INF367: Spring 2020
Exercise 5

Instructions:

You can either return the solutions electronically via MittUiB by Monday 10.00 or show them on paper on Monday's meeting. Grades are awarded for effort so scanned notes are fine if you solve exercises by hand (no need to make fancy latex files).

Students are encouraged to write computer programs to derive solutions whenever appropriate.

Tasks

1. **Poisson-Gamma**

Poisson distribution can be used to model count data. Suppose you have n i.i.d. observations $\mathbf{x} = \{x_i\}_{i=1}^{n}$ from a Poisson(λ) distribution with a rate parameter λ that has a conjugate prior

$$\lambda \sim \text{Gamma}(a, b)$$

with the shape and rate hyperparameters a and b.

1. Derive the posterior distribution $P(\lambda|\mathbf{x})$.
2. Load the data set `exercise5_1.txt` (100 one-dimensional data points). Compute the posterior of λ given this data.
3. Estimate the predictive distribution using Monte Carlo approximation. Plot the histogram of the predictive samples. Compare it to the original data. Do your results make sense? (If you are inclined to answer no then you have probably done something wrong along the way.)
4. Compute 50% and 95% credible intervals for the predictive distribution. Hint: You should start by sorting your samples.

Note: There are several different ways to parameterize the gamma distribution. If you use the gamma distribution from scipy, then $Gamma(a0, b0)$ distribution is specified by `gamma(a=a0, scale=1/b0)`.

2. **Multivariate Gaussian**

Suppose we have N i.i.d. observations $\mathbf{X} = \{\mathbf{x}_i\}_{i=1}^{N}$ from a multivariate Gaussian distribution

$$\mathbf{x}_i \mid \boldsymbol{\mu} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

with unknown mean parameter $\boldsymbol{\mu}$ and a known covariance matrix $\boldsymbol{\Sigma}$. As prior information on the mean parameter we have

$$\boldsymbol{\mu} \sim \mathcal{N}(\mathbf{m_0}, \mathbf{S_0}).$$

- Derive the posterior distribution $P(\boldsymbol{\mu}|\mathbf{X})$ of the mean parameter $\boldsymbol{\mu}$. Hints: It may be more comfortable to work with the log-posterior $\log P(\boldsymbol{\mu}|\mathbf{X})$. You can drop all terms that do not depend on $\boldsymbol{\mu}$. The log-posterior of a multivariate Gaussian is a second degree polynomial so you can use "completing the square".
- Load the data set `exercise5_2.txt`(100 two-dimensional data points). Compute the posterior of $\boldsymbol{\mu}$ given this data.
- (Optional) Plot the posterior in 3D. Do your results make sense?