Monetary and Exchange Rate Policies in a Global Economy*

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Abstract

A consensus in the small open economy literature is that optimal monetary policy and foreign exchange intervention (FXI) separately stabilize inflation and the exchange rate. I develop an analytically tractable two-country framework where FXI balances internal and external objectives. Under international policy cooperation, optimal FXI mitigates the trade-off between domestic inflation and demand faced by monetary authorities, allowing them to stabilize inflation with moderate changes in the interest rate. At the same time, optimal FXI targets world demand since it affects international prices. The model thus suggests an interaction between conventional monetary policy and unconventional exchange rate policy tools and provides a rationale for their combined use. I further show that FXI contributes more to domestic inflation stabilization when all goods traded in international markets are priced in dollars. Finally, a quantitative exercise shows that, in a non-cooperative equilibrium, FXI exacerbates distortions to world prices and demand.

Keywords: Capital flows, International risk-sharing, Foreign exchange intervention, Optimal targeting rules, International policy cooperation.

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1 Introduction

The classical "trilemma" (Mundell 1957, Fleming 1962) suggests that, under free capital mobility, monetary authorities cannot simultaneously pursue inflation/output and exchange rate stabilization. In a modern financially globalized world, countries increasingly rely on unconventional policy tools to manage the capital account and insulate themselves from international spillovers of shocks and crises, to preserve their monetary autonomy (Rey 2015, Kalemli-Özcan 2019, Miranda-Agrippino and Rey 2020). In particular, many countries resort to foreign exchange intervention (FXI), i.e. purchases or sales of foreign currency reserves by central banks, with over 100 countries regularly using FXI (Adler et al. 2023).[1]

There have been recent discussions on the optimal mix of monetary and exchange rate policies. The consensus in the literature is that monetary policy and FXI are two distinct policies and operate separately on their objectives: FXI exclusively stabilizes the exchange rate while monetary policy independently stabilizes inflation (Cavallino 2019, Basu et al. 2020, Itskhoki and Mukhin 2023). This rationale is based on a small-open economy case that takes the international prices of goods as given.

This paper studies two features of FXI that became salient during the COVID-19 pandemic and the Russian-Ukraine war. These features depart from the consensus in the literature. First, central banks use FXI to stabilize inflation. For example, the pandemic and the war caused worldwide inflationary pressure due to supply chain disruption. However, China and India set a record low of around 4% interest rate during the pandemic. Brazil also lowered its rate to 2% and Japan even adopted a negative interest rate policy. Instead, they responded to the crisis by selling the dollar reserves: China and Brazil sold 38 billion and 25 billion dollars in 2020, and India and Japan sold 32 billion and 63 billion dollars in 2022, respectively. This approach contrasts with the conventional method of combating inflation by raising the policy rate. This example suggests that the objectives of monetary policy and FXI are not necessarily independent from each other.[2]

Second, large open economies intervene in the FX market on massive scales. Most notably,

[1]In countries such as Japan, the finance ministry is in charge of FXI instead of the central bank. This paper considers a joint government consisting of the central bank and the finance ministry.

[2]The empirical analysis on the macroeconomic effects of FXI requires a rigorous identification of FXI. As discussed in Fratzscher et al. (2019) and Maggiori (2022), this is a challenging task in the literature and beyond the scope of this paper. As a step toward identification, Gonzalez et al. (2021) and Rodnyansky et al. (2024) combine FXI data with granular bank- or firm-level balance sheet information and study within-country variation of responses to FXI.

China has 3.5 trillion dollars and Japan has 1.4 trillion dollars of FX reserves, accounting for nearly 40% of the world's FX reserves (IMF International Financial Statistics). These countries are on the US monitoring list for accumulating excess FX reserves and gaining an unfair competitive advantage in international trade (US Department of the Treasury 2024). Moreover, large countries, such as China, India, and Japan, sold FX reserves to protect their currency during the pandemic and the war, as noted above. In such a large open-economy context, exchange rate manipulation may lead to global economic fluctuation by changing the international price and the world demand for domestic and foreign goods. This paper studies whether policies should respond to domestic inflation and output gap or global business cycle conditions and imbalances beyond domestic objectives.

This paper constructs a New Keynesian model that studies the optimality, interactions, and trade-offs of monetary and exchange rate policies. The model features two large countries: the US and the local; two policies: monetary policy and FXI; and two frictions: nominal and financial frictions. The frictions I introduce imply a non-neutral role for monetary policy and FXI: sticky prices allow monetary policy to influence the real interest rate (Calvo 1983) and limits to arbitrage in international capital markets allow FXI to influence the exchange rate by changing the demand and supply of bonds in different currencies (Gabaix and Maggiori 2015, Itskhoki and Mukhin 2021).[3]

My first contribution is to provide a full analytical characterization of optimal monetary policy and FXI rules in a large two-country framework. I begin with an international cooperation case, in which central banks maximize global welfare.[4] The key finding is that FXI trades off an internal objective (domestic inflation and demand) and an external objective (foreign inflation and demand).

On the one hand, FXI stabilizes inflation. When a central bank purchases local currency bonds using FXI, the demand for the local currency increases so the local currency appreciates. As a result, local goods become more expensive for US households, making them consume less local goods. This reduced US demand, in turn, leads to lower prices and lower inflation of local goods. Hence, FXI allows the local central banks to stabilize inflation even without tightening

[3] Models of limits to arbitrage are motivated by empirical literature on forward premium puzzle (Fama 1984). Data shows that cross-currency interest differentials are not offset by expected exchange rate depreciation, resulting in positive excess returns on currency carry trades. Without limits to arbitrage, households would obtain an infinite carry-trade profit by investing in currencies with higher returns.

[4] In the later section, I contrast the optimal policies under cooperative and non-cooperative equilibria and study gains from cooperation.

the monetary policy and depressing local consumption demand. This rationalizes why central banks used FXI rather than monetary policy to stabilize inflation during the pandemic and war.

On the other hand, FXI distorts world inflation and demand. To understand the concept, consider an increase in local output, which increases local consumption. At the same time, a higher supply depreciates the local exchange rate and enables US households to import local goods at lower prices. Hence, US consumption also increases. Exchange rates adjust automatically and smooth consumption across countries, even if households cannot trade state-contingent assets internationally.

However, FXI distorts this consumption smoothing by manipulating the exchange rate. When the central bank purchases the local currency, the local currency appreciates. This appreciation has asymmetric effects on local and US households: local households import US goods at lower prices and increase consumption, while US households import local goods at higher prices and decrease consumption. As a result, FXI disproportionately benefits the local households over the US households.

My result shows that FXI has two main objectives. First, FXI stabilizes non-fundamental volatility in capital flows and exchange rates, as pointed out in the literature of small-open economy (Cavallino 2019, Basu et al. 2020, Itskhoki and Mukhin 2023). Second, when local inflation is higher than US inflation, the optimal policy is to buy the local currency, which stabilizes domestic inflation. However, this comes at the cost of destabilizing world demand since FXI by large-open economies affects international prices. Hence, optimal FXI under cooperation balances internal objectives (domestic inflation and consumption demand) and external objectives (relative demand across countries). The key policy implication is that monetary policy and FXI are not completely independent and cannot be optimized in isolation but a policy that encompasses both instruments achieves the optimal outcome.

I conduct a quantitative exercise to validate these analytical findings. I find that, in response to an inflationary cost-push shock in the US, optimal FXI by the local central bank allows nearly full stabilization of its inflation and the output gap with little changes in the monetary policy rate, at the cost of widening the risk-sharing wedge. Moreover, FXI has larger effects on inflation, the output gap, and the exchange rate is larger when the elasticity of substitution between local and US goods is high.

Having highlighted the price stabilization channel of FXI, my second contribution is to establish a novel relationship between capital flow management in international finance and the

US dollar's dominance in international trade. These two strands of literature in international macroeconomics are often discussed in separate contexts. My paper aims to bridge the gap between them. Recent evidence suggests that the majority of world trade is invoiced in a small number of dominant currencies, particularly the US dollar (Gopinath et al. 2020). Motivated by this fact, I consider a case where all goods traded in international markets are priced in dollars (dollar pricing). Under dollar pricing, the following are true. First, an identical local good has different prices in two currencies. When the dollar appreciates, identical local goods are more expensive when denominated in the US dollars than in the local currency. This generates an inefficient cross-currency price dispersion despite the identical marginal costs of production. Second, since both exports and imports are in dollars, changes in the exchange rate affect import prices and consumption for local households but have a muted effect on consumption for US households.

I find that the optimal FXI responds to this price-dispersion wedge under dollar pricing. By manipulating the exchange rate, FXI affects the relative prices of local goods sold locally and in the US, closing the price dispersion. This makes the optimal FXI particularly responsive to shocks under dollar pricing. Moreover, the transmission channel of FXI is asymmetric across countries. Optimal FXI has a large stabilization effect on local inflation without creating a large US inflationary spillover. These results suggest that dollarization in international trade is a key driver of capital flow stabilization policy in international finance.

Finally, as a robustness check, I deviate from the assumption of international policy coordination. The need to fully specify dynamic games poses many challenges to the modern literature on strategic monetary policy coordination. These include the definitions of equilibria (commitment or discretion), the choice of policy instruments (inflation or output gap), and the feasibility of deriving analytical and numerical solutions. In particular, the model is difficult to solve when there are multiple policy instruments: monetary policy and FXI. As a first step toward characterizing FXI in a non-cooperative equilibrium using a quantitative framework, my paper considers a strategic interaction between FXI by the local central bank and the monetary policy of the US central bank (the Fed). I focus on a special case where each central bank initially commits to a state-contingent strategy to maximize its objective and solve the model using a numerical method.

I show that a lack of cooperation results in excess intervention, exacerbating the distortions of world demand. Intuitively, when the local central bank purchases the local currency and sells the dollar, the local currency appreciates, lowering the import price and stabilizing inflation. The local households increase consumption by importing more US goods.

However, unlike the cooperative equilibrium, the local central bank does not take into account the transmission effect of FXI on international price and demand. By appreciating the local currency, FXI increases the US import price and reduces consumption. An over-accumulation of local currency reserves without international coordination results in consumption misallocation in favor of the local households over the US households (beggar-thy-neighbor). Conversely, an excess accumulation of the dollar reserves (competitive devaluation) leads to a consumption misallocation in favor of the US households over the local households (beggar-thy-self). This result provides a rationale for international coordination in the monetary and exchange rate policy designs to achieve a globally optimal outcome.

Literature. First, this paper builds on models of exchange rate determination in an imperfect financial market (Gabaix and Maggiori 2015, Itskhoki and Mukhin 2021, Maggiori 2022, Fukui et al. 2023). Their models have been used to study the effect of FXI (Fanelli and Straub 2021, Davis et al. 2023, Ottonello et al. 2024).[5] Moreover, Cavallino (2019), Amador et al. (2020), Basu et al. (2020), and Itskhoki and Mukhin (2023) study on monetary policy and FXI in a small open economy case, where households take the international price and demand as given.[6] My contribution is to study a global implication of FXI in an analytically tractable two-country framework, in which FXI affects international price and demand, and to provide a full analytical characterization of optimal monetary policy and FXI targeting rules. My model suggests a

[5] The source of non-fundamental exchange rate volatility is modeled in the literature as an exogenous shock to the unhedged carry trade return (Devereux and Engel 2002, Jeanne and Rose 2002, Kollmann 2005) or convenience yield on the dollar bond (Jiang et al. 2021; 2023, Engel and Wu 2023, Kekre and Lenel 2023).

[6] Cavallino (2019) shows that FXI is costly for a central bank since FX purchase lowers the FX return while it is profitable for intermediaries as they take an opposite carry trade position against the central bank. When the domestic households do not own the entire share of the intermediaries, FXI trades off the carry cost with exchange rate stabilization. Amador et al. (2020) show that the zero lower bound of the nominal interest rate generates capital inflow since the expected appreciation of local currency is not offset by the lower interest rate. FXI absorbs the capital flows by accumulating foreign reserves but generates a resource cost. Basu et al. (2020) builds an "integrated policy framework" that jointly studies monetary policy, FXI, capital control, and macroprudential regulation. When banks face a sudden outflow of capital, a lower policy rate relaxes the domestic borrowing constraint but tightens the external borrowing constraint due to currency depreciation. FXI limits this depreciation and improves the monetary trade-off. Itskhoki and Mukhin (2023) show that unrestricted use of monetary policy and FXI stabilize both inflation and exchange rate separately. However, when FXI is constrained, monetary policy faces a trade-off between inflation and exchange rate stabilization.

novel trade-off of FXI between internal objectives (inflation and output) and external objectives (exchange rate and world demand).[7] Moreover, I take the first step to incorporate FXI in a non-cooperative equilibrium and study the strategic interaction between monetary policy and FXI in a large two-country model.

My paper is also based on the large literature on optimal monetary policy. An early strand of literature studies optimal monetary policy in a small open economy (Clarida et al. 2001, Schmitt-Grohé and Uribe 2001, Kollmann 2002, Galí and Monacelli 2005, Faia and Monacelli 2008). Another strand of papers study international monetary policy transmission and cooperation in a large two-country economy (Obstfeld and Rogoff 2000, Corsetti and Pesenti 2001, Clarida et al. 2002, Benigno and Benigno 2003, Corsetti and Pesenti 2005, Benigno and Benigno 2006, Devereux and Engel 2003, Corsetti et al. 2010; 2020; 2023, Engel 2011). These papers study monetary policy independently of FXI. I contribute to the literature by providing a joint configuration of monetary policy and FXI.

A growing empirical evidence documents that FXI is effective in stabilizing the exchange rate (Dominguez and Frankel 1993, Dominguez 2003, Fatum and Hutchison 2010, Blanchard et al. 2015, Kuersteiner et al. 2018, Adler et al. 2019, Fratzscher et al. 2019, Hofmann et al. 2019, Fratzscher et al. 2023, Rodnyansky et al. 2024). My contribution is to provide a normative analysis based on a full analytical characterization of an optimal FXI targeting rule.

This paper is related to recent literature on the dominance of the US dollar in trade invoicing (Gopinath 2016, Gopinath et al. 2020, Gopinath and Stein 2021, Mukhin 2022, Egorov and Mukhin 2023). My paper bridges the gap between the literature on international trade and finance and suggests a novel mechanism where capital flow management policies are motivated by dollar pricing.

Finally, there are discussions on gains from international monetary policy coordination in a large two-country model (Obstfeld and Rogoff 2002, Benigno and Benigno 2006, Benigno and Woodford 2012, Corsetti et al. 2010, Bodenstein et al. 2023). Fanelli and Straub (2021) and Itskhoki and Mukhin (2023) study international coordination on FXI between small open economies. I take the first step to incorporate monetary policy and FXI in a large two-country framework.

[7]Other complementary transmission channels of FXI can create additional trade-offs. Basu et al. (2020), Davis et al. (2023), Rodnyansky et al. (2024) show that FXI mitigates balance-sheet risk when firms or banks have foreign currency debt. Ottonello et al. (2024) show that FXI is used as an industrial policy and helps the convergence to the technological frontier.

The rest of this paper is organized as follows. Section 2 describes the model environment and setup. Section 3 characterizes optimal monetary policy and FXI under cooperation. Section 4 studies the optimal policies under dollar pricing. Section 5 incorporates FXI in a non-cooperative equilibrium. Section 6 concludes.

2 Model Environment: Segmented Currency Markets

The model builds on a standard international real business cycle model (Clarida et al. 2002). The key departure from the literature is that households are restricted from trading bonds denominated in foreign currency. The households' net foreign asset position must be intermediated by global financial intermediaries with limited capacity to bear exchange rate risks. This creates limits to arbitrage opportunities for households, allowing FXI to affect the relative demand and supply of bonds in different currencies (Gabaix and Maggiori 2015, Itskhoki and Mukhin 2021).

There are two symmetric economies, the US and local (the rest of the world). Each country is populated with a continuum of agents of unit mass. Firms in each country are monopolistic suppliers of one type of tradable good and use labor as the only input in production. Households are allowed to trade state-contingent assets domestically but not internationally. In what follows, I describe the setup focusing on the local economy, since similar expressions apply to the US economy. I denote variables related to the US households and firms with an asterisk ($*$) and refer to the US unit of account as the dollar.

Households. In each country, there is a continuum of households that maximize the expected discount value of their lifetime utility. I assume the households have a constant relative risk aversion (CRRA) utility in consumption. The maximization of local households is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \zeta_l \frac{L_t^{1+\eta}}{1+\eta} \right],$$

where C_t and L_t are the consumption and labor supply, σ is the inverse intertemporal elasticity of substitution, β is the discount factor, and η is the inverse Frisch elasticity of labor. The households' consumption basket C_t is a constant elasticity of substitution (CES) aggregator of local and US goods:

$$C_t = \left[a C_{Lt}^{\frac{\phi-1}{\phi}} + (1-a) C_{Ut}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{1-\phi}},$$

where C_{Lt} and C_{Ut} are the consumption of local and US goods, a is the weight of the local good, and ϕ is the elasticity of substitution between the local and US goods. In the limiting case where $\sigma = \phi = 1$ (Cole and Obstfeld 1991), households have log and Cobb-Douglas utility. I assume $a \in (1/2, 1]$ so that households exhibit home bias of consumption. C_{Lt} and C_{Ut} are the bundles of differentiated goods:

$$C_{Lt} = \left[\int_0^1 C_t(l)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{1-\theta}}, \quad C_{Ut} = \left[\int_0^1 C_t(u)^{\frac{\theta-1}{\theta}} du \right]^{\frac{\theta}{1-\theta}},$$

where $C_t(l)$ and $C_t(u)$ are the local households' consumption of the local good l and imported good u, respectively, and θ is the elasticity of substitution between differentiated goods.

The local households' budget constraint is:

$$P_{Lt} C_{Lt} + P_{Ut} C_{Ut} + \frac{B_t}{R_t} = B_{t-1} + W_t L_t + \Pi_t + T_t, \tag{1}$$

where P_{Lt} and P_{Ut} are the prices of local and US goods faced by local households, B_t is the local households' investment in one-period state non-contingent bonds denominated in local currency, R_t is the interest rate on the local currency bond ($1/R_t$ is the bond price), W_t is the wage, Π_t is the lump-sum transfer of firms' profit, and T_t is the government transfer.

The price index of the local good is given by:

$$P_{Lt} = \left[\int_0^1 P_t(l)^{1-\theta} dl \right]^{\frac{1}{1-\theta}},$$

and the consumer price index (CPI) associated with the consumption basket C_t is given by:

$$P_t = \left[a P_{Lt}^{1-\phi} + (1-a) P_{Ut}^{1-\phi} \right]^{\frac{1}{1-\phi}}. \tag{2}$$

Let $\mathcal{E}_t$ denote the nominal exchange rate in terms of the local unit of account per dollar (an increase in $\mathcal{E}_t$ implies a depreciation of the local currency against the dollar). The real exchange rate is defined as the ratio of CPIs expressed in the same currency, i.e., $e_t \equiv \mathcal{E}_t P_t^* / P_t$. The numerator is the CPI faced by US households (P_t^*) expressed in terms of the local currency (multiplied by $\mathcal{E}_t$), while the denominator is the CPI faced by local households (P_t).

The households' intratemporal consumption allocation problem gives the following demand for local and US goods:

$$C_{Lt} = a \left(\frac{P_{Lt}}{P_t} \right)^{-\phi} C_t, \quad C_{Ut} = (1-a) \left(\frac{P_{Ut}}{P_t} \right)^{-\phi} C_t,$$

and the demand for differentiated goods produced within each country:

$$C_t(l) = \left(\frac{P_t(l)}{P_{Lt}} \right)^{-\theta} C_{Lt}, \quad C_t(u) = \left(\frac{P_t(u)}{P_{Ut}} \right)^{-\theta} C_{Ut}.$$

The households' Euler equation and labor supply equation are:

$$\beta R_t E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} = 1,$$
$$C_t^\sigma L_t^\eta = \frac{W_t}{P_t}.$$

Firms. Firms use domestic labor to produce a differentiated good l following a production function:

$$Y_t(l) = A_t L_t(l),$$

where $Y_t(l)$ is the output and $L_t(l)$ is the labor input for the producer of good l. A_t is a technology shock common to all firms and follows an AR(1) process: $\log(A_t) = \rho_a \log(A_{t-1}) + \sigma_a \epsilon_{at}$, where ρ_a and σ_a are the persistence and the standard deviation, respectively. Let $Y_{Lt} = \left[\int_0^1 Y_t(l)^{\frac{\theta-1}{\theta}} dl \right]^{\frac{\theta}{\theta-1}}$ be the final output of the local good. The demand for the differentiated good l is given by:

$$Y_t(l) = \left(\frac{P_t(l)}{P_{Lt}} \right)^{-\theta} Y_{Lt}.$$

Firms are subject to nominal rigidity (Calvo 1983) so that, in each period, firms update their prices with probability ξ_p. To capture the key intuition, I assume producer currency pricing (PCP) so that the export price is sticky in the exporters' currency (Section 4 derives the optimal policy under dollar pricing). The firms' maximization problem is:

$$\max_{\{P_t(l),\,\mathcal{E}_t P_t^*(l)\}} E_t \left\{ \sum_{k=0}^{\infty} Q_{t,t+k} \xi_p^k \begin{pmatrix} (1+\tau_t)\left[P_t(l)Y_{t+k}(l) + \mathcal{E}_t P_t^*(l) Y_{t+k}^*(l)\right] \\ -MC_{t+k}(l)\left[Y_{t+k}(l) + Y_{t+k}^*(l)\right] \end{pmatrix} \right\},$$

where $Q_{t,t+k} = \beta^t \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the households' stochastic discount factor, $Y_t(l)$ and $Y_t^*(l)$ are demand for local goods by local and US households, $P_t(l)$ is the price of local goods in local currency charged to local households, $P_t^*(l)$ is the price of local goods in dollars charged to local and US households, $MC_t = W_t/A_t$ is the marginal cost of production, and τ_t is the sales subsidy.

Solving the maximization problem, the optimal price of local goods charged to local households is:

$$P_t(l) = \frac{\theta}{(\theta-1)(1+\tau_t)} \frac{E_t \sum_{k=0}^{\infty} Q_{t,t+k} \xi_p^k Y_{t+k}(l) MC_{t+k}}{E_t \sum_{k=0}^{\infty} Q_{t,t+k} \xi_p^k Y_{t+k}(l)}.$$

The law of one price (LOOP) implies that local and US households are charged an identical price for local goods when expressed in the same currency, i.e., $P_t^*(l) = P_t(l)/\mathcal{E}_t$.

Assuming symmetry so that all firms that can update the prices choose the same value, the law of motion of local good prices faced by local households can be written as:

$$P_{Lt}^{1-\theta} = \xi_p P_{Lt-1}^{1-\theta} + (1-\xi_p) P_t(l)^{1-\theta}.$$

International Financial Markets. There is a well-known empirical fact that unhedged rates of returns across currencies are not equalized. In other words, the uncovered interest rate parity (UIP) condition does not hold (Fama 1984). Motivated by this fact, I assume that households are restricted from trading foreign currency assets: otherwise, households would gain infinite carry-trade profits by investing in high-return currencies (Jeanne and Rose 2002, Gabaix and Maggiori 2015, Itskhoki and Mukhin 2021). Under limits to arbitrage, FXI can affect the exchange rate by changing the relative demand and supply of bonds in two currencies. Intuitively, when the central bank buys the local currency and sells the dollar bond, households do not immediately take the opposite position to invest in the dollar bond.

Figure 1 shows the basic structure of the international financial market. Households can only trade bonds in their own currency and their net foreign asset position must be intermediated by financiers (global financial intermediaries) who are averse to exchange rate risk.[8] The local central bank has conventional and unconventional policy instruments: it uses monetary policy to set the nominal interest rates and FXI to trade bonds in two currencies.[9] Finally, I introduce UIP shocks, which affect the relative rates of return on bonds across two currencies. The UIP shock can be understood as an exogenous demand for dollar bonds due to their liquidity and safety of dollar bonds. This is a common way in the literature to generate high exchange rate volatility and lack of correlation between exchange rates and macroeconomic fundamentals. [10]

To provide a simple example, consider an increase in demand U_t^* for dollar bonds. Financiers provide these bonds to investors by taking a short position in dollar bonds and a long position in local currency bonds. However, since intermediaries have limited capacity to bear exchange rate risks, the local bond return must be higher than the dollar bond (UIP deviation) to compensate for exchange risks. Despite higher returns on the local currency bonds, households are restricted from taking carry trade positions so they cannot sell dollar bonds to buy local currency bonds. However, if the local central bank buys local bonds and perfectly offsets the investors' demand for dollar bonds, i.e., when $F_t = U_t^*$, the rates of return on bonds are equalized across currencies (UIP condition holds).

I will formalize this intuition in the model. There is a measure m_u of liquidity traders who generate an exogenous UIP shock. The investors hold a zero-net portfolio (U_t, U_t^*) so the investment U_t^* in dollar bonds is matched by the investment $U_t/R_t = -\mathcal{E}_t U_t^*/R_t^*$ in the local currency bonds. The positive U_t^* implies that the liquidity traders take a long position in the dollar and a short position in the local currency, and vice versa. I assume that the liquidity traders' position follows an AR(1) process: $U_t = \rho_u U_{t-1} + \sigma_u \epsilon_{ut}$.

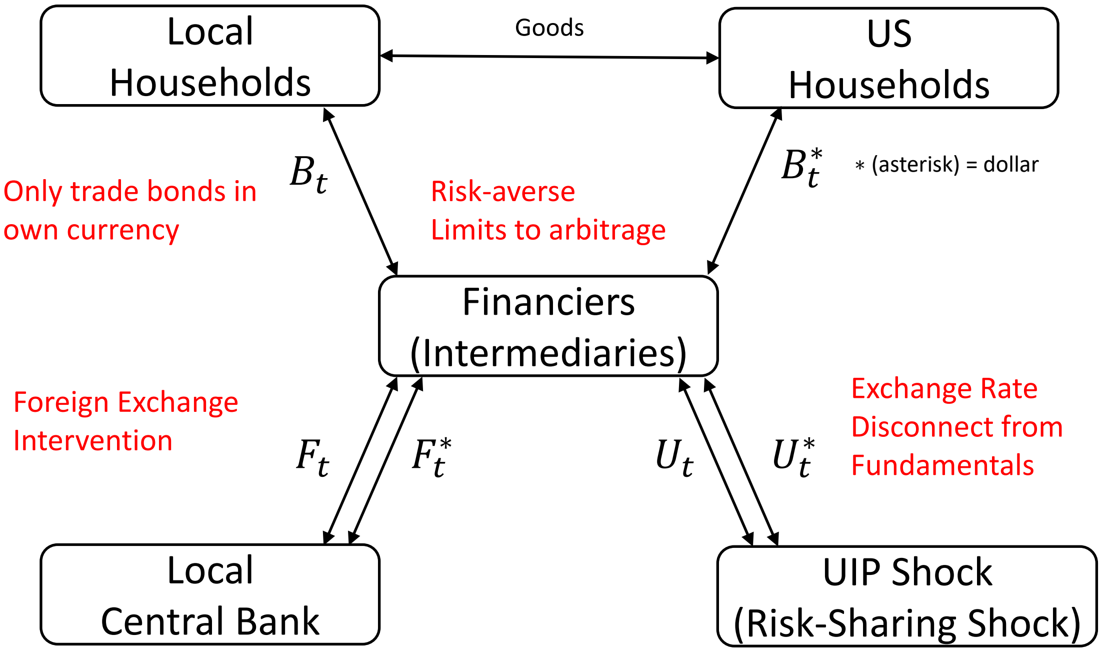
The local central bank uses sterilized intervention and trades bonds in the two currencies. The local central bank holds a zero-net portfolio (F_t, F_t^*) given by $F_t/R_t = -\mathcal{E}_t F_t^*/R_t^*$ and its

[8]For tractability, I assume that households cannot access foreign currency bonds, following Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021). Fukui et al. (2023) generalize this setup so that households and firms can borrow and invest in foreign currency but it is costly to access foreign currency bonds.

[9]I assume that the US central bank (the Fed) does not use FXI since data shows that interventions by the US are infrequent.

[10]Since the focus of this paper is the economic consequence of UIP shocks and the role of monetary and exchange rate policies, the model is agnostic about the source of UIP shocks to keep tractability. There is an extensive discussion on the drivers of UIP shocks, including investors' heterogeneous beliefs (Bacchetta and Van Wincoop 2006) and cognitive bias (Burnside et al. 2011), rare disaster risk (Farhi and Gabaix 2016), interbank friction (Bianchi et al. 2023a), and special role of US Treasury bonds, such as liquidity or collateral values.(Bianchi et al. 2023a;b). How different sources of UIP shocks affect the optimal policy design is beyond the scope of this paper and is left for future research.

Figure 1: Basic Structure of the International Financial Market



Note: The figure shows the basic structure of the international financial market. Local and US households can only trade bonds in their own currency (B_t, B_t^*). The local central bank uses foreign exchange intervention to trade bonds in two currencies (F_t, F_t^*). Liquidity (noise) traders generate an exogenous UIP shock (U_t, U_t^*). Financiers (global financial intermediaries) intermediate the net foreign asset positions of the households, the local central bank, and the liquidity traders.

profits and losses are transferred to the local households in a lump-sum way.[11]

There is a measure m_d of financiers who intermediate the portfolio positions of the households, the liquidity traders, and the local central bank. The financiers hold a zero-net portfolio (D_t, D_t^*) given by $D_t/R_t = -\mathcal{E}_t D_t^*/R_t^*$. Following Itskhoki and Mukhin (2021) and Fukui et al. (2023), I assume that the financiers maximize the following constant absolute risk aversion (CARA) utility:

$$\max_{D_t} E_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \bar{R}_t^* \frac{D_t}{P_t} \right) \right\}, \tag{3}$$

where $\omega \geq 0$ is a risk-aversion parameter and

$$\bar{R}_t = R_t - R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$$

[11] I assume that only the local central bank conducts FXI since data shows that interventions by the Federal Reserve Board are infrequent. Moreover, I assume that FXI is unconstrained for simplicity. In reality, central banks face a zero lower bound on FX reserves, which creates an additional policy trade-off. Davis et al. (2023) show that, when reserves cannot be borrowed, the optimal policy is to accumulate the FX reserves during normal times and sell them during crisis times.

is the unhedged return on the carry trade.[12] In a limiting case where $\omega = 0$, arbitrageurs are risk-neutral and take a carry trade position without charging a risk premium. Hence, the UIP holds and the expected excess return is zero: $E_t\bar{R}_t = 0$. However, when $\omega > 0$, arbitrageurs are risk-averse and require a risk premium for taking the risky carry trade position, which drives the UIP deviation: $E_t\bar{R}_t \neq 0$.

The market clearing conditions for the bond market imply the net demand for local currency and dollar bonds is zero:

$$B_t + U_t + D_t + F_t = 0, \quad \text{and} \quad B_t^* + U_t^* + D_t^* + F_t^* = 0. \tag{4}$$

The competitive equilibrium is defined as the set of prices, quantities, and policy variables that solve the maximization problems of households, firms, and arbitrageurs under the constraints and market clearing conditions.

Role of FXI in a Segmented Financial Market. Having described the key ingredients of the model, I will discuss why FXI can improve allocations in a segmented financial market. To this end, I will introduce the concept of "international risk sharing." Risk sharing implies that the exchange rate adjusts automatically and smooths consumption across countries. Consider an increase in local output so that local consumption increases. At the same time, the local exchange rate depreciates and US households can import local goods at lower prices. Hence, US consumption also increases.

Shocks to capital flows distort this risk sharing. When liquidity traders increase their demand for dollar bonds, the dollar appreciates and US households can import local goods at lower prices. Furthermore, since the rate of return on dollar bonds decreases relative to the return on local bonds, US households have more incentive to consume and less incentive to save. As a result, US consumption increases relative to local consumption. The local central bank can mitigate this risk-sharing distortion using FXI. If the local central bank purchases local currency bonds using FXI and perfectly offsets the traders' demand for dollars, the local

[12]The assumption of a CARA utility improves the traceability since their portfolio decision does not depend on the wealth, allowing us to avoid an additional state variable. The potential ways to microfound the banks' risk-aversion are to introduce occasionally binding borrowing constraints, costs of currency hedging, or liquidity holdings by banks (Bianchi et al. 2023a). Moreover, I assume that the financiers' profit is transferred to the local households as a lump-sum payment. As discussed in Appendix A.2, the profits and losses generated by carry trade positions do not affect the first-order dynamics of the model.

currency appreciates and the rates of return on bonds in two currencies are equalized. Hence, FXI smooths consumption across countries and improves risk-sharing.

I define the risk-sharing wedge $\mathcal{W}_t$ as the ratio of the marginal utility of consumption across the two countries:

$$\mathcal{W}_t \equiv \frac{(C_t^*)^{-\sigma} / \mathcal{E}_t P_t^*}{C_t^{-\sigma} / P_t} = \left(\frac{C_t}{C_t^*}\right)^{\sigma} \frac{1}{e_t}. \tag{5}$$

When $\mathcal{W}_t = 1$, consumption risk is efficiently shared (consumption is smoothed) across the two countries. When $\mathcal{W}_t > 1$, the marginal utility of the local households is lower than that of the US households (the local households have an excess demand or stronger purchasing power) and vice versa when $\mathcal{W}_t < 1$.

Whether the risk-sharing condition is satisfied depends on the asset market structure. If the asset market is complete so that households can trade state-contingent bonds across countries, $\mathcal{W}_t = 1$ always holds. As discussed in Backus and Kehoe (1989), FXI has no effect on the exchange rate under complete asset markets. However, this paper focuses on the incomplete asset market where households can only trade state-non-contingent bonds in the domestic currency, in which case FXI can affect the international risk-sharing and the exchange rate. As discussed later, $\mathcal{W}_t = 1$ does not hold in general under incomplete asset markets except for a special case where the elasticity of substitution between local and US goods equals one ($\sigma\phi = 1$, Cole and Obstfeld (1991)).

Solving the households' and financiers' maximization problems gives the equilibrium relationship between the risk-sharing wedge, UIP deviation, and the demand for bonds in two currencies.

Lemma 1. *The equilibrium condition in the financial market, which is log-linearized under a symmetric steady state, can be written as:*

$$E_t \tilde{\mathcal{W}}_{t+1} - \tilde{\mathcal{W}}_t = \tilde{r}_t - \tilde{r}_t^* - E_t \Delta \tilde{e}_{t+1} = \chi_1 (u_t^* - f_t) - \chi_2 b_t, \tag{6}$$

where $r_t \equiv R_t - E_t \pi_{t+1}$, $r_t^* \equiv R_t^* - E_t \pi_{t+1}^*$, $f_t \equiv F_t / \bar{Y}$, $u_t \equiv U_t / \bar{Y}$, $b_t \equiv B_t / \bar{Y}$, $\chi_1 \equiv m_u (\omega \sigma_{et}^2 / m_d)$ *and* $\chi_2 \equiv \bar{Y} (\omega \sigma_{et}^2 / m_d)$ *for finite* $\omega \sigma_{et}^2 / m_d$*, where* $\bar{Y} \equiv \bar{Y}_L = \bar{Y}_U$ *is GDP under the symmetric steady state and* $\sigma_{et}^2 \equiv var(\Delta \log \mathcal{E}_{t+1})$ *is the standard deviation of the change in log exchange rate* $(\Delta \log \mathcal{E}_{t+1} \equiv \log \mathcal{E}_{t+1} - \log \mathcal{E}_t)$.

Proof. See Appendix A.2.

Intuitively, suppose that the liquidity traders increase their demand for the dollar bond (positive u_t^*). To provide the dollar bonds to liquidity traders, financiers take a short position in the dollar and a long position in the local currency. In a limiting case where $\omega\sigma_{et}^2/m_d \to 0$, financiers' risk-bearing capacity is sufficiently high so that UIP holds in equilibrium. However, when $\omega\sigma_{et}^2/m_d > 0$, financiers have limited risk-bearing capacity and require a risk premium as compensation for exchange rate risk in carry trade.[13] This results in the positive UIP deviation ($\widetilde{UIP}_t \equiv \tilde{r}_t - \tilde{r}_t^* - E_t\Delta\tilde{e}_{t+1} > 0$) so that the rate of return on the local currency bond is higher than that of the dollar bond. Since households are restricted from trading assets internationally, they cannot take an opposite carry trade position against the liquidity traders. This implies that the local households face a higher rate of return on savings, so they have more incentive to invest in bonds and postpone their consumption than the US households. As a result, the home households' demand is expected to increase in the future ($E_t\tilde{\mathcal{W}}_{t+1} - \tilde{\mathcal{W}}_t > 0$). Similarly, the households' net foreign debt position ($b_t < 0$) is associated with the positive UIP deviation. To focus on the role of financial sectors in driving the UIP deviation, I consider the limiting case where $\chi_2 = 0$, so that:[14]

$$E_t\tilde{\mathcal{W}}_{t+1} - \tilde{\mathcal{W}}_t = \tilde{r}_t - \tilde{r}_t^* - E_t\Delta\tilde{e}_{t+1} = \chi_1(u_t^* - f_t). \tag{7}$$

The local central bank can use FXI to eliminate the distortion due to the segmented currency market. If the central bank takes an offsetting position against the liquidity traders and demands the local currency bond ($f_t = u_t^*$), the right-hand side of Equation (6) becomes zero so that the UIP deviation is closed. In other words, FXI effectively shifts the exchange rate risk away from risk-averse financiers to central banks' balance sheets. Since households in the two countries face equal rates of return on savings, the risk-sharing condition holds in expectation ($E_t\tilde{\mathcal{W}}_{t+1} - \tilde{\mathcal{W}}_t = 0$). When $f_t = u_t^*$, the resulting allocation is identical to that when the asset market is incomplete but the currency market is not segmented (Corsetti et al. 2010; 2023).

[13]The risk aversion parameter ω is scaled so that the risk premium $\omega\sigma_{et}^2/m_d$ is finite and nonzero and the variance of the exchange rate σ_{et}^2 affects the first-order dynamics of the model. See discussion by Hansen and Sargent (2011) and Itskhoki and Mukhin (2021).

[14]This assumption can be interpreted so that the size of the financial sector, including liquidity traders (m_u) and financiers (m_d), are sufficiently large relative to the real sector. See Itskhoki and Mukhin (2021).

Even if FXI perfectly offsets the risk-sharing shock, this does not necessarily imply the risk-sharing condition holds for every possible state of the economy but only in expectation. Whether the risk-sharing condition holds in every state or not depends on the elasticity of substitution between the local and US goods. To capture the intuition, it is convenient to focus on (a) the log utility ($\sigma = 1$) and (b) the financial autarky case where no international asset trade is allowed (an extreme form of incomplete asset market). However, we obtain similar results when $\sigma \neq 1$ and households can trade non-contingent bonds internationally.

As discussed in Corsetti et al. (2008), it is possible to write down the relationship between the real exchange rate and the relative consumption as follows:

$$\hat{e}_t = \frac{2a - 1}{2a\phi - 1}(\hat{C}_t - \hat{C}_t^*), \tag{8}$$

and the allocations under the complete market and financial autarky are equalized when $\phi = 1$. The risk-sharing wedge (in log-linearized form with $\sigma = 1$) can be written as $\tilde{\mathcal{W}}_t = \tilde{C}_t - \tilde{C}_t^* - \tilde{e}_t$. Intuitively, when the relative local productivity $\hat{A}_t - \hat{A}_t^*$ increases, the relative consumption $\hat{C}_t - \hat{C}_t^*$ increases. At the same time, a higher supply depreciates the local exchange rate ($\hat{e}_t$ increases) and allows US households to import local goods at lower prices. Hence, the exchange rate adjusts automatically to ensure consumption-smoothing across countries. When the local and US goods are substitutes ($\phi > 1$), the real exchange rate moves less than one-to-one with consumption, so local households have excess demand or higher purchasing power ($\tilde{\mathcal{W}}_t > 0$). When the goods are complements ($\phi < 1$), the real exchange rate moves more than one-to-one, so US households have excess demand ($\tilde{\mathcal{W}}_t < 0$). Only in a special case where households have a Cobb-Douglas preference ($\phi = 1$), consumption is smoothed across countries ($\tilde{\mathcal{W}}_t = 0$).

I define the terms-of-trade as the relative price of imports over exports: $\mathcal{T}_t = P_{Ut}/\mathcal{E}_t P_{Lt}^*$. The numerator is the import price of US goods faced by local households and the denominator is the export price of local goods faced by US households, expressed in terms of the local currency. The real exchange rate can be written in terms of the terms-of-trade:[15]

$$\tilde{e}_t = (2a - 1)\tilde{\mathcal{T}}_t. \tag{9}$$

Under the home bias in consumption ($a > 1/2$), an increase in the US goods prices (higher $\mathcal{T}_t$) leads to an increase in the US consumer prices, which increases the value of the US dollar in

[15] See Appendix A for derivation.

real terms (higher e_t). Hence, an appreciation of the local currency corresponds to an increase in import prices faced by local households.

Finally, a lack of risk-sharing is an important driver of inflation. As discussed in the next section, this has an important implication when studying the inflation-output trade-off of monetary policy. I define $\pi_{Lt} = P_{Lt}/P_{Lt-1} - 1$ as the net inflation rate of local good prices faced by local households and $\pi^*_{Ut} = P^*_{Ut}/P^*_{Ut-1} - 1$ as the net inflation rate of US good prices faced by US households (GDP deflator inflation). Using log-linearization, the New Keynesian Phillips Curves (NKPCs) for local and US firms can be written as:

$$\pi_{Lt} = \beta E_t \pi_{Lt+1} + \kappa\{(\sigma + \eta)\tilde{Y}_{Lt} - (1 - a)[2a(\sigma\phi - 1)\tilde{\mathcal{T}}_t - \tilde{\mathcal{W}}_t] + \mu_t\}, \tag{10}$$

$$\pi^*_{Ut} = \beta E_t \pi^*_{Ut+1} + \kappa\{(\sigma + \eta)\tilde{Y}_{Ut} + (1 - a)[2a(\sigma\phi - 1)\tilde{\mathcal{T}}_t - \tilde{\mathcal{W}}_t] + \mu^*_t\}, \tag{11}$$

where π_{Lt} is the inflation in prices of local goods consumed by local households, π^*_{Ut} is the inflation in prices of US goods consumed by US households, $\kappa = (1 + \tau_t)(\theta - 1)/\nu$ is the slope of the NKPCs, and $\mu_t = \theta/((\theta - 1)(1 + \tau_t))$ is the markup shock. As in the standard New Keynesian model, inflation depends on the expected inflation $E_t\pi_{Lt+1}$ and the output gap $\tilde{Y}_{Lt}$, defined as the deviation of output from its efficient level. In an open economy ($a < 1$), inflation also depends on two additional factors: the terms-of-trade gap $\tilde{\mathcal{T}}_t$, defined as the log deviation of the terms-of-trade from its efficient level, and the risk-sharing wedge $\tilde{\mathcal{W}}_t$.

As discussed in Clarida et al. (2002), the effect of the terms-of-trade gap on inflation depends on whether the local and US goods are substitutes ($\sigma\phi > 1$) or complements ($\sigma\phi < 1$). Consider an increase in the US output, which depreciates the US dollar and decreases the import price of US goods faced by local households. On the one hand, lower import prices increase consumption demand by local households. This increases the marginal cost of production and inflation. On the other hand, higher export prices for local firms increase the marginal benefit of production for local firms and reduce inflation. When local and US goods are substitutes ($\sigma\phi > 1$), the former effect dominates the latter, so π_{Lt} is increasing in $\tilde{\mathcal{T}}_t$ (a local excess demand increases local inflation). When goods are complements, ($\sigma\phi < 1$) the latter effect dominates the former, so π_{Lt} is decreasing in $\tilde{\mathcal{T}}_t$. In a special case where households have a log and Cobb-Douglas preference ($\sigma = \phi = 1$, Cole and Obstfeld 1991), the two effects offset so that risk-sharing wedge has no effects on inflation. In addition, local inflation is increasing in the risk-sharing gap. When $\tilde{\mathcal{W}}_t$ is high, local households have excess demand over US households.

This increases the marginal cost of production by local firms and increases GDP inflation of local goods.

3 Optimal Trade-offs of Monetary Policy and FXI

This section provides a full characterization of optimal policy rules under cooperation and commitment. To build up the intuition step-by-step, in Section 3.1, I start with an analytical characterization of optimal monetary policy rules when FXI is not available (Corsetti et al. 2023). Next, in Section 3.2, I provide a full analytical characterization of optimal monetary policy and FXI rules. Finally, in Section 3.3, I provide a numerical exercise to validate my analytical findings.

I focus on two sources of shocks: productivity and markup, and compare their transmission channels when FXI is available and not available. The case with productivity shocks helps a simple and intuitive understanding of the key transmission mechanism of FXI. However, quantitatively, the volume of optimal FXI and its effect on the exchange rate is larger when there are cost-push shocks.

3.1 Baseline: Optimal Monetary Policy without FXI

I begin with the case where central banks in the two countries can only set the nominal interest rate using the monetary policy instrument but they cannot use FXI to affect the foreign exchange rate reserves. I focus on the case under cooperation and commitment, in which central banks maximize the sum of expected discounted utility in the two countries. This is equivalent to minimizing the quadratic loss function approximated around the efficient equilibrium under a complete asset market and flexible price:[16]

$$\mathcal{L} = -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\begin{array}{l} (\sigma+\eta)\left(\tilde{Y}_{Lt}^2 + \tilde{Y}_{Ut}^2\right) + \frac{\theta}{\kappa}\left(\pi_{Lt}^2 + \pi_{Ut}^{*2}\right) \\ -\frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}\left(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}\right)^2 + \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\tilde{\mathcal{W}}_t^2 \end{array} \right]. \tag{12}$$

Importantly, under cooperation, the loss function not only depends on the internal objective (inflation and output gap in each country) but also the external objective (relative output gap

[16] See Corsetti et al. (2023) for the detailed derivation.

and risk-sharing wedge across countries). The relative output gap can be written in terms of the terms-of-trade gap and the risk-sharing wedge (see Appendix A.1):

$$\tilde{\mathcal{T}}_t = \frac{\tilde{Y}_{Lt} - \tilde{Y}_{Ut} - (2a_H - 1)\tilde{\mathcal{D}}_t}{4a_H(1-a)(\sigma\phi - 1) + 1}. \tag{13}$$

Intuitively, the local export price is higher (lower $\tilde{\mathcal{T}}_t$) when the supply of local goods is low (lower $\tilde{Y}_{Lt}$) and local households have an excess demand (higher $\tilde{\mathcal{W}}_t$).

The following lemma characterizes optimal monetary policy rules under an incomplete asset market.

Lemma 2 (Optimal Monetary Policy Rules without FXI)**.** *Under PCP, cooperation, and commitment, and when FXI is not available, optimal monetary policy rules for the local and US central banks are characterized by:*

$$0 = \theta\pi_{Lt} + \left(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}\right) + \psi_D(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1}), \tag{14}$$

$$0 = \theta\pi^*_{Ut} + \left(\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}\right) - \psi_D(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1}), \tag{15}$$

where:

$$\psi_D = \frac{4a(1-a)\phi}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}} \frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}, \tag{16}$$

which hold without imposing restrictions on σ and ϕ.

Proof. See Appendix A.3.

Intuitively, under an incomplete asset market, an increase in the local excess demand $\tilde{\mathcal{W}}_t$ increases the marginal cost of production for local firms and increases inflation, as suggested by Equation (10). Hence, monetary policy cannot close all gaps in the economy but faces a trade-off between inflation and growth rates in the output gap and risk-sharing wedge.

Next, I characterize the international transmission of shocks. First, the following lemma characterizes the transmission of productivity shocks.

Lemma 3 (Transmission of a Productivity Shock)**.** *Assume PCP, cooperation, and commitment, and suppose that FXI is not available and monetary policy follows the optimal rule in Lemma 2. Consider an increase in the period-0 local productivity A_0.*

- *When* $\sigma\phi > 1$, $\frac{\partial \pi_{L0}}{\partial A_0} > 0 > \frac{\partial \pi^*_{U0}}{\partial A_0}$, $\frac{\partial \tilde{Y}_{L0}}{\partial A_0} < 0 < \frac{\partial \tilde{Y}_{U0}}{\partial A_0}$, *and* $\frac{\partial \tilde{\mathcal{W}}_0}{\partial A_0} > 0$.
- *When* $\sigma\phi < 1$, $\frac{\partial \pi_{L0}}{\partial A_0} < 0 < \frac{\partial \pi^*_{U0}}{\partial A_0}$, $\frac{\partial \tilde{Y}_{L0}}{\partial A_0} > 0 > \frac{\partial \tilde{Y}_{U0}}{\partial A_0}$, *and* $\frac{\partial \tilde{\mathcal{W}}_0}{\partial A_0} < 0$.
- *When* $\sigma\phi = 1$, $\frac{\partial \pi_{L0}}{\partial A_0} = \frac{\partial \pi^*_{U0}}{\partial A_0} = \frac{\partial \tilde{Y}_{L0}}{\partial A_0} = \frac{\partial \tilde{Y}_{U0}}{\partial A_0} = \frac{\partial \tilde{\mathcal{W}}_0}{\partial A_0} = 0$.

Consider the case where local and US goods are substitutes ($\sigma\phi > 1$). As in the standard New Keynesian model under complete asset markets, an increase in local supply creates pressure for deflation and local currency depreciation. The optimal monetary policy is to lower the interest rate (expansionary) to stabilize inflation.

At the same time, since the asset market is incomplete, an increase in local productivity generates an excess consumption demand by local households (higher $\tilde{\mathcal{W}}_t$). This has two main consequences. First, local excess demand increases the local marginal cost of production and inflation, as implied by Equation (10). This high inflation makes the optimal monetary policy less expansionary than the complete market case, allowing for a negative local output gap. Second, since local excess demand mutes the local currency depreciation, US households face higher import prices and reduce demand for local goods. This further widens the negative local output gap, making the optimal monetary policy more expansionary than the complete market case. Hence, the local economy faces positive inflation and a negative output gap. Similarly, the US faces negative inflation and a positive output gap. Whether the optimal monetary policy is more or less expansionary depends on the relative size of the two effects of the risk-sharing wedge.[17]

When local and US goods are complements ($\sigma\phi < 1$), an increase in local productivity generates an excess demand by US households and makes the risk-sharing wedge negative. Hence, inflation and the output gap move in the opposite direction to the case with substitutes. In a special case where the trade elasticity equals one ($\sigma\phi = 1$), productivity shocks have no

[17]It is difficult to solve for the interest rate analytically since the interest rate is determined implicitly to satisfy Equations (14) and (15). However, it is possible to check numerically that when the trade elasticity is sufficiently high, the second effect dominates the first, so the optimal monetary policy in the local economy is to lower the interest rate relative to the complete market case. Intuitively, when the price of local goods increases relative to US goods, US households shift their demand significantly from local to US goods due to a high elasticity of substitution. Under standard calibration (Section 3.3), local and US goods are substitutes so the optimal policy is to lower the interest rate relative to the complete-market case.

effect on the risk-sharing wedge, and optimal monetary policy achieves full stabilization of inflation and the output gap, similarly to the complete-market case.

The next lemma characterizes the transmission of an inefficient cost-push shock under the optimal monetary policy without FXI based on Corsetti et al. (2010). I focus on a case where local and US goods are substitutes since it matches the empirically relevant calibration (Itskhoki and Mukhin 2021) and will provide the most interesting implication of FXI.

Lemma 4 (Transmission of a Cost-Push Shock)**.** *Assume PCP, cooperation, and commitment, and* $\sigma\phi > 1$. *Suppose that FXI is not available and that monetary policy follows the optimal rule in Lemma 2. Consider an inflationary cost-push shock in the US (an increase in* μ_0^**).*

$$\frac{\partial \pi_{U0}^*}{\partial \mu_0^*} > 0 > \frac{\partial \pi_{U1}^*}{\partial \mu_0^*}, \quad \frac{\partial \tilde{Y}_{U0}}{\partial \mu_0^*} < 0, \tag{17}$$

$$\frac{\partial \pi_{L0}}{\partial \mu_0^*} < 0 < \frac{\partial \pi_{L1}}{\partial \mu_0^*}, \quad \frac{\partial \tilde{Y}_{L0}}{\partial \mu_0^*} > 0, \quad \textit{and} \tag{18}$$

$$\frac{\partial \tilde{e}_0}{\partial \mu_0^*} > 0. \tag{19}$$

Proof. See Appendix A.5.

In response to an inflationary US cost-push shock, the optimal US monetary policy is to commit to tightening. Hence, the US faces temporary inflation due to the initial impact of the cost-push shock ($\pi_{U0}^* > 0$), followed by deflation due to monetary tightening ($\pi_{U1}^* < 0$). The US monetary tightening lowers the US demand and makes its output gap negative ($\tilde{Y}_{U0} < 0$).

At the same time, a decline in the US supply increases the price of US goods relative to local goods (higher $\mathcal{T}_0$, from Equation (13)) and depreciates the local currency (higher e_0, from Equation (9)). This has two implications. First, since local households face higher import prices, they reduce consumption demand. This lower demand reduces the marginal cost of production of local goods and causes deflation (Equation (11)). This creates downward pressure on the local monetary policy rate. Hence, the local economy faces temporary deflation ($\pi_{L0} < 0$) followed by inflation ($\pi_{L0} > 0$) and a positive output gap ($\tilde{Y}_{L0} > 0$). Second, since the relative price of local goods decreases, US households shift their demand from US goods to local goods. This substitution channel further increases the local output gap and creates upward pressure on the local monetary policy rate. The combination of these two forces generates a negative correlation between inflation and the output gap in the two countries.[18]

3.2 Optimal Monetary Policy and FXI

Next, I consider the case where both monetary policy and FXI are available. The following proposition provides a full characterization of optimal monetary policy and FXI rules.

Proposition 1 (Optimal Monetary Policy and FXI)**.** *Under PCP, cooperation, and commitment, and both monetary policy and FXI are available, optimal monetary policy rules for the local and US central banks are characterized by:*

$$0 = \theta\pi_{Lt} + \left(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}\right) + \psi_\pi\theta(\pi_{Lt} - \pi^*_{Ut}) + \psi_D(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1}), \tag{20}$$

$$0 = \theta\pi_{Ut} + \left(\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}\right) - \psi_\pi\theta(\pi_{Lt} - \pi^*_{Ut}) - \psi_D(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1}), \tag{21}$$

where:

$$\psi_\pi = (1-a)\frac{2a(\sigma\phi-1)+1}{\sigma+\eta\{4a(1-a)(\sigma\phi-1)+1\}}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1},$$

and ψ_D is given in Equation (16). The optimal FXI for the local central bank is characterized by:

$$f_t = u^*_t + \frac{2a(\sigma\phi-1)+1}{2a\phi\chi_1}\theta(E_t\pi_{Lt+1} - E_t\pi^*_{Ut+1}). \tag{22}$$

These optimal rules hold without imposing the restrictions on σ and ϕ.

Proof. See Appendix A.4.

The proposition suggests that FXI has two main objectives. First, optimal FXI leans against non-fundamental volatility in capital flows and exchange rates generated by UIP shocks. When liquidity traders increase their demand for US dollar bonds (higher u^*_t), the optimal FXI is to take an opposite position and buy local currency bonds (higher f_t). This is consistent with the literature of small open economies (Cavallino 2019, Basu et al. 2020, Itskhoki and Mukhin

[18] It is possible to check numerically that the optimal monetary policy in the local economy is to increase (decrease) the interest rate when local and US goods are substitutes (complements). As shown in Section 3.3, under standard calibration, local and US goods are substitutes, so the optimal policy is to raise the interest rate.

2023). Second, when local inflation is higher than US inflation, the optimal FXI is to buy the local currency and sell the US dollar to stabilize inflation. This is because FXI affects the relative demand (risk-sharing wedge), which in turn changes inflation across countries, as implied by the NKPCs (10) and (11). Full stabilization of UIP ($f_t = u_t^*$) is optimal only in a special case where local and US inflation rates are equal.

To understand how FXI stabilizes inflation, I consider the transmission of productivity and cost-push shocks. I compare the results when both monetary policy and FXI are set optimally (denoted with a superscript FXI) and when only monetary policy is set optimally (denoted without a superscript FXI).

Proposition 2 (Transmission of Productivity Shock when FXI is available)**.** *Assume PCP, cooperation, and commitment, and suppose that the monetary policy and FXI the optimal rule as in Proposition 1. Consider an increase in the period-0 local productivity A_0. When $\sigma\phi > 1$,*

$$\frac{\partial f_0}{\partial A_0} > 0,$$
$$\frac{\partial \pi_{L0}}{\partial A_0} > \frac{\partial \pi_{L0}^{FXI}}{\partial A_0} > 0 > \frac{\partial \pi_{U0}^{FXI*}}{\partial A_0} > \frac{\partial \pi_{U0}^{*}}{\partial A_0},$$
$$\frac{\partial \tilde{Y}_{L0}^{FXI}}{\partial A_0} < \frac{\partial \tilde{Y}_{L0}}{\partial A_0} < 0 < \frac{\partial \tilde{Y}_{U0}}{\partial A_0} < \frac{\partial \tilde{Y}_{U0}^{FXI}}{\partial A_0},$$
$$\frac{\partial \tilde{e}_0}{\partial A_0} > \frac{\partial \tilde{e}_0^{FXI}}{\partial A_0} > 0, \quad \frac{\partial \tilde{\mathcal{W}}_0^{FXI}}{\partial A_0} > \frac{\partial \tilde{\mathcal{W}}_0}{\partial A_0} > 0, \quad \textit{and} \quad \frac{\partial \widetilde{UIP}_0^{FXI}}{\partial A_0} < \frac{\partial \widetilde{UIP}_0}{\partial A_0} < 0,$$

When $\sigma\phi < 1$,

$$\frac{\partial f_0}{\partial A_0} < 0,$$
$$\frac{\partial \pi_{L0}}{\partial A_0} < \frac{\partial \pi_{L0}^{FXI}}{\partial A_0} < 0 < \frac{\partial \pi_{U0}^{FXI*}}{\partial A_0} < \frac{\partial \pi_{U0}^{*}}{\partial A_0},$$
$$\frac{\partial \tilde{Y}_{L0}^{FXI}}{\partial A_0} > \frac{\partial \tilde{Y}_{L0}}{\partial A_0} > 0 > \frac{\partial \tilde{Y}_{U0}}{\partial A_0} > \frac{\partial \tilde{Y}_{U0}^{FXI}}{\partial A_0},$$
$$\frac{\partial \tilde{e}_0}{\partial A_0} < \frac{\partial \tilde{e}_0^{FXI}}{\partial A_0} < 0, \quad \frac{\partial \tilde{\mathcal{W}}_0^{FXI}}{\partial A_0} < \frac{\partial \tilde{\mathcal{W}}_0}{\partial A_0} < 0, \quad \textit{and} \quad \frac{\partial \widetilde{UIP}_0^{FXI}}{\partial A_0} > \frac{\partial \widetilde{UIP}_0}{\partial A_0} > 0,$$

When $\sigma\phi = 1$,

$$\frac{\partial f_0}{\partial A_0} = \frac{\partial \pi_{L0}}{\partial A_0} = \frac{\partial \pi_{U0}^{*}}{\partial A_0} = \frac{\partial \tilde{Y}_{L0}}{\partial A_0} = \frac{\partial \tilde{Y}_{U0}}{\partial A_0} = \frac{\partial \tilde{\mathcal{W}}_0}{\partial A_0} = 0.$$

The proposition shows that a combination of monetary policy and FXI stabilizes inflation in both countries but widens the demand gap and the risk-sharing wedge compared to the case where only monetary policy is available. Intuitively, when the local and US goods are substitutes ($\sigma\phi > 1$), an increase in local productivity generates local excess demand (a positive risk-sharing gap), which increases the local marginal cost of production and inflation. The optimal monetary policy is to lean against inflation, but this comes at the cost of a negative output gap. The local economy faces positive inflation and a negative output gap, while the US faces negative inflation and a positive output gap.

When local inflation is higher than US inflation, the optimal FXI is to buy the local currency and sell the US dollar ($f_0 > 0$). When the central bank buys the local currency, the demand for the local currency increases and the local currency appreciates. Local households face lower import prices and increase consumption demand, while US households face higher import prices and reduce consumption demand. Moreover, local currency purchases increase the local bond prices and reduce the rate of return on investment in local currency bonds relative to dollar bonds, making the UIP deviation more negative. However, since households are restricted from investing in foreign currency assets, they cannot obtain carry-trade profits by buying dollar bonds with higher returns and selling local currency bonds with lower returns. This generates local excess demand and widens the risk-sharing wedge (higher $\tilde{\mathcal{W}}_t$).

This has two main consequences on inflation. First, an increase in the local excess demand raises the marginal cost of production faced by local firms and raises GDP inflation, as implied by Equation (10). Second, since the local currency appreciates, US households face higher import prices and shift their demand from local to US goods. This lower demand for local goods reduces local GDP inflation. As long as the elasticity of substitution between local and US goods is high enough, the second effect dominates the first, so local currency purchases (US dollar sales) using FXI reduce local GDP inflation, making it less positive. By the same logic, local currency purchases increase US GDP inflation, making it less negative. Hence, FXI helps the central banks in the two countries stabilize GDP inflation with moderate changes in the nominal interest rate. Hence, FXI works as a second instrument to stabilize inflation.

However, FXI faces a trade-off between stabilizing inflation and stabilizing the output gap and the risk-sharing wedge. First, as discussed above, local currency purchases widen the risk-sharing wedge, disproportionately benefiting local households over US households. Second, since local currency purchases reduce the demand for local goods and increase the demand for

US goods, the local output gap $\tilde{Y}_{Lt}$ becomes more negative and the US output gap $\tilde{Y}_{Ut}$ becomes more positive.

In a special case where the local and US goods are independent ($\sigma\phi = 1$), productivity shocks do not affect the risk-sharing wedge ($\tilde{W}_t = 0$). In this case, monetary policy and FXI have separate objectives and do not face trade-offs. As in the standard New Keynesian model, the optimal monetary policy is to set the nominal interest rate to close inflation and the output gap ($\pi_{Lt} = \pi^*_{Ut} = \tilde{Y}_{Lt} = \tilde{Y}_{Ut} = 0$). The optimal FXI is to offset the UIP shock ($f_t = u^*_t$), which stabilizes non-fundamental volatility in capital flows and the exchange rate.

The next proposition shows the optimal trade-off of FXI when there is a cost-push shock. I focus on the case where the local and US goods are substitutes.

Proposition 3 (Transmission of Cost-Push Shock when FXI is available)**.** *Assume PCP, cooperation, and commitment, and $\sigma\phi > 1$ and suppose that the monetary policy and FXI follow the optimal rule as in Proposition 1. Consider an inflationary cost-push shock in the US (an increase in μ^*_0).*

$$\frac{\partial f_0}{\partial \mu^*_0} > 0, \tag{23}$$

$$\frac{\partial \pi^{*FXI}_{U0}}{\partial \mu^*_0} < \frac{\partial \pi^*_{U0}}{\partial \mu^*_0} (> 0), \quad \frac{\partial \pi^{*FXI}_{U1}}{\partial \mu^*_0} > \frac{\partial \pi^*_{U1}}{\partial \mu^*_0} (< 0), \quad \frac{\partial \tilde{Y}^{FXI}_{U0}}{\partial \mu^*_0} > \frac{\partial \tilde{Y}_{U0}}{\partial \mu^*_0} (< 0), \tag{24}$$

$$\frac{\partial \pi^{FXI}_{L0}}{\partial \mu^*_0} > \frac{\partial \pi_{L0}}{\partial \mu^*_0} (< 0), \quad \frac{\partial \pi^{FXI}_{L1}}{\partial \mu^*_0} < \frac{\partial \pi_{L1}}{\partial \mu^*_0} (> 0), \quad \frac{\partial \tilde{Y}^{FXI}_{L0}}{\partial \mu^*_0} < \frac{\partial \tilde{Y}_{L0}}{\partial \mu^*_0} (> 0), \tag{25}$$

$$\frac{\partial \tilde{e}^{FXI}_0}{\partial \mu^*_0} < \frac{\partial \tilde{e}_0}{\partial \mu^*_0} (> 0), \quad \frac{\partial \tilde{W}^{FXI}_0}{\partial \mu^*_0} > \frac{\partial \tilde{W}_0}{\partial \mu^*_0} > 0, \quad \textit{and} \quad \frac{\partial \widetilde{UIP}^{FXI}_0}{\partial \mu^*_0} < \frac{\partial \widetilde{UIP}_0}{\partial \mu^*_0} < 0. \tag{26}$$

Proof. See Appendix A.6.

The proposition shows that a combination of monetary policy and FXI smooths inflation and output gap in both countries but widens the risk-sharing wedge compared to the case where only monetary policy is available. Intuitively, if the central bank buys the local currency using FXI, the local currency appreciates against the dollar (lower $\tilde{e}_t$) and the US households shift their demand from local to US goods. This decreases the local output, making the local output gap $\tilde{Y}_{Lt}$ less positive, but this increases the US output, making the US output gap $\tilde{Y}_{Ut}$ less negative. Since FXI partially absorbs the output gap, the monetary policy faces a less trade-off between stabilizing inflation and output gap. In other words, FXI improves the monetary policy trade-off between inflation and output by allowing monetary authorities to stabilize inflation with moderate changes in the interest rate. The local GDP inflation π_{Lt} becomes less negative in period 0 and less positive in period 1, while the US GDP inflation π^*_{Ut} becomes less positive in period 0 and less negative in period 1. However, at the same time, FXI distorts international risk-sharing, disproportionately increasing the consumption demand by local households over the US households. The key policy implication is that FXI faces a trade-off between stabilizing internal objectives (inflation and output in each country) and external objectives (relative demand across countries).

3.3 Numerical Illustration

To validate these analytical predictions, this section provides a numerical illustration of the transmission mechanisms of FXI. First, I discuss the calibration of key model parameters. Then, I study impulse responses to productivity and markup shocks and compare the results when only monetary policy is available and when both monetary policy and FXI are available.

Table 1 shows the calibration of key parameters. I calibrate the model at the quarterly frequency. I set $\beta = 0.995$ to target the annual interest rate of 2%. I set the risk aversion to $\sigma = 2$ and the inverse Frisch elasticity of labor to $\eta = 1$, which are standard values in the international macroeconomics literature. I set the trade openness to $a = 0.88$ to match the trade-to-GDP ratio in large open economies, such as Japan and the United States (Itskhoki and Mukhin 2021). I choose the labor disutility parameter of ζ_l to match the steady-state labor $\bar{L} = 0.33$. There is a wide range of estimates for the trade elasticity ϕ. Estimates using microdata are as high as 4 (Bernard et al. 2003), while values lower than 1 can also be empirically relevant (Corsetti et al. 2008). In the benchmark exercise, I set $\phi = 1.5$, a frequently used value in the international business cycle literature (Backus et al. 1994, Chari et al. 2002, Itskhoki and Mukhin 2021), but I also explore implications of different trade elasticities. I set the elasticity of substitution between differentiated goods to $\theta = 10$ so that the price markup is 11%. I set the Calvo price stickiness to $\xi_p = 0.75$ to target the average duration of price stickiness of four quarters. Finally, I set the persistence of productivity shocks to $\rho_a = 0.97$ following Itskhoki and Mukhin (2021) and the persistence of markup shocks to $\rho_\mu = 0$ following Bodenstein et al. (2019).

Based on these parameters, I study impulse responses to productivity and cost-push (markup) shocks. In addition to validating the analytical findings, I obtain two quantitative results. First,

Table 1: Benchmark Parameters

Value	Description	Notes
$\beta = 0.995$	Discount factor (local)	Annual interest rate = 2%
$\sigma = 2$	Relative risk aversion	Itskhoki and Mukhin (2021)
$\eta = 0.35$	Inverse Frisch elasticity	Bodenstein et al. (2023)
$\zeta_l = 13.3$	Labor disutility (local)	$\bar{L} = 1/3$
$a = 0.88$	Home bias of consumption	Bodenstein et al. (2023)
$\phi = 1.5$	CES Local & US goods	Itskhoki and Mukhin (2021)
$\theta = 10$	CES differentiated goods	Price markup = 11%
$\xi_p = 0.75$	Calvo price stickiness	Duration of four quarters
$\bar{\pi} = 1$	Steady-state inflation	
$\rho_a = 0.97$	Persistence of productivity shock	Itskhoki and Mukhin (2021)
$\rho_\mu = 0.2$	Persistence of markup shock	Bodenstein et al. (2023)

Note: The table shows the parameter settings for impulse response analyses in Section 3.3.

the optimal volume of FXI is larger with markup shocks than productivity shocks. Second, optimal FXI allows the local central bank to achieve nearly full stabilization of inflation and the output gap with little changes in the monetary policy rate. Third, the effect of FXI on inflation, the output gap, and the exchange rate is larger when the trade elasticity between local and US goods is high.

Figure 2 plots the impulse response to a one-percentage increase in local productivity. In panel (a), the red line shows the case where only monetary policy follows the optimal rule but FXI is not available, and the blue line shows the case where both monetary policy and FXI follow the optimal rule. In panel (b), the green line shows the difference between the cases with and without FXI. The result is consistent with Proposition 2. Without FXI, an increase in local productivity generates a local excess demand (a positive risk-sharing wedge), causing positive inflation and a negative output gap in the local economy and negative inflation and a positive output gap in the US. The optimal FXI is to purchase the local currency. This stabilizes inflation in both countries but widens the output gaps in both countries and the risk-sharing wedge. Quantitatively, however, the volume of optimal FXI and its effect on the exchange rate is small.

Next, Figure 3 plots the impulse response to a one-percentage increase in US markup. Consistently with Proposition 3, without FXI, a US cost-push shock leads to temporary inflation followed by persistent deflation in the US and temporary deflation followed by persistent inflation in the local economy. The output gap is negative in the US and positive in the local economy. The optimal FXI is to buy the local currency corresponding to 0.06% of GDP, which is larger than the productivity shock case. FXI mutes the responses of inflation and the output gap in both countries. In particular, FXI allows the local central bank to almost fully stabilize the paths of local inflation and output gap with little change in the monetary policy rate. However, the trade-off is that local currency purchases generate upward pressure on the US rate of return (a negative UIP deviation) and a local excess demand over the US (a positive risk-sharing wedge).

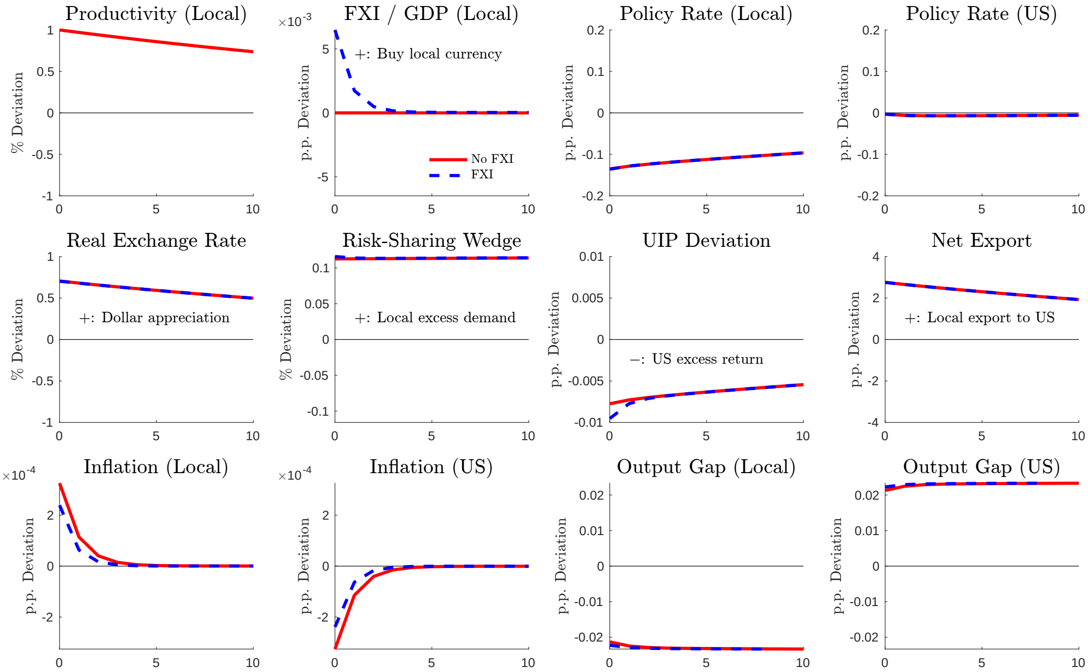
Finally, Figure 4 plots the impulse response to a US inflationary cost-push shock for different trade elasticities. Panels (a) and (b) show the cases with high trade elasticity ($\phi = 4$) and low trade elasticity ($\phi = 0.85$), respectively. Panel (a) shows that the optimal volume of FXI when ϕ is large, as implied by (Equation (22)). Intuitively, when the central bank uses FXI to appreciate the local currency against the dollar, local households face lower import prices of US goods. When the trade elasticity is high, households shift their demand from local to US goods significantly. This leads to a large decline in the local output gap, allowing the monetary policy to better focus on inflation stabilization. Conversely, panel (b) shows that when the trade elasticity is low, the international spillover of a US cost-push shock and the effect of FXI on inflation and the output gap are both limited.

4 Dollar Pricing

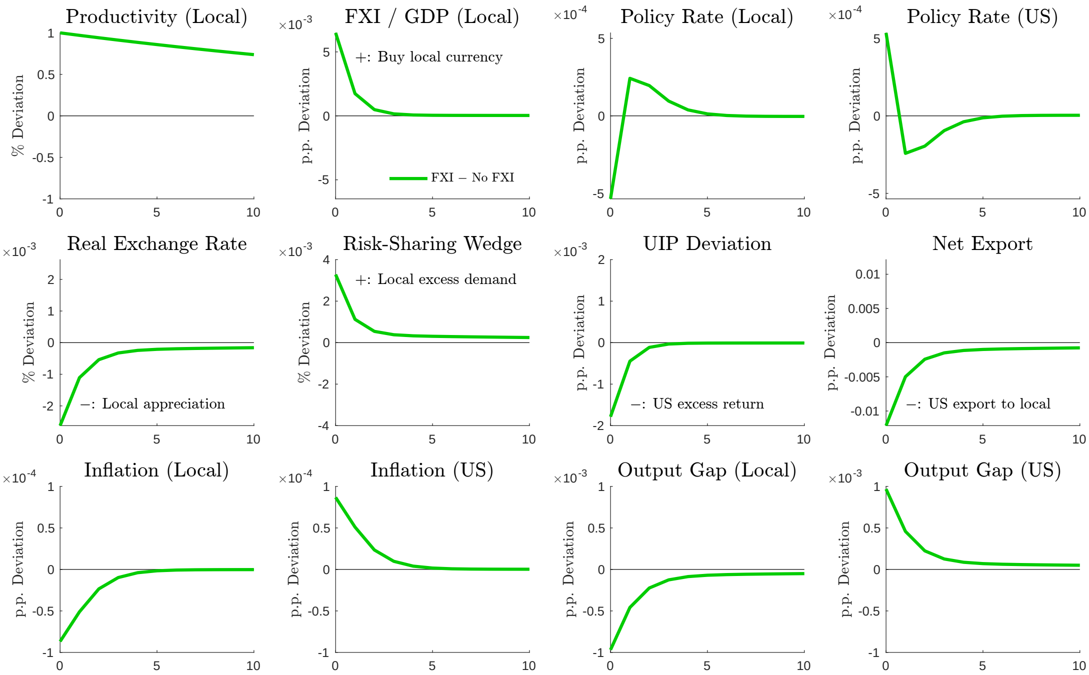
While PCP assumption provides the simplest analytical solution, data suggests that exports and imports are mainly invoiced in US dollars (Gopinath et al. 2020). Motivated by this fact, this section explores the novel interplay between dollar dominance in international trade and capital flow management in international finance. To this end, I introduce dominant currency pricing (DCP) and study optimal FXI and transmission channels. Differently from the previous section, I assume that both exports and imports are denominated in US dollars. This has two major implications. First, identical local goods are priced differently in two currencies, i.e., the LOOP does not hold for local goods. This generates an inefficient cross-currency price dispersion despite the same marginal cost of production (Engel 2011). Second, changes in the exchange rates directly affect import prices for local households, so they change the relative consumption of local and US goods. However, this expenditure-switching effect is muted for

Figure 2: Impulse Response to a Positive Local Productivity Shock

(a) No Intervention vs. Intervention



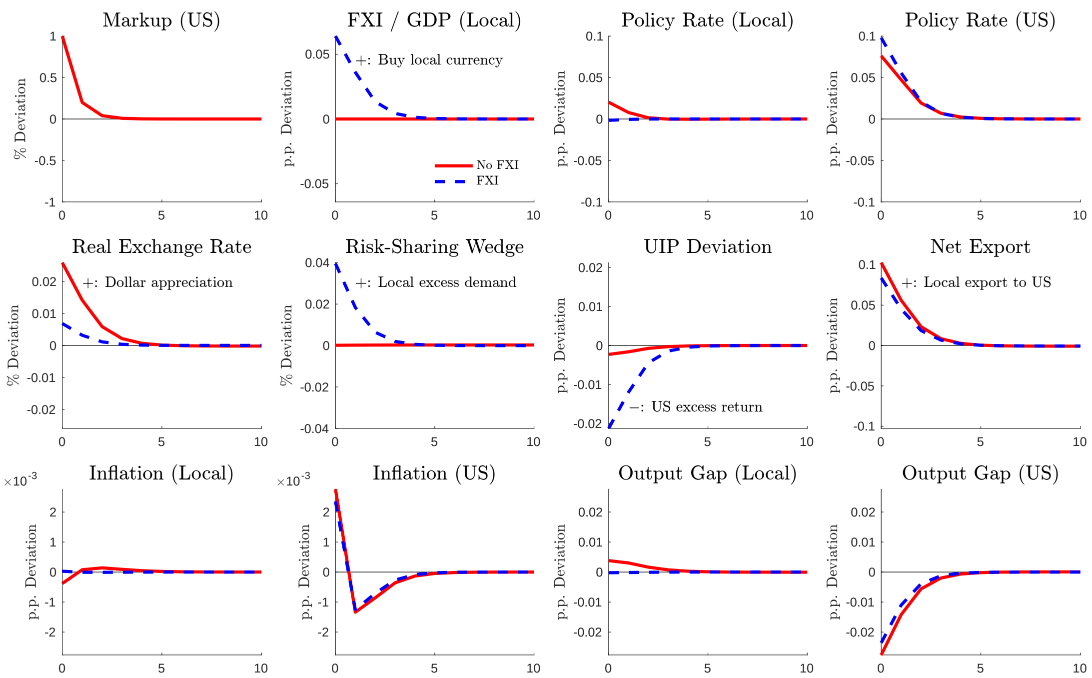
(b) Effect of Intervention



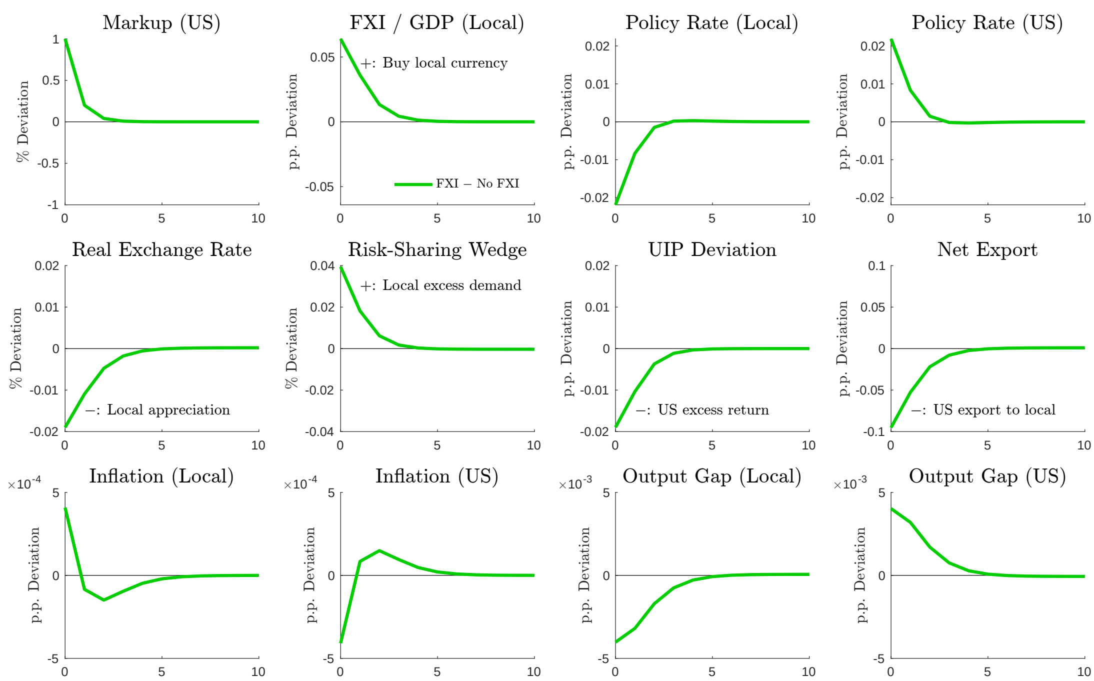
Note: The figure plots the impulse responses to a one-percentage increase in local productivity. Panel (a) plots the impulse responses when only monetary policy follows the optimal rule but FXI is not available (red) and when both monetary policy and FXI follow the optimal rule (blue). Panel (b) takes the difference between cases with and without FXI (green).

Figure 3: Impulse Response to a US Cost-Push Shock

(a) No Intervention vs. Intervention



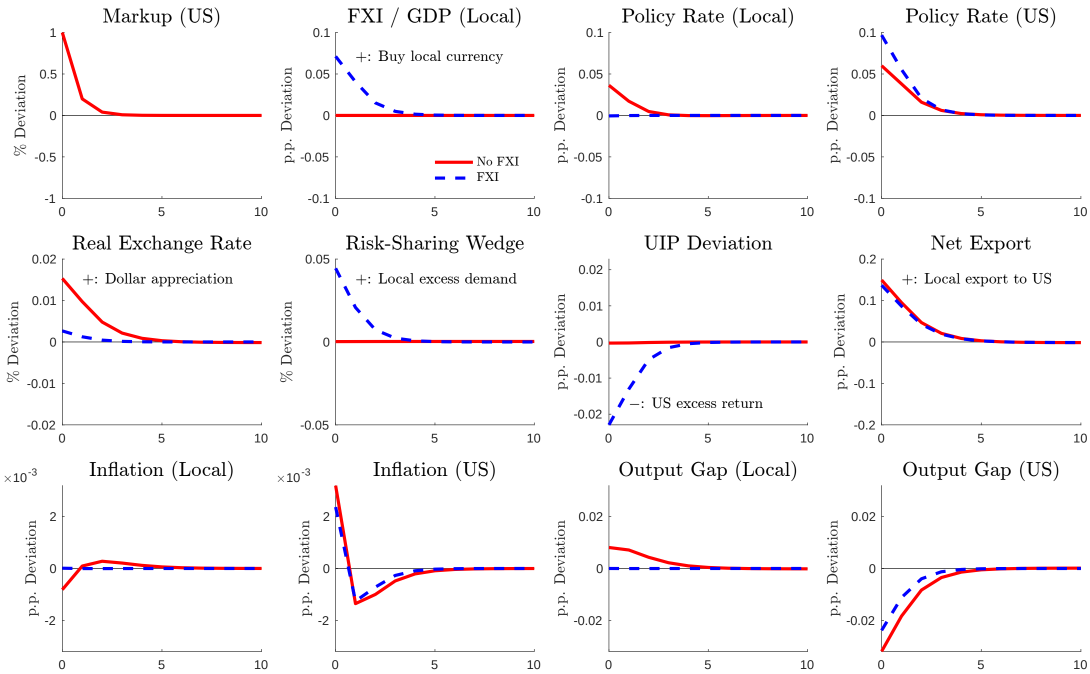
(b) Difference



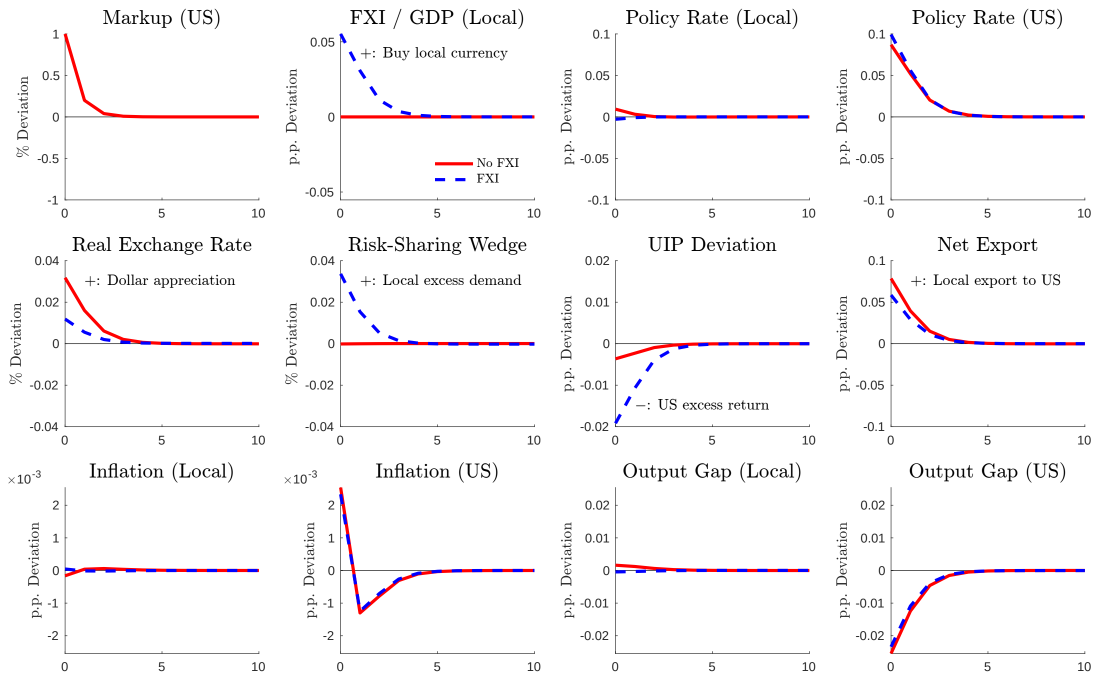
Note: The figure plots the impulse responses to a one-percentage increase in US markup. Panel (a) plots the impulse responses when only monetary policy follows the optimal rule but FXI is not available (red) and when both monetary policy and FXI follow the optimal rule (blue). Panel (b) takes the difference between cases with and without FXI (green).

Figure 4: Impulse Response to a US Cost-Push Shock: Different Trade Elasticities

(a) High Trade Elasticity (ϕ = 4)



(b) Low Trade Elasticity (ϕ = 0.85)



Note: The figure plots the impulse responses to a one-percentage increase in US markup. Panels (a) and (b) plot the cases with high trade elasticity (ϕ = 4) and low trade elasticity (ϕ = 0.85), respectively. The red line plots the impulse responses when only monetary policy follows the optimal rule but FXI is not available. The blue line plots the impulse responses when both monetary policy and FXI follow the optimal rule (blue).

US households since US imports are denominated in dollars.

Under DCP, the price of local goods is sticky in local currency in the local economy $(P_t(l))$ and sticky in the dollars in the US $(P_t^*(l))$. The local firms' maximization problem is:

$$\max_{\{P_t(l), P_t^*(l)\}} E_t \left\{ \sum_{k=0}^{\infty} Q_{t,t+k} \xi_p^k \left(\begin{array}{c} (1+\tau_t) \left[P_t(l) Y_{t+k}(l) + \mathcal{E}_t P_t^*(l) Y_{t+k}^*(l) \right] \\ -MC_{t+k}(l) \left[Y_{t+k}(l) + Y_{t+k}^*(l) \right] \end{array} \right) \right\}.$$

Solving the maximization problem, the optimal prices of local goods charged to local and US households are:

$$\begin{aligned} P_t(l) &= \frac{\theta}{(\theta-1)(1+\tau_t)} \frac{E_t \sum_{k=0}^{\infty} Q_{t,t+k} \xi_p^k Y_{t+k}(l) MC_{t+k}}{E_t \sum_{k=0}^{\infty} Q_{t,t+k} \xi_p^k Y_{t+k}(l)}, \quad \text{and} \\ P_t^*(l) &= \frac{\theta}{(\theta-1)(1+\tau_t)} \frac{E_t \sum_{k=0}^{\infty} Q_{t,t+k} \xi_p^k Y_{t+k}^*(l) MC_{t+k}}{E_t \sum_{k=0}^{\infty} Q_{t,t+k} \xi_p^k \mathcal{E}_{t+k} Y_{t+k}^*(l)}. \end{aligned}$$

The laws of motion of local good prices faced by local and US households can be written as:

$$\begin{aligned} P_{Lt}^{1-\theta} &= \xi_p P_{Lt-1}^{1-\theta} + (1-\xi_p) P_t(l)^{1-\theta}, \quad \text{and} \\ P_{Lt}^{*1-\theta} &= \xi_p P_{Lt-1}^{*1-\theta} + (1-\xi_p) P_t^*(l)^{1-\theta}. \end{aligned}$$

I define $\Delta_{Lt} \equiv \mathcal{E}_t P_{Lt}^* / P_{Lt}$ as the deviation from the LOOP for the local goods. The numerator and the denominator are the prices of local goods charged to local and US households, respectively, expressed in the local currency. Under PCP, the LOOP holds so $\Delta_{Lt} = 1$. However, under DCP, the LOOP does not hold so $\Delta_{Lt} \neq 1$. When P_{Lt} and P_{Lt}^* are sticky, a local depreciation (an increase in $\mathcal{E}_t$) increases Δ_{Lt}, so US households are charged higher prices for the identical local goods than local households despite the same marginal cost of production.

Solving the maximization problem, the NKPCs for local goods can be written as:

$$\pi_{Lt} = \beta \pi_{Lt+1} + \kappa \{ (\sigma+\eta) \tilde{Y}_{Lt} - (1-a)[2a(\sigma\phi-1)(\tilde{\mathcal{T}}_t + \tilde{\Delta}_{Lt}) + (\tilde{\mathcal{W}}_t + \tilde{\Delta}_{Lt})] + \mu_t \}, \tag{27}$$

$$\pi_{Lt}^* = \beta \pi_{Lt+1}^* + \kappa \{ (\sigma+\eta) \tilde{Y}_{Lt} - (1-a)[2a(\sigma\phi-1)(\tilde{\mathcal{T}}_t + \tilde{\Delta}_{Lt}) + (\tilde{\mathcal{W}}_t + \tilde{\Delta}_{Lt})] - \tilde{\Delta}_{Lt} + \mu_t \}, \tag{28}$$

The quadratic loss function in the DCP case can be characterized as:[19]

$$\mathcal{L} = -E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\begin{array}{l} (\sigma+\eta)\left(\tilde{Y}_{Lt}^2 + \tilde{Y}_{Ut}^2\right) + \frac{\theta}{\kappa}\left(a\pi_{Lt}^2 + (1-a)\pi_{Lt}^{*2} + \pi_{Ut}^{*2}\right) \\ -\frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}\left(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}\right)^2 \\ +\frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\left(\tilde{\mathcal{W}}_t + \tilde{\Delta}_{Lt}\right)^2 \end{array} \right]. \tag{29}$$

There are two key differences compared to the PCP case (Equation (12)). First, the central banks take into account the weighted sum of local good inflation in the two countries (π_{Lt}, π_{Lt}^*). Second, the loss depends on the deviation from the LOOP ($\tilde{\Delta}_{Lt}$).

Under DCP, analytically tractable expressions for the optimal policy rule can be derived under the assumption of linear labor disutility (Engel 2011). The following lemma characterizes the optimal monetary policy under dollar pricing when FXI is not available.

Lemma 5 (Optimal Monetary Policy Trade-offs under DCP)**.** *Under DCP, cooperation, and commitment, $\eta = 0$, and when FXI is not available ($f_t = 0$), optimal monetary policy rules for the local and US central banks are characterized by:*

$$0 = \theta a \pi_{Lt} + (\tilde{C}_t - \tilde{C}_{t-1}) + \frac{2a(1-a)\phi}{2a(\phi-1)+1}\frac{\sigma-1}{\sigma}(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1} + \tilde{\Delta}_t - \tilde{\Delta}_{t-1}), \tag{30}$$

$$0 = \theta[(1-a)\pi_{Lt}^* + \pi_{Ut}^*] - (\tilde{C}_t^* - \tilde{C}_{t-1}^*) - \frac{2a(1-a)\phi}{2a(\phi-1)+1}\frac{\sigma-1}{\sigma}(\tilde{\mathcal{W}}_t - \tilde{\mathcal{W}}_{t-1} + \tilde{\Delta}_t - \tilde{\Delta}_{t-1}). \tag{31}$$

Proof. See Appendix A.7.

The result is isomorphic to the one without currency market segmentation (Corsetti et al. 2020). Importantly, the optimal monetary policy rule is asymmetric across countries. The local central bank trades off the stabilization of domestic inflation (π_{Lt}) with growth rates of the risk-sharing wedge and LOOP deviation. In contrast, the US central bank targets the international dollar price, which is the weighted sum of the inflation of local goods prices in the dollars (π_{Lt}^*) and the US-produced goods (π_{Ut}^*).

Next, I study the case where both monetary policy and FXI are available. DCP has two key implications for the design and transmission mechanism of optimal FXI. First, Proposition 4 shows that the optimal FXI closes the inefficient cross-currency dispersion due to incomplete

[19] See Corsetti et al. (2020) for the details.

exchange-rate pass-through. The full derivation of the policy rules under DCP is available in Appendix A.8.

Proposition 4 (Targeting the LOOP Deviation)**.** *Under DCP, cooperation, commitment,* $\sigma = \phi = 1$, $\eta = 0$, *and when both monetary policy and FXI follow the optimal rules,*

1. *Optimal local currency purchase* f_t *is an increasing function of the price dispersion* Δ_{Lt}.

2. *FXI reduces the elasticity of* Δ_{Lt} *to the US cost-push shock.*

3. *The elasticity of optimal local currency purchase to the US cost-push shock is larger under DCP than PCP:*

$$\frac{\partial f_t}{\partial \Delta_{Lt}} > 0, \quad \frac{\partial \Delta_{Lt}^{FXI}}{\partial \mu_t^*} < \frac{\partial \Delta_{Lt}}{\partial \mu_t^*} \; (> 0), \quad \left(\frac{\partial f_t}{\partial \mu_t^*}\right)^{DCP} > \left(\frac{\partial f_t}{\partial \mu_t^*}\right)^{PCP} \; (> 0). \tag{32}$$

Proof. See Appendix A.9.

Statements 1 and 2 show that optimal FXI addresses the inefficient cross-currency price dispersion due to incomplete exchange-rate pass-through. Under DCP, since the local exporters set the price in US dollars, a depreciation of the local currency increases the dollar price relative to the local currency price of an identical locally produced good, causing a deviation from the LOOP. The proposition implies that the optimal FXI is to buy the local currency and respond to its undervaluation. Hence, the optimal FXI rule targets the LOOP deviation in addition to the UIP deviation and the inflation in the two countries. As implied by Statement 3, the optimal FXI volume is larger under DCP than under PCP.

Second, Proposition 5 characterizes the key difference in the transmission mechanism of FXI under different currency paradigms.

Proposition 5 (Asymmetric Transmission)**.** *Assume DCP, cooperation, commitment,* $\sigma = \phi = 1$, $\eta = 0$, *and suppose that both monetary policy and FXI follow the optimal rules. In response to the US cost-push shock, under PCP, optimal FXI decreases the local CPI inflation and increases the US CPI inflation by the same degree:*

$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} = -\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} \; (< 0).$$

Under DCP, optimal FXI decreases the local CPI inflation more and increases the US CPI inflation less than the PCP case:

$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} (< 0),$$
$$\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} (> 0).$$

Proof. See Appendix A.10.

Under PCP, FXI decreases local inflation and increases US inflation symmetrically. However, under DCP, the transmission of FXI is asymmetric across countries. On the one hand, FXI decreases local inflation more under DCP than PCP. Since the optimal FXI is larger under DCP (Proposition 4), FXI reduces the local import price of US goods and thus the local CPI inflation. On the other hand, since the US import price is sticky in dollars, local currency appreciation has a limited effect on the US import price. Hence, by purchasing the local currency, central banks can stabilize local inflation without causing a large upward spillover to US inflation. These results explain why FXI is particularly effective at stabilizing local inflation under DCP.[20]

5 Non-Cooperative Equilibrium: A First Step

Finally, I deviate from international cooperation and study FXI in a strategic non-cooperative equilibrium. As discussed in Corsetti et al. (2010), modern literature on international monetary policy coordination requires a full specification of dynamic games between policymakers. Hence, the literature faces a number of challenges, including the definition of equilibrium (commitment or discretion) and the feasibility of deriving analytical and numerical solutions when the steady state is inefficient. This paper takes the first step in this research and focuses on a case where it is feasible to derive a numerical solution.

In particular, I consider an open-loop Nash equilibrium under commitment and with one strategic policy instrument for each policymaker. Under open-loop Nash equilibrium, in the initial period, each player specifies her state-contingent plans for every future state, and each player's action is the best response to the other player's best response. The model can be solved

[20]Under local currency pricing (LCP) where both exports and imports are invoiced in the destination currency, optimal FXI is larger than the PCP case as it targets the LOOP deviation. However, the transmission is symmetric and FXI has muted effects on the import prices in both countries.

numerically using a second-order perturbation of the equilibrium conditions. To derive the equilibrium conditions analytically, I use the symbolic differentiation toolbox developed by (Bodenstein et al. 2019; 2023).[21] I assume that the local central bank uses FXI and the US central bank uses domestic price inflation as the policy instrument (Benigno and Benigno 2006).

5.1 Definition of Equilibrium

Let $x_t = (\tilde{x}_t', i_{L,t}, i_{U,t})'$ be the $N \times 1$ vector of endogenous variables, where $i_{L,t}$ and $i_{U,t}$ are the strategic policy instrument chosen by the local and US central banks, respectively. Let ϵ_t be the vector of the exogenous shocks. The private optimality and market clearing conditions are summarized by:

$$E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{L,t}, i_{U,t}, \epsilon_t) = 0.$$

5.1.1 Cooperative Equilibrium

Under the cooperative game, the policymakers maximize the weighted average of the local and US households' utility under commitment:

$$\max_{\{\tilde{x}_t', i_{L,t}, i_{U,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha U_{L,t}(\tilde{x}_{t-1}, \tilde{x}_t, \epsilon_t) + (1-\alpha) U_{U,t}(\tilde{x}_{t-1}, \tilde{x}_t, \epsilon_t) \right],$$
$$s.t. \quad E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{L,t}, i_{U,t}, \epsilon_t) = 0.$$

where α and $1-\alpha$ are the weights on the local and US households' utilities, respectively. I refer to the

5.1.2 Non-Cooperative Equilibrium

I consider a non-cooperative interaction between the two central banks under an open-loop Nash game. Let $j = [L, U]$ be the set of players (the local or US central bank). Let $\{i^*_{j,t,-t^*}\}_{t=0}^{\infty}$ be the sequence of policies chosen by player j before and after but not including period t^* and $\{i^*_{-j,t}\}_{t=0}^{\infty}$ be the other player's policies. An open-loop Nash equilibrium is a sequence $\{i^*_{j,t}\}_{t=0}^{\infty}$ such that, for all period t^*, i^*_{j,t^*} maximizes player j's objective function subject to the constraints for given sequences of $\{i^*_{j,t,-t^*}\}_{t=0}^{\infty}$ and $\{i^*_{-j,t}\}_{t=0}^{\infty}$. In each period, each player maximizes her

[21]The toolbox does not currently support the games with multiple strategic instruments per policymaker.

own following objective function given the other player's policies:[22]

$$\max_{\{\tilde{x}'_t, i_{j,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[U_{j,t}(\tilde{x}_{t-1}, \tilde{x}_t, \epsilon_t) \right],$$

$$s.t. \quad E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{L,t}, i_{U,t}, \epsilon_t) = 0, \quad \text{for given } \{i_{-j,t}\}_{t=0}^{\infty}.$$

5.2 Policy Trade-offs under a Non-Cooperative Equilibrium

I compute the cooperative and non-cooperative equilibria numerically and compare the impulse responses to technology and markup shocks. I consider a strategic interaction between the local FXI and the US monetary policy, assuming that the local monetary policy follows a Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\gamma_R} \left(\frac{\bar{\pi}_{Lt}}{\pi}\right)^{\phi_\pi(1-\gamma_R)}.$$

I follow the main parameter settings in Bodenstein et al. (2023). I set $\phi_\pi = 4$ since the policy response to the inflation rate is high enough to ensure the uniqueness of equilibrium. I set $\alpha = 0.5$ so the local and US households' utilities are equally weighted under cooperation. I compare the impulse responses to US productivity and cost-push shocks under cooperative and Nash equilibria.

First, Figure 5, panel (a) plots the impulse response to a one-percentage increase in US productivity. The red and blue lines show the cooperative and Nash equilibria, respectively. Panel (b) shows the difference between the two equilibria. The figure shows that, under Nash equilibria, the optimal FXI is to buy the dollar more (sell the dollar less) and accumulate excess reserves relative to the cooperation case. Intuitively, the dollar purchase has risk-sharing and expenditure-switching effects. First, since the local return increases relative to the US, the local households have a higher marginal propensity to consume, which lowers the local demand and output. This distorts the international risk-sharing and makes the risk-sharing wedge negative. Second, since the local currency depreciates, the US demand for local goods increases via the expenditure switching channel. This increases the inflation rate of the locally produced goods relative to the US. However, the local central bank does not take into account the US incentive to lower its policy rate, which stabilizes its inflation rate and further increases the US demand

[22] I adopt the timeless perspective, which requires an initial pre-commitment so that the optimal policy is time-invariant (Benigno and Woodford 2012).

and output relative to the local. This implies that competitive devaluation via excess reserve accumulation is not only self-defeating as it is matched by the US policy response but also exacerbates the international risk-sharing distortion.

Next, Figure 6 plots the impulse response to a one-percentage increase in US markup. Since the US objective is to stabilize domestic inflation, the optimal US policy is to raise the interest rate higher than the cooperative case. This appreciates the US dollar and increases the demand for local goods, leading to a higher output gap and inflation. The local central bank counteracts against the US dollar by over-accumulating local currency reserves. The local appreciation in turn increases the US import prices. This creates further upward pressure on the US interest rate, stabilizing US inflation and but reducing its output. Hence, a lack of cooperation results in both countries' attempts to stabilize their currency, leading to over-production in the local economy and under-production in the US. Furthermore, since the local currency appreciates due to the accumulation of local currency reserves, the local households can import more at lower prices and increase consumption, resulting in a positive risk-sharing wedge. Hence, over-accumulation of local currency reserves is a beggar-thy-neighbor policy.

Conversely, when there is a decrease in US markup, the local central bank accumulates dollar reserves and the US central bank lowers the monetary policy rate to depreciate their currency relative to the cooperation case. An over-accumulation of dollar reserves is a beggar-thy-self policy since local currency depreciation implies that US households can import local goods at lower prices and increase demand relative to the local households. Monetary policy and FXI

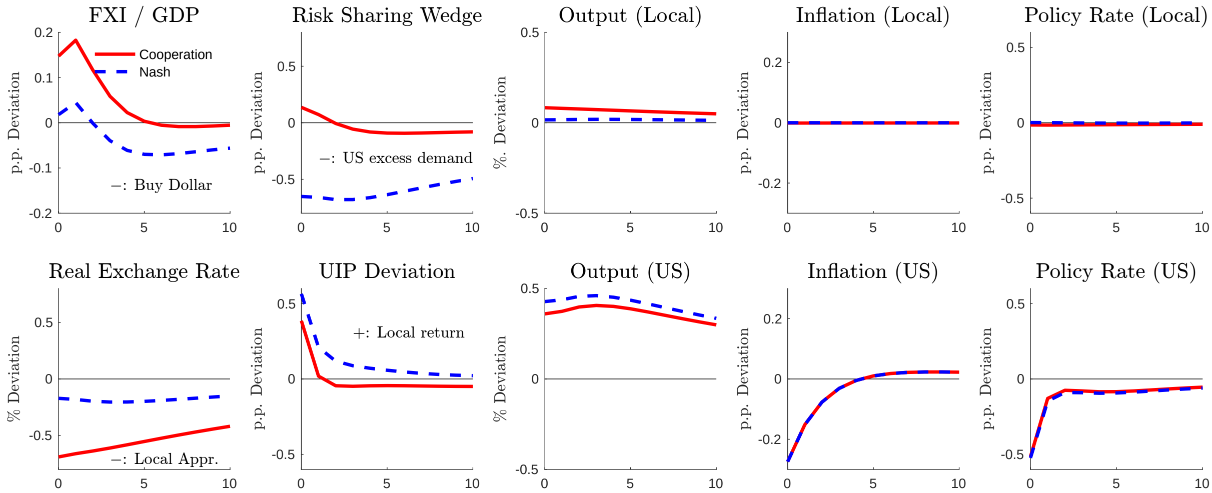
6 Conclusion

Conventional wisdom in the literature is that monetary policy and FXI are two distinct policy tools with separate objectives: optimal monetary policy exclusively targets inflation while optimal FXI independently stabilizes the exchange rate. This consensus is based on a small open economy model where agents take the international price as given. However, this consensus changes drastically in large open economies where FXI can affect international prices and demand. Should policies respond to domestic inflation or global business cycle conditions and imbalances?

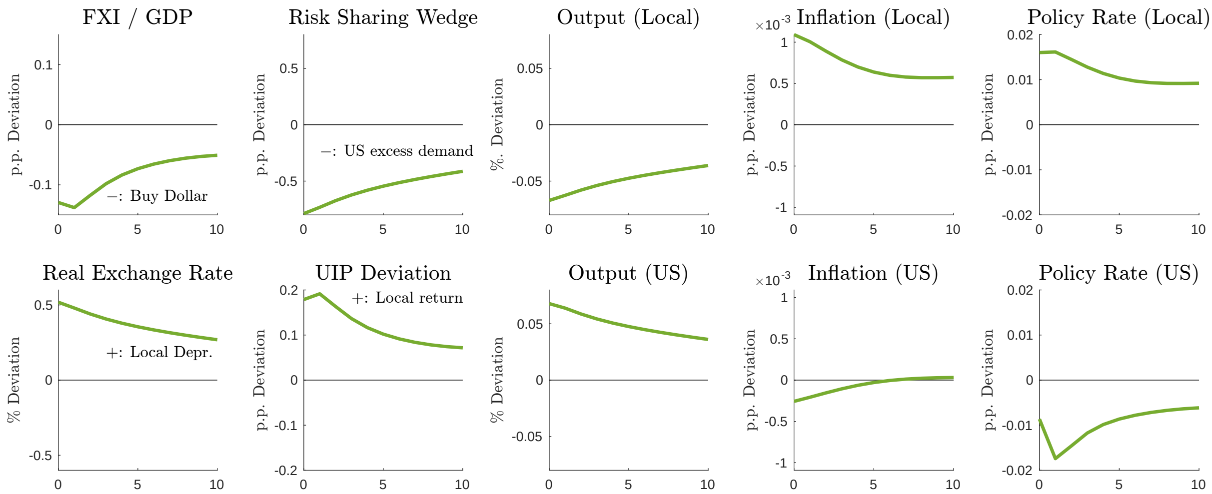
In this paper, I have shown that FXI in large open economies faces a non-trivial trade-off between internal and external objectives. On the one hand, FXI mitigates the inflation-output

Figure 5: Impulse Response to a US Productivity Shock, Cooperation and Nash

(a) Cooperation and Nash



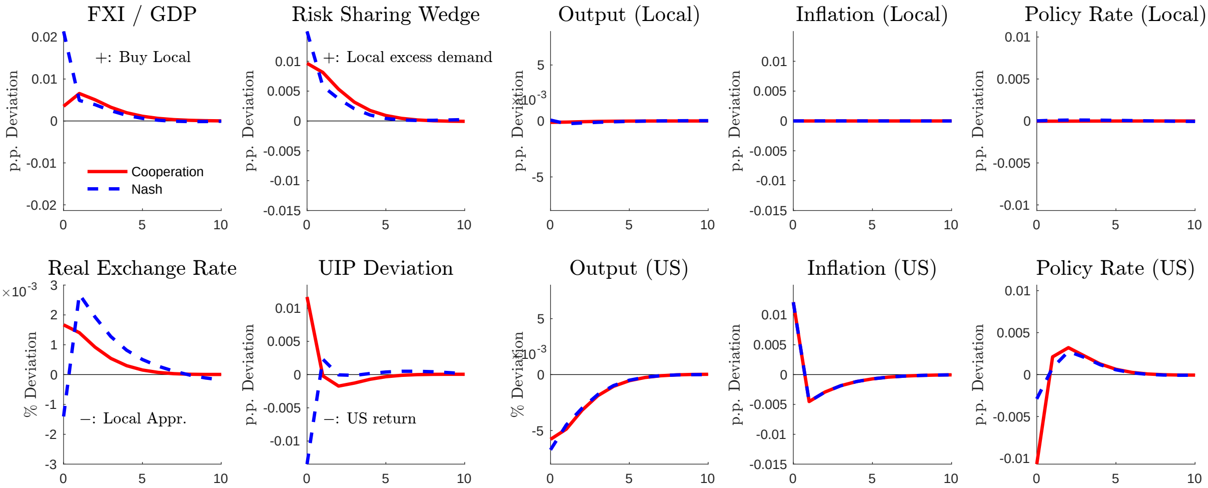
(b) Difference between Cooperation and Nash



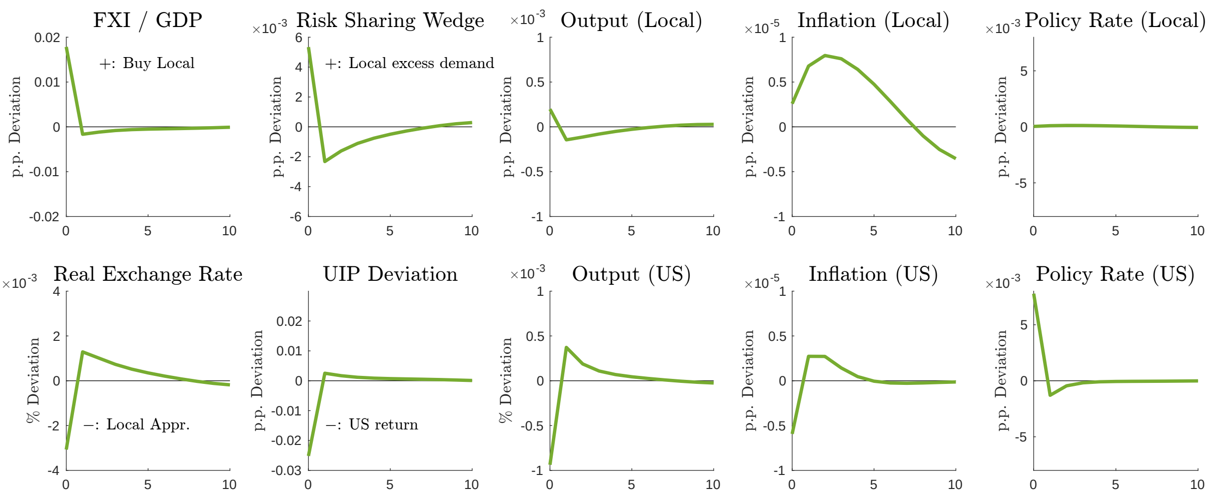
Note: The figure plots the impulse responses to a one-percentage increase in the US productivity under the cooperative equilibrium and the open-loop Nash equilibrium. Panel (a) plots the results under the cooperative equilibrium (red) and the Nash equilibrium (blue). Panel (b) plots the difference between the Nash and the cooperative equilibrium.

Figure 6: Impulse Response to a US Cost-Push Shock, Cooperation and Nash

(a) Cooperation and Nash



(b) Difference between Cooperation and Nash



Note: The figure plots the impulse responses to a one-percentage increase in the US markup under the cooperative equilibrium and the open-loop Nash equilibrium. Panel (a) plots the results under the cooperative equilibrium (red) and the Nash equilibrium (blue). Panel (b) plots the difference between the Nash and the cooperative equilibrium.

trade-off of monetary policy by stabilizing domestic inflation and demand without large changes in policy interest rates. On the other hand, FXI distorts world inflation and demand since FXI affects international prices. This trade-off between internal and external objectives implies that monetary policy and FXI are not completely independent policy instruments, but a policy encompassing both instruments achieves the optimal outcome.

Furthermore, I find that the dominance of the dollar in international trade can be an important driver of capital flow management in international finance. Under dollar pricing, FXI is particularly effective at stabilizing local inflation with limited transmission effects on US inflation. Finally, I take the first step in incorporating FXI in a non-cooperative equilibrium. I find that FXI that targets nationally oriented objectives is a beggar-thy-neighbor policy since it further exacerbates international resource misallocation.

An important and challenging direction for future research is to develop an identification method of FXI and empirically estimate its effects. Fratzscher et al. (2019) compares different identification methods, including reaction function, propensity score matching, instrumental variables, and high-frequency approaches. Recent works exploit granular firm- or bank-level data and combine their balance sheet information with credit supply and employment (Gonzalez et al. 2021) or with stock prices (Rodnyansky et al. 2024).

Another important research agenda is understanding how to combine FXI with other capital account management policies, including capital control and macroprudential policy, in the context of IMF's integrated policy framework (Basu et al. 2020).

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Appendix

A Derivations and Proofs

A.1 Useful Equilibrium Relationships

This section provides equilibrium first-order relationships which are useful for proofs of propositions in Section 2. The derivation follows Corsetti et al. (2010; 2023).

I focus on the LCP case (the PCP case can be derived analogously by setting $\Delta_t = 0$). Let the variables with hat denote the deviation from the steady state. For simplicity, assume symmetry so that $\mathcal{E}_t P^*_{Lt}/P_{Lt} = \mathcal{E}_t P^*_{Ut}/P_{Ut} = \Delta_t$. By the definition of real exchange rate, it is expressed in terms of terms of trade and price dispersion:

$$e_t = \frac{\mathcal{E}_t P^*_t}{P_t} = \frac{\mathcal{E}_t \left[a(P^*_{Lt})^{1-\phi} + (1-a)(P^*_{Ut})^{1-\phi}\right]^{\frac{1}{1-\phi}}}{\left[a(P_{Lt})^{1-\phi} + (1-a)(P_{Ut})^{1-\phi}\right]^{\frac{1}{1-\phi}}}$$
$$= \frac{\left[a\left(\frac{\mathcal{E}_t P^*_{Lt}}{P_{Lt}}\right)^{1-\phi} + (1-a)\left(\frac{\mathcal{E}_t P^*_{Ut}}{P_{Lt}}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}}{\left[a + (1-a)\left(\frac{P_{Ut}}{P_{Lt}}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}}, \tag{A1}$$

where:

$$\frac{\mathcal{E}_t P^*_{Ut}}{P_{Lt}} = \frac{\mathcal{E}_t P^*_{Lt}}{P_{Lt}} \frac{P_{Ut}}{\mathcal{E}_t P^*_{Lt}} \frac{\mathcal{E}_t P^*_{Ut}}{P_{Ut}} = \Delta_t^2 \mathcal{T}_t, \tag{A2}$$
$$\frac{P_{Ut}}{P_{Lt}} = \frac{\mathcal{E}_t P^*_{Lt}}{P_{Lt}} \frac{P_{Ut}}{\mathcal{E}_t P^*_{Lt}} = \Delta_t \mathcal{T}_t. \tag{A3}$$

Log-linearizing Equation (A1), we obtain Equation (9).

Next, I approximate the aggregate demand. Under the assumption of symmetry, we have:

$$\hat{Y}_{Lt} + \hat{Y}_{Ut} = \hat{C}_t + \hat{C}^*_t = 0. \tag{A4}$$

Combining Equations (5) and (A4) gives:

$$\hat{Y}_{Lt} - \hat{C}_t = \hat{C}^*_t - \hat{Y}_{Ut} = \frac{1}{2}[\hat{Y}_{Lt} - \hat{Y}_{Ut} - \sigma^{-1}(\hat{Q}_t + \tilde{\mathcal{D}}_t)]. \tag{A5}$$

Substituting Equation (2) into the aggregate demand $Y_{Lt} = C_{Lt} + C^*_{Lt}$ for local good gives:

$$Y_{Lt} = \left(\frac{P_{Lt}}{P_t}\right)^{-\phi} [aC_t + (1-a)(e_t\Delta_t^{-1})^{\phi} C_t^*]. \tag{A6}$$

Log-linearizing the CPI (2) gives:

$$\hat{P}_t - \hat{P}_{Lt} = (1-a)(\hat{\mathcal{T}}_t + \tilde{\Delta}_t). \tag{A7}$$

Using Equation (A7), (A6) can be log-linearized as:

$$\hat{Y}_{Lt} - \hat{C}_t = (1-a)\sigma^{-1}[\sigma\phi(\hat{e}_t + \hat{\mathcal{T}}_t) - \hat{e}_t - \tilde{\mathcal{D}}_t]. \tag{A8}$$

Using Equations (9), (A8) can be rewritten as:

$$\hat{Y}_{Lt} - \hat{C}_t = (1-a)\sigma^{-1}[2a\sigma\phi(\hat{\mathcal{T}}_t + \tilde{\Delta}_t) - \hat{e}_t - \tilde{\mathcal{D}}_t]. \tag{A9}$$

Combining the two expressions (A5) and (A9) for the aggregate demand, the terms of trade can be expressed as in Equation (13).

A.2 Proof of Lemma 1

The proof follows the appendix of Itskhoki and Mukhin (2021). Differently from their paper, the central bank can use FXI in addition to monetary policy.

To begin with, I show the first equality of Equation (6), which describes the relationship between demand gap and UIP deviation. The Euler equations of local and US households are characterized in log-linearized form:

$$\tilde{r}_t = \sigma E_t\left[\tilde{C}_{t+1} - \tilde{C}_t\right], \tag{A10}$$

$$\tilde{r}^*_t = \sigma E_t\left[\tilde{C}^*_{t+1} - \tilde{C}^*_t\right] \tag{A11}$$

Taking the difference of Equations (A10) and (A11) and subtracting $\Delta\tilde{e}_{t+1} = \tilde{e}_{t+1} - \tilde{e}_t$ from both sides,

$$E_t\left[\sigma\left\{(\tilde{C}_{t+1} - \tilde{C}_t) - (\tilde{C}^*_{t+1} - \tilde{C}^*_t)\right\} - \Delta\tilde{e}_{t+1}\right] = \tilde{r}_t - \tilde{r}^*_t - E_t\Delta\tilde{e}_{t+1}.$$

Using the definition of demand gap (5), we obtain the first equality:

$$E_t \tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = \tilde{r}_t - \tilde{r}_t^* - E_t \Delta \tilde{e}_{t+1}. \tag{A12}$$

Next, I show the second equality of Equation (6), which describes the relationship between financial flows and UIP deviation. The maximization problem (3) of arbitrageurs can be rewritten as:

$$\max_{d_t^*} E_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \bar{R}_t^* (1 - e^{x_t^*}) \frac{D_t^*}{P_t^*} \right) \right\}, \tag{A13}$$

where $x_t^* = \tilde{r}_t^* - \tilde{r}_t - \Delta \tilde{e}_{t+1}$ is the nominal carry trade return. When the time period is short, x_t^* can be expressed as the normal diffusion process:

$$dX_t^* = x_t^* dt + \sigma_{et} dB_t,$$

where $x_t^* = \tilde{r}_t^* - \tilde{r}_t - \Delta \tilde{e}_{t+1}$ is the nominal carry trade return and B_t is a standard Brownian motion. Note that the excess return is equal in nominal and real terms when log-linearized:

$$\begin{aligned}
x_t^* &= \tilde{r}_t^* - \tilde{r}_t - \Delta \tilde{e}_{t+1} \\
&= (\tilde{R}_t - E_t \pi_{t+1}) - (\tilde{R}_t^* - E_t \pi_{t+1}^*) - E_t (\Delta \tilde{e}_{t+1} + \pi_{t+1}^* - \pi_{t+1}) \\
&= \tilde{R}_t^* - \tilde{R}_t - \Delta \tilde{e}_{t+1}.
\end{aligned}$$

The maximization problem (A13) can be rewritten as:

$$\max_{D_t^*} E_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \bar{R}_t^* (1 - e^{dX_t^*}) \frac{D_t^*}{P_t^*} \right) \right\}. \tag{A14}$$

Using Ito's lemma, the objective function can be rewritten as:

$$\begin{aligned}
&E_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \bar{R}_t^* \left(-dX_t^* - \frac{1}{2} (dX_t^*)^2 \right) \frac{D_t^*}{P_t^*} \right) \right\} \\
&= -\frac{1}{\omega} \exp\left(\left[\omega \left(x_t^* + \frac{1}{2} \sigma_{et}^2 \right) \frac{D_t^*}{P_t^*} - \frac{1}{2} \omega^2 \sigma_{et}^2 \left(\frac{D_t^*}{P_t^*} \right)^2 \right] dt \right).
\end{aligned}$$

Solving the maximization problem, the optimal portfolio decision is:

$$\frac{D_t^*}{P_t^*} = -m_d \frac{\tilde{R}_t^* - \tilde{R}_t - \Delta\tilde{e}_{t+1} + \sigma_{et}^2}{\omega\sigma_{et}^2}. \tag{A15}$$

Substituting Equation (A15) and $U_t^* = m_u u_t^*$, $F_t^* = m_u f_t^*$ into the market clearing condition (4) for the dollar bond, we obtain:

$$\frac{B_t^*}{P_t^*} + \frac{1}{P_t^*} m_u u_t^* - m_d \frac{\tilde{R}_t^* - \tilde{R}_t - \Delta\tilde{e}_{t+1} + \sigma_{et}^2}{\omega\sigma_{et}^2} + \frac{1}{P_t^*} m_u f_t^* = 0. \tag{A16}$$

Since the arbitrageurs, noise traders, and central bank (FXI) takes zero net positions,

$$\frac{D_t + U_t + F_t}{R_t} = -\mathcal{E}_t \frac{D_t^* + U_t^* + F_t^*}{R_t^*}.$$

Using (4), we obtain $B_t/R_t + \mathcal{E}_t B_t^*/R_t^* = 0$. Substituting the zero net positions for households and central bank into Equation (A16) yields:

$$\frac{\tilde{R}_t^* - \tilde{R}_t - \Delta\tilde{e}_{t+1} + \sigma_{et}^2}{\omega\sigma_{et}^2/m_d} = \frac{1}{P_t^*} m_u u_t^* - \frac{R_t^*}{R_t}\frac{1}{e_t}\frac{1}{P_t^*} m_u f_t^* - \frac{R_t^*}{R_t}\frac{Y_t}{e_t}\frac{B_t}{P_t Y_t}. \tag{A17}$$

Log-linearizing this gives the second equality:

$$\tilde{r}_t - \tilde{r}_t^* - E_t\Delta\tilde{e}_{t+1} = \chi_1(u_t^* - f_t) - \chi_2 b_t. \tag{A18}$$

Combining Equations (A12) and (A18), we obtain the UIP equation (6). □

Incomes and Losses of Carry Trade Positions. For simplicity, I assume that the profits and losses of carry-trade positions by the financiers and noise traders and interventions by the local central bank are transferred to the local households in a lump-sum way. However, the assumption on the ownership structure does not affect the first-order dynamics of the model, as discussed in Itskhoki and Mukhin (2021). To see this, combining the positions of the financiers, the noise traders, and the local central bank, the total carry trade profit can be written as:

$$\overline{R}_t^*(D_t + U_t + F_t) = -\overline{R}_t^* B_t = \overline{R}_t^* \overline{Y} b_t.$$

The combined carry trade profit equals the product of the UIP deviation ($\overline{R}_t^*$) and the households' net foreign asset position $(\overline{Y}b_t)$. Each of them is first-order but their product is second-order and small enough relative to the size of the countries' budget constraint.

A.3 Proof of Lemma 2

The proof follows the appendix of Corsetti et al. (2023). Under cooperation and commitment, the central banks in the two countries minimize the loss (12) subject to the NKPCs (10) and (11) and the UIP condition (7). Let γ_{Lt}, γ^*_{Ut}, and λ_t be the Lagrange multipliers for the local and US NKPCs and the UIP condition, respectively.

The first-order conditions can be written as:

$$\begin{aligned}\tilde{Y}_{Lt}: \quad 0 =& -(\sigma+\eta)\tilde{Y}_{Lt} + \frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt}-\tilde{Y}_{Ut}) \\ & - \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}\tilde{\mathcal{D}}_t \\ & + \left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa\gamma_{Lt} + \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa\gamma^*_{Ut} \\ & - \frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t-\beta^{-1}\lambda_{t-1}),\end{aligned} \tag{A19}$$

$$\begin{aligned}\tilde{Y}_{Ut}: \quad 0 =& -(\sigma+\eta)\tilde{Y}_{Ut} - \frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt}-\tilde{Y}_{Ut}) \\ & + \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}\tilde{\mathcal{D}}_t \\ & + \left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa\gamma^*_{Ut} + \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa\gamma_{Lt} \\ & + \frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t-\beta^{-1}\lambda_{t-1}),\end{aligned} \tag{A20}$$

$$\pi_{Lt}: \quad 0 = -\frac{\theta}{\kappa}\pi_{Lt} - \gamma_{Lt} + \gamma_{Lt-1}, \tag{A21}$$

$$\pi^*_{Ut}: \quad 0 = -\frac{\theta}{\kappa}\pi^*_{Ut} - \gamma^*_{Ut} + \gamma^*_{Ut-1}, \tag{A22}$$

$$\begin{aligned}\mathcal{B}_t: \quad 0 =& \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}(E_t\tilde{\mathcal{D}}_{t+1}-\tilde{\mathcal{D}}_t) \\ & + (1-a)\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\kappa[(E_t\gamma_{Lt+1}-\gamma_{Lt})-(E_t\gamma^*_{Ut+1}-\gamma^*_{Ut})] \\ & - [(E_t\lambda_{t+1}-\beta^{-1}\lambda_t)-(\lambda_t-\beta^{-1}\lambda_{t-1})].\end{aligned} \tag{A23}$$

Under the assumption of $f_t = u^*_t = 0$, we have $E_t\tilde{\mathcal{D}}_{t+1} - \tilde{\mathcal{D}}_t = 0$. By taking the sum of the FOCs for the output gap in the two countries and combining it with the FOCs for inflation, we obtain:

$$
\begin{aligned}
0 &= \tilde{Y}_{Lt} + \tilde{Y}_{Ut} + \theta(p_{Lt} + p^*_{Ut}) \\
&= (\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}) + (\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}) + \theta(\pi_{Lt} + \pi^*_{Ut}).
\end{aligned} \tag{A24}
$$

Next, taking the difference of FOCs for the output gap,

$$
\begin{aligned}
0 = &\left[\sigma + \eta - \frac{4a(1-a)(\sigma\phi - 1)\sigma}{4a(1-a)(\sigma\phi-1)+1}\right](\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\
&+ \frac{4a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}\tilde{\mathcal{D}}_t \\
&+ \left[\sigma + \eta - \frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\theta(\tilde{p}_{Lt} - \tilde{p}^*_{Ut}) \\
&+ 2\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t - \beta^{-1}\lambda_{t-1}).
\end{aligned}
$$

The FOC for the net foreign asset implies:

$$
-(\lambda_t - \beta^{-1}\lambda_{t-1}) = (1-a)\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\theta(p_{Lt} - p^*_{Ut}).
$$

Combining the FOCs, we obtain the difference rule:

$$
\begin{aligned}
0 = &\left[\sigma + \eta - \frac{4a(1-a)(\sigma\phi - 1)\sigma}{4a(1-a)(\sigma\phi-1)+1}\right][(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}) - (\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}) + \theta(\pi_{Lt} - \pi^*_{Ut})] \\
&+ \frac{4a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1}).
\end{aligned}
$$

Combining the sum and difference rules, we obtain the country-specific monetary policy rules (14) and (15) (Lemma 2).

When $\sigma = \phi = 1$, it is possible to show that when $\sigma = \phi = 1$, the constrained optimal allocation under PCP satisfied $\tilde{\mathcal{D}}_t = 0$ (see Appendix 2.2.2 of Corsetti et al. (2023) for detailed derivation). The monetary policy rules reduce to:

$$
0 = \theta\pi_{Lt} + \left(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}\right), \tag{A25}
$$

$$
0 = \theta\pi^*_{Ut} + \left(\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}\right), \tag{A26}
$$

and the NKPCs with $\tilde{\mathcal{D}}_t = \tilde{\Delta}_t = 0$ reduce to:

$$\pi_{Lt} = \beta\pi_{Lt+1} + \kappa(\sigma + \eta)\tilde{Y}_{Lt}, \tag{A27}$$

$$\pi^*_{Ut} = \beta\pi^*_{Ut+1} + \kappa(\sigma + \eta)\tilde{Y}_{Ut}. \tag{A28}$$

Hence, the equilibrium is the first-best: $\pi_{Lt} = \pi^*_{Ut} = \tilde{Y}_{Lt} = \tilde{Y}_{Ut} = \tilde{\mathcal{D}}_t = 0$. □

A.4 Proof of Proposition 1

The central banks in the two countries face a similar minimization problem to the case without FXI (Appendix A.3), except that the local central bank chooses f_t optimally. The first-order condition for f_t implies:

$$\lambda_t = 0. \tag{A29}$$

First, combining Equation (A29) with Equations (7), (A19), (A20), (A21), and (A22), we obtain the difference rule:

$$\begin{aligned}
0 = &(\tilde{Y}_{Lt} - \tilde{Y}_{Lt-1}) - (\tilde{Y}_{Ut} - \tilde{Y}_{Ut-1}) + \theta(\pi_{Lt} - \pi^*_{Ut}) \\
&+ 2(1-a)\frac{2a(\sigma\phi - 1) + 1}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}}\frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}\theta(\pi_{Lt} - \pi^*_{Ut}) \\
&+ \frac{4a(1-a)\phi}{\sigma + \eta\{4a(1-a)(\sigma\phi - 1) + 1\}}\frac{2a(\sigma\phi - 1) + 1 - \sigma}{2a(\phi - 1) + 1}(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1}).
\end{aligned}$$

Combining this with the sum rule (A24), we obtain the country-specific monetary policy rules (20) and (21). Next, combining Equation (A29) with Equations (A21), (A22), and (A23), we obtain the optimal FXI rule (22). This proves Proposition 1.

In the special case where $\sigma = \phi = 1$, since $\psi_\pi = \psi_D = 0$, the optimal output gap and inflation are pinned down by Equations (A25) through (A28), which are the same as in the no FXI case. Hence, the optimal FXI is to set $f_t = u^*_t$ and the first-best equilibrium $\pi_{Lt} = \pi^*_{Ut} = \tilde{Y}_{Lt} = \tilde{Y}_{Ut} = \tilde{\mathcal{D}}_t = 0$ is achieved.

A.5 Proof of Lemma 4

When $f_t = u^*_t = 0$, the demand gap $\tilde{\mathcal{D}}_t$ is zero on average and at most second order. I first consider the US inflation and output gap. Combining the NKPC (11) for the US firms and the optimal monetary policy rule (15) and assuming the economy is initially at the steady state ($\tilde{Y}_{L,-1} = 0$):

$$\frac{\partial \pi^*_{U0}}{\partial \mu^*_0} = \frac{1}{1+\theta\kappa} > 0, \quad \frac{\partial \tilde{Y}_{U0}}{\partial \mu^*_0} = -\frac{\theta}{1+\theta\kappa} < 0.$$

in period 0 and:

$$\frac{\partial \pi^*_{U0}}{\partial \mu^*_0} = -\frac{\theta\kappa}{(1+\theta\kappa)^{t+1}} < 0, \quad \frac{\partial \tilde{Y}_{U0}}{\partial \mu^*_0} = -\frac{\theta}{(1+\theta\kappa)^{t+1}} < 0.$$

in period $t \geq 1$. This confirms Equation (17).

Next, I consider the transmission of the US cost-push shock to the real exchange rate and the local inflation and output gap. Using Equations 9 (with $\tilde{\Delta}_{Lt} = 0$), the elasticity of the terms-of-trade satisfies:

$$\frac{\partial \tilde{\mathcal{T}}_0}{\partial \mu^*_0} > \frac{\partial \tilde{\mathcal{T}}_1}{\partial \mu^*_0} > \cdots > 0.$$

Using the relationship (13) between the real exchange rate and the terms of trade, we obtain Equation (19). To simplify the proof, I consider the case where $1 + \theta\kappa$ is large enough so that $\mathcal{T}_0$ has a first-order effect on the local inflation while $\mathcal{T}_t$ ($t \geq 1$) does not. As shown in Equation (10) $\sigma\phi > 1$, an increase in $\mathcal{T}_t$ is analogous to a decrease in μ_t. Hence, Equation (18) can be proven similarly to Equation (17). □

A.6 Proof of Proposition 3

From Equations (17), (18), and (22), optimal FXI satisfies $\partial \tilde{f}_t / \partial \mu^*_0 > 0$ for all t when $\mu^*_0 > 0$ and $\mu^*_t = 0$ for all $t \geq 1$. From Equation (6),

$$\frac{\partial \widetilde{UIP}^{FXI}_t}{\partial \tilde{\mathcal{D}}_t} < 0, \quad \text{and} \quad \frac{\partial \tilde{\mathcal{D}}_t}{\partial f_t} > 0,$$

for a given value of $\tilde{\mathcal{D}}_{t+1}$. Since $\partial \tilde{Y}_{Lt} / \partial \mathcal{T}_t = 2a(\phi - 1) + 1$ and:

$$\frac{\partial \tilde{\mathcal{T}}_t}{\partial \tilde{\mathcal{D}}_t} = -\frac{2a-1}{4a(1-a)(\sigma\phi-1)+1} < 0, \tag{A30}$$

from Equation (13), we have:

$$\frac{\partial \tilde{Y}_{Lt}}{\partial \tilde{\mathcal{D}}_t} = \frac{\partial \tilde{Y}_{Lt}}{\partial \tilde{\mathcal{T}}_t}\frac{\partial \tilde{\mathcal{T}}_t}{\partial \tilde{\mathcal{D}}_t} = -\frac{(2a-1)[2a(\phi-1)+1]}{4a(1-a)(\sigma\phi-1)+1}$$

Hence, we have:

$$\frac{\partial \tilde{Y}_{Lt}}{\partial \mu_0^*} > \frac{\partial \tilde{Y}_{Lt}^{FXI}}{\partial \mu_0^*}.$$

Combining this result with the optimal policy rule (14), we obtain:

$$\frac{\partial \pi_{L0}}{\partial \mu_0^*} < \frac{\partial \pi_{L0}^{FXI}}{\partial \mu_0^*}, \quad \frac{\partial \pi_{Lt}}{\partial \mu_0^*} > \frac{\partial \pi_{Lt}^{FXI}}{\partial \mu_0^*}.$$

This proves Equation (18). Equation (17) can be proved analogously. Combining Equations (9) (with $\tilde{\Delta}_{Lt} = 0$) and (A30), we obtain:

$$\frac{\partial \tilde{e}_t}{\partial \mu_0^*} > \frac{\partial \tilde{e}_t^{FXI}}{\partial \mu_0^*}.$$

This proves Equation (26). □

A.7 Proof of Lemma 5

The proof follows (Corsetti et al. 2020; 2023). Under cooperation and commitment, the central banks in the two countries minimize the loss (29) subject to the NKPCs (27), (28), and (11), the UIP condition (7), and the condition that relates the relative price to the terms-of-trade and the LOOP deviation:

$$\pi_{Ut} - \pi_{Lt} = \tilde{\mathcal{T}}_t - \tilde{\mathcal{T}}_{t-1} + \Delta_{Lt} - \Delta_{Lt-1}.$$

Let γ_{Lt}, γ_{Lt}^*, and γ_{Ut}^* be the Lagrange multipliers for the local and US NKPCs, λ_t for the UIP condition, and γ_t for the terms-of-trade equation.

The first-order conditions can be written as:

$$\begin{aligned}\tilde{Y}_{Lt}: \quad 0 = &-(\sigma+\eta)\tilde{Y}_{Lt} + \frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\ &- \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t)\end{aligned}$$

$$
+\left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa(\gamma_{Lt}+\gamma^*_{Lt})+\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa\gamma^*_{Ut}
$$

$$
+\frac{1}{2a(\phi-1)+1}(\beta E_t\gamma_{t+1}-\gamma_t)-\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t-\beta^{-1}\lambda_{t-1}), \tag{A31}
$$

$$
\begin{aligned}
\tilde{Y}_{Ut}: \quad 0 =& -(\sigma+\eta)\tilde{Y}_{Ut}-\frac{2a(1-a)(\sigma\phi-1)\sigma}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt}-\tilde{Y}_{Ut})\\
&+\frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t)\\
&+\left[\sigma+\eta-\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\right]\kappa\gamma^*_{Ut}+\frac{(1-a)(\sigma-1)}{2a(\phi-1)+1}\kappa(\gamma_{Lt}+\gamma^*_{Lt})\\
&-\frac{1}{2a(\phi-1)+1}(\beta E_t\gamma_{t+1}-\gamma_t)+\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\lambda_t-\beta^{-1}\lambda_{t-1}),
\end{aligned} \tag{A32}
$$

$$
\pi_{Lt}: \quad 0=-\frac{\theta}{\kappa}\pi_{Lt}-\gamma_{Lt}+\gamma_{Lt-1}-\gamma_t, \tag{A33}
$$

$$
\pi^*_{Lt}: \quad 0=-\frac{\theta}{\kappa}\pi^*_{Lt}-\gamma^*_{Lt}+\gamma^*_{Lt-1}, \tag{A34}
$$

$$
\pi^*_{Ut}: \quad 0=-\frac{\theta}{\kappa}\pi^*_{Ut}-\gamma^*_{Ut}+\gamma^*_{Ut-1}, \tag{A35}
$$

$$
\begin{aligned}
\mathcal{B}_t: \quad 0 =& \frac{2a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}(E_t\tilde{\mathcal{D}}_{t+1}-\tilde{\mathcal{D}}_t)-\\
&+(1-a)\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\kappa\begin{bmatrix}(E_t\gamma_{Lt+1}-\gamma_{Lt})+(E_t\gamma_{Lt+1}-\gamma_{Lt})\\ -(E_t\gamma^*_{Ut+1}-\gamma^*_{Ut})\end{bmatrix}\\
&-[(E_t\lambda_{t+1}-\beta^{-1}\lambda_t)-(\lambda_t-\beta^{-1}\lambda_{t-1})]\\
&-\frac{2a-1}{4a(1-a)(\sigma\phi-1)+1}[(\beta E_t\gamma_{t+2}-E_t\gamma_{t+1})-(\beta E_t\gamma_{t+1}-\gamma_t)],
\end{aligned} \tag{A36}
$$

$$
\begin{aligned}
\tilde{\Delta}_{Lt}: \quad 0 =& -\frac{2a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t)+\kappa\frac{1}{4a(1-a)(\sigma\phi-1)+1}\\
&\times\frac{1}{2}\begin{bmatrix}(4a(1-a)(\sigma\phi-1)+1)(\gamma_{Lt}-(\gamma^*_{Lt}+\gamma^*_{Ut}))\\ -\left\{(2a-1)-2(1-a)[2a(\phi-1)+1]\dfrac{2a(1-a)(\sigma\phi-1)+1-\phi}{2a(\phi-1)+1}\right\}\\ \times(\gamma_{Lt}+\gamma^*_{Lt}-\gamma^*_{Ut})\end{bmatrix}\\
&-\frac{2a-1}{2a(\phi-1)+1}(\beta E_t\gamma_{t+1}-\gamma_t)\\
&-\frac{2a[2(1-a)(\sigma\phi-1)+1-\phi]}{2a(\phi-1)+1}(\lambda_t-\beta^{-1}\lambda_{t-1}).
\end{aligned} \tag{A37}
$$

Under the assumption of $f_t=u^*_t=0$, we have $E_t\tilde{\mathcal{D}}_{t+1}-\tilde{\mathcal{D}}_t=0$. The FOC for the net foreign asset implies:

$$\lambda_t - \beta^{-1}\lambda_{t-1} = (1-a)\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}\theta(\gamma_{Lt}+\gamma^*_{Lt}-\gamma^*_{Ut}) - \frac{2a-1}{4a(1-a)(\sigma\phi-1)+1}(\beta E_t\gamma_{t+1}-\gamma_t). \tag{A38}$$

The sum rule for the output gaps is given by Equation (A24). Using the symmetry (Equation (A4)), the sum rule can be rewritten as:

$$0 = \theta[a\pi_{Lt} + (1-a)\pi^*_{Lt} + \pi^*_{Ut}] + (\tilde{C}_t - \tilde{C}_{t-1}) + (\tilde{C}^*_t - \tilde{C}^*_{t-1}). \tag{A39}$$

To derive the difference rule, by taking the difference between Equations (A31) and (A32) and substituting Equation (A38), we obtain:

$$2\sigma(\beta E_t\gamma_{t+1}-\gamma_t) = \sigma[(\tilde{Y}_{Lt}-\tilde{Y}_{Ut})] + 4a(1-a)\phi\frac{2a(\sigma\phi-1)+1-\sigma}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t) - \sigma\kappa(\gamma_{Lt}+\gamma^*_{Lt}-\gamma^*_{Ut}). \tag{A40}$$

Moreover, substituting Equation (A38) into Equation (A37) yields:

$$\frac{2a(\sigma\phi-1)+1}{4a(1-a)(\sigma\phi-1)+1}(\tilde{Y}_{Lt}-\tilde{Y}_{Ut}) = -\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t) + \kappa(\gamma_{Lt}-(\gamma^*_{Lt}+\gamma^*_{Ut})) - (2a-1)\kappa\frac{\gamma_{Lt}+\gamma^*_{Lt}-\gamma^*_{Ut}}{4a(1-a)(\sigma\phi-1)+1}. \tag{A41}$$

Combining Equations (A40) and (A41) and rearranging the terms, we obtain:

$$\frac{2a-1}{4a(1-a)(\sigma\phi-1)+1}\sigma(\tilde{Y}_{Lt}-\tilde{Y}_{Ut}) + \left[(2a-1)\frac{2a(\sigma\phi-1)+1-\sigma}{4a(1-a)(\sigma\phi-1)+1}+\sigma\right]\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t) = \sigma\kappa(\gamma_{Lt}-(\gamma_{Lt}+\gamma^*_{Ut})).$$

Using the relationships (9) and (13), the left-hand side can be rewritten as:

$$
\begin{aligned}
&(2a-1)\left[\tilde{\mathcal{T}}_t + \tilde{\Delta}_{Lt} + \frac{(2a-1)(\tilde{\mathcal{D}}_t + \tilde{\Delta}_{Lt})}{4a(1-a)(\sigma\phi-1)+1}\right] \\
&\quad + \left[(2a-1)\frac{2a(\sigma\phi-1)+1-\sigma}{4a(1-a)(\sigma\phi-1)+1} + \sigma\right]\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t + \tilde{\Delta}_{Lt}) \\
&= (\tilde{e}_t - \tilde{\Delta}_{Lt}) + (\tilde{\mathcal{D}}_t + \Delta_{Lt}) - \frac{4a(1-a)\sigma\phi}{4a(1-a)(\sigma\phi-1)+1}(\tilde{\mathcal{D}}_t + \Delta_{Lt}) \\
&\quad + \left[(2a-1)\frac{2a(\sigma\phi-1)+1-\sigma}{4a(1-a)(\sigma\phi-1)+1} + \sigma\right]\frac{4a(1-a)\phi}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t + \tilde{\Delta}_{Lt}) \\
&= \tilde{e}_t + \tilde{\mathcal{D}}_t + \frac{4a(1-a)\phi(\sigma-1)}{2a(\phi-1)+1}(\tilde{\mathcal{D}}_t + \tilde{\Delta}_{Lt}).
\end{aligned}
$$

Using the FOCs for the inflation rates and rearranging the terms, we obtain the difference rule:

$$
\begin{aligned}
0 =& \theta[a\pi_{Lt} - (1-a)\pi^*_{Lt} - \pi^*_{Ut}] + (\tilde{C}_t - \tilde{C}_{t-1}) - (\tilde{C}^*_t - \tilde{C}^*_{t-1}) \\
&+ \frac{4a(1-a)\phi}{2a(\phi-1)+1}\frac{\sigma-1}{\sigma}(\tilde{\mathcal{D}}_t - \tilde{\mathcal{D}}_{t-1} + \tilde{\Delta}_{Lt} - \tilde{\Delta}_{Lt-1}).
\end{aligned} \tag{A42}
$$

Combining the sum and difference rules (Equations (A39) and (A42)), the country-specific monetary policy rules are given by Equations (30) and (31). □

A.8 Optimal Monetary Policy and FXI under DCP

This section provides a full characterization of optimal monetary policy and FXI rules under DCP and provides proofs of Propositions 4 and 5. Let $\gamma_{\Delta t} \equiv 2(\beta E_t \gamma_{t+1} - \gamma_t)$. First, combining the FOCs (A31) and (A32) for the output gap and (A29) for the FXI, the difference rule for the output gap can be written as:

$$
\begin{aligned}
\gamma_{\Delta t} =& \frac{\sigma}{4a(1-a)(\sigma\phi-1)+1}[2a(\phi-1)+1](\tilde{Y}_{Lt} - \tilde{Y}_{Ut}) \\
&+ \frac{4a(1-a)\phi}{4a(1-a)(\sigma\phi-1)+1}[2a(\sigma\phi-1)+1-\sigma](\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) \\
&+ [\sigma(2a(\phi-1)+1) - 2(1-a)(\sigma-1)]\theta[a\pi_{Lt} + (1-a)\pi^*_{Lt} - \pi^*_{Ut}].
\end{aligned} \tag{A43}
$$

Next, from the FOC (A37) for the LOOP deviation and (A29) for the FXI:

$$
\gamma_{\Delta t} = -\frac{4a(1-a)\phi}{2a-1}(\tilde{\Delta}_{Lt} + \tilde{\mathcal{D}}_t) + \theta\frac{1}{4a(1-a)(\sigma\phi-1)+1}\frac{2a(\phi-1)+1}{2a-1}
$$

$$\times\left[\begin{array}{l}(4a(1-a)(\sigma\phi-1)+1)[a\pi_{Lt}-((1-a)\pi^*_{Lt}+\pi^*_{Ut})] \\ -\left\{(2a-1)-2(1-a)[2a(\phi-1)+1]\dfrac{2a(1-a)(\sigma\phi-1)+1-\phi}{2a(\phi-1)+1}\right\} \\ \times[a\pi_{Lt}+(1-a)\pi^*_{Lt}+\pi^*_{Ut}]\end{array}\right] \tag{A44}$$

The optimal monetary policy rules can be characterized by the sum rule (A24), the difference rule (A43), and the optimal LOOP deviation (A44). The general implication is that the monetary policy cannot close all gaps but instead, it faces a trade-off between stabilizing inflation, output, demand gap, and LOOP deviation.

To derive the optimal FXI rule, using the FOC (A36) and the UIP condition (7):

$$\begin{aligned} f_t =& u^*_t + \frac{\theta}{2a\phi\chi_1}[2a(\sigma\phi-1)+1]E_t[a\pi_{Lt+1}+(1-a)\pi^*_{Lt+1}+\pi^*_{Ut+1}] \\ &+\frac{2a-1}{4a(1-a)\phi\chi_1}(E_t\gamma_{\Delta t+t}-\gamma_{\Delta t}), \end{aligned} \tag{A45}$$

where $\gamma_{\Delta t}$ is given in Equation (A44).

Under Cole and Obstfeld (1991) case, the above conditions reduce to:

$$\begin{aligned} 0 =& (\tilde{Y}_{Lt}+\tilde{Y}_{Ut})+\theta[a\pi_{Lt}+(1-a)\pi^*_{Lt}+\pi^*_{Ut}], \\ \gamma_{\Delta_{Lt}}-\gamma_{\Delta_{Lt-1}} =& (\tilde{Y}_{Lt}-\tilde{Y}_{Ut})+\theta[a\pi_{Lt}+(1-a)\pi^*_{Lt}-\pi^*_{Ut}], \\ \gamma_{\Delta_{Lt}} =& -\frac{4a(1-a)}{2a-1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t)+\theta\frac{1}{2a-1} \\ & \times[a\pi_{Lt}-((1-a)\pi^*_{Lt}+\pi^*_{Ut})]-(2a-1)[a\pi_{Lt}+(1-a)\pi^*_{Lt}+\pi^*_{Ut}] \\ f_t =& u^*_t+\frac{\theta}{2a\chi_1}E_t[a\pi_{Lt+1}+(1-a)\pi^*_{Lt+1}+\pi^*_{Ut+1}] \\ &+\frac{2a-1}{2a(1-a)\chi_1}(E_t\gamma_{\Delta t+t}-\gamma_{\Delta t}). \end{aligned}$$

Combining these equations, optimal monetary policy and FXI rules are characterized by:

$$0 = \tilde{Y}_{Lt}+\theta[a\pi_{Lt}+(1-a)\pi^*_{Lt}], \tag{A46}$$

$$0 = \tilde{Y}_{Ut}+\theta\pi^*_{Ut}+\gamma_{\Delta_{Lt}}-\gamma_{\Delta_{Lt-1}}, \tag{A47}$$

$$\begin{aligned} \gamma_{\Delta_{Lt}} =& -\frac{4a(1-a)}{2a-1}(\tilde{\Delta}_{Lt}+\tilde{\mathcal{D}}_t)+\theta\frac{1}{2a-1} \\ & \times[a\pi_{Lt}-((1-a)\pi^*_{Lt}+\pi^*_{Ut})]-(2a-1)[a\pi_{Lt}+(1-a)\pi^*_{Lt}+\pi^*_{Ut}] \end{aligned} \tag{A48}$$

$$f_t = u^*_t+\frac{\theta}{2(1-a)\chi_1}E_t[a\pi_{Lt+1}+(1-a)\pi^*_{Lt+1}+\pi^*_{Ut+1}]$$

$$+\frac{2a-1}{2a(1-a)\chi_1}[(E_t\tilde{Y}_{Lt+1}-\tilde{Y}_{Lt})-(E_t\tilde{Y}_{Ut+1}-\tilde{Y}_{Ut})]. \tag{A49}$$

There are two key implications. First, the optimal monetary policy rule is asymmetric. The local central bank trades off inflation and output growth of locally produced goods. However, the US central bank trades off the US inflation and output growth, as well as LOOP deviation and demand gap. Second, and more importantly, the optimal FXI targets the LOOP deviation, as discussed in the next proposition.

A.9 Proof of Proposition 4

From Equation (A48), $\partial\gamma_{\Delta t}/\partial\tilde{\Delta}_{Lt}<0$. Hence,

$$\frac{\partial f_t}{\partial\tilde{\Delta}_{Lt}}=\frac{\partial f_t}{\partial\gamma_{\Delta t}}\frac{\partial\gamma_{\Delta t}}{\partial\tilde{\Delta}_{Lt}}>0.$$

Thus, the optimal FXI is increasing in Δ_{Lt}. Next, similarly to the PCP case,

$$\frac{\partial\tilde{e}_0}{\partial\mu_0^*}>\frac{\partial\tilde{e}_1}{\partial\mu_0^*}>\cdots>0,\quad\frac{\partial\tilde{e}_t}{\partial\mu_0^*}>\frac{\partial\tilde{e}_t^{FXI}}{\partial\mu_0^*}.$$

Since $\tilde{e}_t$ is close to $\tilde{e}_t$ when the price stickiness is sufficiently high,

$$\frac{\partial\tilde{\Delta}_{L0}}{\partial\mu_0^*}>\frac{\partial\tilde{\Delta}_{L1}}{\partial\mu_0^*}>\cdots>0,\quad\frac{\partial\tilde{\Delta}_{Lt}}{\partial\mu_0^*}>\frac{\partial\tilde{\Delta}_{Lt}^{FXI}}{\partial\mu_0^*}.$$

Hence, the FXI reduces the LOOP deviation. Finally, to show that the optimal FXI is larger under DCP than PCP, the optimal FXI rule under PCP and $\sigma=\phi=1$ is characterized by:

$$f_t=u_t^*+\frac{\theta}{2a\chi_1}E_t(\pi_{Lt+1}-\pi_{Ut+1}^*). \tag{A50}$$

I compare the optimal FXI rules (A50) under PCP and (A49) under DCP. First, for the output gap term in Equation (A49), when $\sigma=\phi=1$, since $\partial\tilde{\mathcal{D}}_t/\partial\tilde{\Delta}_{Lt}=0$, $\partial\tilde{Y}_{Lt}/\partial\tilde{\Delta}_{Lt}=(\partial\tilde{Y}_{Lt}/\partial\tilde{\mathcal{D}}_t)(\partial\tilde{\mathcal{D}}_t/\partial\tilde{\Delta}_{Lt})=0$. Similarly, $\partial\tilde{Y}_{Ut}/\partial\tilde{\Delta}_{Lt}=0$. Next, for the local inflation, from the NKPCs (27) and (28), $\partial\pi_{Lt}/\partial\tilde{\Delta}_{Lt}=0$ under PCP and $\partial(a\pi_{Lt}+(1-a)\pi_{Ut}^*)/\partial\tilde{\Delta}_{Lt}=-2\kappa(1-a)$, which is second-order when the home bias is large enough (large a). For the US inflation,

$\partial\pi^*_{Ut}/\partial\tilde{\Delta}_{Lt} = 0$ under both PCP and DCP. Hence, up to the first order and without FXI,

$$\left(\frac{\partial E_t(\pi_{Lt+1} - \pi^*_{Ut+1})}{\partial\mu^*_0}\right)^{PCP} \doteqdot \left(\frac{\partial E_t(a\pi_{Lt+1} + (1-a)\pi^*_{Lt+1} - \pi^*_{Ut+1})}{\partial\mu^*_0}\right)^{DCP} > 0.$$

The reaction coefficient to the inflation differential is larger under DCP than PCP: [23]

$$\frac{\theta}{2(1-a)\chi_1} > \frac{\theta}{2a\chi_1}.$$

Hence,

$$\left(\frac{\partial f_t}{\partial\mu^*_t}\right)^{DCP} > \left(\frac{\partial f_t}{\partial\mu^*_t}\right)^{PCP}.$$

□

A.10 Proof of Proposition 5

First, I consider the PCP case. Since $\partial f_t/\partial\mu^*_t > 0$ and $\partial\tilde{\mathcal{D}}_t/\partial f_t > 0$, I consider the elasticity of inflation to the demand gap. From the NKPCs for the domestic good inflation in the two countries,

$$\frac{\partial\pi_{Lt}}{\partial\tilde{\mathcal{D}}_t} = -\frac{\partial\pi^*_{Ut}}{\partial\tilde{\mathcal{D}}_t} = \kappa(1-a).$$

Hence, $\partial\pi_{Lt}/\partial\mu^*_t = -\partial\pi^*_{Ut}/\partial\mu^*_t$.

For the imported inflation, from the law of one price,

$$\pi^*_{Ut} = \tilde{e}_t - \tilde{e}_{t-1} + \pi^*_{Ut}, \quad \pi^*_{Lt} = -(\tilde{e}_t - \tilde{e}_{t-1}) + \pi_{Lt}. \tag{A51}$$

Hence,

$$\frac{\partial\pi_{Ut}}{\partial\tilde{\mathcal{D}}_t} = -\frac{\partial\pi^*_{Lt}}{\partial\tilde{\mathcal{D}}_t}, \quad \frac{\partial\pi_{Ut}}{\partial\mu^*_t} = -\frac{\partial\pi^*_{Lt}}{\partial\mu^*_t}$$

[23]The difference between inflation differential terms under PCP and DCP is quantitatively at most second-order. The difference in the optimal FXI volumes under PCP and DCP is mainly because the reaction coefficient to the inflation is larger under DCP, which is due to the deviation from the LOOP.

Hence, the response of CPI inflation is symmetric.

$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} = -\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} \quad (> 0).$$

Next, I consider the DCP case. Since the optimal FXI is larger under DCP than PCP (Equation (32)),

$$\left(\frac{\partial \tilde{e}_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \tilde{e}_t}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \tilde{e}_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \tilde{e}_t}{\partial \mu_t^*}\right)^{PCP} \quad (< 0),$$

For the local imports of US goods, since the LOOP holds,

$$\left(\frac{\partial \pi_{Ut}^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_{Ut}}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_{Ut}^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_{Ut}}{\partial \mu_t^*}\right)^{PCP} \quad (< 0),$$
$$\left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t}{\partial \mu_t^*}\right)^{PCP} \quad (< 0).$$

Next, consider the US imports of local goods. Combining Equations (9) and (13) and using $\partial \tilde{\Delta}_{Lt} / \partial \tilde{\mathcal{D}}_t = 0$ when $\sigma = \phi = 1$,

$$\frac{\partial \tilde{e}_t}{\partial \tilde{\mathcal{D}}_t} = -(2a-1)^2 < 0.$$

When the price stickiness is sufficiently high, $\partial \tilde{e}_t / \partial \tilde{\mathcal{D}}_t \fallingdotseq (2a-1)^2 < 0$.

Under PCP, since π_{Lt}^* is determined by the LOOP condition (A51),

$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{PCP} = -\frac{\partial \tilde{e}_t}{\partial \tilde{\mathcal{D}}_t} + \frac{\partial \tilde{\pi}_{Lt}^*}{\partial \tilde{\mathcal{D}}_t} \fallingdotseq (2a-1)^2 + \kappa(1-a).$$

Under DCP, since the LOOP does not hold and π_{Lt}^* is determined by the NKPC (28),

$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{DCP} = \kappa(1-a).$$

Comparing the PCP and DCP cases,

$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{DCP} - \left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mathcal{D}}_t}\right)^{PCP} = -(2a-1)^2,$$
$$\left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mu}_t^*}\right)^{DCP} - \left(\frac{\partial \pi_{Lt}^*}{\partial \tilde{\mu}_t^*}\right)^{PCP} < 0.$$

The difference is first order when a is sufficiently large. Hence,

$$\left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{DCP} < \left(\frac{\partial \pi_t^{*FXI}}{\partial \mu_t^*} - \frac{\partial \pi_t^*}{\partial \mu_t^*}\right)^{PCP} \quad (> 0).$$

$\square$