# A Guided Tour of Chapter 10: Reinforcement Learning for Control

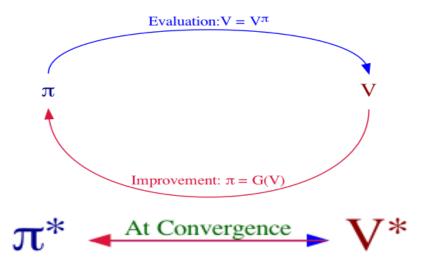
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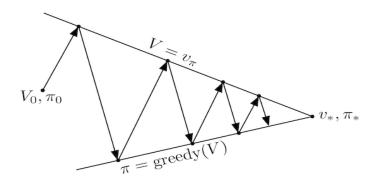
## RL does not have access to a probability model

- ullet DP/ADP assume access to probability model (knowledge of  $\mathcal{P}_R$ )
- Often in real-world, we do not have access to these probabilities
- Which means we'd need to interact with the actual environment
- Actual Environment serves up individual experiences, not probabilities
- Even if MDP model is available, model updates can be challenging
- Often real-world models end up being too large or too complex
- Sometimes estimating a sampling model is much more feasible
- So RL interacts with either actual or simulated environment
- Either way, we receive individual experiences of next state and reward
- We saw how RL Prediction learns from individual experiences
- Now we extend those ideas to RL Control: Learning Optimal VF
- We start with Tabular RL Control

## Let us recall the Policy Iteration algorithm



# The idea of Generalized Policy Iteration (GPI)



- Any Policy Evaluation method, Any Policy Improvement method
- ullet Policy Evaluation estimates  $V^\pi$ , eg: Iterative Policy Evaluation
- ullet Policy Improvement produces  $\pi' \geq \pi$ , eg: Greedy Policy Improvement
- Policy Evaluation and Policy Improvement alternate until convergence

### Natural Idea: GPI with Tabular Monte-Carlo Evaluation

- Let us explore GPI with Tabular Monte-Carlo evaluation
- So we will do Policy Evaluation with Tabular MC evaluation
- And we will do the usual Greedy Policy Improvement
- But Greedy Policy Improvement requires a model of MDP

$$\pi_D'(s) \leftarrow \argmax_{a \in \mathcal{A}} \{\mathcal{R}(s, a) + \gamma \sum_{s' \in \mathcal{N}} \mathcal{P}(s, a, s') \cdot V^{\pi}(s')\}$$

• However, it works if we were working with Action-Value Function

$$\pi'_D(s) \leftarrow \arg\max_{a \in A} Q^{\pi}(s, a)$$

- ullet This means: Policy Evaluation for Action-Value Function  $Q^{\pi}(s,a)$
- Following a policy  $\pi$ , update Q-value for each  $(S_t, A_t)$  each episode:

$$Count(S_t, A_t) \leftarrow Count(S_t, A_t) + 1$$
 
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{1}{Count(S_t, A_t)} \cdot (G_t - Q(S_t, A_t))$$

## *ϵ*-Greedy Policy Improvement

- A full Policy Evaluation with MC takes too long
- So we typically improve policy after each episode
- This can lead to some actions not being tried enough
- Which can lead to premature (greedy) domination of an action
- Which can lead to other actions getting "locked-out"
- Same as Explore v/s Exploit dilemma of Multi-Armed Bandit problem
- Simple solution: Perform an  $\epsilon$ -Greedy Policy Improvement
- ullet All  $|\mathcal{A}|$  actions are tried with non-zero probability (for each state)
- ullet Pick the greedy action with probability  $1-\epsilon$
- ullet With probability  $\epsilon$ , randomly choose one of the  $|\mathcal{A}|$  actions

Stochastic Policy 
$$\pi(s,a) = \begin{cases} \frac{\epsilon}{|\mathcal{A}|} + 1 - \epsilon & \text{if } a = \arg\max_{b \in \mathcal{A}} Q(s,b) \\ \frac{\epsilon}{|\mathcal{A}|} & \text{otherwise} \end{cases}$$

## $\epsilon$ -Greedy improves the policy

#### Theorem

For any  $\epsilon$ -greedy policy  $\pi$ , the  $\epsilon$ -greedy policy  $\pi'$  with respect to  $Q^{\pi}$  is an improvement, i.e.,  $\boldsymbol{V}^{\pi'}(s) \geq \boldsymbol{V}^{\pi}(s)$  for all  $s \in \mathcal{N}$ .

• Applying  ${m B}^{\pi'}$  repeatedly (starting with  ${m V}^{\pi}$ ) converges to  ${m V}^{\pi'}$ :

$$\lim_{i o \infty} ({m{B}}^{\pi'})^i({m{V}}^\pi) = {m{V}}^{\pi'}$$

• So the proof is complete if we prove that:

$$(oldsymbol{\mathcal{B}}^{\pi'})^{i+1}(oldsymbol{\mathcal{V}}^{\pi}) \geq (oldsymbol{\mathcal{B}}^{\pi'})^{i}(oldsymbol{\mathcal{V}}^{\pi})$$
 for all  $i=0,1,2,\ldots$ 

• Increasing tower of Value Functions  $[({m B}^{\pi'})^i({m V}^\pi)|i=0,1,2,\ldots]$  with repeated applications of  ${m B}^{\pi'}$ 

## Base Case of Proof by Induction

The base case of proof by induction is to show that  $m{B}^{\pi'}(m{V}^\pi) \geq m{V}^\pi$ 

$$\begin{split} \boldsymbol{B}^{\pi'}(\boldsymbol{V}^{\pi})(s) &= (\boldsymbol{\mathcal{R}}^{\pi'} + \gamma \cdot \boldsymbol{\mathcal{P}}^{\pi'} \cdot \boldsymbol{V}^{\pi})(s) \\ &= \boldsymbol{\mathcal{R}}^{\pi'}(s) + \gamma \cdot \sum_{s' \in \mathcal{S}} \boldsymbol{\mathcal{P}}^{\pi'}(s, s') \cdot \boldsymbol{V}^{\pi}(s') \\ &= \sum_{a \in \mathcal{A}} \pi'(s, a) \cdot (\mathcal{R}(s, a) + \gamma \cdot \sum_{s' \in \mathcal{S}} \mathcal{P}(s, a, s') \cdot \boldsymbol{V}^{\pi}(s')) \\ &= \sum_{a \in \mathcal{A}} \pi'(s, a) \cdot Q^{\pi}(s, a) \\ &= \frac{\epsilon}{|\mathcal{A}|} \cdot \sum_{a \in \mathcal{A}} Q^{\pi}(s, a) + (1 - \epsilon) \cdot \max_{a \in \mathcal{A}} Q^{\pi}(s, a) \\ &\geq \frac{\epsilon}{|\mathcal{A}|} \cdot \sum_{a \in \mathcal{A}} Q^{\pi}(s, a) + (1 - \epsilon) \cdot \sum_{a \in \mathcal{A}} \frac{\pi(s, a) - \frac{\epsilon}{|\mathcal{A}|}}{1 - \epsilon} \cdot Q^{\pi}(s, a) \\ &= \sum_{a \in \mathcal{A}} \pi(s, a) \cdot Q^{\pi}(s, a) = \boldsymbol{V}^{\pi}(s) \end{split}$$

## Induction Step of Proof by Induction

Induction step is proved by monotonicity of  $\mathbf{B}^{\pi}$  operator (for any  $\pi$ ):

Monotonicity Property of 
$${m B}^\pi: {m X} \geq {m Y} \Rightarrow {m B}^\pi({m X}) \geq {m B}^\pi({m Y})$$

So 
$$({m{\mathcal{B}}}^{\pi'})^{i+1}({m{V}}^\pi) \geq ({m{\mathcal{B}}}^{\pi'})^i({m{V}}^\pi) \Rightarrow ({m{\mathcal{B}}}^{\pi'})^{i+2}({m{V}}^\pi) \geq ({m{\mathcal{B}}}^{\pi'})^{i+1}({m{V}}^\pi)$$

## **GLIE**

#### **Definition**

Greedy in the Limit with Infinite Exploration (GLIE):

All state-action pairs are explored infinitely many times

$$\lim_{k\to\infty} N_k(s,a) = \infty$$

• The policy converges to a greedy policy

$$\lim_{k \to \infty} \pi_k(s, a) = \mathbb{I}_{a = \operatorname{arg} \max_{b \in \mathcal{A}} Q(s, b)}$$

 $\epsilon$ -greedy can be made GLIE if  $\epsilon$  is reduced as:  $\epsilon_k = \frac{1}{k}$ 

#### GLIE Tabular Monte-Carlo Control

- Sample k-th episode using  $\pi$ :  $\{S_0, A_0, R_1, S_1, A_1, \dots, R_T, S_T\} \sim \pi$
- ullet For each state  $S_t$  and action  $A_t$  in episode, updates at *episode-end*:

$$Count(S_t, A_t) \leftarrow Count(S_t, A_t) + 1$$
 
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{1}{Count(S_t, A_t)} \cdot (G_t - Q(S_t, A_t))$$

• Improve policy at end of episode based on updated Q-Value function:

$$\epsilon \leftarrow \frac{1}{k}$$
 $\pi \leftarrow \epsilon\text{-greedy}(Q)$ 

#### Theorem

GLIE Tabular Monte-Carlo Control converges to the Optimal Action-Value function:  $Q(s, a) \rightarrow Q^*(s, a)$ .

#### MC versus TD Control

- TD learning has several advantages over MC learning:
  - Lower variance
  - Online
  - Can work with incomplete traces or continuing traces
  - Generic interface of Iterable of atomic experiences allows for serving up atomic experiences in any order (eg: atomic experience replays)
- So use TD instead of MC in our Control loop
  - Apply TD to Q(S, A) (instead of V(S))
  - Use  $\epsilon$ -greedy Policy Improvement
  - Update Q(S, A) after each atomic experience
  - ullet  $\epsilon$ -greedy policy **automatically updated** after each atomic experience

# Tabular SARSA Algorithm

- Tabular SARSA is our first TD Control algorithm
- ullet Like Tabular MC Control, Policy Improvement is  $\epsilon$ -greedy
- But here Policy Evaluation is with a TD target, as below:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \cdot (R_{t+1} + \gamma \cdot Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t))$$

- Note that Q(S, A) is updated after each atomic experience
- ullet  $\epsilon$ -greedy policy automatically updated after each atomic experience
- Action  $A_t$  is chosen from State  $S_t$  based on  $\epsilon$ -greedy policy
- ullet Action  $A_{t+1}$  is chosen from State  $S_{t+1}$  based on  $\epsilon$ -greedy policy
- Unlike MC Control, trace experience is incrementally generated
- Note: Instead of  $\epsilon$ -greedy, we could employ a more sophisticated exploratory policy derived from Q-value function ( $\epsilon$ -greedy is just our default simple exploratory policy derived from Q-value function)

## SARSA Visualization



## Convergence of Tabular SARSA

#### Theorem

Tabular SARSA converges to the Optimal Action-Value function,  $Q(s, a) \rightarrow Q^*(s, a)$ , under the following conditions:

- GLIE sequence of policies  $\pi_t(s, a)$
- ullet Robbins-Monro sequence of step-sizes  $lpha_t$

$$\sum_{t=1}^{\infty} \alpha_t = \infty$$

$$\sum_{t=1}^{\infty} \alpha_t^2 < \infty$$

## Tabular *n*-step SARSA

Tabular SARSA bootstraps the Q-Value Function with update:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t))$$

• So it's natural to extend this to bootstrapping with 2 steps ahead:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma R_{t+2} + \gamma^2 Q(S_{t+2}, A_{t+2}) - Q(S_t, A_t))$$

• Generalize to bootstrapping with  $n \ge 1$  time steps ahead:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(G_{t,n} - Q(S_t, A_t))$$

•  $G_{t,n}$  (call it *n*-step bootstrapped return) is defined as:

$$G_{t,n} = \sum_{i=t+1}^{t+n} \gamma^{i-t-1} \cdot R_i + \gamma^n \cdot Q(S_{t+n}, A_{t+n})$$
  
=  $R_{t+1} + \gamma \cdot R_{t+2} + \ldots + \gamma^{n-1} \cdot R_{t+n} + \gamma^n \cdot Q(S_{t+n}, A_{t+n})$ 

#### Tabular $\lambda$ -Return SARSA

• Instead of  $G_{t,n}$ , a valid target is a weighted-average target:

$$\sum_{n=1}^{N} u_n \cdot G_{t,n} + u \cdot G_t \text{ where } u + \sum_{n=1}^{N} u_n = 1$$

- ullet Any of the  $u_n$  or u can be 0, as long as they all sum up to 1
- The  $\lambda$ -Return target is a special case of weights  $u_n$  and u

$$u_n = (1 - \lambda) \cdot \lambda^{n-1}$$
 for all  $n = 1, \dots, T - t - 1$   $u_n = 0$  for all  $n \ge T - t$  and  $u = \lambda^{T - t - 1}$ 

• We denote the  $\lambda$ -Return target as  $G_t^{(\lambda)}$ , defined as:

$$G_t^{(\lambda)} = (1 - \lambda) \cdot \sum_{n=1}^{T-t-1} \lambda^{n-1} \cdot G_{t,n} + \lambda^{T-t-1} \cdot G_t$$
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \cdot (G_t^{(\lambda)} - Q(S_t, A_t))$$

# Tabular SARSA $(\lambda)$

- ullet  $\lambda$  can be tuned from SARSA ( $\lambda=0$ ) to MC Control ( $\lambda=1$ )
- Note that for  $\lambda > 0$ ,  $\lambda$ -Return SARSA is an Offline Algorithm
- SARSA( $\lambda$ ) is online "version" of  $\lambda$ -Return SARSA
- Similar to  $TD(\lambda)$  for Prediction,  $SARSA(\lambda)$  uses Eligibility Traces
- Eligibility Trace for a given trace experience at time t is a function

$$E_t: \mathcal{N} imes \mathcal{A} 
ightarrow \mathbb{R}_{\geq 0}$$

$$E_0(s,a) = 0$$
, for all  $s \in \mathcal{N}, a \in \mathcal{A}$ 

$$E_t(s,a) = \gamma \cdot \lambda \cdot E_{t-1}(s,a) + \mathbb{I}_{S_t=s,A_t=a}, \text{ for all } s \in \mathcal{N}, a \in \mathcal{A}, \text{ for all } t$$

• Tabular SARSA( $\lambda$ ) performs following update at each time step t in each trace experience (for each  $s \in \mathcal{N}, a \in \mathcal{A}$ ):

$$Q(s, a) \leftarrow Q(s, a) + \alpha \cdot (R_{t+1} + \gamma \cdot Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)) \cdot E_t(s, a)$$

## Off-Policy Learning

ullet Estimate VF for target policy  $\pi$  while following experience policy  $\mu$ 

$$\{S_0, A_0, R_1, S_1, A_1, \dots, R_T, S_T\} \sim \mu$$

- Why is this important?
  - Learning from observing humans or other agents
  - Re-use experience generated from old policies  $\pi_1, \pi_2, \dots, \pi_{t-1}$
  - Learn about optimal policy while following exploratory policy
  - Learn about multiple policies while following one policy

# Importance Sampling for Off-Policy Learning

- Importance Sampling refers to methods to estimate properties of a distribution P, given access to samples from a different distribution Q
- We can calculate  $\mathbb{E}_{X \sim P}[f(X)]$  given samples from Q as follows:

$$\mathbb{E}_{X \sim P}[f(X)] = \sum_{X \sim P} P(X) \cdot f(X)$$

$$= \sum_{X \sim Q} Q(X) \cdot \frac{P(X)}{Q(X)} \cdot f(X)$$

$$= \mathbb{E}_{X \sim Q} \left[ \frac{P(X)}{Q(X)} \cdot f(X) \right]$$

## Importance Sampling for Off-Policy Monte-Carlo

- ullet Use returns generated from  $\mu$  to estimate Value Function for  $\pi$
- $\bullet$  Weight return  $G_t$  according to similarity between policies
- Multiply importance sampling corrections along whole episode

$$G_t^{\pi/\mu} = \frac{\pi(S_t, A_t)}{\mu(S_t, A_t)} \cdot \frac{\pi(S_{t+1}, A_{t+1})}{\mu(S_{t+1}, A_{t+1})} \cdots \frac{\pi(S_T, A_t)}{\mu(S_T, A_T)} \cdot G_t$$

Update value towards corrected return

$$V(S_t) \leftarrow V(S_t) + \alpha \cdot (G_t^{\pi/\mu} - V(S_t))$$

- Likewise for *Q*-Value Function for MC Control
- ullet Note: We cannot use this method if  $\mu$  is zero when  $\pi$  is non-zero
- Importance sampling can dramatically increase variance

## Importance Sampling for Off-Policy Temporal-Difference

- ullet Use TD targets generated from  $\mu$  to evaluate Value Function for  $\pi$
- Weight TD target  $R + \gamma \cdot V(S')$  with importance sampling
- Here we only need a single importance sampling correction

$$V(S_t) \leftarrow V(S_t) + \alpha \cdot \left(\frac{\pi(S_t, A_t)}{\mu(S_t, A_t)} \cdot (R_{t+1} + \gamma \cdot V(S_{t+1})) - V(S_t)\right)$$

- Likewise for Q-Value Function for TD Control
- This has much lower variance than MC importance sampling
- Policies only need to be similar over a single step

## Tabular Q-Learning

- We now consider off-policy learning of action-values Q(s, a)
- No importance sampling is required
- The next action is chosen using experience policy  $A_{t+1} \sim \mu(S_t, \cdot)$
- But we consider an alternative successor action  $A' \sim \pi(S_t,\cdot)$
- Update  $Q(S_t, A_t)$  towards value of alternative action

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \cdot (R_{t+1} + \gamma \cdot Q(S_{t+1}, A') - Q(S_t, A_t))$$

## Off-Policy Control with Tabular Q-Learning

- We now allow both experience and target policies to improve
- The target (deterministic) policy  $\pi_D$  is **greedy** w.r.t *Q*-Value Function

$$\pi_D(S_{t+1}) = rg \max_{a' \in \mathcal{A}} Q(S_{t+1}, a')$$

- The experience policy  $\mu$  is also improving, eg:  $\epsilon$ -greedy w.r.t. Q
- The *Q*-learning target then simplifies to:

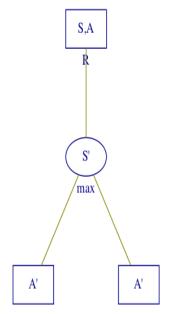
$$\begin{aligned} &R_{t+1} + \gamma \cdot Q(S_{t+1}, A') \\ = &R_{t+1} + \gamma \cdot Q(S_{t+1}, \argmax_{a' \in \mathcal{A}} Q(S_{t+1}, a')) \\ = &R_{t+1} + \gamma \cdot \max_{a' \in \mathcal{A}} Q(S_{t+1}, a') \end{aligned}$$

• Thus the update to *Q*-Value Function after each atomic experience is:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \cdot (R_{t+1} + \gamma \cdot \max_{a' \in \mathcal{A}} Q(S_{t+1}, a') - Q(S_t, A_t))$$

• Convergence proofs require infinite exploration of all (state, action) pairs & appropriate stochastic approximation conditions for step sizes.

## Q-Learning Control Visualization



## RL Control with Function Approximation

- Natural extension of Tabular Control to function approximations
- Instead of parameterized approximation  $V(s; \boldsymbol{w})$ , we have  $Q(s, a; \boldsymbol{w})$
- GPI's Policy Evaluation is Approximate Q-Value Policy Evaluation
- Loss Function for Q(s, a; w) estimate relative to appropriate return
- For MC Control, the loss function for  $(S_t, A_t)$  in an episode is:

$$\mathcal{L}_{(S_t,A_t,G_t)}(\boldsymbol{w}) = \frac{1}{2} \cdot (Q(S_t,A_t;\boldsymbol{w}) - G_t)^2$$

• Therefore, the gradient of the loss function for this  $(S_t, A_t)$  is:

$$\nabla_{\boldsymbol{w}} \mathcal{L}_{(S_t, A_t, G_t)}(\boldsymbol{w}) = (Q(S_t, A_t; \boldsymbol{w}) - G_t) \cdot \nabla_{\boldsymbol{w}} Q(S_t, A_t; \boldsymbol{w})$$

• Therefore, parameters update for this  $(S_t, A_t)$  at episode-end is:

$$\Delta \mathbf{w} = \alpha \cdot (G_t - Q(S_t, A_t; \mathbf{w})) \cdot \nabla_{\mathbf{w}} Q(S_t, A_t; \mathbf{w})$$

ullet  $\epsilon$ -greedy policy improvement is automatic from updated Q estimate

## Bootstrapped Control with Function Approximation

• For SARSA, parameters update after each atomic experience:

$$\Delta \mathbf{w} = \alpha \cdot (R_{t+1} + \gamma \cdot Q(S_{t+1}, A_{t+1}; \mathbf{w}) - Q(S_t, A_t; \mathbf{w})) \cdot \nabla_{\mathbf{w}} Q(S_t, A_t; \mathbf{w})$$

• For Q-Learning, parameters update after each atomic experience:

$$\Delta \mathbf{w} = \alpha \cdot (R_{t+1} + \gamma \cdot \max_{\mathbf{a}' \in \mathcal{A}} Q(S_{t+1}, \mathbf{a}'; \mathbf{w}) - Q(S_t, A_t; \mathbf{w})) \cdot \nabla_{\mathbf{w}} Q(S_t, A_t; \mathbf{w})$$

• For offline  $\lambda$ -Return Control, parameters update at episode-end:

$$\Delta \mathbf{w} = \alpha \cdot (G_t^{(\lambda)} - Q(S_t, A_t; \mathbf{w})) \cdot \nabla_{\mathbf{w}} Q(S_t, A_t; \mathbf{w})$$

ullet For online SARSA( $\lambda$ ), update after each atomic experience:

$$\delta_{t} = R_{t+1} + \gamma \cdot Q(S_{t+1}, A_{t+1}; \mathbf{w}) - Q(S_{t}, A_{t}; \mathbf{w})$$

$$\mathbf{E}_{t} = \gamma \lambda \cdot \mathbf{E}_{t-1} + \nabla_{\mathbf{w}} Q(S_{t}, A_{t}; \mathbf{w})$$

$$\Delta \mathbf{w} = \alpha \cdot \delta_{t} \cdot \mathbf{E}_{t}$$

ullet  $\epsilon$ -greedy policy improvement is automatic from updated Q estimate

# Relationship between DP and TD

	Full Backup (DP)	Sample Backup (TD	
Bellman Expectation	$\begin{array}{c} (-1)^{(i)} \\ x_1 = (V^{(i)}, x_2) \\ \\ x_1 = (V^{(i)}, x_2) \\ \\ y_1 = x_1, y_2 \\ \\ y_1 = x_1, y_2 \\ \\ y_2 = x_2, y_3 \\ \\ y_3 = x_3, y_4 \\ \\ y_4 = x_1 \\ \\ y_{11} = x_1 \\ \\ y_{12} = x_1 \\ \\ y_{12} = x_1 \\ \\ y_{12} = x_1 \\ \\ y_{13} = x_2 \\ \\ y_{14} = x_1 \\ \\ y_{14} = x_2 \\ \\ y_{14} = x_1 \\ \\ y_{14} = x_2 \\ \\ y_{14} = x_1 \\ \\ y_{14} = x_2 \\ \\ y_{14} = x_1 \\ \\ y_{14} = x_2 \\ \\ y_{14} = x_1 \\ \\ y_{14} = x_2 \\ \\ y_{14} = x_1 \\ \\ y_{14} = x_2 \\ \\ y_{14} = x_1 \\ \\ y_{14} = x_2 \\ \\ y_{14} = x_1 \\ \\ y_{14} = x_2 \\ \\ y_{14} = x_1 \\ \\ y_{14} = x_2 \\ \\ y_{14} = x_1 \\ \\$	\( \) \( \)	
Equation for $V^{\pi}(s)$	Iterative Policy Evaluation	TD Learning	
Bellman Expectation Equation for $Q^{\pi}(s,a)$	$\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} $	s A SARSA	
Bellman Optimality Equation for $Q^*(s, a)$	$\begin{array}{c} \begin{array}{c} -\varphi(t_0) \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	Q-Learning	

# Relationship between DP and TD

Full Backup (DP)	Sample Backup (TD)		
Iterative Policy Evaluation: $V(S)$ update	TD Learning: $V(S)$ update		
$\mathbb{E}[R + \gamma V(S') S]$	sample $R + \gamma V(S')$		
Q-Policy Iteration: $Q(S, A)$ update	SARSA: $Q(S, A)$ update		
$\mathbb{E}[R + \gamma Q(S', A') S, A]$	sample $R + \gamma Q(S', A')$		
Q-Value Iteration: $Q(S, A)$ update	Q-Learning: $Q(S, A)$ update		
$\mathbb{E}[R + \gamma \max_{a'} Q(S', a')   S, A]$	sample $R + \gamma \max_{a'} Q(S', a')$		

# Convergence of Prediction Algorithms

On/Off Policy	Algorithm	Tabular	Linear	Non-Linear
On-Policy	MC	✓	✓	✓
	TD(0)	✓	✓	X
	$TD(\lambda)$	$\checkmark$	✓	X
Off-Policy	MC	✓	✓	1
	TD(0)	✓	X	X
	$TD(\lambda)$	✓	X	X

# Gradient Temporal-Difference Learning

- TD does not follow the gradient of any objective function
- This is why TD can diverge:
  - when running off-policy, or
  - when using non-linear function approximation
- Gradient TD follows true gradient of projected Bellman Error

On/Off Policy	Algorithm	Tabular	Linear	Non-Linear
	MC	✓	✓	✓
On-Policy	TD	✓	✓	×
	Gradient TD	✓	✓	$\checkmark$
	MC	✓	✓	✓
Off-Policy	TD	✓	X	×
	Gradient TD	✓	✓	✓

# Convergence of Control Algorithms

Algorithm	Tabular	Linear	Non-Linear
MC Control	✓	( ✓)	Х
SARSA	✓	<b>(</b> ✓)	×
Q-Learning	✓	X	×
Gradient Q-Learning	✓	✓	X

(  $\checkmark$ ) means it chatters around near-optimal Value Function

## Key Takeaways from this Chapter

- RL Control is based on the idea of Generalized Policy Iteration (GPI)
  - Policy Evaluation with Q-Value Function (instead of V)
  - Improved Policy needs to be exploratory, eg:  $\epsilon$ -greedy
- On-Policy versus Off-Policy
- Deadly Triad := [Bootstrapping, Function Approximation, Off-Policy]