Supplementary File of "Performance Analysis of Constrained Evolutionary Multi-Objective Optimization Algorithms on Artificial and Real-World Problems"

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Table S1: Artificial constrained multi-objective optimization problems used in this study.

Problem	m	d	ng	nh	Problem	m	d	ng	nh
LIR-CMOP1	2	6	2	0	MW1	2	6	1	0
LIR-CMOP2	2	6	2	0	MW2	2	6	1	0
LIR-CMOP3	2	6	3	0	MW3	2	6	2	0
LIR-CMOP4	2	6	3	0	MW4	3	6	1	0
LIR-CMOP5	2	6	2	0	MW5	2	6	3	0
LIR-CMOP6	2	6	2	0	MW6	2	6	1	0
LIR-CMOP7	2	6	3	0	MW7	2	6	2	0
LIR-CMOP8	2	6	3	0	MW8	3	6	1	0
LIR-CMOP9	2	6	2	0	MW9	2	6	1	0
LIR-CMOP10	2	6	2	0	MW10	2	6	3	0
LIR-CMOP11	2	6	2	0	MW11	2	6	4	0
LIR-CMOP12	2	6	2	0	MW12	2	6	2	0
LIR-CMOP13	3	6	2	0	MW13	2	6	2	0
LIR-CMOP14	3	6	3	0	MW14	3	6	1	0

Table S2: Real-world constrained multi-objective optimization problems used in this study.

Problem	m	d	ng	nh	Problem	m	d	ng	nh	Problem	m	d	ng	nh
CRE2-3-1	2	3	3	0	RCM1	2	4	2	2	RCM27	2	3	3	0
CRE2-4-2	2	4	4	0	RCM2	2	5	5	0	RCM28	2	7	4	4
CRE2-4-3	2	4	4	0	RCM3	2	4	3	0	RCM29	2	7	9	0
CRE2-7-4	2	7	11	0	RCM4	2	3	4	0	RCM30	2	25	24	0
CRE2-4-5	2	4	1	0	RCM5	2	4	4	0	RCM31	2	25	24	0
CRE3-7-1	3	7	10	0	RCM6	2	7	11	0	RCM32	2	25	24	0
CRE3-6-2	3	6	9	0	RCM7	2	4	1	0	RCM33	2	30	29	0
CRE5-3-1	5	3	7	0	RCM8	3	7	9	0	RCM34	2	30	29	0
					RCM10	2	2	2	0	RCM35	2	30	29	0
					RCM11	5	3	7	0	RCM36	2	28	0	24
					RCM12	2	4	1	0	RCM37	2	28	0	24
					RCM13	3	7	11	0	RCM38	2	28	0	24
					RCM14	2	5	8	0	RCM39	3	28	0	24
					RCM15	2	3	8	0	RCM40	2	34	0	26
					RCM16	2	2	2	0	RCM41	3	34	0	26
					RCM17	3	6	9	0	RCM42	2	34	0	26
					RCM18	2	3	3	0	RCM43	2	34	0	26
					RCM19	3	10	10	0	RCM44	3	34	0	26
					RCM20	2	4	7	0	RCM45	3	34	0	26
					RCM21	2	6	4	0	RCM46	4	34	0	26
					RCM22	2	9	2	4	RCM47	2	18	0	12
					RCM23	2	6	1	4	RCM48	2	18	0	12
					RCM24	3	9	0	6	RCM49	3	18	0	12
					RCM25	2	2	2	0	RCM50	2	6	0	1
					RCM26	2	3	1	1					

Table S3: Average hypervolume of the final population of constraint EMO algorithms on easy CMOPs.

			CDP		(Coevolution	<u> </u>	Two-stage				
Problem	m	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	
LIR-CMOP6	2	0.30	0.31 (≈)	0.34 (+)	$0.32 (\approx)$	0.35 (+)	0.35 (+)	0.35 (+)	0.35 (+)	0.35 (+)	0.35 (+)	
LIR-CMOP7	2	0.43	0.43 (-)	0.43 (-)	0.42 (-)	0.43 (+)	0.43 (+)	0.43 (+)	0.43 (+)	0.43 (+)	0.43 (+)	
LIR-CMOP8	2	0.43	0.43 (-)	0.43 (+)	0.42 (-)	0.43 (+)	0.43 (+)	$0.43 (\approx)$	0.43 (+)	0.43 (+)	0.43 (+)	
LIR-CMOP11	2	0.60	$0.63 \ (\approx)$	$0.64 (\approx)$	0.63 (+)	0.65 (+)	0.65 (+)	0.65 (+)	0.65 (+)	$0.62 (\approx)$	0.65 (+)	
LIR-CMOP12	2	0.54	$0.55~(\approx)$	$0.55~(\approx)$	$0.55~(\approx)$	0.56 (+)	0.56 (+)	0.56 (+)	0.56 (+)	0.56 (+)	0.56 (+)	
MW1	2	0.49	0.49(-)	0.49(-)	0.49 (-)	0.49 (+)	0.49 (+)	0.49 (-)	0.49 (+)	0.49 (+)	0.49(-)	
MW2	2	0.51	0.51 (+)	0.51 (+)	0.51 (+)	0.51 (+)	0.51 (+)	0.47 (-)	0.51 (+)	0.51 (+)	0.48 (-)	
MW3	2	0.47	0.47 (+)	0.47 (+)	0.47 (+)	0.47 (+)	0.47 (+)	$0.47 (\approx)$	0.47 (+)	0.47 (+)	0.47 (+)	
MW5	2	0.20	0.20 (+)	0.20 (-)	0.19 (-)	0.20 (+)	0.20 (+)	0.19 (-)	$0.20 \ (\approx)$	0.20 (-)	$0.20~(\approx)$	
MW6	2	0.21	0.21 (+)	0.21 (+)	0.21 (+)	0.21 (+)	0.21 (+)	0.15 (-)	0.21 (+)	0.21 (+)	0.17(-)	
MW7	2	0.31	0.31 (-)	0.31 (-)	0.31 (-)	0.31 (+)	0.31 (+)	$0.31 (\approx)$	0.31 (+)	0.31 (-)	0.31 (-)	
MW9	2	0.30	0.30 (+)	0.30 (+)	0.29(-)	0.30 (+)	0.30 (+)	0.29 (-)	0.30 (+)	0.30 (+)	$0.30 \ (\approx)$	
MW10	2	0.47	0.48 (+)	$0.49 (\approx)$	$0.49 \ (\approx)$	0.49 (+)	0.49 (+)	0.40 (-)	0.49 (+)	0.49 (+)	0.44 (-)	
MW12	2	0.54	0.55 (+)	0.55 (+)	0.54 (-)	0.55 (+)	0.54 (+)	0.54 (-)	0.54 (+)	0.54 (+)	$0.54~(\approx)$	
MW13	2	0.38	0.38 (-)	0.39 (+)	0.39 (+)	0.39 (+)	0.39 (+)	0.32 (-)	0.39 (+)	0.38 (-)	0.35(-)	
RCM1	2	0.54	0.03(-)	0.55 (+)	0.55 (+)	0.55 (+)	0.55 (+)	0.52 (-)	0.55 (+)	0.42 (-)	0.55 (+)	
RCM5	2	0.82	0.80 (-)	0.82 (+)	0.81 (-)	0.82 (+)	0.82 (+)	$0.82 (\approx)$	0.82 (+)	0.80 (-)	0.82 (+)	
RCM6	2	0.99	0.99(-)	0.99(-)	0.77(-)	0.99(-)	0.99(-)	0.99 (-)	0.99(-)	0.99 (+)	0.99(-)	
RCM7	2	0.49	0.47(-)	0.49 (+)	0.49 (+)	0.49 (+)	0.49 (+)	0.49 (+)	0.49 (+)	0.49 (+)	0.49 (+)	
RCM11	5	1.82	1.21 (-)	1.76 (-)	$1.82 (\approx)$	$1.82 (\approx)$	1.81 (-)	1.58 (-)	1.81 (-)	1.72 (-)	1.64 (-)	
RCM15	2	0.79	0.02 (-)	0.79(-)	0.78 (-)	0.79(-)	0.79(-)	0.79 (-)	0.79(-)	$0.79 \ (\approx)$	0.79 (+)	
RCM18	2	0.64	0.64 (+)	0.64 (+)	0.64 (+)	0.64 (+)	0.64 (+)	0.64 (+)	0.64 (+)	0.64 (+)	0.64 (+)	
RCM21	2	0.59	0.02 (-)	0.60 (+)	0.59 (-)	0.60 (+)	0.60 (+)	0.55 (-)	0.60 (+)	0.60 (+)	0.60 (+)	
RCM25	2	0.40	0.39(-)	0.41 (+)	0.41 (+)	0.41 (+)	0.41 (+)	0.41 (+)	0.41 (+)	0.39(-)	0.41 (+)	
RCM27	2	0.68	0.68 (+)	0.68 (+)	0.68 (+)	0.68 (+)	0.68 (+)	0.68 (+)	0.68 (+)	0.68 (+)	0.68 (+)	
CRE2-4-3	2	0.82	0.80 (-)	0.82 (+)	0.81 (-)	0.82 (+)	0.82 (+)	$0.82 (\approx)$	0.82 (+)	0.79(-)	0.82 (+)	
CRE2-4-5	2	0.82	0.82 (-)	0.82 (-)	0.82 (-)	0.82 (-)	0.82 (-)	0.82 (-)	0.82 (-)	0.82 (-)	0.82 (+)	
CRE3-7-1	3	0.71	0.09(-)	0.73 (+)	0.73 (+)	0.73 (+)	0.73 (+)	0.42 (-)	0.73 (+)	0.74 (+)	0.72 (+)	
CRE3-6-2	3	0.80	0.59 (-)	0.78 (-)	0.79 (-)	0.80 (+)	0.81 (+)	0.79 (-)	$0.80 \ (\approx)$	0.61 (-)	0.80 (-)	
(+/-/≈	:)		(9/17/3)	(17/9/3)	(11/14/4)	(25/3/1)	(25/4/0)	(8/16/5)	(23/4/2)	(17/10/2)	(17/9/3)	

Table S4: Average hypervolume of the final population of each algorithm on CMOPs Type 1 (easy for NSGA-II but difficult for some other algorithms).

			CDP			Coevolution	1	Two-stage				
Problem	m	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	
RCM3	2	0.96	0.06 (-)	0.96 (-)	0.94 (-)	0.96 (-)	0.96 (-)	0.93 (-)	0.96 (-)	0.93 (-)	0.96 (-)	
RCM4	2	0.93	0.03(-)	0.92 (-)	0.92 (-)	0.92 (-)	0.92 (-)	0.92 (-)	0.92 (-)	0.86 (-)	0.92 (-)	
RCM10	2	1.01	0.01 (-)	1.00 (-)	1.00 (-)	1.00 (-)	1.00 (-)	1.01 (-)	1.00 (-)	1.00 (-)	1.00 (-)	
RCM12	2	0.92	0.07 (-)	0.92 (-)	0.92 (-)	0.92 (-)	0.92 (-)	0.90 (-)	0.92 (-)	0.75(-)	0.92 (-)	
RCM14	2	0.87	0.07 (-)	0.87 (-)	0.87 (-)	0.87(-)	0.87(-)	0.82 (-)	0.86 (-)	0.55 (-)	0.86 (-)	
RCM16	2	0.89	0.01 (-)	0.89(-)	0.89 (-)	0.89(-)	0.89(-)	$0.89 (\approx)$	0.89(-)	0.88(-)	0.89(-)	
CRE2-3-1	2	0.96	0.06(-)	0.96 (-)	0.94 (-)	0.96 (-)	0.96 (-)	0.93 (-)	0.96 (-)	0.93 (-)	0.96 (-)	
CRE2-4-2	2	0.94	0.04 (-)	0.93 (-)	0.93 (-)	0.93 (-)	0.93 (-)	0.93 (-)	0.93 (-)	0.87(-)	0.93(-)	
CRE2-7-4	2	0.99	0.99(-)	0.99(-)	0.97 (-)	0.99(-)	0.99(-)	0.99 (-)	0.99(-)	0.98 (-)	0.99(-)	
CRE5-3-1	5	1.83	1.21 (-)	1.77(-)	$1.82 (\approx)$	1.82 (-)	1.81 (-)	1.58 (-)	1.81 (-)	1.70 (-)	1.66 (-)	
(+/-	-/≈)		(0/10/0)	(0/10/0)	(0/9/1)	(0/10/0)	(0/10/0)	(0/9/1)	(0/10/0)	(0/10/0)	(0/10/0)	

Table S5: Average hypervolume of the final population of constraint EMO algorithms on hard CMOPs.

			CDP		C	oevolution	1		Two-	stage	
Problem	m	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
MW11	2	0.55	0.55 (+)	0.55 (+)	0.55 (+)	0.55 (+)	0.55 (+)	0.55 (+)	0.55 (+)	0.55 (+)	0.55 (+)
RCM8	3	0.99	0.06(-)	0.95(-)	1.00 (+)	0.99 (+)	$0.99 (\approx)$	0.47 (-)	$0.99 (\approx)$	1.00 (+)	0.98(-)
RCM13	3	1.20	1.19(-)	1.20 (+)	1.17 (≈)	1.18 (-)	1.18 (-)	0.92 (-)	1.17(-)	1.14(-)	1.17(-)
RCM17	3	0.07	$0.13 \ (\approx)$	0.07 (-)	0.07 (-)	0.06(-)	0.08 (+)	0.20 (+)	0.04(-)	0.07(-)	0.04 (-)
RCM19	3	0.87	0.40 (-)	$0.89 \ (\approx)$	0.18 (-)	0.84(-)	0.73(-)	0.69 (-)	0.85(-)	$0.86 \ (\approx)$	0.99 (+)
RCM20	2	0.66	0.00(-)	0.50 (-)	0.03(-)	0.60 (-)	0.57(-)	0.18 (-)	0.59(-)	0.24 (-)	0.73 (+)
RCM22	2	0.00	$0.02 (\approx)$	$0.00 \ (\approx)$	$0.00 \ (\approx)$	$0.00 \ (\approx)$	0.05 (+)	0.31 (+)	$0.00 \ (\approx)$	$0.01 (\approx)$	0.16 (+)
RCM23	2	0.12	$0.14 (\approx)$	0.00(-)	0.00 (-)	$0.13 \ (\approx)$	0.18 (+)	0.21 (+)	$0.11 (\approx)$	0.18 (+)	0.07(-)
RCM24	3	0.00	$0.00 \ (\approx)$	0.03 (+)	0.04 (+)	$0.00 \ (\approx)$	0.05 (+)	0.06 (+)	$0.00 \ (\approx)$	0.08 (+)	0.10 (+)
RCM28	2	0.00	$0.00 \ (\approx)$	0.21 (+)	$0.00 \ (\approx)$	$0.00 \ (\approx)$	0.04 (+)	0.07 (+)	$0.00 \ (\approx)$	0.23 (+)	0.13 (+)
RCM30	2	0.20	$0.31 (\approx)$	0.08(-)	0.05(-)	0.10 (-)	0.09(-)	0.39 (+)	0.08(-)	0.00(-)	0.06 (-)
RCM31	2	0.12	$0.19 (\approx)$	0.00(-)	0.00 (-)	$0.12 (\approx)$	$0.06 (\approx)$	$0.12 (\approx)$	$0.04~(\approx)$	0.00(-)	$0.03~(\approx)$
RCM32	2	0.21	$0.24~(\approx)$	0.02 (-)	0.00 (-)	$0.10 \ (\approx)$	$0.15 (\approx)$	$0.28~(\approx)$	$0.14~(\approx)$	0.00(-)	0.00(-)
RCM33	2	0.01	0.09 (+)	$0.02 (\approx)$	$0.00 \ (\approx)$	$0.05~(\approx)$	$0.03 \ (\approx)$	$0.02 (\approx)$	$0.02 (\approx)$	$0.00 \ (\approx)$	$0.00 \ (\approx)$
RCM34	2	0.12	$0.09 (\approx)$	0.00(-)	0.00(-)	$0.05~(\approx)$	$0.02 (\approx)$	$0.13 \ (\approx)$	$0.04~(\approx)$	0.00(-)	0.00(-)
RCM35	2	0.06	0.17 (+)	0.00(-)	0.00 (-)	$0.08 \ (\approx)$	$0.00 \ (\approx)$	$0.04 (\approx)$	$0.04~(\approx)$	0.00(-)	$0.00 \ (\approx)$
RCM36	2	0.00	$0.00 \ (\approx)$								
RCM37	2	0.00	$0.00 \ (\approx)$								
RCM38	2	0.00	$0.00 \ (\approx)$								
RCM39	3	0.00	$0.00 \ (\approx)$								
RCM40	2	0.00	$0.00 \ (\approx)$								
RCM41	3	0.00	$0.00 \ (\approx)$	$0.00 \ (\approx)$	$0.00~(\approx)$	$0.00 \ (\approx)$					
RCM42	2	0.00	$0.00 \ (\approx)$	$0.00 \ (\approx)$	$0.00~(\approx)$	$0.00 \ (\approx)$					
RCM43	2	0.00	$0.00 \ (\approx)$								
RCM44	3	0.00	$0.00 \ (\approx)$								
RCM45	3	0.00	$0.00 \ (\approx)$								
RCM46	4	0.00	$0.00 \ (\approx)$								
RCM47	2	0.00	$0.00 \ (\approx)$								
RCM48	2	0.00	$0.00 \ (\approx)$								
RCM49	3	0.00	$0.00 \ (\approx)$								
RCM50	2	0.27	$0.31 (\approx)$	0.02 (-)	0.00 (-)	0.21 (-)	0.11 (-)	$0.25~(\approx)$	$0.23~(\approx)$	0.04 (-)	0.48 (+)
(+/-	·/≈))	(3/4/24)	(4/10/17)	(3/10/18)	(2/6/23)	(6/5/20)	(7/4/20)	(1/5/25)	(5/9/17)	(7/7/17)

Table S6: Average hypervolume of the final population of each algorithm on CMOPs Type 2 (difficult for NSGA-II but easy for some other algorithms).

			CDP		(Coevolution	1	Two-stage					
Problem	m	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10		
LIR-CMOP1	2	0.21	0.26 (+)	0.10 (-)	0.05 (-)	0.18 (≈)	0.25 (+)	0.34 (+)	0.17 (-)	0.13 (-)	0.29 (+)		
LIR-CMOP2	2	0.46	0.55 (+)	$0.49 (\approx)$	$0.50 (\approx)$	0.55 (+)	0.56 (+)	0.68 (+)	$0.47~(\approx)$	0.27(-)	0.57 (+)		
LIR-CMOP3	2	0.15	0.21 (+)	0.08(-)	0.06 (-)	$0.17 (\approx)$	0.23 (+)	0.32 (+)	$0.14 (\approx)$	0.10(-)	0.21 (+)		
LIR-CMOP4	2	0.33	0.47 (+)	0.40 (+)	$0.33 \ (\approx)$	0.40 (+)	0.43 (+)	0.63 (+)	$0.32 (\approx)$	0.22(-)	0.45 (+)		
LIR-CMOP5	2	0.59	$0.59 (\approx)$	$0.66 (\approx)$	$0.65 (\approx)$	0.68 (+)	0.68 (+)	0.68 (+)	0.68 (+)	0.68 (+)	0.68 (+)		
LIR-CMOP9	2	0.37	$0.39 (\approx)$	$0.41 (\approx)$	0.48 (+)	0.50 (+)	0.50 (+)	0.50 (+)	0.50 (+)	0.47 (+)	0.50 (+)		
LIR-CMOP10	2	0.60	0.60 (-)	0.66 (+)	0.65 (+)	0.67 (+)	0.67 (+)	0.67 (+)	0.67 (+)	0.66 (+)	0.67 (+)		
LIR-CMOP13	3	0.65	0.69 (+)	0.69 (+)	0.67 (+)	0.68 (+)	0.68 (+)	0.64 (-)	0.68 (+)	0.68 (+)	0.66 (+)		
LIR-CMOP14	3	0.65	0.69 (+)	0.68 (+)	0.67 (+)	0.68 (+)	0.68 (+)	0.64 (-)	0.68 (+)	0.68 (+)	0.67 (+)		
MW4	3	1.04	1.06 (+)	1.06 (+)	1.06 (+)	1.06 (+)	1.06 (+)	1.02 (-)	1.06 (+)	1.06 (+)	1.05 (+)		
MW8	3	0.65	0.67 (+)	0.65 (+)	0.65 (+)	0.67 (+)	0.67 (+)	0.59(-)	0.67 (+)	0.67 (+)	0.64 (-)		
MW14	3	0.56	0.55 (-)	0.58 (+)	0.58 (+)	0.59 (+)	0.59 (+)	0.49 (-)	0.59 (+)	0.57 (+)	0.58 (+)		
RCM2	2	0.49	0.08(-)	$0.76 (\approx)$	0.39 (≈)	$0.31 (\approx)$	$0.71 (\approx)$	$0.85~(\approx)$	$0.38 \ (\approx)$	$0.55 (\approx)$	0.85 (+)		
RCM26	2	0.30	0.36 (+)	0.23(-)	0.23 (-)	$0.29 \ (\approx)$	$0.27 (\approx)$	0.46 (+)	$0.27 (\approx)$	0.26 (-)	0.45 (+)		
RCM29	2	0.59	$0.60 (\approx)$	0.60 (+)	0.37 (-)	$0.59 (\approx)$	0.60 (+)	0.61 (+)	$0.56 (\approx)$	$0.60 (\approx)$	0.59 (+)		
(+/-/≈)			(9/3/3)	(8/3/4)	(7/4/4)	(10/0/5)	(13/0/2)	(9/5/1)	(8/1/6)	(8/5/2)	(14/1/0)		

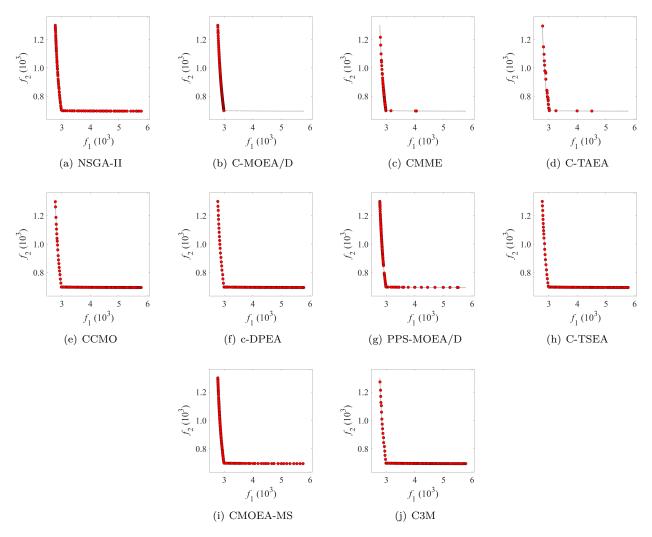


Fig. S1: Final population from a single run with the median hypervolume value among 31 runs for each of ten compared algorithms on LIR-CMOP14.

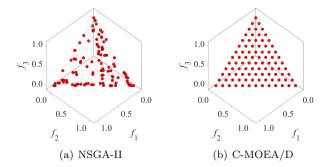


Fig. S2: Final population from a single run with the median hypervolume value among 31 runs for each of NSGA-II and C-MOEA/D on MW4.

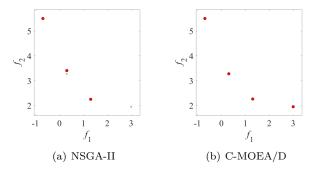


Fig. S3: Final population from a single run with the median hypervolume value among 31 runs for each of NSGA-II and PPS-MOEA/D on RCM29. The gray points are the approximated CPF of RCM29.

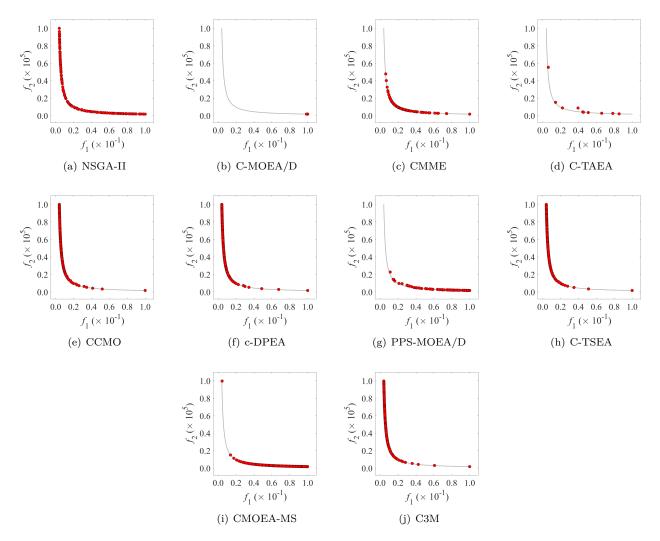


Fig. S4: Final population obtained by the worst run among 31 runs for each of ten compared algorithms on RCM3.

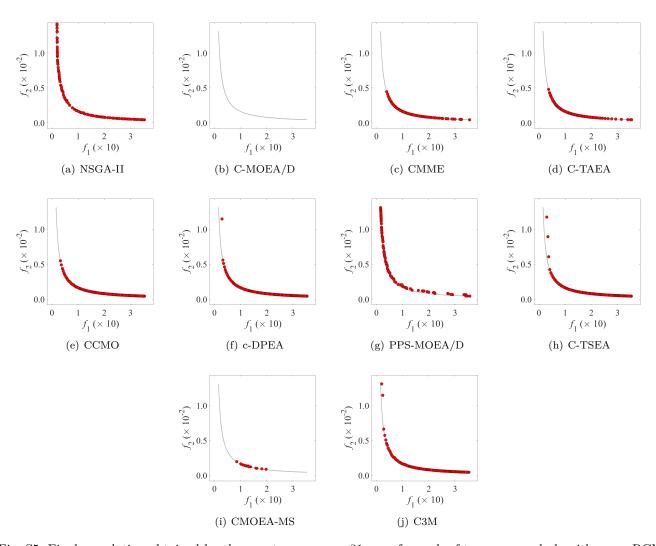


Fig. S5: Final population obtained by the worst run among 31 runs for each of ten compared algorithms on RCM4.

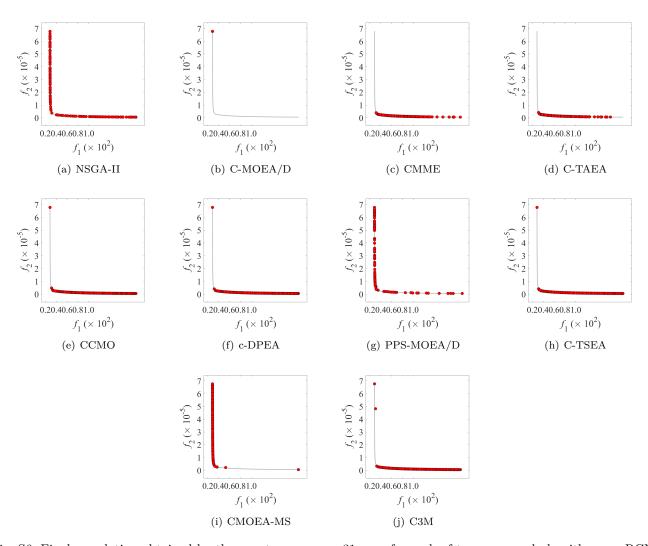


Fig. S6: Final population obtained by the worst run among 31 runs for each of ten compared algorithms on RCM10.

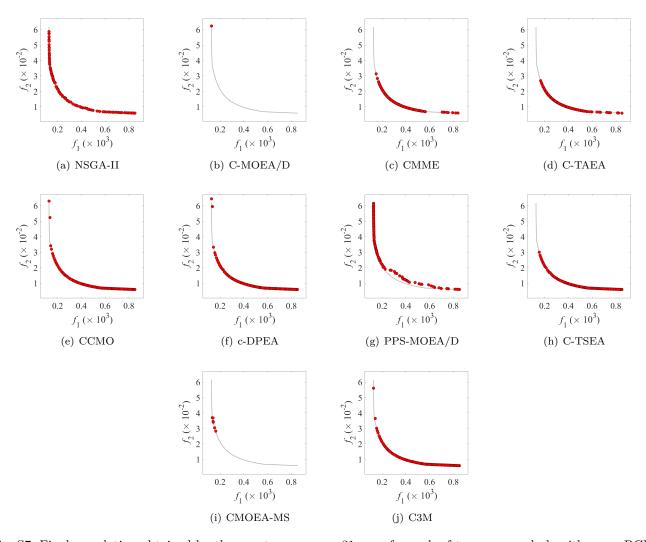


Fig. S7: Final population obtained by the worst run among 31 runs for each of ten compared algorithms on RCM12.

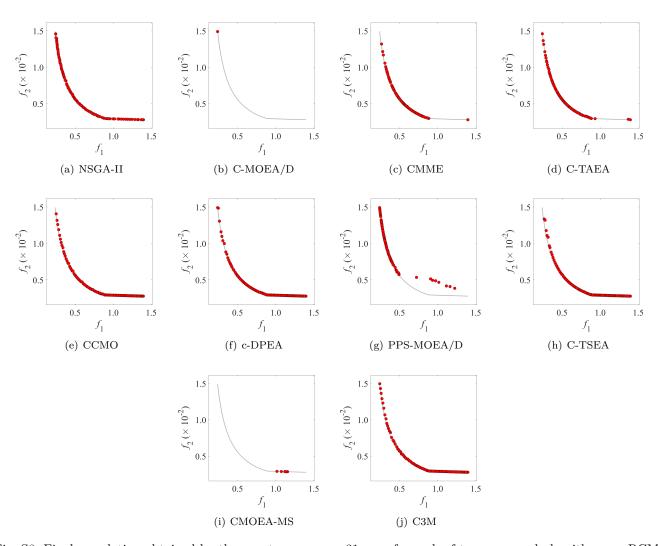


Fig. S8: Final population obtained by the worst run among 31 runs for each of ten compared algorithms on RCM14.

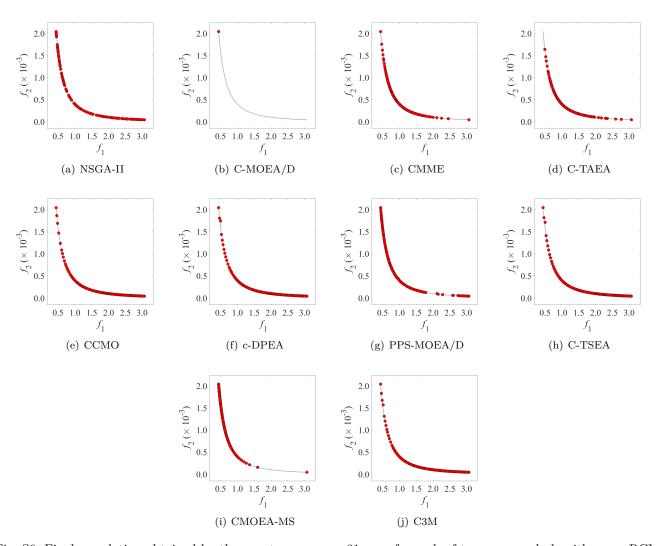


Fig. S9: Final population obtained by the worst run among 31 runs for each of ten compared algorithms on RCM16.

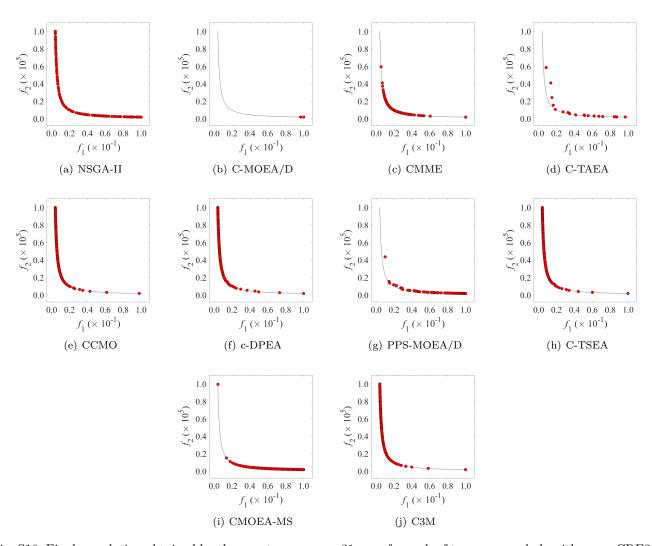


Fig. S10: Final population obtained by the worst run among 31 runs for each of ten compared algorithms on CRE2-3-1.

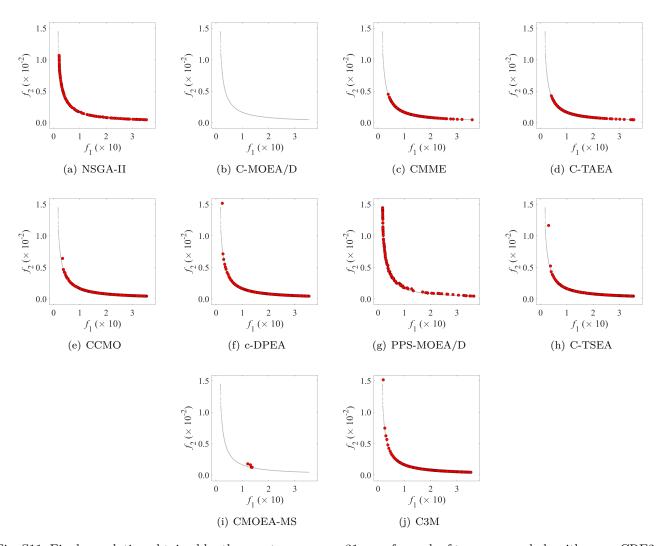


Fig. S11: Final population obtained by the worst run among 31 runs for each of ten compared algorithms on CRE2-4-2.

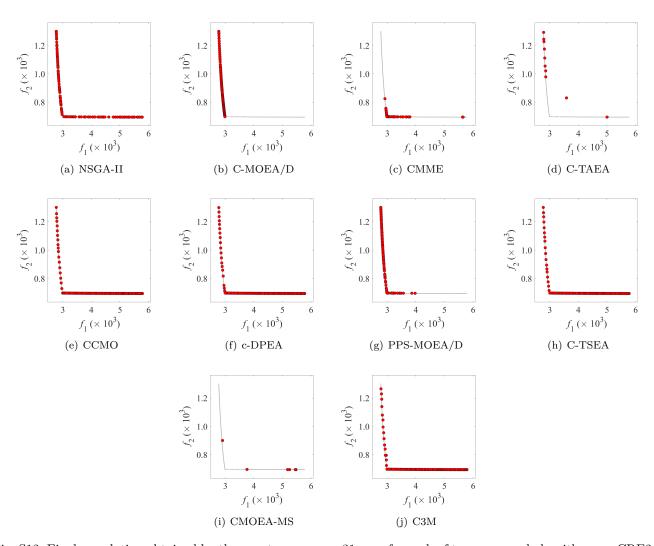


Fig. S12: Final population obtained by the worst run among 31 runs for each of ten compared algorithms on CRE2-7-2.