$$I = \int_0^1 \int_0^{\xi_1} \int_0^{\xi_2} ... \int_0^{\xi_{4999}} \ln(1-\xi_{5000})\, d\xi_{5000} ... d\xi_2\, d\xi_1$$

One approach to this integral is to expand $\ln(1-\xi)$ as a power series, and integrate each polynomial term by interchanging summation and integration and resolving the simple sum at the end. While this is a valid approach, we will apply a slightly cleaner and more elegant solution making use of a lesser known mathematical result known as the Cauchy formula for repeated integration. It states

$$\int_\alpha^x \int_\alpha^{\sigma_1} ... \int_\alpha^{\sigma_{n-1}} f(\sigma_n)\, d\sigma_n ... d\sigma_2\, d\sigma_1 = \frac{1}{(n-1)!} \int_\alpha^x (x-t)^{n-1} f(t)\, dt$$

This result is major in the field of fractional calculus. I will post an inductive proof this week, but feel free to try it for yourself. In the meantime, applying to our integral,

$$I = \frac{1}{4999!} \int_0^1 (1-t)^{4999} \ln(1-t)\, dt$$

Applying reflection identity,

$$= \frac{1}{4999!} \int_0^1 t^{4999} \ln t\, dt$$

We will now apply IBP, differentiating $\ln t$ and integrating t^{4999} to get

$$\begin{aligned} I &= \underbrace{\frac{1}{5000!} \ln t \cdot t^{5000} \Big|_0^1}_{=0} - \int_0^1 \frac{t^{4999}}{5000!}\, dt \\ &= -\frac{1}{5000! \cdot 5000} \end{aligned}$$