Analyzing Transformers in Embedding Space

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Abstract

Understanding Transformer-based models has attracted significant attention, as they lie at the heart of recent technological advances across machine learning. While most interpretability methods rely on running models over inputs, recent work has shown that an inputindependent approach, where parameters are interpreted directly without a forward/backward pass is feasible for some Transformer parameters, and for two-layer attention networks. In this work, we present a conceptual framework where all parameters of a trained Transformer are interpreted by projecting them into the embedding space, that is, the space of vocabulary items they operate on. Focusing mostly on GPT-2 for this paper, we provide diverse evidence to support our argument. First, an empirical analysis showing that parameters of both pretrained and fine-tuned models can be interpreted in embedding space. Second, we present two applications of our framework: (a) aligning the parameters of different models that share a vocabulary, and (b) constructing a classifier without training by "translating" the parameters of a fine-tuned classifier to parameters of a different model that was only pretrained. Overall, our findings show that at least in part, we can abstract away model specifics and understand Transformers in the embedding space.

1 Introduction

Transformer-based models [Vaswani et al., 2017] currently dominate Natural Language Processing [Devlin et al., 2018; Radford et al., 2019; Zhang et al., 2022] as well as many other fields of machine learning [Dosovitskiy et al., 2020; Chen et al., 2020; Baevski et al., 2020]. Consequently, understanding their inner workings has been a topic of great interest. Typically, work on interpreting Transformers relies on feeding inputs to the model and analyzing the resulting activations [Adi et al., 2016; Shi et al., 2016; Clark et al., 2019]. Thus, interpretation involves an expensive forward, and sometimes also a backward pass, over multiple inputs. Moreover, such interpretation methods are conditioned

on the input and are not guaranteed to generalize to all inputs. In the evolving literature on static interpretation, i.e., without forward or backward passes, [Geva et al., 2022b] showed that the value vectors of the Transformer feed-forward module (the second layer of the feed-forward network) can be interpreted by projecting them into the embedding space, i.e., multiplying them by the embedding matrix to obtain a representation over vocabulary items. [Elhage et al., 2021] have shown that in a 2-layer attention network, weight matrices can be interpreted in the embedding space as well. Unfortunately, their innovative technique could not be extended any further.

In this work, we extend and unify the theory and findings of [Elhage et al., 2021] and [Geva et al., 2022b]. We present a zero-pass, input-independent framework to understand the behavior of Transformers. Concretely, we interpret *all* weights of a pretrained language model (LM) in embedding space, including both keys and values of the feed-forward module ([Geva et al., 2020, 2022b] considered just FF values) as well as all attention parameters ([Elhage et al., 2021] analyzed simplified architectures up to two layers of attention with no MLPs).

Our framework relies on a simple observation. Since [Geva et al., 2022b] have shown that one can project hidden states to the embedding space via the embedding matrix, we intuit this can be extended to other parts of the model by projecting to the embedding space and then *projecting back* by multiplying with a right-inverse of the embedding matrix. Thus, we can recast inner products in the model as inner products in embedding space. Viewing inner products this way, we can interpret such products as interactions between pairs of vocabulary items. This applies to (a) interactions between attention queries and keys as well as to (b) interactions between attention value vectors and the parameters that project them at the output of the attention module. Taking this perspective to the extreme, one can view Transformers as operating implicitly in the embedding space. This entails the existence of a single linear space that depends only on the tokenizer,

¹We refer to the unique items of the vocabulary as *vocabulary items*, and to the (possibly duplicate) elements of a tokenized input as *tokens*. When clear, we might use the term *token* for *vocabulary item*.

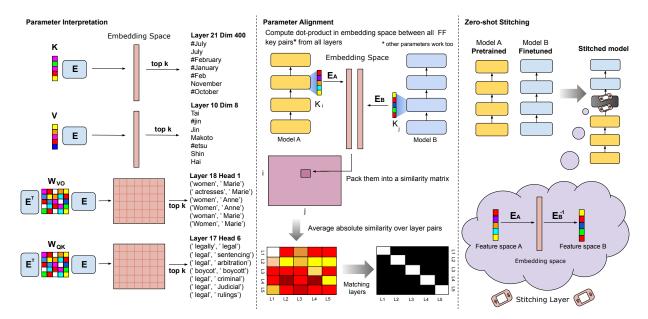


Figure 1: Applications of the embedding space view. Left: interpreting parameters in embedding space. The most active vocabulary items in a feed-forward key (k) and a feed-forward value (v). The most active pairs of vocabulary items in an attention query-key matrix $W_{\rm QK}$ and an attention value-output matrix $W_{\rm VO}$ (see §2). Center: Aligning the parameters of different BERT instances that share a vocabulary. Right: Zero-shot "stitching", where representations of a fine-tuned classifier are translated through the embedding space (multiplying by $E_A E_B^{-1}$) to a pretrained-only model.

in which parameters of different Transformers can be compared. Thus, one can use the embedding space to compare and transfer information across different models that share a tokenizer.

We provide extensive empirical evidence for the validity of our framework, focusing mainly on GPT-2 medium [Radford et al., 2019]. We use GPT-2 for two reasons. First, we do this for concreteness, as this paper is mainly focused on introducing the new framework and not on analyzing its predictions. Second, and more crucially, unlike many other architectures (such as BERT [Devlin et al., 2018], RoBERTa [Liu et al., 2019], and T5 [Raffel et al., 2019]), the GPT family has a *linear* language modeling head (LM head) – which is simply the output embedding matrix. All the other architectures' LM heads are two layer networks that contain non-linearities before the output embedding matrix. Our framework requires a linear language modeling head to work. That being said, we believe in practice this will not be a major obstacle, and we indeed see in the experiments that model alignment works well for BERT in spite of the theoretical difficulties. We leave the non-linearities in the LM head for future work.

On the interpretation front (Fig. 1, Left), we provide qualitative and quantitative evidence that Transformer parameters can be interpreted in embedding space. We also show that when fine-tuning GPT-2 on a sentiment analysis task (over movie reviews), projecting *changes* in parameters into embedding space yields words that characterize sentiment towards movies. Second (Fig. 1, Center), we show that given two distinct instances of BERT pretrained from different random seeds [Sellam et al., 2022], we can align layers of the two instances by casting their weights into the embedding space. We

find that indeed layer i of the first instance aligns well to layer i of the second instance, showing the different BERT instances converge to a semantically similar solution. Last (Fig. 1, Right), we take a model fine-tuned on a sentiment analysis task and "transfer" the learned weights to a different model that was only pretrained by going through the embedding spaces of the two models. We show that in 30% of the cases, this procedure, termed *stitching*, results in a classifier that reaches an impressive accuracy of 70% on the IMDB benchmark [Maas et al., 2011] without any training.

Overall, our findings suggest that analyzing Transformers in embedding space is valuable both as an interpretability tool and as a way to relate different models that share a vocabulary and that it opens the door to interpretation methods that operate in embedding space only. Our code is available at https://github.com/guyd1995/embedding-space.

2 Background

We now present the main components of the Transformer [Vaswani et al., 2017] relevant to our analysis. We discuss the residual stream view of Transformers, and recapitulate a view of the attention layer parameters as *interaction matrices* $W_{\rm VO}$ and $W_{\rm QK}$ [Elhage et al., 2021]. Similar to them, we exclude biases and layer normalization from our analysis.

2.1 Transformer Architecture

The Transformer consists of a stack of layers, each including an attention module followed by a Feed-Forward (FF) module. All inputs and outputs are sequences of N vectors of dimensionality d.

Attention Module takes as input a sequence of representations $X \in \mathbb{R}^{N \times d}$, and each layer L is parameterized by four matrices $W_Q^{(L)}, W_K^{(L)}, W_V^{(L)}, W_O^{(L)} \in \mathbb{R}^{d \times d}$ (we henceforth omit the layer superscript for brevity). The input X is projected to produce queries, keys, and values: $Q_{\rm att} = XW_Q, K_{\rm att} = XW_K, V_{\rm att} = XW_V$. Each one of $Q_{\rm att}, K_{\rm att}, V_{\rm att}$ is split along the columns to H different heads of dimensionality $\mathbb{R}^{N \times \frac{d}{H}}$, denoted by $Q_{\rm att}^i, K_{\rm att}^i, V_{\rm att}^i$ respectively. We then compute H attention maps:

$$A^i = \operatorname{softmax} \left(\frac{Q_{\operatorname{att}}^i K_{\operatorname{att}}^{i\operatorname{T}}}{\sqrt{d/H}} + M \right) \in \mathbb{R}^{N \times N},$$

where $M \in \mathbb{R}^{N \times N}$ is the attention mask. Each attention map is applied to the corresponding value head as $A^i V^i_{\mathrm{att}}$, results are concatenated along columns and projected via W_O . The input to the module is added via a residual connection, and thus the attention module's output is:

$$X + \mathbf{Concat} \left[A^1 V_{\mathrm{att}}^1, \dots, A^i V_{\mathrm{att}}^i, \dots, A^H V_{\mathrm{att}}^H \right] W_O. \tag{1}$$

FF Module is a two-layer neural network, applied to each position independently. Following past terminology [Sukhbaatar et al., 2019; Geva et al., 2020], weights of the first layer are called FF keys and weights of the second layer FF values. This is an analogy to attention, as the FF module too can be expressed as: $f(QK^T)V$, where f is the activation function, $Q \in \mathbb{R}^{N \times d}$ is the output of the attention module and the input to the FF module, and $K, V \in \mathbb{R}^{d_{\overline{g}} \times d}$ are the weights of the first and second layers of the FF module. Unlike attention, keys and values are learnable parameters. The output of the FF module is added to the output of the attention module to form the output of the layer via a residual connection. The output of the i-th layer is called the i-th i-th

Embedding Matrix To process sequences of discrete tokens, Transformers use an embedding matrix $E \in \mathbb{R}^{d \times e}$ that provides a d-dimensional representation to vocabulary items before entering the first Transformer layer. In different architectures, including GPT-2, the same embedding matrix E is often used [Press and Wolf, 2016] to take the output of the last Transformer layer and project it back to the vocabulary dimension, i.e., into the $embedding\ space$. In this work, we show how to interpret all the components of the Transformer model in the embedding space.

2.2 The Residual Stream

We rely on a useful view of the Transformer through its residual connections popularized by [Elhage et al., 2021].² Specifically, each layer takes a hidden state as

input and adds information to the hidden state through its residual connection. Under this view, the hidden state is a *residual stream* passed along the layers, from which information is read, and to which information is written at each layer. [Elhage et al., 2021] and [Geva et al., 2022b] observed that the residual stream is often barely updated in the last layers, and thus the final prediction is determined in early layers and the hidden state is mostly passed through the later layers.

An exciting consequence of the residual stream view is that we can project hidden states in *every* layer into embedding space by multiplying the hidden state with the embedding matrix E, treating the hidden state as if it were the output of the last layer. [Geva et al., 2022a] used this approach to interpret the prediction of Transformer-based language models, and we follow a similar approach.

2.3 W_{OK} and W_{VO}

Following [Elhage et al., 2021], we describe the attention module in terms of interaction matrices $W_{\rm QK}$ and $W_{\rm VO}$ which will be later used in our mathematical derivation. The computation of the attention module (§2.1) can be re-interpreted as follows. The attention projection matrices $W_{\rm Q}, W_{\rm K}, W_{\rm V}$ can be split along the column axis to H equal parts denoted by $W_{\rm Q}^i, W_{\rm K}^i, W_{\rm V}^i \in \mathbb{R}^{d \times \frac{d}{H}}$ for $1 \leq i \leq H$. Similarly, the attention output matrix $W_{\rm O}$ can be split along the row axis into H heads, $W_{\rm O}^i \in \mathbb{R}^{\frac{d}{H} \times d}$. We define the interaction matrices as

$$W_{\mathrm{QK}}^{i} := W_{\mathrm{Q}}^{i} W_{\mathrm{K}}^{i \mathrm{T}} \in \mathbb{R}^{d \times d},$$

$$W_{\mathrm{VQ}}^{i} := W_{\mathrm{V}}^{i} W_{\mathrm{Q}}^{i} \in \mathbb{R}^{d \times d}.$$

Importantly, $W^i_{\rm QK}, W^i_{\rm VO}$ are input-independent. Intuitively, $W_{\rm QK}$ encodes the amount of attention between pairs of tokens. Similarly, in $W^i_{\rm VO}$, the matrices $W_{\rm V}$ and $W_{\rm O}$ can be viewed as a transition matrix that determines how attending to certain tokens affects the subsequent hidden state.

We can restate the attention equations in terms of the interaction matrices. Recall (Eq. 1) that the output of the i'th head of the attention module is $A^iV^i_{\rm att}$ and the final output of the attention module is (without the residual connection):

Concat
$$\left[A^{1}V_{\text{att}}^{1},...,A^{i}V_{\text{att}}^{i},...,A^{H}V_{\text{att}}^{H}\right]W_{\text{O}} = (2)$$

$$\sum_{i=1}^{H}A^{i}(XW_{\text{V}}^{i})W_{\text{O}}^{i} = \sum_{i=1}^{H}A^{i}XW_{\text{VO}}^{i}.$$

Similarly, the attention map A^i at the i'th head in terms of $W_{\rm OK}$ is (softmax is done row-wise):

$$\begin{split} A^i &= \operatorname{softmax} \left(\frac{(XW_{\mathrm{Q}}^i)(XW_{\mathrm{K}}^i)^{\mathrm{T}}}{\sqrt{d/H}} + M \right) & (3) \\ &= \operatorname{softmax} \left(\frac{X(W_{\mathrm{QK}}^i)X^{\mathrm{T}}}{\sqrt{d/H}} + M \right). \end{split}$$

²Originally introduced in [nostalgebraist, 2020].

3 Parameter Projection

In this section, we propose that Transformer parameters can be projected into embedding space for interpretation purposes. We empirically support our framework's predictions in §4-§5.

Given a matrix $A \in \mathbb{R}^{N \times d}$, we can project it into embedding space by multiplying by the embedding matrix E as $\hat{A} = AE \in \mathbb{R}^{N \times e}$. Let E' be a right-inverse of E, that is, $EE' = I \in \mathbb{R}^{d \times d}$. We can reconstruct the original matrix with E' as A = A(EE') = $\hat{A}E'$. We will use this simple identity to reinterpret the model's operation in embedding space. To simplify our analysis we ignore LayerNorm and biases. This has been justified in prior work [Elhage et al., 2021]. Briefly, LayerNorm can be ignored because normalization changes only magnitudes and not the direction of the update. At the end of this section, we discuss why in practice we choose to use $E' = E^{T}$ instead of a seemingly more appropriate right inverse, such as the pseudo-inverse [Moore, 1920; Bjerhammar, 1951; Penrose, 1955]. In this section, we derive our framework and summarize its predictions in Table 1.

Attention Module Recall that $W_{VO}^i := W_V^i W_O^i \in$ $\mathbb{R}^{d \times d}$ is the interaction matrix between attention values and the output projection matrix for attention head i. By definition, the output of each head is: $A^i X W_{VO}^i =$ $A^i \hat{X} E' W_{VO}^i$. Since the output of the attention module is added to the residual stream, we can assume according to the residual stream view that it is meaningful to project it to the embedding space, similar to FF values. Thus, we expect the sequence of $N\ e$ -dimensional vectors $(A^{i}XW_{VO}^{i})E = A^{i}X(E'W_{VO}^{i}E)$ to be interpretable. Importantly, the role of A^i is just to mix the representations of the updated N input vectors. This is similar to the FF module, where FF values (the parameters of the second layer) are projected into embedding space, and FF keys (parameters of the first layer) determine the coefficients for mixing them. Hence, we can assume that the interpretable components are in the term $X(E'W_{VO}^iE)$.

Zooming in on this operation, we see that it takes the previous hidden state in the embedding space (\hat{X}) and produces an output in the embedding space which will be incorporated into the next hidden state through the residual stream. Thus, $E'W_{\text{VO}}^iE$ is a *transition matrix* that takes a representation of the embedding space and outputs a new representation in the same space.

Similarly, the matrix $W_{\rm QK}^i$ can be viewed as a bilinear map (Eq. 2.3). To interpret it in embedding space, we perform the following operation with E':

$$\begin{split} XW_{\text{QK}}^{i}X^{\text{T}} &= (XEE')W_{\text{QK}}^{i}(XEE')^{\text{T}} = \\ (XE)E'W_{\text{OK}}^{i}E'^{\text{T}}(XE)^{\text{T}} &= \hat{X}(E'W_{\text{OK}}^{i}E'^{\text{T}})\hat{X}^{\text{T}}. \end{split}$$

Therefore, the interaction between tokens at different positions is determined by an $e \times e$ matrix that expresses

the interaction between pairs of vocabulary items.

FF Module [Geva et al., 2022b] showed that FF value vectors $V \in \mathbb{R}^{d_f \times d}$ are meaningful when projected into embedding space, i.e., for a FF value vector $v \in \mathbb{R}^d$, $vE \in \mathbb{R}^e$ is interpretable (see §2.1). In vectorized form, the rows of $VE \in \mathbb{R}^{d_{ff} \times e}$ are interpretable. On the other hand, the keys K of the FF layer are multiplied on the left by the output of the attention module, which are the queries of the FF layer. Denoting the output of the attention module by Q, we can write this product as $QK^{T} = \hat{Q}E'K^{T} = \hat{Q}(KE'^{T})^{T}$. Because Q is a hidden state, we assume according to the residual stream view that \hat{Q} is interpretable in embedding space. When multiplying \hat{Q} by $KE^{\prime T}$, we are capturing the interaction in embedding space between each query and key, and thus expect $\tilde{K}\hat{E}'^{\mathrm{T}}$ to be interpretable in embedding space as well.

Overall, FF keys and values are intimately connected – the *i*-th key controls the coefficient of the *i*-th value, so we expect their interpretation to be related. While not central to this work, we empirically show that key-value pairs in the FF module are similar in embedding space in Appendix B.1.

Subheads Another way to interpret the matrices W^i_{VO} and W^i_{QK} is through the *subhead view*. We use the following identity: $AB = \sum_{j=1}^b A_{:,j} B_{j,:}$, which holds for arbitrary matrices $A \in \mathbb{R}^{a \times b}, B \in \mathbb{R}^{b \times c}$, where $A_{:,j} \in \mathbb{R}^{a \times 1}$ are the *columns* of the matrix A and $B_{j,:} \in \mathbb{R}^{1 \times c}$ are the *rows* of the matrix B. Thus, we can decompose W^i_{VO} and W^i_{QK} into a sum of $\frac{d}{H}$ rank-1 matrices:

$$W_{\text{VO}}^{i} = \sum_{j=1}^{\frac{d}{H}} W_{\text{V}}^{i,j} W_{\text{O}}^{i,j}, \quad W_{\text{QK}}^{i} = \sum_{j=1}^{\frac{d}{H}} W_{\text{Q}}^{i,j} W_{\text{K}}^{i,j}^{\text{T}}.$$

where $W_{\mathrm{Q}}^{i,j},W_{\mathrm{K}}^{i,j},W_{\mathrm{V}}^{i,j}\in\mathbb{R}^{d\times 1}$ are columns of $W_{\mathrm{Q}}^{i},W_{\mathrm{K}}^{i},W_{\mathrm{V}}^{i}$ respectively, and $W_{\mathrm{Q}}^{i,j}\in\mathbb{R}^{1\times d}$ are the rows of W_{Q}^{i} . We call these vectors *subheads*. This view is useful since it allows us to interpret subheads directly by multiplying them with the embedding matrix E. Moreover, it shows a parallel between interaction matrices in the attention module and the FF module. Just like the FF module includes key-value pairs as described above, for a given head, its interaction matrices are a sum of interactions between pairs of subheads (indexed by j), which are likely to be related in embedding space. We show this is indeed empirically the case for pairs of subheads in Appendix B.1.

Choosing $E' = E^{\rm T}$ In practice, we do not use an exact right inverse (e.g. the pseudo-inverse). We use the transpose of the embedding matrix $E' = E^{\rm T}$ instead. The reason pseudo-inverse doesn't work is that for interpretation we apply a top-k operation after projecting to embedding space (since it is impractical for humans to read through a sorted list of 50K tokens). So, we only keep the list of the vocabulary items that have the k largest logits, for manageable values of k.

 $^{^3}E'$ exists if $d \le e$ and E is full-rank.

	Symbol	Projection	Approximate Projection
FF values	v	vE	vE
FF keys	k	kE'^{T}	kE
Attention query-key	$W^i_{ m QK}$	$E'W_{ ext{QK}}^{i}E'^{ ext{T}} onumber \ E'W_{ ext{VO}}^{i}E$	$E^{\mathrm{T}}W_{\mathrm{QK}}^{i}E \ E^{\mathrm{T}}W_{\mathrm{VO}}^{i}E$
Attention value-output	$W_{ m VO}^i$	$E'W_{\mathrm{VO}}^{i}E$	$E^{T}W_{VO}^{i}E$
Attention value subheads	$W_{\mathrm{v}}^{i,j}$	$W_{\rm V}^{i,j}E^{\prime { m T}}$	$W^{i,j}_{ m V}E$
Attention output subheads	$W^{i,j}_{\mathrm{O}}$	$W_{0,j}^{i,j}E$	$W^{i,j}_{\mathbf{O}}E$
Attention query subheads	$W_{\mathrm{O}}^{i,j}$	$W_{\mathrm{Q}}^{i,j}E'^{\mathrm{T}} \ W_{\mathrm{Y}}^{i,j}E'^{\mathrm{T}}$	$W^{ec{i},j}_{O}E$
Attention key subheads	$W_{\mathrm{K}}^{i,j}$	$W_{ m K}^{{f i},j} E'^{ m T}$	$W_{\mathbf{K}}^{oldsymbol{i},j}E$

Table 1: A summary of our approach for projecting Transformer components into embedding space. The 'Approximate Projection' shows the projection we use in practice where $E'=E^{T}$.

In Appendix A, we explore the exact requirements for E' to interact well with top-k. We show that the top k entries of a vector projected with the pseudo-inverse do not represent the entire vector well in embedding space. We define keep-k robust invertibility to quantify this. It turns out that empirically $E^{\rm T}$ is a decent keep-k robust inverse for E in the case of GPT-2 medium (and similar models) for plausible values of k. We refer the reader to Appendix A for details.

To give intuition as to why E^{T} works in practice, we switch to a different perspective, useful in its own right. Consider the FF keys for example – they are multiplied on the left by the hidden states. In this section, we suggested to re-cast this as $h^T K = (h^T E)(E'K)$. Our justification was that the hidden state is interpretable in the embedding space. A related perspective (dominant in previous works too; e.g. [Mickus et al., 2022]) is thinking of the hidden state as an aggregation of interpretable updates to the residual stream. That is, schematically, $h=\sum_{i=1}^k \alpha_i r_i$, where α_i are scalars and r_i are vectors corresponding to specific concepts in the embedding space (we roughly think of a concept as a list of tokens related to a single topic). Inner product is often used as a similarity metric between two vectors. If the similarity between a column K_i and h is large, the corresponding i-th output coordinate will be large. Then we can think of K as a *detector* of concepts where each neuron (column in K) lights up if a certain concept is "present" (or a superposition of concepts) in the inner state. To understand which concepts each detector column encodes we see which tokens it responds to. Doing this for all (input) token embeddings and packaging the inner products into a vector of scores is equivalent to simply multiplying by E^{T} on the left (where E is the input embedding in this case, but for GPT-2 they are the same). A similar argument can be made for the interaction matrices as well. For example for W_{VO} , to understand if a token embedding e_i maps to a e_i under a certain head, we apply the matrix to e_i , getting $e_i^T W_{VO}$ and use the inner product as a similarity metric and get the score $e_i^T W_{VO} e_i$.

4 Interpretability Experiments

In this section, we provide empirical evidence for the viability of our approach as a tool for interpreting Transformer parameters. For our experiments, we use

Huggingface Transformers ([Wolf et al., 2020]; License: Apache-2.0).

4.1 Parameter Interpretation Examples

Attention Module We take GPT-2 medium (345M parameters; [Radford et al., 2019]) and manually analyze its parameters. GPT-2 medium has a total of 384 attention heads (24 layers and 16 heads per layer). We take the embedded transition matrices $E'W_{VO}^iE$ for all heads and examine the top-k pairs of vocabulary items. As there are only 384 heads, we manually choose a few heads and present the top-k pairs in Appendix C.1 (k = 50). We observe that different heads capture different types of relations between pairs of vocabulary items including word parts, heads that focus on gender, geography, orthography, particular part-of-speech tags, and various semantic topics. In Appendix C.2 we perform a similar analysis for W_{QK} . We supplement this analysis with a few examples from GPT-2 base and large (117M, 762M parameters – respectively) as proof of concept, similarly presenting interpretable patterns.

A technical note: $W_{\rm VO}$ operates on row vectors, which means it operates in a "transposed" way to standard intuition – which places inputs on the left side and outputs on the right side. It does not affect the theory, but when visualizing the top-k tuples, we take the transpose of the projection $(E'W_{\rm VO}^iE)^{\rm T}$ to get the "natural" format (input token, output token). Without the transpose, we would get the *same* tuples, but in the format (output token, input token). Equivalently, in the terminology of linear algebra, it can be seen as a linear transformation that we represent in the basis of row vectors and we transform to the basis of column vectors, which is the standard one.

FF Module Appendix C.3 provides examples of key-value pairs from the FF modules of GPT-2 medium. We show random pairs (k,v) from the set of those pairs such that when looking at the top-100 vocabulary items for k and v, at least 15% overlap. Such pairs account for approximately 5% of all key-value pairs. The examples show how key-value pairs often revolve around similar topics such as media, months, organs, etc. We again include additional examples from GPT-2 base and large.

Knowledge Lookup Last, we show we can use embeddings to locate FF values (or keys) related to a par-

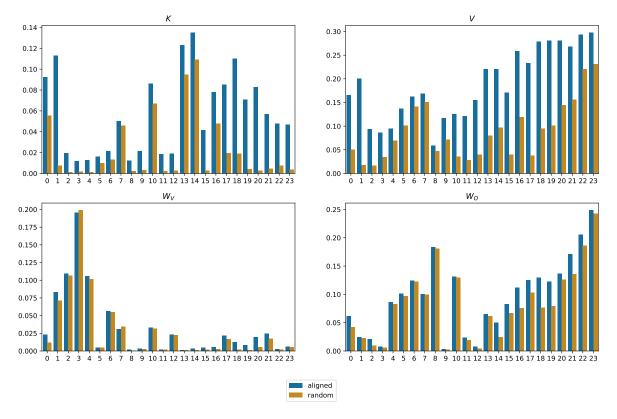


Figure 2: Left: Average R_k score (k=100) across tokens per layer for activated parameter vectors against both the aligned hidden state \hat{h} at the output of the layer and a randomly sampled hidden state \hat{h}_{rand} . Parameters are FF keys (top-left), FF values (top-right), attention values (bottom-left), and attention outputs (bottom-right).

ticular topic. We take a few vocabulary items related to a certain topic, e.g., ['cm', 'kg', 'inches'], average their embeddings,⁴ and rank all FF values (or keys) based on their dot-product with the average. Appendix C.4 shows a few examples of FF values found with this method that are related to programming, measurements, and animals.

4.2 Hidden State and Parameters

One merit of zero-pass interpretation is that it does not require running inputs through the model. Feeding inputs might be expensive and non-exhaustive. In this section and *in this section only*, we run a forward pass over inputs and examine if the embedding space representations of dynamically computed hidden states are "similar" to the representations of the activated static parameter vectors. Due to the small number of examples we run over, the overall GPU usage is still negligible

A technical side note: we use GPT-2, which applies LayerNorm to the Transformer output before projecting it to the embedding space with E. Thus, conservatively, LayerNorm should be considered as part of the projection operation. Empirically, however, we observe that projecting parameters directly without LayerNorm works well, which simplifies our analysis in $\S 3$. Unlike parameters, we apply LayerNorm to hidden states before projection to embedding space to improve interpretability. This nuance was also present in the

code of [Geva et al., 2022a].

Experimental Design We use GPT-2 medium and run it over 60 examples from IMDB (25,000 train, 25,000 test examples; [Maas et al., 2011]). This provides us with a dynamically-computed hidden state h for every token and at the output of every layer. For the projection $\hat{h} \in \mathbb{R}^e$ of each such hidden state, we take the projections of the m most active parameter vectors $\{\hat{x}_i\}_{i=1}^m$ in the layer that computed h and check if they cover the dominant vocabulary items of \hat{h} in embedding space. Specifically, let top-k(wE) be the k vocabulary items with the largest logits in embedding space for a vector $w \in \mathbb{R}^d$. We compute:

$$R_k(\hat{x}_1,...,\hat{x}_m,\hat{h}) = \frac{|\text{top-k}(\hat{h}) \cap \bigcup_{i=1}^m \text{top-k}(\hat{x}_i)|}{k}$$

to capture if activated parameter vectors cover the main vocabulary items corresponding to the hidden state.

We find the m most active parameter vectors separately for FF keys (K), FF values (V), attention value subheads (W_V) (see §3), and attention output subheads (W_O) , where the activation of each parameter vector is determined by the vector's "coefficient" as follows. For a FF key-value pair (k,v) the coefficient is $\sigma(q^Tk)$, where $q \in \mathbb{R}^d$ is an input to the FF module, and σ is the FF non-linearity. For attention, value-output subhead pairs (v,o) the coefficient is x^Tv , where x is the

 $^{^4}$ We subtract the average embedding μ from E before averaging, which improves interpretability.

⁵Note that IMDB was designed for sentiment analysis and we use it here as a general-purpose corpus.

input to this component (for attention head i, the input is one of the rows of $A^{i}X$, see Eq. 3).

Results and Discussion Figure 2 presents the R_k score averaged across tokens per layer. As a baseline, we compare R_k of the activated vectors $\{\hat{x}_i\}_{i=1}^m$ of the correctly-aligned hidden state \hat{h} at the output of the relevant layer (blue bars) against the R_k when randomly sampling \hat{h}_{rand} from all the hidden states (orange bars). We conclude that representations in embedding space induced by activated parameter vector mirror, at least to some extent, the representations of the hidden states themselves. Appendix §B.2 shows a variant of this experiment, where we compare activated parameters throughout GPT-2 medium's layers to the last hidden state, which produces the logits used for prediction.

4.3 Interpretation of Fine-tuned Models

We now show that we can interpret the *changes* a model goes through during fine-tuning through the lens of embedding space. We fine-tune the top-3 layers of the 12-layer GPT-2 base (117M parameters) with a sequence classification head on IMDB sentiment analysis (binary classification) and compute the difference between the original parameters and the fine-tuned model. We then project the difference of parameter vectors into embedding space and test if the change is interpretable w.r.t. sentiment analysis.

Appendix D shows examples of projected differences randomly sampled from the fine-tuned layers. Frequently, the difference or its negation is projected to nouns, adjectives, and adverbs that express sentiment for a movie, such as 'amazing', 'masterpiece', 'incompetence', etc. This shows that the differences are indeed projected into vocabulary items that characterize movie reviews' sentiments. This behavior is present across W_Q , W_K , W_V , K, but not V and W_O , which curiously are the parameters added to the residual stream and not the ones that react to the input directly.

5 Aligning Models in Embedding Space

The assumption Transformers operate in embedding space leads to an exciting possibility – we can relate *different* models to one another so long as they share the vocabulary and tokenizer. In §5.1, we show that we can align the layers of BERT models trained with different random seeds. In §5.2, we show the embedding space can be leveraged to "stitch" the parameters of a fine-tuned model to a model that was not fine-tuned.

5.1 Layer Alignment

Experimental Design Taking our approach to the extreme, the embedding space is a universal space, which depends only on the tokenizer, in which Transformer parameters and hidden states reside. Thus, we can align parameter vectors from different models in this space and compare them even if they come from different models, as long as they share a vocabulary.

To demonstrate this, we use MultiBERTs ([Sellam et al., 2022]; License: Apache-2.0), which contains 25 different instantiations of BERT-base (110M parameters) initialized from different random seeds.⁶ We take parameters from two MultiBERT seeds and compute the correlation between their projections to embedding space. For example, let V_A, V_B be the FF values of models A and B. We can project the values into embedding space: $V_A E_A, V_B E_B$, where E_A, E_B are the respective embedding matrices, and compute Pearson correlation between projected values. This produces a similarity matrix $\tilde{\mathcal{S}} \in \mathbb{R}^{|V_A| \times |V_B|}$, where each entry is the correlation coefficient between projected values from the two models. We bin \hat{S} by layer pairs and average the absolute value of the scores in each bin (different models might encode the same information in different directions, so we use absolute value) to produce a matrix $S \in \mathbb{R}^{L \times L}$, where L is the number of layers – that is, the average (absolute) correlation between vectors that come from layer ℓ_A in model A and layer ℓ_B in Model B is registered in entry (ℓ_A, ℓ_B) of S.

Last, to obtain a one-to-one layer alignment, we use the Hungarian algorithm [Kuhn, 1955], which assigns exactly one layer from the first model to a layer from the second model. The algorithm's objective is to maximize, given a similarity matrix S, the sum of scores of the chosen pairs, such that each index in one model is matched with exactly one index in the other. We repeat this for all parameter groups (W_O, W_K, W_V, W_O, K) .

Results and Discussion Figure 3 (left) shows the resulting alignment. Clearly, parameters from a certain layer in model A tend to align to the same layer in model B across all parameter groups. This suggests that different layers from different models that were trained separately (but with the same training objective and data) serve a similar function. As further evidence, we show that if not projected, the matching appears absolutely random in Figure §3 (right). We show the same results for other seed pairs as well in Appendix B.3.

5.2 Zero-shot Stitching

Model stitching [Lenc and Vedaldi, 2015; Csiszárik et al., 2021; Bansal et al., 2021] is a relatively underexplored feature of neural networks, particularly in NLP. The idea is that different models, even with different architectures, can learn representations that can be aligned through a *linear* transformation, termed *stitching*. Representations correspond to hidden states, and thus one can learn a transformation matrix from one model's hidden states to an equivalent hidden state in the other model. Here, we show that going through embedding space one can align the hidden states of two models, i.e., stitch, *without training*.

Given two models, we want to find a linear stitching transformation to align their representation spaces.

⁶Estimated compute costs: around 1728 TPU-hours for each pre-training run, and around 208 GPU-hours plus 8 TPU-hours for associated fine-tuning experiments.

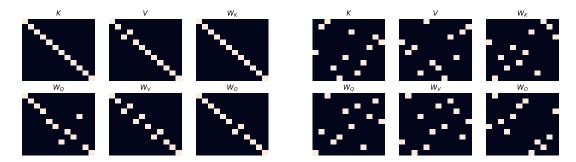


Figure 3: Left: Aligning *in embedding space* the layers of two different BERT models initialized from different random seeds for all parameter groups. Layers that have the same index tend to align with one another. Right: Alignment in feature space leads to unintelligible patterns.

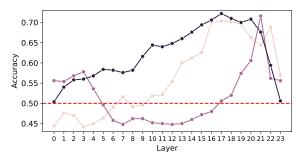


Figure 4: Accuracy on the IMDB evaluation set. We ran stitching randomly 11 times and obtained 3 models with higher than random accuracy when stitching over top layers. Dashed red line indicates random performance.

According to our theory, given a hidden state $v \in \mathbb{R}^{d_1}$ from model A, we can project it to the embedding space as vE_A , where E_A is its embedding matrix. Then, we can re-project to the feature space of model B, with $E_B^+ \in \mathbb{R}^{e \times d_2}$, where E_B^+ is the Penrose-Moore pseudoinverse of the embedding matrix E_B . This transformation can be expressed as multiplication with the kernel $K_{AB} := E_A E_B^+ \in \mathbb{R}^{d_1 \times d_2}$. We employ the above approach to take representations of a fine-tuned classifier, A, and stitch them on top of a model B that was only pretrained, to obtain a new classifier based on B.

Experimental Design We use the 24-layer GPT-2 medium as model A and 12-layer GPT-2 base model trained in §4.3 as model B. We fine-tune the last three layers of model B on IMDB, as explained in §4.3. Stitching is simple and is performed as follows. Given the sequence of N hidden states $H_A^\ell \in \mathbb{R}^{N \times d_1}$ at the output of layer ℓ of model A (ℓ is a hyperparameter), we apply the *stitching layer*, which multiplies the hidden states with the kernel, computing $H_A^\ell K_{AB}$. This results in hidden states $H_B \in \mathbb{R}^{N \times d_2}$, used as input to the three fine-tuned layers from B.

Results and Discussion Stitching produces models with accuracies that are higher than random on IMDB evaluation set, but not consistently. Figure 4 shows the accuracy of stitched models against the layer index from model A over which stitching is performed.

Out of 11 random seeds, three models obtained accuracy that is significantly higher than the baseline 50% accuracy, reaching an accuracy of roughly 70%, when stitching is done over the top layers.

6 Related Work

Interpreting Transformers is a broad area of research that has attracted much attention in recent years. A large body of work has focused on analyzing hidden representations, mostly through probing [Adi et al., 2016; Shi et al., 2016; Tenney et al., 2019; Rogers et al., 2020]. [Voita et al., 2019a] used statistical tools to analyze the evolution of hidden representations throughout layers. Recently, [Mickus et al., 2022] proposed to decompose the hidden representations into the contributions of different Transformer components. Unlike these works, we interpret parameters rather than the hidden representations.

Another substantial effort has been to interpret specific network components. Previous work analyzed single neurons [Dalvi et al., 2018; Durrani et al., 2020], attention heads [Clark et al., 2019; Voita et al., 2019b], and feedforward values [Geva et al., 2020; Dai et al., 2021; Elhage et al., 2022]. While these works mostly rely on input-dependent neuron activations, we inspect "static" model parameters, and provide a comprehensive view of all Transformer components.

Our work is most related to efforts to interpret specific groups of Transformer parameters. [Cammarata et al., 2020] made observations about the interpretability of weights of neural networks. [Elhage et al., 2021] analyzed 2-layer attention networks. We extend their analysis to multi-layer pre-trained Transformer models. [Geva et al., 2020, 2022a,b] interpreted feedforward values in embedding space. We coalesce these lines of work and offer a unified interpretation framework for Transformers in embedding space.

7 Discussion

While our work has limitations (see §8), we think the benefits of our work overshadow its limitations. We provide a simple approach and a new set of tools to interpret Transformer models and compare them. The realm of input-independent interpretation methods is

⁷Since we are not interested in interpretation we use an exact right-inverse and not the transpose.

still nascent and it might provide a fresh perspective on the internals of the Transformer, one that allows to glance intrinsic properties of specific parameters, disentangling their dependence on the input. Moreover, many models are prohibitively large for practitioners to run. Our method requires only a fraction of the compute and memory requirements, and allows interpreting a single parameter in isolation.

Importantly, our framework allows us to view parameters from different models as residents of a canonical embedding space, where they can be compared in model-agnostic fashion. This has interesting implications. We demonstrate two consequences of this observation (model alignment and stitching) and argue future work can yield many more use cases.

8 Limitations

Our work has a few limitations that we care to highlight. First, it focuses on interpreting models through the vocabulary lens. While we have shown evidence for this, it does not preclude other factors from being involved. Second, we used $E'=E^{\rm T}$, but future research may find variants of E that improve performance. Additionally, most of the work focused on GPT-2. This is due to shortcomings in the current state of our framework, as well as for clear presentation. We believe nonlinearities in language modeling are resolvable, as is indicated in the experiment with BERT.

In terms of potential bias in the framework, some parameters might consider terms related to each due to stereotypes learned from the corpus.

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A Rethinking Interpretation

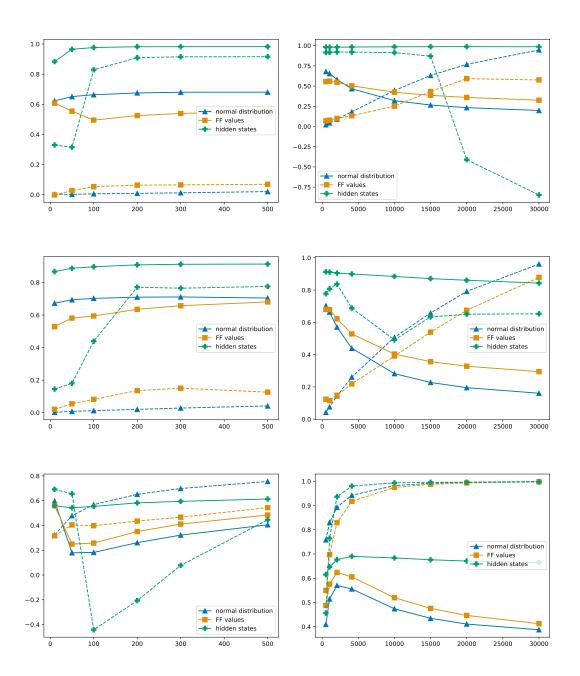


Figure 5: Each row represents a model in the following order from top to bottom: GPT-2 base, GPT-2 medium, GPT-2 large. Left: The keep-k inverse scores for three distributions: normal distribution, hidden states, and FF values, for $k \in \{10, 50, 100, 200, 300, 500\}$. Right: for $k \in \{10, 50, 100, 200, 300, 500\}$.

The process of interpreting a vector v in [Geva et al., 2022b] proceeds in two steps: first the *projection* of the vector to the embedding space (vE); then, we use the list of the tokens that were assigned the largest values in the projected vector, i.e.: top-k(vE), as the *interpretation* of the projected vector. This is reasonable since (a) the most activated coordinates contribute the most when added to the residual stream, and (b) this matches how we eventually decode: we project to the embedding space and consider the top-1 token (or one of the few top tokens, when using beam search).

In this work, we interpret inner products and matrix multiplications in the embedding space: given two vectors $x, y \in \mathbb{R}^d$, their inner product x^Ty can be considered in the embedding space by multiplying with E and then by one of its right inverses (e.g., its pseudo-inverse E^+ [Moore, 1920; Bjerhammar, 1951; Penrose, 1955]): $x^Ty = x^TEE^+y = (x^TE)(E^+y)$. Assume xE is interpretable in the embedding space, crudely meaning that it represents logits over vocabulary items. We expect y, which interacts with x, to also be interpretable in the embedding

space. Consequently, we would like to take E^+y to be the projection of y. However, this projection does not take into account the subsequent interpretation using top-k. The projected vector E^+y might be harder to interpret in terms of its most activated tokens. To alleviate this problem, we need a different "inverse" matrix E' that works well when considering the top-k operation. Formally, we want an E' with the following "robustness" guarantee: $\ker \mathbb{E} = \mathbb{E} \times \mathbb{E} = \mathbb{E} \times \mathbb$

This is a stronger notion of inverse – not only is $EE' \approx I$, but even when truncating the vector in the embedding space we can still reconstruct it with E'.

We claim that E^{T} is a decent instantiation of E' and provide some empirical evidence. While a substantive line of work [Ethayarajh, 2019; Gao et al., 2019; Wang et al., 2020; Rudman et al., 2021] has shown that embedding matrices are not isotropic (an isotropic matrix E has to satisfy $EE^{\mathrm{T}} = \alpha I$ for some scalar α), we show that it is isotropic enough to make E^{T} a legitimate compromise. We randomly sample 300 vectors drawn from the normal distribution $\mathcal{N}(0,1)$, and compute for every pair x,y the cosine similarity between $x^{\mathrm{T}}y$ and $k = -k(x^{\mathrm{T}}E)k = -k(E'y)$ for k = 1000, and then average over all pairs. We repeat this for $E' \in \{E^+, E^{\mathrm{T}}\}$ and obtain a score of 0.10 for E^+ , and 0.83 for E^{T} , showing the E^{T} is better under when using top-k. More globally, we compare $E' \in \{E^+, E^{\mathrm{T}}\}$ for $k \in \{10, 50, 100, 200, 300, 500\}$ with three distributions:

- x,y drawn from the normal $\mathcal{N}(0,1)$ distribution
- x, y chosen randomly from the FF values
- x, y drawn from hidden states along Transformer computations.

In Figure 5 we show the results, where dashed lines represent E^+ and solid lines represent $E^{\rm T}$. The middle row shows the plots for GPT-2 medium, which is the main concern of this paper. For small values of k (which are more appropriate for interpretation), $E^{\rm T}$ is superior to E^+ across all distributions. Interestingly, the hidden state distribution is the only distribution where E^+ has similar performance to $E^{\rm T}$. Curiously, when looking at higher values of k the trend is reversed ($k = \{512, 1024, 2048, 4096, 10000, 15000, 20000, 30000\}$) - see Figure 5 (Right).

This settles the deviation from findings showing embedding matrices are not isotropic, as we see that indeed as k grows, $E^{\rm T}$ becomes an increasingly bad approximate right-inverse of the embedding matrix. The only distribution that keeps high performance with $E^{\rm T}$ is the hidden state distribution, which is an interesting direction for future investigation.

For completeness, we provide the same analysis for GPT-2 base and large in Figure 5. We can see that GPT-2 base gives similar conclusions. GPT-2 large, however, seems to show a violent zigzag movement for E^+ but for most values it seems to be superior to $E^{\rm T}$. It is however probably best to use $E^{\rm T}$ since it is more predictable. This zigzag behavior is very counter-intuitive and we leave it for future work to decipher.

B Additional Material

B.1 Corresponding Parameter Pairs are Related

We define the following metric applying on vectors after projecting them into the embedding space:

$$\mathrm{Sim}_k(\hat{x},\hat{y}) = \frac{|\mathtt{top-k}(\hat{x}) \cap \mathtt{top-k}(\hat{y})|}{|\mathtt{top-k}(\hat{x}) \cup \mathtt{top-k}(\hat{y})|}$$

where $t \circ p - k(v)$ is the set of k top activated indices in the vector v (which correspond to tokens in the embedding space). This metric is the Jaccard index [Jaccard, 1912] applied to the top-k tokens from each vector. In Figure 6, Left, we demonstrate that FF key vectors and their corresponding value vectors are more similar (in embedding space) than two random key and value vectors. In Figure 6, Right, we show a similar result for attention value and output vectors. In Figure 6, Bottom, the same analysis is done for attention query and key vectors. This shows that there is a much higher-than-chance relation between corresponding FF keys and values (and the same for attention values and outputs).

B.2 Final Prediction and Parameters

We show that the final prediction of the model is correlated in embedding space with the most activated parameters from each layer. This implies that these objects are germane to the analysis of the final prediction in the embedding space, which in turn suggests that the embedding space is a viable choice for interpreting these vectors. Figure 7 shows that just like §4.2, correspondence is better when hidden states are not randomized, suggesting their parameter interpretations have an impact on the final prediction.

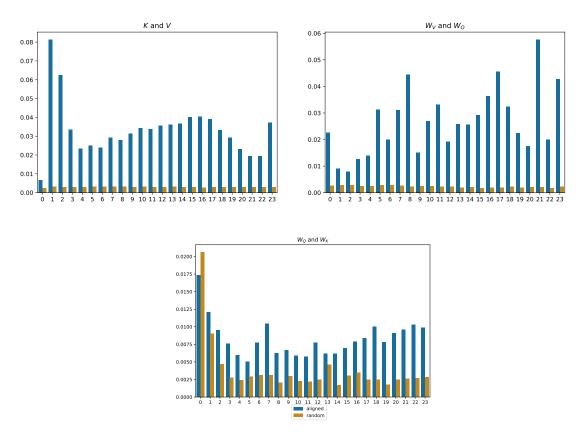


Figure 6: Average $\mathrm{Sim}_k(\hat{x},\hat{y})$ for k=100 by layer, where blue is when matching pairs are aligned, and orange is when pairs are shuffled within the layer. Top Left: FF keys and FF values. Top Right: The subheads of W_O and W_V . Bottom: The subheads of W_Q and W_K .

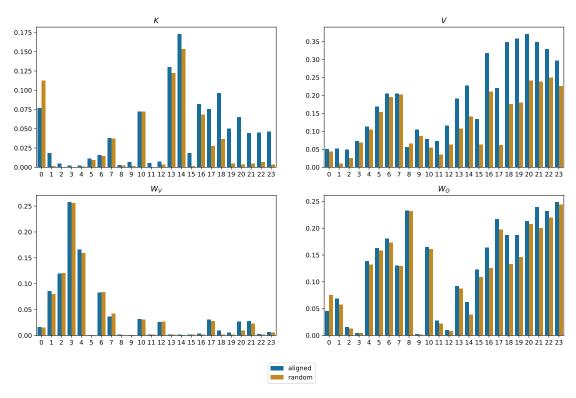
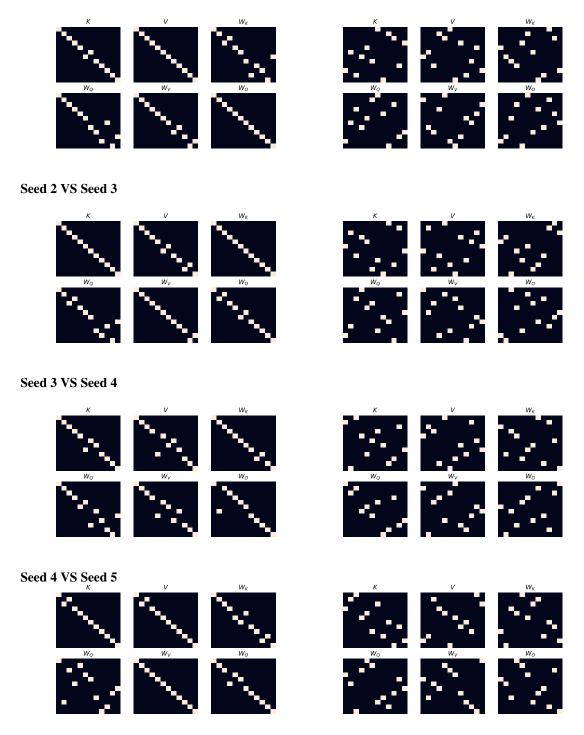


Figure 7: Left: Average R_k score (k=100) across tokens per layer for activated parameter vectors against both the aligned hidden state \hat{h} at the output of the *final* layer and a randomly sampled hidden state \hat{h}_{rand} . Parameters are FF keys (top-left), FF values (top-right), attention values (bottom-left), and attention outputs (bottom-right).

B.3 Parameter Alignment Plots for Additional Model Pairs

Alignment in embedding space of layers of pairs of BERT models trained with different random seeds for additional model pairs.

Seed 1 VS Seed 2



C Example Cases

C.1 Wvo Matrices

Below we show output-value pairs from different heads of GPT-2 medium. For each head, we show the 50 pairs with the largest values in the $e \times e$ transition matrix. There are 384 attention heads in GPT-2 medium from which we manually choose a subset. Throughout the section some lists are marked with asterisks indicating the way this particular list was created:

- * pairs of the form (x,x) were excluded from the list
- ** pairs where both items are present in the corpus (we use IMDB training set).

Along with GPT-2 medium, we also provide a few examples from GPT-2 base and GPT-2 large.

C.1.1 Low-Level Language Modeling

GPT-2 Medium - Layer 21 Head 7*

```
('NF', 'FN'),
('Ram', ' Ramos'),
('Hug', ' Hughes'),
('nug', 'nughes'),
('gran', 'GR'),
('FN', 'NF'),
('CLA', 'CL'),
('McC', 'McCain'),
('Marsh', 'Marshall'),
('Hughes', 'Hug'),
('Tan', 'Tanner'),
('nih', 'NH'),
('NRS', 'NR'),
(' Bowman', 'Bow'),
(' Marshall', 'Marsh'),
('Jac', 'Jacobs'),
('Hay', 'Hayes'),
(' Hayes', 'Hay'),
('McC', 'McCorm'),
('NI', 'NR'),
(' sidx', ' Dawson'),
(' Tanner', 'Tan'),
('gra', 'GR'),
('JA', 'jac'),
('zos', 'zo'),
('NI', 'NF'),
('McC', 'McCull'),
(' Jacobs', 'Jac'),
(' Beetle', ' Beet'),
('GF', 'FG'),
('jas', 'ja'),
('Wil', 'Wilkinson'),
('Ramos', 'Ram'),
('GRE', 'GR'),
('NF', 'FN'),
('McCorm', 'McC'),
('Scar', 'Scarborough'),
('Baal', 'Ba'),
('FP', 'FG'),
('FH', 'FN'),
(' Garfield', 'Gar'),
('jas', 'jac'),
('nuts', 'nut'),
('WI', 'Wis'),
(' Vaughn', ' Vaughan'),
('FP', 'PF'),
```

```
('RNA', 'RN'),

(' Jacobs', 'jac'),

('FM', 'FN'),

(' Knox', 'Kn'),

('NI', 'nic')
```

GPT-2 Medium - Layer 19 Head 13 (first letter/consonant of the word and last token of the word)

```
(' R', 'senal'),
                                         arsenal
 ('senal', 'R'),
 (' G', 'vernment'),
                                         government
 (' Madness', ' M'),
 (' M', ' Mayhem'),
(' W', 'nesday'),
                                         wednesday
 ('vernment', 'G'),
 ('M', 'Madness'),
('N', 'lace'),
                                         necklace
 ('nesday', 'W'),
 ('Rs', 'senal'),
('g', 'vernment'),
('N', 'farious'),
                                         nefarious
 ('eneg', ' C'),
 (' r', 'senal'),
(' F', 'ruary'),
                                         february
 ('senal', 'RIC'),
 (' R', 'ondo'),
(' N', ' Mandela'),
                                         nelson
 (' Mayhem', 'M'),
 (' RD', 'senal'),
(' KD', 'senal',
(' C', 'estine'),
('Gs', 'vernment'),
('RF', 'senal'),
(' N', 'esis'),
(' N', 'Reviewed'),
(' C', 'arette'),
                                         cigarette
('rome', 'N'),
('N', 'theless'),
                                         nonetheless
('lace', 'N'),
('H', 'DEN'),
('V', 'versa'),
('P', 'bably'),
                                         probably
 ('vernment', 'GF'),
 ('g', 'vernment'),
 ('GP', 'vernment'),
('C', 'ornia'),
                                         california
 ('ilipp', ' F'),
('ilipp', ' F'),
(' N', 'umbered'),
(' C', 'arettes'),
('RS', 'senal'),
(' N', 'onsense'),
('RD', 'senal'),
('RAL', 'senal'),
('F', 'uci'),
('R', 'ondo'),
 (' RI', 'senal'),
(' H', 'iday'),
                                     # holiday
 ('senal', ' Rx'),
 (' F', 'odor')
```

GPT-2 Medium - Layer 20 Head 9

```
('On', ' behalf'),
(' On', ' behalf'),
(' on', ' behalf'),
('during', ' periods'),
('within', ' bounds'),
(' inside', ' envelope'),
('outside', 'door'),
('inside', ' envelope'),
(' Under', ' regime'),
```

```
(' during', ' periods'),
(' LIKE', 'lihood'),
(' on', ' occasions'),
('Under', ' regime'),
('inside', 'door'),
('during', 'period'),
('Like', 'lihood'),
                                                                                                                       (' documentaries', ' films')
(' microphone', ' microphones')
(' cameras', ' camera')
('Journal', ' journals')
(' restrooms', ' bathrooms')
(' tasks', ' chores')
                                                                                                                         (' perspectives', ' viewpoints')
                                                                                                                       (' perspectives', ' viewpoin
(' shelf', ' shelves')
(' rooms', ' bedrooms')
(' hurdle', ' hurdles')
(' barriers', ' fences')
(' magazines', ' journals')
(' journals', 'Magazine')
(' sources', ' source')
(' manuals', ' textbooks')
('During', 'periods'),
('Inside', 'envelope'),
('for', 'sake'),
('inside', 'doors'),
('under', 'regime'),
('ON', 'behalf'),
('for', 'purposes'),
('On', 'occasions'),
                                                                                                                        ( sources', ' source')
(' manuals', ' textbooks')
(' story', ' stories')
(' labs', ' laboratories')
 ('inside', ' doors'),
(' on', ' basis'),
                                                                                                                        ('tales', 'Stories')
('chores', 'duties')
('roles', 'role')
('Under', 'regimes'),
('outside', 'doors'),
('inside', 'Osc'),
('During', 'periods'),
                                                                                                                        (' ceilings', ' walls')
('inside', 'door'),
('UNDER', 'regime'),
('under', 'regimes'),
                                                                                                                       (' microphones', ' microphone')
                                                                                                                        (' pathway', ' pathways')
( under', ' regimes'),
('Under', ' regimes'),
('inside', 'doors'),
('inside', 'zx'),
('during', ' period'),
                                                                                                                       GPT-2 Large - Layer 27 Head 6
                                                                                                                       (' where', 'upon'),
                                                                                                                      ('where', 'upon'),
('where', 'upon'),
('with', 'regards'),
('with', 'regards'),
('with', 'regards'),
('Where', 'upon'),
('Like', 'lihood'),
('of', 'course'),
('inside', 'ascript'),
('Inside', 'door'),
('On', 'occasions'),
 ('BuyableInstoreAndOnline', 'ysc'),
('Inside', 'envelope'),
('during', 'pauses'),
('under', 'regime'),
('on', 'occasion'),
                                                                                                                        (' with', ' regard'),
                                                                                                                        (' LIKE', 'lihood'),
                                                                                                                      ('Where', 'upon'),
('from', 'afar'),
('with', 'stood'),
('on', 'occasion'),
('outside', 'doors'),
('UNDER', 'banner'),
('within', 'envelope'),
('here', 'abouts'),
('during', 'duration')
                                                                                                                       (' FROM', ' afar'),
                                                                                                                       (' like', 'lihood'),
                                                                                                                      (' WHERE', 'upon'),
                                                                                                                     ('Like', 'lihood'),
\textit{GPT-2 Base} - Layer 10 Head 11^{**}
                                                                                                                     ('with', 'stood'),
('of', 'course'),
('of', 'course'),
('Of', 'course'),
 (' sources', 'ources')
 (' repertoire', ' reperto')
(' repertoire', ' reperto')
(' tales', ' stories')
(' stories', ' tales')
(' journals', ' magazines')
('stories', ' tales')
(' journal', ' journals')
(' magazines', 'Magazine')
(' magazines', ' newspapers')
(' reperto', ' repertoire')
(' cameras' ' Camer')
                                                                                                                    (' from', ' afar'),
(' WITH', ' regard'),
                                                                                                                     ('where', 'abouts'),
('with', 'impunity'),
                                                                                                                     ('WITH', 'regards'),
('With', 'stood'),
('for', 'purposes'),
 ('cameras', 'Camer')
('source', 'sources')
                                                                                                                     ('with', ' respect'),
                                                                                                                     ('With', 'stood'),
('like', 'lihood'),
('Of', 'course'),
 (' newspapers', ' magazines')
(' position', ' positions')
                                                                                                                      ('With', ' regard'),
 (' tale', ' tales')
                                                                                                                    ('With', ' regard'),
(' With', ' regard'),
('where', 'abouts'),
(' WITH', 'stood'),
('With', ' regards'),
(' OF', ' course'),
(' From', ' afar'),
(' with', ' impunity'),
(' With', ' regards'),
(' with', ' regards')
('positions', 'position')
('obstacles', 'hurdles')
('chores', 'tasks')
('journals', 'papers')
 (' role', ' roles')
(' hurdles', ' obstacles')
(' journals', ' journal')
(' windows', ' doors')
                                                                                                                     ('with', 'regards'),
('with', 'respect'),
('From', 'afar'),
('with', 'standing'),
('on', 'behalf'),
 (' ceiling', ' ceilings')
 ('loophole', 'loopholes')
('Sources', 'ources')
('source', 'sources')
```

```
(' by', 'products'),
(' for', ' purposes'),
(' or', 'acle'),
('for', ' sake'),
(' with', 'standing')
                                                                                             (' herself', 'Maria'),
                                                                                             (' her', 'Maria'),
                                                                                             (' herself', ' Anne'),
                                                                                             ('She', 'Maria'),
                                                                                             ('hers', 'Louise'),
('herself', 'Louise'),
('hers', 'Anne'),
('hers', 'pher'),
('she', 'Maria'),
C.1.2 Gender
GPT-2 Medium - Layer 18 Head 1
                                                                                             ('actress', 'actresses'),
('herself', 'Isabel'),
('herself', 'pher'),
('women', ' Marie'),
(' actresses', ' Marie'),
                                                                                             (' she', 'Maria'),
(' SHE', ' Marie'),
('women', 'Anne'),
('Women', 'Anne'),
('woman', 'Marie'),
('Women', 'Marie'),
                                                                                             (' herself', ' Gloria'),
(' herself', ' Amanda'),
(' Ivanka', ' Ivanka'),
(' her', ' Louise'),
('woman', 'Anne'),
('Woman', 'Marie'),
                                                                                              (' herself', ' Kate'),
(' actresses', ' Anne'),
(' heroine', ' Marie'),
                                                                                             (' her', 'pher'),
(' her', 'Anne'),
('Women', 'Jane'),
                                                                                             (' she', 'pher'),
('she', ' Louise'),
(' heroine', ' Anne'),
('meroine', 'Anne'),
('women', 'Jane'),
('Women', 'actresses'),
('Woman', 'Anne'),
('Women', 'Esther'),
('women', 'Esther'),
('girls', 'Marie'),
('Mrs', 'Anne'),
                                                                                             (' herself', 'Kate'),
                                                                                            (' she', ' Louise'),
(' she', ' Anne'),
(' She', ' Marie'),
(' she', ' Gloria'),
('She', ' Louise'),
                                                                                             (' hers', ' Gloria'),
(' actress', ' Marie'),
('women', 'actresses'),
('Woman', 'Jane'),
('girls', 'Marie'),
('actresses', 'Jane'),
                                                                                             (' herself', ' Diana'),
                                                                                             ('She', 'Gloria'),
('she', 'Anne'),
                                                                                            ('she', 'Anne'),

('she', 'pher'),

('Her', 'Marie'),

('she', 'Gloria'),

('Paleo', 'Paleo'),

('hers', 'Diana')
('Woman', 'Anne'),
('Girls', 'Marie'),
('women', 'Anne'),
('women', 'Anne'),
('Girls', 'Anne'),
('Woman', 'actresses'),
('Women', 'Marie'),
('Women', 'Anne'),
('girls', 'Anne'),
('girl', 'Anne'),
                                                                                              GPT-2 Base - Layer 9 Head 7**
                                                                                             ('her', 'herself')
('She', 'herself')
('she', 'herself')
('she', 'herself')
('Her', 'herself')
('Women', 'Anne'),
('Woman', 'Women'),
('girls', 'Anne'),
                                                                                             ('She', 'herself')
('actresses', 'Anne'),
                                                                                             (' SHE', ' herself')
('their', ' themselves')
('women', ' Michelle'),
                                                                                            ('tneir', ' themselves')
(' hers', ' herself')
('Their', ' themselves')
(' Her', ' herself')
(' Their', ' themselves')
(' THEIR', ' themselves')
(' Actress', ' Marie'),
('girl', ' Marie'),
(' Feminist', ' Anne'), (' women', ' Marie'),
('Women', 'Devi'),
('Women', 'Elizabeth'),
                                                                                              (' HER', ' herself')
                                                                                              (' their', ' themselves')
('They', ' themselves')
('His', ' himself')
(' actress', ' Anne'),
('Mrs', 'Anne'),
('answered', 'Answer'),
('woman', 'Anne'),
('Woman', 'maid'),
('women', 'Marie')
                                                                                              (' herself', 'erest')
                                                                                              ('they', 'themselves')
('his', 'himself')
                                                                                              ('Their', 'selves')
(' They', ' themselves')
GPT-2 Large - Layer 27 Head 12
(' herself', ' Marie'),

(' hers', ' Marie'),

('she', ' Marie'),

(' she', ' Marie'),

(' her', ' Marie'),

('She', ' Marie'),
                                                                                             (' herself', ' Louise')
                                                                                             ('their', 'selves')
('her', 'herself')
('his', 'himself')
                                                                                             (' herself', ' Marie')
                                                                                            ('He', ' himself')
('She', ' Louise')
(' they', ' themselves')
(' hers', 'Maria'),
(' actresses', ' actresses'),
```

```
(' Melbourne', ' Nadu')
(' Adelaide', ' Nadu')
(' Cambod', ' Nguyen')
 ('their', 'chairs')
(' herself', ' dow')
(' herself', 'eva')
(' THEY', ' themselves')
                                                                                                            (' Vietnamese', ' Nguyen')
(' herself', ' Mae')
(' His', ' himself')
 ('clinton', 'enegger')
                                                                                                           GPT-2 Medium - Layer 16 Head 6*
 ('She', 'erest')
(' her', ' Louise')
                                                                                                          ('Chennai', 'Mumbai'),
('India', 'Mumbai'),
('Mumbai', 'Chennai'),
(' herself', ' Devi')
(' Their', 'selves')
('Their', 'chairs')
                                                                                                            (' Queensland', ' Tasmania'),
                                                                                                            ('India', 'Rahul'), ('India', 'Gujar'),
(' Himself', 'enegger')
(' she', ' Louise')
(' herself', ' Anne')
                                                                                                            ('Chennai', 'Bangalore'),
('England', 'Scotland'),
('Chennai', 'Kerala'),
('Delhi', 'Mumbai'),
('Britain', 'Scotland'),
 ('Its', 'itself')
('her', 'erest')
 (' herself', ' Christina')
 ('she', 'erest')
                                                                                                            ('Bangalore', 'Mumbai'),
('Pakistan', 'India'),
('Scotland', 'Ireland'),
('Mumbai', 'Bangalore'),
 ('their', 'selves')
C.1.3 Geography
                                                                                                            ('Bangalore', 'Chennai'),
('Aadhaar', 'Gujar'),
('Mumbai', 'Maharashtra'),
GPT-2 Base - Layer 11 Head 2**
('Halifax', 'Scotia')
('Saudi', 'Arabia')
('Nova', 'Scotia')
('Tamil', 'Nadu')
('Finnish', 'onen')
('Saudi', 'Arabia')
('Pitt', 'sburgh')
('Dutch', 'ijk')
                                                                                                            (' Maharashtra', ' Gujarat'),
                                                                                                            ('Gujarat', 'Gujar'),
('Australian', 'Australia'),
                                                                                                            ('India', 'Gujarat'), ('Rahul', 'Gujar'),
                                                                                                            (' Maharashtra', ' Mumbai'),
                                                                                                            ('Britain', 'England'), ('India', 'Chennai'),
('Schwartz', 'enegger')
('Afghans', 'Kabul')
('Icelandic', 'sson')
('Finland', 'onen')
                                                                                                          ('India', 'Chennai'),
('Mumbai', 'Bombay'),
('Tamil', 'Kerala'),
('Hindi', 'Mumbai'),
('Tasmania', 'Tasman'),
('Mumbai', 'India'),
('Hindi', 'Gujar'),
('Mobarachtra', 'Cujar')
 ('Pitt', 'enegger')
('Czech', 'oslov')
('Manitoba', 'Winnipeg')
('Malaysian', 'Lumpur')
('Swedish', 'borg')
                                                                                                         ('Maharashtra', 'Gujar'),
('Australians', 'Austral'),
('Maharashtra', 'Kerala'),
 (' Saskatchewan', ' Sask')
(' Saskatchewan', ' Sask')
(' Chennai', ' Nadu')
(' Argentine', ' Aires')
(' Iceland', ' Icelandic')
(' Swedish', 'sson')
(' Tasman', ' Nadu')
('Houston', ' Astros')
('Colorado', ' Springs')
(' Kuala', ' Lumpur')
('Tai', 'pport')
                                                                                                          ('India', 'Bangalore'),
('India', 'Kerala'),
('India', 'Bombay'),
                                                                                                          ('Australia', 'Austral'),
('Aadhaar', 'India'),
('Sharma', 'Mumbai'),
                                                                                                          ('Australian', 'Austral'),
('Mumbai', 'Kerala'),
('Scotland', 'England'),
('Mumbai', 'Gujar'),
('Rahul', 'Mumbai'),
 ('Tai', 'pport')
('Houston', 'Dynamo')
('Manitoba', 'Marginal')
('Afghan', 'Kabul')
('Buenos', 'Aires')
                                                                                                            (' Queensland', ' Tasman'),
                                                                                                           ('Tamil', 'Chennai'),
('Gujarat', 'Maharashtra'),
('India', 'Modi')
 ('Alberta', 'Calgary')
('Stockholm', 'sson')
('Stocknorm, Sson,
('Sweden', 'borg')
('Brazil', 'Paulo')
('Iceland', 'sson')
('Winnipeg', 'Manitoba')
('Sweden', 'sson')
                                                                                                            GPT-2 Medium - Layer 16 Head 2*
                                                                                                           ('Austral', ' Australians'), ('Australia', 'Austral'),
                                                                                                          ('Australia', 'Austral'),
('Canberra', 'Austral'),
('Austral', 'Canberra'),
('Winnipeg', 'Edmonton'),
('Australian', 'Austral'),
('Alberta', 'Edmonton'),
 (' Carolina', ' Hurricanes')
('Dutch', 'ijk')
('Swed', 'borg')
('Aki', 'pport')
(' Winnipeg', 'Marginal')
(' Argentine', ' pes')
(' Halifax', 'imore')
(' Brisbane', 'enegger')
                                                                                                          ('Australia', ' Australians'),
                                                                                                          (' Australians', 'Austral'),
                                                                                                            ('Ukraine', 'ovych'),
```

```
(' Quebec', ' Canad'),
                                                                                           ('Brazil', ' Brazilian'),
('Quebec', 'Canad'),
('Australian', 'Australians'),
('Winnipeg', 'Manitoba'),
('Manitoba', 'Winnipeg'),
('Canadian', 'Canada'),
('Moscow', 'Bulgar'),
                                                                                          ('Bulgarian', 'Bulgar'),
('Malaysian', 'Malays'),
                                                                                          ('Ankara', 'oglu'),
('Bulgaria', 'Bulgarian'),
('Malays', 'Indones'),
                                                                                           (' Taiwanese', ' Tai'),
(' Manitoba', ' Edmonton'),
                                                                                           ('Turkey', 'oglu'),
('Brazil', ' Janeiro'),
('Italian', 'zzi'),
(' Kuala', ' Malays'),
('berra', 'Austral'),
('Austral', 'Australian'),
(' Ukrainians', 'ovych'),
('Canada', ' Canadians'),
                                                                                          ('Kuala', ralays',
('Japanese', 'Fuk'),
('Jakarta', 'Indonesian'),
('Taiwanese', 'Taiwan'),
('Erdogan', 'oglu'),
('Viet', 'Nguyen'),
(' Canberra', ' Australians'),
('Canada', 'Canadian'),
(' Yanukovych', 'ovych'),
('Canada', 'Trudeau'), ('Dmitry', 'Bulgar'),
                                                                                           ('Philippine', 'Filipino'),
('Jakarta', 'Indonesia'),
('Koreans', 'Jong'),
('Filipino', 'Duterte'),
('Australia', 'Austral'),
('Mulcair', 'Canad'),
('berra', 'Canberra'),
('Turkish', 'oglu'),
('udeau', 'Canada'),
                                                                                            (' Azerbaijan', ' Azerbai'),
('Edmonton', 'Oilers'),
('Australia', 'Canberra'),
('Canada', 'Edmonton'),
                                                                                            (' Bulgar', ' Bulgarian')
                                                                                           GPT-2 Large - Layer 23 Head 5
('Edmonton', 'Calgary'),
('Alberta', 'Calgary'),
('udeau', 'Trudeau'),
                                                                                           ('Canada', ' Trudeau'),
                                                                                          ('Canadians', 'Trudeau'),
('Canadian', 'Trudeau'),
(' Calgary', ' Edmonton'),
                                                                                          (' Queensland', ' Tasman'),
                                                                                         (' Queensland', ' lasman'),
(' Tasman', ' Tasman'),
(' Canada', ' Trudeau'),
(' Canberra', ' Canberra'),
(' Winnipeg', ' Winnipeg'),
(' Canberra', ' Tasman'),
('Canadian', 'Canada'),
'' Canadian', 'Trudeau')
('Canadian', 'Trudeau'),
('Australian', 'Canberra'),
('Vancouver', 'Canucks'),
('Australia', 'Australian'),
('Vancouver', 'Fraser'),
('Canadian', 'Edmonton'),
('Austral', 'elaide'),
                                                                                           ('Canadian', 'Trudeau'),
('Brisbane', 'Brisbane'),
('Tex', 'Braz'),
('Canada', 'RCMP'),
                                                                                          ('Brisbane', 'Brisbane'),
('Quebec', 'Trudeau'),
('Canadian', 'Canadian'),
('Brisbane', 'Tasman'),
('Tasmania', 'Tasman'),
('Canadian', 'Canadians'),
('RCMP', 'Trudeau'),
('Manitaba', 'Trudeau')
('Moscow', 'sov'),
('Russia', 'Bulgar'),
(' Canadians', 'Canada')
GPT-2 Medium - Layer 21 Head 12*
                                                                                           ('Manitoba', 'Trudeau'),
('Queensland', 'Brisbane'),
('Queensland', 'Canberra'),
(' Indonesian', ' Indones'),
(' Vietnamese', ' Nguyen'),
(' Indonesian', ' Jakarta'),
(' Indonesian', ' Indonesia'),
                                                                                           ('Canada', 'Saskatchewan'),
('Canadian', 'Saskatchewan'),
('Canada', 'Canadian'),
('RCMP', 'Saskatchewan'),
('Turkish', 'oglu'),
('Indonesia', 'Indones'),
('Jakarta', 'Indones'),
('Korean', 'Koreans'),
                                                                                           ('Canberra', 'Brisbane'),
('Canadians', 'Canada'),
('Winnipeg', 'Trudeau'),
('Canadian', 'Canada'),
('Canada', 'Canadians'),
('Turkish', 'oglu'),
('Taiwan', 'Taiwanese'),
('Thai', 'Nguyen'),
('Brazilian', 'Brazil'),
('Indones', 'Indonesia'),
                                                                                            ('Australian', ' Canberra'),
('Tai', ' Taiwanese'),
                                                                                            (' Melbourne', ' Canberra'),
('Istanbul', 'oglu'),
('Indones', 'Indonesian'),
('Indones', 'Jakarta'),
('Laos', 'Nguyen'),
                                                                                            (' RCMP', ' Canad'),
                                                                                           (' Canadians', ' Canadians'),
                                                                                           ('CBC', 'Trudeau'),
                                                                                           ('Canadian', 'Canadian'), ('Canadian', 'Winnipeg'),
('Slovenia', 'Sloven'),
('Koreans', 'Korean'),
('Cambod', 'Nguyen'),
                                                                                           (' Australians', ' Canberra'),
                                                                                           (' Quebec', 'Canada'),
('Italy', 'zzi'),
                                                                                           (' Canadian', 'Canada'),
                                                                                          ('NSW', 'Canberra'),
('Toronto', 'Canada'),
('Canada', 'Canada'),
('NSW', 'Tasman'),
('RCMP', 'RCMP'),
(' Taiwanese', 'Tai'),
(' Indonesia', ' Jakarta'),
('Indonesia', 'Jakarta'),
('Indonesia', 'Indonesian'),
('Bulgarian', 'Bulgaria'),
('Iceland', 'Icelandic'),
('Korea', 'Koreans'),
                                                                                            (' Canadian', ' Canadians'),
```

```
(' Saskatchewan', ' Saskatchewan'),
(' Canadians', ' Saskatchewan'),
('Canadian', ' Canad'),
(' Ottawa', ' Winnipeg')
                                                                                                                       (' unbiased', ' equitable'),
('failed', ' inconsistent'),
                                                                                                                         (' liberated', ' emanc'),
(' humane', ' equitable'),
                                                                                                                          (' liberating', ' liberated'),
                                                                                                                     ('failed', 'incompatible'),
('failed', 'incompatible'),
('miracles', 'mirac'),
('peacefully', 'consensual'),
('unconditional', 'uncond'),
('unexpectedly', 'unexpected'),
('untouched', 'unconditional'),
('healthier', 'Better'),
('unexpected', 'unexpectedly'),
('peacefully', 'graceful'),
('emancipation', 'emanc'),
('seamlessly', 'effortlessly'),
('peacefully', 'honorable'),
('uncond', 'unconditional'),
('excuses', 'rubbish'),
('liberating', 'emanc'),
('peacefully', 'equitable'),
('gracious', 'Feather'),
('liberated', 'emancipation'),
('nuances', 'nuanced'),
('avoids', 'icable'),
                                                                                                                          ('failed', 'incompatible'),
C.1.4 British Spelling
GPT-2 Medium - Layer 19 Head 4
(' realise', ' Whilst'),
(' Whilst', ' Whilst'),
(' Whilst', ' Whilst'),
(' realised', ' Whilst'),
(' organise', ' Whilst'),
(' recognise', ' Whilst'),
(' civilisation', ' Whilst'),
(' organisation', ' Whilst'),
 (' whilst', ' Whilst'),
(' organising', ' Whilst'),
(' organised', ' Whilst'),
(' organis', ' Whilst'),
(' util', ' Whilst'),
(' apologise', ' Whilst'),
(' emphas', ' Whilst'),
(' analyse', ' Whilst'),
('analyse', whilst'),
('organisations', 'Whilst'),
('recognised', 'Whilst'),
('flavours', 'Whilst'),
('colour', 'Whilst'),
                                                                                                                      ('nuances', 'nuanced',
('avoids', 'icable'),
('freeing', 'liberated'),
('freeing', 'liberating'),
('lousy', 'inconsistent'),
('failed', 'lousy'),
(' colour', ' Whilst'),
('colour', ' Whilst'),
(' Nasa', ' Whilst'),
(' Nato', ' Whilst'),
(' analys', ' Whilst'),
(' flavour', ' Whilst'),
(' colourful', ' Whilst'),
(' colours', ' Whilst'),
(' realise', ' organising'),
(' behavioural', ' Whilst')
                                                                                                                          (' unaffected', ' unconditional'),
                                                                                                                        ('ivable', ' equitable'), ('Honest', ' equitable'),
                                                                                                                        (' principled', 'erning'),
                                                                                                                          ('surv', ' survival'),
                                                                                                                        (' lackluster', 'ocre'),
(' liberating', ' equitable'),
(' behavioural', ' Whilst'),
(' coloured', ' Whilst'),
(' learnt', ' Whilst'),
                                                                                                                       ('Instead', 'Bah'),
(' inappropriate', ' incompatible'),
                                                                                                                       (' emanc', ' emancipation'),
 (' favourable', ' Whilst'),
                                                                                                                       ('unaffected', 'unchanged'),
('peaceful', 'peacefully'),
('safer', 'equitable'),
 ('isation', ' Whilst'),
 (' programmes', ' Whilst'),
 (' realise', ' organis'),
                                                                                                                         (' uninterrupted', ' unconditional')
('authorised', 'Whilst'),
('practise', 'Whilst'),
('criticised', 'Whilst'),
                                                                                                                        GPT-2 Medium - Layer 12 Head 14*
                                                                                                                     (' died', ' perished'),
(' dies', ' perished'),
(' testifying', ' testify'),
(' interven', ' intervened'),
(' advising', ' advises'),
(' disband', ' disbanded'),
(' perished', 'lost'),
(' perished', ' died'),
(' applaud', ' applauded'),
(' dictate', ' dictates'),
(' prevailed', ' prev'),
 (' organisers', ' Whilst'),
 (' organise', ' organising'),
(' analysed', ' Whilst'),
 ('programme', 'Whilst'),
('behaviours', 'Whilst'),
('humour', 'Whilst'),
('isations', 'Whilst'),
('tyres', 'Whilst'),
(' tyres', ' WHILLSC',,
(' aluminium', ' Whilst'),
(' realise', ' organised'),
(' favour', ' Whilst'),
(' ageing', ' Whilst'),
(' organise', ' organis')
                                                                                                                         (' prevailed', ' prev'),
(' advising', ' advise'),
                                                                                                                         ('thood', 'shed'),
('orsi', 'Reviewed'),
                                                                                                                         (' perished', ' dies'),
(' publishes', 'published'),
(' prevail', ' prevailed'),
(' dies', ' died'),
C.1.5 Related Words
\textit{GPT-2 Medium} - Layer 13 Head 8^*
                                                                                                                        (' testifying', ' testified'),
(' testify', ' testifying'),
(' governs', ' dictates'),
 (' miraculous', ' mirac'),
(' miracle', ' mirac'),
(' nuance', ' nuanced'),
(' smarter', 'Better'),
                                                                                                                      (' complicity', ' complicit'),
(' dictate', ' dictated'),
 (' healthier', ' equitable'),
(' liberated', ' liberating'),
(' untouched', ' unaffected'),
                                                                                                                      ('CHO', 'enough'),
('independence', 'skelet'),
```

```
(' prescribe', ' Recomm'),
(' perished', 'essential'),
                                                                                                                                 (' adequately', ' adequate'),
(' correctly', ' properly'),
(' Worse', ' hurting'),
   ('CHO', 'noticed'),
  ('approving', 'avorable'),
('perished', 'perish'),
('oversee', 'overseeing'),
                                                                                                                                  (' worse', 'nutring'),
(' correctly', ' Proper'),
(' fails', ' fail'),
(' incorrectly', ' mistaken'),
(' adversely', ' harming')
  ('shed', ' skelet'),
('chart', 'EY'),
(' overseeing', ' presiding'),
                                                                                                                                    GPT-2 Large - Layer 24 Head 9
                                                                                                                                (' interviewer', ' interviewer'),
(' lectures', ' lectures'),
(' lecture', ' lecture'),
  ('pees', ' fundament'), ('appro', ' sanction'),
   (' prevailed', ' prevail'),
(' regulates', ' governs'),
                                                                                                                                (' interview', 'Interview'),
(' interview', ' interview'),
(' interview', ' interviewer'),
(' interviewing', ' interviewing'),
  ('shed', 'tails'),
('chart', 'Period'),
('hower', 'lihood'),
                                                                                                                                 (' interviewing', ' interviewing')
(' magazine', ' magazine'),
(' Reviews', ' Reviews'),
(' reviewer', ' reviewer'),
(' reviewers', ' reviewers'),
(' lectures', ' lecture'),
(' testers', ' testers'),
(' editors', ' editors'),
(' interviewer', ' interview'),
(' Interviewer', ' Interview'),
(' interviewer', ' Interview').
  ('prevail', 'prev'),
('helps', 'aids'),
('dict', 'dictated'),
  (' dictates', ' dictated'),
  ('itta', ' Dise'),
('CHO', 'REC'),
  ('ORTS', 'exclusive'),
('helps', 'Helpful'),
('ciples', 'bart')
                                                                                                                                     (' interviewer', 'Interview'),
  GPT-2 Medium - Layer 14 Head 1*
                                                                                                                                    ('Interview', 'Interview'), (' lecture', ' lectures'),
  (' incorrectly', ' misunderstand'),
(' properly', ' Proper'),
(' incorrectly', ' inaccur'),
(' wrongly', ' misunderstand'),
                                                                                                                                    (' interviewing', ' interviewer'),
                                                                                                                                  (' journal', ' journal'),
(' interviewer', ' interviewing'),
  (' wrongly', ' misunderstand'),
(' incorrectly', ' misinterpret'),
(' incorrectly', ' incorrect'),
(' incorrectly', ' mistakes'),
(' incorrectly', ' misunderstanding'),
(' properly', ' proper'),
(' incorrectly', ' fail')
                                                                                                                                   (' blogs', ' blogs'),
(' editorial', ' editorial'),
                                                                                                                                   (' tests', ' tests'),
(' presentations', ' presentations'),
                                                                                                                                  (' Editorial', ' Editorial'),
(' interview', ' Interview'),
(' reviewer', ' reviewers'),
  (' properly', properly,
(' incorrectly', 'fail'),
(' incorrectly', ' faulty'),
(' incorrectly', ' misrepresent'),
                                                                                                                              (' reviewer', reviewer's,',
(' interviews', 'Interview'),
(' interview', ' interviewing'),
(' interviewer', ' Interview'),
(' interviews', ' interview'),
(' Interviews', ' Interviews'),
  (' fails', ' failing'),
  (' incorrectly', ' inaccurate'),
(' incorrectly', ' errors'),
(' incorrectly', ' errors'),
(' worse', ' harmful'),
(' wrong', ' misunderstand'),
(' improperly', ' misunderstand'),
(' incorrectly', 'wrong'),
(' incorrectly', ' harmful'),
(' incorrectly', ' mistake'),
(' incorrectly', ' mis'),
(' fails', 'fail'),
(' worse', ' detrimental'),
(' properly', ' rightful'),
(' interview', ' interviewer'),
(' diterview', ' interviewer'),
(' magazine', ' magazines'),
(' editorial', ' Editorial'),
(' interview', ' interviews'),
(' interview', ' interviews'),
(' interview', ' interviews'),
(' interview', ' interview')
  (' properly', ' rightful'),
(' inappropriately', ' misunderstand'),
(' unnecessarily', ' harmful'),
(' unnecessarily', ' neglect'),
                                                                                                                                    (' interview', ' interview'),
(' Interview', ' interview'),
(' interviews', ' interviews'),
  (' properly', ' correctly'),
(' Worse', ' Worst'),
(' fails', ' failure'),
                                                                                                                                     (' tests', 'tests'),
                                                                                                                                    (' interviews', ' interviewing'),
('Interview', ' interview')
  (' adequately', ' satisfactory'),
(' incorrectly', ' defective'),
(' mistakenly', ' misunderstand'),
                                                                                                                                  GPT-2 Medium - Layer 14 Head 13*
   ('Worse', 'harming'),
                                                                                                                                   (' editorial', ' editors'),
                                                                                                                               (' editorial', ' editors'),
(' broadcasting', ' broadcasters'),
(' broadcasts', ' broadcasting'),
(' broadcasts', ' broadcast'),
(' broadcasters', ' Broadcasting'),
(' Editorial', ' editors'),
(' broadcast', ' broadcasters'),
(' broadcast', ' Broadcasting'),
(' lecture', ' lectures'),
  (' Worse', ' harming'),
(' incorrectly', ' mishand'),
(' adequately', 'adequ'),
(' incorrectly', ' misuse'),
(' fails', 'Failure'),
(' Worse', ' hurts'),
('wrong', ' misunderstand'),
(' incorrectly', ' mistakenly')
   (' incorrectly', ' mistakenly'),
   (' fails', ' failures'),
```

```
(' broadcasting', ' Broadcast'),
(' broadcaster', ' broadcasters'),
(' broadcasts', ' broadcasters'),
(' publishing', ' Publishers'),
(' broadcast', ' broadcasting'),
(' Broadcasting', ' broadcasters'),
(' Publishing', ' Publishers'),
(' lectures', ' lecture'),
(' editorial', ' Editors'),
(' broadcasting', ' broadcast'),
                                                                                                         (' Billion', ' 1934'),
(' Eric', 'Larry'),
(' 2015', 'Released'),
                                                                                                               (' Copyright', 'Rat'),

(' tomorrow', ' postp'),

(' 2017', 'Latest'),
                                                                                                                ('previous', 'obin'),
                                                                                                                ('controversial', 'Priv'),
                                                                                                                (' recently', ' nightly'),
 (' broadcasting', ' broadcast'),
(' broadcasts', ' Broadcasting'),
                                                                                                                 ('Base', ' LV'),
                                                                                                                 (' recently', 'Project'),
                                                                                                                (' historically', ' globalization'),
(' recently', ' vulner'),
(' tonight', 'Wednesday'),
 (' broadcasters', ' broadcasting'),
(' journalistic', ' journalism'),
 ('Journal', 'reports'),
                                                                                                                ('Copyright', 'Abstract'),
('Tuesday', 'Friday'),
('Anthony', 'Born'),
('Budget', 'Premium'),
('tonight', 'Welcome'),
 ('Broadcasting', 'Broadcast'),
('Publisher', 'Publishers'),
 (' Broadcasting', 'azeera'),
 ('Journal', 'Reporting'),
(' journalism', ' journalistic'),
(' broadcaster', ' Broadcasting'),
(' broadcaster', ' broadcasting'),
                                                                                                                 ('yle', 'lite'),
                                                                                                                (' Wednesday', 'Latest'),
(' broadcaster', ' broadcasting'),
(' broadcasting', ' broadcaster'),
(' publication', ' editors'),
('journal', ' journalism'),
('Journal', ' Journalists'),
(' documentaries', ' documentary'),
                                                                                                                (' wednesday', 'Latest
(' Latest', 'show'),
(' B', ' pione'),
(' Copyright', 'cop'),
(' Pablo', ' Dia'),
(' recent', 'Latest')
 (' filmed', ' filming'),
(' publishing', ' publishers'),
('Journal', 'journalism'),
('broadcasts', 'Broadcast'),
('broadcasters', 'broadcast'),
                                                                                                                 GPT-2 Medium - Layer 22 Head 1
                                                                                                                (' usual', ' usual'),
(' occasional', ' occasional'),
 ('Journal', 'articles'),
('reports', 'reporting'),
                                                                                                                ('aforementioned', 'aforementioned'),
                                                                                                               ('general', 'usual'),
('usual', 'slightest'),
('agn', 'ealous'),
('traditional', 'usual'),
('manuscript', 'manuscripts'),
('publishing', 'publish'),
('broadcasters', 'azeera'),
('publication', 'Publishers'),
('publications', 'Publishers'),
                                                                                                                ('free', 'amina'),
('major', 'major'),
('frequent', 'occasional'),
('generous', 'generous'),
 (' Newsp', ' newspapers'),
(' broadcasters', ' Broadcast'),
 ('Journal', ' Readers')
                                                                                                                   (' free', 'lam'),
                                                                                                                   (' regular', ' usual'),
(' standard', ' usual'),
C.2 Query-Key Matrices
                                                                                                                   (' main', ' usual'),
                                                                                                                   (' complete', ' Finished'),
GPT-2 Large - Layer 19 Head 7**
                                                                                                                   (' main', 'liest'),
 (' tonight', 'Friday'),
(' Copyright', 'Returns'),
                                                                                                                  (' traditional', ' traditional'),
                                                                                                                (' traditional', ' traditional',
(' latest', ' aforementioned'),
(' current', ' aforementioned'),
(' normal', ' usual'),
(' dominant', ' dominant'),
(' free', 'ministic'),
(' brief', ' brief'),
'' biggest' 'liest').
 ('TM', 'review'),
('Weekend', 'Preview'),
('tonight', 'Thursday'),
('recently', 'Closure'),
('Copyright', 'Contents'),
(' Copyright', 'Contents'),
(' Copyright', 'Wisconsin'),
(' Copyright', 'Methods'),
(' tonight', 'Sunday'),
(' tomorrow', ' postpone'),
(' tomorrow', ' tonight'),
(' recently', 'acerb'),
(' Copyright', 'Rated'),
(' myself', ' my'),
(' Copyright', 'Cop'),
(' Wednesday', 'Closure'),
                                                                                                                 ('biggest', 'liest'),
('usual', 'usual'),
('rash', 'rash'),
('regular', 'occasional'),
                                                                                                                  (' specialized', ' specialized'),
                                                                                                                (' specialized', ' specialize
(' free', 'iosis'),
(' free', 'hero'),
(' specialty', ' specialty'),
(' general', 'iosis'),
(' nearby', ' nearby'),
(' best', 'liest'),
(' Copyright', 'Cop'),
(' Wednesday', 'Closure'),
(' Billion', ' 1935'),
(' tonight', 'Saturday'),
(' tonight', ' celebr'),
(' tomorrow', ' postponed'),
                                                                                                                 (' officially', ' formal'),
(' immediate', 'mediate'),
(' special', ' ultimate'),
('Copyright', 'Show'),
('Wednesday', 'Friday'),
('Copyright', 'Earn'),
                                                                                                                 (' free', 'otropic'),
                                                                                                                 (' rigorous', ' comparative'),
(' actual', ' slightest'),
```

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('54', '88'),
('156', '39'),
('212', '79'),
('59', '28'),
('57', '27'),
('156', '29'),
('36', '27'),
('217', '79'),
('59', '38'),
('63', '27'),
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('156', '27'),
('156', '27'),
('156', '38'),
('63', '26'),
('59', '25'),
('138', '27'),
(' complete', ' comparative'),
(' typical', ' usual'),
(' modern', ' modern'),
(' best', ' smartest'),
(' free', ' free'),
(' highest', ' widest'),
(' specialist', ' specialist'),
(' appropriate', ' slightest'),
(' usual', 'liest')
 (' usual', 'liest')
GPT-2 Large - Layer 20 Head 13 **
(' outdoors', ' outdoors'),
(' outdoor', ' outdoors'),
 (' Gre', 'burg'),
(' healing', ' healing'),
(' indoor', ' outdoors'),
(' Hemp', 'burg'),
 (' Ticket', ' Ticket'),
 ('accommodations', 'accommodations'),
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('59', '25'),
('138', '27'),
('217', '38'),
('72', '27'),
('54', '27'),
('36', '29'),
('37', '26'),
('37', '26'),
('37', '26'),
('54', '38'),
('59', '29'),
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('37', '28'),
('37', '29'),
('37', '29'),
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('89', '27'),
('561', '79'),
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('183', '27'),
('183', '27'),
('183', '27'),
('183', '27'),
('183', '27'),
('183', '27'),
('183', '27'),
('eco', 'aco'),
('prem', 'otti')
 (' Candy', 'cott'),
 (' decorative', ' ornament'),
('yan', 'ava'),
(' deadlines', ' schedule'),
 (' Lor', 'ian'),
 ('architectural', 'ornament'),
 (' Ratings', ' Ratings'),
('Bod', 'za'),
('exotic', 'exotic'),
('food', 'baths'),
' LOOG', ' baths'),
(' Marketplace', ' Marketplace'),
(' heal', ' healing'),
(' Ex', 'ilus'),
(' indeers', '
('indoors', 'outdoors'),
('therm', 'therm'),
('bleach', 'coated'),
 (' Sod', 'opol'),
('District', 'Metropolitan'),
('Anonymous', 'Rebell'),
 (' Corn', 'burg'),
 (' indoor', ' indoors'),
                                                                                                          ('183', '27'),
('54', '29')
(' R', 'vale'),
('rom', 'otti'),
 (' ratings', ' Ratings'),
                                                                                                         GPT-2 Medium - Layer 17 Head 6*
 (' attendance', ' attendance'),
(' destinations', ' destinations'),
                                                                                                         (' legally', ' legal'),
(' legal', ' sentencing'),
(' legal', ' arbitration'),
(' boycot', ' boycott'),
(' legal', ' criminal'),
(' legal', ' Judicial'),
(' legal', ' rulings'),
(' judicial', ' sentencing'),
(' marketing', ' advertising'),
(' legal', ' confidential').
 (' VIDEOS', ' VIDEOS'),
 ('yan', 'opol'),
 (' Suffolk', 'ville'),
(' retali', ' against'),
 ('mos', 'oli'),
 (' pacing', ' pacing'),
(' Spectrum', ' QC'),
(' Il', 'ian'),
(' archived', ' archived'),
(' Pledge', ' Pledge'),
                                                                                                         ('legal', 'confidential'),
('protesting', 'protest'),
('recruited', 'recruit'),
('recruited', 'recruits'),
('judicial', 'criminal'),
('legal', 'exemptions'),
('demographics', 'demographics')
 ('alg', 'otti'),
 (' Freedom', 'USA'),
 ('anto', 'ero'),
(' decorative', ' decoration')
                                                                                                           (' demographics', ' demographic'),
                                                                                                           (' boycott', ' boycot'),
GPT-2 Medium - Layer 0 Head 9
                                                                                                           (' boycott', boycot',
(' sentencing', ' criminal'),
(' recruitment', ' recruits'),
(' recruitment', ' recruit'),
 ('59', '27'),
 ('212', '39'),
 ('212', '38'),
                                                                                                           (' Constitutional', ' sentencing'),
('217', '39'),
('37', '27'),
('59', '26'),
                                                                                                           (' Legal', ' sentencing'),
                                                                                                           (' constitutional', ' sentencing'),
                                                                                                           (' legal', ' subpoena'),
```

```
(' injury', ' injuries'),
(' FOIA', ' confidential'),
(' legal', ' licenses'),
                                                                                                                           ('aroo', 'anny'),
                                                                                                                          ('aroo', 'anny'),
('academic', 'academia'),
('Congress', 'Amendments'),
('academic', 'academics'),
('student', 'academic'),
('committee', 'convened'),
('",','..."'),
('ove', 'idia')
(' legal', ' licenses'),
(' donation', ' donations'),
(' disclosure', ' confidential'),
(' negotiation', ' negotiating'),
(' Judicial', ' legal'),
(' legally', ' criminal'),
(' legally', ' confidential'),
(' legal', ' jur'),
(' legal', ' enforcement'),
(' legal', ' lawyers'),
(' legally', ' enforcement')
                                                                                                                            GPT-2 Medium - Layer 16 Head 13
                                                                                                                         (' sugg', ' hindsight'),
(' sugg', ' anecdotal'),
(' unsuccessfully', ' hindsight'),
 (' legally', ' enforcement'),
(' regarry, ' enrorement',
(' recruitment', ' recruiting'),
(' recruiting', ' recruit'),
(' criminal', ' sentencing'),
(' legal', ' attorneys'),
                                                                                                                          ('didn', ' hindsight'),
                                                                                                                         ('orously', 'staking'),
('illions', 'uries'),
('until', 'era'),
(' lobbied', ' hindsight'),
 (' negotiations', ' negotiating'),
(' negotiations', ' negotiating'),
(' legally', ' arbitration'),
(' recruited', ' recruiting'),
(' legally', ' exemptions'),
(' legal', ' judicial'),
(' voting', ' Vote'),
(' negotiated', ' negotiating'),
(' legislative', ' veto'),
(' funding', ' funded')
                                                                                                                          (' incorrectly', ' incorrect'),
(' hesitate', ' hindsight'),
                                                                                                                           ('ECA', ' hindsight'),
                                                                                                                           (' regret', ' regrets'),
                                                                                                                           ('inventoryQuantity', 'imore'),
                                                                                                                          ('consider', ' anecdotal'),
(' errone', ' incorrect'),
(' someday', ' eventual'),
('illions', 'Murray'),
GPT-2 Medium - Layer 17 Head 7
                                                                                                                          ('recently', 'recent'),
('Learned', 'hindsight'),
('before', 'hindsight'),
('lately', 'ealous'),
 ('tar', 'idia'),
('[...]', '..."'),

('lecture', 'lectures'),

('Congress', 'senate'),

('staff', 'staffers'),
                                                                                                                         ('upon', 'rity'),
('ja', ' hindsight'),
(' regretted', ' regrets'),
('Scholarship', 'collegiate'),
('executive', 'overseeing'),
('Scholarship', 'academic'),
                                                                                                                         (' unsuccessfully', 'udging'),
                                                                                                                       (' unsuccessfully', 'udgin-
(' lately', 'dated'),
(' sugg', 'anecd'),
(' inform', 'imore'),
(' lately', 'recent'),
(' anecd', 'anecdotal'),
('orously', 'hindsight'),
(' postwar', 'Era'),
(' lately', 'recent'),
(' skept', 'cynicism'),
(' sugg', 'informed'),
(' unsuccessfully', 'ealou
(' academ', ' academic'),
('."', '..."'),
('[', '..."'),
('";', '..."'),
 (' Memorial', 'priv'),
(' festival', 'conference'),
 ('crew', ' supervisors'),
('crew', 'supervisors'),
('certification', 'grading'),
('scholarship', 'academic'),
('rumored', 'Academic'),
('Congress', 'delegated'),
('staff', 'technicians'),
('Plex', 'CONS'),
('congress', 'sepate')
                                                                                                                           (' unsuccessfully', 'ealous'),
                                                                                                                           ('ebin', ' hindsight'),
                                                                                                                         (' underest', ' overest'), (' Jinn', ' hindsight'),
('Plex', 'CONS'),
(' congress', ' senate'),
(' university', ' tenure'),
(' Congress', ' appointed'),
(' Congress', ' duly'),
(' investigative', ' investig'),
(' legislative', ' senate'),
                                                                                                                         ('someday', '2019'),
('recently', 'turned'),
('sugg', 'retrospect'),
                                                                                                                           ('unsuccessfully', 'didn'),
('unsuccessfully', 'gged'),
('mistakenly', 'incorrect'),
 ('ademic', 'academic'), ('bench', 'academic'),
                                                                                                                            ('assment', ')</'),
                                                                                                                           ('ja', 'didn'),
                                                                                                                         ('ja', 'didn','
('illions', ' hindsight'),
(' sugg', ' testimony'),
('jri', ' hindsight')
 (' scholarship', ' tenure'),
('campus', 'campuses'),
('staff', 'Facilities'),
('Editorial', 'mn'),
('clinic', 'laboratory'),
('crew', 'crews'),
                                                                                                                           GPT-2 Medium - Layer 12 Head 9
                                                                                                                           (' PST', ' usual'),
('etimes', ' foreseeable'),
 (' Scholarship', ' academ'),
(' scholarship', ' academ'),
(' staff', ' staffer'),
('icken', 'oles'),
('?"', '..."'),
(' Executive', ' overseeing'),
(' academic', ' academ'),
(' Congress', 'atra'),
                                                                                                                            ('uld', 'uld'),
(' Der', ' Mankind'),
                                                                                                                           (' statewide', ' yearly'),
(' guarantees', ' guarantees'),
                                                                                                                           ('Flynn', 'Logged'),
('borne', 'foreseeable'),
```

```
(' contiguous', ' contiguous'),
(' exceptions', ' exceptions'),
                                                                                         (' Ratings', ' organisations'),
('vernment', 'spons'),
                                                                                         ('..."', '),"'),
(' Caucas', ' commodity'),
(' redist', ' costly'),
(' downstream', ' day'),
                                                                                         ('dictators', 'governments'),
('istration', 'sponsor'),
('iquette', 'acron'),
('Announce', 'answ'),
('Journalism', 'empowering'),
(' ours', ' modern'),
(' foreseeable', ' foreseeable'),
(' Posted', ' Posted'),
(' anecdotal', ' anecdotal'),
(' moot', ' costly'),
(' successor', ' successor'),
                                                                                          ('Media', 'bureaucr'),
('Discrimination', 'organizations'),
(' successor', ' successor', (' any', ' ANY'), (' generational', ' modern'), (' temporarily', ' costly'),
                                                                                         (' Journalism', 'Online'),
                                                                                        ('FAQ', 'sites'),
('antitrust', 'Governments'),
('..."', '..."'),
('Questions', 'acron'),
('rities', 'organisations'),
(' overall', ' overall'),
(' offective', ' incentiv'),
(' future', ' tomorrow'),
(' ANY', ' lifetime'),
(' dispatch', ' dispatch'),
(' legally', ' WARRANT'),
                                                                                         (' Editorial', ' institutional'),
                                                                                         (' tabl', ' acron'),
(' legally', ' WARRANI'),
(' guarantees', ' incentiv'),
(' listed', ' deductible'),
(' CST', ' foreseeable'),
                                                                                         ('antitrust', 'governments'),
('Journalism', 'Everyday'),
                                                                                         ('icter', ' Lieberman'),
(' defect', 'SPONSORED'),
('anywhere', 'any'),
('guaranteed', 'incentiv'),
('successors', 'successor'),
('weekends', 'day'),
                                                                                          (' Journalists', ' organisations')
                                                                                          GPT-2 Medium - Layer 22 Head 5 (names and parts of names
('iquid', ' expensive'),
(' Trib', ' foreseeable'),
(' phased', ' modern'),
(' constitutionally', ' foreseeable'),
                                                                                          seem to attend to each other here)
                                                                                         ('Smith', 'ovich'),
('Jones', 'ovich'),
('Jones', 'Jones'),
('Smith', 'Williams'),
('Rogers', 'opoulos'),
('any', 'anybody'),
('anywhere', 'ANY'),
('veto', 'precedent'),
('veto', 'recourse'),
                                                                                         ('Jones', 'ovich'),
(' Jones', 'inez'),
(' veto', ' recourse'),
(' hopefully', ' hopefully'),
(' potentially', ' potentially'),
(' ANY', ' ANY'),
(' substantive', ' noteworthy'),
                                                                                         ('ug', ' Ezek'),
                                                                                       (' Moore', 'ovich'),
                                                                                       ('orn', 'roit'),
('van', 'actionDate'),
('morrow', ' day'),
('ancial', ' expensive'),
('listed', ' breastfeeding'),
(' holiday', ' holidays')
                                                                                         ('Jones', 'inelli'),
('Edwards', 'opoulos'),
('Jones', 'Lyons'),
('Williams', 'opoulos'),
                                                                                         ('Moore', 'ovich'),
GPT-2 Medium - Layer 11 Head 10
                                                                                         (' Rodriguez', 'hoff'),
                                                                                         ('North', 'suburbs'),
('Smith', 'chio'),
('Smith', 'ovich'),
('Smith', 'opoulos'),
(' Journalism', ' acron'),
(' democracies', ' governments'),
('/-', 'verty'),
(' legislatures', ' governments'),
('ocracy', ' hegemony'),
('osi', ' RAND'),
                                                                                         ('Mc', 'opoulos'),
                                                                                         ('Johnson', 'utt'),
(' Jones', 'opoulos'),
('Ross', 'Downloadha'),
('pet', 'ilage'),
(' Organizations', ' organisations'),
('ellectual', ' institutional'),
(' Journalists', ' acron'),
('eworks', ' sponsors'),
(' Inqu', ' reviewer'),
('ocracy', ' diversity'),
(' careers', ' Contributions'),
                                                                                          (' Everett', ' Prairie'),
                                                                                         ('Cass', 'isma'),
('Jones', 'zynski'),
('Jones', 'Jones'),
                                                                                        (' McCl', 'elman'),
(' Smith', 'Jones'),
('gency', '\\-'),
('ellectual', ' exceptions'),
(' Profession', ' specializing'),
                                                                                       (' Simmons', 'opoulos'), (' Smith', 'brown'),
('online', 'Online'),
('Publications', 'authorised'),
                                                                                         (' Mc', 'opoulos'),
('Online', 'Online'),
('sidx', 'Lazarus'),
('eworks', 'Networks'),
('Groups', 'organisations'),
                                                                                         (' Jones', 'utt'),
                                                                                      ('Richards', 'Davis'),
('Johnson', 'utt'),
('Ross', 'bred'),
('McG', 'opoulos'),
('Governments', 'governments'),
('democracies', 'nowadays'),
('psychiat', 'Mechdragon'),
('educ', 'Contributions'),
                                                                                       (' Stevens', 'stadt'),
                                                                                         ('ra', 'abouts'),
                                                                                         (' Johnson', 'hoff'),
```

```
(' North', ' Peninsula'),
(' Smith', 'Smith'),
('Jones', 'inez'),
                                                                                                                                                                  #Els
                                                                                                                          #annels
                                                                                                                          #netflix
                                                                                                                                                                  #osi
                                                                                                                          telev
                                                                                                                                                                  #mpeg
 (' Hernandez', 'hoff'),
                                                                                                                                                                  #vous
                                                                                                                          # t. v
(' Lucas', 'Nor'),
(' Agu', 'hoff'),
('Jones', 'utt')
                                                                                                                          #avi
                                                                                                                                                                  #iane
                                                                                                                          #flix
                                                                                                                                                                  transmitter
                                                                                                                         Television Sinclair #outube Streaming
GPT-2 Medium - Layer 19 Head 12
                                                                                                                          #channel
                                                                                                                                                                  #channel
                                                                                                                         Vid
                                                                                                                                                                  mosqu
 (' 2015', 'ADVERTISEMENT'),
                                                                                                                         #Channel
                                                                                                                                                                broadcaster
(' 2014', '2014'),
                                                                                                                         documentaries airs
(' 2014', 2014'),

(' 2015', '2014'),

(' 2015', 'Present'),

(' 2013', '2014'),

(' 2017', 'ADVERTISEMENT'),
                                                                                                                          #videos Broadcasting
                                                                                                                         Hulu
                                                                                                                                                                  broadcasts
                                                                                                                         channels streams the vision channels byDs the channels the channels by the channels the channels
('2016', 'ADVERTISEMENT'),
('itor', 'Banner'),
('2015', 'Bulletin'),
                                                                                                                                                              broadcasters
                                                                                                                         broadcasts broadcasting
#azeera #RAFT
                                                                                                                          #azeera
('2012', 'Bulletin'),
('2014', 'Bulletin'),
                                                                                                                         MPEG
                                                                                                                                                                  #oded
('2014', 'Bulletin'),

('Airl', 'Stream'),

('2016', 'Bulletin'),

('2016', '2014'),

('2017', 'Bulletin'),

('2013', '2014'),

('2012', '2014'),
                                                                                                                         televised htt
                                                                                                                                                                 transmissions
                                                                                                                         aired
                                                                                                                         broadcasters playback
Streaming Instruction
                                                                                                                        Streaming lnsc
                                                                                                                         #TV
                                                                                                                                                                  Sirius
                                                                                                                         Kodi
                                                                                                                                                                  viewership
 (' stadiums', 'ventions'),
                                                                                                                         TTV
                                                                                                                                                                  radio
 (' 2015', ' Bulletin'),
('2013', 'Bulletin')

('2013', 'Bulletin'),

('2017', '2014'),

('2011', '2011'),

('2014', '2014'),

('2011', '2009'),
                                                                                                                         #ovies
                                                                                                                                                                  #achers
                                                                                                                         channel
                                                                                                                                                                  channel
                                                                                                                          GPT-2 Medium - Layer 3 Dim 2711
                                                                                                                         purposes
                                                                                                                                                              purposes
                                                                                                                         sake
                                                                                                                                                             sake
 (' mile', 'eming'),
 (' 2013', 'ADVERTISEMENT'),
                                                                                                                        purpose
                                                                                                                                                           reasons
                                                                                                                         reasons
                                                                                                                                                         purpose
(' 2014', '2015'),
(' 2014', 'Present'),
                                                                                                                         convenience ages
                                                                                                                          reason
                                                                                                                                                             reason
 (' 2011', '2014'),
                                                                                                                                                            #ummies
                                                                                                                         Seasons
 (' 2011', '2009'),
(' 2015', ' 2014'),

(' 2013', ' Bulletin'),

(' 2015', '2015'),
                                                                                                                          #Plex
                                                                                                                                                           #going
                                                                                                                         Reasons
                                                                                                                                                            foreseeable
                                                                                                                                                             Reasons
                                                                                                                          #ummies
('2011', '2003'),

('2011', '2010'),

('2017', 'Documents'),

('2017', 'iaries'),
                                                                                                                          #asons
                                                                                                                                                             #reason
                                                                                                                         #lation
                                                                                                                                                           #pur
                                                                                                                         #alsh
                                                                                                                                                         Developers
                                                                                                                          #agos
                                                                                                                                                             #akers
('2013', '2015'),
('2017', 'Trend'),
('2011', '2011'),
                                                                                                                          #ACY
                                                                                                                                                             transl
                                                                                                                                                          Reason
                                                                                                                          STATS
                                                                                                                          #itas
                                                                                                                                                          consideration
(' 2016', 'Present'),
(' 2011', ' 2014'),
                                                                                                                          ages
                                                                                                                                                             #purpose
                                                                                                                          #purpose
                                                                                                                                                         beginners
(' years', 'years'),
('Plug', 'Stream'),
                                                                                                                          #=[
                                                                                                                                                             awhile
                                                                                                                          #gencies
                                                                                                                                                           Pur
                                                                                                                         Millennium #benefit
(' 2014', 'ADVERTISEMENT'), ('2015', 'Present'),
                                                                                                                          Brewers
                                                                                                                                                              #atel
('2013', Flesent')

('2018', 'thora'),

('2017', 'thora'),

('2012', '2011'),

('2012', '2014')
                                                                                                                          Festival
                                                                                                                                                             #t.un
                                                                                                                          EVENT
                                                                                                                                                              pur
                                                                                                                          #payment
                                                                                                                                                              Ages
                                                                                                                                                              preservation
                                                                                                                          #=-
                                                                                                                          #printf
                                                                                                                                                              Metatron
                                                                                                                          beginners
                                                                                                                                                             11m
                                                                                                                          Expo
                                                                                                                                                              #KEN
C.3 Feedforward Keys and Values
```

Key-value pairs, (k_i, v_i) , where at least 15% of the top-k vocabulary items overlap, with k = 100. We follow our forerunner's convention of calling the index of the value in the layer "dimension" (Dim).

Here again we use two asterisks (**) to represent lists where we discarded tokens outside the corpus vocabulary. GPT-2 Medium - Layer 0 Dim 116

GPT-2 Medium - Layer 4 Dim 621

#ovie headlined newspapers pestic television dime editorial describ #journal Afric broadcasters broadcasts

#Journal	# ('
publication	#umbnails
Newsweek	#adish
Zeit	#uggest
columnist	splash
Editorial	#ZX
newsletter	objectiona

able #article cartoon #eport Bucc telev #London radio reprint headlined #azine #ribune Giov BBC #ender reprint headline sitcom #oops reprinted #articles broadcast snipp tabloid Ajax documentaries marqu journalist # (" TV #otos headline mast #idem news

GPT-2 Medium - Layer 7 Dim 72

sessions session dinners sessions #cation #cation #iesta session dinner Booth #eteria screenings Dinner booked #Session #rogram rehears vacation baths baths Lunch #pleasant #hops meetings #Session visits Session greet #session #athon meetings Sessions boarding chatting lunch rituals chats booking festivities Grape boarding #miah #workshop #session #rooms Pars #tests simulated seated Dispatch visit. Extras appointments toile Evening #vu #rations showers #luaj abroad

GPT-2 Medium - Layer 10 Dim 8

Miy	Tai
#imaru	#jin
Gong	Jin
Jinn	Makoto
Xia	#etsu
Makoto	Shin
Kuro	Hai
Shin	Fuj
#Tai	Dai
Yamato	Miy
Tai	#iku
Ichigo	Yun

#Shin Ryu Shu #atsu Haku Hua Suzuki Chun #ku Yang Xia Qing #Shin Tsuk Hua #iru Jiang Yu Nanto #yu manga Chang Yosh Nan Qian yen Osaka #hao Qian Fuk #uku Chun #iku Yong Yue #Tai

GPT-2 Medium - Layer 11 Dim 2

progressing	toward
#Progress	towards
#progress	Pace
#osponsors	progression
#oppable	#inness
advancement	onward
progress	canon
Progress	#progress
#senal	pace
#venge	#peed
queue	advancement
#pun	advancing
progression	progressing
#wagon	ladder
advancing	path
#cknowled	honoring
#Goal	ranks
momentum	standings
#zag	goal
#hop	#grand
pursuits	momentum
#encing	#ometer
#Improve	timetable
STEP	nearing
#chini	quest
standings	spiral
#eway	trajectory
#chie	progress
#ibling	accelerating
Esports	escal

GPT-2 Medium - Layer 15 Dim 4057

EDITION	copies
versions	Version
copies	#edition
version	#Version
Version	version
edition	#download
editions	download
reprint	versions
#edition	#Download
EDIT	сору
Edition	#release
reproduce	#version
originals	release
#edited	#сору
VERS	VERS
#Versions	#pub
#Publisher	Download
reprodu	#released

#uploads editions playthrough edition Printed reprint reproduction Release #Available #Reviewed #published сору #Version #Published paperback EDITION preview print surv #Quantity #Download #available circulate RELEASE

GPT-2 Medium - Layer 16 Dim 41

#duino alarm #Battery alarms Morse signal alarms circuit GPTO GPIO LEDs timers batteries volt.age #toggle signals signal circuitry circuitry electrical #PsyNetMessage circuits alarm LEDs standby autop signalling signalling #volt signaling volt liahts signals Idle voltage triggers LED batteries electrom Morse timers LED #LED malfunction amplifier button radios Signal wiring timer #Alert wiring signaling buzz #Clock disconnect arming Arduino Arduino triggered

GPT-2 Medium - Layer 17 Dim 23

responsibility responsibility Responsibility respons responsibilities responsibilities #ipolar Responsibility #responsible oversee duties #respons #respons duties supervision superv supervision superv #abwe stewards Adin chore respons oversight oversee oversees responsible ent.rust.ed overseeing #responsible handling helicop presided handles overseen overseeing #dyl chores responsible manage #ADRA managing reins duty #accompan Respons chores charge

oversees reins supervised handle blame oversaw CONTROL #archment RESP tasks

GPT-2 Medium - Layer 19 Dim 29

subconscious thoughts thoughts thought #brain Thoughts #Brain minds memories mind OCD thinking flashbacks #thought brainstorm imagination Anxiety Thinking #mind Thought fantas imagin amygdala thinker #thinking impuls Thinking #mind #Memory memories Thoughts #think dreams imagining #ocamp impulses #Psych fantasies #mares think mentally urges desires #mental mind dreams #thinking delusions #Mind subconscious #dream emotions psyche imag prefrontal #dream PTSD conscience Memories visions

GPT-2 Medium - Layer 20 Dim 65

exercises volleyball #Sport tennis #athlon sports Exercise sport. #ournaments #basketball volleyball Tennis Recre soccer Mahjong golf #basketball playground exercise Golf bowling athletics skating #athlon spar athletic skiing rugby amusement gymn #sports gymn sled drills #Training #Sport tournaments cricket sled Soccer Volunte amuse Activities skate golf recreational #Pract Ski activities dunk #hower basketball athletics #games sport skating hockey Solitaire #BALL #sports

GPT-2 Medium - Layer 21 Dim 86

number identifiers #number surname #Number surn Number identifier NUM initials numbers #Registered Numbers NAME #Numbers address #names #address pseudonym #codes #Num #NUM nomine names addresses username Address #TDs identifier ID #Address registration #num #76561 TD #soDeliveryDate numbering #ADRA IDs CLSID #ID

numbering identifiers #ername identification

#address numer
addresses digits
codes #numbered
#Names numerical
regist Ident
name numeric
Names Identification

GPT-2 Medium - Layer 21 Dim 400

#July Oct July Feb #February Sept #January Dec #Feb Jan November Nov #October Aug January #Oct Feb May October #Nov #September Apr September March #June April #Sept #Sept February June #November #Aug #April October April #Feb June July #December December August Sep #March November Sept #Jan December #May Aug August March Jul #August Jun #Aug September

GPT-2 Medium - Layer 23 Dim 166

January

February

#k #k #ks #K #kish #ks #K #KS

#wcs

Apr

#kat k #kus #k+ #KS #ked #kr #kr #kl #kish #kB #kan #kos #kw #king #ket #ked #king #kie #kb #KB #kos #kk #kHz #kowski #kk #KR #kick #KING #kers #KT #kowski #KK #KB #KC #krit #kw #KING #kb #kt #Ka #ksh #krit #kie #KN #ky #kar #kh #KY #ket #ku

GPT-2 Medium - Layer 23 Dim 907

hands hand hand #Hand #hands Hand #hand #hand fingers hands #feet Hands fingertips fist claws #hands paw finger handed paws metab thumb fingers palms fingert foot #Hand #handed fists paw wrists handing levers #finger thumbs #hander fingertips tentacles feet claw limb fingert slider #Foot #handed Stick #dimension arm jaws #Accessory

skelet #fing
lapt Foot
ankles index
weap toe
foot #auntlet

GPT-2 Large - Layer 25 Dim 2685**

#manager engineering #Engineers Marketing chemist #engineering humanities Communications #communications sciences anthropology anthropology lingu Engineering #engineering lingu psychologist psychology Coordinator neurolog

Analyst Economics #iologist designer accountant sociology communications strategist #ographer marketing curator pharmac sciences Engineers archae economics Accounting Designer Editing #econom chemist biologist #ologist merch psychologists pharm economist architect theolog Marketing #Manager engineer Architect Architects sociology #technical engineer architects physicist logistics

GPT-2 Large - Layer 21 Dim 3419**

#overtv impoverished #wana poverty povertv poorest #Saharan poorer poorest Yemen Poverty families malnutrition Poverty marginalized Senegal impoverished refugees #poor subsistence Gujar displaced homelessness hardship Homeless refugee #heid households Ramadan migrant mlgrand disadvantaged #Palest poorer Sudan Rahman oppressed #amily socioeconomic illiter peasant Mahmoud homeless Haitian poor #advertisement Ethiopian #hva Kaf #African Rw wealthier #poor Africans Αf caste rural homeless #fam Hait needy

GPT-2 Large - Layer 25 Dim 2442**

Tracker tracking Tracker gau charts tracker tracker Tracking quant #Measure measurement #Stats measuring gau GPS #Tracker Track gauge estimating tracking tally Tracking #Monitor #ometers #chart tracked Meter calculate #HUD calculating

#ometers measurement gauge surve estimation #Stats #Statistics monitoring calculate #stats #tracking Measure quant track #asuring measuring Calculator Monitoring #ometer #Detailed calculator #ometer Monitoring estim #Maps stats pione chart.s timet timet

GPT-2 Base - Layer 9 Dim 1776

radios cable antennas modem radio wireless WiFi modem voltage wired transformer broadband Ethernet Et.hernet. telev radios #Radio power electricity radio loudspe Cable kW Wireless #radio telephone broadband network volt signal microphones Networks telecommunications networks cable electricity Telephone wifi amplifier #levision wifi coax broadcasting transmit transistor transmitter Radio TV wireless Network LTE television watts transmission microwave router telephone cables amps amplifier

GPT-2 Base - Layer 9 Dim 2771

arous increase freeing increasing incent accelerating stimulate allev induce exped discourage enhanced inducing aggrav mitigating enhance stimulating inhib emanc improving alleviate infl empowering #oint preventing alien #ufact alter #HCR enabling influencing incre handc indu disadvant #Impro #roying intens arresting improve allev easing

elevate encouraging depri dissu accelerate impede enlarg convol energ encouraging accent #xiety acceler #akening depri lowering elong

GPT-2 Base - Layer 1 Dim 2931

week

evening

#shows evening night night #sets morning #lav afternoon month afternoon #/+ #'s Night #naissance Loll #genre Kinnikuman semester Weekend #ched morning #ague #enna weekend latest Saturday Sunday #cher #EST week Blossom #icter #Night happens #atto dav #vertising happened #spr #essim #Sunday Masquerade #morning #ished #Thursday sounded Week #ching Panc pesky Evening #chy #allerv t.rope **#ADVERTISEMENT #feature** #Street #fy

GPT-2 Base - Layer 0 Dim 1194

Pav receipts depos #Pay refund Deposit police deduct #pay #milo #igree #paying #Tax #eln debit levied PayPal deposit #enforcement ATM endot cops #soType tax paperwork ID #payment deposits payment loopholes checkout waivers #police receipt agents waive DMV loophole application arresting card commissioner applications Forms office transporter arrested Dupl confisc #paid

Clapper

#ventures

pay

#tax

RCMP #Tax

whistleblowers PAY

APPLIC #ADRA

GPT-2 Base - Layer 9 Dim 2771

lurking weaknesses failings dangers vulnerabilities scams inaccur shortcomings pitfalls scams shortcomings injust flawed faults glitches flawed pitfalls abuses inconsistencies imperfect lurking wrongdoing rigged biases deficiencies corruption weaknesses inaccur discrepancies inadequ hypocrisy fraud inequ rigging deceptive weakness misinformation scam #urities hazards problematic lur hoax imperfect regress danger #abase failings #errors problems #lived injustice abuses plagiar misinterpret plaq suspic deceptive

C.4 Knowledge Lookup

Given a few seed embeddings of vocabulary items we find related FF values by taking a product of the average embeddings with FF values.

Seed vectors:
["python", "java", "javascript"]

Layer 14 Dim 1215 (ranked 3rd)

filesystem debugging Windows HTTP configure Python debug config Linux Java

configuration

cache Unix lib runtime kernel plugins virtual FreeBSD hash plugin header file server

PHP

GNU headers Apache initialization Mozilla

percent years

Seed vectors: ["cm", "kg", "inches"] Layer 20 Dim 2917 (ranked 1st)

hours minutes million seconds inches months miles weeks pounds # % kilometers ounces kilograms grams kilometres metres centimeters thousand days km yards Years meters #million acres kg

#years inch

Seed vectors: ["horse", "dog", "lion"] Layer 21 Dim 3262 (ranked 2nd)

animal animals Animal dogs horse wildlife Animals birds horses dog mammal bird mammals predator beasts Wildlife species #Animal #animal Dogs fish rabbits deer elephants wolves pets veterinary

canine beast predators reptiles rodent primates hunting livestock creature rabbit rept elephant creatures human hunters hunter shark Rept cattle wolf Humane tiger lizard

D Sentiment Analysis Fine-Tuning Vector Examples

This section contains abusive language

Classification Head Parameters

Below we show the finetuning vector of the classifier weight. "POSITIVE" designates the vector corresponding to the label "POSITIVE", and similarly for "NEGATIVE".

POSITIVE	NEGATIVE
#yssey #knit	bullshit
#etts	lame
passions	crap incompetent
#etooth	incompetent
#iscover	bland
pioneers	incompetence
#emaker	idiots
Pione	crappy
#raft	shitty
#uala	idiot
prosper	pointless
#izons	retarded
#encers	worse
#joy	garbage
cherish	CGI
loves	FUCK
#accompan	Nope
strengthens	useless
#nect	shit
comr	mediocre
honoured	poorly
insepar	stupid
embraces	inept
battled	lousy
#Together	fuck
intrig	sloppy
#jong	Worse
friendships	Worst
#anta	meaningless

In the following sub-sections, we sample 4 difference vectors per each parameter group (FF keys, FF values; attention query, key, value, and output subheads), and each one of the fine-tuned layers (layers 9-11). We present the ones that seemed to contain relevant patterns upon manual inspection. We also report the number of "good" vectors among the four sampled vectors for each layer and parameter group.

FF Keys

Layer 9

4 out of 4

diff -diff		-diff	diff	-diff
amazing sei:		seiz	reperto	wrong
movies coerc		coerc	congratulatio	ns unreasonable
wonderful	L	Citiz	Citation	horribly
love		#cffff	thanks	inept
movie		#GBT	Recording	worst
cinematio	2	targ	rejo	egregious
enjoyable	€	looph	Profile	#wrong
wonderful	lly	Procedures	Tradition	unfair
beautiful		#iannopoulos	canopy	worse
enjoy	_	#Leaks	#ilion	atro
films		#ilon	extracts	stupid
comedy		grievance	descendant	egreg
fantastic	2	#merce	#cele	bad
awesome		Payments	enthusiasts	terribly
#Enjoy		#RNA	:-)	ineffective
cinem		Registrar	#photo	nonsensical
film		Regulatory	awaits	awful
loving		immobil	believer	#worst
enjoyment	_	#bestos	#IDA	incompetence
masterpie	ece	#SpaceEngineers	welcomes	#icably
diff	-di	ff	diff	-diff
movie	sei:	 Z	incompetence	#knit
movie fucking	sei:	z ongh	incompetence bullshit	#knit #Together
movie fucking really	sei: Stro	z ongh ooth	incompetence bullshit crap	#knit #Together Together
movie fucking really movies	sei: Stro #eto #20	z ongh ooth 439	incompetence bullshit crap useless	#knit #Together Together versatile
movie fucking really movies damn	seis Stro #eto #20 #Seo	z z ongh ooth 439 cure	incompetence bullshit crap useless pointless	#knit #Together Together versatile #Discover
movie fucking really movies damn funny	seis Stro #eto #200 #Seo Regi	z z ongh ooth 439 cure ulation	incompetence bullshit crap useless pointless incompetent	#knit #Together Together versatile #Discover richness
movie fucking really movies damn funny shit	sei: Stro #eto #20 #Seo Regn Qua:	z z ongh ooth 439 cure ulation rterly	incompetence bullshit crap useless pointless incompetent idiots	#knit #Together Together versatile #Discover richness #iscover
movie fucking really movies damn funny shit kinda	sei: Stro #eto #Seo Regn Qua: cono	z z ongh ooth 439 cure ulation rterly cess	incompetence bullshit crap useless pointless incompetent idiots incompet	#knit #Together Together versatile #Discover richness #iscover forefront
movie fucking really movies damn funny shit kinda REALLY	sei: Stro #eto #20 #Seo Qua: cono Reco	z z ongh ooth 439 cure ulation rterly cess ep	incompetence bullshit crap useless pointless incompetent idiots incompet garbage	#knit #Together Together versatile #Discover richness #iscover forefront inspiring
movie fucking really movies damn funny shit kinda REALLY Movie	sei: Stro #eto #20 #Seo Regn Qua: cono Reco	z z ongh ooth 439 cure ulation rterly cess ep	incompetence bullshit crap useless pointless incompetent idiots incompet garbage meaningless	#knit #Together Together versatile #Discover richness #iscover forefront inspiring pioneering
movie fucking really movies damn funny shit kinda REALLY Movie stupid	sei: Stro #20 #Seo Reg Qua: cono Reco #al:	z z ongh ooth 439 cure ulation rterly cess ep igned	incompetence bullshit crap useless pointless incompetent idiots incompet garbage meaningless stupid	#knit #Together Together versatile #Discover richness #iscover forefront inspiring pioneering #accompan
movie fucking really movies damn funny shit kinda REALLY Movie stupid #movie	sei: Stro #eto #20 #Seo Regn Qua: cono Reco #al: taro	z z ongh ooth 439 cure ulation rterly cess ep igned g	incompetence bullshit crap useless pointless incompetent idiots incompet garbage meaningless stupid crappy	#knit #Together Together versatile #Discover richness #iscover forefront inspiring pioneering #accompan unparalleled
movie fucking really movies damn funny shit kinda REALLY Movie stupid #movie goddamn	sei: Stro #eto #20 #Seo Regg Qua cono Reco #al: taro moso #ve:	z z ongh ooth 439 cure ulation rterly cess ep igned g qu rning	incompetence bullshit crap useless pointless incompetent idiots incompet garbage meaningless stupid crappy shitty	#knit #Together Together versatile #Discover richness #iscover forefront inspiring pioneering #accompan unparalleled #Explore
movie fucking really movies damn funny shit kinda REALLY Movie stupid #movie goddamn crap	sei: Stro #eto #20 #Seo Reg Qua: cono Reco #al: taro moso #ve: Free	z z ongh ooth 439 cure ulation rterly cess ep igned g qu rning eBSD	incompetence bullshit crap useless pointless incompetent idiots incompet garbage meaningless stupid crappy shitty nonexistent	#knit #Together Together versatile #Discover richness #iscover forefront inspiring pioneering #accompan unparalleled #Explore powerfully
movie fucking really movies damn funny shit kinda REALLY Movie stupid #movie goddamn crap shitty	sei: Stro #eto #20 #Seo Reg Qua: cono Reco #al: taro moso #ve: Free Psyl	z z z z z z z z z z z z z	incompetence bullshit crap useless pointless incompetent idiots incompet garbage meaningless stupid crappy shitty nonexistent worthless	#knit #Together Together versatile #Discover richness #iscover forefront inspiring pioneering #accompan unparalleled #Explore powerfully #"},{"
movie fucking really movies damn funny shit kinda REALLY Movie stupid #movie goddamn crap shitty film	sei: Stro #eto #20 #Seo Regg Qua: cono Recc #al: taro moso #ve: Free Psyl Fac:	z z z z z z z z z z z z z	incompetence bullshit crap useless pointless incompetent idiots incompet garbage meaningless stupid crappy shitty nonexistent worthless Worse	#knit #Together Together versatile #Discover richness #iscover forefront inspiring pioneering #accompan unparalleled #Explore powerfully #"},{" #love
movie fucking really movies damn funny shit kinda REALLY Movie stupid #movie goddamn crap shitty film crappy	sei: Stro #20 #Seo Reg Qua: Cono Recc #al: taro moso #ve: Free Psyl Fac: #Lac	z z z z z z z z z z z z z	incompetence bullshit crap useless pointless incompetent idiots incompet garbage meaningless stupid crappy shitty nonexistent worthless Worse lame	#knit #Together Together versatile #Discover richness #iscover forefront inspiring pioneering #accompan unparalleled #Explore powerfully #"},{" #love admired
movie fucking really movies damn funny shit kinda REALLY Movie stupid #movie goddamn crap shitty film crappy damned	sei: Strr #et. #See Regg Qua: con Recc #al tare mose #ve Free Psyl Fac. #Lae	z z z z z z z z z z z z z z z z z z z	incompetence bullshit crap useless pointless incompetent idiots incompet garbage meaningless stupid crappy shitty nonexistent worthless Worse lame worse	#knit #Together Together versatile #Discover richness #iscover forefront inspiring pioneering #accompan unparalleled #Explore powerfully #"},{" #love admired #uala
movie fucking really movies damn funny shit kinda REALLY Movie stupid #movie goddamn crap shitty film crappy	sei: Stro #20 #Seo Reg Qua: Cono Recc #al: taro moso #ve: Free Psyl Fac: #Lac	z congh coth 439 cure ulation rterly cess ep igned g qu rning eBSD Net ilities go gister],"	incompetence bullshit crap useless pointless incompetent idiots incompet garbage meaningless stupid crappy shitty nonexistent worthless Worse lame	#knit #Together Together versatile #Discover richness #iscover forefront inspiring pioneering #accompan unparalleled #Explore powerfully #"},{" #love admired

Layer 10 4 out of 4

diff	-diff	diff	-diff
quotas #RNA cessation subsidy #SpaceEngineers placebo exemptions treadmill Labs receipt moratorium designation	wonderfully wonderful beautifully amazing	isEnabled guiActiveUnfoc #igate waivers expires expire reimb expired #rollment #Desktop prepaid #verning	wonderfully
ineligible reimbursement roundup Articles PubMed waivers Citiz landfill	love marvelous vividly terrific memorable #Enjoy loving fascinating	#andum reimbursement Advisory permitted #pta issuance Priebus #iannopoulos diff	movie comedic hilarious #movie #Amazing scenes Amazing enjoyable
horror # whim # subconscious [unrealistic # imagination # viewers wenjoyment # nostalgia mabsolute sentimental # unreal # Kubrick awe # inspiration mubtle cinematic perfection comedic #	deals	#Leaks quotas #RNA subsidy #?'" Penalty #iannopoulos #>] discredited #conduct #pta waivers Authorization #admin HHS arbitrarily #arantine #ERC memorandum	loving love loved lovers wonderful lover nostalgic alot beautiful amazing great passionate admire passion lovely loves unforgettable proud inspiration

Layer 11 4 out of 4

diff	-diff	diff	-diff
inco pointless Nope bullshit crap useless nonsense futile anyways anyway meaningless clueless lame wasting bogus vomit nonsensical retarded idiots	cherish #knit #terday #accompan prosper versatile friendships #uala Lithuan cherished redes inspires Proud friendship exceptional #beaut #ngth pioneering pioneers	#SpaceEngines nuisance #erous #aband Brist racket Penalty bystand #iannopoulos Citiz Codec courier #>] #termination incapac #interstitial fugitive breaching targ	definitely always wonderful loved wonderfully cherish loves truly enjoy really #olkien beautifully #love great
shit diff	nurt -diff	thug diff	amazing -diff
#accompan Pione celebrate #Discover #knit pioneering recogn reunited comr thriving #iscover commemorate Remem ecstatic forefront enthusi renewed colle Inspired #uala	bad crap inefficient stupid worse mistake incompetence mistakes incompetent miser garbage retarded #bad poor ineffective retard Poor bullshit inept errors	#knit passions #accompan #ossom #Explore welcomes pioneering forefront embraces pioneers intertw #izons #iscover unparalleled evolving Together vibrant prosper strengthens #Together	bullshit crap idiots goddamn stupid shitty shit garbage fuck incompetence crappy bogus useless idiot #shit pointless stupidity fucking nonsense FUCK

FF Values

Layer 9
0 out of 4
Layer 10
0 out of 4
Layer 11
0 out of 4

$W_{\mathbf{Q}}$ Subheads

Layer 9 3 out of 4

diff	-diff	diff	-diff
#ARGET #idal #+ Prev #enger #iannopoulos #report #RELATED issuance #earcher Previous Legislation #astical #iper #>[# Vendor #" #phrine #wcsstore	kinda alot amazing interesting wonderful definitely unbelievable really amazingly pretty nice absolutely VERY wonderfully incredible hilarious funny fantastic quite defin	bullshit bogus faux spurious nonsense nonsensical inept crap junk shitty fake incompetence crappy phony sloppy dummy mediocre lame outrage inco	strengthens Also #helps adjusts #ignt evolves helps grew grows #cliffe recognizes #assadors regulates flourished improves welcomes embraces gathers greets prepares
diff	-diff		
alot kinda amazing definitely pretty tho hilarious VERY really lol wonderful thats dont pics doesnt underrated funny REALLY #love alright	Provision coerc Marketable contingency #Dispatch seiz #verning #iannopoulos #Reporting #unicip Fiscal issuance provision #Mobil #etooth policymakers credential Penalty #activation #Officials		

diff	-diff		diff	-diff
crap	#Register		love	Worse
shit	Browse		unforgettak	ole Nope
bullshit	#etooth		beautiful	#Instead
stupid	#ounces		loved	Instead
shitty	#verning		#love	#Unless
horrible	#raft		loving	incompetence
awful	#egu		amazing	incapable
fucking	#Lago		#joy	Unless
comedic	Payments		inspiring	#failed
crappy	#orsi		passion	incompet
cheesy	Coinbase		adventure	incompetent
comedy	#ourse		loves	ineffective
fuck	#iann		excitement	#Fuck
mediocre	#"}],"		joy	#Wr
terrible	#onductor		LOVE	inept
movie	#obil		together	spurious
bad	#rollment		memories	#Failure
gimmick	#ivot		wonderful	worthless
filler	#Secure		enjoyment	obfusc
inept	#ETF		themes	inadequate
diff		-diff	diff	-diff
 #knit		crap	crap	 #equ
		crap bullshit	crap bullshit	#egu #etooth
 #knit #"},{" #"}],"		bullshit	-	_
#"},{"		-	bullshit	#etooth
#"},{" #"}],"		bullshit stupid	bullshit shit	<pre>#etooth #verning</pre>
#"},{" #"}]," #estones		bullshit stupid inept	bullshit shit :(lol	<pre>#etooth #verning #ounces</pre>
#"},{" #"}]," #estones #Learn		bullshit stupid inept shit	bullshit shit :(<pre>#etooth #verning #ounces #accompan</pre>
#"},{" #"}]," #estones #Learn #ounces		bullshit stupid inept shit idiots	bullshit shit :(lol stupid	<pre>#etooth #verning #ounces #accompan coh</pre>
#"},{" #"}]," #estones #Learn #ounces #egu		bullshit stupid inept shit idiots shitty	bullshit shit :(lol stupid filler	<pre>#etooth #verning #ounces #accompan coh #assadors</pre>
<pre>#"},{" #"}]," #estones #Learn #ounces #egu #Growing #ributes</pre>	Action	bullshit stupid inept shit idiots shitty crappy	bullshit shit :(lol stupid filler shitty	<pre>#etooth #verning #ounces #accompan coh #assadors #pherd</pre>
<pre>#"},{" #"}]," #estones #Learn #ounces #egu #Growing #ributes</pre>	Action	bullshit stupid inept shit idiots shitty crappy incompetence	bullshit shit :(lol stupid filler shitty fucking	<pre>#etooth #verning #ounces #accompan coh #assadors #pherd #acio</pre>
<pre>#"},{" #"}]," #estones #Learn #ounces #egu #Growing #ributes #external</pre>	Action	bullshit stupid inept shit idiots shitty crappy incompetence fuck	bullshit shit :(lol stupid filler shitty fucking pointless	<pre>#etooth #verning #ounces #accompan coh #assadors #pherd #acio #uchs</pre>
<pre>#"},{" #"}]," #estones #Learn #ounces #egu #Growing #ributes #external #encers</pre>	Action	bullshit stupid inept shit idiots shitty crappy incompetence fuck pointless	bullshit shit :(lol stupid filler shitty fucking pointless idiots	<pre>#etooth #verning #ounces #accompan coh #assadors #pherd #acio #uchs strengthens</pre>
<pre>#"},{" #"}]," #estones #Learn #ounces #egu #Growing #ributes #external #encers Browse</pre>	Action	bullshit stupid inept shit idiots shitty crappy incompetence fuck pointless nonsense	bullshit shit :(lol stupid filler shitty fucking pointless idiots anyways	<pre>#etooth #verning #ounces #accompan coh #assadors #pherd #acio #uchs strengthens #reprene</pre>
<pre>#"},{" #"}]," #estones #Learn #ounces #egu #Growing #ributes #external #encers Browse jointly</pre>	Action	bullshit stupid inept shit idiots shitty crappy incompetence fuck pointless nonsense nonsensical	bullshit shit :(lol stupid filler shitty fucking pointless idiots anyways nonsense	<pre>#etooth #verning #ounces #accompan coh #assadors #pherd #acio #uchs strengthens #reprene Scotia</pre>
<pre>#"},{" #"}]," #estones #Learn #ounces #egu #Growing #ributes #external #encers Browse jointly Growing</pre>	Action	bullshit stupid inept shit idiots shitty crappy incompetence fuck pointless nonsense nonsensical stupidity	bullshit shit :(lol stupid filler shitty fucking pointless idiots anyways nonsense anyway	<pre>#etooth #verning #ounces #accompan coh #assadors #pherd #acio #uchs strengthens #reprene Scotia #rocal</pre>
<pre>#"},{" #"}]," #estones #Learn #ounces #egu #Growing #ributes #external #encers Browse jointly Growing #ossom</pre>		bullshit stupid inept shit idiots shitty crappy incompetence fuck pointless nonsense nonsensical stupidity gimmick	bullshit shit :(lol stupid filler shitty fucking pointless idiots anyways nonsense anyway crappy	<pre>#etooth #verning #ounces #accompan coh #assadors #pherd #acio #uchs strengthens #reprene Scotia #rocal reciprocal</pre>
<pre>#"},{" #"}]," #estones #Learn #ounces #egu #Growing #ributes #external #encers Browse jointly Growing #ossom honoured</pre>		bullshit stupid inept shit idiots shitty crappy incompetence fuck pointless nonsense nonsensical stupidity gimmick inco	bullshit shit :(lol stupid filler shitty fucking pointless idiots anyways nonsense anyway crappy stupidity	<pre>#etooth #verning #ounces #accompan coh #assadors #pherd #acio #uchs strengthens #reprene Scotia #rocal reciprocal Newly</pre>
#"},{" #"}]," #estones #Learn #ounces #egu #Growing #ributes #external #encers Browse jointly Growing #ossom honoured #accompan		bullshit stupid inept shit idiots shitty crappy incompetence fuck pointless nonsense nonsensical stupidity gimmick inco lame	bullshit shit :(lol stupid filler shitty fucking pointless idiots anyways nonsense anyway crappy stupidity fuck	<pre>#etooth #verning #ounces #accompan coh #assadors #pherd #acio #uchs strengthens #reprene Scotia #rocal reciprocal Newly fost</pre>

diff	-diff	diff	-diff
<pre>#utterstock #ARGET #cffff #etooth #Federal POLITICO #Register #Registration #rollment #ETF #ulia Payments #IRC Regulatory Alternatively #RN #pta Regulation #GBT #":""},{"</pre>	amazing movie alot scenes comedy movies cinematic greatness wonderful storytelling film tho masterpiece films Kubrick realism comedic cinem #movie genre	<pre>#also #knit helps strengthens :) broaden #ossom incorporates #Learn incorporate #"},{" enjoy enjoyed complementary #etts enhances integrates #ospons differs #arger</pre>	meaningless incompetence inco pointless incompetent Worse inept nonsensical coward unint obfusc excuses panicked useless bullshit stupid incompet incomprehensibl stupidity lifeless
diff	-diff	#arger	IIIeIeSS
amazing beautifully love wonderful wonderfully unforgettable beautiful loving #love #beaut enjoyable #Beaut inspiring fantastic defin incredible memorable greatness amazingly timeless	#iannopoulos expired ABE Yiannopoulos liability #SpaceEngineers #isance Politico waivers #utterstock excise #Stack phantom PubMed #ilk impunity ineligible Coulter issuance IDs		

$W_{\mathbf{K}}$ Subheads

Layer 9

3 out of 4

diff	-dif	f	diff	-diff
enclave #. #; #omial apiece #assian #. #ulent #,[#eria #ourse exerc #\/ #Wire #arium #icle #.[#/\$ #API #ium</td <td>very shit nice many wond genu</td> <td>ing h ible py nge ything ty erful inely tiful</td> <td>Then Instead Unfortunately Why Sometimes Secondly #Then But Luckily Anyway And Suddenly Thankfully Eventually Somehow Fortunately Meanwhile What Obviously Because</td> <td>any #ady #imate #cussion #ze appreci #raq currently #kers #apixel active significant #ade #imal specific #ability anyone #ker #unction reap</td>	very shit nice many wond genu	ing h ible py nge ything ty erful inely tiful	Then Instead Unfortunately Why Sometimes Secondly #Then But Luckily Anyway And Suddenly Thankfully Eventually Somehow Fortunately Meanwhile What Obviously Because	any #ady #imate #cussion #ze appreci #raq currently #kers #apixel active significant #ade #imal specific #ability anyone #ker #unction reap
diff		-diff	Decause	reap
bullshit anyway crap anyways unless nonsense #falls fuck #. fallacy #tics #punk damned #fuck stupidit shit commerci because despite movies	y als	#avorite #ilyn #xtap #insula #cedented #aternal #lyak #rieve #uana #accompan #ashtra #icer #andum Mehran #ardise #racuse #assadors #Chel rall #abella		

diff	-diff	diff	-diff
#,	Nope	#sup	#etting
work	Instead	Amazing	#liness
#icle	Thankfully	#airs	#ktop
#.	Surely	awesome	#ulkan
outdoors	#Instead	Bless	#enthal
inspiring	Fortunately	Loving	#enance
exped	Worse	my	#yre
ahead	Luckily	#OTHER	#eeds
together	#Thankfully	#BW	omission
touches	Unless	#perfect	#reys
out	Apparently	#-)	#lihood
personalized	Perhaps	amazing	#esian
#joy	#Unless	#adult	#holes
#unction	#Fortunately	perfect	syndrome
warm	Sorry	welcome	grievance
exceptional	Secondly	Rated	offenders
experience	#Luckily	#Amazing	#wig
lasting	#Rather	#anch	#hole
integ	Hence	FANT	#creen
#astic	Neither	#anche	#pmwiki

Layer 11 2 out of 4

diff	-diff	diff	-diff
shots shit bullshit stuff tits crap boobs creepy noises spectacle boring things everything noise #anim ugly garbage stupidity	#Kind suscept Fathers #Footnote concess #accompan Strait #orig #ESE #ufact Founder #iere #HC #Prev #alias participated #Have #coe	#ly storytelling sounding spectacle #ness #hearted cinematic #est portrayal quality paced combination juxtap representation mixture #!!!!! filmmaking enough	#say actionGroup prefers #ittees #reon presumably waivers #aucuses #Phase #racuse #arge #hers #sup #later expired stricter #onds #RELATED
visuals selfies	#Father strugg	thing rendition	<pre>#rollment #orders</pre>

 $W_{
m V}$ Subheads Layer 9 4 out of 4

diff	-diff	diff	-diff
#":""},{"	honestly	crap	jointly
#etooth	definitely	shit	#verning
#ogenesis	hilarious	bullshit	#pora
#verning	alot	fucking	#rocal
broker	amazing	idiots	#raft
#ounces	funn	fuck	#etooth
threatens	cinem	goddamn	#estead
#astical	Cinem	stupid	#ilitation
foothold	comedic	FUCK	#ourse
intruder	Absolutely	#fuck	migr
#vernment	comedy	shitty	#ourses
#activation	absolutely	damn	#iership
#Oracle	amazingly	#shit	Pione
fugitive	satire	lol	#iscover
visitor	underrated	fuckin	pioneering
#assian	really	nonsense	#egu
barrier	fantastic	crappy	#ivities
#":[enjoyable	kinda	neighbourhood
#vier	REALLY	Fuck	pioneer
#oak	wonderful	idiot	nurt
diff	-diff	diff	-diff
crap	Pione	anime	#rade
crap bullshit	Pione pioneers	anime kinda	#rade #jamin
-			"
bullshit	pioneers	kinda	#jamin
bullshit shit	pioneers complementary	kinda stuff	#jamin #ounces
bullshit shit vomit	pioneers complementary pioneering	kinda stuff shit	#jamin #ounces #pherd
bullshit shit vomit nonsense	<pre>pioneers complementary pioneering #knit</pre>	kinda stuff shit lol	#jamin #ounces #pherd Unable
bullshit shit vomit nonsense stupid	<pre>pioneers complementary pioneering #knit #raits</pre>	kinda stuff shit lol tho	#jamin #ounces #pherd Unable #pta
bullshit shit vomit nonsense stupid idiots	<pre>pioneers complementary pioneering #knit #raits Browse</pre>	kinda stuff shit lol tho realism	#jamin #ounces #pherd Unable #pta Roche
bullshit shit vomit nonsense stupid idiots fucking	<pre>pioneers complementary pioneering #knit #raits Browse #iscover</pre>	kinda stuff shit lol tho realism damn	#jamin #ounces #pherd Unable #pta Roche Payments
bullshit shit vomit nonsense stupid idiots fucking #shit	<pre>pioneers complementary pioneering #knit #raits Browse #iscover strengthened</pre>	kinda stuff shit lol tho realism damn :)	#jamin #ounces #pherd Unable #pta Roche Payments Gupta
bullshit shit vomit nonsense stupid idiots fucking #shit idiot	<pre>pioneers complementary pioneering #knit #raits Browse #iscover strengthened #rocal</pre>	kinda stuff shit lol tho realism damn :) fucking	#jamin #ounces #pherd Unable #pta Roche Payments Gupta #odan
bullshit shit vomit nonsense stupid idiots fucking #shit idiot fuck	pioneers complementary pioneering #knit #raits Browse #iscover strengthened #rocal prosper	kinda stuff shit lol tho realism damn :) fucking alot	#jamin #ounces #pherd Unable #pta Roche Payments Gupta #odan #uez
bullshit shit vomit nonsense stupid idiots fucking #shit idiot fuck gimmick	pioneers complementary pioneering #knit #raits Browse #iscover strengthened #rocal prosper Communities	kinda stuff shit lol tho realism damn :) fucking alot movie	#jamin #ounces #pherd Unable #pta Roche Payments Gupta #odan #uez #adr
bullshit shit vomit nonsense stupid idiots fucking #shit idiot fuck gimmick stupidity	pioneers complementary pioneering #knit #raits Browse #iscover strengthened #rocal prosper Communities neighbourhoods	kinda stuff shit lol tho realism damn :) fucking alot movie funny	#jamin #ounces #pherd Unable #pta Roche Payments Gupta #odan #uez #adr #ideon #Secure
bullshit shit vomit nonsense stupid idiots fucking #shit idiot fuck gimmick stupidity goddamn	pioneers complementary pioneering #knit #raits Browse #iscover strengthened #rocal prosper Communities neighbourhoods #Learn strengthens	kinda stuff shit lol tho realism damn :) fucking alot movie funny anyways	#jamin #ounces #pherd Unable #pta Roche Payments Gupta #odan #uez #adr #ideon #Secure
bullshit shit vomit nonsense stupid idiots fucking #shit idiot fuck gimmick stupidity goddamn shitty	pioneers complementary pioneering #knit #raits Browse #iscover strengthened #rocal prosper Communities neighbourhoods #Learn strengthens	kinda stuff shit lol tho realism damn :) fucking alot movie funny anyways enjoyable crap comedy	<pre>#jamin #ounces #pherd Unable #pta Roche Payments Gupta #odan #uez #adr #ideon #Secure #raught</pre>
bullshit shit vomit nonsense stupid idiots fucking #shit idiot fuck gimmick stupidity goddamn shitty incompetence	pioneers complementary pioneering #knit #raits Browse #iscover strengthened #rocal prosper Communities neighbourhoods #Learn strengthens #iscovery	kinda stuff shit lol tho realism damn :) fucking alot movie funny anyways enjoyable crap	<pre>#jamin #ounces #pherd Unable #pta Roche Payments Gupta #odan #uez #adr #ideon #Secure #raught Bei</pre>
bullshit shit vomit nonsense stupid idiots fucking #shit idiot fuck gimmick stupidity goddamn shitty incompetence lame	pioneers complementary pioneering #knit #raits Browse #iscover strengthened #rocal prosper Communities neighbourhoods #Learn strengthens #iscovery #ributes	kinda stuff shit lol tho realism damn :) fucking alot movie funny anyways enjoyable crap comedy	#jamin #ounces #pherd Unable #pta Roche Payments Gupta #odan #uez #adr #ideon #Secure #raught Bei sovere

diff	-diff	diff	-diff
#knit	crap	#"}],"	crap
welcomes	bullshit	#verning	stupid
Together	idiots	#etooth	shit
Growing	stupid	#"},{"	fucking
#Explore	shitty	Browse	fuck
pioneering	incompetence	#Register	shitty
complementary	pointless	#Lago	bullshit
milestone	goddamn	#raft	crappy
pioneer	retarded	#egu	idiots
#Together	lame	jointly	horrible
strengthens	Worse	#iership	stupidity
#ossom	crappy	strengthens	kinda
pioneers	incompet	Scotia	goddamn
#Learn	shit	#ounces	awful
jointly	stupidity	#uania	mediocre
#Growing	fucking	#iann	pathetic
embraces	Nope	workspace	#fuck
#"},{"	FUCK	seiz	damn
sharing	incompetent	Payments	FUCK
#Discover	pathetic	#Learn	damned
diff	-diff	diff	-diff
bullshit	inspiring	bullshit	Pione
bullshit incompetence	inspiring unforgettable	bullshit	Pione pioneers
bullshit incompetence Worse	inspiring unforgettable #knit	bullshit crap stupid	Pione pioneers pioneering
bullshit incompetence Worse idiots	inspiring unforgettable #knit #love	bullshit crap stupid nonsense	Pione pioneers pioneering complementary
bullshit incompetence Worse idiots crap	inspiring unforgettable #knit #love passions	bullshit crap stupid nonsense incompetence	Pione pioneers pioneering complementary #knit
bullshit incompetence Worse idiots crap dummy	inspiring unforgettable #knit #love passions cherish	bullshit crap stupid nonsense incompetence idiots	Pione pioneers pioneering complementary #knit #Learn
bullshit incompetence Worse idiots crap dummy incompetent	inspiring unforgettable #knit #love passions cherish richness	bullshit crap stupid nonsense incompetence idiots shit	Pione pioneers pioneering complementary #knit #Learn #accompan
bullshit incompetence Worse idiots crap dummy incompetent Nope	inspiring unforgettable #knit #love passions cherish richness timeless	bullshit crap stupid nonsense incompetence idiots shit stupidity	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid	inspiring unforgettable #knit #love passions cherish richness timeless loves	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid retarded	inspiring unforgettable #knit #love passions cherish richness timeless loves passionate	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless inco	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable #ossom
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid retarded lame	inspiring unforgettable #knit #love passions cherish richness timeless loves passionate beautifully	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless inco retarded	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable #ossom #Together
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid retarded lame nonexistent	inspiring unforgettable #knit #love passions cherish richness timeless loves passionate beautifully overcoming	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless inco retarded idiot	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable #ossom #Together Browse
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid retarded lame nonexistent wasting	inspiring unforgettable #knit #love passions cherish richness timeless loves passionate beautifully overcoming unique	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless inco retarded idiot vomit	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable #ossom #Together Browse versatile
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid retarded lame nonexistent wasting #Fuck	inspiring unforgettable #knit #love passions cherish richness timeless loves passionate beautifully overcoming unique highs	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless inco retarded idiot vomit lame	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable #ossom #Together Browse versatile welcomes
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid retarded lame nonexistent wasting #Fuck bogus	inspiring unforgettable #knit #love passions cherish richness timeless loves passionate beautifully overcoming unique highs nurture	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless inco retarded idiot vomit lame meaningless	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable #ossom #Together Browse versatile welcomes #"},{"
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid retarded lame nonexistent wasting #Fuck bogus worse	inspiring unforgettable #knit #love passions cherish richness timeless loves passionate beautifully overcoming unique highs nurture unparalleled	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless inco retarded idiot vomit lame meaningless goddamn	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable #ossom #Together Browse versatile welcomes #"},{" admired
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid retarded lame nonexistent wasting #Fuck bogus worse nonsense	inspiring unforgettable #knit #love passions cherish richness timeless loves passionate beautifully overcoming unique highs nurture unparalleled vibrant	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless inco retarded idiot vomit lame meaningless goddamn nonsensical	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable #ossom #Together Browse versatile welcomes #"},{" admired jointly
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid retarded lame nonexistent wasting #Fuck bogus worse nonsense ineligible	inspiring unforgettable #knit #love passions cherish richness timeless loves passionate beautifully overcoming unique highs nurture unparalleled vibrant #beaut	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless inco retarded idiot vomit lame meaningless goddamn nonsensical garbage	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable #ossom #Together Browse versatile welcomes #"},{" admired jointly Sharing
bullshit incompetence Worse idiots crap dummy incompetent Nope stupid retarded lame nonexistent wasting #Fuck bogus worse nonsense	inspiring unforgettable #knit #love passions cherish richness timeless loves passionate beautifully overcoming unique highs nurture unparalleled vibrant	bullshit crap stupid nonsense incompetence idiots shit stupidity pointless inco retarded idiot vomit lame meaningless goddamn nonsensical	Pione pioneers pioneering complementary #knit #Learn #accompan pioneer invaluable #ossom #Together Browse versatile welcomes #"},{" admired jointly

diff	-diff	diff	-diff
Provision	alot	crap	#rocal
issuance	amazing	fucking	#verning
Securities	kinda	bullshit	#etooth
#ogenesis	fucking	fuck	#uania
Holdings	awesome	goddamn	caches
Regulatory	funny	shit	Browse
indefinitely	damn	#fuck	#"},{"
Advisory	REALLY	stupidity	#imentary
designation	hilarious	pathetic	exerc
unilaterally	tho	spoiler	#Lago
Province	unbelievable	stupid	#"}j,"
Regulation	fuckin	inept	#cium
#Lago	wonderful	blah	#enges
issued	doesnt	FUCK	#ysis
Recep	definitely	awful	quarterly
Advis	thats	shitty	#iscover
#verning	veah	trope	Scotia
broker	fantastic	Godd	#resso
#Mobil	badass	inco	#appings
Policy	dont	incompetence	jointly
-			
diff	-diff	diff	-diff
pioneers	bullshit	Worse	#knit
pioneers pioneering	bullshit crap	Worse bullshit	#knit pioneers
pioneers pioneering Browse	bullshit crap shit	Worse bullshit Nope	#knit pioneers pioneering
pioneers pioneering Browse Pione	bullshit crap shit idiots	Worse bullshit Nope crap	#knit pioneers pioneering inspiring
pioneers pioneering Browse	bullshit crap shit	Worse bullshit Nope	#knit pioneers pioneering inspiring #iscover
pioneers pioneering Browse Pione complementary #knit	bullshit crap shit idiots stupid vomit	Worse bullshit Nope crap incompetence idiots	#knit pioneers pioneering inspiring #iscover complementary
pioneers pioneering Browse Pione complementary #knit prosper	bullshit crap shit idiots stupid vomit incompetence	Worse bullshit Nope crap incompetence idiots incompetent	#knit pioneers pioneering inspiring #iscover complementary pioneer
pioneers pioneering Browse Pione complementary #knit	bullshit crap shit idiots stupid vomit incompetence nonsense	Worse bullshit Nope crap incompetence idiots incompetent stupid	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom
pioneers pioneering Browse Pione complementary #knit prosper #raits	bullshit crap shit idiots stupid vomit incompetence nonsense gimmick	Worse bullshit Nope crap incompetence idiots incompetent stupid incompet	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate
pioneers pioneering Browse Pione complementary #knit prosper #raits #Trend	bullshit crap shit idiots stupid vomit incompetence nonsense	Worse bullshit Nope crap incompetence idiots incompetent stupid	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate passions
pioneers pioneering Browse Pione complementary #knit prosper #raits #Trend #ributes #Learn	bullshit crap shit idiots stupid vomit incompetence nonsense gimmick stupidity idiot	Worse bullshit Nope crap incompetence idiots incompetent stupid incompet pointless inco	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate passions journeys
pioneers pioneering Browse Pione complementary #knit prosper #raits #Trend #ributes #Learn strengthen	bullshit crap shit idiots stupid vomit incompetence nonsense gimmick stupidity	Worse bullshit Nope crap incompetence idiots incompetent stupid incompet pointless	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate passions
pioneers pioneering Browse Pione complementary #knit prosper #raits #Trend #ributes #Learn	bullshit crap shit idiots stupid vomit incompetence nonsense gimmick stupidity idiot shitty	Worse bullshit Nope crap incompetence idiots incompetent stupid incompet pointless inco Stupid	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate passions journeys unique
pioneers pioneering Browse Pione complementary #knit prosper #raits #Trend #ributes #Learn strengthen strengthened	bullshit crap shit idiots stupid vomit incompetence nonsense gimmick stupidity idiot shitty fucking lame	Worse bullshit Nope crap incompetence idiots incompetent stupid incompet pointless inco Stupid meaningless	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate passions journeys unique embraces
pioneers pioneering Browse Pione complementary #knit prosper #raits #Trend #ributes #Learn strengthen strengthened #ossom	bullshit crap shit idiots stupid vomit incompetence nonsense gimmick stupidity idiot shitty fucking	Worse bullshit Nope crap incompetence idiots incompetent stupid incompet pointless inco Stupid meaningless nonsense	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate passions journeys unique embraces admired
pioneers pioneering Browse Pione complementary #knit prosper #raits #Trend #ributes #Learn strengthen strengthened #ossom pioneer	bullshit crap shit idiots stupid vomit incompetence nonsense gimmick stupidity idiot shitty fucking lame crappy	Worse bullshit Nope crap incompetence idiots incompetent stupid incompet pointless inco Stupid meaningless nonsense lame	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate passions journeys unique embraces admired forefront
pioneers pioneering Browse Pione complementary #knit prosper #raits #Trend #ributes #Learn strengthen strengthened #ossom pioneer #iscover	bullshit crap shit idiots stupid vomit incompetence nonsense gimmick stupidity idiot shitty fucking lame crappy goddamn	Worse bullshit Nope crap incompetence idiots incompetent stupid incompet pointless inco Stupid meaningless nonsense lame idiot	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate passions journeys unique embraces admired forefront richness
pioneers pioneering Browse Pione complementary #knit prosper #raits #Trend #ributes #Learn strengthen strengthened #ossom pioneer #iscover #Growing	bullshit crap shit idiots stupid vomit incompetence nonsense gimmick stupidity idiot shitty fucking lame crappy goddamn pointless inco	Worse bullshit Nope crap incompetence idiots incompetent stupid incompet pointless inco Stupid meaningless nonsense lame idiot worse	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate passions journeys unique embraces admired forefront richness invaluable
pioneers pioneering Browse Pione complementary #knit prosper #raits #Trend #ributes #Learn strengthen strengthened #ossom pioneer #iscover #Growing prosperity	bullshit crap shit idiots stupid vomit incompetence nonsense gimmick stupidity idiot shitty fucking lame crappy goddamn pointless inco	Worse bullshit Nope crap incompetence idiots incompetent stupid incompet pointless inco Stupid meaningless nonsense lame idiot worse #Fuck	#knit pioneers pioneering inspiring #iscover complementary pioneer #ossom passionate passions journeys unique embraces admired forefront richness invaluable prosper

$W_{\mathbf{O}}$ Subheads

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Layer 10
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Layer 11
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