CME 241: Foundations of Reinforcement Learning with Applications in Finance

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Meet your Instructor

- Joined Stanford ICME as Adjunct Professor in Fall 2018
- Research Interests: A.I. for Dynamic Decisioning under Uncertainty
- Technical mentor to ICME students, partnerships with industry
- Educational background: Algorithms Theory & Abstract Algebra
- 10 years at Goldman Sachs (NY) *Rates/Mortgage Derivatives* Trading
- 4 years at Morgan Stanley as *Managing Director - Market Modeling*
- Currently co-founder of an early-stage Tech Startup
- Teaching experience: Pure & Applied Math, CompSci, Finance, Mgmt

Requirements and Setup

- Pre-requisites:
  - Undergraduate-level background in Applied Mathematics (Multivariate Analysis, Linear Algebra, Probability, Optimization)
  - Background in data structures/algorithms, fluency with numpy
  - Basic familiarity with Pricing, Portfolio Mgmt and Algo Trading, but we will do an overview of the requisite Finance/Economics
  - No background required in MDP, DP, RL (we will cover from scratch)
- Here's a sample exam to get a sense of course difficulty
- Register for the course on Ed Discussion
- Install Python 3 and supporting IDE/tools (eg: PyCharm, Jupyter)
- Install LaTeX/Markdown and supporting editor for tech writing
- Get the course textbook or download the PDF version
- Assignments and code in the textbook based on this open-source code
- *Fork* this repo and get set up to use this code in assignments
- Create separate directories for each assignment for CA Amil to review and grade - send Amil your forked repo URL and *git push* by due dates

Housekeeping

- Lectures: Wed & Fri 3:00pm-4:20pm. Lane History Corner, 205.
- Office Hours:
    1. Ashwin: 1:00pm-2:30pm Fri (or by appointment) in ICME Mezzanine, room M05 (within Huang Engg Bldg)
    2. Amil: poll will be sent out on Ed (or by appointment)
- Course Web Site: cme241.stanford.edu
- Ask Questions and engage in Discussions on Ed Discussion
- My e-mail: ashwin.rao@stanford.edu

Resources

- Course based on my RL For Finance book
- I prepare slides for each lecture ("guided tour" of respective chapter)
- Code in my book based on this open-source code
- Reading this code as important as the reading of the theory
- We will go over some classical papers on the Finance applications
- Some supplementary/optional papers from Finance/RL
- All resources organized on the course web site ("source of truth")

Grading

- Grade based on:
  - 40% *Exam* (1.5 hour exam on Theory, Modeling, Programming)
  - 30% *Group Assignments* (Technical Writing and Programming)
  - 30% *Course Project* (Same groups as assignments)
- Assignments
  - Can be completed in groups of up to 3
  - Grade more on effort than for correctness
  - Designed to take 3-5 hours outside of class
- Exam
  - Individual take-home exam, covering topics from weeks 1-7
  - Practice exams from prior years available
  - 2 hours within 48 hour window (tentatively February 20-22)
- Project
  - Open-ended research or implementation project
  - Choose topics of interest, a list of ideas will be released by Week 5
  - Deliverables will be a presentation and code turn-in

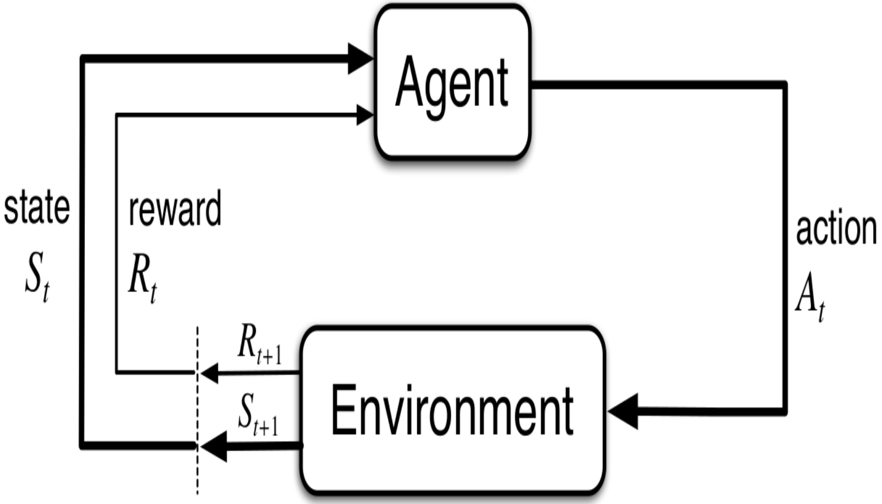
Stanford Honor Code - For Assignments versus Exams

- *Assignments*: You can discuss solution approaches with other students
  - Because assignments are graded more for effort than correctness
  - Writing (answers/code) should be your own (don't copy/paste)
  - You can invoke the core modules I have written (as instructed)
- *Exams*: You cannot engage in any conversation with other students
  - Write to the CA if a question is unclear
  - Exams are graded on correctness and completeness
  - *Don't ask for help* on how to solve exam questions
  - Open-internet Exams: Search for concepts, not answers to exam Qs
  - If you accidentally run into a strong hint/answer, state it honestly

A.I. for Dynamic Decisioning under Uncertainty

- Let's browse some terms used to characterize this branch of A.I.
- *Stochastic*: Uncertainty in key quantities, evolving over time
- *Optimization*: A well-defined metric to be maximized ("The Goal")
- *Dynamic*: Decisions need to be a function of the changing situations
- *Control*: Overpower uncertainty by persistent steering towards goal
- Jargon overload due to confluence of Control Theory, O.R. and A.I.
- For language clarity, let's just refer to this area as *Stochastic Control*
- The core framework is called *Markov Decision Processes* (MDP)
- *Reinforcement Learning* is a class of algorithms to solve MDPs

The MDP Framework



Components of the MDP Framework

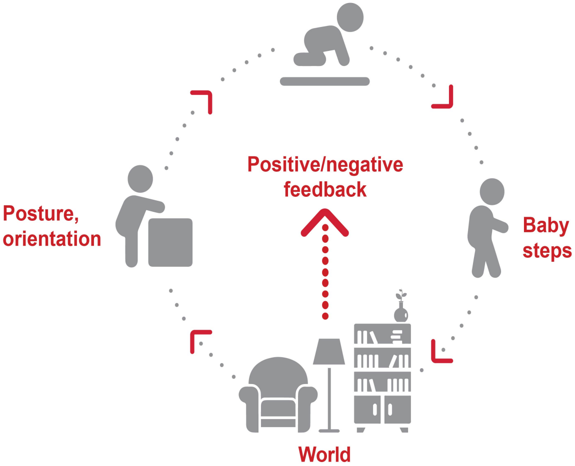
- The *Agent* and the *Environment* interact in a time-sequenced loop
- *Agent* responds to [*State*, *Reward*] by taking an *Action*
- *Environment* responds by producing next step's (random) *State*
- *Environment* also produces a (random) scalar denoted as *Reward*
- Each *State* is assumed to have the *Markov Property*, meaning:
    - Next *State/Reward* depends only on Current *State* (for a given *Action*)
    - Current *State* captures all relevant information from *History*
    - Current *State* is a sufficient statistic of the future (for a given *Action*)
- Goal of *Agent* is to maximize *Expected Sum* of all future *Reward*s
- By controlling the ($Policy : State \rightarrow Action$) function
- This is a dynamic (time-sequenced control) system under uncertainty

Formal MDP Framework

The following notation is for discrete time steps. Continuous-time formulation is analogous (often involving Stochastic Calculus)

- Time steps denoted as $t = 1, 2, 3, \ldots$
- Markov States $S_t \in \mathcal{S}$ where $\mathcal{S}$ is the State Space
- Actions $A_t \in \mathcal{A}$ where $\mathcal{A}$ is the Action Space
- Rewards $R_t \in \mathbb{R}$ denoting numerical feedback
- Transitions $p(r, s'|s, a) = \mathbb{P}[(R_{t+1} = r, S_{t+1} = s')|S_t = s, A_t = a]$
- $\gamma \in [0, 1]$ is the Discount Factor for Reward when defining *Return*
- Return $G_t = R_{t+1} + \gamma \cdot R_{t+2} + \gamma^2 \cdot R_{t+3} + \ldots$
- Policy $\pi(a|s)$ is probability that Agent takes action a in states s
- The goal is find a policy that maximizes $\mathbb{E}[G_t|S_t = s]$ for all $s \in \mathcal{S}$

How a baby learns to walk



Many real-world problems fit this MDP framework

- Self-driving vehicle (speed/steering to optimize safety/time)
- Game of Chess (Boolean *Reward* at end of game)
- Complex Logistical Operations (eg: movements in a Warehouse)
- Make a humanoid robot walk/run on difficult terrains
- Manage an investment portfolio
- Control a power station
- Optimal decisions during a football game
- Strategy to win an election (high-complexity MDP)

Self-Driving Vehicle

Why are these problems hard?

- *State* space can be large or complex (involving many variables)
- Sometimes, *Action* space is also large or complex
- No direct feedback on "correct" *Actions* (only feedback is *Reward*)
- Time-sequenced complexity (*Actions* influence future *States/Actions*)
- *Action*s can have delayed consequences (late *Reward*s)
- *Agent* often doesn't know the *Model* of the *Environment*
- "Model" refers to probabilities of state-transitions and rewards
- So, *Agent* has to learn the *Model* AND solve for the Optimal *Policy*
- *Agent Action*s need to tradeoff between "explore" and "exploit"

Value Function and Bellman Equations

- Value function (under policy π) $V_\pi(s) = \mathbb{E}[G_t | S_t = s]$ for all $s \in \mathcal{S}$

$$V_\pi(s) = \sum_a \pi(a|s) \sum_{r,s'} p(r, s'|s, a) \cdot (r + \gamma V_\pi(s')) \text{ for all } s \in \mathcal{S}$$

- Optimal Value Function $V_*(s) = \max_\pi V_\pi(s)$ for all $s \in \mathcal{S}$

$$V_*(s) = \max_a \sum_{r,s'} p(r, s'|s, a) \cdot (r + \gamma V_*(s')) \text{ for all } s \in \mathcal{S}$$

- *There exists an Optimal Policy* π_* achieving $V_*(s)$ for all $s \in \mathcal{S}$
- Determining $V_\pi(s)$ known as *Prediction*, and $V_*(s)$ known as *Control*
- The above recursive equations are called *Bellman equations*
- In continuous time, refered to as *Hamilton-Jacobi-Bellman (HJB)*
- The algorithms based on Bellman equations are broadly classified as:
  - Dynamic Programming
  - Reinforcement Learning

Dynamic Programming

- When Probabilities Model is known $\Rightarrow$ *Dynamic Programming* (DP)
- DP Algorithms take advantage of knowledge of probabilities
- So, DP Algorithms do not require interaction with the environment
- In the Language of AI, DP is a type of *Planning Algorithm*
- DP algorithms are iterative algorithms based on Fixed-Point Theorem
- Finding a *Fixed Point* of Operator based on Bellman Equation
- Why is DP not effective in practice?
  - Curse of Dimensionality
  - Curse of Modeling
- Curse of Dimensionality can be partially cured with Approximate DP
- To resolve both curses effectively, we need RL

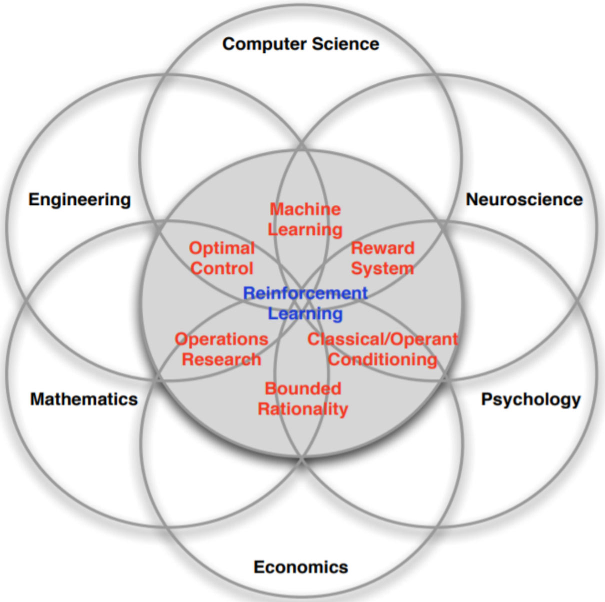
Reinforcement Learning

- Typically in real-world, we don't have access to a Probabilities Model
- All we have is access to an environment serving individual transitions
- Even if MDP model is available, model updates can be challenging
- Often real-world models end up being too large or too complex
- Sometimes estimating a *sampling model* is much more feasible
- RL interacts with either *actual* or *simulated* environment
- Either way, we receive *individual transitions* to next state and reward
- RL is a "trial-and-error" approach linking *Actions* to *Returns*
- Try different actions & learn what works, what doesn't
- This is hard because actions have overlapping reward sequences
- Also, sometimes Actions result in *delayed Rewards*

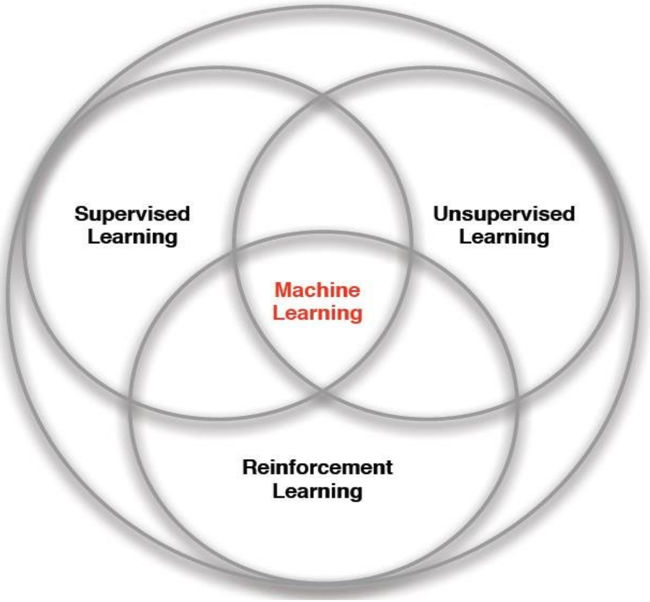
RL: Learning Value Function Approximation from Samples

- RL incrementally learns the Value Function from transitions data
- Appropriate Approximation of Value Function is key to success
- Deep Neural Networks are typically used for function approximation
- Big Picture: Sampling and Function Approximation come together
- RL algorithms are clever about balancing "explore" versus "exploit"
- Most RL Algorithms are founded on the Bellman Equations
- **Promise of modern A.I. is based on success of RL algorithms**
- Potential for automated decision-making in many industries
- In 10-20 years: Bots that act or behave more optimal than humans
- RL already solves various low-complexity real-world problems
- RL might soon be the most-desired skill in the technical job-market
- Possibilities in Finance are endless (we cover 5 important problems)
- Studying RL is a lot of fun! (interesting in theory as well as coding)

Many Faces of Reinforcement Learning



Vague (but in-vogue) Classification of Machine Learning



Overview of the Course

- Theory of Markov Decision Processes (MDPs)
- Dynamic Programming (DP) Algorithms
- Approximate DP and Backward Induction Algorithms
- Reinforcement Learning (RL) Algorithms
- Plenty of Python implementations of models and algorithms
- Apply these algorithms to 5 Financial/Trading problems:
    - (Dynamic) Asset-Allocation to maximize Utility of Consumption
    - Pricing and Hedging of Derivatives in an Incomplete Market
    - Optimal Exercise/Stopping of Path-dependent American Options
    - Optimal Trade Order Execution (managing Price Impact)
    - Optimal Market-Making (Bids and Asks managing Inventory Risk)
- By treating each of the problems as MDPs (i.e., Stochastic Control)
- We will go over classical/analytical solutions to these problems
- Then introduce real-world considerations, and tackle with RL (or DP)
- Course blends Theory/Math, Algorithms/Coding, Real-World Finance

Optimal Asset Allocation to Maximize Consumption Utility

- You can invest in (allocate wealth to) a collection of assets
- Investment horizon is a fixed length of time
- Each risky asset characterized by a probability distribution of returns
- Periodically, you are re-allocate your wealth to the various assets
- Transaction Costs & Constraints on trading hours/quantities/shorting
- Allowed to consume a fraction of your wealth at specific times
- Dynamic Decision: Time-Sequenced Allocation & Consumption
- To maximize horizon-aggregated *Risk-Adjusted Consumption*
- *Risk-Adjustment* involves a study of *Utility Theory*

MDP for Optimal Asset Allocation problem

- *State* is [Current Time, Current Holdings, Current Prices]
- *Action* is [Allocation Quantities, Consumption Quantity]
- *Action*s limited by various real-world trading constraints
- *Reward* is Utility of Consumption less Transaction Costs
- *State*-transitions governed by risky asset movements

Derivatives Pricing and Hedging in an Incomplete Market

- Classical Pricing/Hedging Theory assumes "frictionless market"
- Technically, refered to as arbitrage-free and complete market
- *Complete market* means derivatives can be perfectly replicated
- But real world has transaction costs and trading constraints
- So real markets are incomplete where classical theory doesn't fit
- How to price and hedge in an *Incomplete Market*?
- Maximize "risk-adjusted-return" of the derivative plus hedges
- Similar to Asset Allocation, this is a stochastic control problem
- Deep Reinforcement Learning helps solve when framed as an MDP

MDP for Pricing/Hedging in an Incomplete Market

- *State* is [Current Time, PnL, Hedge Qtys, Hedge Prices]
- *Action* is Units of Hedges to be traded at each time step
- *Reward* only at termination, equal to Utility of terminal PnL
- *State*-transitions governed by evolution of hedge prices
- Optimal Policy $\Rightarrow$ Derivative Hedging Strategy
- Optimal Value Function $\Rightarrow$ Derivative Price

Optimal Exercise of Path-dependent American Options

- An American option can be exercised anytime before option maturity
- Key decision at any time is to exercise or continue
- The default algorithm is Backward Induction on a tree/grid
- But it doesn't work for American options with complex payofss
- Also, it's not feasible when state dimension is large
- Industry-Standard: Longstaff-Schwartz's simulation-based algorithm
- RL is an attractive alternative to Longstaff-Schwartz
- RL is straightforward once Optimal Exercise is modeled as an MDP

MDP for Optimal American Options Exercise

- *State* is [Current Time, History of Underlying Security Prices]
- *Action* is Boolean: Exercise (i.e., Payoff and Stop) or Continue
- *Reward* always 0, except upon Exercise (= Payoff)
- *State*-transitions governed by Underlying Prices' Stochastic Process
- Optimal Policy $\Rightarrow$ Optimal Stopping $\Rightarrow$ Option Price
- Can be generalized to other Optimal Stopping problems

Optimal Trade Order Execution (controlling Price Impact)

- You are tasked with selling a large qty of a (relatively less-liquid) stock
- You have a fixed horizon over which to complete the sale
- Goal is to maximize aggregate sales proceeds over horizon
- If you sell too fast, *Price Impact* will result in poor sales proceeds
- If you sell too slow, you risk running out of time
- We need to model temporary and permanent *Price Impact*s
- Objective should incorporate penalty for variance of sales proceeds
- Again, this amounts to maximizing Utility of sales proceeds

MDP for Optimal Trade Order Execution

- *State* is [Time Remaining, Stock Remaining to be Sold, Market Info]
- *Action* is Quantity of Stock to Sell at current time
- *Reward* is Utility of Sales Proceeds (i.e., Variance-adjusted-Proceeds)
- *Reward* & *State*-transitions governed by *Price Impact Model*
- Real-world *Model* can be quite complex (*Order Book Dynamics*)

Optimal Market-Making (controlling Inventory Buildup)

- Market-maker's job is to submit bid and ask prices (and sizes)
- On the Trading *Order Book* (which moves due to other players)
- Market-maker needs to adjust bid/ask prizes/sizes appropriately
- By anticipating the *Order Book Dynamics*
- Goal is to maximize *Utility of Gains* at the end of a suitable horizon
- If Buy/Sell LOs are too narrow, more frequent but small gains
- If Buy/Sell LOs are too wide, less frequent but large gains
- Market-maker also needs to manage potential unfavorable inventory (long or short) buildup and consequent unfavorable liquidation
- This is a classical stochastic control problem

MDP for Optimal Market-Making

- *State* is [Current Time, Mid-Price, PnL, Inventory of Stock Held]
- *Action* is Bid & Ask Prices & Sizes at each time step
- *Reward* is Utility of Gains at termination
- *State*-transitions governed by probabilities of hitting/lifting Bid/Ask
- Also governed by Order Book Dynamics (can be quite complex)

Week by Week (Tentative) Schedule

- W1: Markov Decision Processes
- W2: Bellman Equations & Dynamic Programming Algorithms
- W3: Backward Induction and Approximate DP Algorithms
- W4: Optimal Asset Allocation & Derivatives Pricing/Hedging
- W5: Options Exercise, Order Execution, Market-Making
- W6: RL For Prediction (MC, TD, TD(λ))
- W7: RL for Control (SARSA, Q-Learning)
- W8: Batch Methods (DQN, LSTD/LSPI) and Gradient TD
- W9: Policy Gradient, Model-based RL, Explore v/s Exploit
- W10: Read-World RL and Guest Lecture by an Industry leader

Some Landmark Papers we cover in this course

- Merton's solution for Optimal Portfolio Allocation/Consumption
- Longstaff-Schwartz Algorithm for Pricing American Options
- Almgren-Chriss paper on Optimal Order Execution
- Avellaneda-Stoikov paper on Optimal Market-Making
- Original DQN paper and Nature DQN paper
- Lagoudakis-Parr paper on Least Squares Policy Iteration
- Sutton, McAllester, Singh, Mansour's Policy Gradient Theorem
- Chang, Fu, Hu, Marcus' AMS origins of Monte Carlo Tree Search

Similar Courses offered at Stanford

- AA 228/CS 238 (Mykel Kochenderfer)
- CS 234 (Emma Brunskill)
- CS 332 (Emma Brunskill)
- MS&E 338 (Ben Van Roy)
- EE 277 (Ben Van Roy)
- MS&E 251 (Edison Tse)
- MS&E 348 (Gerd Infanger)
- MS&E 351 (Ben Van Roy)
- MS&E 339 (Ben Van Roy)

Salient/Distinguishing features of this Course

- Emphasis on Foundations and Core Concepts
- More about *why* and *how*, versus *what*
- Balance between mathematical precision and intuitive understanding
- Coding from scratch, avoiding standard packages
- Encourages *Creator/Builder* mindset, versus *User* mindset
- Emphasis on code design driven by mathematical concepts/structures
- Key purpose of coding: Enables long-term retention of key learnings
- Several financial trading applications (and a couple from Retail)
- Coverage of continuous-time versions (elegant, analytical)
- I will dispel some common myths about industry versus academia