Analyzing large-scale spike trains data with spatio-temporal constraints

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1 Context

Recent experimental advances have made it possible to record several hundred neurons simultaneously in the retina as well as in the cortex. Analyzing such a huge amount of data requires to elaborate statistical, mathematical and numerical methods, to describe both the spatio-temporal structure of the population activity and its relevance to sensory coding. Among these methods, the maximum entropy principle has been used to describe the statistics of spike trains. Recall that the maximum entropy principle consists of fixing a set of constraints, determined with the empirical average of quantities ("observables") measured on the raster: for example average firing rate of neurons, or pairwise correlations. Maximising the statistical entropy given those constraints provides a probability distribution, called a Gibbs distribution, that provides a statistical model to fit the data and extrapolate phenomenological laws. Most approaches were restricted to instantaneous observables i.e. quantities corresponding to spikes occurring at the same time (singlets, pairs, triplets and so on. See [6],[5] and [4]). The corresponding Gibbs distributions are only successful in predicting the probability of patterns lying within one time bin [2] but fail to predict the temporal statistics of the neural activity.

2 Extension of the maximum entropy to spatio-temporal statistics

We first define the Gibbs potential as:

$$\mathcal{H}_\beta = \sum_{k=1}^{K} \beta_k \mathcal{O}_k, \tag{1}$$

where the β_ks are real numbers and free parameters. $\mathcal{O}_k s$ are the observables that defines the constraints. Fitting the data to this Gibbs potential model aims

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to reproduce the probability distribution of the data, i.e, finding a theoretical distribution whose observables probabilities are close to the empirical probabilities measured on the raster, using the maximum entropy principle. We have extended this principle considering spatio temporal constraints (i.e. neurons interactions with time lags) and allowing the characterization of spatio-temporal patterns statistics. This approach, based on a transfer matrix technique, avoids the computation of the normalization factor (partition function) and directly provides the moment generating function (free energy density). We first developed a numerical method to learn the parameters of a maximum entropy model with spatio-temporal constraints on real spike train data [7]. However, this exact method could only be applied to small subsets of neurons, since it required construction of huge matrices whose size increases exponentially with the number of neurons.

To circumvent this combinatorial explosion, we then adopted a Monte-Carlo approach to compute the observable averages for a given set of model parameters (see [3] for more details). We first tested the efficiency of this new algorithm in the case where a raster plot is generated from a known Gibbs distribution [Fig. 1]. We then make the algorithm find back this Gibbs distribution from that raster. This allowed us to quantify both convergence rate and model accuracy as a function of the number of neurons. The main advantage of this new technique is its computational speed. Fig. 2 shows the difference between the computation time taken using transfer matrix technique and the Monte-Carlo technique: The computation time with Monte-Carlo increases linearly with the number of neurons while it increases exponentially with transfer matrix based technique.

3 Learning the parameters β_k

We have also developed an efficient algorithm to compute the parameters β_k defining the Gibbs distribution, even for large data sets. We adopted the method used in [1] and we extended it to the spatio-temporal case. Their method is based on the minimization a criterion, the negative likelihood, given by (Eq. 2)

$$L_{\pi_\omega^{(T)}}(\beta) = -\pi_\omega^{(T)}[\log \mu_\beta], \tag{2}$$

where $\beta_k s$ are the parameters to be inferred, μ_β is the probability distribution estimated using Monte-Carlo technique, $\pi_\omega^{(T)}$ is the empirical distribution and finally. $L_{\pi_\omega^{(T)}}(\beta)$ represent the likelihood between the data and the model, to be minimized. We have tested the method considering large spatio temporal data sets where the number N of neurons is large ($N \sim 100$) as well as the model range R, where the exact transfer matrix computation becomes intractable. In Fig. 3 we present the maximal distance (Eq. 3) between the exact value of the coefficient and the estimated value, averaged on a set of 10 random Gibbs potentials.

The error is given by the following formula:

$$Error = \frac{1}{L} \sum_{k=1\ldots L} \left| \beta_k - \hat{\beta_k} \right| \tag{3}$$

where β_k is the exact value of coefficient k, $\hat{\beta_k}$ is the estimated value and L is the number of terms in the potential.

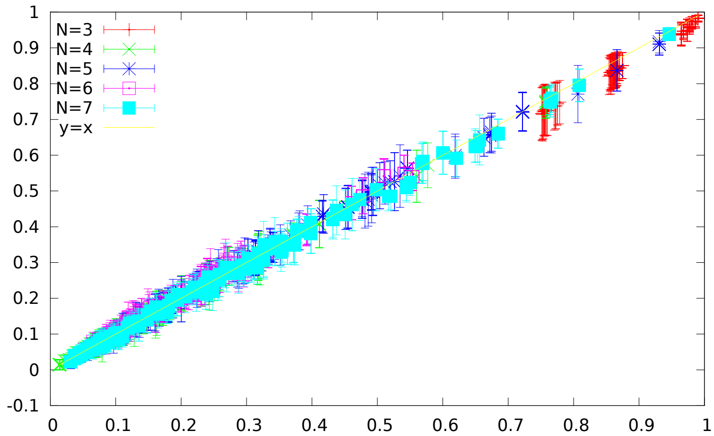

Figure 1: Comparison between the real ($x - axis$) and estimated ($y - axis$) values of observable averages (Points represent the mean value of the observable averages and segments represents the standard deviation, over 10 trials).Given a random Gibbs potential, we computed the probability distribution of observables with the transfer-matrix based technique and with the Monte-Carlo based technique. Since the transfer matrix cannot handle for large sets, we were limited to a small number of neurons in order compare the empirical and theoretical averages. The values obtained with Monte-Carlo are closed to those obtained with the transfer matrix technique since they are all aligned to the $y = x$ axis.

4 Conclusion

We have extended the maximum entropy technique to handle spatio-temporal temporal constraint and developped a Monte-Carlo based method which allows to infer statistical properties for large data set, in less time than the transfer matrix based technique. This work will help to analyse spike train sorted from large MEA (Multi-Electrode array) recordings and discover the structure of the hidden statistics behind the data. The choice of the Gibbs potential form (what are the terms that we put in the potential) remains a challenge and will be subject for future studies.

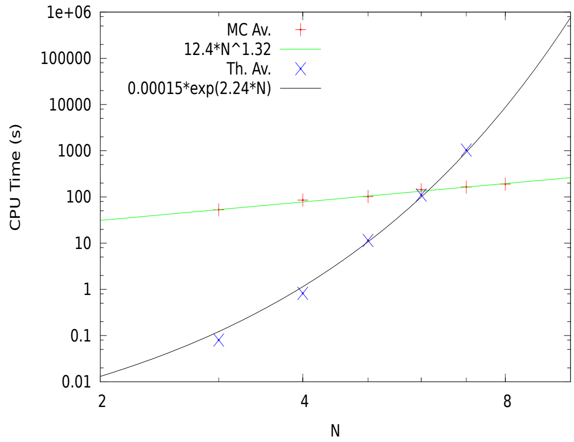

Figure 2: The CPU time necessary to obtain the observable averages presented in Fig. 1, for the Monte-Carlo Average (MC Av.) and for the theoretical (Th. Av.), as a function of N (Here, the theoretical average is exact since the potential is generated synthetically with known probability distribution). The full lines correspond to fit. The computation time with Monte-Carlo increases linearly with the number of neurons while it increases exponentially with transfer matrix based technique.

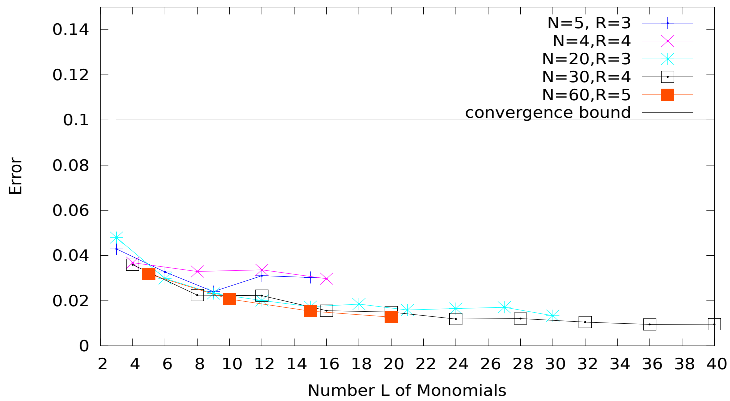

Figure 3: Max distance between the exact values of coefficients and the estimated values, averaged on a set of 10 random potentials for several values of N, R. Since for each (N, R) value the potential is not the same, then the maximum number of monomials (L) will not be the same and by consequence we observe that the curves do not have all the same number of points.

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