$$\int \tan(x)\,dx$$

Swipe For Solution.

$\bullet \Rightarrow$

We can make this easier solve by breaking down tangent of x into functions we can work with.

$$\begin{aligned} \sin x &= \frac{opp}{hyp} \\ \cos x &= \frac{adj}{hyp} \end{aligned}$$

If we take the ratio between sine and cosine of x we see that $\frac{\sin x}{\cos x}$ is equal to $\frac{\frac{opp}{hyp}}{\frac{adj}{hyp}}$ which by further simplification is equivalent to $\frac{opp}{adj}$ which is why

$$\frac{\sin x}{\cos x} = \frac{\frac{opp}{hyp}}{\frac{adj}{hyp}} = \tan x$$

Therefore our integral is equivalent to $\int \frac{\sin x}{\cos x} dx$

$$\int \tan(x) dx = \int \frac{\sin x}{\cos x} dx$$

We can solve this integral with a simple u sub. Let u be equal to $\cos x$ then the following is obvious.

$$\begin{aligned} du &= -\sin x dx \\ -du &= \sin x dx \end{aligned}$$

Our integral becomes

$$\int \tan(x) = -1 \int \frac{1}{u} du$$
$$\int \tan(x) = -1 \ln|u|$$

according to the laws of logarithms we can change our integral to

$$\int \tan(x) = \ln|(u)^{-}1|$$
$$\int \tan(x) = \ln|\frac{1}{u}|$$
$$\int \tan(x) = \ln|\frac{1}{\cos x}|$$
$$\int \tan(x) = \ln|\sec x| + c$$

Almost forgot the + c :0

$$\int \tan(x) = \ln|\sec x| + c$$