

Monetary and Exchange Rate Policies in a Global Economy

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What I Do

- Data and central banks' and currency officials' statements suggest:
 - ① **Large countries** use FXI on a massive scale
 - ② Countries **cooperate** to achieve exchange rate stability
 - Self-oriented intervention can cause disagreements across countries
 - ③ Countries **combine** MP and FXI to stabilize inflation
 - ④ In countries with FXI, **exports and imports are mainly in dollars**, but assets and liabilities are not necessarily in dollars
- I build a **general macro-framework with both monetary and exchange rate policies** and study optimality, interaction, and trade-offs

What I Do

► Intuition (summary)

- 2-country (local, US), 2-policy (MP, FXI), 2-friction (nominal, financial)
 - Price is costly to adjust \Rightarrow MP affects the output (Rotemberg'82)
 - Limits to arbitrage \Rightarrow FXI affects the exchange rate (Gabaix/Maggiori'15) ¹
- Optimal MP and FXI targeting rules under cooperation & commitment
 - ① MP \Rightarrow first-best inflation & output, FXI \Rightarrow first-best exchange rate
 - ② MP faces inflation-output trade-off (inefficiencies $>$ policies):
 - **FXI improves the MP trade-off** by stabilizing inflation but **distorts the exchange rate** by improving the local purchasing power
 - MP and FXI should be **combined** to stabilize the inflation

¹Empirically, limits to arbitrage are reflected in the excess carry trade return (deviation from uncovered interest rate parity).

What I Do

- Capital flow management in international finance can be driven by **dollar dominance in international trade** (Gopinath et al., 2020)
- Now in progress: Nash equilibrium (e.g. competitive devaluation), gains from cooperation (Bodenstein et al., 2019, 2024)

Literature

▶ Table

▶ Relationship with Literature

▶ FXI volume

▶ FXI over GDP share

● Theory on foreign exchange intervention

- Gabaix/Maggiore'15, Fanelli/Straub'21: FXI independently of MP
- Cavallino'19, Amador/etal'20, Basu/etal'21, Itskhoki/Mukhin'23:
MP and FXI in a small open economy

⇒ New trade-off between internal and external objectives,
Cooperative MP and FXI in a large two-country model

● Empirical evidence on the effectiveness of FXI

- Fatum/Hutchison'10, Kuesteiner/Phillips/Villamizar-Villegas'18, Fratzscher/etal'19,
Adler/Mano'21, Rodnyansky/Timmer/Yago'24, Dao/Gourinchas/Mano/Yago'24

⇒ Normative implication of FXI

● Exchange rate disconnect from macro fundamentals

- Itskhoki/Mukhin'21, Jiang/Krishnamurthy/Lustig'21,23, Devereux/Engel/Wu'23,
Engel/Wu'22, Kekre/Lenel'23, Fukui/Nakamura/Steinsson'23

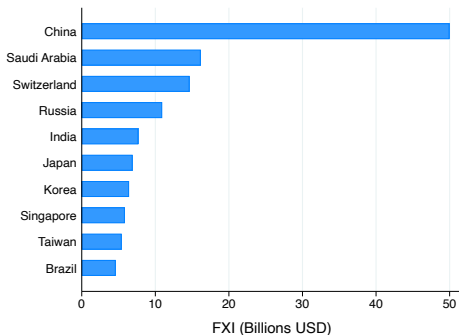
⇒ Role of FXI in stabilizing capital flows and exchange rate

FXI Volume by Countries

► GDP share

- Fact 1. Large emerging and advanced economies conduct FXI on massive scales (1.2% of GDP on average).

Figure 1: The 10 Largest FXI Volumes by Country (Billions of US Dollars)



Note: Quarterly FXI in 122 countries, 2000-24 average (Adler et al., 2022). Excludes countries that mainly intervene against the euro.

Cooperation on Exchange Rate Stabilization

- Fact 2. Countries cooperate to promote exchange rate stability.
 - Plaza Accord (1985): US, UK, Germany, France, Japan
 - Recent pandemic & war \Rightarrow worldwide depreciation & inflation

*We will continue to **cooperate** ... also continue to **consult closely** on foreign exchange market developments in line with our existing G20 commitments, while acknowledging serious concerns of Japan and the Republic of Korea about the recent **sharp depreciation of the Japanese yen and the Korean won.***

— Japan - Korea - United States Trilateral Ministerial Joint Press Statement, 2024/04/17

Cooperation on Exchange Rate Stabilization

- Self-oriented intervention can cause disagreements across countries.

*The Treasury chief said **China's yuan is among the currencies that she monitors, along with the euro and yen.** Yellen continued to register her **discomfort with government intervention in currency markets** — especially among Group of Seven countries. ... **"It should happen very rarely and be communicated to trade partners."** (Bloomberg)*

— Janet Yellen, Secretary of the Treasury, United States, 2024/05/13

Combination of MP and FXI

- Fact 3. FXI is combined with MP to stabilize inflation.

South Korea's central bank is ready to take steps, including intervention to stabilize the won against the dollar. "This depreciation pressure due to the dollar strength is a bad factor for our inflation, because our imported prices increase a lot." (Reuters)

— Rhee Changyong, Governor of the Bank of Korea, 2024/04/28

"The combination of rising interest rates and foreign currency sales was effective in quickly bringing inflation back into the range of price stability. Without the use of foreign currency sales, the SNB would have had to raise the policy rate to a higher level." (ICMB Public Lecture)

— Martin Schlegel, Vice Chairman of the Governing Board, Swiss National Bank, 2024/04/09

Step-by-step construction of a two-country model

- 1 Optimal MP, nominal friction
⇒ First-best allocation
- 2 Optimal MP & FXI, nominal & financial frictions
⇒ First-best allocation
- 3 Optimal MP & FXI, nominal & financial frictions, cost-push shock
⇒ FXI improves the MP trade-off between inflation and output
- 4 Dollar pricing
⇒ FXI has a strong effect on local inflation stabilization

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Optimal Monetary Policy

► Lagrangian

► Local Currency Pricing loss function

- Cooperation = maximize the sum of welfare in the two countries
= minimize the loss function:²³

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\underbrace{\frac{\theta}{\kappa} (\pi_{Lt}^2 + \pi_{Ut}^{*2})}_{\text{Inflation}} + \underbrace{\tilde{Y}_{Lt}^2 + \tilde{Y}_{Ut}^2}_{\text{Output gap}} \right] + a(1-a) \underbrace{\tilde{\mathcal{D}}_t^2}_{\text{Demand gap}}$$

- $\tilde{\mathcal{D}}_t \equiv \tilde{C}_t - \tilde{C}_t^* - \tilde{Q}_t$: demand gap = relative marginal utility⁴
 - $\tilde{\mathcal{D}}_t = 0$: consumption smoothing across countries (risk-sharing)
 - $\tilde{\mathcal{D}}_t > 0$: Local economy has excess demand (purchasing power) over US

² κ : slope of NKPC, θ : CES between differentiated goods, a : home bias, \tilde{Q}_t : real exchange rate ($\tilde{Q}_t \uparrow$ = local depreciation)

³ π_{Lt}, π_{Ut}^* : inflation of local (US) goods consumed by local (US) households

⁴ $\mathcal{D}_t \equiv (U'(C_t^*)/U'(C_t))/\tilde{Q}_t = (C_t/C_t^*)/\tilde{Q}_t$ with log utility

Optimal Monetary Policy

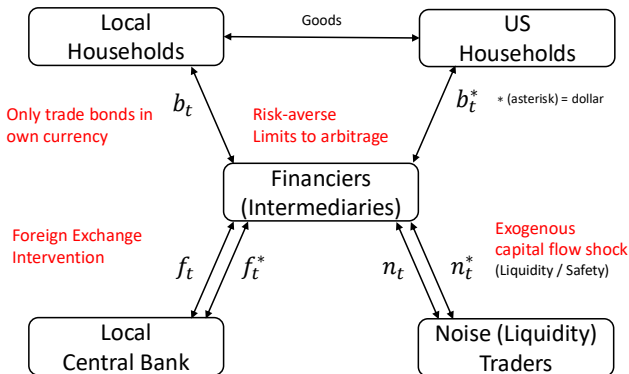
Lemma 1 (Optimal Monetary Policy)

With log and Cobb-Douglas utility and without cost-push shocks, optimal monetary policy closes all gaps: $\pi_{Lt} = \pi_{Ut}^ = \tilde{Y}_{Lt} = \tilde{Y}_{Ut} = \tilde{D}_t = 0$.*

- MP: natural (flexible-price) level interest rate
→ No inflation and output gap
- Output $\uparrow \rightarrow$ Price \downarrow (exchange rate depreciates) proportionally
→ Relative wealth is unchanged ($P_{Lt} Y_{Lt} = P_{Ut} Y_{Ut}$), No demand gap
(Corsetti/Dedola/Leduc'10)

Step-by-step construction of a two-country model

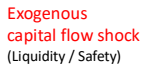
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⇒ FXI has a strong effect on local inflation stabilization



Example:

- Investors **buy \$ bonds** ($n_t^* \uparrow$) \Rightarrow financiers short \$ and long local currency
- Local bond return** \uparrow due to risk premium (UIP deviation)
- However, HHs cannot sell \$ bond to buy local bond

- ▶ UIP condition



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Optimal Monetary Policy and FXI

► Lagrangian

Proposition 1 (Optimal MP and FXI: “Dichotomy” Case)

With log and Cobb-Douglas utility and without cost-push shocks, optimal monetary policy closes the inflation and output gap:

$$\pi_{Lt} = \pi_{Ut}^* = \tilde{Y}_{Lt} = \tilde{Y}_{Ut} = 0$$

and optimal FXI ($f_t = n_t^$) closes the demand gap:*

$$\tilde{\mathcal{D}}_t = 0$$

- Conventional **Dichotomy** between monetary policy and FXI
 - Two separate targets: MP \Rightarrow inflation, FXI \Rightarrow exchange rate

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Optimal Policies when the First-Best Cannot be Achieved

- ① Cost-push (markup) shock (μ_t^*) (example: pandemic, war)
 \Rightarrow MP trades off inflation and output gap [▶ Details](#)

- ② General CRRA & CES preference⁵ [▶ Details](#)

$$U(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma}, \quad C_t = \left[a^{\frac{1}{\phi}} C_{Lt}^{\frac{\phi-1}{\phi}} + (1-a)^{\frac{1}{\phi}} C_{Ut}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{1-\phi}}$$

- Local and US goods are **substitutes** when $\sigma\phi > 1$

⁵CO preference is the limiting case where $\sigma = \phi = 1$.

International Transmission of Cost-Push Shock (No FXI)

Lemma 3 (Transmission of shock: No FXI)

Suppose that the local and US goods are substitutes ($\sigma\phi > 1$), FXI is not available, and monetary policy follows the optimal rule.

In response to a period-0 US cost-push shock,

- *US: inflation \Rightarrow deflation, output gap < 0*
- *Local: deflation \Rightarrow inflation, output gap > 0*
- *Local currency depreciates*

Optimal MP and FXI

[► Details](#)

- Case 2: Both MP and FXI are available

Proposition 2 (Optimal Monetary Policy and FXI)

Optimal monetary policy rules are:

$$0 = \theta \pi_{Lt} + (\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}) + \xi_{\pi}(\pi_{Lt} - \pi_{Ut}^*) + \xi_D(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1}),$$

$$0 = \theta \pi_{Ut}^* + (\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}) - \xi_{\pi}(\pi_{Lt} - \pi_{Ut}^*) - \xi_D(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1})$$

Optimal FXI rule is:

$$f_t = n_t^* + \xi_f E_t(\pi_{Lt+1} - \pi_{U,t+1})$$

- Zero-UIP deviation ($f = n^*$) is no longer an optimal FXI
- When the local inflation is higher than the US ($\pi_L > \pi_U^*$), optimal FXI is to buy the local currency ($f \uparrow$).

International Transmission of Cost-Push Shock (FXI)

Proposition 3 (Transmission of shock: both MP and FXI)

Suppose that (i) local and US goods are substitutes ($\sigma\phi > 1$) and (ii) both MP and FXI follow the optimal rule.

Comparing the responses to a US cost-push shocks with and without FXI,

- Optimal FXI is to *buy the local currency*.
- *FXI smooths the paths of inflation, output gap, and real exchange rate.*
- However, *UIP deviation becomes negative* (local bond return \downarrow) and *demand gap becomes positive* (local purchasing power \uparrow).

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⇒ **FXI has a strong effect on local inflation stabilization**

Dollar Pricing

[▶ NKPC and loss function](#)

- Exports and imports are both denominated in dollars (Gopinath/etal'20)
 - Law of one price (LOOP) does not hold for local goods
 - $\Delta_{Lt} \equiv \mathcal{E}_t P_{Lt}^* / P_{Lt}$: Price of local goods in \$ / local currency
 - $\Delta_{Lt} \neq 1$: inefficiency due to price dispersion despite the same marginal cost
 - Loss function depends on Δ_{Lt} (Engel'11)
- Expenditure switching mainly works via local imports from US

Dollar Pricing

► Optimal MP

► Optimal MP and FXI

Proposition 4 (Optimal FXI under Dollar Pricing)

Under DCP, log and Cobb-Douglas preference, and when both MP and FXI follow the optimal rules,

- ① *Optimal local currency purchase is increasing in $\Delta_{Lt} \equiv \mathcal{E}_t P_{Lt}^* / P_{Lt}$.*
 - ② *When the US cost-push shock hits,*
 - *Optimal FXI is larger under DCP than PCP.*
 - *Price dispersion Δ_{Lt} increases less with FXI than without FXI.*
- Optimal FXI targets the price dispersion.
 - US cost-push inflation ($\mu_t^* \uparrow$) \Rightarrow \$ appreciates ($\mathcal{E}_t \uparrow$)
 \Rightarrow Local good is expensive in \$ ($\Delta_{Lt} \uparrow$) \Rightarrow buy local currency ($f_t \uparrow$)

Dollar Pricing

Proposition 5 (Asymmetric Transmission)

Under PCP, optimal FXI decreases the local CPI inflation ($\pi_t \Downarrow$) and increases the US CPI inflation ($\pi_t^ \Uparrow$) by the same degree (symmetry).*

*Under DCP, optimal FXI **decreases the local CPI inflation more** and **increases the US CPI inflation less** than the PCP case (asymmetry):*

- Buy local currency \Rightarrow local appreciation \Rightarrow
 - Local import price from US \Downarrow (as optimal FXI is large)
 - But limited effects on US import price from local (as dollar-priced)
- Explains why FXI is often used in DCP economies

Nash Equilibrium & Gains from Cooperation (In Progress)

- In reality, central banks **do not necessarily coordinate** on policies.
 - Inward-looking MP (Taylor rule)
 - Reserve accumulation & competitive devaluation using FXI
- Focus on the **open-loop Nash equilibrium** (Bodenstein et al., 2019, 2024)
 - In period 0, each player specifies his/her state-contingent plans for every possible future state.
 - Each player's action is the best response to the other player's best response.

▶ Formal Definition
- The steady state is not efficient in general.
- Assume Taylor-rule MP + compare cooperative & non-cooperative FXI
 - Remark: the computational toolbox supports one strategic instrument per player.

Non-Cooperative Monetary Policy (Bodenstein/etal'19,24)

- Consider a decline in the local markup \Rightarrow local inflation \downarrow , output \uparrow
- Both local and US monetary policies are more expansionary under Nash (**competitive devaluation**).
- Spillover on US output depends on the substitutability between the local and US goods
- Comparing Nash with cooperation,
 - **Substitutes** ($\sigma\phi > 1$): US output \downarrow (demand for local good \uparrow)
 - **Complements** ($\sigma\phi < 1$): US output \uparrow (demand for US good \uparrow)
 - **Independent** ($\sigma\phi = 1$): No international spillover
- Study how non-cooperative FXI depends on the substitutability.

Conclusion

- A general framework to study monetary and exchange rate policies
- MP and FXI are not two separate policy tools but central banks should combine them to stabilize the exchange rate and inflation
- Bridges the gap between dollar pricing in international trade and capital flow management in international finance
- Now in progress: Nash equilibrium, gains from cooperation
- Important to understand how to combine FXI with other policies
 - Capital control, macroprudential policy
 - IMF's integrated policy framework

Appendix

Intuition

▶ Back

- Different transmission channels of MP and FXI
- Raise interest rate → **inflation ↓ and consumption ↓**
- Buy LC, sell \$ bond → **inflation ↓ but consumption ↑**
 - FXI appreciates the local currency
 - US demand for local good ↓ → domestic good inflation ↓
 - Cheaper \$ → imported good inflation ↓
 - Local bond return ↓ relative to \$ bond return
 - Limits to arbitrage: local households cannot invest in US bonds⁶
 - Local currency stronger + consumption demand ↑ relative to the US
- Central banks should combine MP and FXI appropriately

⁶In a standard model without financial friction, FXI has no effect on exchange rate (Backus/Kehoe'89).

Evidence for Limits to Arbitrage: UIP Deviation

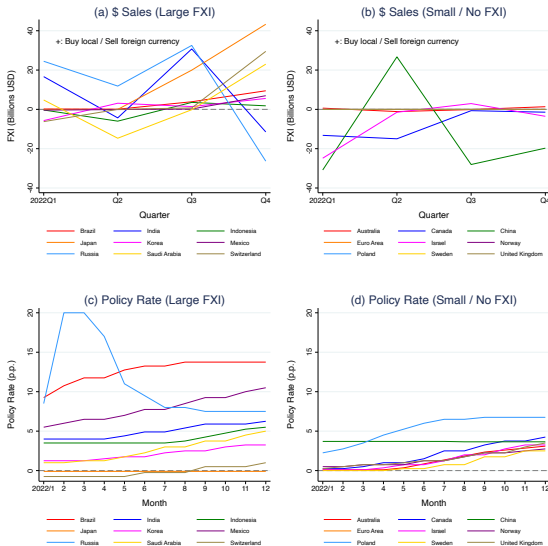
Country	Currency	α_0	(s.e.)	β_1	(s.e.)	$\chi^2(\alpha_0 = \beta_1 = 0)$	R^2
Australia	AUD	-0.001	(0.002)	-1.63***	(0.48)	16.3***	0.014
Austria	ATS	0.002	(0.002)	-1.75***	(0.58)	9.5***	0.023
Belgium	BEF	-0.0002	(0.002)	-1.58***	(0.39)	17.5***	0.025
Canada	CAD	-0.003	(0.001)	-1.43***	(0.38)	19.1***	0.013
Denmark	DKK	-0.001	(0.001)	-1.51***	(0.32)	25.4***	0.025
France	FRF	-0.001	(0.002)	-0.84	(0.63)	1.9	0.007
Germany	DEM	0.002	(0.001)	-1.58***	(0.57)	7.9**	0.015
Ireland	IEP	-0.002	(0.002)	-1.32***	(0.38)	12.3***	0.020
Italy	ITL	-0.002	(0.002)	-0.79**	(0.33)	7.0**	0.013
Japan	JPY	0.006***	(0.002)	-2.76***	(0.51)	28.9***	0.038
Netherlands	NLG	0.003	(0.002)	-2.34***	(0.59)	16.0***	0.041
Norway	NOK	-0.0003	(0.001)	-1.15***	(0.39)	10.4***	0.013
New Zealand	NZD	-0.001	(0.002)	-1.74***	(0.39)	28.3***	0.038
Portugal	PTE	-0.002	(0.002)	-0.45**	(0.20)	5.9*	0.019
Spain	ESP	0.002	(0.003)	-0.19	(0.46)	2.8	0.001
Sweden	SEK	0.0001	(0.001)	-0.42	(0.50)	0.9	0.002
Switzerland	CHF	0.005***	(0.002)	-2.06***	(0.55)	13.9***	0.026
UK	GBP	-0.003**	(0.001)	-2.24***	(0.60)	14.2***	0.028
Panel, pooled		0.0002	(0.001)	-0.79***	(0.15)	22.3***	
Panel, fixed eff.				-1.01***	(0.21)	19.1***	

Source: Valchev (2015).

- If households can invest freely in two currencies, the excess return is zero (uncovered interest rate parity holds). However, UIP does not hold in data.
- Fama (1985) regression: $e_{t+1} - e_t - (i_t - i_t^*) = \alpha_0 + \beta_1(i_t - i_t^*) + \epsilon_t$
- When $\beta_1 < 0$, high interest rate currency appreciates in future = positive return

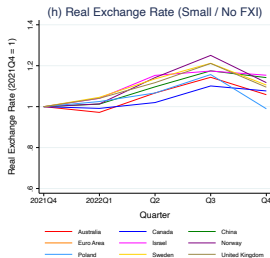
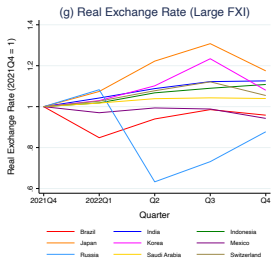
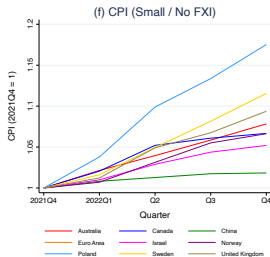
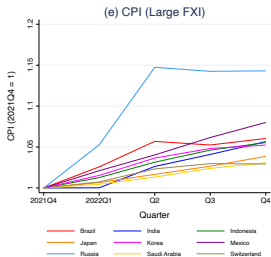
Raw Data (FXI Volume, Policy Rate)

▶ Back



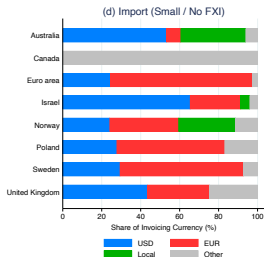
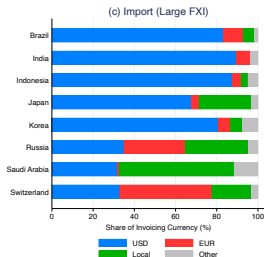
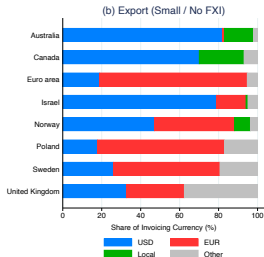
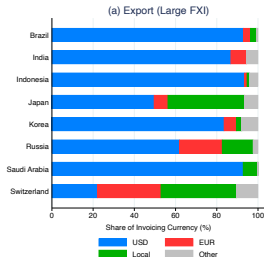
Raw Data (Inflation, Real Exchange Rate)

▶ Back



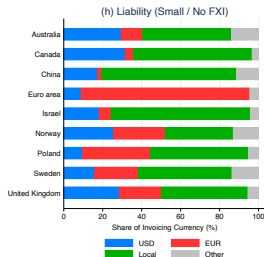
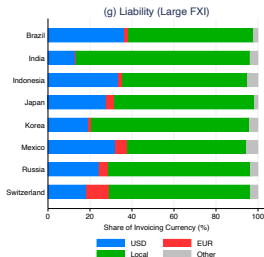
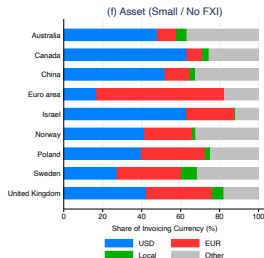
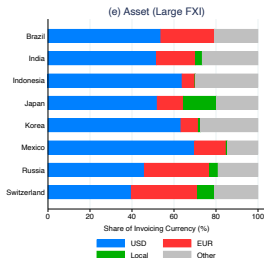
Raw Data (Export, import)

▶ [Back](#)



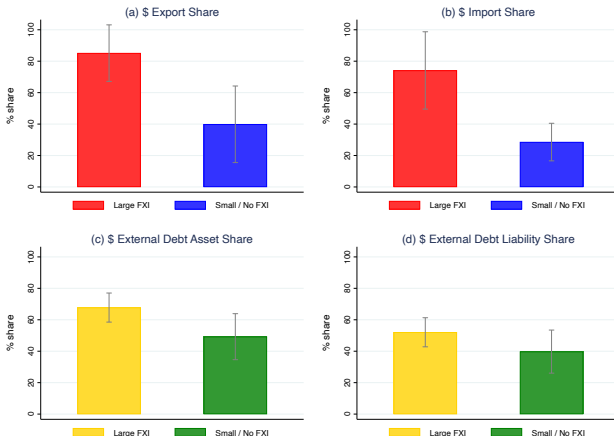
Raw Data (Asset, Liability)

▶ Back



Invoicing Currency (Debt Asset & Liability)

▶ Back



- Export and import: more dollarized in large FXI countries
- Debt Asset and liability: small difference

Households (Details)

- CRRA, CES bundle of local and US goods

$$U(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \iota_l \frac{L_t^{1+\eta}}{1+\eta}, \quad C_t = \left[a^{\frac{1}{\phi}} C_{Lt}^{\frac{\phi-1}{\phi}} + (1-a)^{\frac{1}{\phi}} C_{Ut}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{1-\phi}}$$

$$C_{Lt} = \left[\int_0^1 C_t(l)^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\zeta}{1-\zeta}}, \quad C_{Ut} = \left[\int_0^1 C_t(u)^{\frac{\zeta-1}{\zeta}} du \right]^{\frac{\zeta}{1-\zeta}}$$

- $\sigma = \phi = 1$: log and Cobb-Douglas utility (Cole/Obstfeld '91)
- Budget constraint:

$$P_{Lt} C_{Lt} + P_{Ut} C_{Ut} + \frac{B_t}{R_t} = B_{t-1} + W_t L_t + \Pi_t + T_t$$

► Households (simple)

► Households (general)

► Full quantitative model

Solution to Households' Problem

- Euler equation for the local bond: $\beta R_t E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} = 1$
- Labor supply equation: $C_t^\sigma L_t^\eta = \frac{W_t}{P_t}$
- Demand for local and US goods:

$$C_{Lt} = a \left(\frac{P_{Lt}}{P_t} \right)^{-\phi} C_t, \quad C_{Ut} = (1 - a) \left(\frac{P_{Ut}}{P_t} \right)^{-\phi} C_t$$

- Demand for differentiated goods produced within each country:

$$C_t(l) = \left(\frac{P_t(l)}{P_{Lt}} \right)^{-\zeta} C_{Lt}, \quad C_t(u) = \left(\frac{P_t(u)}{P_{Lt}} \right)^{-\zeta} C_{Ut}$$

Firms' Maximization Problem

- Producer currency pricing (PCP): law of one price $P_t(l) = \varepsilon_t P_t^U(l)$
- Firms set prices subject to price adjustment cost (Rotemberg '82)

$$\max \sum_{t=0}^{\infty} \beta^t \frac{U'(C_{t+k})}{U'(C_t)} \left[P_t(l) \left(\frac{P_t(l)}{P_{Lt}} \right)^{-\zeta} Y_{Lt} - \frac{W_t}{A_t} \frac{1}{1-\tau_t} \left(\frac{P_t(l)}{P_{Lt}} \right)^{-\zeta} Y_{Lt} - \frac{AC_p}{2} \left(\frac{P_t(l)}{P_{t-1}(l)} - 1 \right)^2 P_{Lt} Y_{Lt} \right]$$

- New Keynesian Phillips Curve:

$$\pi_{Lt} = \beta E_t \pi_{Lt+1} + \kappa \left\{ \underbrace{(\sigma + \eta) \tilde{Y}_{Lt}}_{\text{Output gap}} + \underbrace{2a(\sigma\phi - 1)(\tilde{P}_{Lt} - \tilde{P}_{Ut})}_{\text{Relative price}} + \underbrace{(1-a)\tilde{D}_t}_{\text{Demand gap}} + \underbrace{\mu_t}_{\text{Cost-push}} \right\}$$

- Productivity: $\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_{at}$, $\epsilon_{at} \sim N(0, \sigma_a^2)$
- Markup shock: $\mu_t = \frac{\zeta}{(\zeta-1)(1-\tau_t)}$

Lagrangian for Cooperative Policy

[▶ CO Solution](#)
[▶ CO Solution \(FXI\)](#)

$$\begin{aligned}
 \mathcal{L}_t^W = & -\frac{1}{2} \left[\tilde{Y}_{Lt}^2 + \tilde{Y}_{Ut}^2 + \frac{\zeta}{\kappa} ((\pi_t^{CPI})^2 + (\pi_t^{CPI*})^2) \right] + \frac{a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1} (\tilde{Y}_{Lt} - \tilde{Y}_{Ut})^2 \\
 & - \frac{a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1} (\hat{\Delta}_t + \tilde{\mathcal{D}}_t)^2 \\
 & + \gamma_{Lt}^J \left[\begin{aligned} & -\pi_{Lt} + \beta E_t \pi_{Lt+1} \\ & + \kappa \left\{ (\sigma + \eta) \tilde{Y}_{Lt} - (1-a)[2a(\sigma\phi-1)(\tilde{T}_t + \hat{\Delta}_t) - (\hat{\Delta}_t + \tilde{\mathcal{D}}_t)] \right\} \end{aligned} \right] \\
 & + \gamma_{Lt}^U \left[\begin{aligned} & -\pi_{Lt}^* + \beta E_t \pi_{Lt+1}^U \\ & + \kappa \left\{ (\sigma + \eta) \tilde{Y}_{Lt} - (1-a)[2a(\sigma\phi-1)(\tilde{T}_t + \hat{\Delta}_t) - (\hat{\Delta}_t + \tilde{\mathcal{D}}_t) - \Delta_t] \right\} \end{aligned} \right] \\
 & + \gamma_{Ut}^U \left[\begin{aligned} & -\pi_{Ut}^* + \beta E_t \pi_{Ut+1}^U \\ & + \kappa \left\{ (\sigma + \eta) \tilde{Y}_{Ut} - (1-a)[2a(\sigma\phi-1)(\tilde{T}_t + \hat{\Delta}_t) - (\hat{\Delta}_t + \tilde{\mathcal{D}}_t)] \right\} \end{aligned} \right] \\
 & + \gamma_{Ut}^J \left[\begin{aligned} & -\pi_{Ut} + \beta E_t \pi_{Ut+1}^J \\ & + \kappa \left\{ (\sigma + \eta) \tilde{Y}_{Ut} - (1-a)[2a(\sigma\phi-1)(\tilde{T}_t + \hat{\Delta}_t) - (\hat{\Delta}_t + \tilde{\mathcal{D}}_t) + \Delta_t] \right\} \end{aligned} \right] \\
 & + \gamma_t \left[-\pi_{Lt} + \pi_{Ut} - \hat{T}_t + \hat{T}_{t-1} - \hat{\Delta}_t + \hat{\Delta}_{t-1} \right] \\
 & + \lambda_t \left[-E_t \tilde{\mathcal{D}}_{t+1} + \tilde{\mathcal{D}}_t + n_t^* - f_t \right]
 \end{aligned}$$

- Under PCP, $\hat{\Delta}_t = 0$ and no constraint for TOT misalignment

[▶ Back to Loss Function](#)

Financiers' Problem

▶ Back

▶ Asset market clearing

- I model financiers based on [Itskhoki and Mukhin \(2021, 2023\)](#).⁷⁸
- Financiers trade both local and US bonds but they are risk-averse.

$$\max_{d_t^*} E_t \left\{ -\frac{1}{\omega} \exp(-\omega \Pi_t) \right\}, \quad \Pi_t = \bar{R}_t^* \frac{D_t^*}{P_t^*}$$

- $\omega > 0$: risk aversion
- $\bar{R}_t^* = R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$: nominal carry trade return ($\neq 0$ when $\omega > 0$)
- D_t^* : carry trade position

⁷I assume that the financiers' profit is transferred to the local households in a lump-sum way. $d_t^* = D_t^* / \bar{Y}$.

⁸The CARA utility can potentially be micro-founded using occasionally binding borrowing constraints or costs of hedging.

International Asset Market

▶ Back

▶ UIP general case

$$\underbrace{E_t \tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t}_{\Delta \text{ Demand gap}} = \underbrace{\tilde{r}_t - \tilde{r}_t^* - (E_t \tilde{Q}_{t+1} - \tilde{Q}_t)}_{\substack{\text{UIP deviation (real)} \\ \text{(LC - dollar return)}}} = \underbrace{\chi(n_t^* - f_t)}_{\substack{\text{Noise trader buys \$ } (n_t^*) \\ \text{- CB buys LC } (f_t)}}$$

- \$ demand ($n_t^* \uparrow$) \rightarrow local return $\tilde{r}_t \uparrow$, depreciation $\tilde{Q}_t \uparrow$
 $\rightarrow UIP > 0$, local savings \uparrow , future demand $E_t \tilde{\mathcal{D}}_{t+1} \uparrow$
- Central banks' local currency demand = noise traders' \$ demand
 $\Rightarrow UIP = 0$ and $\tilde{\mathcal{D}}_t = 0$ (risk sharing holds).⁹¹⁰

⁹Without log and Cobb-Douglas preference, $E_t \tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = 0$: risk sharing holds only in expectation but not for every state of the economy. I assume that the size of the financial sector is large enough relative to the households.

¹⁰ $\chi = \omega \sigma_e^2 / \beta$ is increasing in financiers' risk-aversion (ω) and exchange rate volatility (σ_e^2).

Asset Market Clearing

- B_t, N_t, D_t, F_t : aggregate demand for local bond
- Zero net position:

$$B_t/R_t + \mathcal{E}_t B_t^*/R^* = 0, \quad N_t/R_t + \mathcal{E}_t N_t^*/R^* = 0$$

$$D_t/R_t + \mathcal{E}_t D_t^*/R^* = 0, \quad F_t/R_t + \mathcal{E}_t F_t^*/R^* = 0$$

- Market clearing for local and US bonds:

$$B_t + N_t + D_t + F_t = 0, \quad B_t^* + N_t^* + D_t^* + F_t^* = 0$$

UIP condition (General Case)

► UIP simple case

► Quantitative model

- The maximization problem for intermediaries implies:

$$\underbrace{E_t \tilde{D}_{t+1} - \tilde{D}_t}_{\Delta \text{ Demand gap}} = \underbrace{\tilde{r}_t - \tilde{r}_t^* - E_t \Delta \tilde{e}_{t+1}}_{\text{UIP deviation (LC - \$ return)}} = \underbrace{\chi_1 (n_t^* - f_t)}_{\substack{\text{Noise trader buys \$ } (n_t^*) \\ \text{- CB buys LC } (f_t)}} - \underbrace{\chi_2 b_t}_{\text{HHs' savings}}$$

where $\chi_1 \equiv m_n(\omega\sigma_e^2/m_d)$, $\chi_2 \equiv \bar{Y}(\omega\sigma_e^2/m_d)$ for finite $(\omega\sigma_e^2/m_d)$.

- [Itskhoki and Mukhin \(2021, 2023\)](#) scale the risk aversion ω so that $\omega\sigma_e^2$ is finite and nonzero and risk premium is first-order.¹¹
- When deriving analytical result, I assume $\chi_1 = 1$ and $\chi_2 = 0$ for tractability. Assume financial sector (m_d financiers and m_n noise traders) is larger than HHs.

¹¹See [Hansen and Sargent \(2011\)](#).

Optimal MP and FXI: Details

► Optimal Rules

► IRF

Optimal monetary policies for local and US central banks are:

$$0 = \zeta \pi_{Lt} + (\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}) + \xi_{\pi}(\pi_{Lt} - \pi_{Ut}^*) + \xi_D(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1})$$

$$0 = \zeta \pi_{Ut} + (\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}) - \xi_{\pi}(\pi_{Lt} - \pi_{Ut}^*) - \xi_D(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1})$$

where

$$\xi_{\pi} = (1-a) \frac{2a(\sigma\phi - 1) + 1}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}} \frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1} \zeta$$

$$\xi_D = \frac{2a(1-a)\phi}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}} \frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}.$$

Optimal FXI is:

$$f_t = n_t^* + \xi_f E_t(\pi_{Lt+1} - \pi_{U,t+1}^U)$$

where

$$\xi_f = \frac{1-a}{\chi} \frac{2a(\sigma\phi - 1) + 1}{2a(1-a)\phi} \text{ and } \chi = \frac{\omega\sigma_e^2}{\beta} \left(\xi_f > 0 \text{ if } \sigma\phi > 1 - \frac{1}{2a} \right).$$

NKPC and Loss Function under Dollar Pricing

► Back

- NKPCs for local goods in LC (π_{Lt}) and \$ (π_{Lt}^*), US goods in \$ (π_{Ut}^*)
 - Local good inflation depends on the LOOP deviation (Δ_{Lt})

$$\pi_{Lt} = \beta\pi_{Lt+1} + \kappa\{(\sigma + \eta)\tilde{Y}_{Lt} - (1 - a)[2a(\sigma\phi - 1)(\tilde{T}_t + \tilde{\Delta}_{Lt}) + (\tilde{D}_t + \tilde{\Delta}_{Lt})] + \mu_t\}$$

$$\pi_{Lt}^* = \beta\pi_{Lt+1}^* + \kappa\{(\sigma + \eta)\tilde{Y}_{Lt} - (1 - a)[2a(\sigma\phi - 1)(\tilde{T}_t + \tilde{\Delta}_{Lt}) + (\tilde{D}_t + \tilde{\Delta}_{Lt})] - \tilde{\Delta}_{Lt} + \mu_t^*\}$$

$$\pi_{Ut}^* = \beta\pi_{Ut+1}^* + \kappa\{(\sigma + \eta)\tilde{Y}_{Ut} + (1 - a)[2a(\sigma\phi - 1)\tilde{T}_t - \tilde{D}_t] + \mu_t^*\}$$

- Loss function depends on the LOOP deviation (Δ_{Lt}):

$$\mathcal{L} = -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\begin{aligned} &(\sigma + \eta) (\tilde{Y}_{Lt}^2 + \tilde{Y}_{Ut}^2) + \frac{\zeta}{\kappa} (a\pi_{Lt}^2 + (1 - a)\pi_{Lt}^{*2} + \pi_{Ut}^{*2}) \\ &- \frac{2a(1 - a)(\sigma\phi - 1)\sigma}{4a(1 - a)(\sigma\phi - 1) + 1} (\tilde{Y}_{Lt} - \tilde{Y}_{Ut})^2 \\ &+ \frac{2a(1 - a)\phi}{4a(1 - a)(\sigma\phi - 1) + 1} (\tilde{D}_t + \Delta_{Lt})^2 \end{aligned} \right]$$

Dollar Pricing: Optimal Monetary Policy

▶ Back

Optimal monetary policy under DCP when FXI is not available:

$$0 = \theta a \pi_{L_t} + (\tilde{C}_t - \tilde{C}_{t-1}) + \frac{2a(1-a)\phi}{2a(\phi-1)+1} \frac{\sigma-1}{\sigma} (\tilde{D}_t - \tilde{D}_{t-1} + \tilde{\Delta}_t - \tilde{\Delta}_{t-1})$$

$$0 = \theta[(1-a)\pi_{L_t}^* + \pi_{U_t}^*] - (\tilde{C}_t^* - \tilde{C}_{t-1}^*) - \frac{2a(1-a)\phi}{2a(\phi-1)+1} \frac{\sigma-1}{\sigma} (\tilde{D}_t - \tilde{D}_{t-1} + \tilde{\Delta}_t - \tilde{\Delta}_{t-1})$$

- Local: trades off **local inflation** and demand growth.
- US: trades off **international dollar price inflation** and demand growth.
- When $\sigma \neq 1$, MP also trades off the LOOP deviation.

Dollar Pricing: Optimal Monetary Policy and FXI

► Back

Optimal monetary policy and FXI under DCP:

$$\begin{aligned}
 0 &= \tilde{Y}_{Lt} + \theta[a\pi_{Lt} + (1-a)\pi_{Lt}^*], \\
 0 &= \tilde{Y}_{Ut} + \theta\pi_{Ut}^* + \gamma\Delta_{Lt} - \gamma\Delta_{Lt-1}, \\
 \gamma\Delta_{Lt} &= -\frac{4a(1-a)}{2a-1}(\tilde{\Delta}_{Lt} + \tilde{D}_t) + \theta\frac{1}{2a-1} \\
 &\quad \times [a\pi_{Lt} - ((1-a)\pi_{Lt}^* + \pi_{Ut}^*)] - (2a-1)[a\pi_{Lt} + (1-a)\pi_{Lt}^* + \pi_{Ut}^*] \\
 f_t &= n_t^* + \frac{\theta}{2a\chi_1}E_t[a\pi_{Lt+1} + (1-a)\pi_{Lt+1}^* + \pi_{Ut+1}^*] \\
 &\quad + \frac{2a-1}{2a(1-a)\chi_1}(E_t\gamma\Delta_{t+1} - \gamma\Delta_t).
 \end{aligned}$$

- Local: MP trades off local inflation and output growth.
- US: MP trades off US inflation, output growth, LOOP deviation, and demand gap.
- FXI responds to the LOOP deviation.

Calibration (Optimal Policy)

Table 1: Benchmark Parameters

	Description	Value	Notes
β	Discount factor (local)	0.995	Annual interest rate = 2%
σ	Relative risk aversion	5	Cole and Obstfeld (1991)
η	Inverse Frisch elasticity	1.5	Itskhoki and Mukhin (2021)
ζ_l	Labor disutility (local)	1	$\bar{L} = 1$
a	Home bias of consumption	0.88	Bodenstein et al. (2023)
ϕ	CES Local & US goods	1.5	Cole and Obstfeld (1991)
θ	CES differentiated goods	10	Ottonello and Winberry (2020)
ρ_a	Persistence of productivity shock	0.95	Bodenstein et al. (2023)
χ_1	Elasticity of UIP to FXI	0.43	$\Delta \log \text{UIP}_t / \Delta \log \text{FXI}_t$
χ_2	Elasticity of UIP to NFA	0.001	UIP/NFA ratio

Note: The table shows the parameter settings for the CO case (Cole/Obstfeld'91) where $\sigma = \phi = 1$ and non-CO case where $\sigma, \phi \neq 1$.

Definition of Open-Loop Nash Equilibrium

[▶ Back](#)

- $x_t = (\tilde{x}_t, i_{1,t}, i_{2,t})$: endogenous variables, ζ_t : exogenous variables
- $i_{1,t}, i_{2,t}$: policy instrument of player $j = [1, 2]$

Definition 1

An open-loop Nash equilibrium is a sequence $\{i_{j,t}^*\}_{t=0}^{\infty}$ such that, for all period t^* , i_{j,t^*}^* maximizes player j 's objective function subject to the constraints for given sequences of $\{i_{j,t,-t^*}^*\}_{t=0}^{\infty}$ (own action except for period t^*) and $\{i_{-j,t}^*\}_{t=0}^{\infty}$ (the other player's action).

- Each player j maximizes for given $\{i_{j,-t}^*\}_{t=0}^{\infty}$,

$$\max_{\tilde{x}_t, \{i_{j,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) \quad \text{s.t.} \quad E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0.$$

- Timeless perspective: the Lagrange multiplier λ_{-1} is chosen so that agents face the same FOCs for all $t \geq 0$.