

Hidden Golden Ratio Integral (@solvingtogether)

Shreenabh Agrawal

June 28, 2020

1 Question

If $I =$

$$\int_0^1 \frac{1-x^2}{x^2+(x^2+1)^2} dx$$

Then find $\tan(I) + \sec(I)$

2 Solution

Let us take substitution $x = \tan\theta$ so that $dx = \sec^2\theta d\theta$, thus our integral becomes,

$$\int_0^{\pi/4} \frac{1-\tan^2\theta}{\tan^2\theta + \sec^4\theta} \sec^2\theta d\theta$$

Now converting $\tan\theta$ and $\sec\theta$ to respective \sin and \cos functions, the integral becomes,

$$\int_0^{\pi/4} \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta \cos^2\theta + 1} d\theta$$

Using double angle properties, the integral can be rewritten as:

$$\int_0^{\pi/4} \frac{\cos 2\theta}{\frac{\sin^2 2\theta}{4} + 1} d\theta$$

Taking a final substitution $\sin 2\theta = u$ so that $2 \cos 2\theta d\theta = du$,

$$\int_0^1 \frac{2}{4+u^2} du$$

Plugging in the limits, we get,

$$I = \tan^{-1}\left(\frac{1}{2}\right)$$

For the final answer,

$$\tan I + \sec I = \frac{1}{2} + \frac{\sqrt{5}}{2}$$

$\tan I + \sec I = \phi$

Where ϕ is the Golden Ratio.