Model Predictive Control

Chapter 4: Constrained Finite Time Optimal Control

Prof. Melanie Zeilinger

ETH Zurich

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Coauthors: Prof. Manfred Morari, University of Pennsylvania
Prof. Colin Jones, EPFL
Prof. Francesco Borrelli, UC Berkeley

Constrained Infinite Time Optimal Control (what we would like to solve)

$$
\begin{aligned}
J^\star_\infty(x(0)) = \min_{u(\cdot)} & \sum_{i=0}^{\infty} l(x_i, u_i) \\
\text{subj. to } & x_{i+1} = Ax_i + Bu_i, \;\; i = 0, \ldots, \infty \\
& x_i \in \mathcal{X}, u_i \in \mathcal{U}, i = 0, \ldots, \infty \\
& x_0 = x(0)
\end{aligned}
$$

- **Stage cost** $l(x, u)$: "cost" of being in state x and applying input u
- Optimizing over a trajectory provides a **tradeoff between short- and long-term benefits** of actions
- We'll see that such a control law has many beneficial properties...
  ... but we can't compute it: there are an **infinite number of variables**

Constrained Finite Time Optimal Control (what we can sometimes solve)

$$
\begin{aligned}
J^{\star}_{k\to k+N|k}(x(k)) = \min_{U_{k\to k+N|k}} \; & l_f(x_{k+N|k}) + \sum_{i=0}^{N-1} l(x_{k+i|k}, u_{k+i|k}) \\
\text{subj. to } \; & x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k}, \;\; i = 0, \ldots, N-1 \\
& x_{k+i|k} \in \mathcal{X}, u_{k+i|k} \in \mathcal{U}, i = 0, \ldots, N-1 \\
& x_{k+N|k} \in \mathcal{X}_f \\
& x_{k|k} = x(k)
\end{aligned} \qquad (1)
$$

where $U_{k\to k+N|k} = \{u_{k|k}, \ldots, u_{k+N-1|k}\}$.

Truncate after a finite horizon:

- $l_f(x_{k+N|k})$: Approximates the 'tail' of the cost
- $\mathcal{X}_f$: Approximates the 'tail' of the constraints