INF-367A V20 EXERCISE 3

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1.a) $P(\text{overslept}=\text{yes} \mid \text{Alarm}=\text{Yes}) = \dfrac{P(\text{overslept}=\text{yes}, \text{Alarm}=\text{Yes})}{P(\text{Alarm}=\text{Yes})}$

$$= \frac{P(\text{Alarm}=\text{Yes})\, P(\text{Overslept}=\text{Yes} \mid \text{Alarm}=\text{Yes})}{P(\text{Alarm} \mid \text{Yes})} = \frac{0.9 \cdot 0.1}{0.9} = \underline{\underline{0.1}}$$

1b) $P(\text{overslept}=\text{Yes}) = P(\text{overslept}=\text{Yes}, \text{Alarm}=\text{No}) + P(\text{Overslept}=\text{Yes}, \text{Alarm}=\text{Yes})$

$$= P(\text{Alarm}=\text{No})\, P(\text{overslept}=\text{Yes} \mid \text{Alarm}=\text{No}) + P(\text{Alarm}=\text{Yes})\, P(\text{overslept}=\text{Yes} \mid \text{Alarm}=\text{Yes})$$

$$= 0.1 \cdot 0.9 + 0.9 \cdot 0.1 = \underline{\underline{0.18}}$$

1c) $P(\text{overslept}=\text{Yes} \mid \text{on time}=\text{No}) = \dfrac{P(\text{overslept}=\text{Yes}, \text{on time}=\text{No})}{P(\text{on time}=\text{No})}$

Let OS = overslept, OT = on time, Yes = 1, No = 0, AO = Alarm on, BL = Bus late

$$\frac{P(OS=1, OT=0)}{P(OT=0)} = \frac{\sum_{BL}\sum_{AO} P(OS=1, OT=0, BL, AO)}{\sum_{BL}\sum_{AO}\sum_{OS} P(OT=0, OS, BL, AO)}$$

$$= \frac{\sum_{BL}\sum_{AO} P(OS=1 \mid AO)\, P(AO)\, P(OT=0 \mid BL, \overset{\overset{1}{\shortparallel}}{OS})\, P(BL)}{\sum_{BL}\sum_{AO}\sum_{OS} P(OT=0 \mid BL, OS)\, P(BL)\, P(AO)\, P(OS \mid AO)}$$

$$= \frac{\sum_{AO} P(OS=1 \mid AO) P(AO) \sum_{BL} P(OT=0 \mid BL, \overset{\overset{1}{\shortparallel}}{OS}) P(BL)}{\text{— ” —}} = \frac{P(OS=1) \sum_{BL} P(OT=0 \mid BL, OS=1) P(BL)}{\text{— ” —}}$$

$$= \frac{0.18 \cdot (0.2 \cdot 0.9 + 0.8 \cdot 0.7)}{\text{— ” —}} = \frac{0.18 \cdot 0.74}{\text{— ” —}} = \frac{0.1332}{\text{— ” —}}$$

Want Want: $P(OS=1 \mid OT=0) = \underbrace{P(OT=0 \mid OS=1)\, P(OS=1)}_{0.1332} \Big/ \big(P(OT=0 \mid OS=1) \cdot P(OS=1) + P(OT=0 \mid OS=0)\, P(OS=0)\big)$

$P(OT=0 \mid OS=0)\, P(OS=0) = P(OT=0, OS=0) = P(BL=0)\, P(OT=0 \mid BL=0, OS=0) + P(BL=1)\, P(OT=0 \mid BL=1, OS=0)$

$= 0.8 \cdot 0.1 + 0.2 \cdot 0.8 = 0.24$

OS=0

$$\Rightarrow P(OS=1 \mid OT=0) = \frac{0.1332}{0.1332 + 0.24 \cdot (1-0.18)} = \underline{\underline{0.403}}$$

1d)

$$P(OS=1 \mid OT=0, BL=1) = \frac{P(OS=1)\, P(OT=0 \mid BL=1, OS=1)\, P(BL=1)}{P(OS=1)\, P(OT=0, BL=1 \mid OS=1) + P(OS=0)\, P(OT=0, BL=1 \mid OS=0)}$$

$$= \frac{P(OS=1)\, P(OT=0 \mid BL=1, OS=1)\, \cancel{P(BL=1)}}{P(OS=1)\, P(OT=0 \mid BL=1, OS=1)\, \cancel{P(BL=1)} + P(OS=0)\, P(OT=0 \mid BL=1 \mid OS=0)\, \cancel{P(BL=1)}}$$

$$= \frac{0.18 \cdot 0.9}{0.18 \cdot 0.9 + 0.82 \cdot 0.8} = \underline{\underline{0.198}}$$

1e)

$$P(OT=0 \mid OS=1) = \frac{P(OT=0, OS=1)}{P(OS=1)} = \frac{P(OS=1 \mid OT=0)\, P(OT=0)}{P(OS=1)}$$

$$= \frac{0.403 \cdot 033}{0.18} = \underline{\underline{0.7388}}$$

2a) $P(A,B,C,D,E,F,G,H) = P(A)P(B)P(C)\,P(D|A,B)\,P(E|C,D)\,P(F|D)\,P(G|F)\,P(H|F)$

$P(D|A=0,B=1,C=1,E=0,F=1) \propto P(D=d, A=0, B=1, \overset{C=1}{E=0}, F=1)$

$P(D=d, A=0, B=1, \underset{C=1}{,} E=0, F=1) = \sum_G \sum_H P(A=0, B=1, C=1, D=d, E=0, F=1, G, H)$

$= P(A=0)P(B=1)P(C=1)\,P(D=d|A=0,B=1)\,P(E=0|C=1,D=d)\,P(F=1|D=d)\overbrace{\sum_G P(G|F=1)}^{1}\overbrace{\sum_H P(H|F=1)}^{1}$

$P(D=0, A=0, B=1, C=1, E=0, F=1) = 0.3 \cdot 0.8 \cdot 0.4 \cdot 0.3 \cdot 0.9 \cdot (1-0.5) = 0.01296$

$P(D=1, - \;\; '' \;\; -) = 0.3 \cdot 0.8 \cdot 0.4 \cdot (1-0.3) \cdot 0.3 \cdot (1-0.7) = 0.006048$

Normalization constant: $P(A=0, B=1, C=1, E=0, F=1)$. Obtain by marginalizing away D:

$P(A=0, B=1, C=1, E=0, F=1) = P(A=0)P(B=1)P(C=1)\sum_D P(D|A=0,B=1)\,P(E=0|C=1,D)\,P(F=1|D)$

$= P(D=0, \ldots) + P(D=1, \ldots) = 0.01296 + 0.006048 = 0.019008$

$P(D=0|\ldots) = \dfrac{0.01296}{0.019008} = 0.6818$

$\underline{\underline{P(D=1|\ldots) = 1 - 0.6818 = 0.3181}}$

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2b) $P(A|B=T, D=T, H=T)$

Observe that $MB(A) = \{B, D\}$

$\Rightarrow P(A|B=T, D=T, H=T) = P(A|B=T, D=T)$

State of all variables outside of $MB(A)\setminus A$ can be ignored.

$$P(A|B=T, D=T) = \frac{P(A=a, B=T, D=T)}{P(B=T, D=T)}$$

Observe that B does not have ancestor i.e $P(B)$ is just a constant

$$\Rightarrow P(A|B=T, D=T) = P(A|D=T) = \frac{P(A=a, D=T)}{P(D=T)}$$

$$= \frac{P(A=a)\, P(D=T|A=a, B=T)}{P(A=0)\, P(D=T|A=0, B=T) + P(A=T)\, P(D=T|A=T, B=T)}$$

$P(A=0)\, P(D=T|A=0, B=T) = 0.3 \cdot (1-0.3) = 0.21$

$P(A=T)\, P(D=T|A=T, B=T) = 0.7 \cdot (1-0.8) = 0.14$

$$P(A=T|B=T, D=T, H=T) = \frac{0.14}{0.21+0.14} = \underline{\underline{0.4}}$$

$P(A=0|B=T, D=T, H=T) = 1-0.4 = \underline{\underline{0.6}}$

3a) $P(x_1) = P(x_1 = x, x_2, \dots, x_d, C)$

$$= \sum_C \sum_{x_2} \dots \sum_{x_d} P(C) P(\underset{\overset{\shortparallel}{x}}{x_1}|C) P(x_2|C) \dots P(x_d|C)$$

$$= \sum_C P(C) P(x_1|C) \underbrace{\sum_{x_2} P(x_2|C)}_{1} \underbrace{\dots}_{1,1\dots} \underbrace{\sum_{x_d} P(x_d|C)}_{1}$$

$$= \sum_C P(C) P(x_1|C) \Rightarrow C \text{ is binary, } 2 \text{ multiplications } 1 \text{ addition}$$

3b) $$\dots = \sum_{x_2} \dots \sum_{x_d} \sum_C P(C) P(x_1 = x|C) P(x_2|C) \dots P(x_d|C)$$

$$= \underbrace{\sum_{x_2} \dots \sum_{x_d}}_{2^{d-1} \text{ combinations}} \underbrace{\left[P(C=0) P(x_1 = x|C=0) P(x_2|C=0) \dots P(x_d|C=0) + P(C=1) P(x_1 = x|C=1) P(x_2|C=1) \dots P(x_d|C=1) \right]}_{\gamma_C(x_2, x_3 \dots) \quad 2 \cdot d \text{ multiplications, } 1 \text{ addition}}$$

Multiplications: $2^{d-1} \cdot 2 \cdot d = d 2^d$

Additions: 2^{d-1}