

Solution to Solving Together's Integral

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Evaluate

$$\int_0^1 \left(\frac{x^x}{(1-x)^{1-x}} - \frac{(1-x)^{1-x}}{x^x} \right) dx$$

Solution

Let

$$I = \int_0^1 \left(\frac{x^x}{(1-x)^{1-x}} - \frac{(1-x)^{1-x}}{x^x} \right) dx$$

Substitute $x \rightarrow 1-x$

$$I = \int_0^1 \left(\frac{(1-x)^{1-x}}{x^x} - \frac{x^x}{(1-x)^{1-x}} \right) dx$$

Therefore,

$$\begin{aligned} 2I &= \int_0^1 \left(\frac{x^x}{(1-x)^{1-x}} - \frac{x^x}{(1-x)^{1-x}} \right) - \left(\frac{(1-x)^{1-x}}{x^x} - \frac{(1-x)^{1-x}}{x^x} \right) dx \\ &\Rightarrow \int_0^1 \left(\frac{x^x}{(1-x)^{1-x}} - \frac{(1-x)^{1-x}}{x^x} \right) dx = 0 \end{aligned}$$