CSPC
ASSIGNMENT-6

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Linear Recurrence - Worksheet

1) Solve the following recurrence problem.

(i) $a_n = a_{n-1} + n$

$a_n - a_{n-1} = n$ — ①

Homogeneous part:

$a_n - a_{n-1} = 0$

Let $a_n = A\alpha^n$

$A\alpha^n - A\alpha^{n-1} = 0$

$A\alpha^{n-1}(\alpha - 1) = 0$

$\Rightarrow$ $\boxed{\alpha = 1}$ is characteristic equation.

$a_n(h) = A(1)^n = A.$

$a_n(p)$: $f(n) = n.1^n$

Assume $a_n(p) = n(P_0 + P_1 n)(1)^n$

Substitute in eq ①,

$n(P_0 + P_1 n) - (n-1)(P_0 + P_1(n-1)) = n$

equating coefficients of 'n'

$P_0 - P_0 + P_1 + P_1 = 1$

$2P_1 = 1 \Rightarrow P_1 = 1/2$

Now by equating constants

$P_0 - P_1 = 0$

$P_0 = P_1 = 1/2$

$a_n(h) = A$

$a_n(P) = \frac{n}{2}(1+n)$

$a_n = a_n(h) + a_n(P)$

$a_n = A + \frac{n(n+1)}{2}$

ii) $f(n) = f(n/2) + n$

$a_n = a_{n/2} + n$

$a_n(h)$:

$a_n - a_{n/2} = 0.$

Let $a_n = A\alpha^n$.

$A\alpha^n - A\alpha^{n/2} = 0$

$A\alpha^{n/2}(\alpha^{n/2} - 1) = 0.$

$\alpha^{n/2} - 1 = 0$ → characteristic equation

$\alpha^{n/2} = 1$

$\alpha = 1$

$n = 0$
Not possible for all 'n'.

$\boxed{a_n = A}$

$a_n(p)$:

$f(n) = n$

$= n(1)^n$

$= n(P_0 + P_1 n)(1)^n.$

$a_n(p) = n(P_0 + P_1 n)$

2) $a_n - 3a_{n-1} - 4a_{n-2} = 4^n$.

Let $a_n = A\alpha^n$

$a_n(h)$: $A\alpha^n - 3A\alpha^{n-1} - 4A\alpha^{n-2} = 0$

$A\alpha^{n-2}[\underbrace{\alpha^2 - 3\alpha - 4}] = 0$.

$(\alpha+1)(\alpha-4) = 0$.

$\alpha = -1, 4$

$a_n(h) = A_1(-1)^n + A_2(4)^n$

$a_n(p)$:

Assume $a_n = nP_0 \cdot 4^n$

Substitute,

$nP_0 4^n - 3(n-1)P_0 4^{n-1} - 4(n-2)P_0 4^{(n-2)} = 4^n$.

$P_0 \cancel{4^{n-2}}[n(16) - 3(n-1)4 - 4(n-2)] = \cancel{4^n}\, 16$.

$P_0[\cancel{16n} - \cancel{12n} + 12 - \cancel{4n} + 8] = 16$.

$P_0 = \frac{16}{20} = \frac{4}{5}$.

equating coefficients of n,

$P_0 - \frac{3}{4}P_0 - \frac{P_0}{4} = 0$

$0 = 0$.

$a_n = a_n(h) + a_n(p)$

$a_n = A_1(-1)^n + A_2(4)^n + \frac{n \cdot 4^{n+1}}{5}$

Substituting,

$n(P_0 + P_1 n) - \frac{n}{2}\left(P_0 + \frac{P_1 n}{2}\right) = n$

Equating n-terms,

$P_0 - P_0/2 = 1$

$\frac{P_0}{2} = 1 \Rightarrow \boxed{P_0 = 2}$

equating n^2 terms on both sides

$P_1 - \frac{P_1}{4} = 0 \Rightarrow \boxed{P_1 = 0}$

$a_n(P) = n(2 + 0n)$

$= \boxed{2n}$

$a_n = a_n(h) + a_n(P)$

$a_n = A + 2n$

iii) $f(n) = 2f(n/2) + n$

Homogeneous:

$a_n(h)$:

$a_n - 2a_{n/2} = 0$

Let $a_n = A\alpha^n$

$A\alpha^n - 2A\alpha^{n/2} = 0.$

$A\alpha^{n/2}[\alpha^{n/2} - 2] = 0.$

characteristic equation.

$\alpha^{n/2} = 2$

$\hookrightarrow \alpha = 2^{2/n}.$

(3) $a_n - 2a_{n-1} + a_{n-2} = 7$

Let $a_n = A\alpha^n$

$a_n(h)$:

$$A\alpha^n - 2A\alpha^{n-1} + A\alpha^{n-2} = 0$$

$$A\alpha^{n-2}(\alpha^2 - 2\alpha + 1) = 0$$

$(\alpha - 1)^2 = 0 \rightarrow$ characteristic equation.

$\alpha = 1, 1$

$$a_n = (A_1 + A_2 n)(1)^n$$

$$= A_1 + A_2 n$$

$a_n(p)$:

$$f(n) = 7.(1)^n$$

Assume $a_n = n^2 P_0 . 1^n$

$$= n^2 P_0.$$

Substitute,

$$n^2 P_0 - 2(n-1)^2 P_0 + (n-2)^2 P_0 = 7.$$

equating constants,

$$0 - 2(1)P_0 + 4P_0 = 7.$$

$$P_0 = 7/2.$$

$$a_n = a_n(h) + a_n(p)$$

$$a_n = A_1 + A_2 n + \frac{7n^2}{2}.$$

(4) Solve:

$$a_{n+2} - 2a_{n+1} + a_n = 0$$

Homogeneous

Let $a_n = A\alpha^n$

$$A\alpha^n(\alpha^2 - 2\alpha + 1) = 0.$$

$$(\alpha - 1)^2 = 0.$$

$$\alpha = 1, 1$$

$$a_n = (A_1 + A_2 n)(1)^n.$$

$$= A_1 + A_2 n.$$

$f(0) = 2$

$$a_0 = 2 = A_1 + A_2(0) = A_1$$

$$\boxed{A_1 = 2}$$

$f(1) = 30$

$$30 = A_1 + A_2(1)$$

$$30 = 2 + A_2.$$

$$\boxed{A_2 = 28}$$

$$a_n = 2 + 28n.$$

(5) Solve:

$$a_{n+2} - 3a_{n+1} + 2a_n = 2^n.$$

Homogeneous part:

$a_n(h)$: $a_{n+2} - 3a_{n+1} + 2a_n = 0$

Let $a_n = A\alpha^n$.

$$A\alpha^n[\alpha^2 - 3\alpha + 2] = 0.$$

$$(\alpha - 1)(\alpha - 2) = 0.$$

$$\alpha = 1, 2$$

$$a_n = A_1(1)^n + A_2(2)^n.$$

$a_n(P)$:

$f(n) = 2^n$.

Assume. $a_n = n P_0 2^n$.

$(n+2)P_0 2^{(n+2)} - 3(n+1)P_0 2^{(n+1)} + 2n P_0 2^n = 2^n$

Dividing by 2^n we will get:

$(n+2)4P_0 - 3(n+1).2P_0 + 2nP_0 = 1$.

equate n terms,

$4P_0 - 6P_0 + 2P_0 = 0$

$0 = 0$.

equating constants:

$8P_0 - 3(2)P_0 = 1$.

$2P_0 = 1$.

$P_0 = 1/2$.

$a_n(P) = \frac{n.2^n}{2}$.

$a_n = a_n(h) + a_n(P)$

$a_n = A_1(1)^n + A_2(2)^n + n.2^{n-1}$

6 $a_n - 5a_{n-1} + 6a_{n-2} = 2^n + n$ → 1

$a_n(h)$:

Let $a_n = A\alpha^n$.

$A\alpha^n - 5A\alpha^{n-1} + 6A\alpha^{n-2} = 0$.

$A\alpha^{n-2}[\alpha^2 - 5\alpha + 6] = 0$.

$(\alpha-2)(\alpha-3) = 0$.

$\alpha = 2, 3$

$a_n = A_1(2)^n + A_2(3)^n$.

$a_n(P)$:

$f(n) = 2^n + n$

Assume $a_n = n' P_0 2^n + (P_1 + P_2 n)$

$a_n = n P_0 2^n + P_1 + P_2 n.$

Substituting in eq.①, we get

$(nP_0 2^n + P_1 + P_2 n) - 5((n-1)P_0 2^{n-1} + P_1 + P_2(n-1))$
$+ 6\left[(n-2)P_0 2^{n-2} + P_1 + P_2(n-2)\right] = 2^n + n$

Equating coefficient of n,

$P_2 - 5P_2 + 6P_2 = 1.$

$2P_2 = 1 \quad ; P_2 = 1/2.$

Equating constants,

$P_1 + 5P_2 + 6P_1 - 5P_1 - 12P_2 = 0$

$2P_1 - 7P_2 = 0.$

$2P_1 = 7/2. \Rightarrow P_1 = 7/4.$

Equating 2^n terms,

$\frac{5P_0}{2} - \frac{12P_0}{4} = 1.$

$\frac{-2P_0}{4} = 1$

$P_0 = -2.$

$a_n(P) = -2n.2^n + 7/4 + n/2.$

$a_n = a_n(h) + a_n(P)$

$a_n = A_1(2)^n + A_2(3)^n - n.2^{n+1} + 7/4 + n/2.$