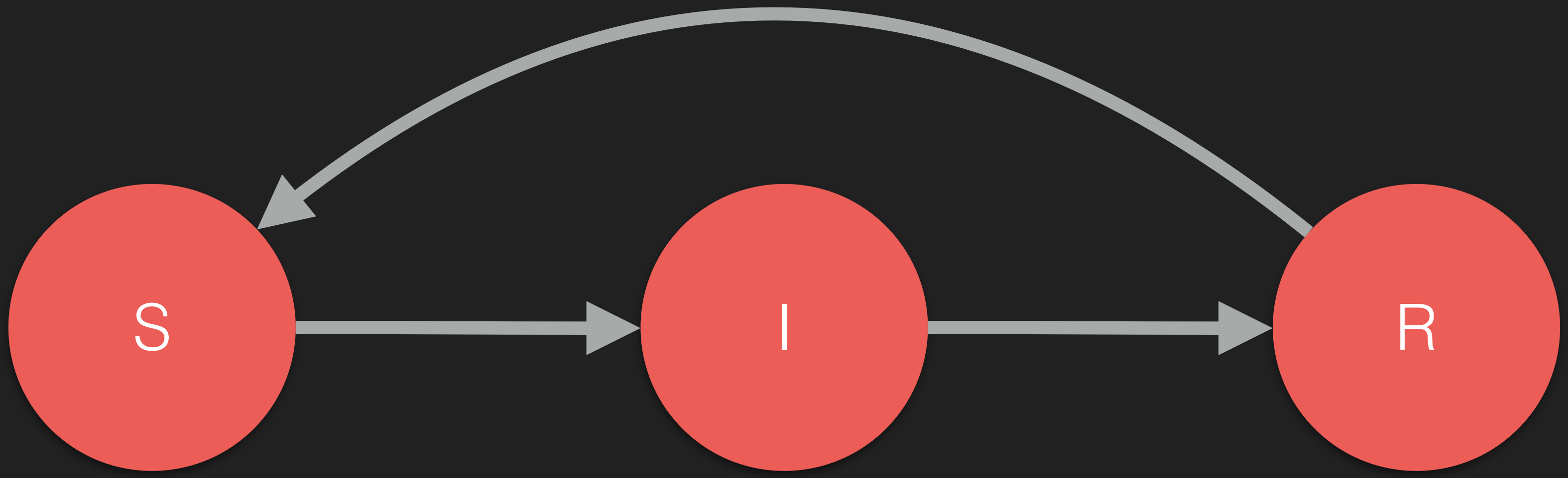
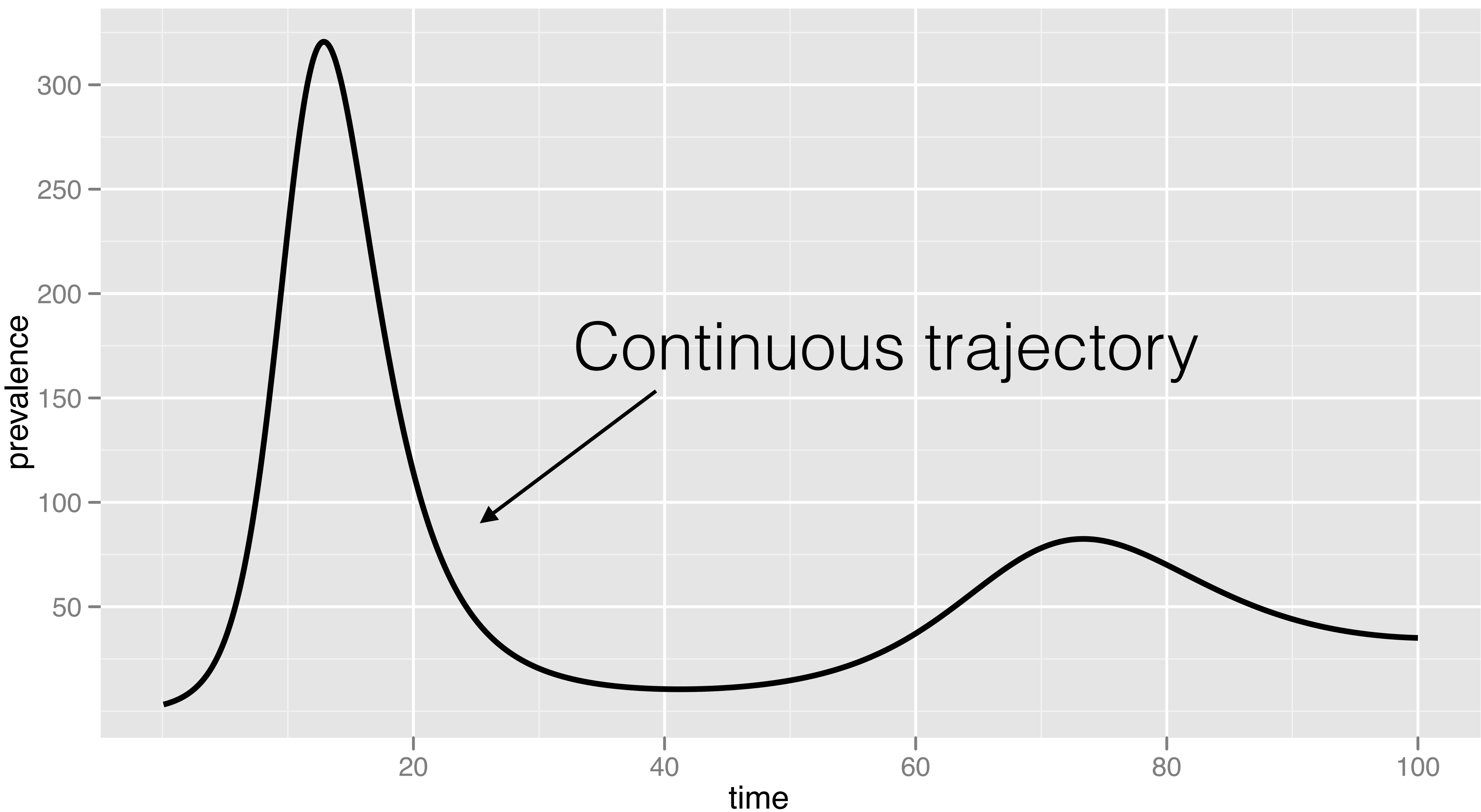
Deterministic models



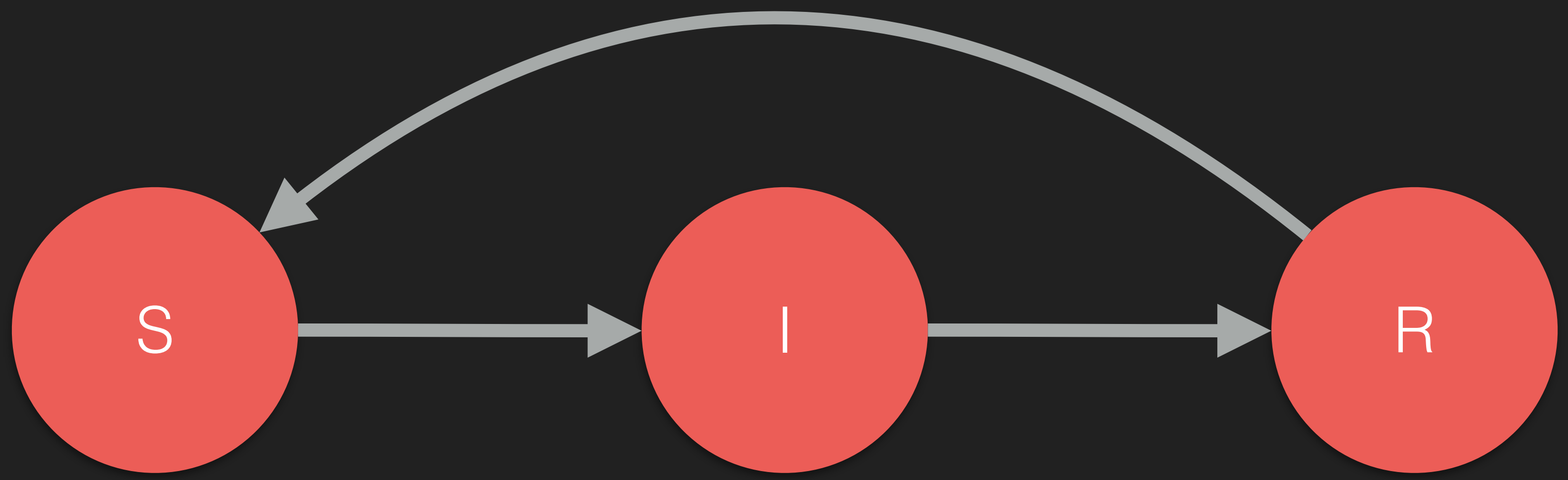
$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{\beta}{N}SI + \gamma(N - S - I)$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\beta}{N}SI - \nu I$$

One $\boldsymbol{\Theta}$ = One trajectory

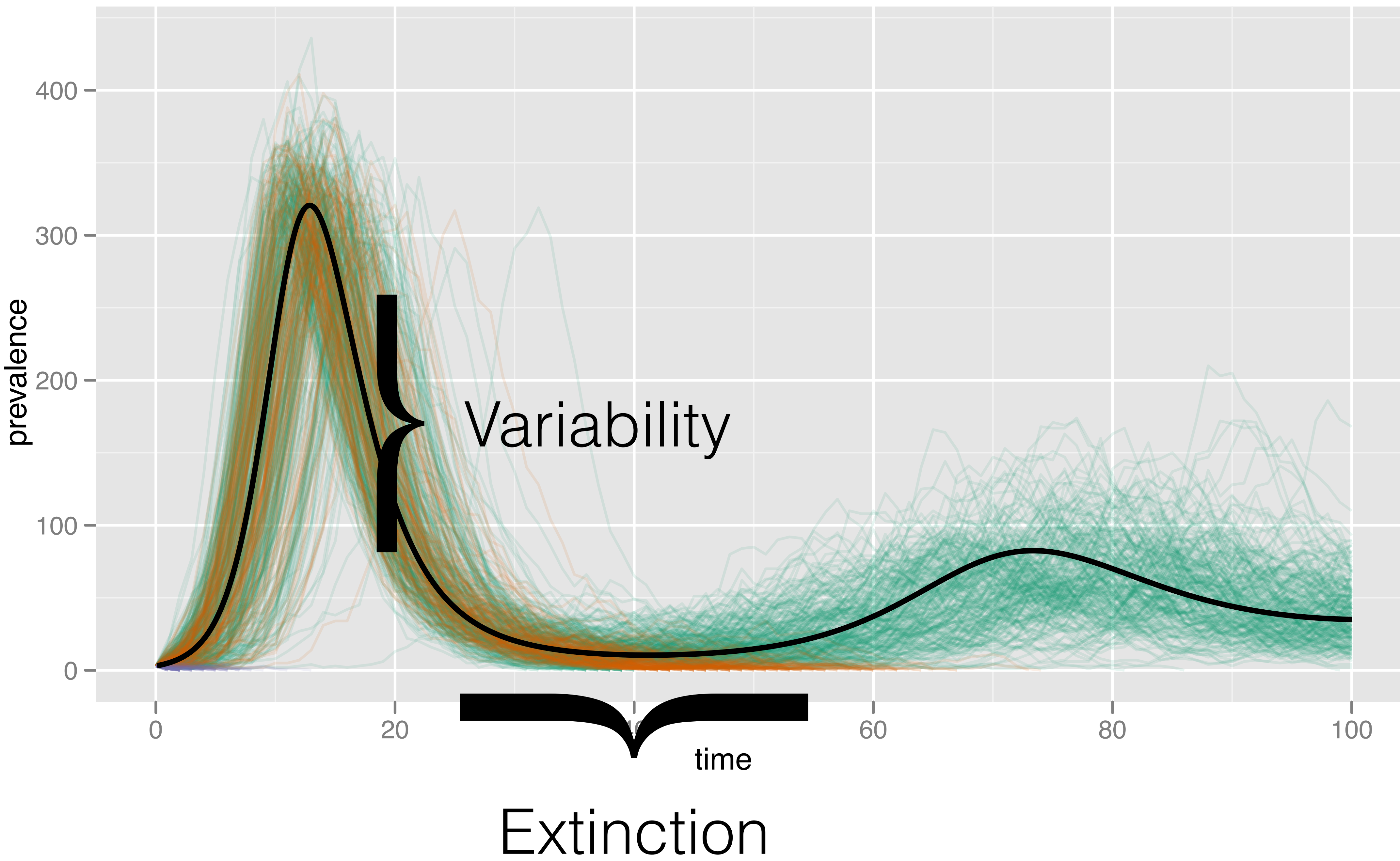


Life is discrete & stochastic



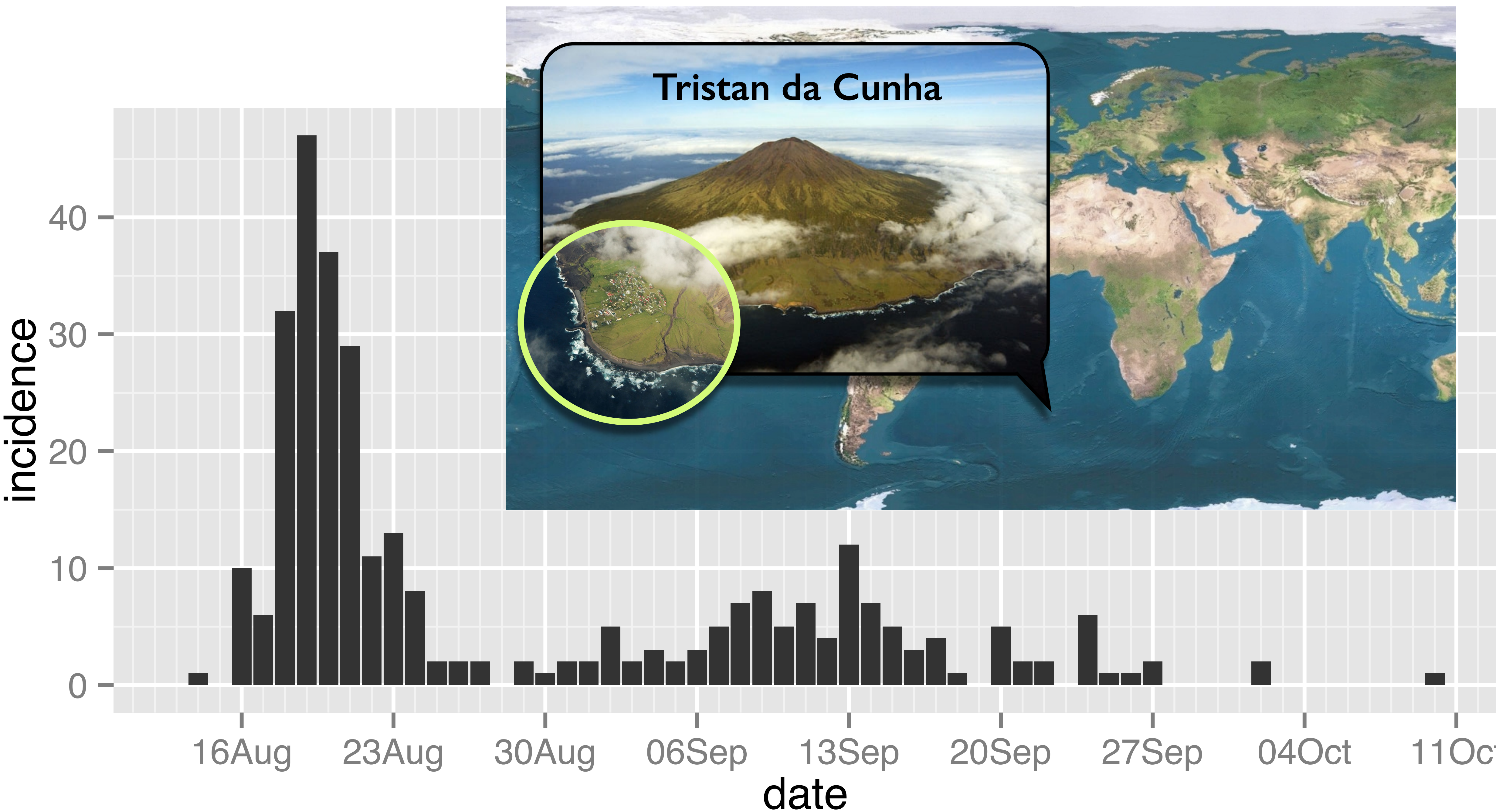
Event	Transition	Jump intensity
Infection	$(s,i) \to (s-1, i+1)$	$\beta si/N$
Recovery	$(s,i) \to (s, i-1)$	νi
Loss of immunity	$(s,i) \to (s+1, i)$	$\gamma(N-s-i)$

One Θ = Many trajectories

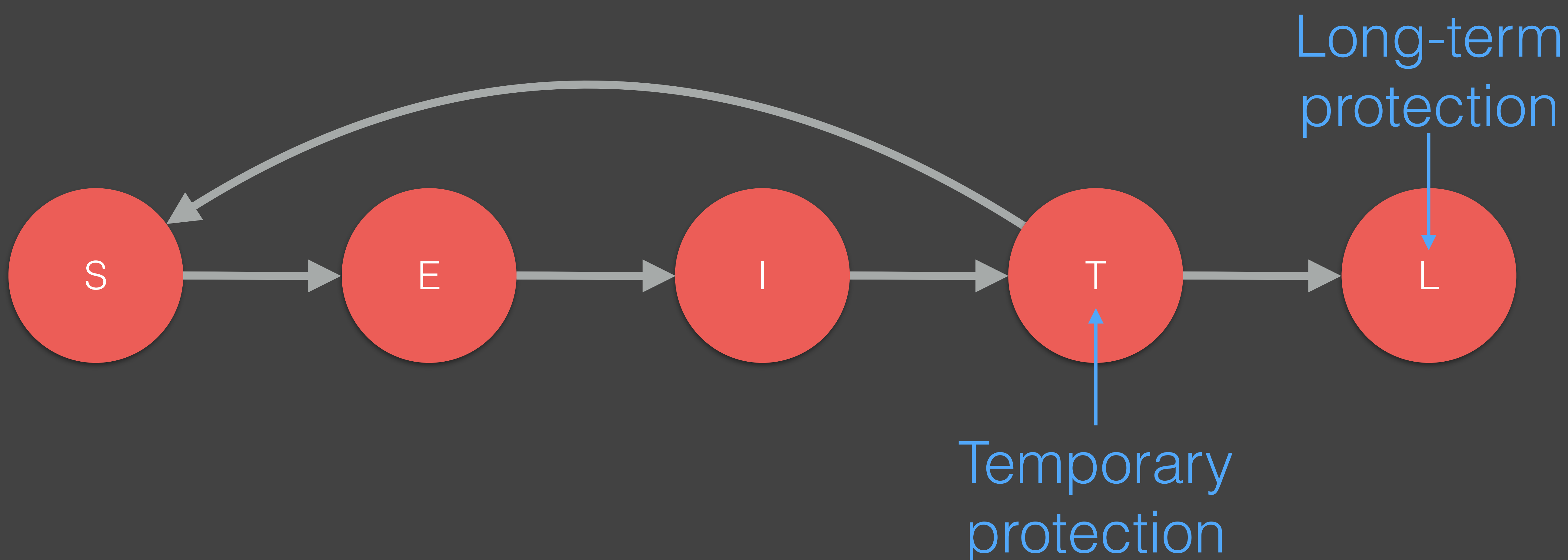


Inference for small population outbreaks

284 habs - 32% reinfected



One possible model...



Already implemented as a fitmodel so let's play!

Inference

Posterior

Prior

$$p(\theta|y) \propto p(y|\theta) \times p(\theta)$$

Marginal likelihood

Marginal likelihood

$$p(y|\theta) = \sum_X p(y|x, \theta) \times p(x|\theta)$$

All possible trajectories of the model

Deterministic case

$$p(y|\theta) = \sum_X p(y|x,\theta) \times 1_{x=f(\theta)}$$

Perfectly known

Deterministic case

$$p(y|\theta) = p(y|x = f(\theta), \theta) \times 1$$

ODE integration

That's what the function **`trajLogLike`** does.

Marginal likelihood

$$p(y|\theta) = \sum_{X} p(y|x,\theta) \times p(x|\theta)$$

All possible trajectories of the model

Stochastic case

$$p(y|\theta) = \sum_{X} p(y|x,\theta) \times p(x|\theta)$$

Can be billions!

No longer known

Stochastic case

Trajectory of particle j

$$p(y|\theta) \approx \sum_J p(y|x_j, \theta) \times p(x_j|\theta)$$

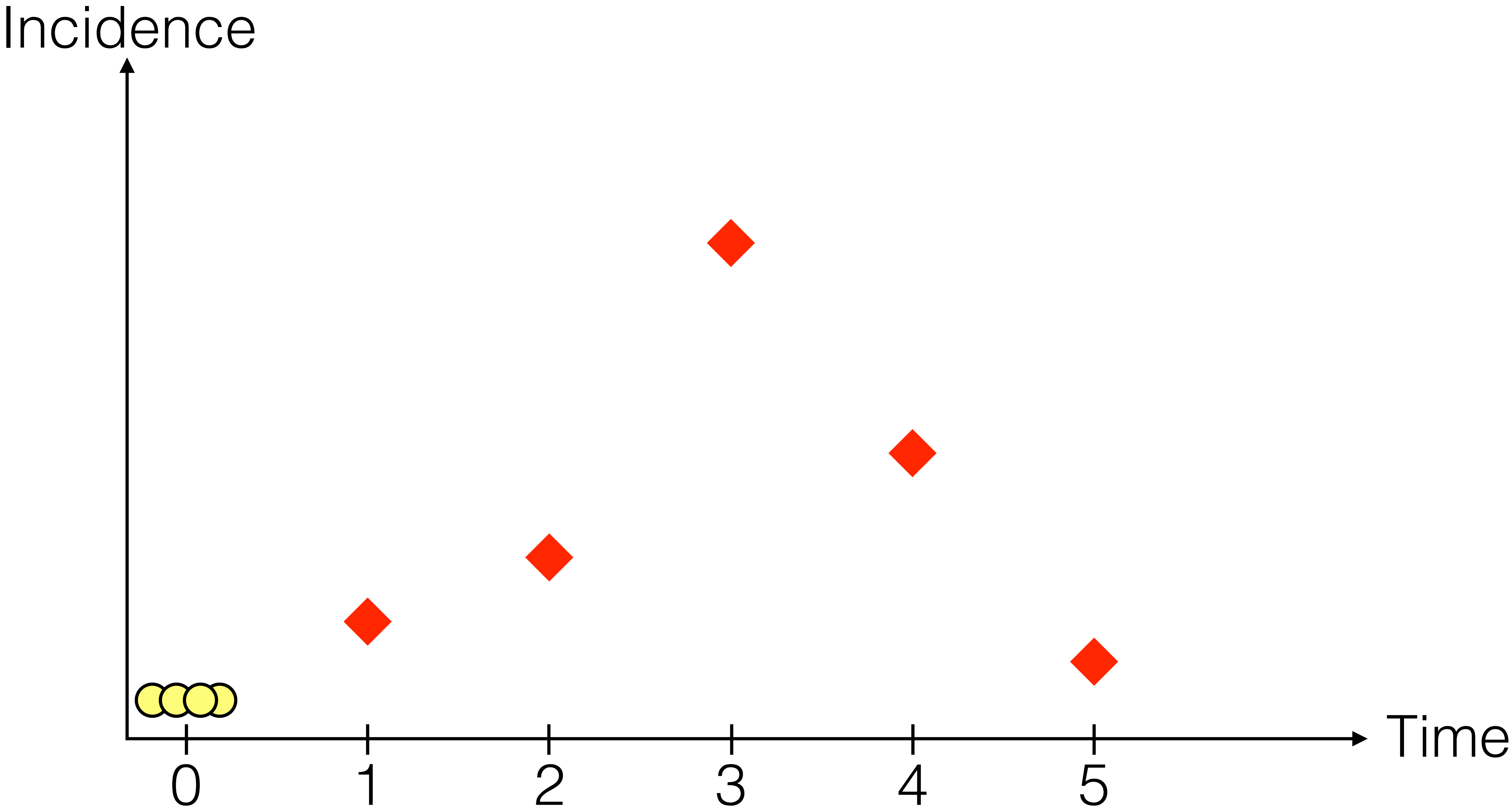
J particles

Monte-Carlo approximation

Sequential Monte-Carlo

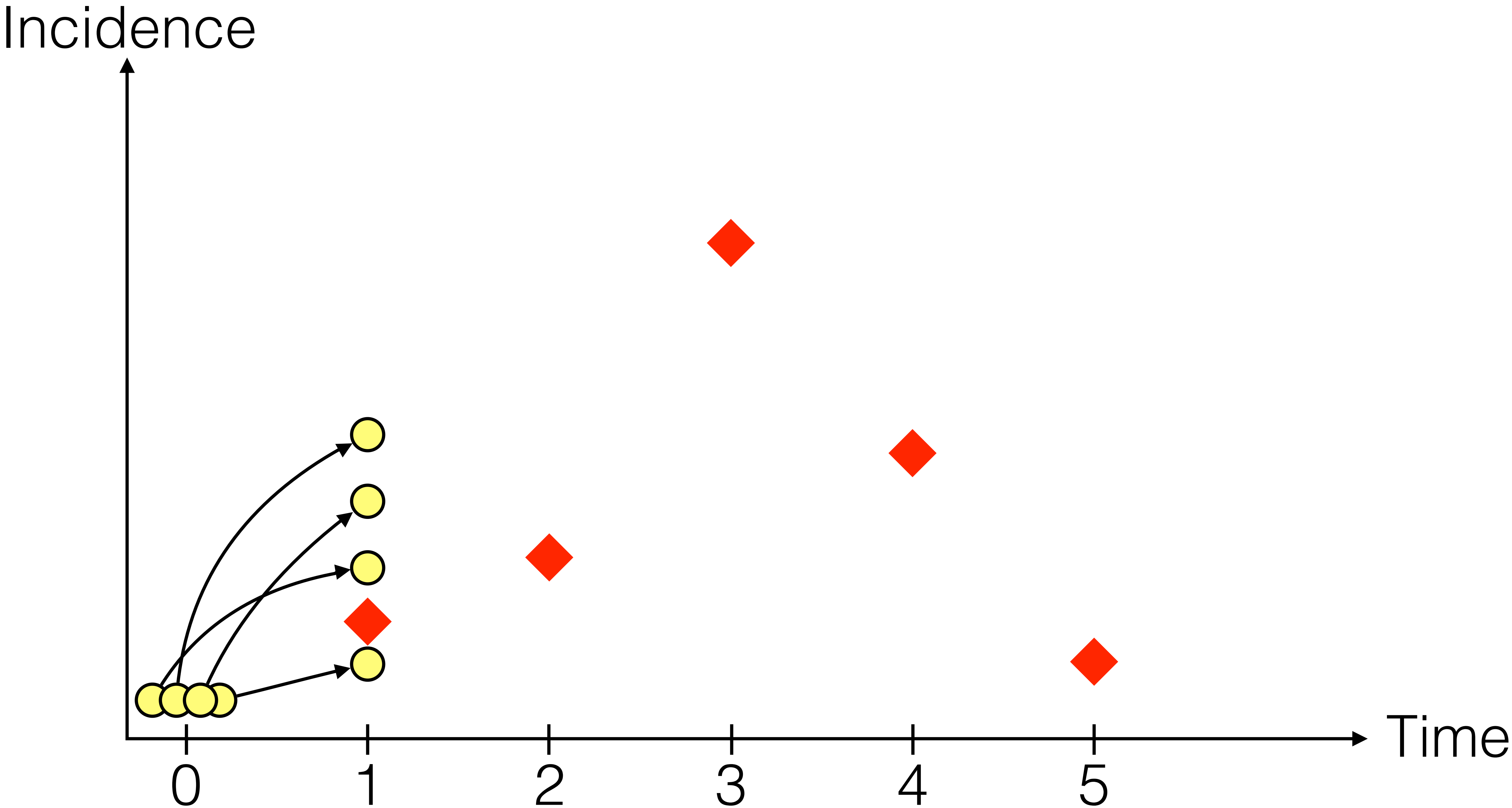
aka

Particle Filtering



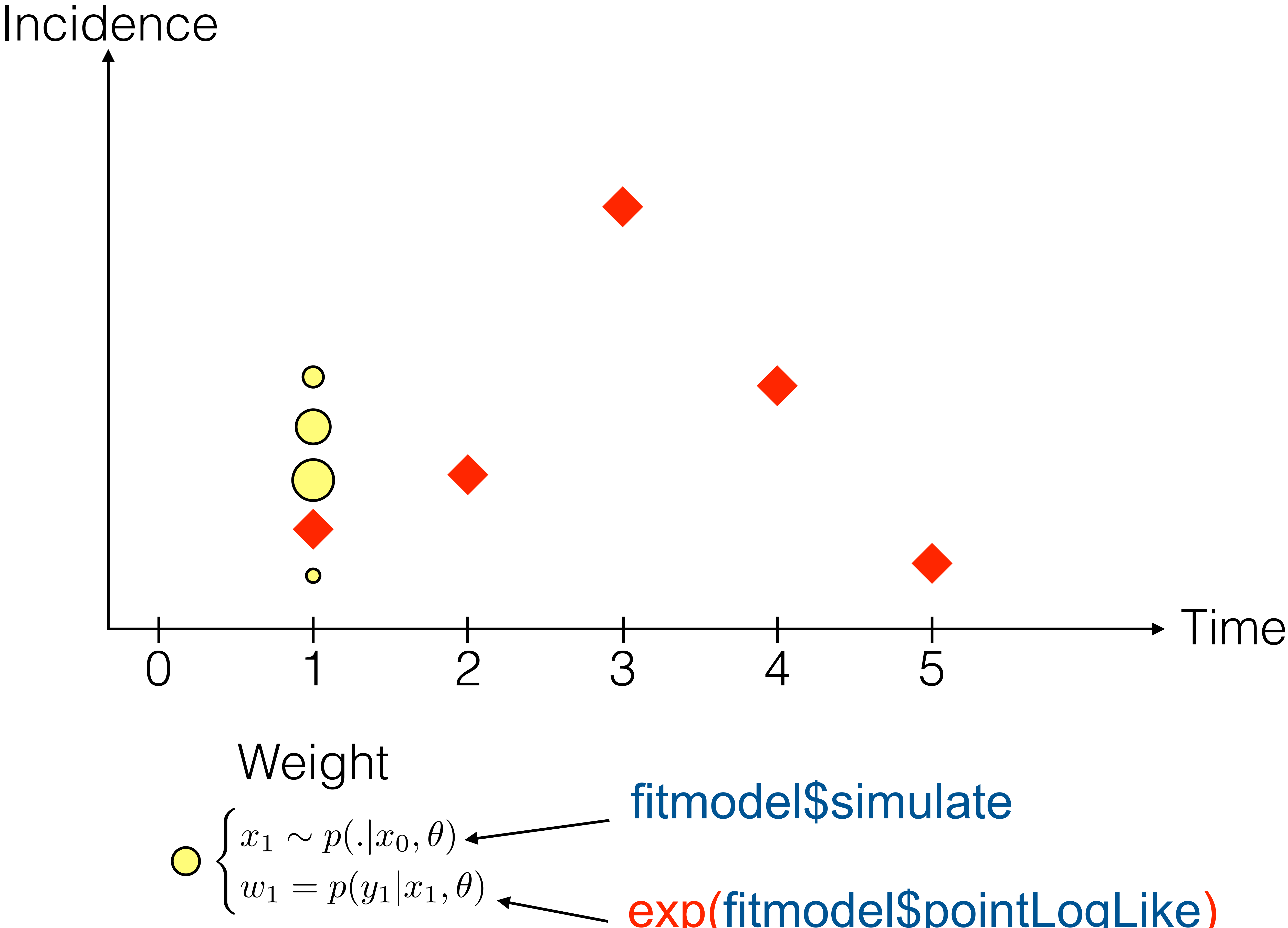
Initialise

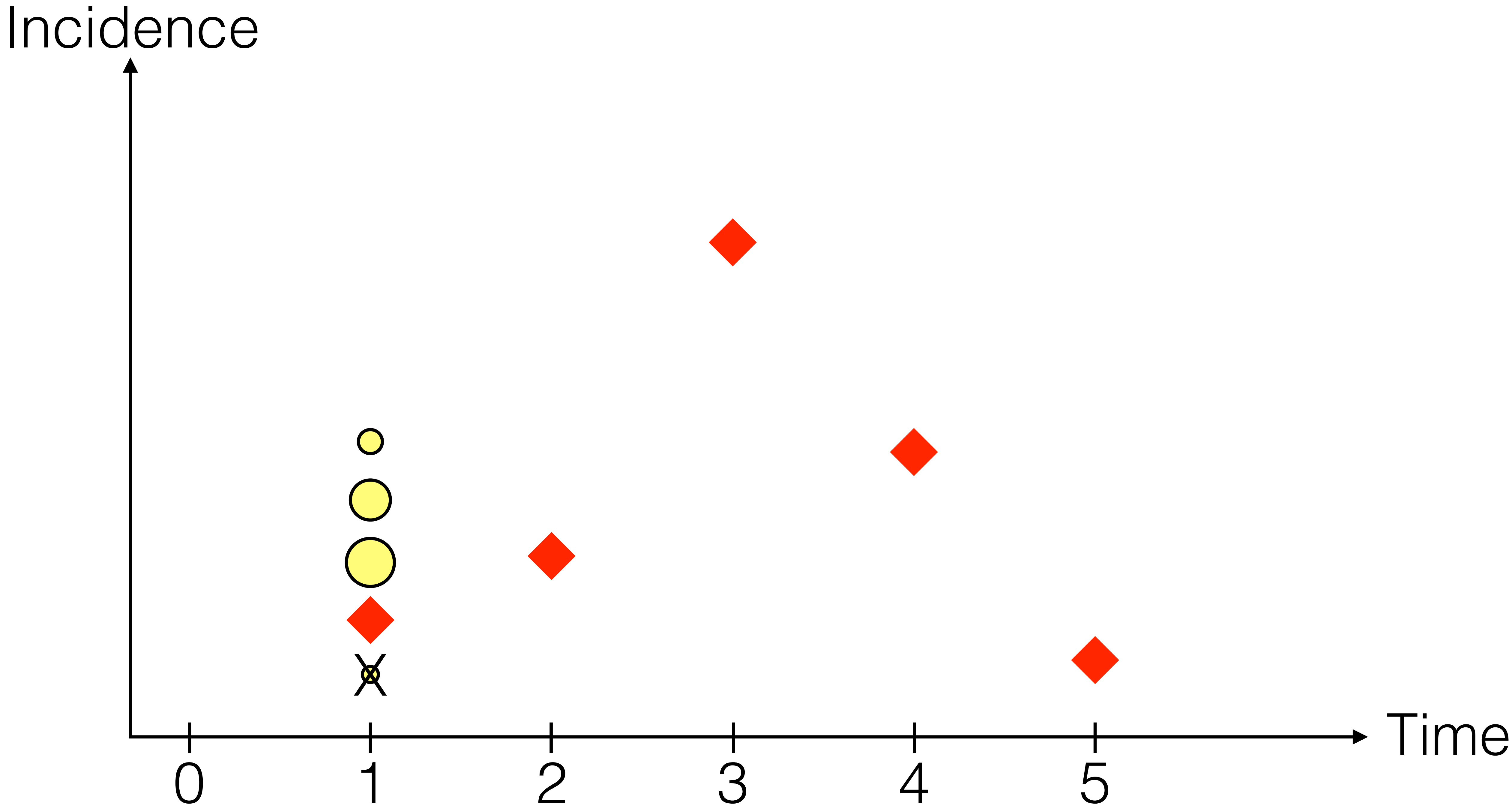
$$\begin{cases} x_0 \sim p(.|\theta) \\ w_0 = 1/J \end{cases}$$



Propagate

$$\begin{cases} x_1 \sim p(.|x_0, \theta) \\ \ldots \end{cases}$$

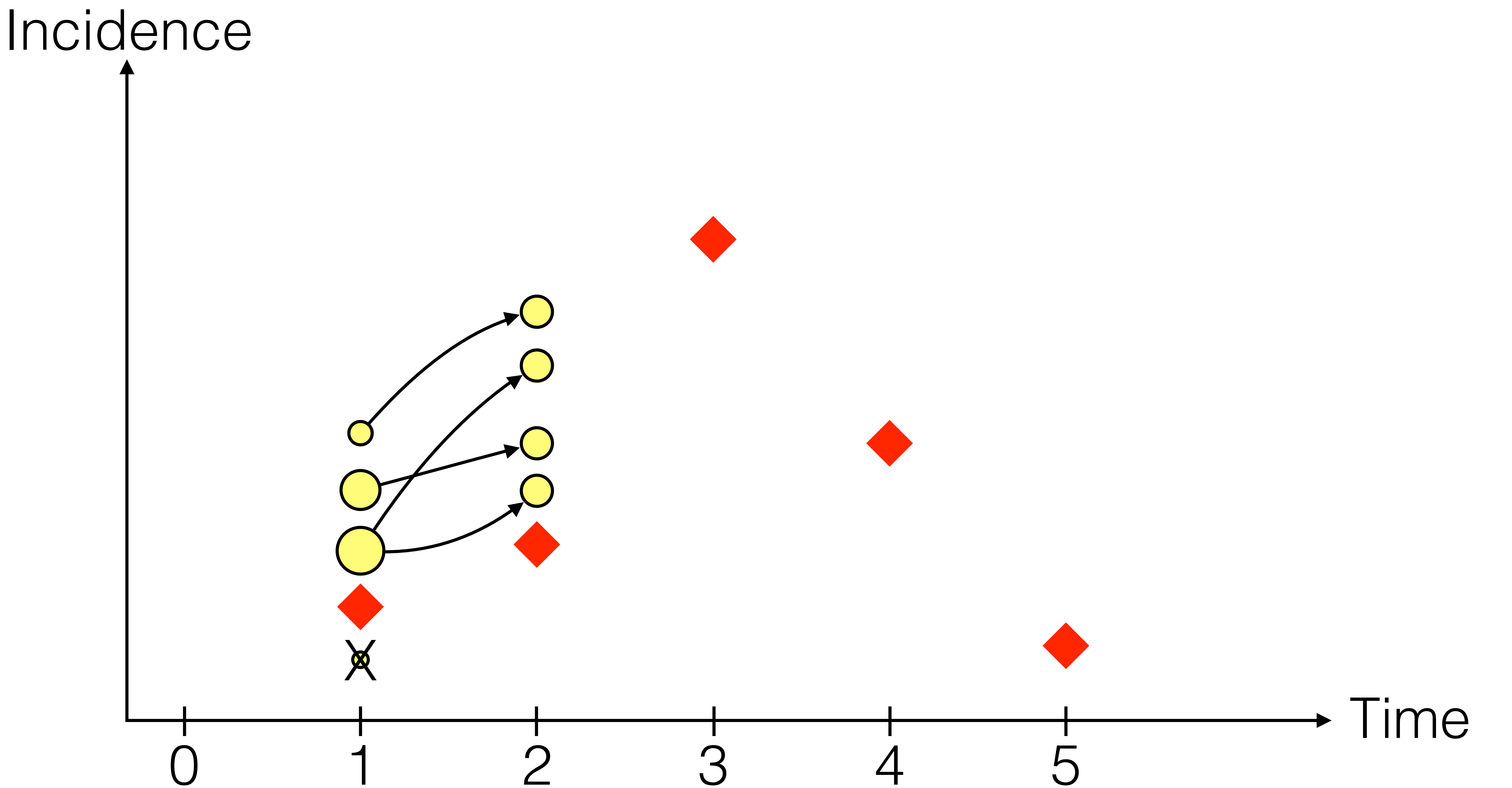

Incidence
Time
0
1
2
3
4
5
Weight
$x_1 \sim p(.|x_0, \theta)$
$w_1 = p(y_1|x_1, \theta)$
fitmodel$simulate
exp(fitmodel$pointLogLike)



Resample

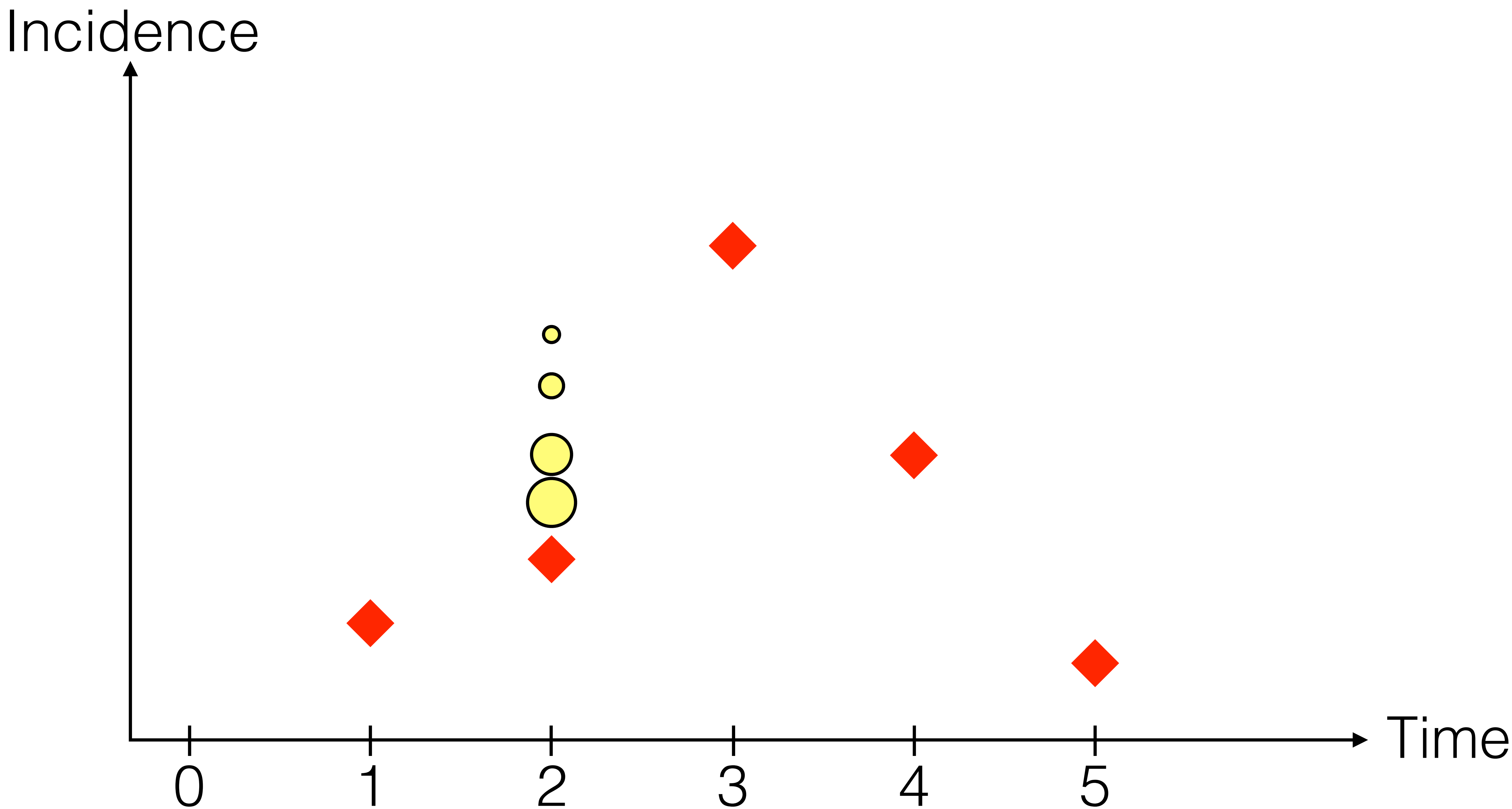
$\propto w_1$

Use the R function sample(...)



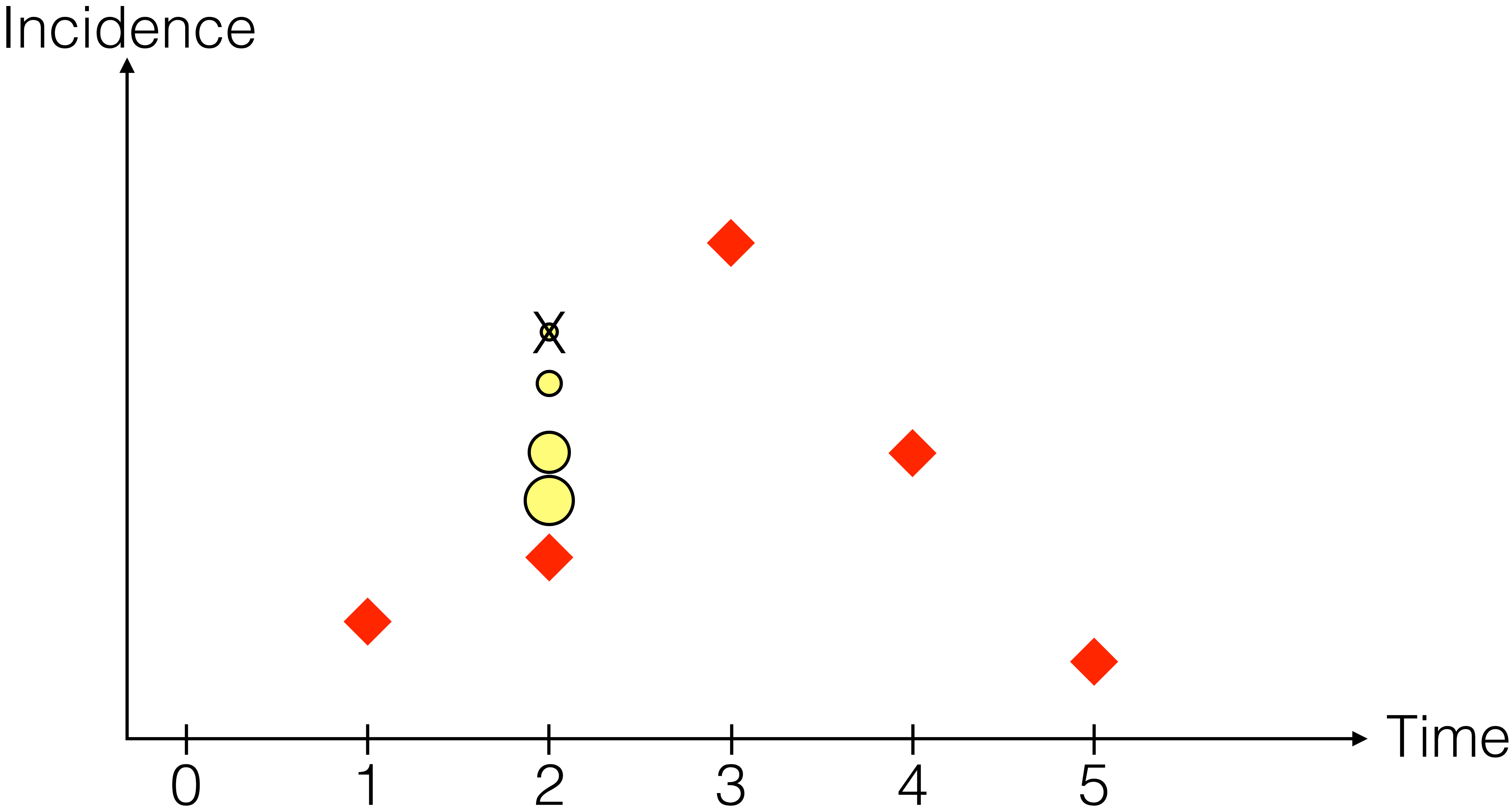
Propagate

$$\begin{cases} x_2 \sim p(.|x_1, \theta) \\ \dots \end{cases}$$



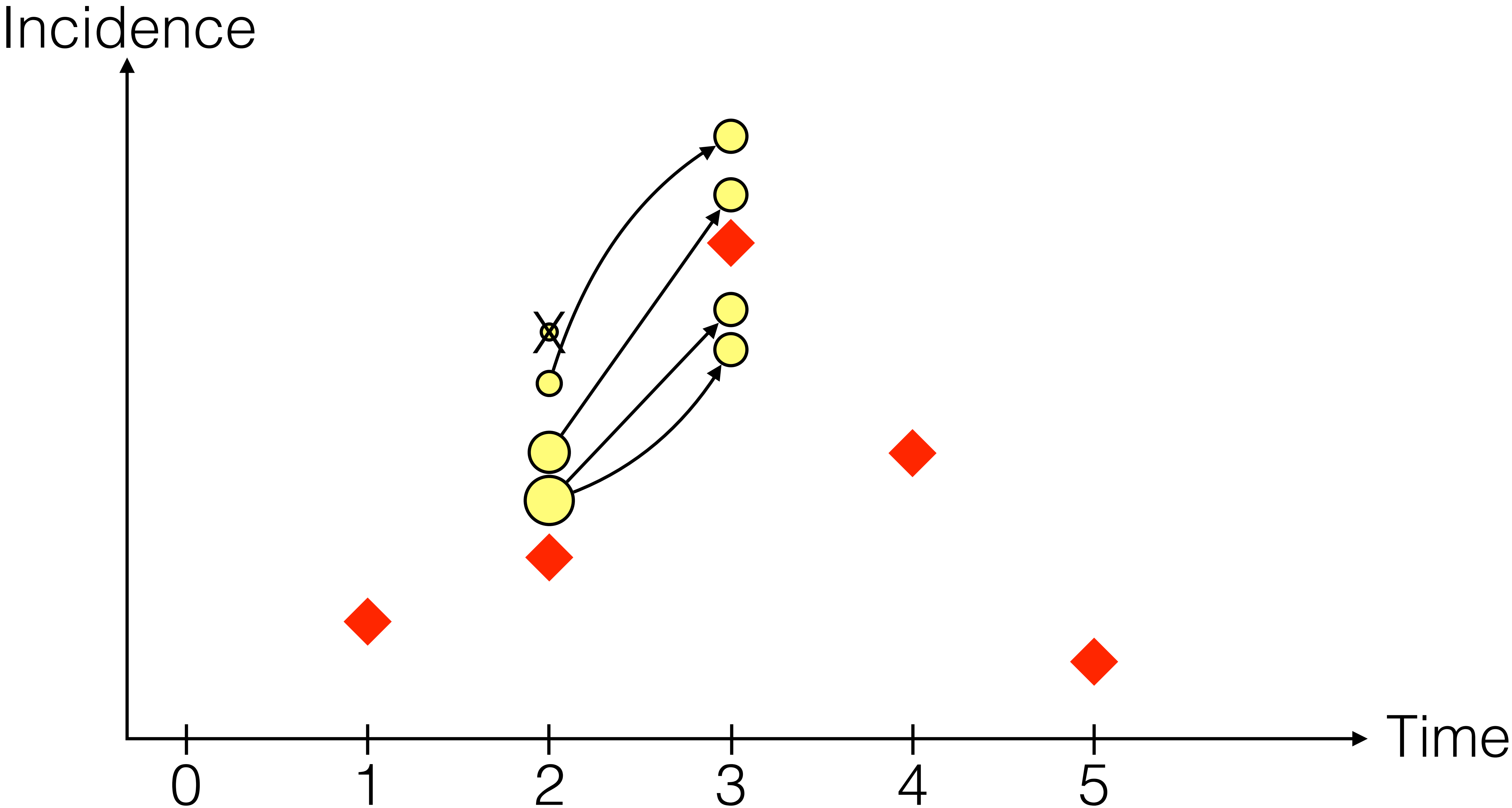
Weight

$$\begin{cases} x_2 \sim p(.|x_1, \theta) \\ w_2 = p(y_2|x_2, \theta) \end{cases}$$



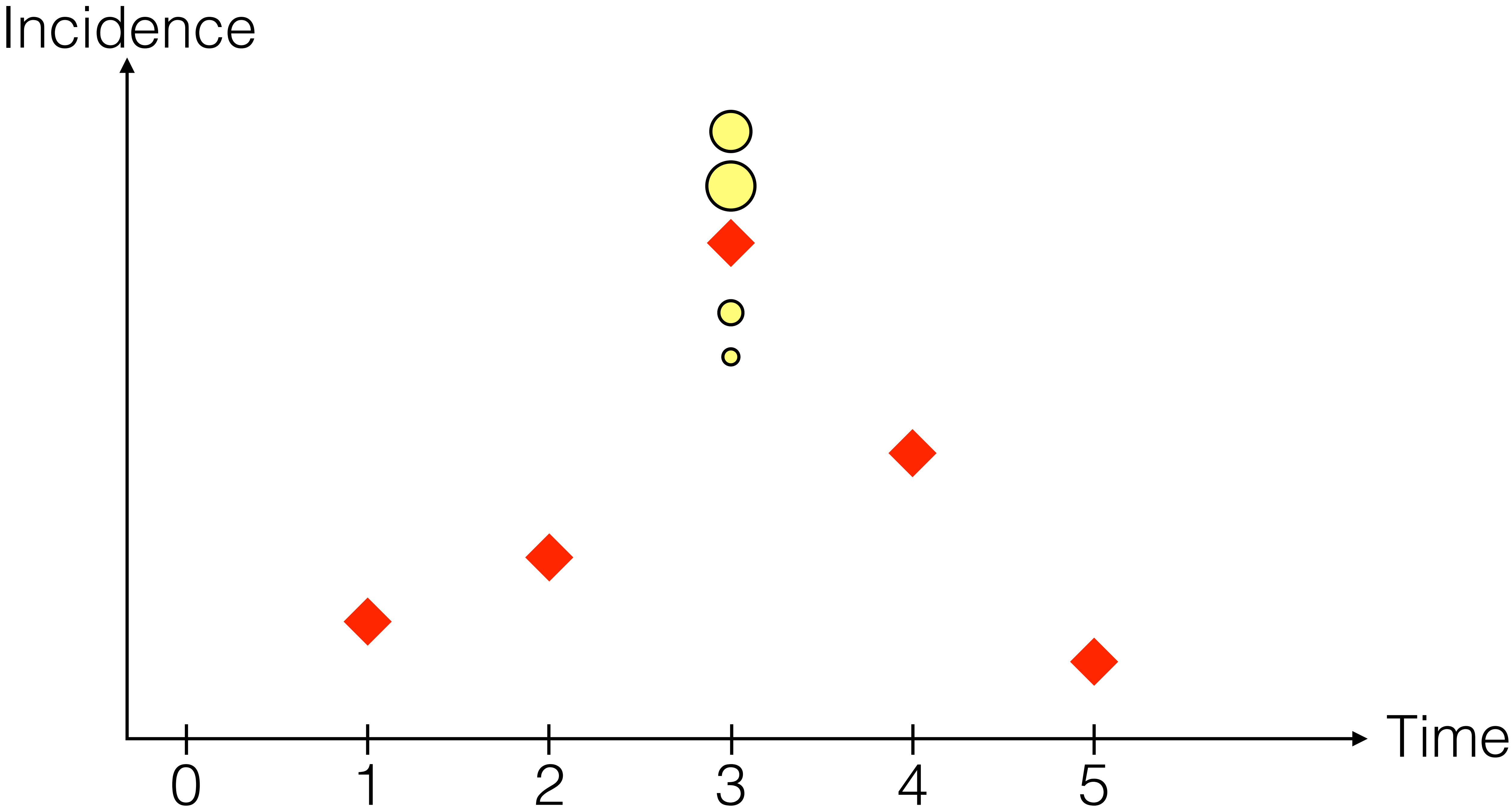
Resample

$\bigcirc \propto w_2$



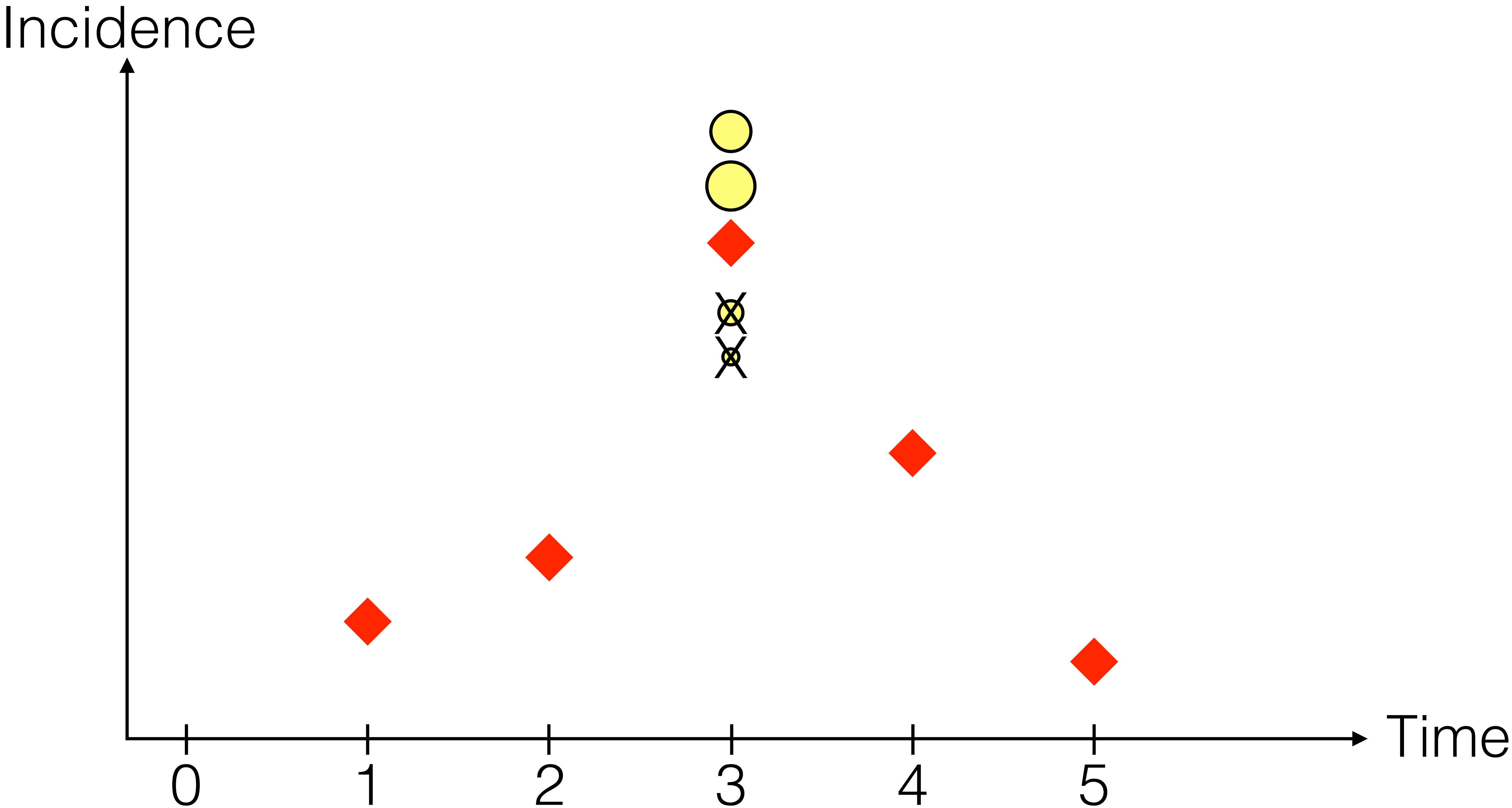
Propagate

$$\begin{cases} x_3 \sim p(.|x_2,\theta) \\ \dots \end{cases}$$



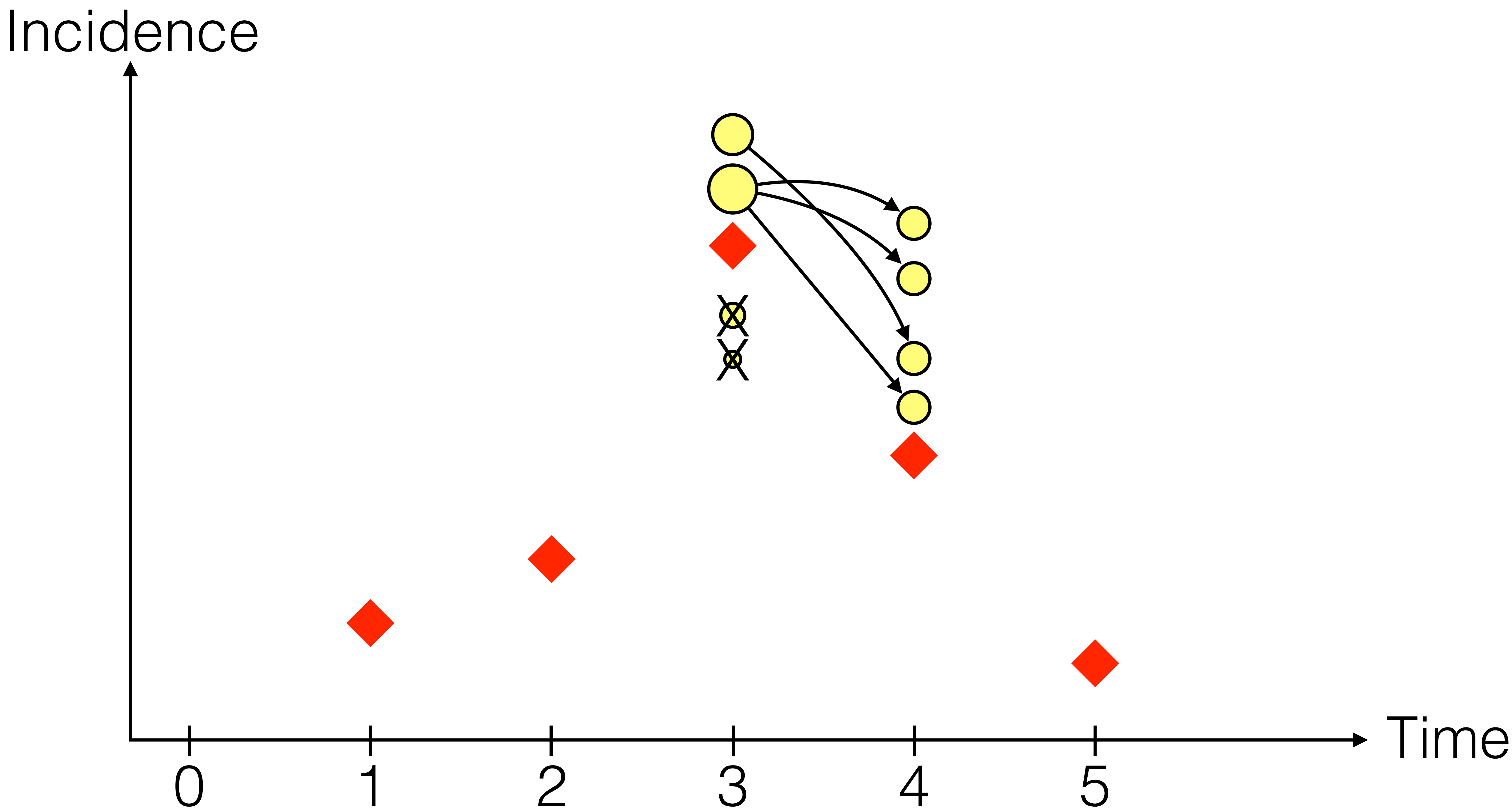
Weight

$$\begin{cases} x_3 \sim p(.|x_2, \theta) \\ w_3 = p(y_3|x_3, \theta) \end{cases}$$



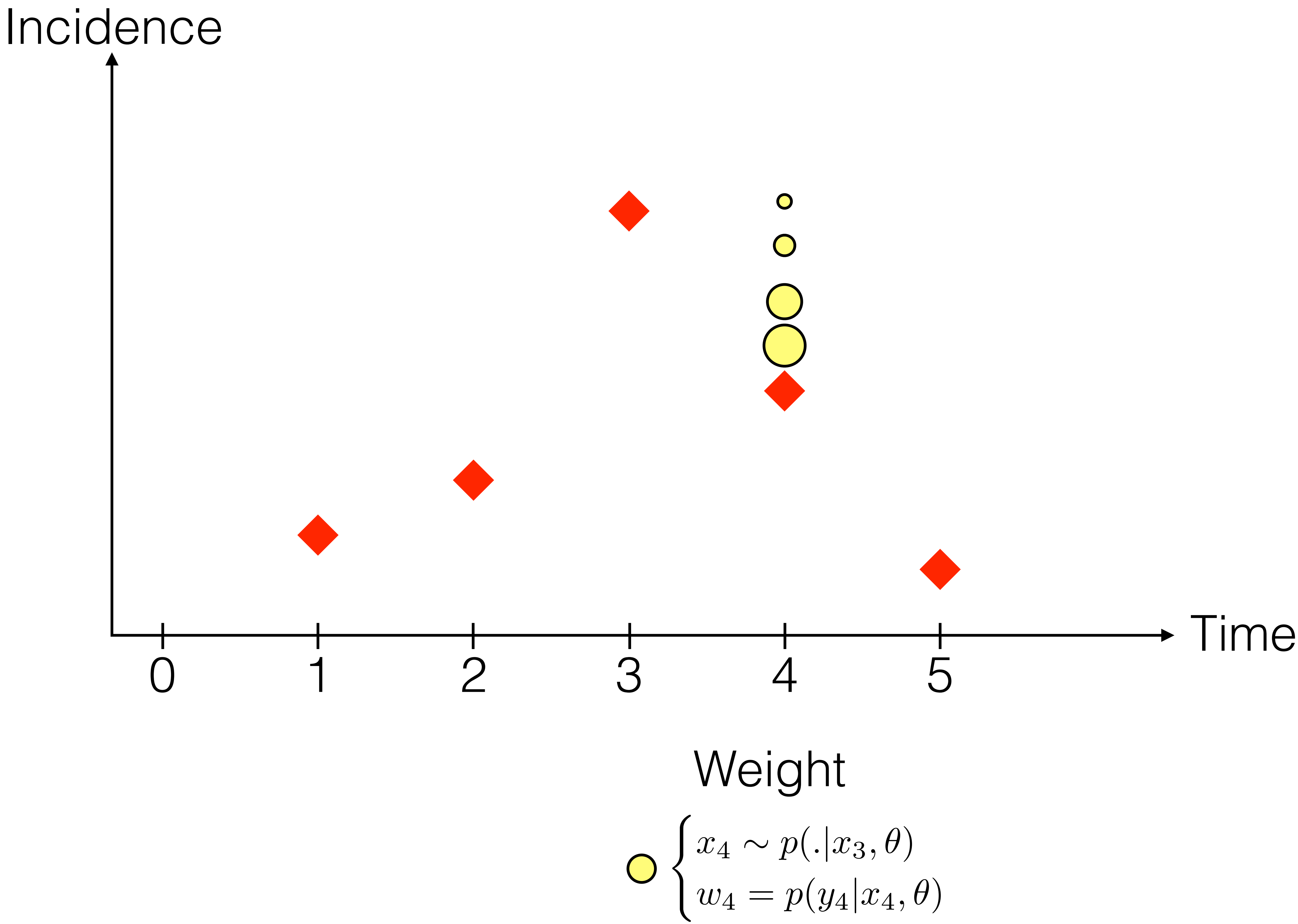
Resample

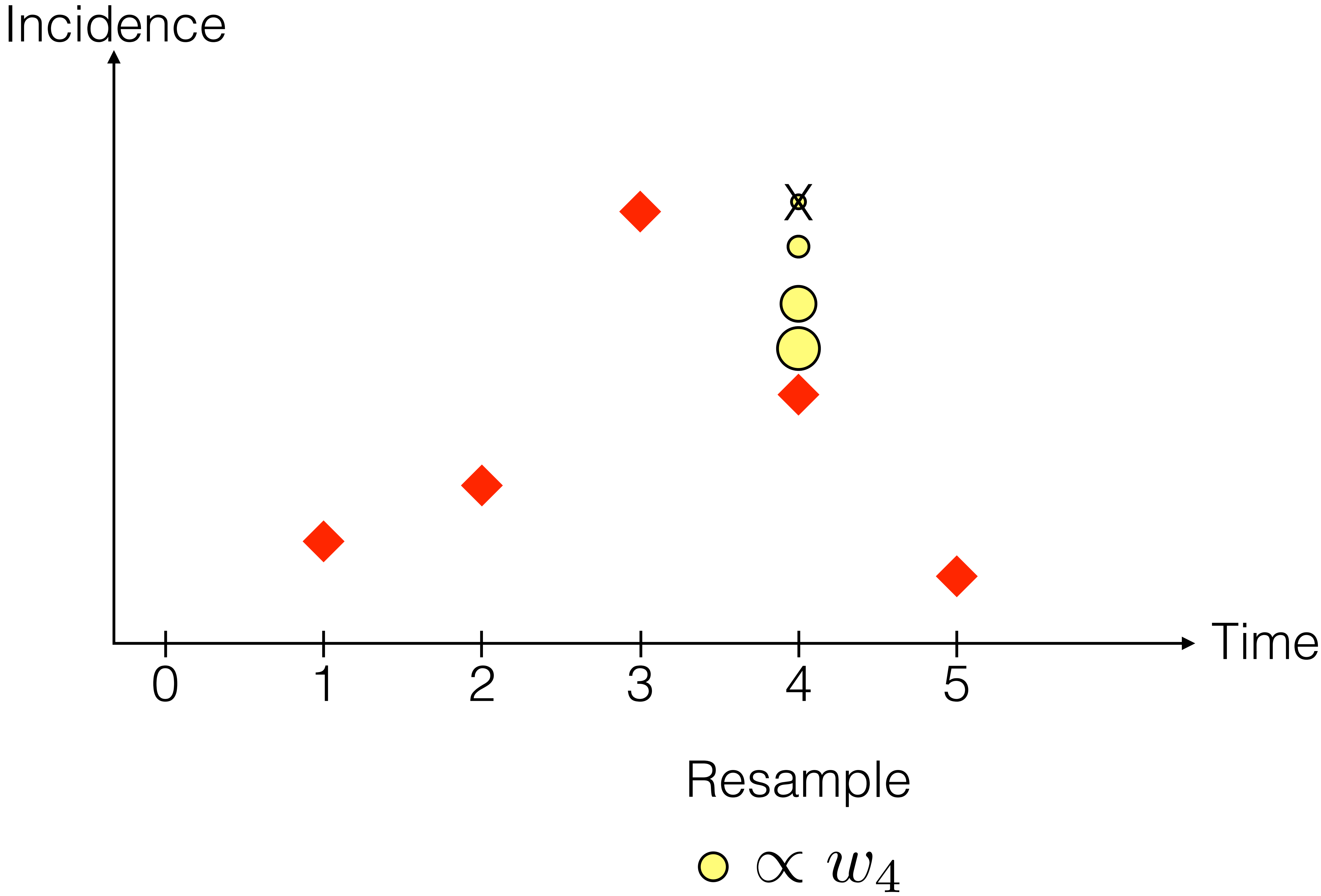
$\propto w_3$

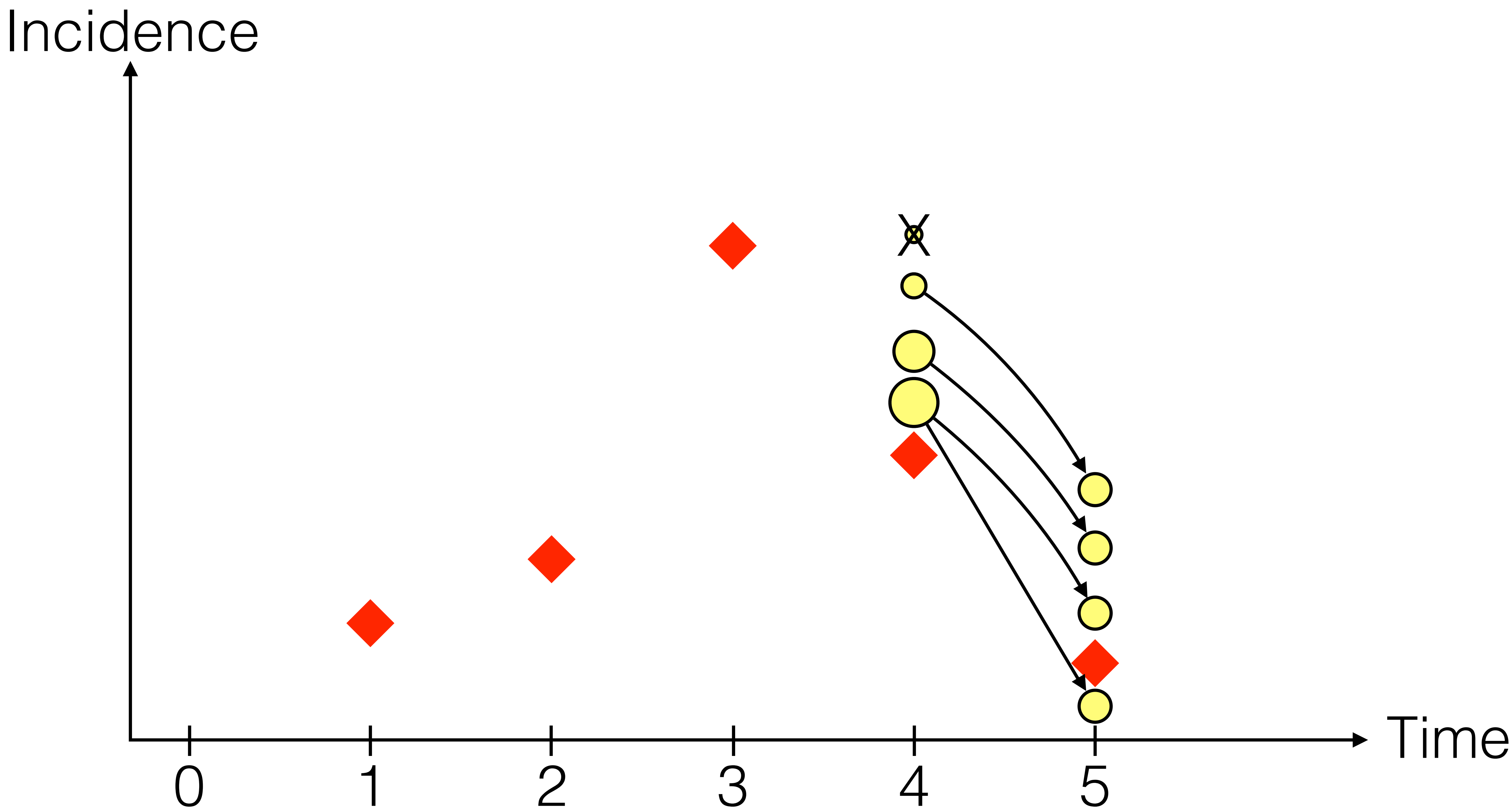


Propagate

$$\begin{cases} x_4 \sim p(.|x_3, \theta) \\ \ldots \end{cases}$$

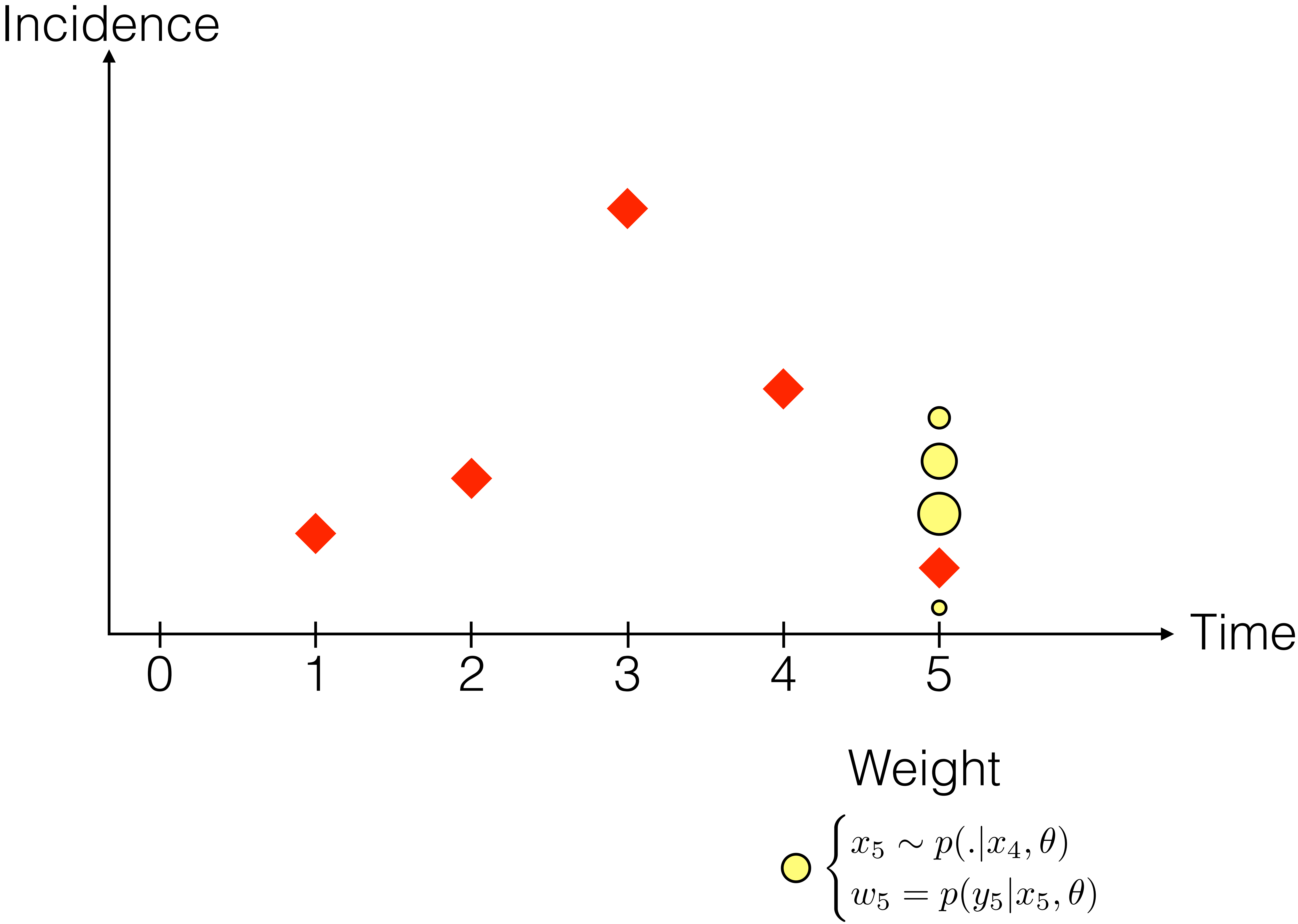

Incidence
Time
0
1
2
3
4
5
Weight
$x_4 \sim p(.|x_3, \theta)$
$w_4 = p(y_4|x_4, \theta)$


Incidence
Time
0
1
2
3
4
5
Resample
$\propto w_4$

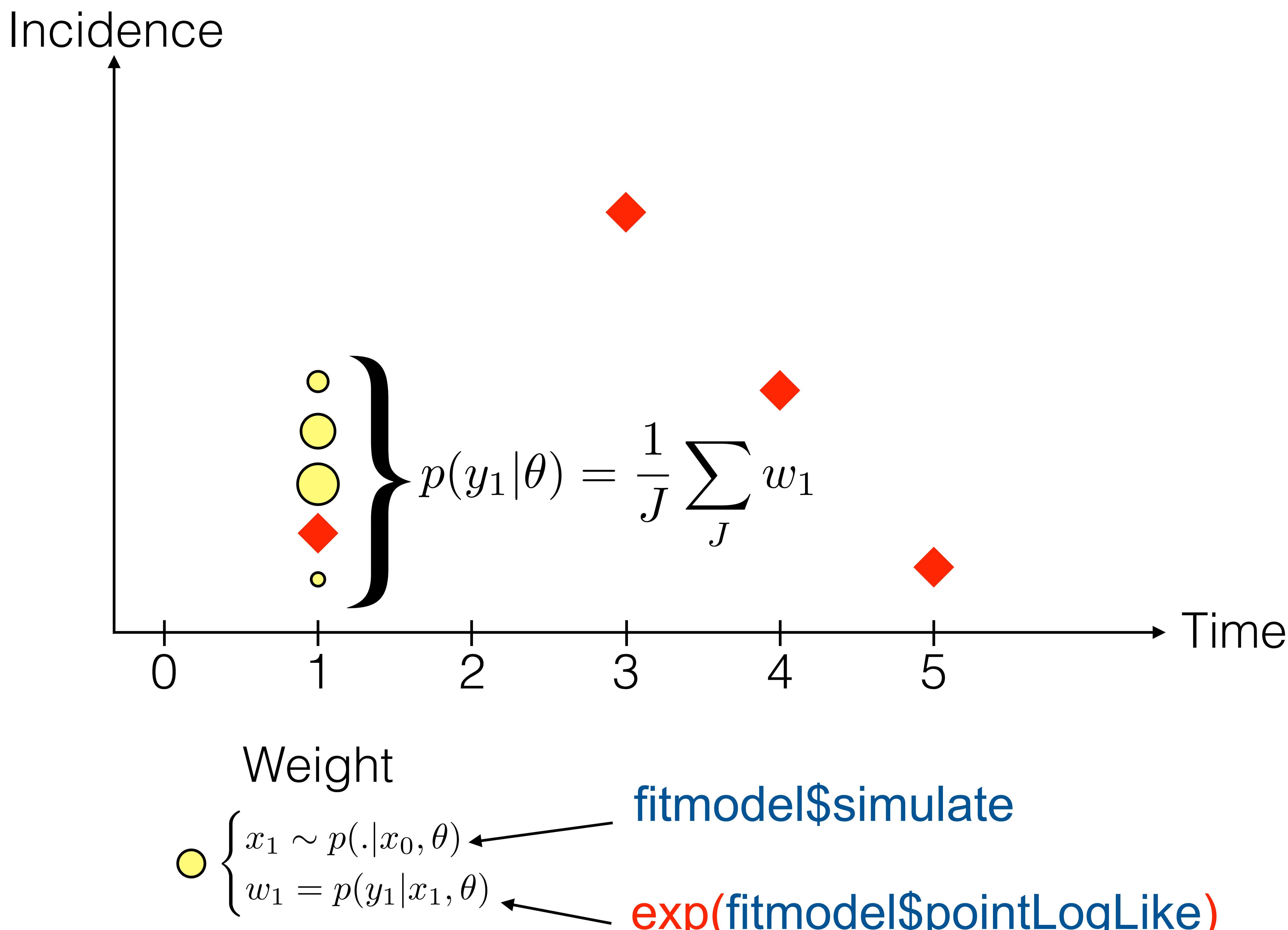


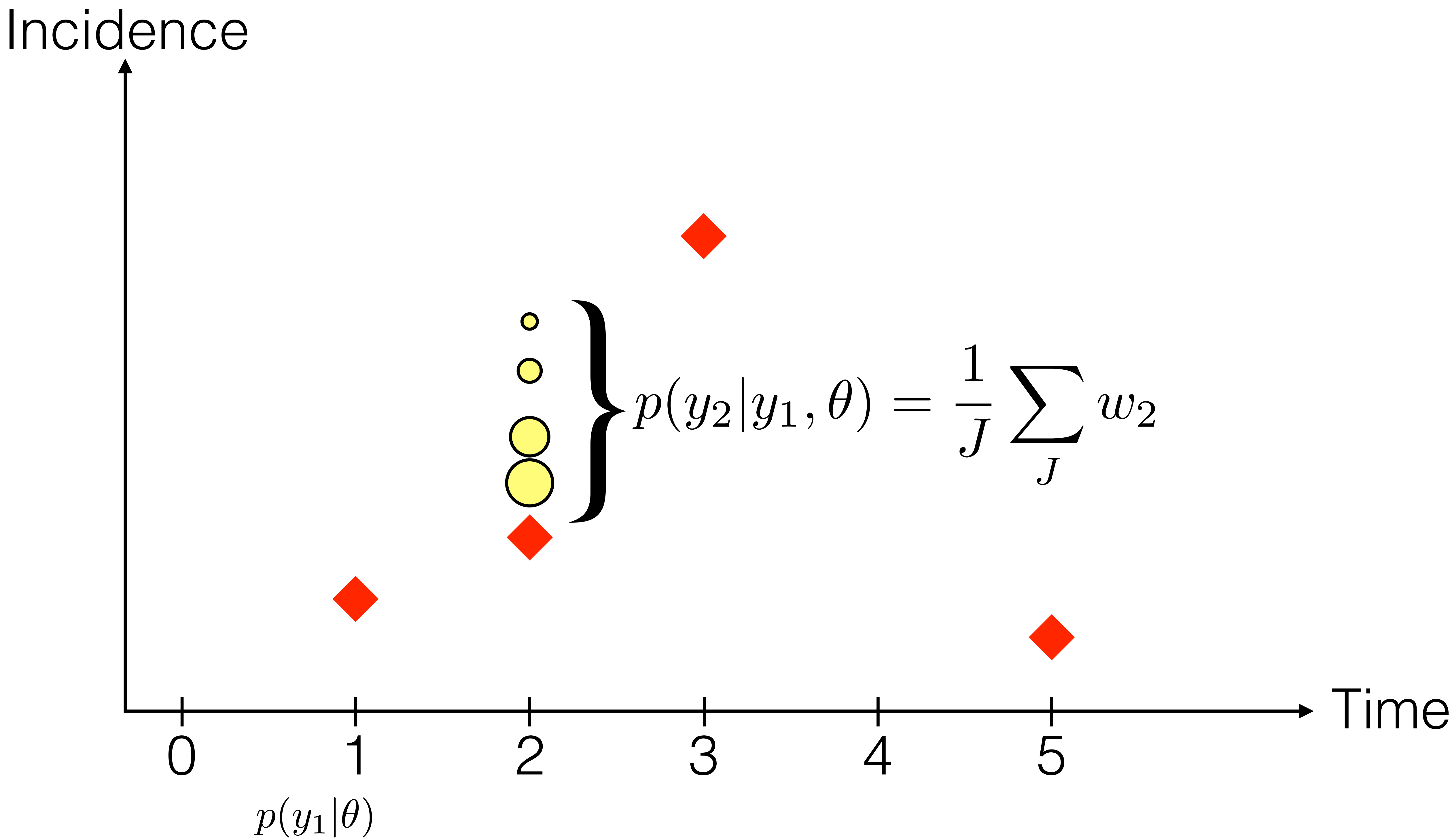
Propagate

$$\begin{cases} x_5 \sim p(.|x_4, \theta) \\ \ldots \end{cases}$$

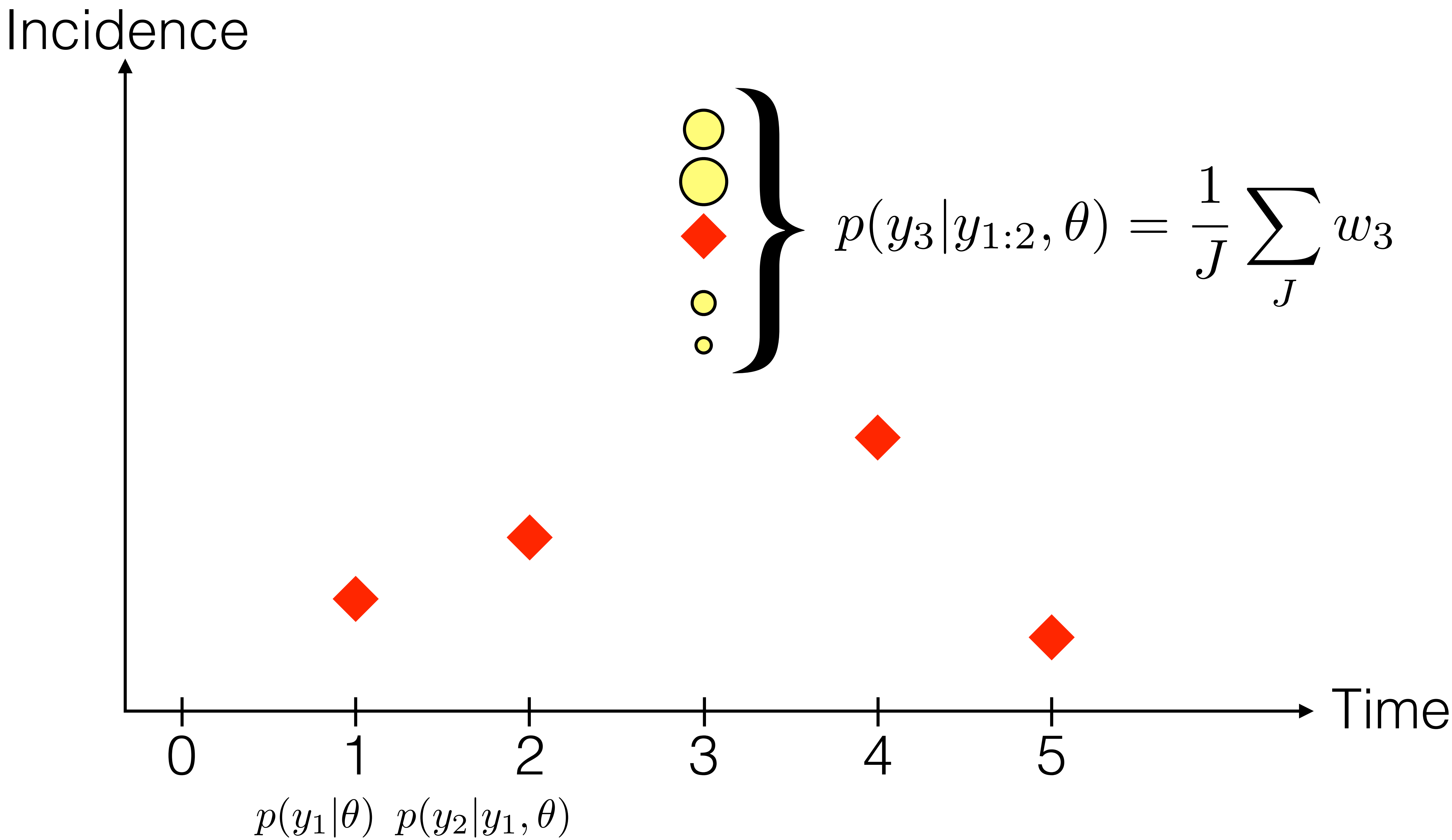

Incidence
Time
0
1
2
3
4
5
Weight
$x_5 \sim p(.|x_4, \theta)$
$w_5 = p(y_5|x_5, \theta)$

So how can I get the likelihood from this particle filter?

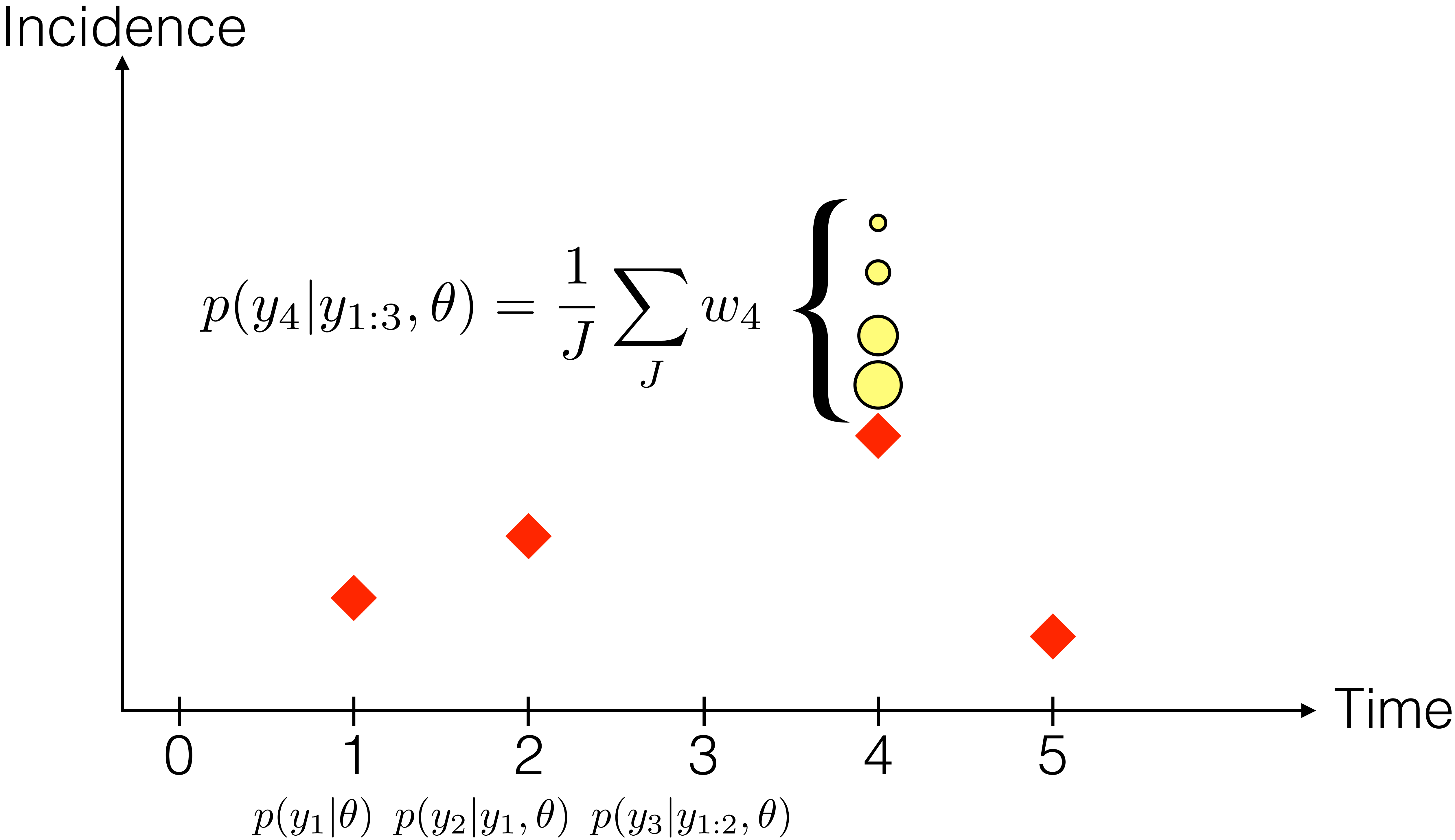

Incidence
Time
0
1
2
3
4
5
$p(y_1|\theta) = \frac{1}{J}\sum_J w_1$
Weight
$x_1 \sim p(.|x_0, \theta)$
$w_1 = p(y_1|x_1, \theta)$
fitmodel$simulate
exp(fitmodel$pointLogLike)



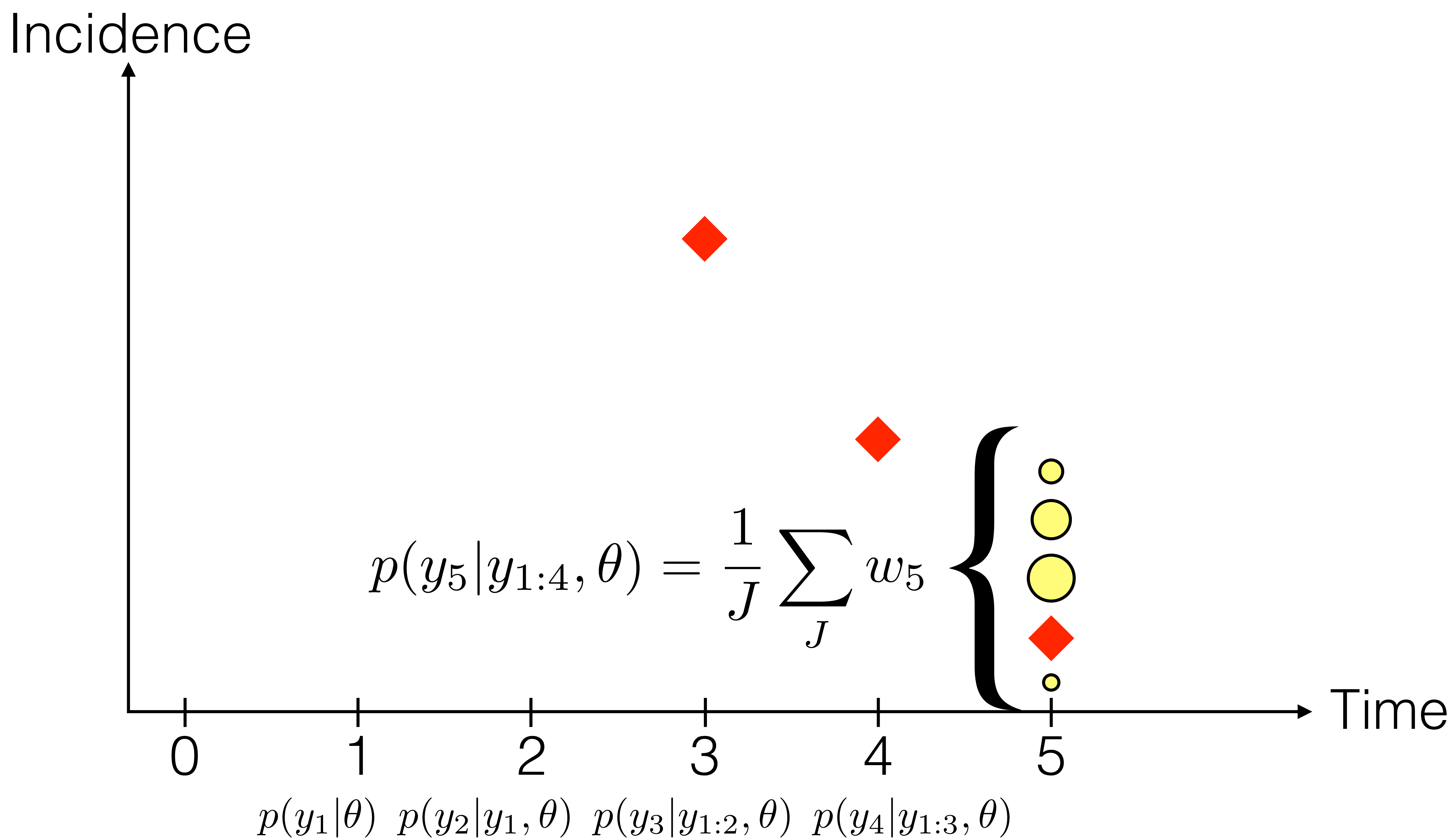
Incidence
$p(y_2|y_1,\theta) = \frac{1}{J}\sum_J w_2$
Time
0
1
2
3
4
5
$p(y_1|\theta)$

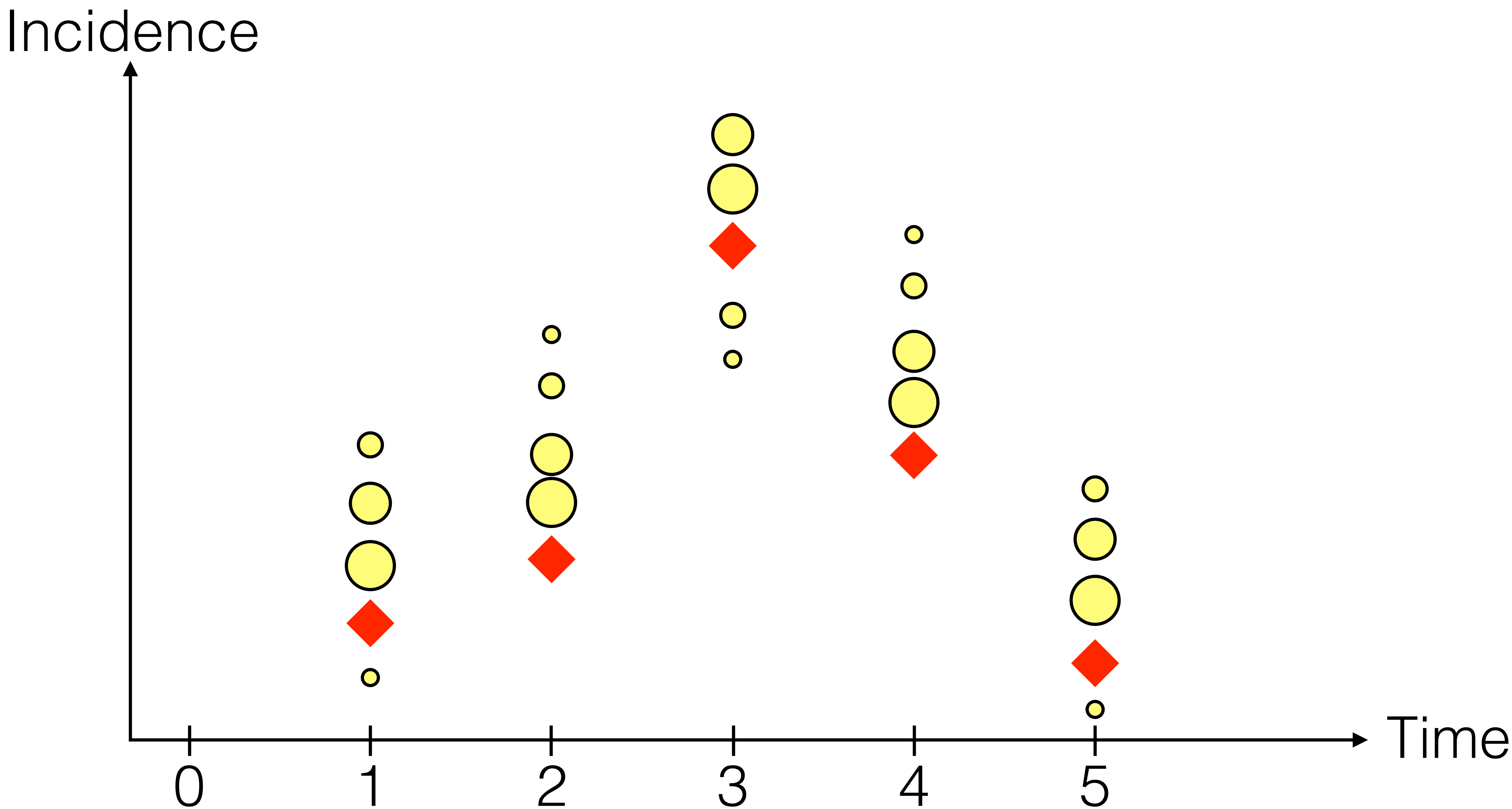


Incidence
$p(y_3|y_{1:2},\theta) = \frac{1}{J}\sum_J w_3$
Time
0
1
2
3
4
5
$p(y_1|\theta)$
$p(y_2|y_1,\theta)$



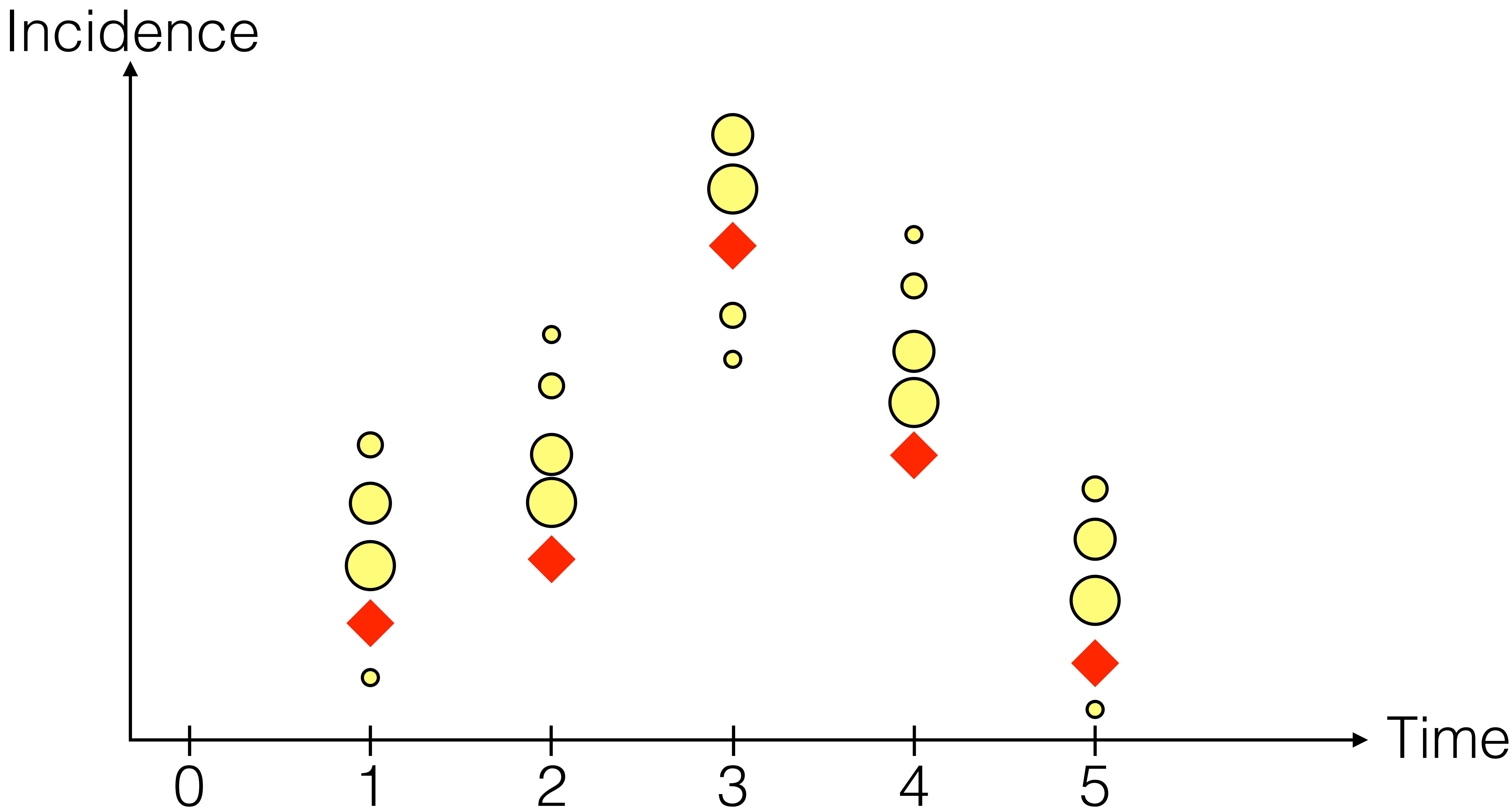
Incidence
$p(y_4|y_{1:3},\theta) = \frac{1}{J}\sum_J w_4$
Time
0
1
2
3
4
5
$p(y_1|\theta)$
$p(y_2|y_1,\theta)$
$p(y_3|y_{1:2},\theta)$


Incidence
Time
0
1
2
3
4
5
$p(y_5|y_{1:4},\theta) = \frac{1}{J}\sum_J w_5$
$p(y_1|\theta)$
$p(y_2|y_1,\theta)$
$p(y_3|y_{1:2},\theta)$
$p(y_4|y_{1:3},\theta)$



$p(y_1|\theta) \times p(y_2|y_1,\theta) \times p(y_3|y_{1:2},\theta) \times p(y_4|y_{1:3},\theta) \times p(y_5|y_{1:4},\theta)$

Likelihood: $p(y_{1:T}|\theta) = \prod_T p(y_t|y_{1:t-1},\theta)$



$p(y_1|\theta) \times p(y_2|y_1,\theta) \times p(y_3|y_{1:2},\theta) \times p(y_4|y_{1:3},\theta) \times p(y_5|y_{1:4},\theta)$

Log-Likelihood: $\log\{p(y_{1:T}|\theta)\} = \sum_T \log\{p(y_t|y_{1:t-1},\theta)\}$

Implement your own particle filter

Go to the pMCMC practical

Pseudocode for the particle filter

1. For each particle j = 1 … J

2. initialise the sate of particle j

3. initialise the weight of particle j

4. For each observation time t = 1 … T

5. resample particles

6. **For each particle j = 1 … J**

7. propagate particle j to next observation time

8. weight particle j