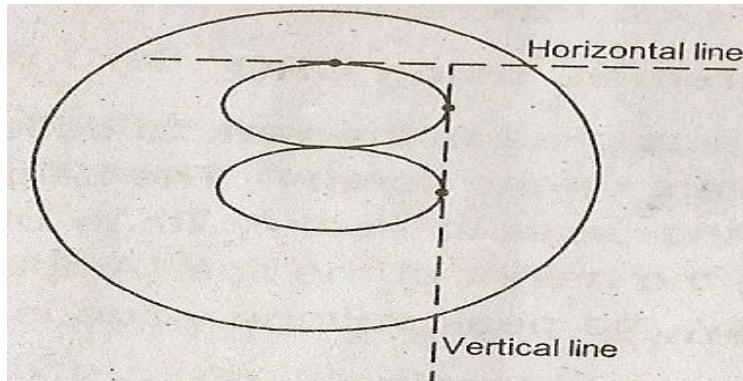


Fig.4.19

Frequency on the vertical plate ( $f_V$ )      number of loops cut by the horizontal line

----- = -----

Frequency on the horizontal plate ( $f_H$ )      number of loops cut by the vertical line

Let us assume that frequency applied to horizontal plates is 1000 Hz. If we draw horizontal vertical line, it cuts horizontal line once and vertical line twice,

$f_V$       number of loops cut by the horizontal line

----- = -----

$f_H$       number of loops cut by the vertical line

$f_V$       1

----- = -----

$f_H$       2

1

$$f_V = 1000 \times \frac{1}{2} = 50 \text{ Hz}$$

2

i.e, unknown frequency is 500 Hz

## 2. Phase Measurement using CRO

To measure, the relative phase angle of two sinusoidal voltages of equal amplitude and same frequency, they are applied to X and Y deflecting plates of a CRO. An ellipse form is obtained. Value A and B are found from the ellipse as shown in fig.4.20

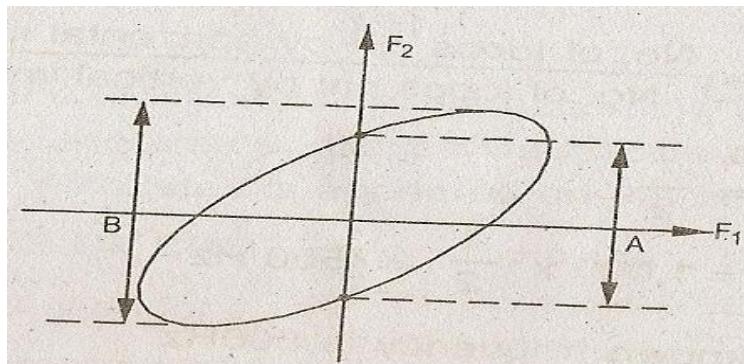


Fig.4.20

The phase angle is given by the expression.

A

$$\sin \alpha = \frac{A}{\sqrt{A^2 + B^2}}$$

B

### 3. Voltage Measurement using CRO

The signal is applied to the vertical deflecting plates. A vertical line appears on the screen. The height of the line is proportional to the peak to peak voltage of the applied voltage. Multiplying the height of the line (amplitude), by the deflecting sensitivity, peak voltage value is obtained.

### 4. Current Measurement using CRO

To measure the current, the signal is passed through a known resistor. The potential developed across the resistor is measured by CRO as explained in voltage measurement using CRO. Then by applying ohm's law, the current is calculated.

V

$$I = \frac{V}{R}$$

I

## Study of Waveforms using CRO

The Signal under study is applied to the vertical input terminal of the CRO. The sweep circuit is set to internal, so that saw tooth wave is applied to the horizontal input. The various control are adjusted to obtain sharp and well defined signal waveform on the screen.

## TYPES OF CRO

There are number of types of oscilloscopes are used for special application, they are

1. Multiple trace oscilloscope
2. Multiple beam oscilloscope
3. Digital storage oscilloscope
4. Sampling oscilloscope

## DUAL TRACE CRO

A dual trace oscilloscope can display two traces on the screen at the same time, allowing comparison of two different signals, such as the input and output of an amplifier. Always consider buying and using a dual trace CRO.

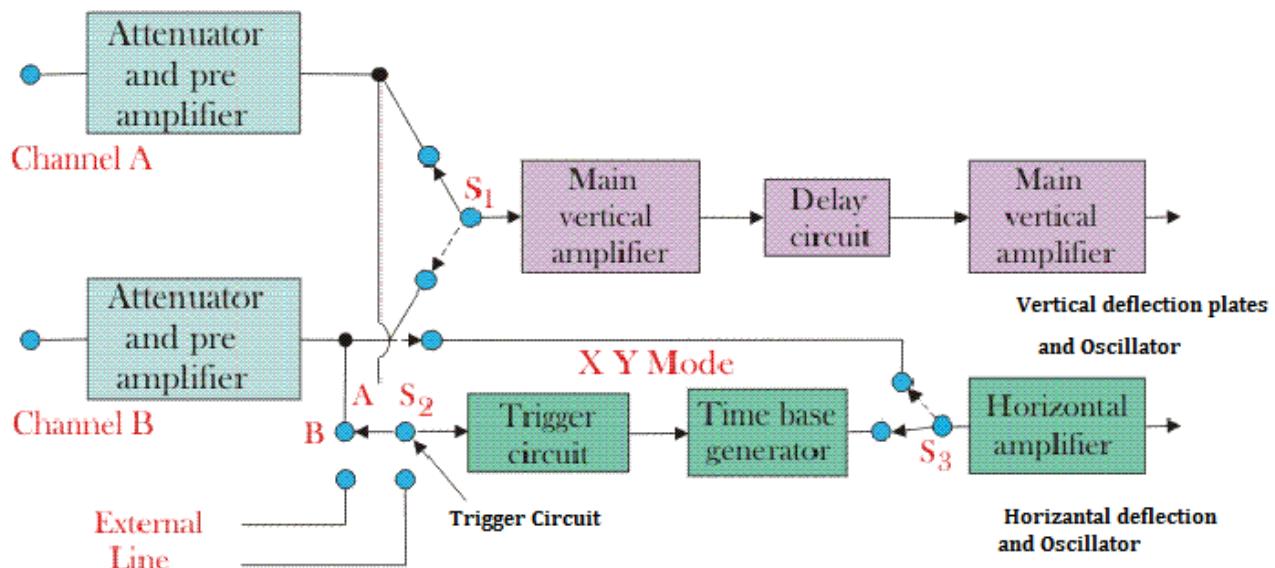


Fig.4.21

The block diagram dual trace CRO is shown in fig.4.21. There are two separate vertical input channels A and B. A separate preamplifier and attenuator stage exist for each channel. Hence amplitude of each input can be individually controlled. After preamplifier stage, both the signals are fed to an electronic switch. The switch has an ability to pass one channel at a time through delay line to the vertical amplifier. The time base circuit uses a trigger selector switch  $S_2$  which allows the circuit to be triggered on either A or B channel. The horizontal amplifier is fed from the sweep generator or the B channel through switch  $S_1$  and  $S_3$ .

The X-Y mode means, the oscilloscope operates from the channel A as the vertical signal and the channel B as the horizontal signal.

When the display mode is in X,Y position, the sweep generator is disconnected and channel B is connected to the horizontal amplifier. When the display mode selector is in the B position the electronics switch connects the input of the main vertical amplifier.

Depending on the selecting of front pane controls several modes of operation can be selected such as channel A only, channel B only, channel A and B as separate traces.

## DUAL BEAM CRO

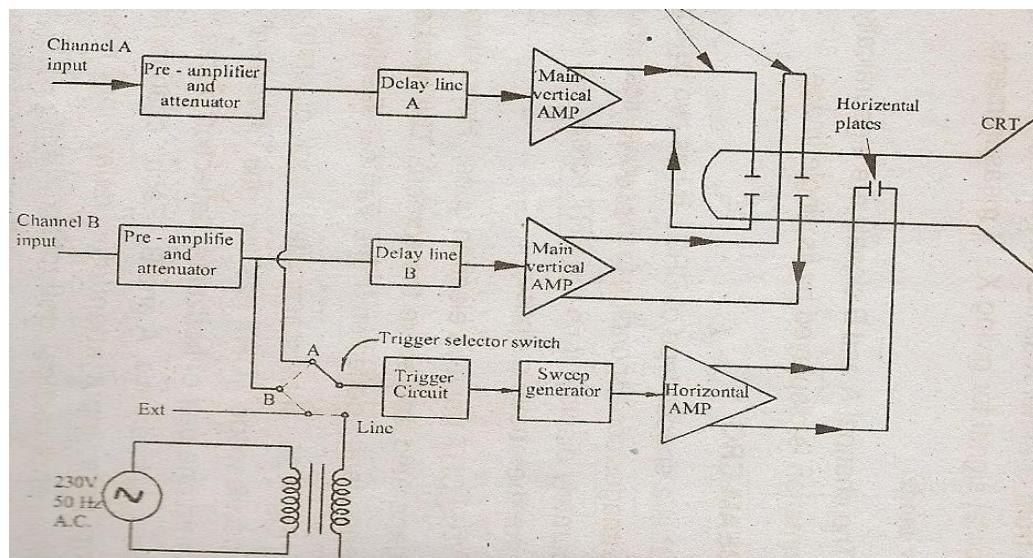


Fig.4.22

It is a special type of CRO. The dual-beam analog oscilloscope can display two signals simultaneously. A special dual-beam CRT generates and deflects two separate beams. Although multi-trace analog oscilloscopes can simulate a dual-beam display with chop and alternate sweeps.

The CTR has two electron guns. Hence two electron beam generated and strikes the screen. The two beam can be independently deflected in the vertical direction.

A dual-beam oscilloscope was a type of oscilloscope once used to compare one signal with another. There were two beams produced in a special type of CRT. It consists of two pairs of vertical deflection plates, one pair of horizontal deflection plates, two electron guns, two preamplifier, trigger circuit and sweep generator. The block diagram of dual beam CRO is shown in fig.4.22. CRT has two pairs of vertical deflection plates and one pair of horizontal deflection plates. There are two identical channels, A and B. Each channel has its own preamplifier, input attenuator, delay line, main vertical deflection plates. The time base generator generates saw tooth voltage, which drives the single set of horizontal plates. Hence both the vertical signals are swept on the screen at the same time. Because of the special arrangements the dual beam CRO is costlier.

### **Comparison of Dual Trace and Dual Beam**

S. No	Dual Trace CRO	Dual Beam CRO
1	Single electron gun is used	Two electron gun is used
2	Common time base is used for channel A and B	They may have common time base or independent time base circuits for two channels.
3	Single set of vertical deflection plates are used.	Separate sets of vertical deflection plates are used.
4	Only one vertical amplifier is used	Two separate vertical amplifiers are used for two channels.
5	Low cost	High cost

### **Digital Storage Oscilloscope**

#### **Introduction**

The digital storage oscilloscope, or DSO for short, is now the preferred type for most industrial applications, although simple analog CROs are still used by hobbyists. It replaces the unreliable storage method used in analog storage scopes with digital memory which can store data as long as required without degradation. It also allows complex processing of the signal by high-speed digital signal processing circuits.

## Block Diagram

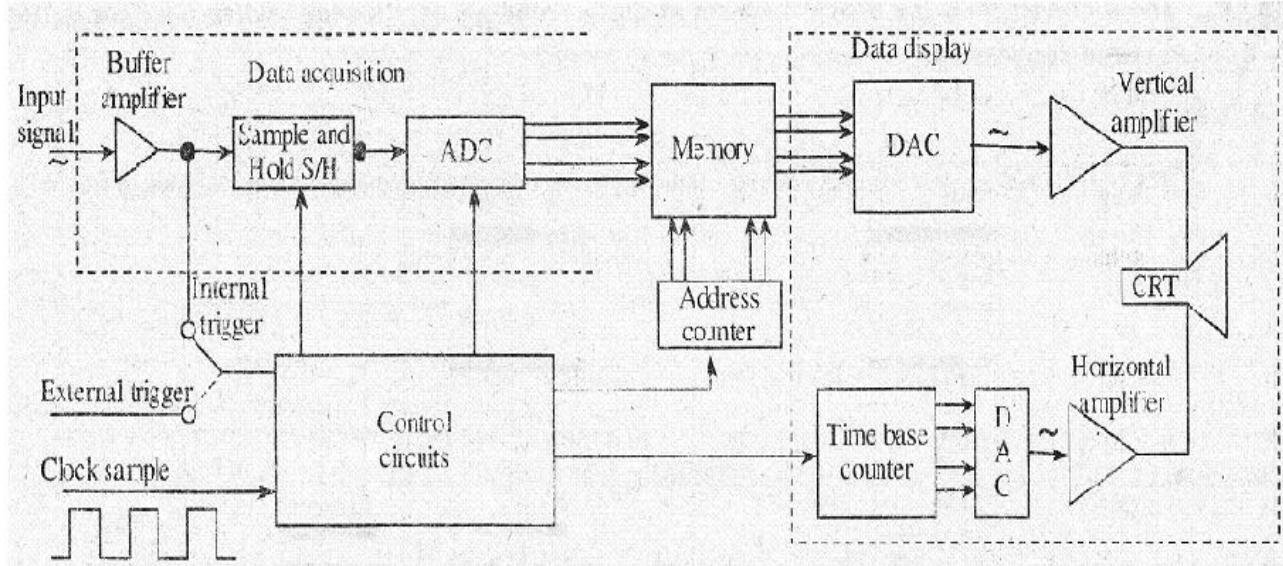


Fig. 4.23

In this oscilloscope the waveform to be stored is digitized and then stored in a digital memory. The conventional cathode ray tube is used In this oscilloscope. The stored image can be displayed indefinitely as long as power is supplied to memory. Once the waveform is digitized then it can be further loaded into the computer and can be analyzed in detail.

The block diagram of digital storage oscilloscope is shown in fig.4.23. The input signal is applied to the amplifier and attenuator section. The attenuated signal is then applied to the vertical amplifier. To digitize the analog signal, the analog to digital (A/D) converter is used. The output of the vertical amplifier is applied to the A/D converter section. The digitized output needed only in the binary form.

Digitizing the analog input signal means taking samples at periodic interval of the input signal. The rate of sampling should be at least twice as fast as the highest frequency present in the input signal. This ensures no loss of information. The rate of sampling rates as high as 100,000 samples per second is used. This requires very fast conversion rates of A / D converter. The sampling rate and memory size are selected depending upon the duration and waveform to be recorded. Once the input signal is sampled, A / D converter digitises it. The signal is then stored in the memory. Once it is stored in the memory, then the data is read out from the memory.

### Mode of operation

This mode means that the oscilloscope can display what happened before a trigger input is applied. This mode of operation is useful when a failure occurs.

The digital storage oscilloscope has three mode of operation.

### **Roll mode**

Very fast varying signals are displayed clearly in this mode. In this mode the input signal is not triggered at all.

### **Store Mode**

This is called refresh. In this case input initiates the trigger circuit. Memory writer cycle starts with trigger pulse. When memory is full, write cycle stops. Then digital to analog converter, the stored signal is converted to analog and displayed. When next trigger is occurs, the memory is **refreshed**.

### **Hold or Save Mode**

This is automatic refresh mode. When new sweep signal is generated by time base generator, the old contents get over written by new one. By pressing hold or save button, overwriting can be stopped and previously saved signal gets locked.

### **Advantages**

1. Has more capacity
2. The characters can be displayed on screen along with the waveform and its information such as minimum, maximum , frequency, amplitude etc.
3. The X-Y plots, B-H curve , P-V diagrams can be displayed.
4. The pretrigger viewing feature allows to display the waveform before trigger pulse.
5. Keeping the records is possible by transmitting the data to computer system where the further processing is possible.

### **Function Generator**

A function generator is a signal source that has the capability of producing different types of waveforms as its output signal. The most common output waveforms are sine-waves, triangular waves, square waves, and saw tooth waves. The frequencies of such waveforms may be adjusted from a fraction of a hertz to several hundred kHz.

## Block Diagram

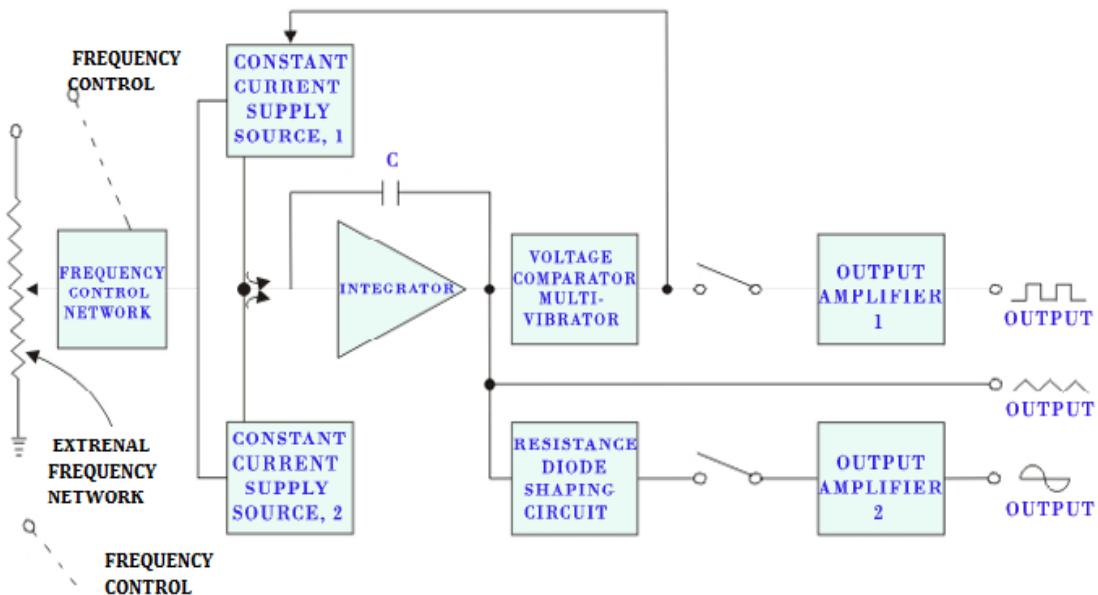


Fig. 4.24

The block diagram of a function generator is given in figure 4.24. In this instrument the frequency is controlled by varying the magnitude of current that drives the integrator. This instrument provides different types of waveforms (such as sinusoidal, triangular and square waves) as its output signal with a frequency range of 0.01 Hz to 100 kHz.

The frequency controlled voltage regulates two current supply sources. Current supply source 1 supplies constant current to the integrator whose output voltage rises linearly with time. An increase or decrease in the current increases or reduces the slope of the output voltage and thus controls the frequency.

The voltage comparator multivibrator changes state at a predetermined maximum level, of the integrator output voltage. This change cuts-off the current supply from supply source 1 and switches to the supply source 2. The current supply source 2 supplies a reverse current to the integrator so that its output drops linearly with time. When the output attains a predetermined level, the voltage comparator again changes state and switches on to the current supply source. The output of the integrator is a triangular wave whose frequency depends on the current supplied by the constant current supply sources.

The comparator output provides a square wave of the same frequency as output. The resistance diode network changes the slope of the triangular wave as its amplitude changes and produces a sinusoidal wave with less than 1% distortion.

## UNIT V

### **TRANSDUCERS, SENSORS AND TEST INSTRUMENTS**

#### **TRANSDUCER**

A device which converts one form of energy into another form of energy is called transducer. The input quantity for most instrumentation system is non-electrical. In order to use electrical methods and techniques for measurement, the non-electrical quantity is converted in to a proportional electrical signal by a device called transducer.

#### **Classification of Transducers**

(1) Based on physical phenomenon

- a) Primary Transducer
- b) Secondary Transducer

(2) Based on the power type

- a) Active Transducer
- b) Passive Transducer

(3) Based on the type of output

- a) Analog Transducer
- b) Digital Transducer

(4) Based on electrical

- a) Resistive Transducer
- b) Capacitive Transducer
- c) Inductive Transducer
- d) Photoelectric Transducer
- e) Photovoltaic Transducer

(5) Based on non- electrical phenomenon

- a) Linear Displacement Transducer (LVDT)
- b) Rotary Displacement Transducer (RVDT)

#### **1. Primary Transducer**

The primary transducers are detectors which sense a physical phenomenon. The example is 'thermocouple'. The thermocouple's hot junction senses the radiant heat energy and directly converts it to an analogous electrical output which is a voltage.

## 2. Secondary Transducer

The output signal from the primary transducer is converted subsequently into a useable output by the 'strain gauges' and it is known as secondary transducers.

## 3. Active Transducer

The active transducers are also known as self generating type transducers. It develops their own voltage or current. The example is 'piezoelectric transducers'.

## 4. Passive Transducer

The passive transducer is known as externally powered transducer. This transducer drives the power required for energy conversion from an external power source.

## 5. Analog Transducer

The analog transducer converts the input physical phenomenon into an analogous output, which is continuous function of time. The examples for this type are 'LVDT' 'thermistor' etc.

## 6. Digital Transducer

The digital transducer converts the input physical phenomenon into an electrical output which is in the form of pulses

## **STRAIN GAUGE**

Strain gauge is a passive transducer that uses the variation in electrical resistance in wires to sense the strain produced by a force on the wires. The principle is used in strain gauge. Strain can be measured more easily using variable resistance transducers.

Stress (force / unit area) and strain (elongation or compression / unit area) in a member or portion of any object under pressure is directly related to the modulus of elasticity. Since strain can be measured more easily by using variable resistance transducers.

There will be a change in the value of resistivity of the conductor, when it is subjected to strain. That property is called ' piezo- resistive effect '. The strain gauge employs piezo-resistive effect.

## **Types of Strain Gauges**

The electrical strain gauges are of different types. They are

- a) Wire strain gauge

- b) Foil strain gauge, and
- c) Semiconductor strain gauge

## Wire Strain Gauge

Wire strain gauge (or) Resistance wire gauge is used in two basic forms. They are

- a) Bonded resistance wire strain gauge
- b) Unbonded resistance wire strain gauge

### Bonded Resistance Wire Strain Gauge

Different types of Bonded Resistance Wire Strain Gauge is shown in fig. 5.1. A bonded wire strain gauge consists of a grid of fine resistance wire of diameter of about  $25\mu\text{m}$ . The wire is connected to a base. The base may be a thin sheet of paper or a very thin Bakelite sheet. The wire is covered with a thin sheet of material so that it is not damaged mechanically. The base is bonded to the structure with an adhesive material. It acts as a bonding material. It permits as good transfer of strain from base to wire.

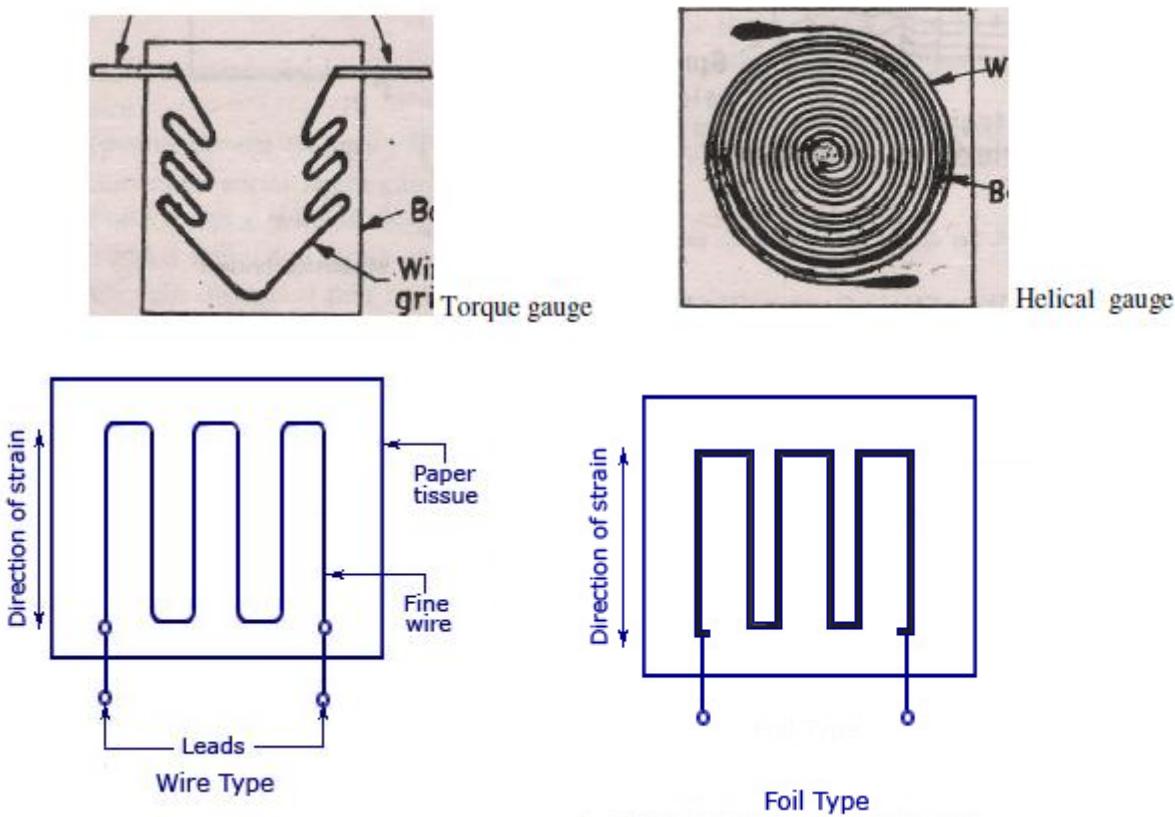


Fig.5.1

## Working

When the load is applied to the surface, the surface is strained. Since the strain gauge is bonded with the surface it is also strained. The resistance of the wire changes on account of the changes in length and diameter of the wire. This change in resistance is proportional to the applied strain and it is measured by very sensitive instrument. There are two circuit arrangements for strain measurement.

1. Potentiometric strain gauge
2. Wheat stone bridge strain gauge

### 1. Potentiometric Strain Gauge

The circuit arrangement of potentiometric strain gauge is shown fig. 5.2. This system is easily affected by A C interference. Hence precautions must be taken. This potentiometric circuit is also called 'ballast circuit'. It consists of D.C voltage  $E_1$  applied across the resistance network  $R_s$  and  $R_g$  in which one or both can be strain gauges. The voltage  $E_0$  that appears across  $R_g$  is given as

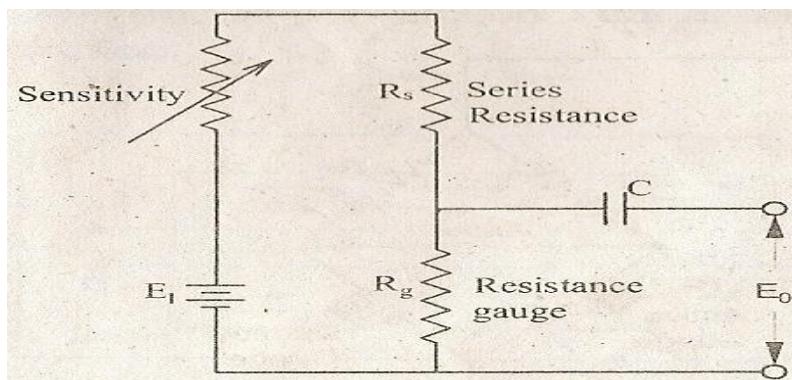


Fig.5.2

$$E_0 = E_1 \cdot \frac{R_g}{R_s + R_g}$$

Hence the output voltage is changed, when gauge resistance changed. That is the output voltage is proportional to strain gauge resistance.

### 1. Wheatstone Bridge Type Strain Gauge

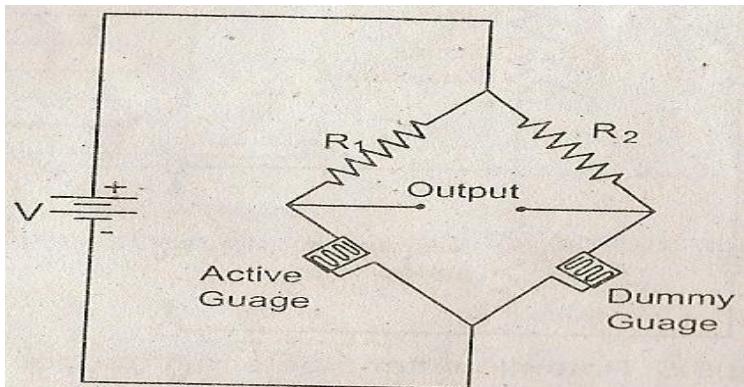


Fig.5.3

The strain gauge is normally used in wheatstone bridge arrangement, in which the gauge forms one arm of the bridge . A simple DC arrangement for bridge type strain gauge is shown fig.5.3

In this arrangement, only one of the gauges is an active element producing an output proportional to the strain. The other(dummy) gauge is not strained, but simply balances the bridge. The purpose of the dummy gauge is to cancel out any temperature related resistance variation in the active gauge. When both the gauges are affected by same temperature related the resistance variations appear in both the gauges. Therefore the bridge remains in balanced condition . In this way error due to temperature variations can be eliminated. If the two resistors  $R_1$ and  $R_2$  have negligible temperature coefficients, the bridge retains its balance under conditions of no-strain, at any temperature within its operating range.

### **Unbonded Type Strain Gauge**

An unbonded strain gauge is used to measure force, acceleration and pressure. It consists of a wire stretched between two points in an insulating medium as shown fig 5.4(a). The diameter of the wire used about  $25\mu\text{m}$ . The wires are kept under tension, so that there is no sag and no free vibration.

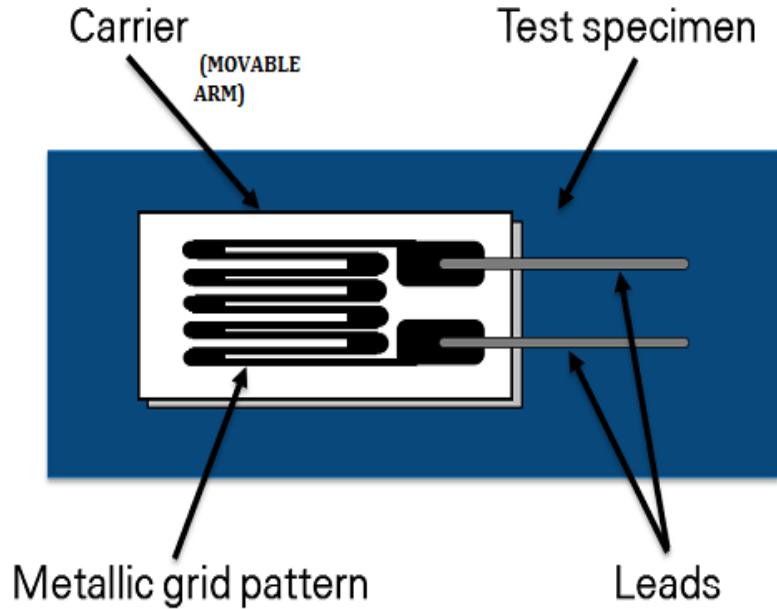


Fig.6.4(a)

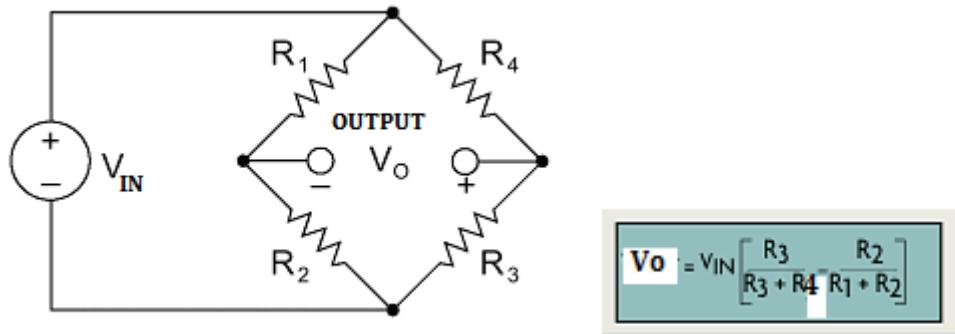


Fig.5.4(b)

Unbonded strain gauge are usually connected in bridge circuit. The bridge is balanced with no load applied as shown fig.5.4(b).

When an external load is applied, the resistance of the strain gauge changes, causing an unbalance of the bridge circuit resulting in an output voltage. This voltage is proportional to the strain. A displacement of the order of 50  $\mu\text{m}$  can be detected with these strain gauges.

### Application of Strain Gauge

- 1) For analyzing the dynamic strain in complex structures such as
  - i) stress and strain in bridges
  - ii) automobiles

- iii) roads etc
- 2) Measurement of tension, torque, stress in structures
- 3) To measure force by strain produced in load rings.

### **Photo Electric Transducer**

These transducers convert the light energy into electrical signal. These transducers make use of the properties of a photo emissive cell or photo tube or photo conductive cell.

Photo emissive cells one in which light energy causes the emission of electrons from a metal surface, thereby providing an electric current. The test circuit is shown in fig.5.5(a). The anode and cathode are placed in high vacuum glass envelope. When voltage is applied between the anode and cathode, the current produced by the photo tube is directly proportional to the amount of light falling on the cathode. The current produced by the tube is small in the range of microampere. Hence the photo tube is usually connected with electron multiplier to provide higher output.

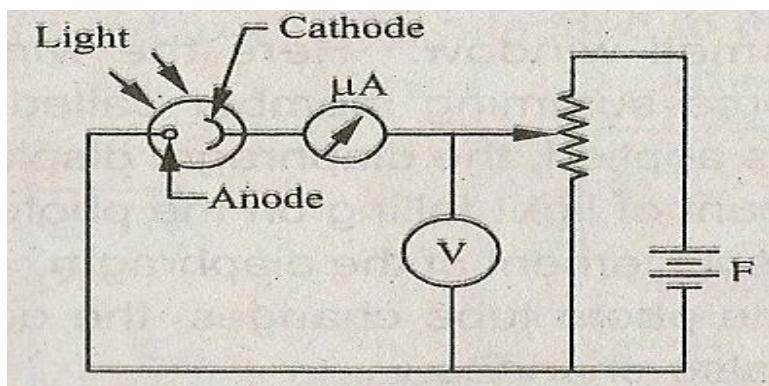


Fig.5.5(a)

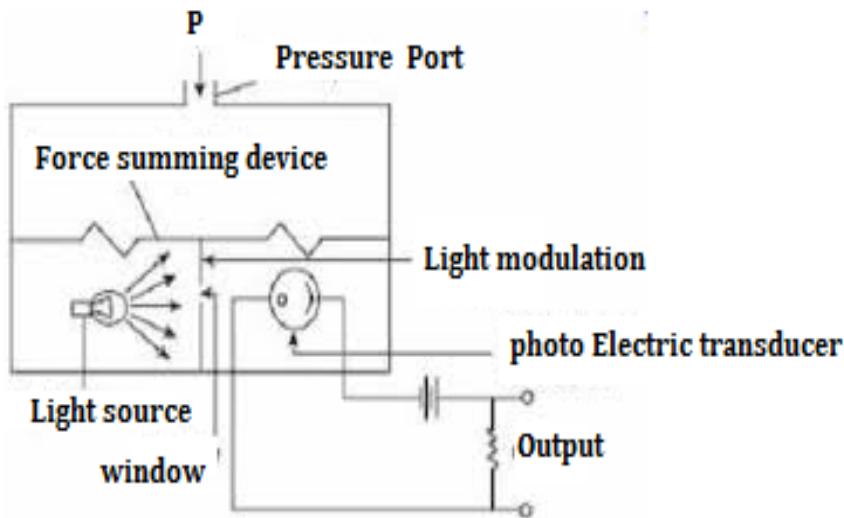


Fig .5.5(b)

In fig.5.5(b) photo tube and a light source separated by a small window. Here the window gap is controlled by a force summing member called diaphragm. When a pressure is applied the diaphragm displaces from its position. The amount of light falling on the photo tube varies according to the displacement of the diaphragm. Hence when a light falling on the photo tube changes, the output current of the photo tube also changes.

### Advantages

1. Have higher efficiency.
2. Can be used for both static and dynamic condition.

### Disadvantages

1. It does not respond to high frequency light variations.
2. It requires a large displacement of the force summing member.

### Linear Variable Differential Transformer (LVDT)

Linear variable differential transformer (LVDT) is a passive inductive transducer. It is used to translate the linear motion into electrical signal.

#### Construction

The fig.5(a) shows the Linear variable differential transformer. It consists of a single primary winding (P) and two secondary windings  $S_1$  and  $S_2$  wound on a hollow cylindrical former. The secondaries have an equal number of turns but they are connected in series opposition so that the emfs induced in the coils opposes each other. The primary winding is

connected to an AC source. A moveable soft iron core slides inside the former for displacement. The displacement to be measured is applied to an arm attached to the core.

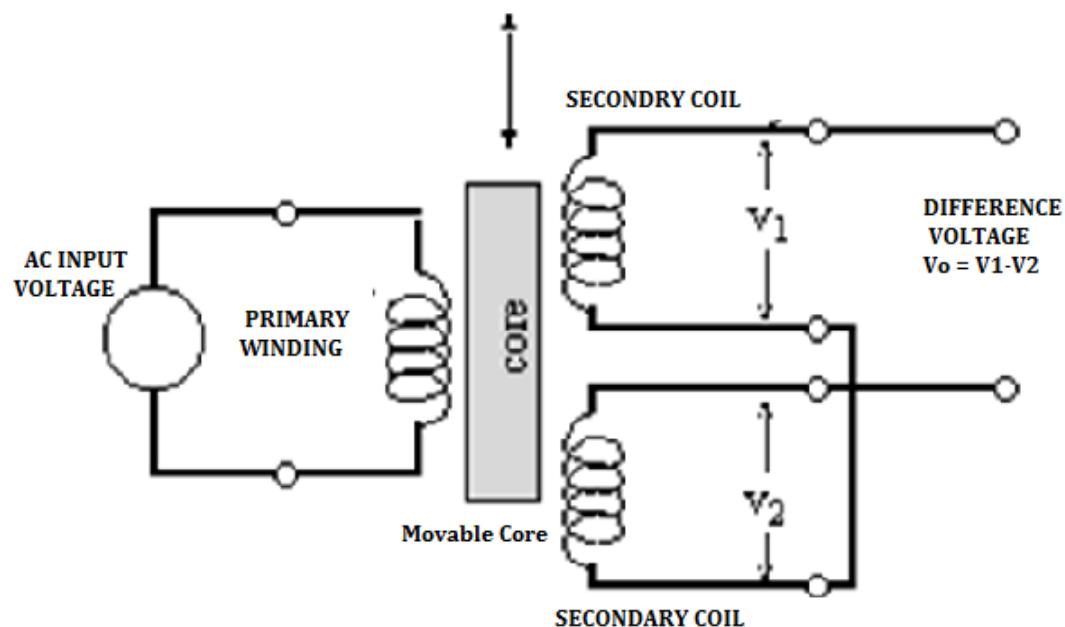
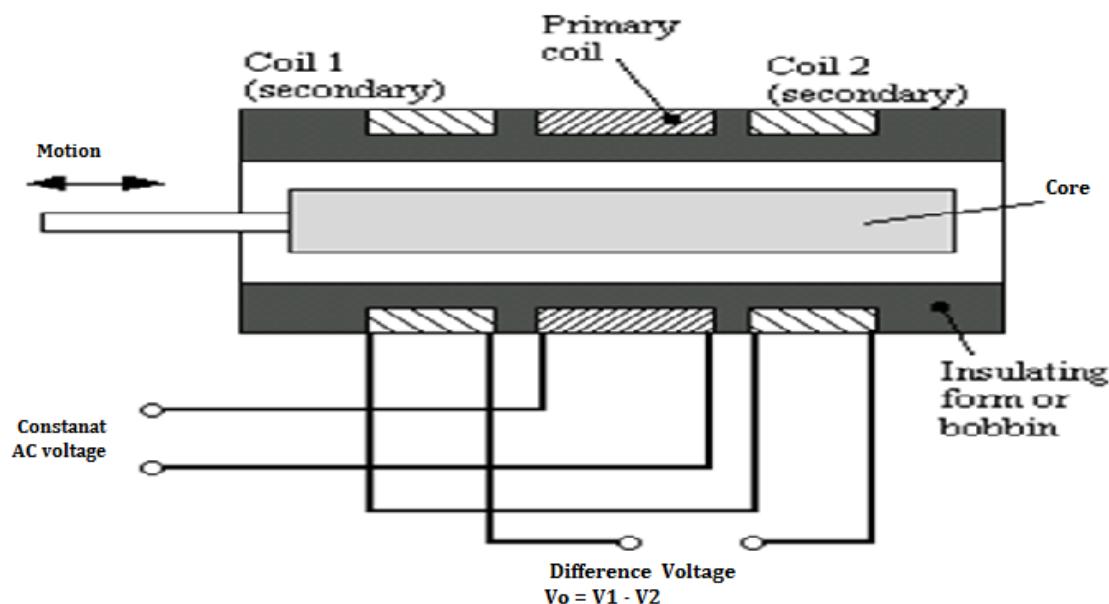


Fig.5.6 (a)

## Operation

When the core is at the center of the former the induced emfs in the secondaryes are equal i.e  $E_{s1} = E_{s2}$ . Since these two emfs oppose each other, the output voltage will be  $E_{s1} - E_{s2} = 0$

i.e,  $E_0 = \text{zero at null position}$

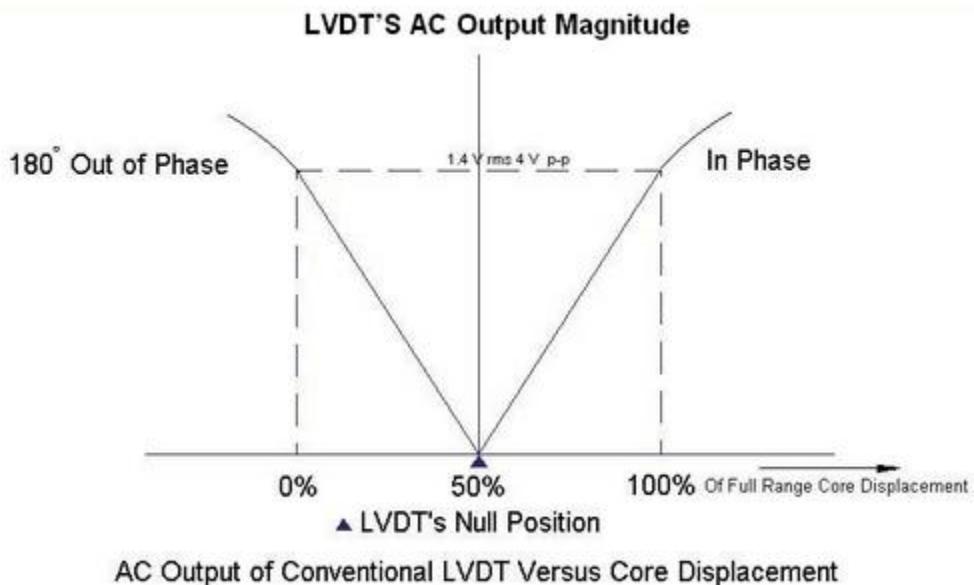


Fig.5.6 (b)

When an externally applied force moves the core to the left hand position ,more magnetic flux links the winding  $S_1$  and less with the winding  $S_2$  hence the output voltage  $E_{s1}$  is greater than  $E_{s2}$ . Therefore the magnitude of the output voltage of the secondary is  $E_{s1} - E_{s2}$  and it is in phase with the  $E_{s1}$ .

i.e.,  $E_0 = \text{positive when the core is at left of the null position.}$

Similarly when the core is moved to the light of the null position. the flux linking with the winding  $S_2$  becomes greater than that link with  $S_1$ . This result in  $E_{s2}$  becoming greater than  $E_{s1}$ . Therefore the output voltage in this case is  $E_0 = E_{s2} - E_{s1}$  and it is in phase with  $E_{s2}$ .

This output voltage is  $180^\circ$  out of phase with the output when the core is at left end.

i.e.,  $E_0 = \text{negative when the core is at right of null position}$

i.e , $E_0 = \text{positive when the core is moved to the light right of the null position.}$

Phase angle =  $180^\circ$

Fig.5.6 (b) shows the variation of output voltage against displacement for various position of the core.

Therefore the amplitude of output voltage depends upon the distance of core moves. The polarity or phase of the voltage indicates the direction of motion

### **Advantages Of LVDT**

1. Linear for displacement up to 5mm
2. Infinite resolution
3. High output
4. High sensitivity
5. Ruggedness
6. Less friction
7. Low hysteresis error
8. Low power consumption.

### **Disadvantages of LVDT**

1. Requires large displacements for appreciable differential output
2. Affected by external magnetic field.
3. Temperature affects the performance.

### **Applications of LVDT**

1. LVDT is used in all applications where displacement ranging from fraction of mm to a few cm has to be measured.
2. It is used to measure force ,weight ,pressure thickness etc

## LOAD CELL

Load cell is a transducer that is used in measurement of load acting linearly along a string or a wire. Load cells are used to measure extremely heavy loads.

### Strain Gauge Load Cell

Strain gauge load cell is an electro mechanical transducer which converts changes in force or weight into changes in voltage. This change in voltage is a measure of applied force or load to the cell

### Construction

Fig.5.7 (a) shows the construction of strain gauge load cell. It consists of strain gauge A and B , a lengthy steel bar, and base. The steel bar is used as an active element. The weight of the load applies a particular stress to the bar. The amount of strain which results in the bar different values of applied stress is determined. The strain may be used as a direct measure of the stress causing it.

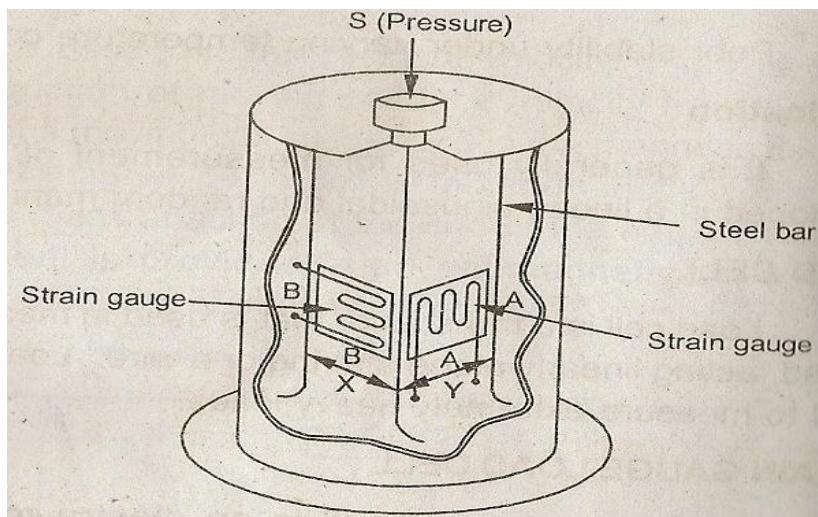


Fig.5.7(a)

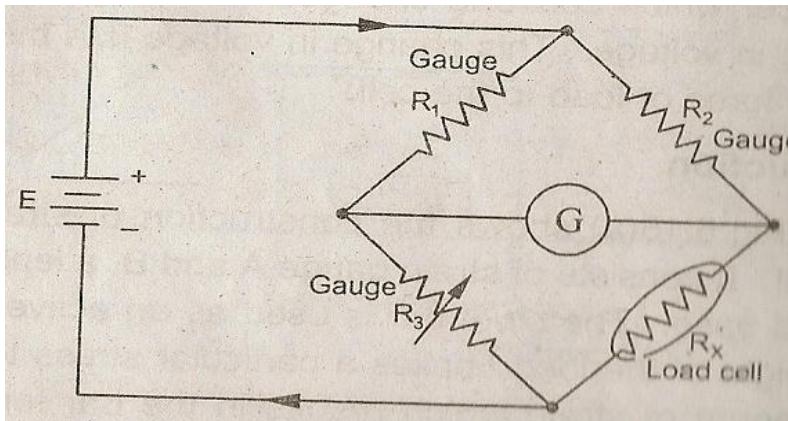


Fig.5.7(b)

### Working

When the stress is applied along the direction S the steel bar experiences a compression along that axis and an expansion along the x and y axes. As a result gauge A experiences a decrease in resistance, while gauge B undergoes an increase in resistance. These two gauges A and B and the gauges on the two remaining sides of the steel are connected to form a bridge circuit are shown fig.5.7 (b)

At first ,make the circuit under balance condition by adjusting the resistance R3. After Appling load, the bridge goes to unbalance condition. Varying the resistor R3 the bridge is again brought to balance condition.

Then

$$R_x = \frac{R_2 R_3}{R_1}$$

Thus the unknown resistance Rx can be calculated. Where R<sub>1</sub>, R<sub>2</sub> ,R<sub>3</sub>, are the known resistance of the strain gauge.

### Thermocouple

Thermocouple is the most commonly used electrical device for temperature measurement. A thermocouple consists of two dissimilar material wire joined together at one end forming a hot junction. The two wires are terminated at the other end. The junction is called reference or cold junction. It is shown in the fig.5.8. The cold junction is maintained at a known constant temperature called reference temperature.

The scientist Thomson see back discovered that when a temperature different exists between the hot and cold junction, an emf is produced. This emf causes a current in the circuit.

When a meter or a recorder is connected to the cold junction as shown in fig.5.8. The meter indication will be proportional to the temperature difference between the hot junction and cold junction. This thermo electric effect caused by constant potential at the junctions. It is known as the see back effect. The magnitude of emf depends on the wire material used and also on the temperature difference between the junctions.

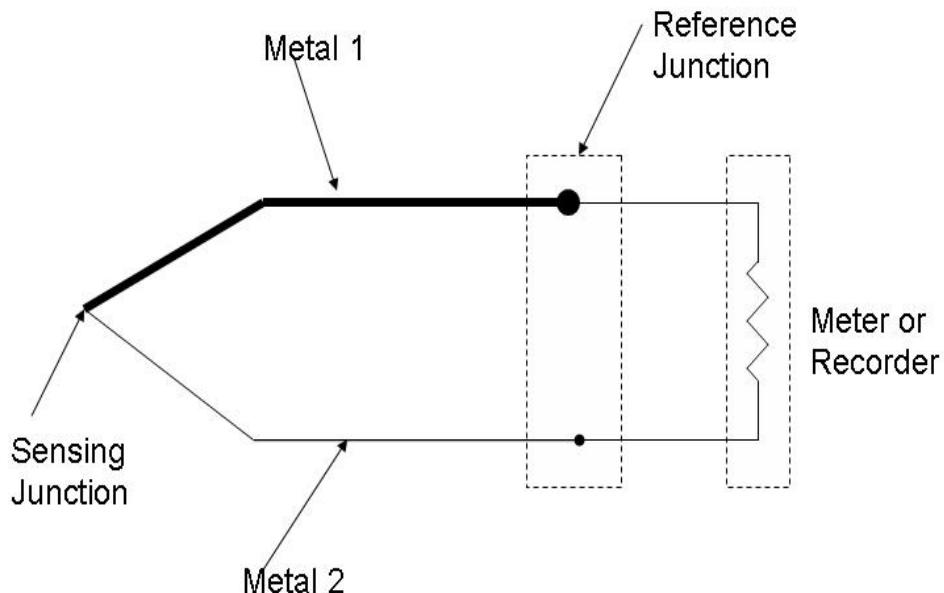


Fig.5.8

### Operation

Fig.5.9 shows the temperature measurement using thermocouple. Hot junction is placed near the heat source, whose temperatures to be measured. A sensitive millvolt meter is directly connected across the cold junction. Now the deflection of the meter is directly proportional to the difference in temperature between the hot junction and the cold junction. Then the voltmeter reading is converted into temperature.

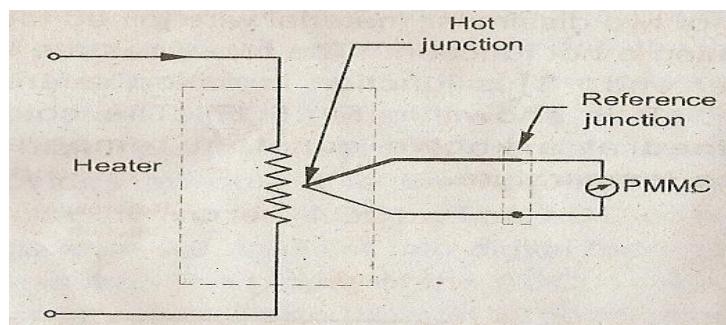


Fig.5.9

### Advantages

1. Rugged construction

1. Can make measurement over a very wide range.
2. No bridge circuit is required.
3. Good accuracy.

### **Disadvantages**

1. Reference temperature can be kept constant.
2. As the output voltage is less than 10 mV a very sensitive meter is required.

### **Applications**

1. Used to measure temperature range from  $270^{\circ}\text{C}$  to  $2800^{\circ}\text{C}$ .
2. Used in industrial furnaces.

### **Thermister**

Thermistors or thermal resistors are THERMally sensitive resistor. It means its resistance changes with temperature.

### **Thermister - symbol and samples**

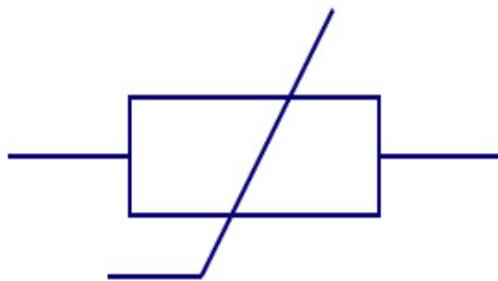


Fig.5.10

The type of thermistor are

1. Negative temperature co-efficient resistor (NTC)
2. Positive temperature co-efficient resistor (PTC)

## NTC Thermistor

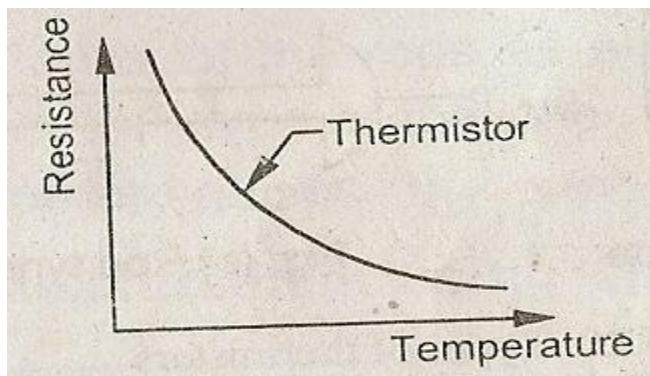


Fig.5.11

In this type the resistance of thermistor decreases with increase in temperature. The relation between resistance and temperature of a NTC thermistor is shown in fig. Fig.5.11.

### Positive Temperature Co-efficient Resistor (PTC)

In this type the resistance of thermistor increases with increase in temperature. These are usually made from barium titanate. The characteristics more complex than NTC thermistors. These thermisors are designed for temperature between  $20^{\circ}\text{C}$  and  $200^{\circ}\text{C}$ .

## Construction

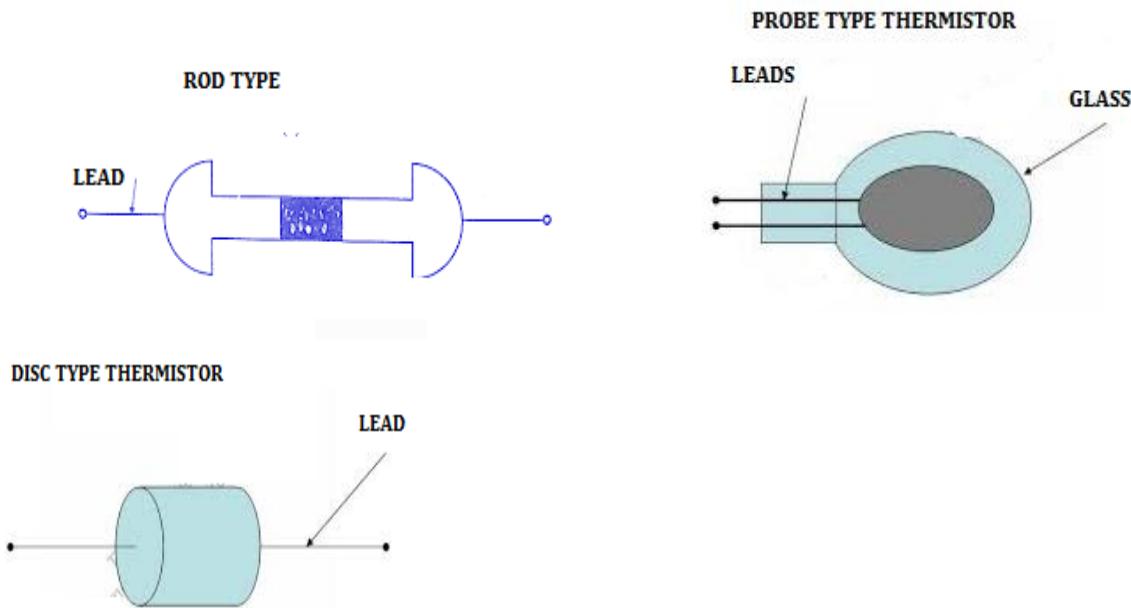


Fig.5.12

Thermistor are composed of sintered mixture of metallic oxides such as manganese, nickel, cobalt, copper, iron and uranium. They are available in verity of sizes and shapes. Thermistor may be in the form of beads, probes, rods and discs. Some of the commercial form are shown in fig.5.12.

### **1. Bead Thermistor**

It is the smallest thermistor in the form of beads with a diameter of 0.15 mm of 1.25 mm. This is the most familiar type of thermistor usually glass coated as shown in fig.5.20(a)

### **2. Probe Thermistor**

Beads may be sealed in the tips of solid glass rod to form probes as shown in the fig.5.20(b). Glass probes have a diameter of about 2.5mm. The probes are used for measuring the temperature of liquids.

### **3. Disc Thermistor**

Discs are made by pressing material under high pressure into cylindrical flat shapes either a diameter ranging from 2.5 mm to 25 mm and a thickness of 0.25 mm to 0.75 mm. These are sintered and coated with silver on two flat surfaces as shown in fig.5.20(c). These thermistors have the resistance of from 1 Ohm to 1 Mega ohm.

### **4. Washer Type Thermistor**

They are just like disk thermistor but they have a hole in the center. This hole is suitable for mounting on a bolt ,as shown in fig.5.20 (d).

### **5. Rod Type Thermistor**

They are usually like long cylindrical unit with 4.25 mm diameter and 12.5 to 50 mm long. Leads are attached to the ends of the rods as shown in the fig.5.13. Tthe advantage of this type is that they produce high resistance under moderate power.

## Temperature Measurement Using Thermistor

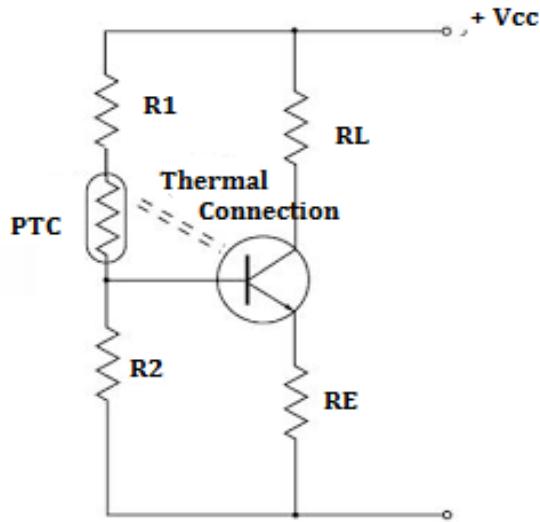


Fig.5.13

Initially the thermistor is placed near the environment whose temperature is to be measured. Then the thermistor is connected in a series simple circuit consisting of battery and micro ammeter as shown in fig.5.13. Any change in temperature causes a change in resistance of the thermistor. Hence the circuit current changes. By directly calibrating the micro ammeter in terms of temperature, we can measure the temperature.

### Advantages

1. Low cost
2. High sensitivity
3. Smaller in size
4. Good stability
5. High output signal

### Disadvantages

1. Not suitable for high temperature measurement.
2. Requires external power supply.

### Applications

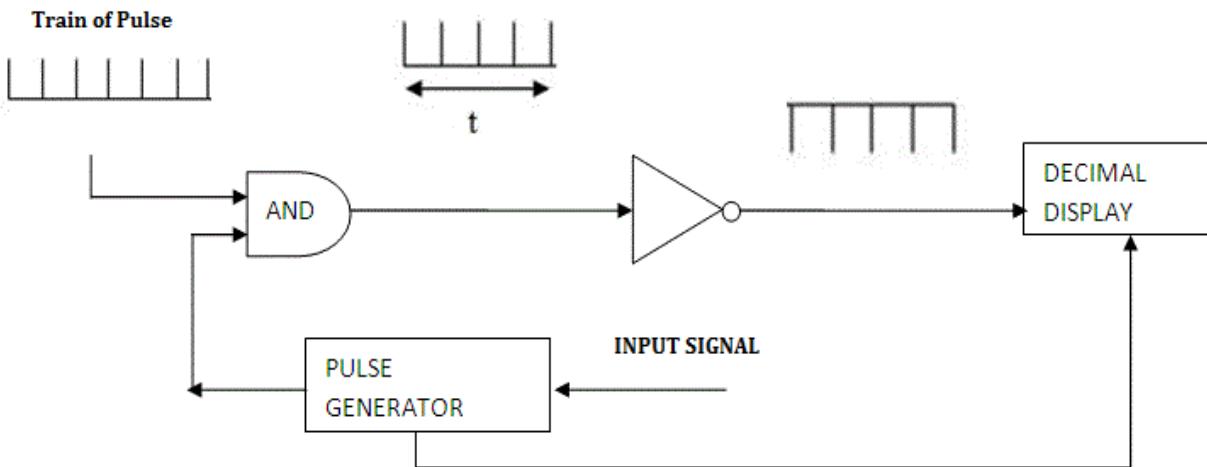
1. Used for measurement and control of temperature.
2. Used for providing time delay.
3. Used as temperature compensation.
4. Used for measuring thermal conductivity.
5. Used for measurement of level, flow and pressure of liquid.
6. Used for measuring power at high frequencies.

## **DIGITAL VOLTMETER (DVM)**

Digital voltmeter (DVM) convert the analog signal into digital signal and display the voltage to be measured as discrete numerical instead of pointer deflection ,on the digital displays. Such voltmeters can be used to measure the A.C and D.C voltages. They are also used to measure the quantities like pressure ,temperature, stress etc using proper transducer and signal conditioning circuit.

### **Block Diagram of DVM**

The block diagram of a simple volt meter is shown in fig.5.14. The unknown voltage signal is fed to the pulse generator which generates a pulse whose width is directly proportional to the input unknown voltage. The output of the pulse generator is applied to one leg of an AND gate. The input signal to the other leg of the AND gate is a train of pulses. The output of the AND gate is thus a positive trigger train of duration 't' seconds (in AND gate when both the inputs are high, the output is high). The inverter converts the positive trigger train from the AND gate into a negative trigger train. The counter counts the number of triggers in 't' seconds which is proportional to the voltage under measurement. Thus the counter can be calibrated to indicate voltage in volts directly. Thus we see that the DVM described above is an ADC which converts an analog signal into a train of pulse.



**Fig.5.14**

### Types of DVM

Digital voltmeters are classified mainly based on the analog to digital conversion. they are

1. Integrated type DVM
2. Successive approximation type DVM
3. Ramp type DVM
4. Potentiometric type DVM
5. Continuous type balance type DVM.

### Digital Frequency Meter (Or) Counter

A digital instrument which is used to measure the frequency of any periodic waveform is known as frequency meter. It can perform a wide range of functions, such as frequency measurement, period measurement, ratio of frequencies measurement, time interval measurement etc.

The block diagram of a basic frequency meter is shown in fig.5.26. The frequency measurement is performed by totaling the number of input cycles for a known period of time. The resulting total count is proportional to the unknown input frequency. The time reference is obtain from a precision, high stability quartz crystal oscillator along with a Schmitt trigger and a set of dividers

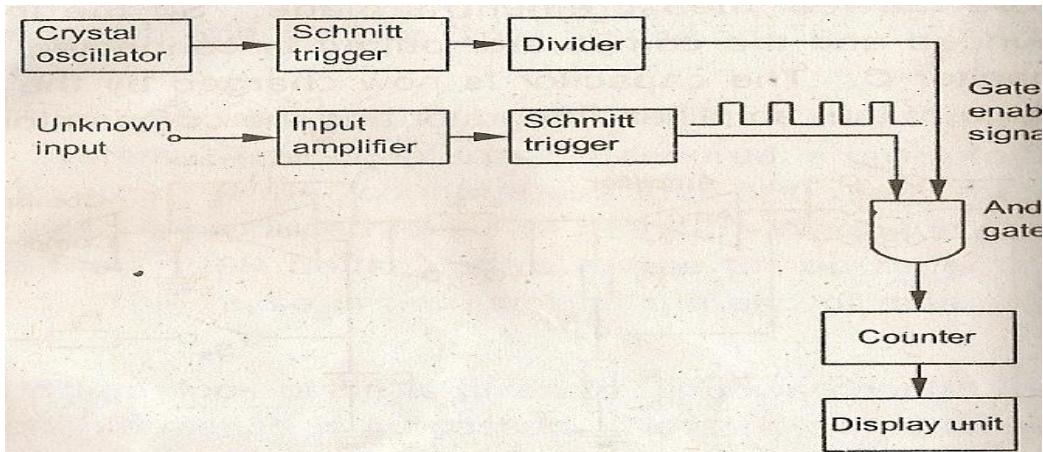


Fig.5.16

The input signal is amplified and applied to a Schmitt trigger circuit so that it will provide uniform pulse. This pulse train is given to the counter through an AND gate and this gate is controlled by the time reference. The number of pulses added in the counter for the selected time period 't' represents the input signal frequency. Finally it is displayed on the numeric display device at the output of the counter. The counted frequency is displayed on the display devices for a finite time until a new sample is taken. The display time of the frequency measurement is determined by the sample rate control. After resetting the counter, next measurement cycle is initiated by the sample rate control.

For example, if the time reference is one second and the input signal is a 50Hz square wave, at the end of 1 second the counter will have counted up to 50 pulses which is exactly the frequency of the input signal.

### Digital Multimeter

Digital Multimeter is used to measure AC voltage, AC current, DC voltage and DC current, resistance etc. Fig.5.17 shows the block diagram of digital multimeter. The basic meter of digital multimeter is always a DC voltmeter.

In AC voltage mode, the applied input is fed to the precision AC/DC converter through a calibrated, compensated attenuator. the precision AC/ DC converter is nothing but a full wave rectifier circuit followed by a ripple reduction filter. The resulting DC voltage fed to the ADC and the display system.

In the AC current mode, the drop across the shunt is converted into DC by means of AC/DC converter. Then the D.C output is fed to ADC and displayed in the display system in digital form.

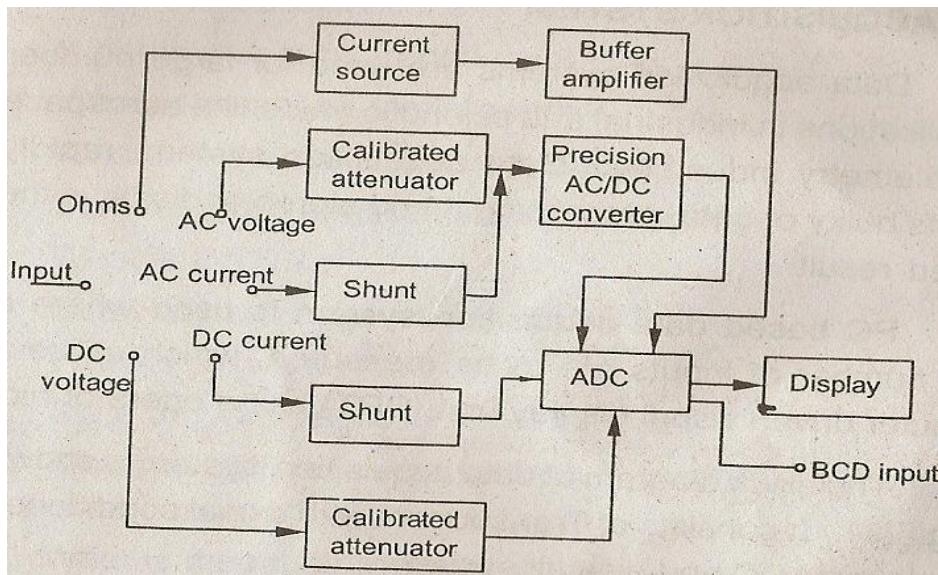


Fig.5.17

In the DC current mode, they drop across an internal calibrated shunt is measured directly by the ADC and is displayed.

In DC voltage mode, the applied input is fed to the ADC through a calibrated compensated attenuator and is displayed.

In resistance mode, the meter includes a precision low current source that is applied across the unknown resistance which gives a DC voltage drop. This voltage is converted into digital signal and read out in ohm.

### Data Acquisition System

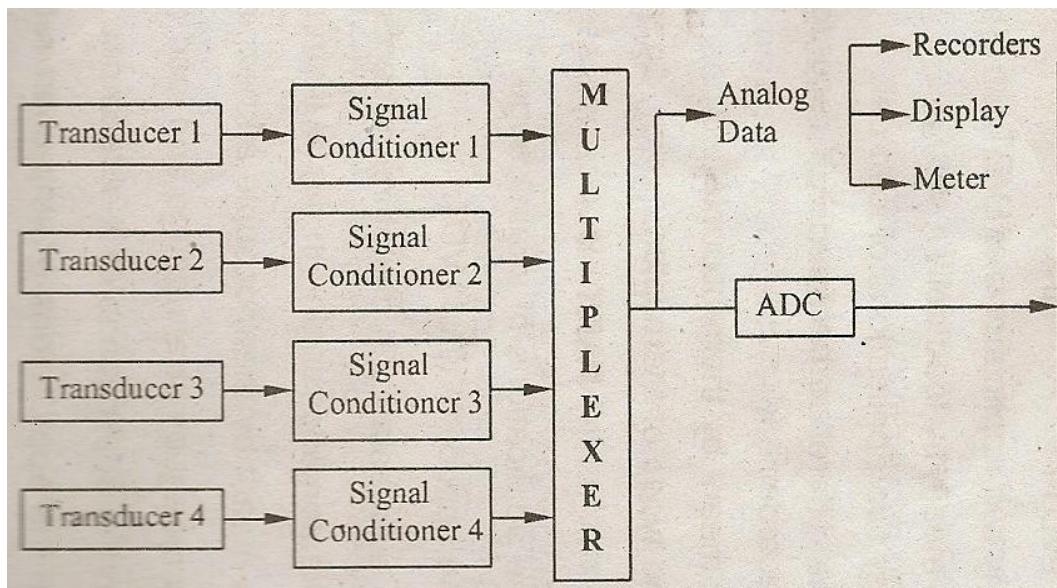


Fig.5.18

Data acquisition systems are used for large number of applications in industrial and scientific areas like aerospace and telemetry industries. Data acquisition system, rapidly collects bulky of data ,processing it and stores or displays the desired result.

PC based data acquisition system is used where a large number of inputs are to be measured, which utilize a computer driven visual display unit (CRT) as an operator aid.

The block diagram of data acquisition system is shown in fig.5.18. It consists of transducers with signal conditioner, multiplexer, ADC and display system.

Transducer converts physical parameters such as temperature, pressure, acceleration, weight, displacement etc., into electrical signal which are acceptable by the acquisition system. Signal conditioner amplifies the transducer output signal.

The signals are sequentially selected by the multiplexer. The selected signal is given to the input of the ADC. The ADC converts analog signal to digital signal, which is stored in the computer's memory for data processing.