The Gaussian Integral

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$$

Swipe For EZ Proof $\Rightarrow$

Let's start off with a simple u-substitution.

$$I = \int_0^\infty e^{-x^2}\, dx$$

$$x^2 \Rightarrow u \qquad 2x\, dx = \, du$$

As $x = \sqrt{u}$, we have $dx = \frac{1}{2}u^{-\frac{1}{2}}\, du$

$$I = \frac{1}{2}\int_0^\infty u^{\frac{1}{2}-1}e^{-u}\, du = \frac{1}{2}\cdot\Gamma\left(\frac{1}{2}\right)$$

Recall from my last post that

$$\Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin(\pi s)}$$

Let's plug in $s = \frac{1}{2}$ here to give us.

$$\left[\Gamma\left(\frac{1}{2}\right)\right]^2 = \frac{\pi}{\sin\left(\frac{\pi}{2}\right)} \Rightarrow \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$I = \frac{1}{2}\cdot\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2}$$

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$$

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