INF-367A V20 EXERCISE 2
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1. Def 1.1: A path between variables A and B is blocked by a set of variables C, if either of the cases below are true:
   i) there is a collider in the path s.t. neither the collider nor any of its descendants is in the conditioning set C.
   OR
   ii) there is a non-collider in the path that is in the conditioning set C.

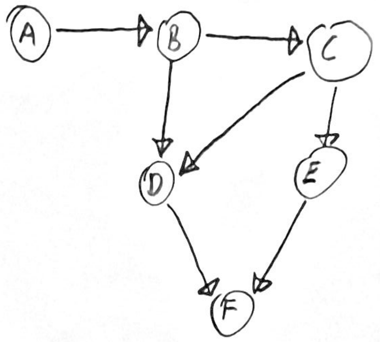
Def 1.2: Sets of variables A and B are d-separated by C if all paths between A and B are blocked by C.
   - d-separation implies $A \perp\!\!\!\perp B \mid C$

Def 1.3: Markov blanket of a node X, denoted by MB(x), consists of the children, parents and spouses (co-parents of children) of X.

Claim: $X \perp N \setminus MB(x) \mid MB(x)$

Proof: Any path from X to $N \setminus MB(x)$ has a non-collider formation at the last node before leaving MB(x). Given that MB(x) has been observed, then by case ii) in Def 1.1 we know that every paths are blocked, which implies that X and $N \setminus MB(x)$ are d-separated, which again implies that namely $X \perp N \setminus MB(x) \mid MB(x)$ is true.

2. List all d-separations in the following DAG:



We have:

$A \perp C \mid B$ $\quad B \perp E \mid C$

$A \perp D \mid B$ $\quad B \perp F \mid (D \wedge C)$

$A \perp E \mid B$ $\quad C \perp F \mid (D \wedge E)$

$A \perp F \mid B$ $\quad D \perp E \mid C$

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3. Class

1: $\emptyset$

2: $\{XZ, ZX\}$

3: $\{XY, YX\}$

4: $\{YZ, ZY\}$

5. $\{XY, ZY\}$

6. $\{XZ, YZ\}$

7. $\{YX, ZX\}$

8. $\{YZ, XY\}, \{YX, ZY\}, \{YX, YZ\}$

9. $\{ZY, XZ\}, \{ZX, ZY\}, \{ZX, YZ\}$

10. $\{YX, XZ\}, \{XY, ZX\}, \{XZ, XY\}$

11. $\{XZ, ZY, XY\}, \{XY, YZ, XZ\}, \{YX, YZ, XZ\}, \{YX, YZ, ZX\}, \{ZX, ZY, XY\}, \{ZX, ZY, YX\}$

This is called a tedious task. Please no more tedious tasks.

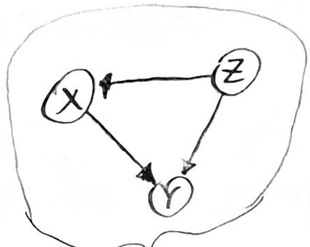
Networks in the same class are markov equivalence:
- same undirected graph (skeleton)
- Same v-structures (immoralities)

4a) Let G be a DAG with a covered edge $X \to Y$ and G' the graph that results by reversing the edge $X \to Y$, but leaving everything else unchanged. Prove that G and G' are Markov equivalent.

Criteria for Markov eq: 1. Same skeleton 2. Same v-structures

Proof: 1. Changing from $X \to Y$ to $X \leftarrow Y$ will obviously not alter the skeleton, so G and G' have same skeleton.

2. Since we know that the $X \to Y$ edge is covered we know that the structure looks like this:



not a v-structure

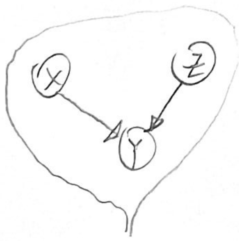
Reversing $X \to Y$ to $Y \to X$ will change any v-structures in G, implying G has same v-structures as G'.

Behold! Both criteria for Markov eq. is fulfilled.

4b) Provide counterexample where $X \to Y$ is not covered:

First part of proof above still holds.

Second part will not. The case is now:



v-structure

Obviously reversing $X \to Y$ to $Y \to X$ alters the v-structure. So criterions for Markov eq. is not fulfilled.

5a) Number of params needed to define the distribution $P(X_1, X_2, X_3, X_4, X_5)$ if no assumptions are made:

Generally, let d be the number of random vars, then the probability table is of size $2^d - 1$. In our case: $2^5 - 1 = \underline{\underline{31}}$

5b) How many parameters are needed for:

$$X_1 \to X_2 \to X_3 \to X_4 \to X_5$$

We'll need $P(X_1)$, $P(X_2)$, $P(X_2|X_1)$, $P(X_3)$, $P(X_3|X_2)$, $P(X_4)$, $P(X_4|X_3)$, $P(X_5)$, $P(X_5|X_4)$ $\Rightarrow$ $\underline{\underline{9 \text{ params}}}$

5c) How many params to remember if know that

$P(X_{i+1}|X_i) = P(X_i|X_{i-1})$, $i = 2, 3, 4$?

Then only need to know one of them. Since we have 4 things, we only need 1.

In total we must know $\underline{\underline{6 \text{ params}}}$.