

Infinite Radical Limit

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1 Question

Find x for which

$$\lim_{x \rightarrow \infty} \sqrt{1 + \sqrt{x + \sqrt{x^2 + \dots + \sqrt{x^n}}}} = 2$$

2 Solution

The key observation here is that

$$\begin{aligned} t + 1 &= \sqrt{t^2 + 2t + 1} \\ &= \sqrt{t^2 + \sqrt{(2t + 1)^2}} \\ &= \sqrt{t^2 + \sqrt{4t^2 + 4t + 1}} \\ &= \sqrt{t^2 + \sqrt{4t^2 + \sqrt{(4t + 1)^2}}} \\ &= \sqrt{t^2 + \sqrt{4t^2 + \sqrt{16t^2 + \sqrt{64t^2 + \dots}}}} \end{aligned}$$

Now putting $t = 1$, we get,

$$2 = \sqrt{1 + \sqrt{4 + \sqrt{16 + \sqrt{64 + \dots}}}}$$

Hence, the answer is

$$\boxed{x = 4}$$