Monetary and Exchange Rate Policies in a Global Economy*

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October 16, 2024

Abstract

Foreign exchange intervention (FXI) is often justified to stabilize exchange rates in small-open economies. However, in large economies, FXI may have international spillover effects. I find that, in a large two-country model, monetary policy and FXI no longer separately target inflation and exchange rate. Instead, under cooperation, optimal FXI mitigates the inflation-output trade-off of monetary policy and improves monetary autonomy by allowing central banks to stabilize inflation without large changes in the monetary policy rate. However, this comes at the cost of distortions to international consumption allocation, disproportionately strengthening the local purchasing power and demand while depressing the US demand. This trade-off between internal and external objectives renders the combination of monetary policy and FXI the optimal policy. Moreover, under dollar pricing, FXI stabilizes local inflation with limited spillover effects on US inflation. Finally, without cooperation, nationally oriented interventions can further exacerbate the international risk-sharing distortion.

Keywords: Capital flows, International risk-sharing, Foreign exchange intervention, Optimal targeting rules, International policy cooperation.

JEL Classification Codes: E58, F31, F32, F41, F42.

*Most recent version available here. I am grateful to Vasco Carvalho, Giancarlo Corsetti, and Chryssi Giannitsarou for their invaluable guidance and support. I acknowledge helpful comments from Florin Bilbiie, Markus Brunnermeier, Edouard Challe, Sebastian Graves, John Jones, Simon Lloyd, Lucio Sarno, Yannick Timmer, François Verde, Jesús Fernández-Villaverde, and seminar participants at the 5th Emerging Market Macroeconomics Workshop, MMF PhD Conference, North American Meeting of the Econometric Society, EEA Annual Conference, MMF Annual Conference, Doctoral Workshop on Quantitative Dynamic Economics, Hitotsubashi University, and the University of Cambridge. I acknowledge financial support from the Keynes Fund.

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1 Introduction

A classical theory by Mundell (1957) and Fleming (1962) suggests a "trilemma", where monetary policy faces a trade-off between inflation and exchange rate stabilization under free capital mobility. In a modern financially globalized world, countries use a mixed approach of conventional and unconventional policy tools to manage the capital account and insulate themselves from international spillovers of shocks and crises (Rey 2015, Kalemli-Özcan 2019, Miranda-Agrippino and Rey 2020). In particular, foreign exchange intervention (FXI), i.e. purchases or sales of foreign currency reserves by central banks, is a popular policy tool among central banks, with more than 100 countries regularly using FXI (Adler et al. 2023).

There have been recent discussions on optimal monetary policy and FXI, and the consensus is that they are two separate policy instruments: monetary policy should target inflation, while FXI should target the exchange rate. However, this rationale is solely based on a small-open economy case, taking the international price of goods as given. In reality, large open economies are also active users of FXI. Most notably, China has 3.5 trillion dollars and Japan has 1.4 trillion dollars of FX reserves, accounting for nearly 40% of the world's FX reserves (IMF International Financial Statistics). These countries are on the US monitoring list for accumulating excess FX reserves and gaining an unfair competitive advantage in international trade (US Department of the Treasury 2024). Moreover, during COVID-19 and the Russian-Ukraine war, which caused worldwide inflation and currency depreciation against the dollar, large countries sold FX reserves to protect their currency. For instance, Australia and Brazil sold around 20 dollars in 2020, and India and Japan sold 32 billion and 63 billion dollars in 2022, respectively. In such a large open-economy context, exchange rate manipulation can change the international price and the world demand for domestic and foreign goods. This cross-border spillover can in turn affect the optimal designs of monetary policy and FXI, giving rise to a challenging policy trade-off between stabilizing internal objectives (inflation and output) and external objectives (exchange rate and capital flows).

This paper constructs a macroeconomic framework that studies the optimality, interactions, and trade-offs of monetary and exchange rate policies. The model features two large countries: the US and the local (rest of the world); two policies: monetary policy and FXI; and two frictions: nominal and financial frictions. The frictions I introduce imply a non-neutral role for monetary policy and FXI: sticky prices allow the central bank to influence the real interest rate (Rotemberg 1982) and limits to arbitrage in international capital markets imply policy interventions to change the demand and supply of bonds, affecting the exchange rate (Gabaix and Maggiori 2015, Itskhoki and Mukhin 2021).[1]

My first contribution is to provide a full characterization of optimal monetary policy and FXI rules. To provide a sharp analytical characterization of policy trade-offs, the main focus of this paper is an international cooperation case, in which central banks maximize global welfare. The key finding is that FXI trades off an internal objective (inflation) and an external objective (international resource allocation).

On the one hand, FXI stabilizes inflation. When a central bank purchases local currency bonds using FXI, the demand for the local currency increases so the local currency appreciates. As a result, local goods become more expensive for US households, making them consume less local goods. This reduced demand, in turn, leads to lower prices and lower inflation of local goods. Hence, FXI allows the central banks to stabilize inflation even without tightening the monetary policy. For example, during COVID-19 and the Russian-Ukraine war, Brazil set a record low of 2% interest rate and Australia and Japan even adopted a near-zero interest rate policy despite high inflation rates. Instead, they responded to the crisis by selling the dollar reserves, as noted above. Although this approach contrasts with the conventional method of combating inflation by raising the policy rate, the theoretical framework I develop provides a rationale behind it.[2]

On the other hand, FXI distorts international resource allocation. To understand the concept, consider an increase in local output, which increases local consumption. At the same time, a higher supply enables US households to import local goods at cheaper prices. Hence, US consumption also increases. Exchange rates adjust automatically and smooth consumption across countries, even if households cannot trade state-contingent assets internationally.

However, FXI distorts this consumption smoothing by manipulating the exchange rate. When the central bank purchases the local currency, the local currency appreciates. This allows local households to import US goods at cheaper prices and increase their consumption but causes US households to reduce imports and consumption. This resource misallocation across

[1]Models of limits to arbitrage are motivated by empirical literature on forward premium puzzle (Fama 1984). Data shows that cross-currency interest differentials are not offset by expected exchange rate depreciation, resulting in positive excess returns on currency carry trades.

[2]The empirical analysis on the macroeconomic effects of FXI requires a rigorous identification of FXI. As discussed in Fratzscher et al. (2019) and Maggiori (2022), this is a challenging task in the literature and beyond the scope of this paper. As a step toward identification, Gonzalez et al. (2021) and Rodnyansky et al. (2024) combine FXI data with granular bank- or firm-level balance sheet information and study within-country variation of responses to FXI.

countries, driven by the negative spillover effects of FXI, is a unique feature of my two-country model. The key policy implication is that FXI faces a non-trivial trade-off between stabilizing inflation and international resource allocation. Monetary policy and FXI are not two separate policy tools but they should be combined to stabilize inflation and the exchange rate.

Having highlighted the price stabilization channel of FXI, my second contribution is to establish a novel relationship between capital flow management in international finance and the US dollar's dominance in international trade (Gopinath et al. 2020). These two strands of literature in international macroeconomics are often discussed in separate contexts. My paper aims to bridge the gap between them. Recent evidence suggests that the majority of world trade is invoiced in a small number of dominant currencies, particularly the US dollar. Motivated by this fact, I introduce dollar pricing into the model. Under dollar pricing, the following are true. First, an identical local good has different prices in two currencies. When the dollar appreciates, identical local goods are more expensive when denominated in the US dollars than in the local currency. This generates an inefficient cross-currency price dispersion wedge despite the identical marginal costs of production. Second, since both exports and imports are in dollars, changes in the exchange rate affect the relative consumption of local and US goods for local households but do not affect the consumption for US households.

I find that the optimal FXI responds to the price dispersion wedge under dollar pricing. By manipulating the exchange rate, FXI affects the relative prices of local goods sold locally and in the US, closing the price-dispersion wedge. This makes the optimal FXI particularly responsive to shocks under dollar pricing. Moreover, the transmission channel of FXI is asymmetric across countries. Optimal FXI has a large stabilization effect on local inflation without creating a large US inflationary spillover. These results suggest that dollarization in international trade is a key driver of capital flow stabilization policy in international finance.

Finally, as a robustness check, I deviate from the assumption of international policy coordination. The need to fully specify dynamic games poses many challenges to the modern literature on strategic monetary policy coordination. These include the definitions of equilibria (commitment or discretion), the choice of policy instruments (inflation or output gap), and the feasibility of deriving analytical and numerical solutions. In particular, the model is difficult to solve when there are multiple policy instruments: monetary policy and FXI. As a first step toward incorporating FXI in a non-cooperative equilibrium, my paper considers a strategic interaction between FXI by the local central bank and the monetary policy of the US central bank (the

Fed). I focus on a special case where each central bank initially commits to a state-contingent strategy to maximize its objective and solve the model using a numerical method.

The result shows that a lack of cooperation results in excess intervention, exacerbating the international resource misallocation. Intuitively, when the local central bank purchases the local currency and sells the dollar, the local currency appreciates, lowering the import price and stabilizing inflation. The local households increase consumption by importing more US goods.

However, the local central bank does not take into account the international spillover effect of FXI. By appreciating the local currency, FXI increases the US import price and inflation and reduces consumption. An over-accumulation of local currency reserves without international coordination results in consumption misallocation in favor of the local households over the US households (beggar-thy-neighbor). Conversely, an excess accumulation of the dollar reserves, as in the China and Japan cases mentioned above, leads to a consumption misallocation in favor of the US households over the local households (beggar-thy-self). This result provides a rationale for international coordination in the monetary and exchange rate policy designs.

Literature. First, this paper builds on models of exchange rate determination in an imperfect financial market (Gabaix and Maggiori 2015, Itskhoki and Mukhin 2021, Maggiori 2022, Fukui et al. 2023). Their models have been used to study the effect of FXI (Fanelli and Straub 2021, Davis et al. 2023, Ottonello et al. 2024).[3] More recent literature studies both monetary policy and FXI in a special small open economy case, where households take the international price and demand as given (Cavallino 2019, Amador et al. 2020, Basu et al. 2020, Itskhoki and Mukhin 2023).[4] My contribution to the literature is to provide a full analytical characterization

[3]The deviation from uncovered or covered interest rate parity is modeled in the literature as an exogenous shock to the unhedged carry trade return (Devereux and Engel 2002, Jeanne and Rose 2002, Kollmann 2005) or convenience yield on the dollar bond (Jiang et al. 2021; 2023, Engel and Wu 2023, Kekre and Lenel 2023).

[4]Cavallino (2019) shows that FXI is costly for a central bank since FX purchase lowers the FX return while it is profitable for intermediaries as they take an opposite carry trade position against the central bank. When the domestic households do not own the entire share of the intermediaries, FXI trades off the carry cost with exchange rate stabilization. Amador et al. (2020) show that the zero lower bound of the nominal interest rate generates capital inflow since the expected appreciation of local currency is not offset by the lower interest rate. FXI absorbs the capital flows by accumulating foreign reserves but generates a resource cost. Basu et al. (2020) builds an "integrated policy framework" that jointly studies monetary policy, FXI, capital control, and macroprudential regulation. When banks face a sudden outflow of capital, a lower policy rate relaxes the domestic borrowing constraint but tightens the external borrowing constraint due to currency depreciation. FXI limits this depreciation and improves the monetary trade-off. Itskhoki and Mukhin (2023) show that unrestricted use of monetary policy and

of monetary policy and FXI targeting rules under a two-country framework.[5] The unique feature of my two-country model suggests a novel trade-off of FXI between internal objectives (inflation and output) and external objectives (exchange rate and international resource misallocation). Moreover, I take the first step to incorporate FXI in a non-cooperative equilibrium and study the strategic interaction between monetary policy and FXI in a large two-country model.

This paper is also based on the large literature on optimal monetary policy. A large body of literature studies optimal monetary policy in a small open economy (Clarida et al. 2001, Schmitt-Grohé and Uribe 2001, Kollmann 2002, Galí and Monacelli 2005, Faia and Monacelli 2008). Another strand of papers study international monetary policy transmission and cooperation in a large two-country economy (Obstfeld and Rogoff 2000, Corsetti and Pesenti 2001, Clarida et al. 2002, Benigno and Benigno 2003, Corsetti and Pesenti 2005, Benigno and Benigno 2006, Devereux and Engel 2003, Corsetti et al. 2010; 2020; 2023, Engel 2011). These papers study monetary policy independently of FXI. My paper contributes to the literature by providing a joint configuration of monetary policy and FXI.

The effectiveness of FXI is backed by its empirical analysis (Dominguez and Frankel 1993, Dominguez 2003, Fatum and Hutchison 2010, Blanchard et al. 2015, Kuersteiner et al. 2018, Adler et al. 2019, Fratzscher et al. 2019, Hofmann et al. 2019, Fratzscher et al. 2023, Rodnyansky et al. 2024). My contribution is to provide a normative analysis based on a full analytical characterization of an optimal FXI targeting rule.

This paper is related to recent literature on the dominance of the US dollar in trade invoicing (Gopinath 2016, Gopinath et al. 2020, Gopinath and Stein 2021, Mukhin 2022, Egorov and Mukhin 2023). My contribution is to suggest a bridge the gap between the literature on international trade and finance and suggests a novel mechanism where capital flow management policies are motivated by dollar pricing.

Finally, there is a strand of literature on gains from international monetary policy coordination (Obstfeld and Rogoff 2002, Benigno and Benigno 2006, Benigno and Woodford 2012, Corsetti et al. 2010, Bodenstein et al. 2023). Fanelli and Straub (2021) and Itskhoki and Mukhin

FXI stabilize both inflation and exchange rate separately. However, when FXI is constrained, monetary policy faces a trade-off between inflation and exchange rate stabilization.

[5] Literature studies different rationales for FXI. Basu et al. (2020), Davis et al. (2023), Rodnyansky et al. (2024) show that FXI mitigates balance-sheet risk when firms or banks have foreign currency debt. Ottonello et al. (2024) show that FXI is used as an industrial policy and helps the convergence to the technological frontier. The price stabilization channel of FXI in my paper is complementary to these channels.

(2023) study international coordination on FXI between small open economies. While they focus on monetary policy only, I study strategic interaction between monetary policy and FXI.

2 Optimal Policy

To capture the key intuition, I conduct a step-by-step construction of the model. First, I focus on the optimal monetary policy (without FXI) under nominal friction and explain the inflation-output trade-off generated by imperfect risk-sharing. Second, I derive the optimal monetary policy and FXI under nominal and financial frictions and show the trade-off of FXI between inflation and risk-sharing. Third, I introduce DCP and compare the result with the PCP case. Finally, I compare the cooperative and non-cooperative equilibria.

2.1 Optimal Monetary Policy

I begin the analysis with the simplest possible case with only monetary policy and nominal friction (no FXI and financial friction). The results in this section follow Corsetti et al. (2010; 2023). There are two symmetric large open economies, the US and local (rest of the world), and the US variables are denoted with an asterisk. I refer to the US unit of account as the dollar.

Households. In each country, there is a continuum of households that maximize the expected discount value of their lifetime utility. I assume the households have a constant relative risk aversion (CRRA) utility in consumption. The maximization of local households is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \zeta_l \frac{L_t^{1+\eta}}{1+\eta} \right],$$

where C_t and L_t are the consumption and labor supply, σ is the inverse intertemporal elasticity of substitution, β is the discount factor, and η is the inverse Frisch elasticity of labor. The households' consumption basket C_t is a constant elasticity of substitution (CES) aggregator of local and US goods:

$$C_t = \left[a C_{Lt}^{\frac{\phi-1}{\phi}} + (1-a) C_{Ut}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{1-\phi}},$$

where C_{Lt} and C_{Ut} are the consumption of local and US goods, a is the weight of the local good, and ϕ is the elasticity of substitution between the local and US goods. In the limiting case where $\sigma = \phi = 1$ (Cole and Obstfeld (1991), CO preference), households have log and Cobb-Douglas utility.

I assume $a \in (1/2, 1]$ so that households exhibit home bias of consumption. C_{Lt} and C_{Ut} are the bundles of differentiated goods:

$$C_{Lt} = \left[\int_0^1 C_t(l)^{\frac{\zeta-1}{\zeta}} dj\right]^{\frac{\zeta}{1-\zeta}}, \quad C_{Ut} = \left[\int_0^1 C_t(u)^{\frac{\zeta-1}{\zeta}} du\right]^{\frac{\zeta}{1-\zeta}}$$

where $C_t(l)$ and $C_t(u)$ are the local households' consumption of the local good l and imported good u, respectively.

The local households' budget constraint is:

$$P_{Lt}C_{Lt} + P_{Ut}C_{Ut} + \frac{B_t}{R_t} + \frac{\mathcal{E}_t B_{Ut}}{R_t^*} = B_{t-1} + \mathcal{E}_t B_{Ut-1} + W_t L_t + \Pi_t + T_t, \tag{1}$$

where P_{Lt} and P_{Ut} are the prices of local and US goods faced by local households, B_t and B_{Ut} are the local households' investments in one-period state non-contingent bonds denominated in local currency and US dollars, R_t and R_t^* are the interest rates on the local and US bonds ($1/R_t$ and $1/R_t^*$ are the bond prices), $\mathcal{E}_t$ is the nominal exchange rate in terms of the local unit of account per dollar (an increase in $\mathcal{E}_t$ implies a depreciation of the local currency), W_t is the wage, Π_t is the lump-sum transfer of firms' profit, and T_t is the government transfer.

The price index of the local good is given by:

$$P_{Lt} = \left[\int_0^1 P_t(l)^{1-\zeta} dl\right]^{\frac{1}{1-\zeta}},$$

and the consumer price index (CPI) associated with the consumption basket C_t is given by:

$$P_t = \left[aP_{Lt}^{1-\phi} + (1-a)P_{Ut}^{1-\phi}\right]^{\frac{1}{1-\phi}}. \tag{2}$$

The real exchange rate is defined as the ratio of CPIs: $e_t \equiv \mathcal{E}_t P_t^* / P_t$. The terms-of-trade is defined as the relative price of local imports over exports: $\mathcal{T}_t = P_{Lt}/\mathcal{E}_t P_{Ut}^*$.

The households' intratemporal consumption allocation problem gives the following demand for local and US goods:

$$C_{Lt} = a \left(\frac{P_{Lt}}{P_t} \right)^{-\phi} C_t, \quad C_{Ut} = (1-a) \left(\frac{P_{Ut}}{P_t} \right)^{-\phi} C_t,$$

and the demand for differentiated goods produced within each country:

$$C_t(l) = \left(\frac{P_t(l)}{P_{Lt}} \right)^{-\zeta} C_{Lt}, \quad C_t(u) = \left(\frac{P_t(u)}{P_{Ut}} \right)^{-\zeta} C_{Ut}.$$

The households' Euler equation for local currency bond and labor supply equation:

$$\beta R_t E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} = 1,$$
$$C_t^{\sigma} L_t^{\eta} = \frac{W_t}{P_t},$$

Since households can trade bonds in two currencies, combining the Euler equations for the local currency bond faced by the local and US households, I obtain:

$$E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] = E_t \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{\mathcal{E}_t P^*_t}{\mathcal{E}_{t+1} P^*_{t+1}} \right]. \tag{3}$$

When the asset market is complete and the households can trade state-contingent bonds, Equation (3) holds state-by-state. However, when the market is incomplete and households can only trade state non-contingent bonds, Equation (3) only holds in expectation but not for each state of the world. I define the risk-sharing wedge $\mathcal{W}_t$ as the ratio of the marginal utility of consumption across the two countries:

$$\mathcal{W}_t \equiv \frac{(C^*_t)^{-\sigma} / \mathcal{E}_t P^*_t}{C_t^{-\sigma} / P_t} = \left(\frac{C_t}{C^*_t} \right)^{\sigma} \frac{1}{e_t}. \tag{4}$$

When $\mathcal{W}_t = 1$, consumption risk is efficiently shared across the two countries. When $\mathcal{W}_t > 1$, the marginal utility of the local households is lower than that of the US households (the local households have an excess demand or stronger purchasing power) and vice versa when $\mathcal{W}_t < 1$.

The risk-sharing gap is determined by the substitutability between the local and US goods. To understand this, it is convenient to focus on (a) the log utility ($\sigma = 1$) and (b) the financial autarky case where no international asset trade is allowed (an extreme form of incomplete market), but we obtain similar results under a bond economy. It is possible to show that the relationship between the real exchange rate and the relative consumption can be expressed as:

$$\hat{e}_t = \frac{2a - 1}{2a\phi - 1}(\hat{C}_t - \hat{C}_t^*), \tag{5}$$

and the allocations under the complete market and financial autarky are equalized when $\phi = 1$. The risk-sharing wedge (in log-linearized form with $\sigma = 1$) can be written as $\tilde{\mathcal{W}}_t = \tilde{C}_t - \tilde{C}_t^* - \tilde{e}_t$. Intuitively, when the relative local productivity $\hat{A}_t - \hat{A}_t^*$ increases, the relative consumption $\hat{C}_t - \hat{C}_t^*$ increases but the real exchange rate depreciates ($\hat{e}_t$ increases) to insure consumption risk across countries. When the local and US goods are substitutes ($\phi > 1$), the real exchange rate moves less than one-to-one with consumption so that the risk-sharing wedge $\tilde{W}_t$ is positive. When the goods are complements ($\phi < 1$), the real exchange rate moves more than one-to-one so that $\tilde{\mathcal{W}}_t$ is negative. As discussed later, imperfect risk-sharing plays a crucial role in shaping the inflation-output trade-off of monetary policy.

Firms. Firms use domestic labor to produce a differentiated good l following a production function:

$$Y_t(l) = A_t L_t(l),$$

where $Y_t(l)$ is the output and $L_t(l)$ is the labor input for the producer of good l. A_t is a technology shock common to all firms and follows an AR(1) process: $\log(A_t) = \rho_a \log(A_{t-1}) + \sigma_a \epsilon_{at}$, where ρ_a and σ_a are the persistence and the standard deviation, respectively. Let $Y_{Lt} = \left[\int_0^1 Y_t(l)^{\frac{\zeta-1}{\zeta}} dl\right]^{\frac{\zeta}{\zeta-1}}$ be the final output of the local good. The demand for the differentiated good l is given by:

$$Y_t(l) = \left(\frac{P_t(l)}{P_{Lt}}\right)^{-\zeta} Y_{Lt}.$$

Firms are subject to nominal rigidity (Rotemberg 1982) so that firms set the price $P_t(l)$ but must pay a quadratic adjustment cost $\frac{\nu}{2}\left(\frac{P_t(l)}{P_{t-1}(l)} - 1\right)^2 P_t Y_t$. To capture the key intuition, I assume producer currency pricing (PCP) so that the export price is sticky in the exporters' currency (Section 3 derives the optimal policy under dollar pricing). The firms' maximization problem is as follows:

$$\max_{\{P_t(l)\}_{t=0}^{\infty}} E_0 Q_{0,t}\left[(1+\tau)P_t(l)Y_t(l) - W_t L_t(l) - \frac{\nu}{2}\left(\frac{P_t(l)}{P_{t-1}(l)} - 1\right)^2 P_{Lt}Y_{Lt}\right], \tag{6}$$

where $Q_{0,t} = \beta^t \left(\frac{C_t}{C_0}\right)^{-\sigma} \frac{P_0}{P_t}$ is the households' stochastic discount factor and τ_t is the sales subsidy. In a symmetric steady state where all firms choose the same price ($P_t(l) = P_{Lt}$), defining $\pi_{Lt} = P_{Lt}/P_{Lt-1} - 1$ as the net inflation, the New Keynesian Phillips Curve (NKPC) can be written as:

$$\pi_{Lt}(1+\pi_{Lt}) = \beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{Y_{Lt+1}}{Y_{Lt}} \pi_{Lt+1}(1+\pi_{Lt+1})\right] + \frac{\zeta - 1}{\nu}\left[\frac{\zeta}{\zeta - 1}\frac{W_t}{A_t P_{Lt}} - (1+\tau_t)\right].$$

Using log-linearization, the NKPC for the local firms (and similarly for the US firms) can be written in terms of the terms-of-trade gap and the risk-sharing wedge in addition to the output gap:

$$\pi_{Lt} = \beta\pi_{Lt+1} + \kappa\{(\sigma+\eta)\tilde{Y}_{Lt} - (1-a)[2a(\sigma\phi - 1)\tilde{\mathcal{T}}_t - \tilde{\mathcal{W}}_t] + \mu_t\}, \tag{7}$$

$$\pi^*_{Ut} = \beta\pi^*_{Ut+1} + \kappa\{(\sigma+\eta)\tilde{Y}_{Ut} + (1-a)[2a(\sigma\phi - 1)\tilde{\mathcal{T}}_t - \tilde{\mathcal{W}}_t] + \mu^*_t\}, \tag{8}$$

where π_{Lt} and π^*_{Ut} are the inflation of local (US) goods faced by the local (US) households, respectively, $\kappa = (1+\tau_t)(\zeta - 1)/\nu$ is the slope of the NKPCs, and $\mu_t = \zeta/((\zeta - 1)(1+\tau_t))$ is the markup shock. As in the standard New Keynesian model, inflation depends on the expected inflation and the output gap, defined as the deviation of output from its efficient level. In an open economy ($a < 1$), the inflation depends on two additional factors: the terms-of-trade gap $\tilde{\mathcal{T}}_t$, defined as the deviation of the terms-of-trade from its efficient level, and the risk-sharing wedge $\tilde{\mathcal{W}}_t$. As discussed in Clarida et al. (2002), the effect of the terms-of-trade gap on inflation depends on whether the local and US goods are substitutes ($\sigma\phi > 1$) or complements ($\sigma\phi < 1$). Consider an increase in the US output, which leads to local appreciation and a decline in import price (lower $\tilde{\mathcal{T}}_t$). When $\sigma\phi > 1$, lower import price increases local consumption via risk-sharing, which increases the marginal cost of production and the inflation rate. When $\sigma\phi < 1$, an increase in export price lowers the marginal cost and the inflation. The two effects are canceled out when $\sigma\phi = 1$. Moreover, when the risk-sharing wedge is positive, a lower marginal utility for the local households increases the marginal cost and the inflation.

Central Banks. Central banks in the two countries use monetary policy to set the nominal interest rate. I focus on the cooperation and commitment case, in which central banks maximize the sum of expected discounted utility in the two countries.[6] This is equivalent to minimizing the quadratic loss function which is approximated around the efficient flexible-price equilibrium:[7]

$$\mathcal{L} = -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\begin{array}{l} (\sigma+\eta)\left(\tilde{Y}_{Lt}^2 + \tilde{Y}_{Ut}^2\right) + \frac{\zeta}{\kappa}\left(\pi_{Lt}^2 + \pi_{Ut}^{*2}\right) \\ -\frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}\left(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}\right)^2 + \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\tilde{\mathcal{W}}_t^2 \end{array} \right]. \tag{9}$$

Importantly, under cooperation, the loss function not only depends on the internal objective (inflation and output gap in each country) but also the external objective (relative output gap and risk-sharing wedge across countries).

Optimal Policy. This section studies the optimal monetary policy rules and transmission channels of shocks when FXI is not available. First, the following lemma characterizes optimal monetary policy rules under an incomplete asset market.

Lemma 1 (Optimal Monetary Policy Rules without FXI). *Under PCP, cooperation, and commitment, and when FXI is not available, optimal monetary policy rules for the local and US central banks are characterized by:*

$$0 = \theta\pi_{Lt} + \left(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}\right) + \psi_D(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1}), \tag{10}$$

$$0 = \theta\pi_{Ut}^* + \left(\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}\right) - \psi_D(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1}), \tag{11}$$

where:

$$\psi_D = \frac{4a(1-a)\phi}{\sigma + \eta\{4a(1-a)(\sigma\phi-1)+1\}} \frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}, \tag{12}$$

which hold without imposing restrictions on σ and ϕ.

Proof. See Appendix A.2.

[6] In countries such as Japan, the Ministry of Finance is in charge of FXI instead of central banks. This paper considers a joint government consisting of the central bank and the finance ministry.

[7] See Corsetti et al. (2023) for the detailed derivation.

Intuitively, as suggested by Equation (7), an increase in the risk-sharing wedge generates endogenous cost-push inflation under incomplete asset markets, even without assuming an exogenous markup shock. Hence, monetary policy faces a trade-off between inflation and growth rates in the output gap and risk-sharing wedge.

Next, I characterize the international transmission of shocks. First, the following lemma characterizes the transmission of productivity shocks.

Lemma 2 (Transmission of Productivity Shock)**.** *Assume PCP and suppose that FXI is not available and monetary policy follows the optimal rule in Lemma 1.*

- *When* $\sigma\phi > 1$, $\frac{\partial \pi_{Lt}}{\partial A_t} > 0 > \frac{\partial \pi^*_{Ut}}{\partial A_t}$, $\frac{\partial \tilde{Y}_{Lt}}{\partial A_t} < 0 < \frac{\partial \tilde{Y}_{Ut}}{\partial A_t}$, *and* $\frac{\partial \tilde{\mathcal{W}}_t}{\partial A_t} > 0$.
- *When* $\sigma\phi < 1$, $\frac{\partial \pi_{Lt}}{\partial A_t} < 0 < \frac{\partial \pi^*_{Ut}}{\partial A_t}$, $\frac{\partial \tilde{Y}_{Lt}}{\partial A_t} > 0 > \frac{\partial \tilde{Y}_{Ut}}{\partial A_t}$, *and* $\frac{\partial \tilde{\mathcal{W}}_t}{\partial A_t} < 0$.
- *When* $\sigma\phi = 1$, $\frac{\partial \pi_{Lt}}{\partial A_t} = \frac{\partial \pi^*_{Ut}}{\partial A_t} = \frac{\partial \tilde{Y}_{Lt}}{\partial A_t} = \frac{\partial \tilde{Y}_{Ut}}{\partial A_t} = \frac{\partial \tilde{\mathcal{W}}_t}{\partial A_t} = 0$.

Consider an increase in local productivity (or a decrease in US productivity), which depreciates the local exchange rate. When local and US goods are substitutes, the local consumption increases more than the exchange rate depreciates, so the local households have lower marginal utility. This increases the local inflation but decreases the US inflation. The optimal policy is to tighten the policy rate, which leads to a negative output gap. The opposite pattern holds when the two goods are complements. When the two goods are independent, there is no policy trade-off and monetary policy perfectly stabilizes inflation and output gap.

The next lemma characterizes the transmission of an inefficient cost-push shock under the optimal monetary policy without FXI based on Corsetti et al. (2010). I focus on a case where local and US goods are substitutes since it matches the empirically relevant calibration (Itskhoki and Mukhin 2021) and will provide the most interesting implication of FXI.

Lemma 3 (Transmission of Cost-Push Shock)**.** *Suppose that* $\sigma\phi > 1$*, FXI is not available, and monetary policy follows the optimal rule in Lemma 1. Up to the first order, the elasticities of inflation, output gap, and real exchange rate to a period-0 US cost-push shock satisfy:*

$$\frac{\partial \pi^*_{U0}}{\partial \mu^*_0} > 0, \quad \frac{\partial \pi^*_{U1}}{\partial \mu^*_0} < \frac{\partial \pi^*_{U2}}{\partial \mu^*_0} < \cdots < 0, \quad \frac{\partial \tilde{Y}_{U0}}{\partial \mu^*_0} < \frac{\partial \tilde{Y}_{U1}}{\partial \mu^*_0} < \cdots < 0, \tag{13}$$

$$\frac{\partial \pi_{L0}}{\partial \mu^*_0} < 0, \quad \frac{\partial \pi_{L1}}{\partial \mu^*_0} > \frac{\partial \pi_{L2}}{\partial \mu^*_0} > \cdots > 0, \quad \frac{\partial \tilde{Y}_{L0}}{\partial \mu^*_0} > \frac{\partial \tilde{Y}_{L1}}{\partial \mu^*_0} > \cdots > 0, \tag{14}$$

$$\frac{\partial \tilde{e}_0}{\partial \mu_0^*} > \frac{\partial \tilde{e}_1}{\partial \mu_0^*} > \cdots > 0. \tag{15}$$

Proof. See Appendix A.5.

In response to a US cost-push shock, the optimal US monetary policy is to commit to tightening, which lowers the inflation expectation and output gap over time. Hence, the United States faces temporary inflation due to the initial impact of the cost-push shock, followed by mild and persistent deflation due to the monetary tightening. At the same time, the decrease in the US output depreciates the local currency and worsens the local terms of trade. As shown in the NKPC (7) for the local firms, as long as the local and US goods are substitutes ($\sigma\phi > 1$), the local terms-of-trade worsening (an increase in $\mathcal{T}_0$) has a similar transmission mechanism to a negative local cost-push shock (a decrease in μ_t) and generates a negative comovement of inflation and output gap across countries. The local currency depreciation causes an increase in demand for local goods, so the local output gap is positive. The optimal local monetary policy is to commit to tightening, so the local economy faces temporary deflation due to tightening, followed by mild and persistent inflation due to higher demand. Figure 3 provides a graphical representation of this transmission mechanism. I plot the impulse response to a one-percentage increase in the US markup μ_0^*. The parameter values for the benchmark calibration are listed in Table 1.[8]

2.2 Optimal Monetary Policy and FXI

Next, I consider the two-policy (monetary policy and FXI) and two-friction (nominal and financial) environment. I model the financial sector based on Jeanne and Rose (2002), Gabaix and Maggiori (2015), and Itskhoki and Mukhin (2021). Figure 2 shows the basic model structure. The key departure from the previous section is currency market segmentation. Households can only trade bonds in their own currency and their net foreign asset position must be intermediated

[8] There is a wide range of estimates for the trade elasticity. Bernard et al. (2003) estimate the value around 4 using US plant-level data, while Corsetti et al. (2008) use 0.85 to generate an empirically relevant correlation between the real exchange rate and the relative consumption across countries (Backus and Smith 1993). I follow Backus et al. (1994) and set $\phi = 1.5$, which is widely used in international finance literature. I set the elasticity of UIP to FXI at $\chi_1 = 0.43$ to match the observed response of the UIP deviation to Japan's dollar sales from September to October 2022. Following Itskhoki and Mukhin (2021), I set the elasticity of UIP to households' net foreign asset position in Equation (18) at $\chi_2 = 0.001$ to match its observed persistence.

by financiers (global financial intermediaries) who are averse to exchange rate risk, which creates limits to arbitrage.[9] The local central bank has two instruments. In addition to setting the nominal interest rate using monetary policy, they can use FXI to trade bonds in two currencies to affect their relative demand and supply. Finally, UIP shocks (or risk-sharing shocks) generate volatile exchange rates and capital flows relative to the macroeconomic fundamentals (exchange rate disconnect) and the lack of international risk-sharing. The UIP shock can be understood as an exogenous demand for dollar bonds due to their liquidity and safety.[10]

International Financial Market. I will provide a detailed description of the model. The local households can invest only in the local currency bond (B_t) and the US households can invest only in the dollar bond (B_t^*). However, due to currency market segmentation, the local and US households cannot directly trade any assets with each other.

In addition to the households, there is a measure m_u of investors who generate an exogenous capital flow (UIP) shock. The liquidity traders hold a zero-net portfolio (U_t, U_t^*) so the investment U_t^* in dollar bonds is matched by the investment $U_t/R_t = -\mathcal{E}_t U_t^*/R_t^*$ in the local currency bonds. The positive U_t^* implies that the liquidity traders take a long position in the dollar and a short position in the local currency, and vice versa. I assume that the liquidity traders' position follows an AR(1) process: $U_t = \rho_u U_{t-1} + \sigma_u \epsilon_{ut}$.

The local central bank uses sterilized intervention and trades bonds in the two currencies. The local central bank holds a zero-net portfolio (F_t, F_t^*) given by $F_t/R_t = -\mathcal{E}_t F_t^*/R_t^*$ and its profits and losses are transferred to the local households in a lump-sum way.[11]

There is a measure m_d of financiers who intermediate the portfolio positions of the house-

[9] For tractability, I assume that households cannot access foreign currency bonds ($B_{Ut} = 0$ in Equation (1)), following Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021). Fukui et al. (2023) generalize this setup so that households and firms can borrow and invest in foreign currency but it is costly to access foreign currency bonds.

[10] Since the focus of this paper is the economic consequence of UIP shocks and the role of monetary and exchange rate policies, the model is agnostic about the source of UIP shocks to keep tractability. There is an extensive discussion on the drivers of UIP shocks, including investors' heterogeneous beliefs (Bacchetta and Van Wincoop 2006) and cognitive bias (Burnside et al. 2011), rare disaster risk (Farhi and Gabaix 2016), interbank friction (Bianchi et al. 2023a), and special role of US Treasury bonds, such as liquidity or collateral values.(Bianchi et al. 2023a;b). How different sources of UIP shocks affect the optimal policy design is beyond the scope of this paper and is left for future research.

[11] I assume that only the local central bank conducts FXI since data shows that interventions by the Federal Reserve Board are infrequent. Moreover, I assume that FXI is unconstrained for simplicity. In reality, central banks face a zero lower bound on FX reserves, which creates an additional policy trade-off. Davis et al. (2023) show that, when reserves cannot be borrowed, the optimal policy is to accumulate the FX reserves during normal times and sell them during crisis times.

holds, the liquidity traders, and the local central bank. The financiers hold a zero-net portfolio (D_t, D_t^*) given by $D_t/R_t = -\mathcal{E}_t D_t^*/R_t^*$. Following Itskhoki and Mukhin (2021) and Fukui et al. (2023), I assume that the financiers maximize the following constant absolute risk aversion (CARA) utility:

$$\max_{D_t^*} E_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \bar{R}_t^* \frac{D_t^*}{P_t^*} \right) \right\}, \tag{16}$$

where $\omega \geq 0$ is a risk-aversion parameter and

$$\bar{R}_t^* = R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}},$$

is the unhedged return on the carry trade.[12] In a limiting case where $\omega = 0$, arbitrageurs are risk-neutral and take a carry trade position without charging a risk premium. Hence, the UIP holds and the expected excess return is zero: $E_t \bar{R}_t^* = 0$. However, when $\omega > 0$, arbitrageurs are risk-averse and require a risk premium for taking the risky carry trade position, which drives the UIP deviation: $E_t \bar{R}_t^* \neq 0$.

The market clearing conditions for the bond market imply the net demand for local currency and dollar bonds is zero:

$$B_t + U_t + D_t + F_t = 0, \quad \text{and} \quad B_t^* + U_t^* + D_t^* + F_t^* = 0. \tag{17}$$

The competitive equilibrium is defined as the set of prices, quantities, and policy variables that solve the maximization problems of households, firms, and arbitrageurs under the constraints and market clearing conditions.

Solving the households' and financiers' maximization problems gives the equilibrium relationship between the risk-sharing wedge, UIP deviation, and the demand for bonds in two currencies.

Lemma 4. *The equilibrium condition in the financial market, which is log-linearized under a*

[12]The assumption of a CARA utility improves the traceability since their portfolio decision does not depend on the wealth, allowing us to avoid an additional state variable. The potential ways to microfound the banks' risk-aversion are to introduce occasionally binding borrowing constraints, costs of currency hedging, or liquidity holdings by banks (Bianchi et al. 2023a). Moreover, I assume that the financiers' profit is transferred to the local households as a lump-sum payment. As discussed in Appendix A.3, the profits and losses generated by carry trade positions do not affect the first-order dynamics of the model.

symmetric steady state, can be written as:

$$E_t \tilde{\mathcal{W}}_{t+1} - \tilde{\mathcal{W}}_t = \tilde{r}_t - \tilde{r}_t^* - E_t \Delta \tilde{e}_{t+1} = \chi_1 (u_t^* - f_t) - \chi_2 b_t, \tag{18}$$

where $r_t \equiv R_t - E_t \pi_{t+1}$, $r_t^* \equiv R_t^* - E_t \pi_{t+1}^*$, $f_t \equiv F_t / \overline{Y}$, $u_t \equiv U_t / \overline{Y}$, $b_t \equiv B_t / \overline{Y}$, $\chi_1 \equiv m_u (\omega \sigma_{et}^2 / m_d)$ *and* $\chi_2 \equiv \overline{Y} (\omega \sigma_{et}^2 / m_d)$ *for finite* $\omega \sigma_{et}^2 / m_d$*, where* $\overline{Y} \equiv \overline{Y}_L = \overline{Y}_U$ *is GDP under the symmetric steady state and* $\sigma_{et}^2 \equiv var(\Delta \log \mathcal{E}_{t+1})$ *is the standard deviation of the change in log exchange rate* $(\Delta \log \mathcal{E}_{t+1} \equiv \log \mathcal{E}_{t+1} - \log \mathcal{E}_t)$.

Proof. See Appendix A.3.

Intuitively, suppose that the liquidity traders increase their demand for the dollar bond (positive u_t^*). To provide the dollar bonds to liquidity traders, financiers take a short position in the dollar and a long position in the local currency. In a limiting case where $\omega \sigma_{et}^2 / m_d \to 0$, financiers' risk-bearing capacity is sufficiently high so that UIP holds in equilibrium. However, when $\omega \sigma_{et}^2 / m_d > 0$, financiers have limited risk-bearing capacity and require a risk premium as compensation for exchange rate risk in carry trade.[13] This results in the positive UIP deviation ($\widetilde{UIP}_t \equiv \tilde{r}_t - \tilde{r}_t^* - E_t \Delta \tilde{e}_{t+1} > 0$) so that the rate of return on the local currency bond is higher than that of the dollar bond. Since households are restricted from trading assets internationally, they cannot take an opposite carry trade position against the liquidity traders. This implies that the local households face a higher rate of return on savings, so they have more incentive to invest in bonds and postpone their consumption than the US households. As a result, the home households' demand is expected to increase in the future ($E_t \tilde{\mathcal{W}}_{t+1} - \tilde{\mathcal{W}}_t > 0$). Similarly, the households' net foreign debt position ($b_t < 0$) is associated with the positive UIP deviation. To focus on the role of financial sectors in driving the UIP deviation, I consider the limiting case where $\chi_2 = 0$, so that:[14]

$$E_t \tilde{\mathcal{W}}_{t+1} - \tilde{\mathcal{W}}_t = \tilde{r}_t - \tilde{r}_t^* - E_t \Delta \tilde{e}_{t+1} = \chi_1 (u_t^* - f_t). \tag{19}$$

[13] The risk aversion parameter ω is scaled so that the risk premium $\omega \sigma_{et}^2 / m_d$ is finite and nonzero and the variance of the exchange rate σ_{et}^2 affects the first-order dynamics of the model. See discussion by Hansen and Sargent (2011) and Itskhoki and Mukhin (2021).

[14] This assumption can be interpreted so that the size of the financial sector, including liquidity traders (m_u) and financiers (m_d), are sufficiently large relative to the real sector. See Itskhoki and Mukhin (2021). In the later quantitative section, I will relax this assumption and consider the case where both χ_1 and χ_2 are positive.

The local central bank can use FXI to eliminate the distortion due to the segmented currency market. If the central bank takes an offsetting position against the liquidity traders and demands the local currency bond ($f_t = u_t^*$), the right-hand side of Equation (18) becomes zero so that the UIP deviation becomes zero. In other words, FXI effectively shifts the exchange rate risk away from risk-averse financiers to central banks' balance sheets. Since households in the two countries face equal rates of return on savings, the risk-sharing wedge is zero in expectation ($E_t\tilde{\mathcal{W}}_{t+1} - \tilde{\mathcal{W}}_t = 0$). The resulting allocation is identical to that when the asset market is incomplete but the currency market is not segmented (Corsetti et al. 2010; 2023). This does not necessarily imply $\tilde{\mathcal{W}}_t = 0$ in general but under the assumption of CO preference, the real exchange rate automatically insures the countries from consumption risk so that $\tilde{\mathcal{W}}_t = 0$ holds for every state of the economy, as discussed in the previous section.

Optimal Policy. Having established the model, I will discuss the optimal trade-offs and international transmission mechanisms. The following proposition provides a full analytical characterization of optimal monetary policy and FXI targeting rules.

Proposition 1 (Optimal Monetary Policy and FXI)**.** *Under PCP, cooperation, and commitment, and both monetary policy and FXI are available, optimal monetary policy rules for the local and US central banks are characterized by:*

$$0 = \theta\pi_{Lt} + \left(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}\right) + \psi_\pi\theta(\pi_{Lt} - \pi^*_{Ut}) + \psi_D(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1}), \tag{20}$$

$$0 = \theta\pi_{Ut} + \left(\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}\right) - \psi_\pi\theta(\pi_{Lt} - \pi^*_{Ut}) - \psi_D(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1}), \tag{21}$$

where:

$$\psi_\pi = (1-a)\frac{2a(\sigma\phi - 1) + 1}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}}\frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1},$$

and ψ_D is given in Equation (12). The optimal FXI for the local central bank is characterized by:

$$f_t = u_t^* + \frac{(1-a)}{\chi_1}\frac{2a(\sigma\phi - 1) + 1}{2a(1-a)\phi}\theta(E_t\pi_{Lt+1} - E_t\pi^*_{Ut+1}). \tag{22}$$

These optimal rules hold without imposing the restrictions on σ and ϕ.

Proof. See Appendix A.4.

The key difference from Lemma 1 is that, in general, optimal FXI does not perfectly offset the UIP shock, i.e., $f_t = u_t^*$ is no longer optimal. Instead, when the local inflation expectation is higher than the US ($E_t\pi_{Lt+1} > E_t\pi^*_{Ut+1}$), the optimal FXI is to buy the local currency and sell the US dollar.

The next two propositions show how optimal FXI trade-offs depend on the source of shocks. In particular, I focus on productivity and cost-push shocks.

Proposition 2 (Transmission of Productivity Shock when FXI is available)**.** *Assume PCP, cooperation, and commitment, and suppose that the monetary policy follows the optimal rule. When $\sigma\phi > 1$, comparing the cases where FXI follows the optimal rule and FXI is not available,*

$$\frac{\partial f_t}{\partial A_t} > 0, \quad \frac{\partial \tilde{\mathcal{W}}_t^{FXI}}{\partial A_t} > \frac{\partial \tilde{\mathcal{W}}_t}{\partial A_t} > 0, \quad \frac{\partial \pi_{Lt}}{\partial A_t} > \frac{\partial \pi_{Lt}^{FXI}}{\partial A_t} > 0 > \frac{\partial \pi_{Ut}^{FXI*}}{\partial A_t} > \frac{\partial \pi^*_{Ut}}{\partial A_t},$$
$$\text{and} \quad \frac{\partial \tilde{Y}_{Lt}^{FXI}}{\partial A_t} < \frac{\partial \tilde{Y}_{Lt}}{\partial A_t} < 0 < \frac{\partial \tilde{Y}_{Ut}}{\partial A_t} < \frac{\partial \tilde{Y}_{Ut}^{FXI}}{\partial A_t}.$$

When $\sigma\phi < 1$,

$$\frac{\partial f_t}{\partial A_t} < 0, \quad \frac{\partial \tilde{\mathcal{W}}_t^{FXI}}{\partial A_t} < \frac{\partial \tilde{\mathcal{W}}_t}{\partial A_t} < 0, \quad \frac{\partial \pi_{Lt}}{\partial A_t} < \frac{\partial \pi_{Lt}^{FXI}}{\partial A_t} < 0 < \frac{\partial \pi_{Ut}^{FXI*}}{\partial A_t} < \frac{\partial \pi^*_{Ut}}{\partial A_t},$$
$$\text{and} \quad \frac{\partial \tilde{Y}_{Lt}^{FXI}}{\partial A_t} > \frac{\partial \tilde{Y}_{Lt}}{\partial A_t} > 0 > \frac{\partial \tilde{Y}_{Ut}}{\partial A_t} > \frac{\partial \tilde{Y}_{Ut}^{FXI}}{\partial A_t}.$$

When $\sigma\phi = 1$,

$$\frac{\partial f_t}{\partial A_t} = \frac{\partial \pi_{Lt}}{\partial A_t} = \frac{\partial \pi^*_{Ut}}{\partial A_t} = \frac{\partial \tilde{Y}_{Lt}}{\partial A_t} = \frac{\partial \tilde{Y}_{Ut}}{\partial A_t} = \frac{\partial \tilde{\mathcal{W}}_t}{\partial A_t} = 0.$$

Intuitively, when the local and US goods are substitutes ($\sigma\phi > 1$), an increase in local productivity (or a decrease in US productivity) increases local inflation and risk-sharing wedge and reduces the output, as discussed in Lemma 2. The optimal FXI is to buy the local currency, which creates a non-trivial policy trade-off. On the one hand, the appreciation of the local currency reduces the US demand for local goods and the local inflation rate. On the other hand, not only does this lower demand reduce the output further but local appreciation also widens the risk-sharing wedge. Vice versa, when the local and US goods are complements ($\sigma\phi < 1$), the optimal FXI is to buy the US dollar. Hence, in general, monetary policy and FXI are not two independent policy tools, but they should be used jointly to stabilize inflation.

In the special case where the local and US goods are independent ($\sigma\phi = 1$), productivity shock has no effect on the risk-sharing wedge and the inflation rate. Hence, the optimal FXI is to perfectly offset the UIP shock ($f_t = u_t^*$), and the optimal monetary policy is to set the interest rate at the natural level and close the inflation and output gap in the two countries. This result is a well-known dichotomy in the open economy since the monetary policy and FXI have two separate targets.

The next proposition shows the optimal trade-off of FXI when there is a cost-push shock. Similarly to Lemma 3, I focus on the case where the local and US goods are substitutes.

Proposition 3 (Transmission of Cost-Push Shock when FXI is available)**.** *Suppose $\sigma\phi > 1$. Comparing the cases where FXI follows the optimal rule and FXI is not available,*

$$\frac{\partial \pi_{U0}^{*FXI}}{\partial \mu_0^*} < \frac{\partial \pi_{U0}^*}{\partial \mu_0^*} (> 0), \quad \frac{\partial \pi_{Ut}^{*FXI}}{\partial \mu_0^*} > \frac{\partial \pi_{Ut}^*}{\partial \mu_0^*} (< 0), \quad \frac{\partial \tilde{Y}_{Ut}^{FXI}}{\partial \mu_0^*} > \frac{\partial \tilde{Y}_{Ut}}{\partial \mu_0^*} (< 0), \tag{23}$$

$$\frac{\partial \pi_{L0}^{FXI}}{\partial \mu_0^*} > \frac{\partial \pi_{L0}}{\partial \mu_0^*} (< 0), \quad \frac{\partial \pi_{Lt}^{FXI}}{\partial \mu_0^*} < \frac{\partial \pi_{Lt}}{\partial \mu_0^*} (> 0), \quad \frac{\partial \tilde{Y}_{Lt}^{FXI}}{\partial \mu_0^*} < \frac{\partial \tilde{Y}_{Lt}}{\partial \mu_0^*} (> 0), \tag{24}$$

$$\frac{\partial \tilde{e}_t^{FXI}}{\partial \mu_0^*} < \frac{\partial \tilde{e}_t}{\partial \mu_0^*} (> 0), \quad \frac{\partial \widetilde{UIP}_t^{FXI}}{\partial \mu_0^*} < \frac{\partial \widetilde{UIP}_t}{\partial \mu_0^*} (\fallingdotseq 0). \tag{25}$$

Proof. See Appendix A.6.

The proposition shows that, by combining monetary policy and FXI, both inflation and output gap are smoothed out in both countries. If the central bank buys the local currency using FXI, the local currency appreciates. Hence, the US households decrease the relative demand for local goods via expenditure switching channels. This change in demand composition narrows down the positive local output gap and the negative US output gap. Since FXI partially absorbs the output gap, the monetary policy can focus more on inflation stabilization. In other words, FXI improves the monetary policy trade-off between inflation and output gap stabilization.

However, at the same time, by buying the local currency using FXI, the local bond price increases, and its return decreases relative to the dollar bond. Due to limits to arbitrage, local households cannot invest in the dollar bond despite its higher return. Since the local households face a lower rate of return on savings than the US households, the local households enjoy lower marginal utility than the US households (the risk-sharing wedge becomes positive). Hence, FXI faces a trade-off in stabilizing the internal objective (inflation and output gap in each country) and the external objective (risk-sharing wedge across countries).

Figure 3 shows a graphical representation of this proposition. The figure compares the impulse response to a US cost-push shock with and without FXI. The red line shows the case where only monetary policy is available (same as panel a) and the blue line shows the case where both monetary policy and FXI are available.

3 Dollar Pricing

While PCP assumption provides the simplest analytical solution, data suggests that exports and imports are mainly invoiced in US dollars (Gopinath et al. 2020). Motivated by this fact, this section explores the novel interplay between dollar dominance in international trade and capital flow management in international finance. To this end, I introduce DCP and study its implication for FXI. Differently from the PCP case, I assume that both exports and imports are denominated in US dollars, so the law of one price (LOOP) does not hold for the local goods. This creates an inefficient cross-currency price dispersion despite the same marginal cost of production. The expenditure switching mainly works via local imports as US imports are dollar-priced.

For the local firms, the price-setting problem in local currency is given by Equation (6). The price-setting problem in the US dollar is:

$$\max_{\{P_t^*(l)\}_{t=0}^{\infty}} E_0 Q_{0,t}\left[(1+\tau)\mathcal{E}_t P_t^*(l) Y_t^*(l) - W_t L_t(l) - \frac{\nu}{2}\left(\frac{P_t^*(l)}{P_{t-1}^*(l)} - 1\right)^2 \mathcal{E}_t P_{Lt}^* Y_{Lt}^*\right], \tag{26}$$

where $P_t^*(l)$ and $Y_t^*(l)$ are the dollar price and quantity of local good l sold in the US. Let $\Delta_{Lt} \equiv \mathcal{E}_t P_{Lt}^*/P_{Lt}$ be the relative price of a local good denominated in the US dollar over the local currency. Under DCP, the price of local goods is sticky in local currency (P_{Lt}) in the local economy and sticky in the dollars (P_{Ut}^*) in the US. Hence, a local depreciation (higher $\mathcal{E}_t$) increases the dollar price relative to the local currency price (higher Δ_{Lt}). Solving the firms' maximization problem,

$$\pi_{Lt} = \beta\pi_{Lt+1} + \kappa\{(\sigma+\eta)\tilde{Y}_{Lt} - (1-a)[2a(\sigma\phi-1)(\tilde{\mathcal{T}}_t + \tilde{\Delta}_{Lt}) + (\tilde{\mathcal{W}}_t + \tilde{\Delta}_{Lt})] + \mu_t\}, \tag{27}$$

$$\pi_{Lt}^* = \beta\pi_{Lt+1}^* + \kappa\{(\sigma+\eta)\tilde{Y}_{Lt} - (1-a)[2a(\sigma\phi-1)(\tilde{\mathcal{T}}_t + \tilde{\Delta}_{Lt}) + (\tilde{\mathcal{W}}_t + \tilde{\Delta}_{Lt})] - \tilde{\Delta}_{Lt} + \mu_t^*\}, \tag{28}$$

and the NKPC for the US firms is given by Equation (8).

The quadratic loss function in the DCP case can be characterized as:[15]

$$\mathcal{L} = -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\begin{array}{l} (\sigma+\eta)\left(\tilde{Y}_{Lt}^2 + \tilde{Y}_{Ut}^2\right) + \frac{\zeta}{\kappa}\left(a\pi_{Lt}^2 + (1-a)\pi_{Lt}^{*2} + \pi_{Ut}^{*2}\right) \\ -\frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}\left(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}\right)^2 \\ +\frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\left(\tilde{\mathcal{W}}_t + \tilde{\Delta}_{Lt}\right)^2 \end{array} \right]. \tag{29}$$

There are two key differences compared to the PCP case (Equation (9)). First, the central banks take into account the weighted sum of local good inflation in the two countries (π_{Lt}, π_{Lt}^*). Second, the loss depends on the deviation from the LOOP ($\tilde{\Delta}_{Lt}$).

Under DCP, analytically tractable expressions for the optimal policy rule can be derived under the assumption of linear labor disutility (Engel 2011). The following lemma characterizes the optimal monetary policy under dollar pricing when FXI is not available.

Lemma 5 (Optimal Monetary Policy Trade-offs under DCP). *Under DCP, cooperation, and commitment, $\eta = 0$, and when FXI is not available ($f_t = 0$), optimal monetary policy rules for the local and US central banks are characterized by:*

$$0 = \theta a \pi_{Lt} + (\tilde{C}_t - \tilde{C}_{t-1}) + \frac{2a(1-a)\phi}{2a(\phi-1)+1}\frac{\sigma-1}{\sigma}(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1} + \tilde{\Delta}_t - \tilde{\Delta}_{t-1}), \tag{30}$$

$$0 = \theta[(1-a)\pi_{Lt}^* + \pi_{Ut}^*] - (\tilde{C}_t^* - \tilde{C}_{t-1}^*) - \frac{2a(1-a)\phi}{2a(\phi-1)+1}\frac{\sigma-1}{\sigma}(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1} + \tilde{\Delta}_t - \tilde{\Delta}_{t-1}). \tag{31}$$

Proof. See Appendix A.7.

The result is isomorphic to the one without currency market segmentation (Corsetti et al. 2020). Importantly, the optimal monetary policy rule is asymmetric across countries. The local central bank trades off the stabilization of domestic inflation (π_{Lt}) with growth rates of the risk-sharing wedge and LOOP deviation. In contrast, the US central bank targets the international dollar price, which is the weighted sum of the inflation of local goods prices in the dollars (π_{Lt}^*) and the US-produced goods (π_{Ut}^*).

Next, I study the case where both monetary policy and FXI are available. DCP has two key implications for the design and transmission mechanism of optimal FXI. First, Proposition 4

[15] See Corsetti et al. (2020) for the details.

shows that the optimal FXI closes the inefficient cross-currency dispersion due to incomplete exchange-rate pass-through. The full derivation of the policy rules under DCP is available in Appendix A.8.

Proposition 4 (Targeting the LOOP Deviation)**.** *Under DCP, cooperation, commitment,* $\sigma = \phi = 1$, $\eta = 0$, *and when both monetary policy and FXI follow the optimal rules,*

1. *Optimal local currency purchase* f_t *is an increasing function of the price dispersion* Δ_{Lt}*.*
2. *FXI reduces the elasticity of* Δ_{Lt} *to the US cost-push shock.*
3. *The elasticity of optimal local currency purchase to the US cost-push shock is larger under DCP than PCP:*

$$\frac{\partial f_t}{\partial \Delta_{Lt}} > 0, \quad \frac{\partial \Delta_{Lt}^{FXI}}{\partial \mu_t^*} < \frac{\partial \Delta_{Lt}}{\partial \mu_t^*} \; (> 0), \quad \left(\frac{\partial f_t}{\partial \mu_t^*}\right)^{DCP} > \left(\frac{\partial f_t}{\partial \mu_t^*}\right)^{PCP} \; (> 0). \tag{32}$$

Proof. See Appendix A.9.

Statements 1 and 2 show that optimal FXI addresses the inefficient cross-currency price dispersion due to incomplete exchange-rate pass-through. Under DCP, since the local exporters set the price in US dollars, a depreciation of the local currency increases the dollar price relative to the local currency price of an identical locally produced good, causing a deviation from the LOOP. The proposition implies that the optimal FXI is to buy the local currency and respond to its undervaluation. Hence, the optimal FXI rule targets the LOOP deviation in addition to the UIP deviation and the inflation in the two countries. As implied by Statement 3, the optimal FXI volume is larger under DCP than under PCP.

Second, Proposition 5 characterizes the key difference in the transmission mechanism of FXI under different currency paradigms.

Proposition 5 (Asymmetric Transmission)**.** *Assume DCP, cooperation, commitment,* $\sigma = \phi = 1$, $\eta = 0$, *and suppose that both monetary policy and FXI follow the optimal rules. In response to the US cost-push shock, under PCP, optimal FXI decreases the local CPI inflation and increases the US CPI inflation by the same degree:*

$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} = -\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} \; (< 0).$$

Under DCP, optimal FXI decreases the local CPI inflation more and increases the US CPI inflation less than the PCP case:

$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} \quad (< 0),$$
$$\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} \quad (> 0).$$

Proof. See Appendix A.10.

Under PCP, FXI decreases local inflation and increases US inflation symmetrically. However, under DCP, the transmission of FXI is asymmetric across countries. On the one hand, FXI decreases local inflation more under DCP than PCP. Since the optimal FXI is larger under DCP (Proposition 4), FXI reduces the local import price of US goods and thus the local CPI inflation. On the other hand, since the US import price is sticky in dollars, local currency appreciation has a limited effect on the US import price. Hence, by purchasing the local currency, central banks can stabilize local inflation without causing a large upward spillover to US inflation. These results explain why FXI is particularly effective at stabilizing local inflation under DCP.[16]

4 Non-Cooperative Equilibrium

Finally, I deviate from international cooperation and study FXI in a strategic non-cooperative equilibrium. As discussed in Corsetti et al. (2010), modern literature on international monetary policy coordination requires a full specification of dynamic games between policymakers. Hence, the literature faces a number of challenges, including the definition of equilibrium (open- or closed-loop, commitment or discretion) and the feasibility of deriving analytical and numerical solutions when the steady state is inefficient. This paper takes the first step in this research and focuses on a case where it is feasible to derive a numerical solution. In particular, I consider an open-loop Nash equilibrium under commitment and with one strategic policy instrument for each policymaker. The model can be solved numerically using a second-order

[16]Under local currency pricing (LCP) where both exports and imports are invoiced in the destination currency, optimal FXI is larger than the PCP case as it targets the LOOP deviation. However, the transmission is symmetric and FXI has muted effects on the import prices in both countries.

perturbation method of the welfare function developed by Bodenstein et al. (2019; 2023).[17] I consider a strategic interaction between FXI by the local central bank and monetary policy by the US.

4.1 Definition of Equilibrium

Let $x_t = (\tilde{x}_t', i_{L,t}, i_{U,t})'$ be the $N \times 1$ vector of endogenous variables, where $i_{L,t}$ and $i_{U,t}$ are the strategic policy instrument chosen by the local and US central banks, respectively. Let ϵ_t be the vector of the exogenous shocks. The private optimality and market clearing conditions are summarized by:

$$E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{L,t}, i_{U,t}, \epsilon_t) = 0.$$

4.1.1 Cooperative Equilibrium

Under the cooperative game, the policymakers maximize the weighted average of the local and US households' utility under commitment:

$$\max_{\{\tilde{x}_t', i_{L,t}, i_{U,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha U_{L,t}(\tilde{x}_{t-1}, \tilde{x}_t, \epsilon_t) + (1-\alpha) U_{U,t}(\tilde{x}_{t-1}, \tilde{x}_t, \epsilon_t) \right],$$
$$s.t. \quad E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{L,t}, i_{U,t}, \epsilon_t) = 0.$$

where α and $1-\alpha$ are the weights on the local and US households' utilities, respectively. I refer to the

4.1.2 Non-Cooperative Equilibrium

I consider a non-cooperative interaction between the two central banks under an open-loop Nash game. Under open-loop Nash equilibrium, in the initial period, each player specifies her state-contingent plans for every future state, and each player's action is the best response to the other player's best response.

More formally, let $j = [L, U]$ be the set of players (the local or US central bank). Let $\{i^*_{j,t,-t^*}\}_{t=0}^{\infty}$ be the sequence of policies chosen by player j before and after but not including period t^* and $\{i^*_{-j,t}\}_{t=0}^{\infty}$ be the other player's policies. An open-loop Nash equilibrium is a

[17] The toolbox does not currently support the games with multiple strategic instruments per policymaker.

sequence $\{i^*_{j,t}\}_{t=0}^{\infty}$ such that, for all period t^*, i^*_{j,t^*} maximizes player j's objective function subject to the constraints for given sequences of $\{i^*_{j,t,-t^*}\}_{t=0}^{\infty}$ and $\{i^*_{-j,t}\}_{t=0}^{\infty}$. In each period, each player maximizes her own following objective function given the other player's policies:[18]

$$\max_{\{\tilde{x}'_t, i_{j,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[U_{j,t}(\tilde{x}_{t-1}, \tilde{x}_t, \epsilon_t) \right],$$

$$s.t. \quad E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{L,t}, i_{U,t}, \epsilon_t) = 0, \quad \text{for given } \{i_{-j,t}\}_{t=0}^{\infty}.$$

4.2 Policy Trade-offs under a Non-Cooperative Equilibrium

I compute the cooperative and non-cooperative equilibria numerically and compare the impulse responses to technology and markup shocks. I consider a strategic interaction between the local FXI and the US monetary policy, assuming that the local monetary policy follows a Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\gamma_R} \left(\frac{\bar{\pi}_{Lt}}{\pi}\right)^{\phi_\pi(1-\gamma_R)}.$$

I follow the main parameter settings in Bodenstein et al. (2023). I set $\phi_\pi = 4$ since the policy response to the inflation rate is high enough to ensure the uniqueness of equilibrium. I set $\alpha = 0.5$ so the local and US households' utilities are equally weighted under cooperation. I compare the impulse responses to US productivity and cost-push shocks under cooperative and Nash equilibria.

First, Figure 4, panel (a) plots the impulse response to a one-percentage increase in US productivity. The red and blue lines show the cooperative and Nash equilibria, respectively. Panel (b) shows the difference between the two equilibria. The figure shows that, under Nash equilibria, the optimal FXI is to buy the dollar more (sell the dollar less) and accumulate excess reserves relative to the cooperation case. Intuitively, the dollar purchase has risk-sharing and expenditure-switching effects. First, since the local return increases relative to the US, the local households have a higher marginal propensity to consume, which lowers the local demand and output. This distorts the international risk-sharing and makes the risk-sharing wedge negative. Second, since the local currency depreciates, the US demand for local goods increases via the expenditure switching channel. This increases the inflation rate of the locally produced goods

[18] I adopt the timeless perspective, which requires an initial pre-commitment so that the optimal policy is time-invariant (Benigno and Woodford 2012).

relative to the US. However, the local central bank does not take into account the US incentive to lower its policy rate, which stabilizes its inflation rate and further increases the US demand and output relative to the local. This implies that competitive devaluation via excess reserve accumulation is not only self-defeating as it is matched by the US policy response but also exacerbates the international risk-sharing distortion.

Next, Figure 5 plots the impulse response to a one-percentage increase in US markup. Under Nash equilibria, the optimal FXI is to buy the local currency more and over-stabilize the exchange rate. The risk-sharing effect implies that the local currency purchase reduces the local return on savings so the local households enjoy a lower marginal propensity to consume. Second, the expenditure-switching effect implies that the local appreciation increases the US demand for its own goods. However, the local central bank does not take into account the US incentive to raise the interest rate and counter the local appreciation, which stabilizes its price but lowers the output. This implies that excess stabilization of the domestic currency is a beggar-thy-neighbor policy since it distorts the relative output and the risk-sharing in favor of the domestic economy over the foreign.

5 Conclusion

This paper develops a unified macroeconomic framework incorporating both monetary and exchange rate policies. I study the optimal policy mix and transmission channels when frictions in both goods and financial markets create a non-trivial interaction and trade-offs between different policy tools.

According to conventional wisdom, monetary and exchange rate policies should separately target the inflation and exchange rate. FXI should be used to fully offset the exchange rate disconnect from macroeconomic fundamentals and monetary policy should respond to the exchange rate only to the extent that it affects the domestic inflation and output gap. However, I find that this conventional “dichotomy” is valid only in a special case where a unitary trade elasticity ensures efficient international risk-sharing even under imperfect financial markets. In general, a lack of risk-sharing generates an inflation-output trade-off of monetary policy. I find that, when the optimal monetary policy no longer features full inflation stabilization, optimal FXI faces a trade-off between the internal and external objectives. The expenditure-switching mechanism of FXI helps improve the inflation-output trade-off, but this comes at the cost of distorting the international risk-sharing in favor of the domestic economy over the foreign. This trade-off implies that monetary policy and FXI are interrelated policy instruments and they should be combined to stabilize the inflation and exchange rates.

Having studied the monetary policy aspect of FXI, the model further shows that capital flow management in international finance can be driven by dollar pricing in international trade. The optimal FXI is larger under DCP than PCP since it additionally targets the price-dispersion wedge due to incomplete exchange-rate pass-through. Moreover, asymmetric pass-through implies that FXI is particularly effective at stabilizing domestic inflation without causing a large US inflationary pressure.

Finally, I consider a strategic interaction between domestic FXI and US monetary policy. I find that an excessive reserve accumulation is not only self-defeating as it is matched by the US monetary easing but also exacerbates the international risk-sharing distortion.

An important and challenging direction for future research is to develop an identification method of FXI and empirically estimate its effects. Fratzscher et al. (2019) compares different identification methods, including reaction function, propensity score matching, instrumental variables, and high-frequency approaches. Recent works exploit granular firm- or bank-level data and combine their balance sheet information with credit supply and employment (Gonzalez et al. 2021) or with stock prices (Rodnyansky et al. 2024).

Another important research agenda is to understand how to combine FXI with other capital account management policies, including capital control and macroprudential policy, in the context of IMF's integrated policy framework (Basu et al. 2020).

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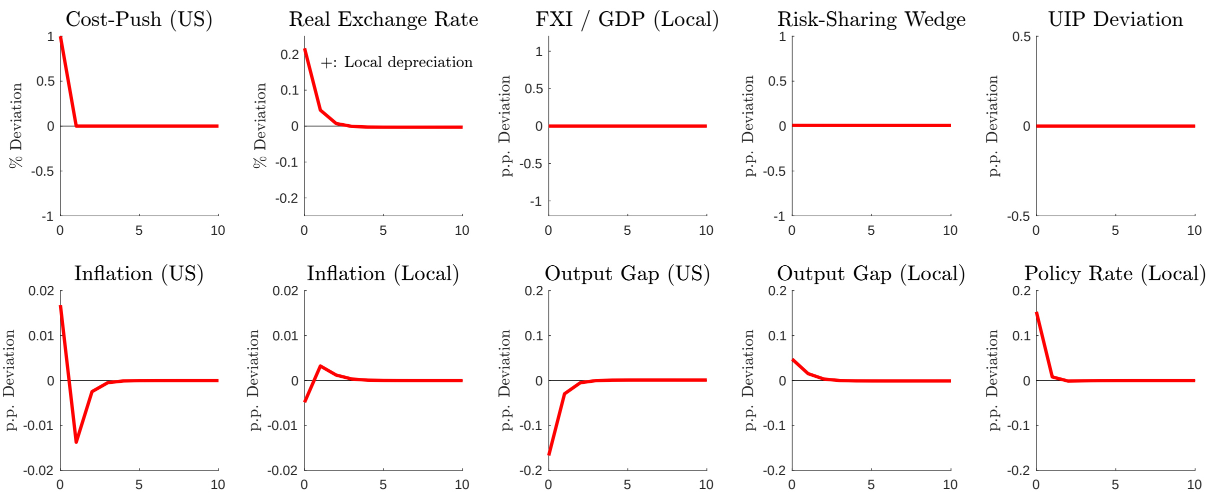
Table 1: Benchmark Parameters

	Description	Value	Notes
β	Discount factor (local)	0.995	Annual interest rate = 2%
σ	Relative risk aversion	5	Cole and Obstfeld (1991)
η	Inverse Frisch elasticity	1.5	Itskhoki and Mukhin (2021)
ζ_l	Labor disutility (local)	1	$\overline{L} = 1$
a	Home bias of consumption	0.88	Bodenstein et al. (2023)
ϕ	CES Local & US goods	1.5	Cole and Obstfeld (1991)
θ	CES differentiated goods	10	Ottonello and Winberry (2020)
ρ_a	Persistence of productivity shock	0.95	Bodenstein et al. (2023)
χ_1	Elasticity of UIP to FXI	0.43	$\Delta \log \text{UIP}_t / \Delta \log \text{FXI}_t$
χ_2	Elasticity of UIP to NFA	0.001	UIP/NFA ratio

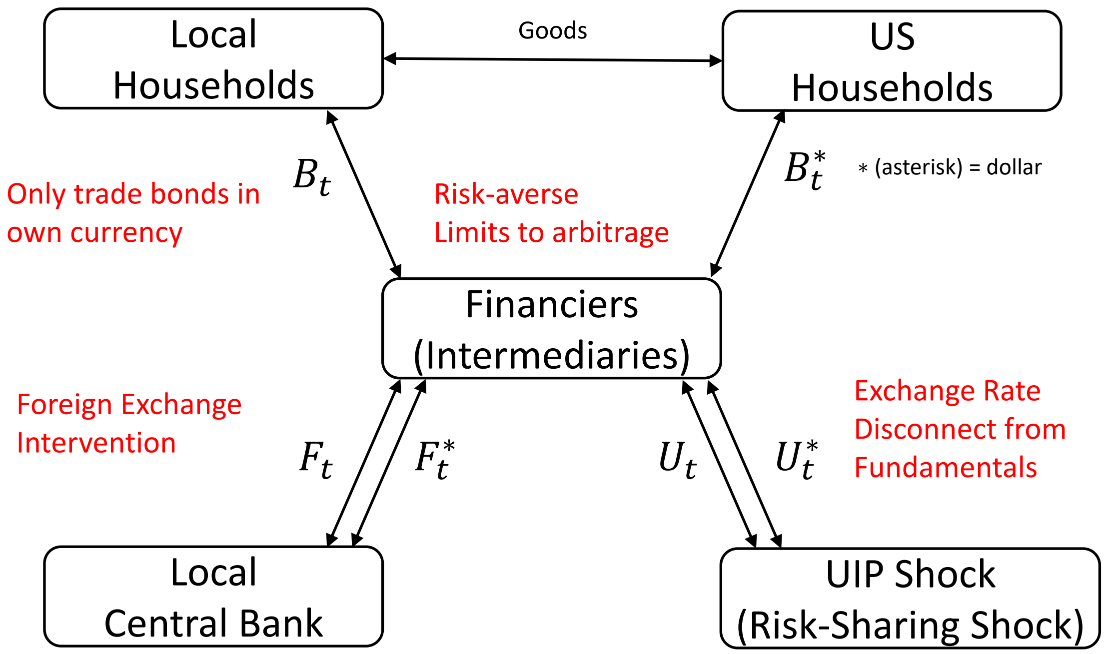
Note: The table shows the benchmark parameter settings for Figure 3.

Figure 1: Impulse Response to a US Cost-Push Shock (No Intervention)



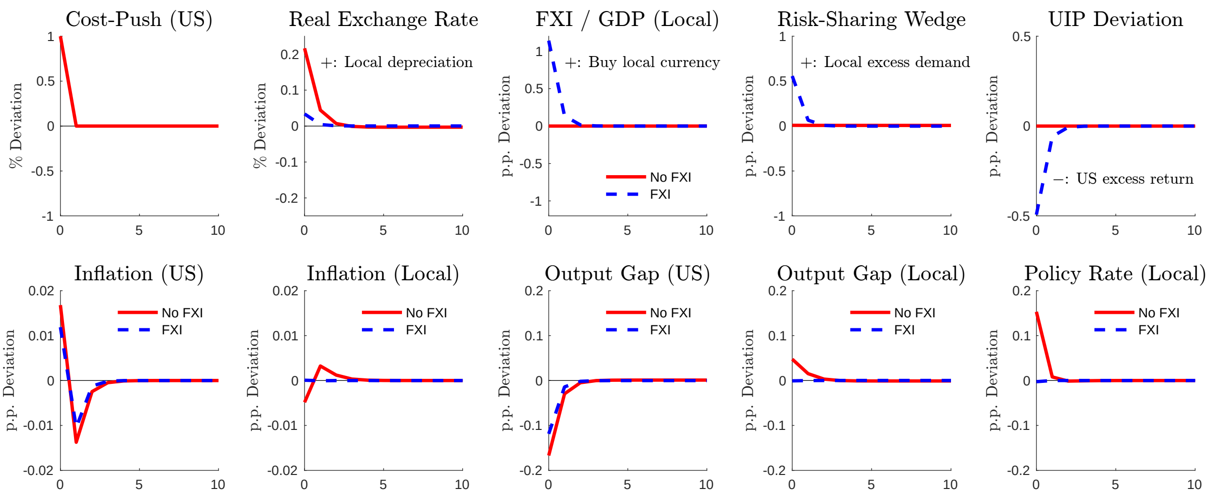
Note: The figure plots the impulse responses to a one-percentage increase in the US markup when FXI is not available and monetary policy follows optimal rules in Lemma 1.

Figure 2: The Basic Model Structure



Note: The figure shows the basic structure of the model with both nominal and financial frictions. Local and US households can only trade bonds in their own currency (B_t, B_t^*). The local central bank uses foreign exchange intervention to trade bonds in two currencies (F_t, F_t^*). Liquidity (noise) traders generate an exogenous UIP shock (U_t, U_t^*). Financiers (global financial intermediaries) intermediate the net foreign asset positions of the households, the local central bank, and the liquidity traders.

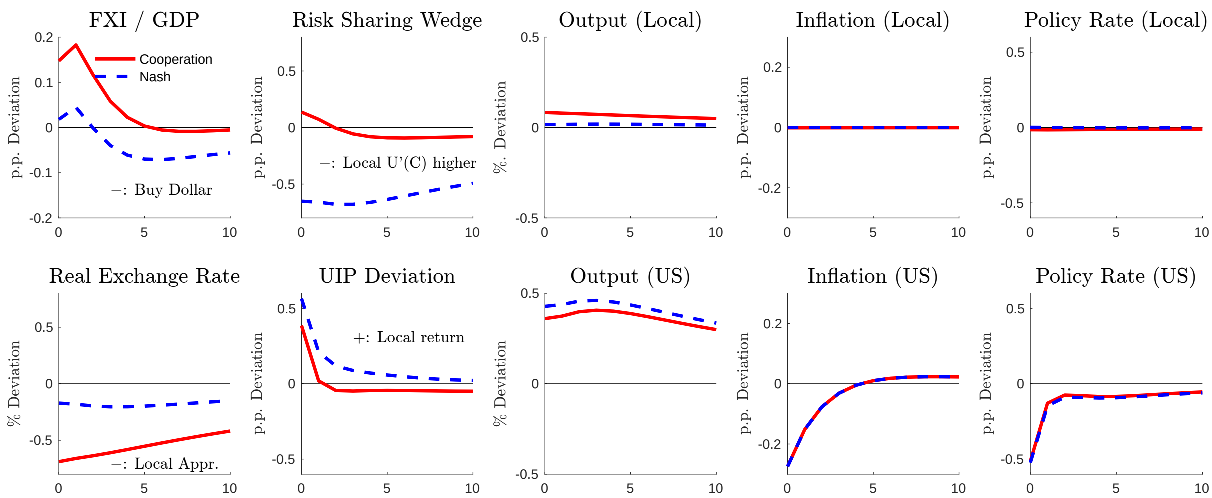
Figure 3: Impulse Response to a US Cost-Push Shock (Intervention)



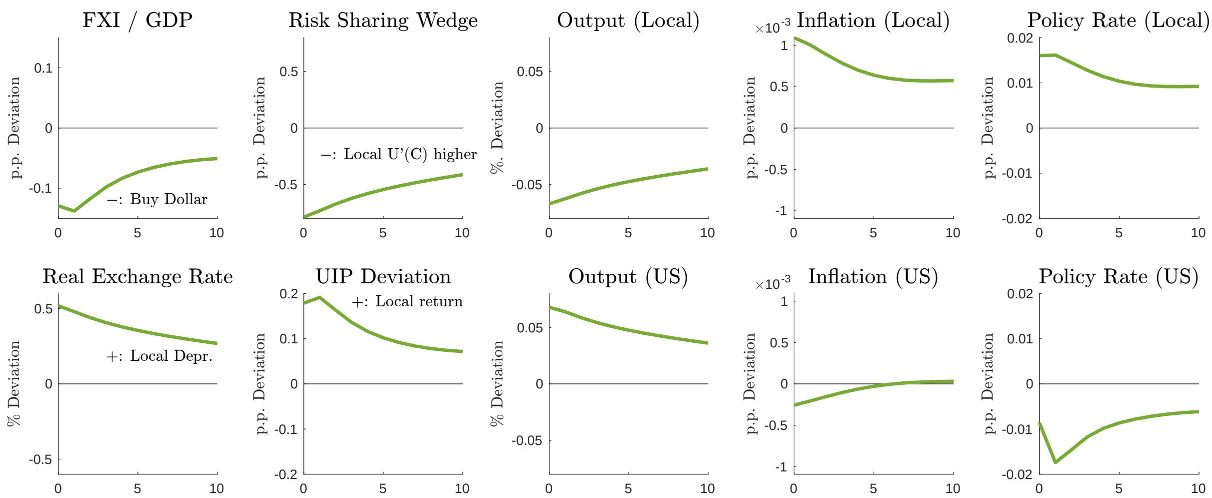
Note: The figure plots the impulse responses to a one-percentage increase in the US markup when both monetary policy and FXI follow optimal rules in Proposition 1.

Figure 4: Impulse Response to a US Productivity Shock, Cooperation and Nash

(a) Cooperation and Nash



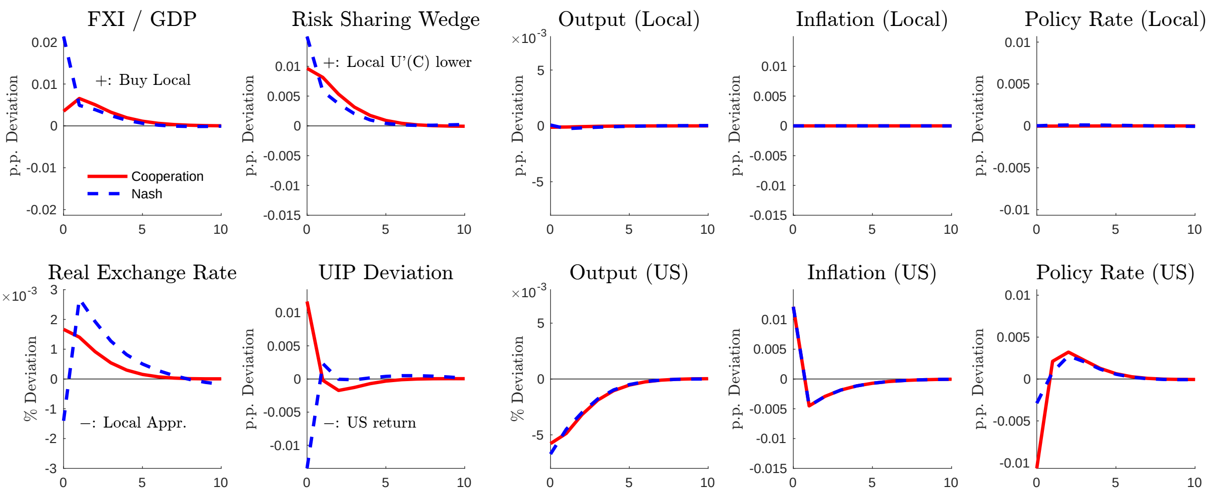
(b) Difference between Cooperation and Nash



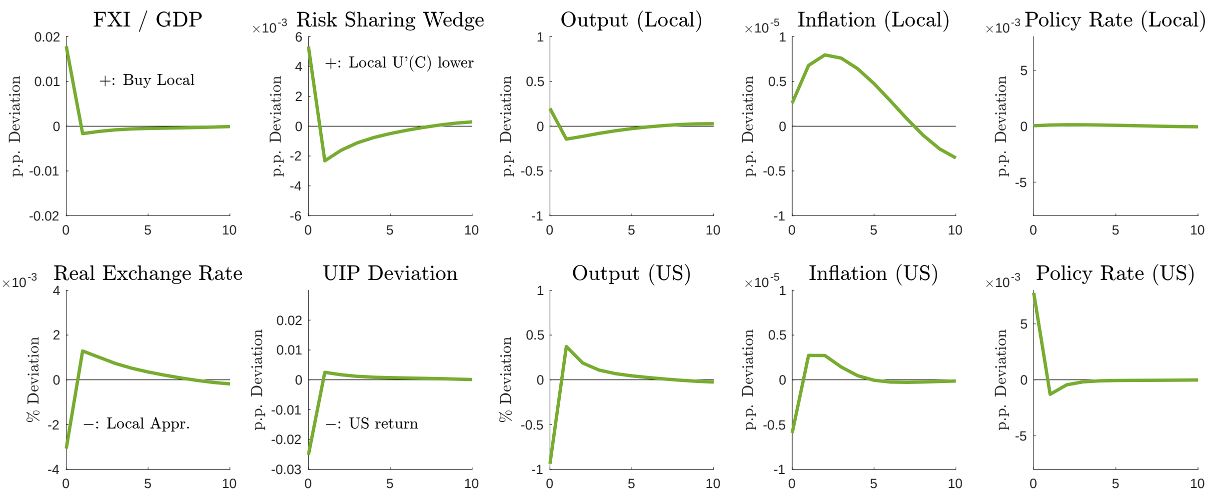
Note: The figure plots the impulse responses to a one-percentage increase in the US productivity under the cooperative equilibrium and the open-loop Nash equilibrium. Panel (a) plots the results under the cooperative equilibrium (red) and the Nash equilibrium (blue). Panel (b) plots the difference between the Nash and the cooperative equilibrium.

Figure 5: Impulse Response to a US Cost-Push Shock, Cooperation and Nash

(a) Cooperation and Nash



(b) Difference between Cooperation and Nash



Note: The figure plots the impulse responses to a one-percentage increase in the US markup under the cooperative equilibrium and the open-loop Nash equilibrium. Panel (a) plots the results under the cooperative equilibrium (red) and the Nash equilibrium (blue). Panel (b) plots the difference between the Nash and the cooperative equilibrium.

Appendix

A Derivations and Proofs

A.1 Useful Equilibrium Relationships

This section provides equilibrium first-order relationships which are useful for proofs of propositions in Section 2. The derivation follows Corsetti et al. (2010; 2023).

I focus on the LCP case (the PCP case can be derived analogously by setting $\Delta_t = 0$). Let the variables with hat denote the deviation from the steady state. For simplicity, assume symmetry so that $\mathcal{E}_t P^*_{Lt}/P_{Lt} = \mathcal{E}_t P^*_{Ut}/P_{Ut} = \Delta_t$. By the definition of real exchange rate, it is expressed in terms of terms of trade and price dispersion:

$$e_t = \frac{\mathcal{E}_t P^*_t}{P_t} = \frac{\mathcal{E}_t \left[a(P^*_{Lt})^{1-\phi} + (1-a)(P^*_{Ut})^{1-\phi}\right]^{\frac{1}{1-\phi}}}{\left[a(P_{Lt})^{1-\phi} + (1-a)(P_{Ut})^{1-\phi}\right]^{\frac{1}{1-\phi}}}$$

$$= \frac{\left[a\left(\frac{\mathcal{E}_t P^*_{Lt}}{P_{Lt}}\right)^{1-\phi} + (1-a)\left(\frac{\mathcal{E}_t P^*_{Ut}}{P_{Lt}}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}}{\left[a + (1-a)\left(\frac{P_{Ut}}{P_{Lt}}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}}, \tag{A1}$$

where:

$$\frac{\mathcal{E}_t P^*_{Ut}}{P_{Lt}} = \frac{\mathcal{E}_t P^*_{Lt}}{P_{Lt}} \frac{P_{Ut}}{\mathcal{E}_t P^*_{Lt}} \frac{\mathcal{E}_t P^*_{Ut}}{P_{Ut}} = \Delta_t^2 \mathcal{T}_t, \tag{A2}$$

$$\frac{P_{Ut}}{P_{Lt}} = \frac{\mathcal{E}_t P^*_{Lt}}{P_{Lt}} \frac{P_{Ut}}{\mathcal{E}_t P^*_{Lt}} = \Delta_t \mathcal{T}_t. \tag{A3}$$

Log-linearizing Equation (A1), we obtain:

$$\hat{Q}_t = (2a_H - 1)\hat{\mathcal{T}}_t + 2a_H \tilde{\Delta}_t. \tag{A4}$$

Next, I approximate the aggregate demand. Under the assumption of symmetry, we have:

$$\hat{Y}_{Lt} + \hat{Y}_{Ut} = \hat{C}_t + \hat{C}^*_t = 0. \tag{A5}$$

Combining Equations (4) and (A5) gives:

$$\hat{Y}_{Lt} - \hat{C}_t = \hat{C}_t^* - \hat{Y}_{Ut} = \frac{1}{2}[\hat{Y}_{Lt} - \hat{Y}_{Ut} - \sigma^{-1}(\hat{Q}_t + \tilde{\mathcal{D}}_t)]. \tag{A6}$$

Substituting Equation (2.1) into the aggregate demand $Y_{Lt} = C_{Lt} + C_{Lt}^*$ for local good gives:

$$Y_{Lt} = \left(\frac{P_{Lt}}{P_t}\right)^{-\phi} [a_H C_t + (1-a)(e_t \Delta_t^{-1})^{\phi} C_t^*]. \tag{A7}$$

Log-linearizing the CPI (2) gives:

$$\hat{P}_t - \hat{P}_{Lt} = (1-a)(\hat{\mathcal{T}}_t + \tilde{\Delta}_t). \tag{A8}$$

Using Equation (A8), (A7) can be log-linearized as:

$$\hat{Y}_{Lt} - \hat{C}_t = (1-a)\sigma^{-1}[\sigma\phi(\hat{e}_t + \hat{\mathcal{T}}_t) - \hat{e}_t - \tilde{\mathcal{D}}_t]. \tag{A9}$$

Using Equations (A4), (A9) can be rewritten as:

$$\hat{Y}_{Lt} - \hat{C}_t = (1-a)\sigma^{-1}[2a_H\sigma\phi(\hat{\mathcal{T}}_t + \tilde{\Delta}_t) - \hat{e}_t - \tilde{\mathcal{D}}_t]. \tag{A10}$$

Combining the two expressions (A6) and (A10) for the aggregate demand, the terms of trade can be expressed as:

$$\hat{\mathcal{T}}_t + \tilde{\Delta}_t = \frac{\hat{Y}_{Lt} - \hat{Y}_{Ut} - (2a_H - 1)(\tilde{\mathcal{D}}_t + \tilde{\Delta}_t)}{4a_H(1-a)(\sigma\phi - 1) + 1}. \tag{A11}$$

A.2 Proof of Lemma 1

The proof follows the appendix of Corsetti et al. (2023). Under cooperation and commitment, the central banks in the two countries minimize the loss (9) subject to the NKPCs (7) and (8) and the UIP condition (19). Let γ_{Lt}, γ_{Ut}^*, and λ_t be the Lagrange multipliers for the local and US NKPCs and the UIP condition, respectively.

The first-order conditions can be written as:

$$\tilde{Y}_{Lt}: \quad 0 = -(\sigma + \eta)\tilde{Y}_{Lt} + \frac{2a(1-a)(\sigma\phi - 1)\sigma}{4a(1-a)(\sigma\phi - 1) + 1}(\tilde{Y}_{Lt} - \tilde{Y}_{Ut})$$

$$
\begin{aligned}
&- \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}\tilde{\mathcal{D}}_t \\
&+ \left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa\gamma_{Lt} + \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa\gamma^*_{Ut} \\
&- \frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t - \beta^{-1}\lambda_{t-1}),
\end{aligned} \tag{A12}
$$

$$
\begin{aligned}
\tilde{Y}_{Ut}: \quad 0 =& -(\sigma+\eta)\tilde{Y}_{Ut} - \frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt}-\tilde{Y}_{Ut}) \\
&+ \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}\tilde{\mathcal{D}}_t \\
&+ \left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa\gamma^*_{Ut} + \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa\gamma_{Lt} \\
&+ \frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t - \beta^{-1}\lambda_{t-1}),
\end{aligned} \tag{A13}
$$

$$
\pi_{Lt}: \quad 0 = -\frac{\theta}{\kappa}\pi_{Lt} - \gamma_{Lt} + \gamma_{Lt-1}, \tag{A14}
$$

$$
\pi^*_{Ut}: \quad 0 = -\frac{\theta}{\kappa}\pi^*_{Ut} - \gamma^*_{Ut} + \gamma^*_{Ut-1}, \tag{A15}
$$

$$
\begin{aligned}
\mathcal{B}_t: \quad 0 =& \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}(E_t\tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t) \\
&+ (1-a)\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\kappa[(E_t\gamma_{Lt+1}-\gamma_{Lt}) - (E_t\gamma^*_{Ut+1}-\gamma^*_{Ut})] \\
&- [(E_t\lambda_{t+1} - \beta^{-1}\lambda_t) - (\lambda_t - \beta^{-1}\lambda_{t-1})].
\end{aligned} \tag{A16}
$$

Under the assumption of $f_t = u^*_t = 0$, we have $E_t\tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = 0$. By taking the sum of the FOCs for the output gap in the two countries and combining it with the FOCs for inflation, we obtain:

$$
\begin{aligned}
0 &= \tilde{Y}_{Lt} + \tilde{Y}_{Ut} + \theta(p_{Lt} + p^*_{Ut}) \\
&= (\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}) + (\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}) + \theta(\pi_{Lt} + \pi^*_{Ut}).
\end{aligned} \tag{A17}
$$

Next, taking the difference of FOCs for the output gap,

$$
\begin{aligned}
0 =& \left[\sigma+\eta-\frac{4a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}\right](\tilde{Y}_{Lt}-\tilde{Y}_{Ut}) \\
&+ \frac{4a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}\tilde{\mathcal{D}}_t \\
&+ \left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\theta(\tilde{p}_{Lt} - \tilde{p}^*_{Ut})
\end{aligned}
$$

$$+2\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t-\beta^{-1}\lambda_{t-1}).$$

The FOC for the net foreign asset implies:

$$-(\lambda_t-\beta^{-1}\lambda_{t-1})=(1-a)\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\theta(p_{Lt}-p^*_{Ut}).$$

Combining the FOCs, we obtain the difference rule:

$$\begin{aligned}0=&\left[\sigma+\eta-\frac{4a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}\right][(\tilde{Y}_{Lt}-\tilde{Y}_{Lt-1})-(\tilde{Y}_{Ut}-\tilde{Y}_{Ut-1})+\theta(\pi_{Lt}-\pi^*_{Ut})]\\&+\frac{4a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t-\tilde{\mathcal{D}}_{t-1}).\end{aligned}$$

Combining the sum and difference rules, we obtain the country-specific monetary policy rules (10) and (11) (Lemma 1).

When $\sigma=\phi=1$, it is possible to show that when $\sigma=\phi=1$, the constrained optimal allocation under PCP satisfied $\tilde{\mathcal{D}}_t=0$ (see Appendix 2.2.2 of Corsetti et al. (2023) for detailed derivation). The monetary policy rules reduce to:

$$0=\theta\pi_{Lt}+\left(\tilde{Y}_{Lt}-\tilde{Y}_{Lt-1}\right), \tag{A18}$$

$$0=\theta\pi^*_{Ut}+\left(\tilde{Y}_{Ut}-\tilde{Y}_{Ut-1}\right), \tag{A19}$$

and the NKPCs with $\tilde{\mathcal{D}}_t=\tilde{\Delta}_t=0$ reduce to:

$$\pi_{Lt}=\beta\pi_{Lt+1}+\kappa(\sigma+\eta)\tilde{Y}_{Lt}, \tag{A20}$$

$$\pi^*_{Ut}=\beta\pi^*_{Ut+1}+\kappa(\sigma+\eta)\tilde{Y}_{Ut}. \tag{A21}$$

Hence, the equilibrium is the first-best: $\pi_{Lt}=\pi^*_{Ut}=\tilde{Y}_{Lt}=\tilde{Y}_{Ut}=\tilde{\mathcal{D}}_t=0$. □

A.3 Proof of Lemma 4

The proof follows the appendix of Itskhoki and Mukhin (2021). Differently from their paper, the central bank can use FXI in addition to monetary policy.

To begin with, I show the first equality of Equation (18), which describes the relationship between demand gap and UIP deviation. The Euler equations of local and US households are characterized in log-linearized form:

$$\tilde{r}_t = \sigma E_t \left[\tilde{C}_{t+1} - \tilde{C}_t\right], \tag{A22}$$

$$\tilde{r}^*_t = \sigma E_t \left[\tilde{C}^*_{t+1} - \tilde{C}^*_t\right] \tag{A23}$$

Taking the difference of Equations (A22) and (A23) and subtracting $\Delta\tilde{e}_{t+1} = \tilde{e}_{t+1} - \tilde{e}_t$ from both sides,

$$E_t \left[\sigma \left\{(\tilde{C}_{t+1} - \tilde{C}_t) - (\tilde{C}^*_{t+1} - \tilde{C}^*_t)\right\} - \Delta\tilde{e}_{t+1}\right] = \tilde{r}_t - \tilde{r}^*_t - E_t\Delta\tilde{e}_{t+1}.$$

Using the definition of demand gap (4), we obtain the first equality:

$$E_t\tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = \tilde{r}_t - \tilde{r}^*_t - E_t\Delta\tilde{e}_{t+1}. \tag{A24}$$

Next, I show the second equality of Equation (18), which describes the relationship between financial flows and UIP deviation. The maximization problem (16) of arbitrageurs can be rewritten as:

$$\max_{d^*_t} E_t \left\{-\frac{1}{\omega}\exp\left(-\omega\bar{R}^*_t(1 - e^{x^*_t})\frac{D^*_t}{P^*_t}\right)\right\}, \tag{A25}$$

where $x^*_t = \tilde{r}^*_t - \tilde{r}_t - \Delta\tilde{e}_{t+1}$ is the nominal carry trade return. When the time period is short, x^*_t can be expressed as the normal diffusion process:

$$d\mathcal{X}^*_t = x^*_t dt + \sigma_{et} dB_t,$$

where $x^*_t = \tilde{r}^*_t - \tilde{r}_t - \Delta\tilde{e}_{t+1}$ is the nominal carry trade return and B_t is a standard Brownian motion. Note that the excess return is equal in nominal and real terms when log-linearized:

$$\begin{aligned}
x^*_t &= \tilde{r}^*_t - \tilde{r}_t - \Delta\tilde{e}_{t+1} \\
&= (\tilde{R}_t - E_t\pi_{t+1}) - (\tilde{R}^*_t - E_t\pi^*_{t+1}) - E_t(\Delta\tilde{e}_{t+1} + \pi^*_{t+1} - \pi_{t+1}) \\
&= \tilde{R}^*_t - \tilde{R}_t - \Delta\tilde{e}_{t+1}.
\end{aligned}$$

The maximization problem (A25) can be rewritten as:

$$\max_{D_t^*} E_t \left\{ -\frac{1}{\omega} \exp \left(-\omega \bar{R}_t^* (1 - e^{dX_t^*}) \frac{D_t^*}{P_t^*} \right) \right\}. \tag{A26}$$

Using Ito's lemma, the objective function can be rewritten as:

$$\begin{aligned} & E_t \left\{ -\frac{1}{\omega} \exp \left(-\omega \bar{R}_t^* \left(-dX_t^* - \frac{1}{2}(dX_t^*)^2 \right) \frac{D_t^*}{P_t^*} \right) \right\} \\ = & -\frac{1}{\omega} \exp \left(\left[\omega \left(x_t^* + \frac{1}{2}\sigma_{et}^2 \right) \frac{D_t^*}{P_t^*} - \frac{1}{2}\omega^2 \sigma_{et}^2 \left(\frac{D_t^*}{P_t^*} \right)^2 \right] dt \right). \end{aligned}$$

Solving the maximization problem, the optimal portfolio decision is:

$$\frac{D_t^*}{P_t^*} = -m_d \frac{\tilde{R}_t^* - \tilde{R}_t - \Delta \tilde{e}_{t+1} + \sigma_{et}^2}{\omega \sigma_{et}^2}. \tag{A27}$$

Substituting Equation (A27) and $U_t^* = m_u u_t^*$, $F_t^* = m_u f_t^*$ into the market clearing condition (17) for the dollar bond, we obtain:

$$\frac{B_t^*}{P_t^*} + \frac{1}{P_t^*} m_u u_t^* - m_d \frac{\tilde{R}_t^* - \tilde{R}_t - \Delta \tilde{e}_{t+1} + \sigma_{et}^2}{\omega \sigma_{et}^2} + \frac{1}{P_t^*} m_u f_t^* = 0. \tag{A28}$$

Since the arbitrageurs, noise traders, and central bank (FXI) takes zero net positions,

$$\frac{D_t + U_t + F_t}{R_t} = -\mathcal{E}_t \frac{D_t^* + U_t^* + F_t^*}{R_t^*}.$$

Using (17), we obtain $B_t/R_t + \mathcal{E}_t B_t^*/R_t^* = 0$. Substituting the zero net positions for households and central bank into Equation (A28) yields:

$$\frac{\tilde{R}_t^* - \tilde{R}_t - \Delta \tilde{e}_{t+1} + \sigma_{et}^2}{\omega \sigma_{et}^2 / m_d} = \frac{1}{P_t^*} m_u u_t^* - \frac{R_t^*}{R_t} \frac{1}{e_t} \frac{1}{P_t^*} m_u f_t^* - \frac{R_t^*}{R_t} \frac{Y_t}{e_t} \frac{B_t}{P_t Y_t}. \tag{A29}$$

Log-linearizing this gives the second equality:

$$\tilde{r}_t - \tilde{r}_t^* - E_t \Delta \tilde{e}_{t+1} = \chi_1 (u_t^* - f_t) - \chi_2 b_t. \tag{A30}$$

Combining Equations (A24) and (A30), we obtain the UIP equation (18). □

Incomes and Losses of Carry Trade Positions. For simplicity, I assume that the profits and losses of carry-trade positions by the financiers and noise traders and interventions by the local central bank are transferred to the local households in a lump-sum way. However, the assumption on the ownership structure does not affect the first-order dynamics of the model, as discussed in Itskhoki and Mukhin (2021). To see this, combining the positions of the financiers, the noise traders, and the local central bank, the total carry trade profit can be written as:

$$\bar{R}_t^*(D_t + U_t + F_t) = -\bar{R}_t^* B_t = \bar{R}_t^* \bar{Y} b_t.$$

The combined carry trade profit equals the product of the UIP deviation ($\bar{R}_t^*$) and the households' net foreign asset position ($\bar{Y} b_t$). Each of them is first-order but their product is second-order and small enough relative to the size of the countries' budget constraint.

A.4 Proof of Propositions 1

$$\lambda_t = 0. \tag{A31}$$

First, combining Equation (A31) with Equations (19), (A12), (A13), (A14), and (A15), we obtain the difference rule:

$$\begin{aligned} 0 = &(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}) - (\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}) + \theta(\pi_{Lt} - \pi_{Ut}^*) \\ &+ 2(1-a)\frac{2a(\sigma\phi - 1) + 1}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}} \frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}\theta(\pi_{Lt} - \pi_{Ut}^*) \\ &+ \frac{4a(1-a)\phi}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}} \frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1}). \end{aligned}$$

Combining this with the sum rule (A17), we obtain the country-specific monetary policy rules (20) and (21). Next, combining Equation (A31) with Equations (A14), (A15), and (A16), we obtain the optimal FXI rule (22). This proves Proposition 1.

In the special case where $\sigma = \phi = 1$, since $\psi_\pi = \psi_D = 0$, the optimal output gap and inflation are pinned down by Equations (A18) through (A21), which are the same as in the no FXI case. Hence, the optimal FXI is to set $f_t = u_t^*$ and the first-best equilibrium $\pi_{Lt} = \pi_{Ut}^* = \tilde{Y}_{Lt} = \tilde{Y}_{Ut} = \tilde{\mathcal{D}}_t = 0$ is achieved.

A.5 Proof of Lemma 3

When $f_t = u_t^* = 0$, the demand gap $\tilde{\mathcal{D}}_t$ is zero on average and at most second order. I first consider the US inflation and output gap. Combining the NKPC (8) for the US firms and the optimal monetary policy rule (11) and assuming the economy is initially at the steady state ($\tilde{Y}_{L,-1} = 0$):

$$\frac{\partial \pi_{U0}^*}{\partial \mu_0^*} = \frac{1}{1+\theta\kappa} > 0, \quad \frac{\partial \tilde{Y}_{U0}}{\partial \mu_0^*} = -\frac{\theta}{1+\theta\kappa} < 0.$$

in period 0 and:

$$\frac{\partial \pi_{U0}^*}{\partial \mu_0^*} = -\frac{\theta\kappa}{(1+\theta\kappa)^{t+1}} < 0, \quad \frac{\partial \tilde{Y}_{U0}}{\partial \mu_0^*} = -\frac{\theta}{(1+\theta\kappa)^{t+1}} < 0.$$

in period $t \geq 1$. This confirms Equation (13).

Next, I consider the transmission of the US cost-push shock to the real exchange rate and the local inflation and output gap. Using Equations A4 (with $\tilde{\Delta}_{Lt} = 0$), the elasticity of the terms-of-trade satisfies:

$$\frac{\partial \tilde{\mathcal{T}}_0}{\partial \mu_0^*} > \frac{\partial \tilde{\mathcal{T}}_1}{\partial \mu_0^*} > \cdots > 0.$$

Using the relationship (A11) between the real exchange rate and the terms of trade, we obtain Equation (15). To simplify the proof, I consider the case where $1 + \theta\kappa$ is large enough so that $\mathcal{T}_0$ has a first-order effect on the local inflation while $\mathcal{T}_t$ $(t \geq 1)$ does not. As shown in Equation (7) $\sigma\phi > 1$, an increase in $\mathcal{T}_t$ is analogous to a decrease in μ_t. Hence, Equation (14) can be proven similarly to Equation (13). □

A.6 Proof of Proposition 3

From Equations (13), (14), and (22), optimal FXI satisfies $\partial \tilde{f}_t / \partial \mu_0^* > 0$ for all t when $\mu_0^* > 0$ and $\mu_t^* = 0$ for all $t \geq 1$. From Equation (18),

$$\frac{\partial \widetilde{UIP}_t^{FXI}}{\partial \tilde{\mathcal{D}}_t} < 0, \quad \text{and} \quad \frac{\partial \tilde{\mathcal{D}}_t}{\partial f_t} > 0,$$

for a given value of $\tilde{\mathcal{D}}_{t+1}$. Since $\partial\tilde{Y}_{Lt}/\partial\tilde{\mathcal{T}}_t = 2a(\phi-1)+1$ and:

$$\frac{\partial\tilde{\mathcal{T}}_t}{\partial\tilde{\mathcal{D}}_t} = -\frac{2a-1}{4a(1-a)(\sigma\phi-1)+1} < 0, \tag{A32}$$

from Equation (A11), we have:

$$\frac{\partial\tilde{Y}_{Lt}}{\partial\tilde{\mathcal{D}}_t} = \frac{\partial\tilde{Y}_{Lt}}{\partial\tilde{\mathcal{T}}_t}\frac{\partial\tilde{\mathcal{T}}_t}{\partial\tilde{\mathcal{D}}_t} = -\frac{(2a-1)[2a(\phi-1)+1]}{4a(1-a)(\sigma\phi-1)+1}$$

Hence, we have:

$$\frac{\partial\tilde{Y}_{Lt}}{\partial\mu_0^*} > \frac{\partial\tilde{Y}_{Lt}^{FXI}}{\partial\mu_0^*}.$$

Combining this result with the optimal policy rule (10), we obtain:

$$\frac{\partial\pi_{L0}}{\partial\mu_0^*} < \frac{\partial\pi_{L0}^{FXI}}{\partial\mu_0^*}, \quad \frac{\partial\pi_{Lt}}{\partial\mu_0^*} > \frac{\partial\pi_{Lt}^{FXI}}{\partial\mu_0^*}.$$

This proves Equation (14). Equation (13) can be proved analogously. Combining Equations (A4) (with $\tilde{\Delta}_{Lt}=0$) and (A32), we obtain:

$$\frac{\partial\tilde{e}_t}{\partial\mu_0^*} > \frac{\partial\tilde{e}_t^{FXI}}{\partial\mu_0^*}.$$

This proves Equation (25). □

A.7 Proof of Lemma 5

The proof follows (Corsetti et al. 2020; 2023). Under cooperation and commitment, the central banks in the two countries minimize the loss (29) subject to the NKPCs (27), (28), and (8), the UIP condition (19), and the condition that relates the relative price to the terms-of-trade and the LOOP deviation:

$$\pi_{Ut} - \pi_{Lt} = \tilde{\mathcal{T}}_t - \tilde{\mathcal{T}}_{t-1} + \Delta_{Lt} - \Delta_{Lt-1}.$$

Let γ_{Lt}, γ_{Lt}^*, and γ_{Ut}^* be the Lagrange multipliers for the local and US NKPCs, λ_t for the UIP condition, and γ_t for the terms-of-trade equation.

The first-order conditions can be written as:

$$\begin{aligned}
\tilde{Y}_{Lt}: \quad 0 =& -(\sigma+\eta)\tilde{Y}_{Lt} + \frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt}-\tilde{Y}_{Ut}) \\
&- \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t) \\
&+ \left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa(\gamma_{Lt}+\gamma^*_{Lt}) + \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa\gamma^*_{Ut} \\
&+ \frac{1}{2a(\phi-1)+1}(\beta E_t\gamma_{t+1}-\gamma_t) - \frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t-\beta^{-1}\lambda_{t-1}),
\end{aligned} \tag{A33}$$

$$\begin{aligned}
\tilde{Y}_{Ut}: \quad 0 =& -(\sigma+\eta)\tilde{Y}_{Ut} - \frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt}-\tilde{Y}_{Ut}) \\
&+ \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t) \\
&+ \left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa\gamma^*_{Ut} + \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa(\gamma_{Lt}+\gamma^*_{Lt}) \\
&- \frac{1}{2a(\phi-1)+1}(\beta E_t\gamma_{t+1}-\gamma_t) + \frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t-\beta^{-1}\lambda_{t-1}),
\end{aligned} \tag{A34}$$

$$\pi_{Lt}: \quad 0 = -\frac{\theta}{\kappa}\pi_{Lt} - \gamma_{Lt} + \gamma_{Lt-1} - \gamma_t, \tag{A35}$$

$$\pi^*_{Lt}: \quad 0 = -\frac{\theta}{\kappa}\pi^*_{Lt} - \gamma^*_{Lt} + \gamma^*_{Lt-1}, \tag{A36}$$

$$\pi^*_{Ut}: \quad 0 = -\frac{\theta}{\kappa}\pi^*_{Ut} - \gamma^*_{Ut} + \gamma^*_{Ut-1}, \tag{A37}$$

$$\begin{aligned}
\mathcal{B}_t: \quad 0 =& \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}(E_t\tilde{\mathcal{D}}_{t+1}-\tilde{\mathcal{D}}_t) - \\
&+ (1-a)\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\kappa\left[\begin{array}{l}(E_t\gamma_{Lt+1}-\gamma_{Lt})+(E_t\gamma_{Lt+1}-\gamma_{Lt}) \\ -(E_t\gamma^*_{Ut+1}-\gamma^*_{Ut})\end{array}\right] \\
&- [(E_t\lambda_{t+1}-\beta^{-1}\lambda_t)-(\lambda_t-\beta^{-1}\lambda_{t-1})] \\
&- \frac{2a-1}{4a(1-a)(\sigma\phi-1)+1}[(\beta E_t\gamma_{t+2}-E_t\gamma_{t+1})-(\beta E_t\gamma_{t+1}-\gamma_t)],
\end{aligned} \tag{A38}$$

$$\begin{aligned}
\tilde{\Delta}_{Lt}: \quad 0 =& -\frac{2a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t) + \kappa\frac{1}{4a(1-a)(\sigma\phi-1)+1} \\
&\times\frac{1}{2}\left[\begin{array}{l}(4a(1-a)(\sigma\phi-1)+1)(\gamma_{Lt}-(\gamma^*_{Lt}+\gamma^*_{Ut})) \\ -\left\{(2a-1)-2(1-a)[2a(\phi-1)+1]\dfrac{2a(1-a)(\sigma\phi-1)+1-\phi}{2a(\phi-1)+1}\right\} \\ \times(\gamma_{Lt}+\gamma^*_{Lt}-\gamma^*_{Ut})\end{array}\right]
\end{aligned}$$

$$
\begin{aligned}
&- \frac{2a-1}{2a(\phi-1)+1}(\beta E_t \gamma_{t+1} - \gamma_t) \\
&- \frac{2a[2(1-a)(\sigma\phi-1)+1-\phi]}{2a(\phi-1)+1}(\lambda_t - \beta^{-1}\lambda_{t-1}).
\end{aligned} \tag{A39}
$$

Under the assumption of $f_t = u_t^* = 0$, we have $E_t\tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = 0$. The FOC for the net foreign asset implies:

$$
\begin{aligned}
\lambda_t - \beta^{-1}\lambda_{t-1} = (1-a)&\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\theta(\gamma_{Lt} + \gamma_{Lt}^* - \gamma_{Ut}^*) \\
&- \frac{2a-1}{4a(1-a)(\sigma\phi-1)+1}(\beta E_t \gamma_{t+1} - \gamma_t).
\end{aligned} \tag{A40}
$$

The sum rule for the output gaps is given by Equation (A17). Using the symmetry (Equation (A5)), the sum rule can be rewritten as:

$$
0 = \theta[a\pi_{Lt} + (1-a)\pi_{Lt}^* + \pi_{Ut}^*] + (\tilde{C}_t - \tilde{C}_{t-1}) + (\tilde{C}_t^* - \tilde{C}_{t-1}^*). \tag{A41}
$$

To derive the difference rule, by taking the difference between Equations (A33) and (A34) and substituting Equation (A40), we obtain:

$$
\begin{aligned}
2\sigma(\beta E_t \gamma_{t+1} - \gamma_t) = \sigma&[(\tilde{Y}_{Lt} - \tilde{Y}_{Ut})] \\
&+ 4a(1-a)\phi\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) \\
&- \sigma\kappa(\gamma_{Lt} + \gamma_{Lt}^* - \gamma_{Ut}^*).
\end{aligned} \tag{A42}
$$

Moreover, substituting Equation (A40) into Equation (A39) yields:

$$
\begin{aligned}
&\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\
&= -\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) + \kappa(\gamma_{Lt} - (\gamma_{Lt}^* + \gamma_{Ut}^*)) - (2a-1)\kappa\frac{\gamma_{Lt} + \gamma_{Lt}^* - \gamma_{Ut}^*}{4a(1-a)(\sigma\phi-1)+1}.
\end{aligned} \tag{A43}
$$

Combining Equations (A42) and (A43) and rearranging the terms, we obtain:

$$
\begin{aligned}
&\frac{2a-1}{4a(1-a)(\sigma\phi-1)+1}\sigma(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\
&+ \left[(2a-1)\frac{2a(\sigma\phi-1)+1-\sigma}{4a(1-a)(\sigma\phi-1)+1} + \sigma\right]\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t)
\end{aligned}
$$

$$= \sigma\kappa(y_{Lt} - (y_{Lt} + y^*_{Ut})).$$

Using the relationships (A4) and (A11), the left-hand side can be rewritten as:

$$\begin{aligned}
&(2a-1)\left[\tilde{\mathcal{T}}_t + \tilde{\Delta}_{Lt} + \frac{(2a-1)(\tilde{\mathcal{D}}_t + \tilde{\Delta}_{Lt})}{4a(1-a)(\sigma\phi-1)+1}\right] \\
&\quad + \left[(2a-1)\frac{2a(\sigma\phi-1)+1-\sigma}{4a(1-a)(\sigma\phi-1)+1} + \sigma\right]\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t + \tilde{\Delta}_{Lt}) \\
&= (\tilde{e}_t - \tilde{\Delta}_{Lt}) + (\tilde{\mathcal{D}}_t + \Delta_{Lt}) - \frac{4a(1-a)\sigma\phi}{4a(1-a)(\sigma\phi-1)+1}(\tilde{\mathcal{D}}_t + \Delta_{Lt}) \\
&\quad + \left[(2a-1)\frac{2a(\sigma\phi-1)+1-\sigma}{4a(1-a)(\sigma\phi-1)+1} + \sigma\right]\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t + \tilde{\Delta}_{Lt}) \\
&= \tilde{e}_t + \tilde{\mathcal{D}}_t + \frac{4a(1-a)\phi(\sigma-1)}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t + \tilde{\Delta}_{Lt}).
\end{aligned}$$

Using the FOCs for the inflation rates and rearranging the terms, we obtain the difference rule:

$$\begin{aligned}
0 =& \theta[a\pi_{Lt} - (1-a)\pi^*_{Lt} - \pi^*_{Ut}] + (\tilde{C}_t - \tilde{C}_{t-1}) - (\tilde{C}^*_t - \tilde{C}^*_{t-1}) \\
&+ \frac{4a(1-a)\phi}{2a(\phi-1)+1}\frac{\sigma-1}{\sigma}(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1} + \tilde{\Delta}_{Lt} - \tilde{\Delta}_{Lt-1}).
\end{aligned} \tag{A44}$$

Combining the sum and difference rules (Equations (A41) and (A44)), the country-specific monetary policy rules are given by Equations (30) and (31). □

A.8 Optimal Monetary Policy and FXI under DCP

This section provides a full characterization of optimal monetary policy and FXI rules under DCP and provides proofs of Propositions 4 and 5. Let $\gamma_{\Delta t} \equiv 2(\beta E_t \gamma_{t+1} - \gamma_t)$. First, combining the FOCs (A33) and (A34) for the output gap and (A31) for the FXI, the difference rule for the output gap can be written as:

$$\begin{aligned}
\gamma_{\Delta t} =& \frac{\sigma}{4a(1-a)(\sigma\phi-1)+1}[2a(\phi-1)+1](\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\
&+ \frac{4a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}[2a(\sigma\phi-1)+1-\sigma](\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) \\
&+ [\sigma(2a(\phi-1)+1) - 2(1-a)(\sigma-1)]\theta[a\pi_{Lt} + (1-a)\pi^*_{Lt} - \pi^*_{Ut}].
\end{aligned} \tag{A45}$$

Next, from the FOC (A39) for the LOOP deviation and (A31) for the FXI:

$$\gamma_{\Delta t} = -\frac{4a(1-a)\phi}{2a-1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) + \theta\frac{1}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\phi-1)+1}{2a-1}$$
$$\times\left[\begin{array}{l}(4a(1-a)(\sigma\phi-1)+1)[a\pi_{Lt} - ((1-a)\pi^*_{Lt} + \pi^*_{Ut})] \\ -\left\{(2a-1) - 2(1-a)[2a(\phi-1)+1]\dfrac{2a(1-a)(\sigma\phi-1)+1-\phi}{2a(\phi-1)+1}\right\} \\ \times[a\pi_{Lt} + (1-a)\pi^*_{Lt} + \pi^*_{Ut}]\end{array}\right] \tag{A46}$$

The optimal monetary policy rules can be characterized by the sum rule (A17), the difference rule (A45), and the optimal LOOP deviation (A46). The general implication is that the monetary policy cannot close all gaps but instead, it faces a trade-off between stabilizing inflation, output, demand gap, and LOOP deviation.

To derive the optimal FXI rule, using the FOC (A38) and the UIP condition (19):

$$\begin{aligned} f_t =& u^*_t + \frac{\theta}{2a\phi\chi_1}[2a(\sigma\phi-1)+1]E_t[a\pi_{Lt+1} + (1-a)\pi^*_{Lt+1} + \pi^*_{Ut+1}] \\ &+ \frac{2a-1}{4a(1-a)\phi\chi_1}(E_t\gamma_{\Delta t+t} - \gamma_{\Delta t}), \end{aligned} \tag{A47}$$

where $\gamma_{\Delta t}$ is given in Equation (A46).

Under Cole and Obstfeld (1991) case, the above conditions reduce to:

$$\begin{aligned} 0 =& (\tilde{Y}_{Lt} + \tilde{Y}_{Ut}) + \theta[a\pi_{Lt} + (1-a)\pi^*_{Lt} + \pi^*_{Ut}], \\ \gamma_{\Delta_{Lt}} - \gamma_{\Delta_{Lt-1}} =& (\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) + \theta[a\pi_{Lt} + (1-a)\pi^*_{Lt} - \pi^*_{Ut}], \\ \gamma_{\Delta_{Lt}} =& -\frac{4a(1-a)}{2a-1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) + \theta\frac{1}{2a-1} \\ &\times[a\pi_{Lt} - ((1-a)\pi^*_{Lt} + \pi^*_{Ut})] - (2a-1)[a\pi_{Lt} + (1-a)\pi^*_{Lt} + \pi^*_{Ut}] \\ f_t =& u^*_t + \frac{\theta}{2a\chi_1}E_t[a\pi_{Lt+1} + (1-a)\pi^*_{Lt+1} + \pi^*_{Ut+1}] \\ &+ \frac{2a-1}{2a(1-a)\chi_1}(E_t\gamma_{\Delta t+t} - \gamma_{\Delta t}). \end{aligned}$$

Combining these equations, optimal monetary policy and FXI rules are characterized by:

$$0 = \tilde{Y}_{Lt} + \theta[a\pi_{Lt} + (1-a)\pi^*_{Lt}], \tag{A48}$$

$$0 = \tilde{Y}_{Ut} + \theta\pi^*_{Ut} + \gamma_{\Delta_{Lt}} - \gamma_{\Delta_{Lt-1}}, \tag{A49}$$

$$
\begin{aligned}
\gamma_{\Delta_{Lt}} =& -\frac{4a(1-a)}{2a-1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t)+\theta\frac{1}{2a-1} \\
&\times[a\pi_{Lt}-((1-a)\pi^*_{Lt}+\pi^*_{Ut})]-(2a-1)[a\pi_{Lt}+(1-a)\pi^*_{Lt}+\pi^*_{Ut}] \qquad \text{(A50)} \\
f_t =& u^*_t+\frac{\theta}{2(1-a)\chi_1}E_t[a\pi_{Lt+1}+(1-a)\pi^*_{Lt+1}+\pi^*_{Ut+1}] \\
&+\frac{2a-1}{2a(1-a)\chi_1}[(E_t\tilde{Y}_{Lt+1}-\tilde{Y}_{Lt})-(E_t\tilde{Y}_{Ut+1}-\tilde{Y}_{Ut})]. \qquad \text{(A51)}
\end{aligned}
$$

There are two key implications. First, the optimal monetary policy rule is asymmetric. The local central bank trades off inflation and output growth of locally produced goods. However, the US central bank trades off the US inflation and output growth, as well as LOOP deviation and demand gap. Second, and more importantly, the optimal FXI targets the LOOP deviation, as discussed in the next proposition.

A.9 Proof of Proposition 4

From Equation (A50), $\partial\gamma_{\Delta t}/\partial\tilde{\Delta}_{Lt}<0$. Hence,

$$
\frac{\partial f_t}{\partial\tilde{\Delta}_{Lt}}=\frac{\partial f_t}{\partial\gamma_{\Delta t}}\frac{\partial\gamma_{\Delta t}}{\partial\tilde{\Delta}_{Lt}}>0.
$$

Thus, the optimal FXI is increasing in Δ_{Lt}. Next, similarly to the PCP case,

$$
\frac{\partial\tilde{e}_0}{\partial\mu^*_0}>\frac{\partial\tilde{e}_1}{\partial\mu^*_0}>\cdots>0, \qquad \frac{\partial\tilde{e}_t}{\partial\mu^*_0}>\frac{\partial\tilde{e}^{FXI}_t}{\partial\mu^*_0}.
$$

Since $\tilde{e}_t$ is close to $\tilde{e}_t$ when the price stickiness is sufficiently high,

$$
\frac{\partial\tilde{\Delta}_{L0}}{\partial\mu^*_0}>\frac{\partial\tilde{\Delta}_{L1}}{\partial\mu^*_0}>\cdots>0, \qquad \frac{\partial\tilde{\Delta}_{Lt}}{\partial\mu^*_0}>\frac{\partial\tilde{\Delta}^{FXI}_{Lt}}{\partial\mu^*_0}.
$$

Hence, the FXI reduces the LOOP deviation. Finally, to show that the optimal FXI is larger under DCP than PCP, the optimal FXI rule under PCP and $\sigma=\phi=1$ is characterized by:

$$
f_t=u^*_t+\frac{\theta}{2a\chi_1}E_t(\pi_{Lt+1}-\pi^*_{Ut+1}). \qquad \text{(A52)}
$$

I compare the optimal FXI rules (A52) under PCP and (A51) under DCP. First, for the output gap term in Equation (A51), when $\sigma=\phi=1$, since $\partial\tilde{\mathcal{D}}_t/\partial\tilde{\Delta}_{Lt}=0$, $\partial\tilde{Y}_{Lt}/\partial\tilde{\Delta}_{Lt}=(\partial\tilde{Y}_{Lt}/\partial\tilde{\mathcal{D}}_t)(\partial\tilde{\mathcal{D}}_t/\partial\tilde{\Delta}_{Lt})=0$. Similarly, $\partial\tilde{Y}_{Ut}/\partial\tilde{\Delta}_{Lt}=0$. Next, for the local inflation, from the

NKPCs (27) and (28), $\partial\pi_{Lt}/\partial\tilde{\Delta}_{Lt} = 0$ under PCP and $\partial(a\pi_{Lt}+(1-a)\pi^*_{Ut})/\partial\tilde{\Delta}_{Lt} = -2\kappa(1-a)$, which is second-order when the home bias is large enough (large a). For the US inflation, $\partial\pi^*_{Ut}/\partial\tilde{\Delta}_{Lt} = 0$ under both PCP and DCP. Hence, up to the first order and without FXI,

$$\left(\frac{\partial E_t(\pi_{Lt+1} - \pi^*_{Ut+1})}{\partial\mu^*_0}\right)^{PCP} \fallingdotseq \left(\frac{\partial E_t(a\pi_{Lt+1} + (1-a)\pi^*_{Lt+1} - \pi^*_{Ut+1})}{\partial\mu^*_0}\right)^{DCP} > 0.$$

The reaction coefficient to the inflation differential is larger under DCP than PCP: [19]

$$\frac{\theta}{2(1-a)\chi_1} > \frac{\theta}{2a\chi_1}.$$

Hence,

$$\left(\frac{\partial f_t}{\partial\mu^*_t}\right)^{DCP} > \left(\frac{\partial f_t}{\partial\mu^*_t}\right)^{PCP}.$$

□

A.10 Proof of Proposition 5

First, I consider the PCP case. Since $\partial f_t/\partial\mu^*_t > 0$ and $\partial\tilde{\mathcal{D}}_t/\partial f_t > 0$, I consider the elasticity of inflation to the demand gap. From the NKPCs for the domestic good inflation in the two countries,

$$\frac{\partial\pi_{Lt}}{\partial\tilde{\mathcal{D}}_t} = -\frac{\partial\pi^*_{Ut}}{\partial\tilde{\mathcal{D}}_t} = \kappa(1-a).$$

Hence, $\partial\pi_{Lt}/\partial\mu^*_t = -\partial\pi^*_{Ut}/\partial\mu^*_t$.

For the imported inflation, from the law of one price,

$$\pi^*_{Ut} = \tilde{e}_t - \tilde{e}_{t-1} + \pi^*_{Ut}, \quad \pi^*_{Lt} = -(\tilde{e}_t - \tilde{e}_{t-1}) + \pi_{Lt}. \tag{A53}$$

Hence,

$$\frac{\partial\pi_{Ut}}{\partial\tilde{\mathcal{D}}_t} = -\frac{\partial\pi^*_{Lt}}{\partial\tilde{\mathcal{D}}_t}, \quad \frac{\partial\pi_{Ut}}{\partial\mu^*_t} = -\frac{\partial\pi^*_{Lt}}{\partial\mu^*_t}$$

[19] The difference between inflation differential terms under PCP and DCP is quantitatively at most second-order. The difference in the optimal FXI volumes under PCP and DCP is mainly because the reaction coefficient to the inflation is larger under DCP, which is due to the deviation from the LOOP.

Hence, the response of CPI inflation is symmetric.

$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} = -\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} \quad (> 0).$$

Next, I consider the DCP case. Since the optimal FXI is larger under DCP than PCP (Equation (32)),

$$\left(\frac{\partial \tilde{e}_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \tilde{e}_t}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \tilde{e}_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \tilde{e}_t}{\partial \mu_t^*}\right)^{PCP} \quad (< 0),$$

For the local imports of US goods, since the LOOP holds,

$$\left(\frac{\partial \pi_{Ut}^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_{Ut}}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_{Ut}^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_{Ut}}{\partial \mu_t^*}\right)^{PCP} \quad (< 0),$$
$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} \quad (< 0).$$

Next, consider the US imports of local goods. Combining Equations (A4) and (A11) and using $\partial\tilde{\Delta}_{Lt}/\partial\tilde{\mathcal{D}}_t = 0$ when $\sigma = \phi = 1$,

$$\frac{\partial \tilde{e}_t}{\partial \tilde{\mathcal{D}}_t} = -(2a-1)^2 < 0.$$

When the price stickiness is sufficiently high, $\partial\tilde{e}_t/\partial\tilde{\mathcal{D}}_t \fallingdotseq (2a-1)^2 < 0$.

Under PCP, since π_{Lt}^* is determined by the LOOP condition (A53),

$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{PCP} = -\frac{\partial \tilde{e}_t}{\partial \tilde{\mathcal{D}}_t} + \frac{\partial \tilde{\pi}_{Lt}^*}{\partial \tilde{\mathcal{D}}_t} \fallingdotseq (2a-1)^2 + \kappa(1-a).$$

Under DCP, since the LOOP does not hold and π_{Lt}^* is determined by the NKPC (28),

$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{DCP} = \kappa(1-a).$$

Comparing the PCP and DCP cases,

$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{DCP} - \left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{PCP} = -(2a-1)^2,$$
$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mu}_t^*}\right)^{DCP} - \left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mu}_t^*}\right)^{PCP} < 0.$$

The difference is first order when a is sufficiently large. Hence,

$$\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} \quad (> 0).$$

$\square$