INF-367 V20 EXCERCISE 1

NAPHAT AMUNDSEN

1. Axiom 1: The probability of an event is a real number greater than or equal to 0.

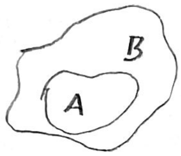
Axiom 2: The probability that at least one of all the possible outcomes of a process (such as rolling a die) will occur is 1.

Axiom 3: If two events A and B are mutually exclusive, then the probability of either A or B occuring is the probability of A occuring plus the probability of B occuring.

Prove statements

1a) $\boxed{P(\emptyset)=0}$: Axiom 2 states that the prob. of something happening is 1, which implies you will guarented get an outcome, hence the probability of getting none of the outcomes, $\emptyset$, is 0.

1b) $\boxed{\text{If } A \subseteq B, \text{ then } P(A) \leq P(B)}$:



Observe that $P(B) = P(A \cup (B \setminus A))$. Axiom 3 states that $P(B) = P(A) + P(B \setminus A)$. Axiom 1 states that probabilities are non-negative, implying $P(A) \leq P(B)$

1c) $\boxed{P(A \setminus B) = P(A) - P(A \cap B)}$:

$P(A \setminus B)$ is equal to $P(A \cap \neg B)$. Note that $P(A) = \underbrace{P(A \cap B) \cup P(A \cap \neg B)}_{\text{disjoint}}$.

Then by axiom 3: $P(A) = P(A \cap B) + P(A \cap \neg B)$

$\Leftrightarrow \quad P(A \cap \neg B) = P(A) - P(A \cap B)$

1d) $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$: $A \cup B$ can be expressed as the disjoint union $A \cup B = (A \cap \neg B) \cup (A \cap B) \cup (B \cap \neg A)$. By axiom 3:

$$\begin{aligned} P(A \cup B) &= P(A \cap \neg B) + P(A \cap B) + P(B \cap \neg A) \\ &= P(A) - \cancel{P(A \cap B)} + \cancel{P(A \cap B)} + P(B) - P(A \cap B) \\ &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

2. Joint prob. dist $P(X,Y)$ of two vars. is given as follows:

	X=0	X=1	X=2	X=3
Y=0	1/16	1/16	1/16	0
Y=1	2/16	3/16	3/16	1/16
Y=2	0	1/16	2/16	1/16

2a) Calculate marginal dists of X and Y:

	X=0	X=1	X=2	X=3	
Y=0	1/16	1/16	1/16	0	3/16
Y=1	2/16	3/16	3/16	1/16	9/16
Y=2	0	1/16	2/16	1/16	4/16
	3/16	5/16	6/16	2/16	1

Marginal dist for Y

Marginal dist for X

2b) $P(X|Y=2) = 4/16$

2c) $P(Y|X=1) = 5/16$

2d) Events A and B are independent if $P(A \cap B) = P(A)P(B)$. Essentially, the probability in each cell must be the product the marginal probabilities of its row and column.

The table does not satisfy this, hence X and Y are NOT independent vars.

2e) $W = X+Y$, outcome space: $\{0,1,2,3,5\}$. $P(W=w)$ can simply be calculated by adding the probabilities of $X=x$ and $Y=y$ such that you get $W=w$:

$P(W=0) = P(X=0) \cap P(Y=0) = 1/16$

$P(W=1) = (P(X=0) \cap P(Y=1)) \cup (P(X=1) \cap P(Y=0)) = \frac{2}{16} + \frac{2}{16} = 4/16$

Change of notation → $P(W=2) = P(X=0, Y=2) \cup P(X=2, Y=0) \cup P(X=1, Y=1) = 0 + 1/16 + 3/16 = 4/16$

$P(W=3) = P(X=3, Y=0) \cup P(X=2, Y=1) \cup P(X=1, Y=2) = 0 + 3/16 + \frac{1}{16} = 4/16$

$P(W=4) = P(X=3, Y=1) \cup P(X=2, Y=2) = 1/16 + 2/16 = 3/16$

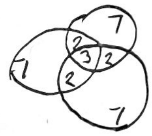
$P(W=5) = P(X=3, Y=2) = 1/16$

3a) $P(\text{sick} \mid \text{vaccinated}) = \dfrac{\#(\text{sick} \mid \text{vacc})}{\#\text{ vaccinated people}} = \dfrac{3}{276} = \underline{\underline{\dfrac{1}{92}}}$

3b) $P(\text{vacc} \mid \neg\text{sick}) = \dfrac{\#(\text{vacc} \mid \neg\text{sick})}{\#(\text{vacc} \mid \neg\text{sick}) + \#(\neg\text{vacc} \mid \neg\text{sick})} = \dfrac{276-3}{(276-3)+(818-69-273)} = \underline{\underline{\dfrac{39}{107}}}$

4a) The fact that $A \cap B \subseteq A$ implies $P(A \cap B) \leq P(A)$. Since $P(A) = 0$ $P(A \cap B)$ must also be $0 \Rightarrow P(A \cap B) = P(A) = P(A) \cdot P(B) = 0$
By definition independent since $P(A \cap B) = P(A) \cdot P(B)$.

4b) i) A, B, C are independent, compute $P(A \cup B \cup C)$:



$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap C) - P(A \cap B) - P(B \cap C) + P(A \cap B \cap C)$
$= \underline{\underline{P(A) + P(B) + P(C) - P(A) \cdot P(C) - P(A) \cdot P(B) - P(B) \cdot P(C) + P(A) \cdot P(B) \cdot P(C)}}$

ii) A, B, C are disjoint, -||- :
$\underline{\underline{P(A \cup B \cup C) = P(A) + P(B) + P(C)}}$

5

Information:

$P(\text{white car} \mid \text{Alice}) = W_A = 0.9$

$P(\text{sushi} \mid \text{Alice}) = S_A = ?$

$P(\text{white car} \mid \text{Bella}) = W_B = ?$

$P(\text{sushi} \mid \text{Bella}) = S_B = 0.9$

It's either Alice or Bella, the distribution of outcomes is

$P(\text{Alice}) = 0{,}5 \quad P(\text{Bella}) = 0.5 = P(\neg\text{Alice})$

$$P(\text{Alice} \mid \neg\text{Sushi}, \text{white car}) \overset{\text{Bayes}}{=} \frac{P(\text{Alice}) \cdot P(\neg\text{Sushi}, \text{white car} \mid \text{Alice})}{P(\text{Alice}) P(\neg\text{sushi}, \text{white car} \mid \text{Alice}) + P(\neg\text{Alice}) P(-\text{||}- \mid \neg\text{Alice})}$$

$$\overset{P(\text{Alice}) = P(\neg\text{Alice})}{\Longrightarrow} P(\text{Alice} \mid -\text{||}-) = \frac{P(\neg\text{Sushi}, \text{white car} \mid \text{Alice})}{P(\neg\text{Sushi}, \text{white car} \mid \text{Alice}) + P(\neg\text{Sushi}, \text{white car} \mid \neg\text{Alice})}$$

$$\overset{\text{indep}}{=} \frac{\neg S_A \cdot W_A}{\neg S_A \cdot W_A + \neg S_B \cdot W_B} = \frac{\neg S_A \cdot 0{,}9}{\neg S_A \cdot 0{,}9 + 0{,}1 \cdot W_B}$$

Outcome space for S_A, W_B is that person either has/likes or not, i.e two possible outcomes. Assume that either outcomes are equal: $S_A = 0.5 \quad W_B = 0.5$

$$\Rightarrow P(\text{Alice} \mid \neg\text{Sushi}, \text{white car}) = \frac{0{,}5 \cdot 0{,}9}{0{,}5 \cdot 0{,}9 + 0{,}1 \cdot 0{,}5} = \underline{\underline{0{,}9}}$$