Mobile Robots - Localization memento

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Jacobian matrix:

Let $f : \Re_n \mapsto \Re_p$ such that $f = [f_1(x_1, \dots, x_n), \dots, f_p(x_1, \dots, x_n)]^T$. The Jacobian matrix of f with respect to vector $X = [x_1, \dots, x_n]^T$ is:

$J = \frac{\partial f}{\partial X} = \left[\frac{\partial f_i}{\partial x_j}\right]_{p*n}$

Linear relation between random vectors:

Let X be a random vector of $\Re_n$ and Y a random vector of $\Re_p$ bound by a linear relation $Y = K * X$. Then the covariance matrix of Y can be determined from the covariance matrix of X by the relation:

$P_Y = K P_X K^T$

Non linear relation between random vectors:

Let X be a random vector of $\Re_n$ and Y a random vector of $\Re_p$ bound by a non linear relation $Y = f(X)$ and let J be the Jacobian matrix of f with respect to X. J is a linear approximation of f around the point where it is calculated. Then as an approximation it is possible to calculate the covariance matrix of Y from the covariance matrix of X around a given value of X as:

$P_Y = J P_X J^T$

Posture errors in the absolute frame vs. in the robot frame

The posture error in the absolute reference frame is simply:

$$^0P_e = {}^0P_{real} - {}^0P_{est} = \begin{bmatrix} x_{real} - x_{est} \\ y_{real} - y_{est} \\ \theta_{real} - \theta_{est} \end{bmatrix}$$

$^{est}P_e$ is obtained by the following equations:

$$^{est}P_e = \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = {}^{est}R_0\left(\theta_{est}\right)\left[{}^0P_{real} - {}^0P_{est}\right]$$

$$^{est}P_e = \begin{bmatrix} \left(x_{real} - x_{est}\right)\cos\theta_{est} + \left(y_{real} - y_{est}\right)\sin\theta_{est} \\ -\left(x_{real} - x_{est}\right)\sin\theta_{est} + \left(y_{real} - y_{est}\right)\cos\theta_{est} \\ \theta_{real} - \theta_{est} \end{bmatrix}$$

Equations of the Extended Kalman Filter

$$\begin{cases} X_{k+1} = f\left(X_k, U_k\right) + \alpha_k \\ Y_k = g\left(X_k\right) + \gamma_k \end{cases}$$

Prediction phase:

$$\begin{cases} \widehat{X}_{k+1/k} = f\left(\widehat{X}_{k/k}, U_k\right) \\ P_{k+1/k} = A_k P_{k/k} A_k^T + Q_\alpha \end{cases}$$

With:

$$A_k = \frac{\partial f}{\partial X}\left(\widehat{X}_{k/k}, U_k\right)$$

Estimation phase:

$$\begin{cases} \widehat{X}_{k+1/k+1} = \widehat{X}_{k+1/k} + K_k\left[Y_k - g\left(\widehat{X}_{k+1/k}\right)\right] \\ P_{k+1/k+1} = \left(I - K_k C_k\right) P_{k+1/k} \end{cases}$$

With:

$$\begin{cases} C_k = \dfrac{\partial g}{\partial X}\left(\widehat{X}_{k+1/k}\right) \\ K_k = P_{k+1/k} C_k^T \left(C_k P_{k+1/k} C_k^T + Q_\gamma\right)^{-1} \end{cases}$$

Coherence tests can be based on the Mahalanobis distance between the measurement Y_k and predicted measurement $\widehat{Y}_{k+1/k} = g\left(\widehat{X}_{k+1/k}\right)$:

$$d^2 = (Y_k - \widehat{Y}_{k+1/k})^T \left(C_k P_{k+1/k} C_k^T + Q_\gamma\right)^{-1} (Y_k - \widehat{Y}_{k+1/k})$$

Extended Kalman Filter with noisy input

$$\begin{cases} X_{k+1} = f\left(X_k, U_k\right) + \alpha_k \\ U_k^* = U_k + \beta_k \\ Y_k = g\left(X_k\right) + \gamma_k \end{cases}$$

The prediction phase becomes:

$$\begin{cases} \widehat{X}_{k+1/k} = f\left(\widehat{X}_{k/k}, U_k^*\right) \\ P_{k+1/k} = A_k \cdot P_{k/k} \cdot A_k^T + B_k \cdot Q_\beta \cdot B_k^T + Q_\alpha \end{cases}$$

With A_k and B_k defined as:

$$A_k = \frac{\partial f}{\partial X}\left(\widehat{X}_{k/k}, U_k^*\right) \quad and \quad B_k = \frac{\partial f}{\partial U}\left(\widehat{X}_{k/k}, U_k^*\right)$$

In the calculation of the predicted state and of matrices A and B, we have no other choice but to use the measured input, as the real input is unknown. The estimation phase and coherence tests are unchanged.

Lie derivative - Observability

Applies to continuous systems of the form:

$$
\begin{cases} \dot{x} = f(x,u) \ , \ x \in \mathbb{R}^n \\ y = g(x) \ , \ y \in \mathbb{R}^p \end{cases} \tag{1}
$$

Gradient vector of a real-valued function h of n variables:

$$
dh = \left[\frac{\partial h}{\partial x_1} \cdots \frac{\partial h}{\partial x_n} \right]^T \tag{2}
$$

Rule 1. *Let h be a differentiable real-valued function and f a real vector function, then the Lie derivative of h with respect to f is the vector field :*

$$
L_f h = \frac{\partial h}{\partial x} f \tag{3}
$$

Rule 2. *Let $g : \mathbb{R}^n \to \mathbb{R}^p$. Its components (which are real-valued functions) are written:*

$$
g = \left[\begin{array}{ccc} g^1 & \dots & g^p \end{array} \right]^T \tag{4}
$$

Let the observability matrix O be defined by:

$$
O = \left[\begin{array}{cccccccc} dg^1 & dL_f g^1 & \dots & dL_f^{n-1} g^1 & \dots & dg^p & \dots & dL_f^{n-1} g^p \end{array} \right] \tag{5}
$$

If the rank of O is full (equal to n), then the system is observable.