INF367: Spring 2020
Exercise 4

Instructions:

You can either return the solutions electronically via MittUiB by Monday 10.00 or show them on paper on Monday's meeting. Grades are awarded for effort so scanned notes are fine if you solve exercises by hand (no need to make fancy latex files).

Students are encouraged to write computer programs to derive solutions whenever appropriate.

Tasks

1. **Bent coins**

There are two bent coins (c_1 and c_2) with different properties, and your objective is to guess which coin was used (i.e. the value of random variable $C \in \{c_1, c_2\}$), after learning the results of a sequence of coin tosses (i.e. a coin toss is a random variable $X \in \{h, t\}$ where h denoteds heads and t denotes tails).

We know the probabilities of each coin resulting in tails: $P(X = t \mid C = c_1) = \theta_1$ and $P(X = t \mid C = c_2) = \theta_2$. In addition, the prior probability for using coin c_1 is known: $P(C = c_1) = \pi$.

a) Give the posterior probability of coin c_1 being used for a single toss, $P(C = c_1 \mid X)$, in terms of θ_1 , θ_2 and π, for both $X = t$ and $X = h$.

b) Generalize the formulation in a) to compute posterior distribution $P(C = c_1 \mid D)$ where $D = \{X_i\}_{i=1}^n$ is a sequence of n coin tosses (using the same coin) consisting of n_t tails and n_h heads.

2. **Bayesian belief updating**

Assume we know that $\theta_1 = 0.7$ and $\theta_2 = 0.4$.

a) Specify a prior distribution for C by fixing the value of the parameter π.

b) Assume that we observe the following sequence of 5 tosses: $D_1 = htthh$. What is the posterior distribution $P(C \mid D_1)$ given the prior you specified in a)?

c) Later, we observe five more tosses: $D_2 = tttht$. Update your beliefs based on this new information. In other words, use the posterior from the previous step as your prior and compute the new posterior. That is, compute

$$P(C \mid D_1, D_2) = \frac{P(D_2 \mid C)P(C \mid D_1)}{P(D_2 \mid D_1)} = \frac{P(D_2 \mid C)P(C \mid D_1)}{\sum_C P(D_2 \mid C)P(C \mid D_1)}$$

d) Assume that instead of observing two sequences of 5 tosses we would have observed just one sequence of ten tosses. That is, $D = htthhttht$. Compute the posterior distribution $P(C \mid D)$ using the prior you specified in a). Is the posterior same as in c)?

3. **Predictive distribution**

Given the posterior distribution computed in 2.b, what is our belief (probability) that the next coin toss results in t?

4. **Effect of the prior**

Supoose that there are two students, Alice and Bob, who have strongly disagree about whether the coin c_1 will be tossed. Specifically, Alice is almost sure that the coin c_1 will be tossed and thus her prior distribution is specified by $\pi_A = 0.99$. On the other hand, Bob is almost certain that the coin c_2 will be used so his prior is specified by $\pi_B = 0.01$.

a) Alice and Bob observe a sequence of coin tosses D_3 resulting in 3 tails and 7 heads. What are their posteriors?

b) Alice and Bob observe a sequence of coin tosses D_4 resulting in 36 tails and 64 heads. What are their posteriors?

c) What happened to the beliefs of Alice and Bob when they observed more data?