INF367: Spring 2020
Exercise 2

Instructions:

You can either return the solutions electronically via MittUiB by Monday 10.00 or show them on paper on Monday's meeting. Grades are awarded for effort so scanned notes are fine if you solve exercises by hand (no need to make fancy latex files).

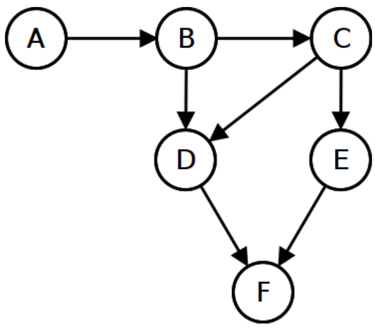
Tasks

1.

Show by using the d-separation criterion that a node in a Bayesian network is conditionally independent of all the other nodes, given its Markov blanket.

2.

List all d-separations in the following DAG (for each pair of d-separated variables, it is enough to give one set that d-separates them):



3.

List all 3-node Bayesian networks, and partition them into equivalence classes. For the notation, let us call the nodes X, Y and Z, and let XY means that there is a directed arc from X to Y . For example, $\{\}$ is an empty network with no arcs, and $\{XY, XZ, YZ\}$ is a fully connected network with arcs $X \to Y$, $X \to Z$ and $Y \to Z$.

4.

[Friedman and Koller 3.18] An edge $X \to Y$ in graph G is said to be *covered* if $Pa_G(Y) = Pa_G(X) \cup \{X\}$. Covered edges have the nice property that for every pair of Markov equivalent networks G and G', there exists a sequence of covered edge reversal operations that converts G to G'.

a) Let G be a DAG with a covered edge $X \to Y$ and G' the graph that results by reversing the edge $X \to Y$ to produce $Y \to X$, but leaving everything else unchanged. Prove that G and G' are Markov equivalent.

b) Provide a counterexample to this result in the case where $X \to Y$ is not a covered edge.

5.

Consider a distribution of five binary variables x_i.

(a) What is the number of parameters needed to define the distribution $P(x_1, x_2, x_3, x_4, x_5)$ if no assumptions are made, i.e. P is an arbitrary distribution.

(b) How about if the Bayesian network in Figure 1 is assumed, i.e. P factorizes as implied by the graph.

(c) And how about if, additionally to (b), we assume that the conditional distributions are shared, i.e. $P(x_{i+1} \mid x_i) = P(x_i \mid x_{i-1})$, $i = 2, 3, 4$?

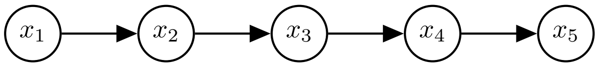


Figure 1: Model for Problem 5.