

INF367: Spring 2020

Exercise 1

Instructions:

You can either return the solutions electronically via MittUiB by Monday 10.00 or show them on paper on Monday's meeting. Grades are awarded for effort so scanned notes are fine if you solve exercises by hand (no need to make fancy latex files).

Tasks

1. Prove the following properties using the axioms of probability.

- a) $P(\emptyset) = 0$.
- b) If $A \subseteq B$, then $P(A) \leq P(B)$.
- c) $P(A \setminus B) = P(A) - P(A \cap B)$.
- d) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

2. The joint probability distribution $P(X, Y)$ of two variables X and Y is given as follows:

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$Y = 0$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	0
$Y = 1$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$
$Y = 2$	0	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

- (a) Calculate the marginal distribution of X and Y , i.e., $P(X)$ and $P(Y)$.
- (b) Calculate $P(X|Y = 2)$.
- (c) Calculate $P(Y|X = 1)$.
- (d) Justify, if X and Y are independent.
- (e) Suppose $W = X + Y$, find the distribution of W .

3. A population consists of 818 persons, out of which 276 are vaccinated against a certain epidemic. 69 persons got sick and 3 of them were vaccinated.

- a) What is the probability that a person got sick given he was vaccinated?
- b) What is the probability that a person was vaccinated given he didn't get sick?

4. Independencies

- a) Prove that if $P(A) = 0$ (or $= 1$), then A is independent of all events B (at the same sample space).
- b) Let A , B , and C be events such that $P(A) = P(B) = P(C) = 0.25$. Compute $P(A \cup B \cup C)$, when we in addition know that
 - i) A , B , and C are independent,
 - ii) A , B and C are disjoint.

Hint: 1.d) may be useful

5. [Barber 1.18] Sally is new to the area and listens to some friends discussing about another female friend. Sally knows that they are talking about either Alice or Bella but doesn't know which. From previous conversations Sally knows some independent pieces of information: She's 90% sure that Alice has a white car, but doesn't know if Bella's car is white or black. Similarly, she's 90% sure that Bella likes sushi, but doesn't know if Alice likes sushi. Sally hears from the conversation that the person being discussed hates sushi and drives a white car. What is the probability that the friends are talking about Alice?