INF367, Spring 2020
Exercise 9: Solutions

Gibbs sampler:

$$P(\mathbf{w}\,|\,y, \mathbf{x}, \beta) \quad \propto \quad N(\mathbf{w}\,|\,\mathbf{0}, \alpha^{-1}\mathbf{I}) \prod_{i=1}^{n} N(y_i\,|\,\mathbf{w}^T\mathbf{x}_i, \beta^{-1})$$

Gaussian likelihood is the conjugate prior for Gaussian prior. It follows that $P(\mathbf{w}\,|\,y, \mathbf{x}, \alpha, \beta) = N(\mathbf{w}\,|\,\mathbf{m}, \mathbf{S})$ where

$$\mathbf{S} = (\beta \sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i^T + \alpha \mathbf{I})^{-1}$$

and

$$\mathbf{m} = \mathbf{S}(\beta \sum_{i=1}^{n} y_i \mathbf{x}_i).$$

$$P(\beta\,|\,y, \mathbf{x}, \mathbf{w}, \alpha) \quad \propto \quad Gamma(\beta\,|\,a_0, b_0) \prod_{i=1}^{n} N(y_i\,|\,\mathbf{w}^T\mathbf{x}_i, \beta^{-1})$$

We notice that there are Gamma prior for the precision of Gaussian likelihood with known mean. It follows that the probability of β follows a Gamma distribution

$$Gamma(\beta\,|\,a_0 + \frac{n}{2}, b_0 + \frac{\sum_{i=1}^{n}(y_i - \mathbf{w}^T\mathbf{x}_i)^2}{2}).$$