Solution to Number theory #2

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1 Problem

Is there a power of 2 (> 8) such that that its digits can be rearranged to form another power of 2? (zeros are not allowed in leading digits, e.g. 032 is not allowed)

2 solution

We list out some simple and obvious facts here to help us with the problem:

1. Rearranging digits does not change the total number of digits.
2. At most 4 consecutive powers of 2 can have the same number of digits.
3. A number is equal to its digit sum modulo 9.

Using these, we rephrase the question:

Is there a power of 2 such that there is another power of 2 with the same number of digits and the same remainder modulo 9?

Observing the remainders modulo 9 of some powers of 2, we conjecture that the remainders repeat themselves every 6 powers.

$$\because 64 \equiv 1 \mod 9$$

$$\therefore 2^{n+6} \equiv 2^n \times 64 \equiv 2^n \mod 9$$

Since the remainders modulo 9 repeat themselves every 6 powers and there can only be at most 4 powers of 2 can have the same digits, the answer to the question is NO.