Tree 2

Tree

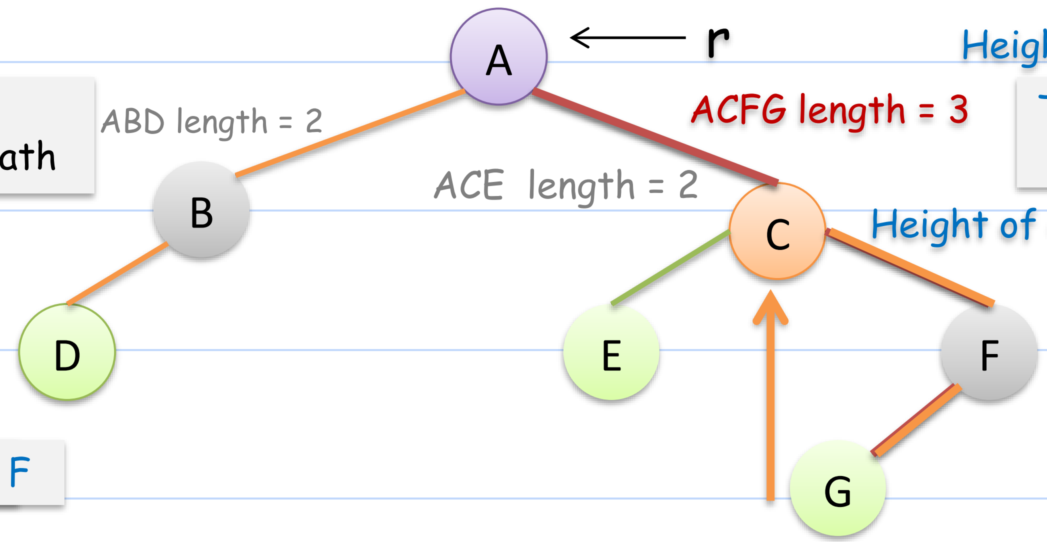
1. Tree Definitions
2. Binary Tree
   - Traversals
   - Binary Search Tree
   - Representations
   - Application : Expression Tree

3. AVL Tree
4. Which Represention[s] ?
5. n-ary Tree
6. Generic Tree
7. Multiway Search Tree
8. B-Trees

Height

Height of node = longest path length from node to leaf
path ที่ยาวที่สุดจาก node นั้น ถึง leaf

path length =
จำนวน branch ใน path



Height of A = ? = 3

Tree Height= 3
= Root's Height

Height of C = ? = 2

Whose height = 1 ? B, F

Leaf Height = 0

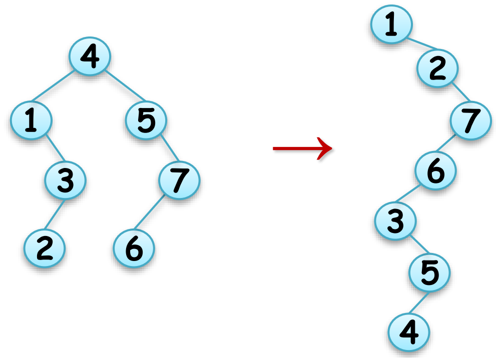
Empty Tree Height = ? = -1

Height:
จาก leaf ขึ้นไปยัง node

Why Balanced Tree ?

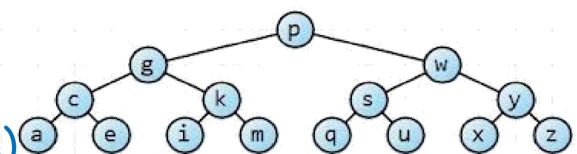
Tree Efficiency (searching, inserting, deleting,....) → O(Height)

Binary Search Tree :
sometimes turn degenerated



Linear Linked list or near
Linear Search O(n)

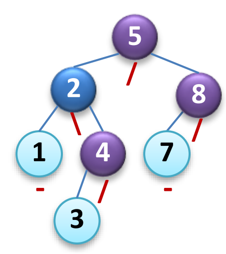
Binary Search Tree :
(Perfect) Balanced Tree
Every leaf is at the same level (very rear case)



Balanced Binary Search Tree

$O(\log_2 n)$

Binary Search Tree :
Hight Balanced Tree → AVL Maximum height <= 1.44 $\log_2 n$
Every node's subtrees' heights differ <= 1



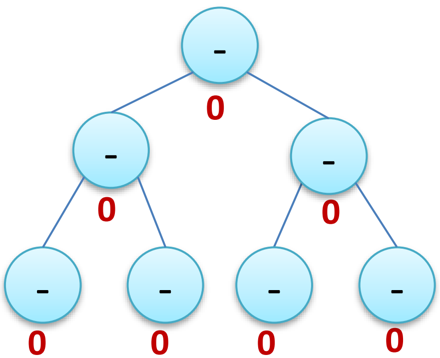
Height Balanced

$O(\log_2 n)$

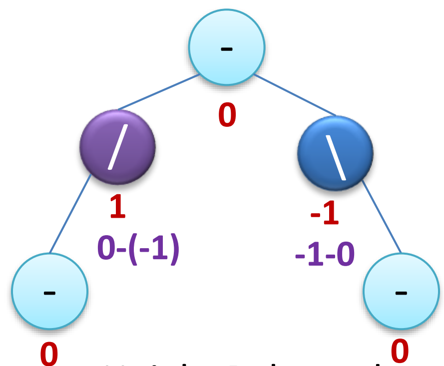
Balance Factor

Height Balanced Tree : Every node's subtrees' heights differ <= 1
(Nearly Balanced) Balance factor of every node is 0 or 1 or -1

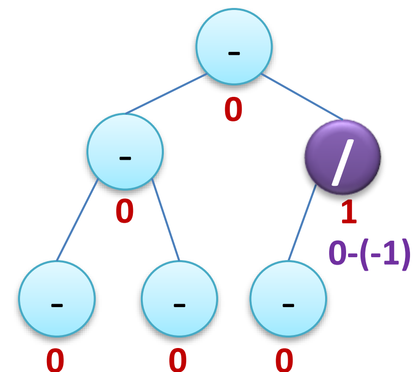
Balance factor of a node : different of left & right subtrees' hights
= **Height(Left Subtree) – Height(Right Subtree)**



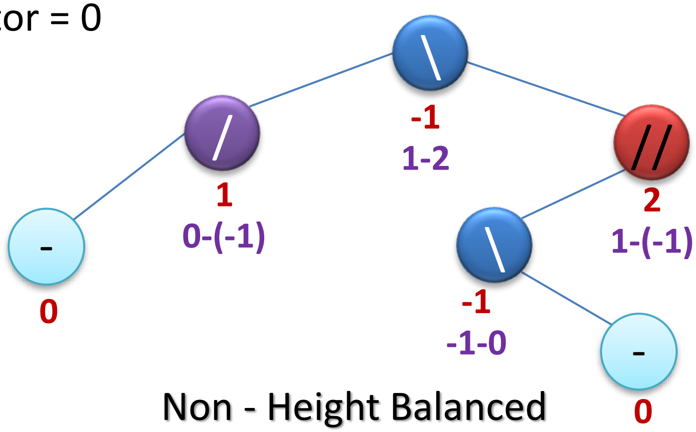
Height Balanced
Every node's balance factor = 0



Height Balanced



Height Balanced



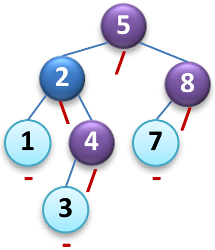
Non - Height Balanced

AVL (Height Balanced) Tree

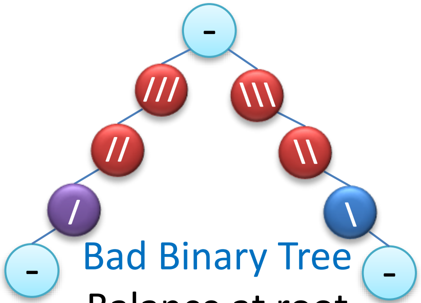
AVLTree Self- rebalancing binary search tree to keep height balance.

- Maximum height <= 1.44 $\log_2 n$: near the ideal complete binary tree.
- Search, insertion, deletion (both average & worst cases) $O(\log_2 n)$
- Insertions & deletions may require 1 or > 3 rotations.
- Worst case not much slower than random binary search tree.

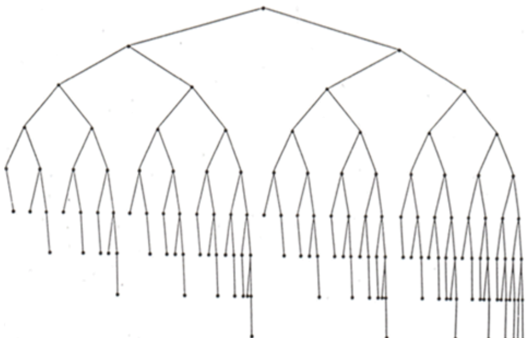
Adelson-Velskii Landis
(1962 Soviet inventors)



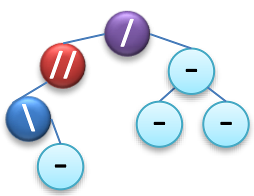
Height Balanced



Bad Binary Tree
Balance at root
is not enough.



Smallest AVL tree of height 9



Non-Height Balanced

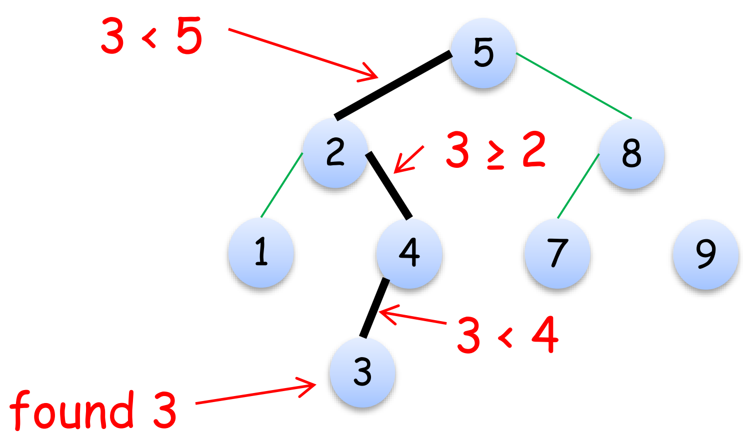
Searching in AVL = Searching in BST

AVL is a binary search tree.

Review Searching a BST (Binary Search Tree).

- Compare search key against key in node start from root.
- Follow associated link (recursively).
  - Left link if search key < key in node
  - Right link if search key ≥ key in node

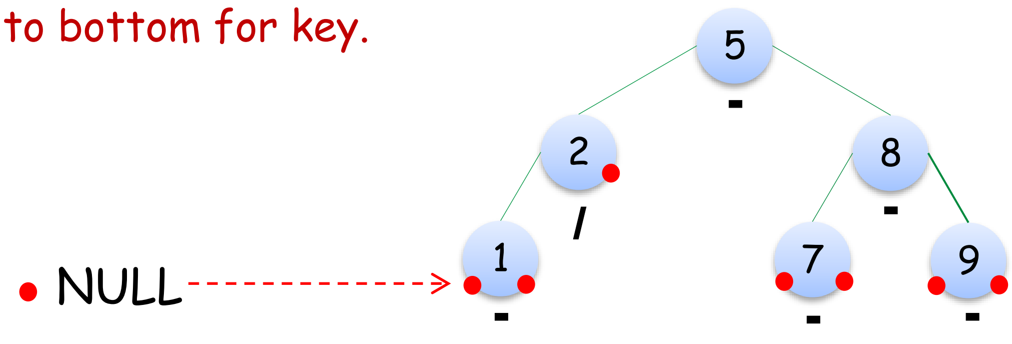
Ex. Search for 3



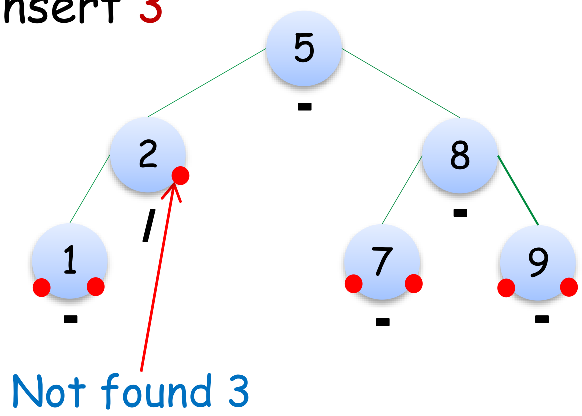
Review Inserting in BST

Review Inserting in BST

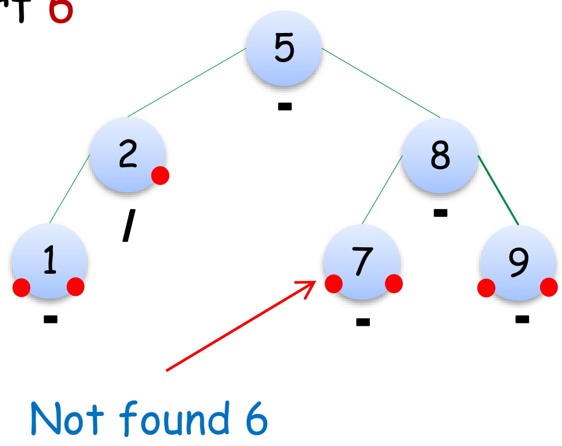
- Search to bottom for key.



Ex. Insert 3



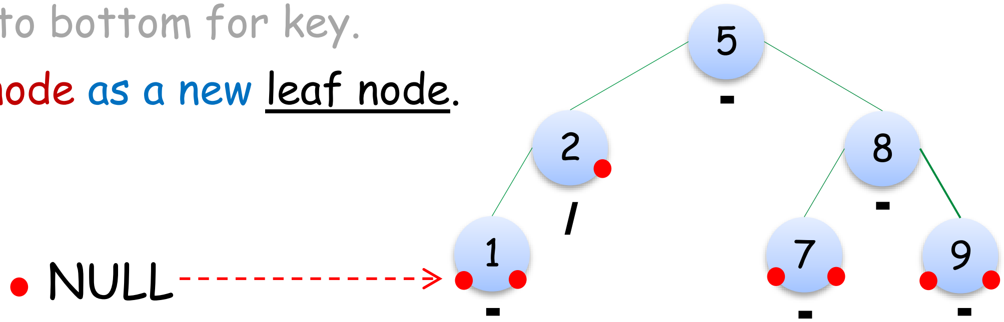
Ex. ert 6



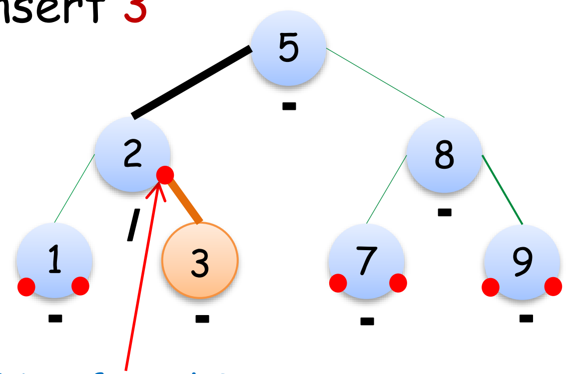
Review Inserting in BST

Review Inserting in BST

- Search to bottom for key.
- Insert node as a new leaf node.

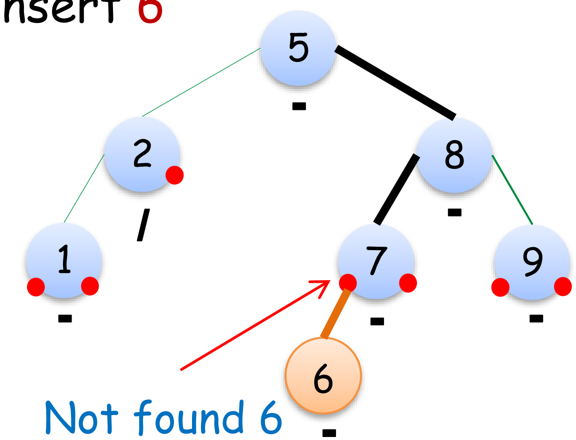


Ex. Insert 3



Not found 3
Insert 3 as a new leaf

Ex. Insert 6



Not found 6
Insert 6 as a new leaf

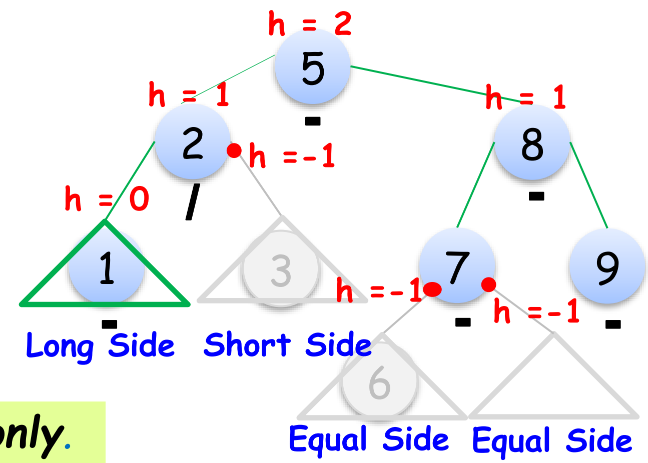
Inserting in AVL without Rebalancing

Inserting at short side or equal side

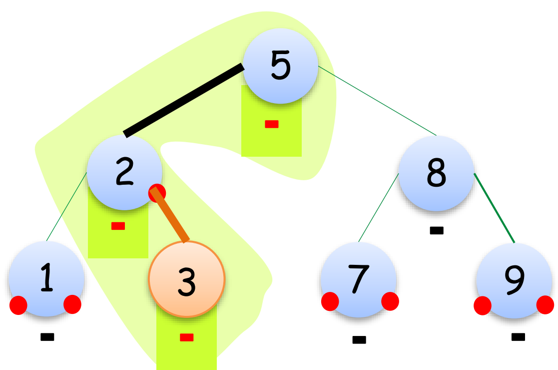
→ AVL 's still height balanced

- Insert BST new leaf node.
- Inserting node will
  change the height of some nodes (*increase* by 1)
  on the path from the newly inserted node to the root ***only***.

  Change balance factors along the path to the root.

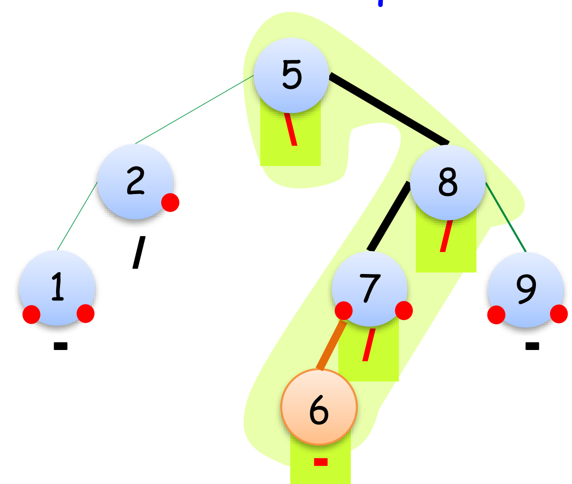


Ex. Insert 3 at short side.



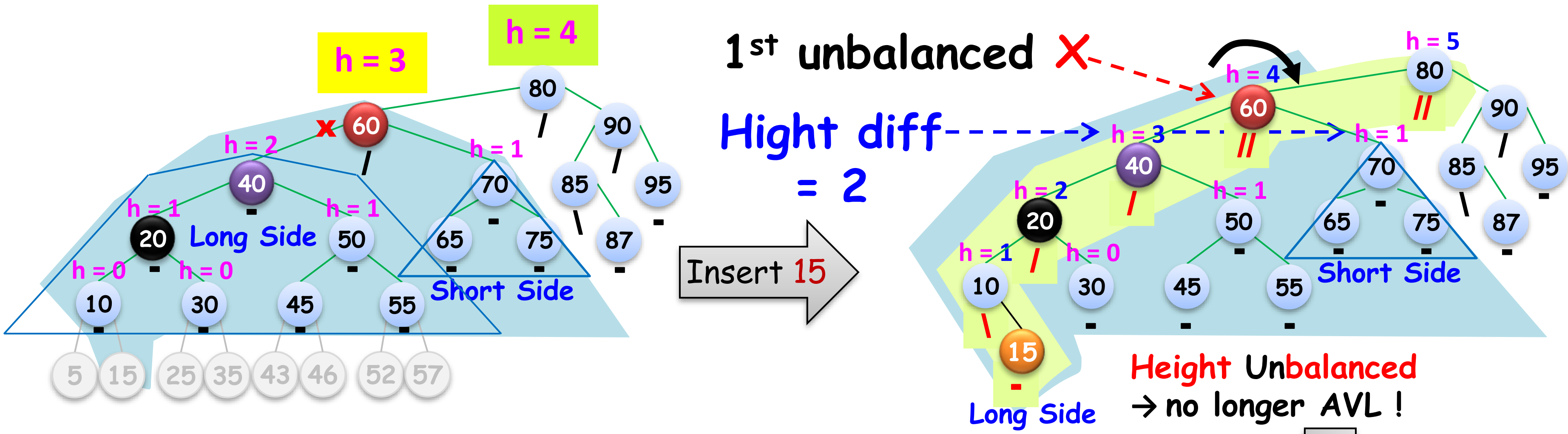
insert 3 at short side
still height balance

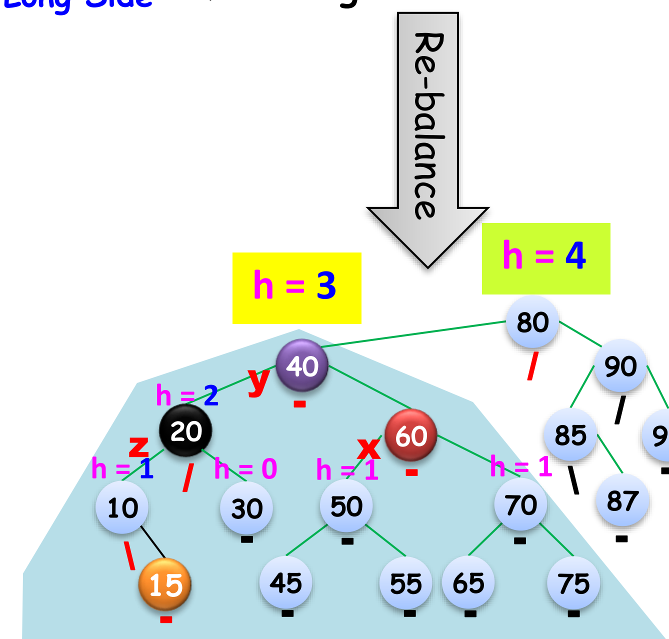
Ex. Insert 6 at equal side.



insert 6 at equal side
still height balance

Insertion Re-balancing in AVL





Height balanced again→ AVL

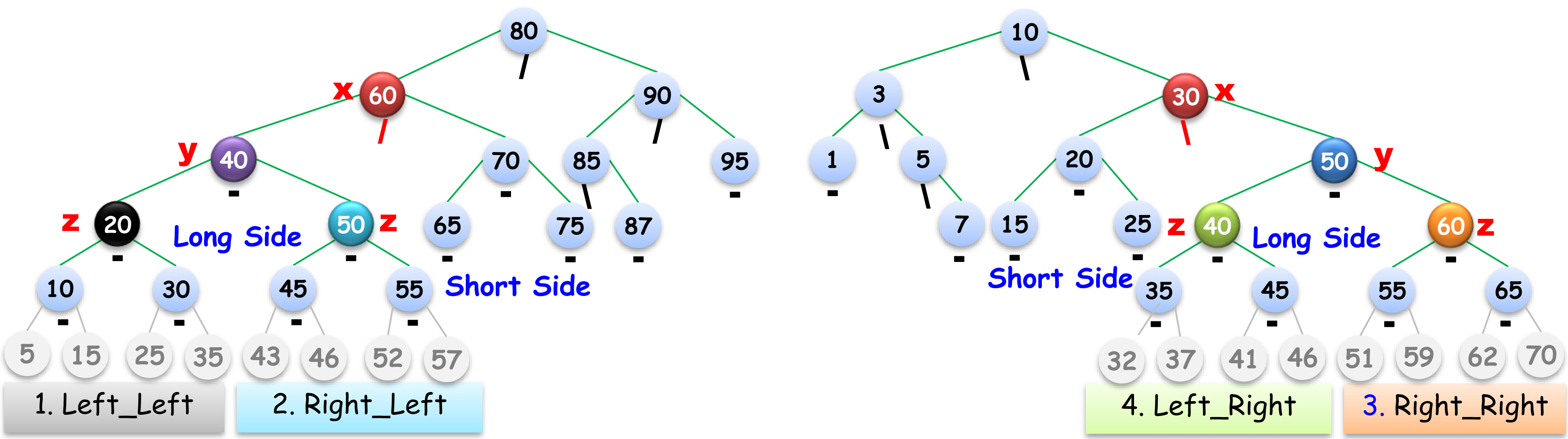
Inserting AVL in a **long side**.

1. Causes some node(s) unbalanced (height diff = 2) ***only*** up the inserted path.
    - x = 1st unbalanced node
2. Need rebalancing the (blue) shaded subtree.

   Rebalancing does not change :
    - height of the subtree
    - → ie. Nor height of root.
    - → Only some part of the shaded subtree's balance facters need changing.

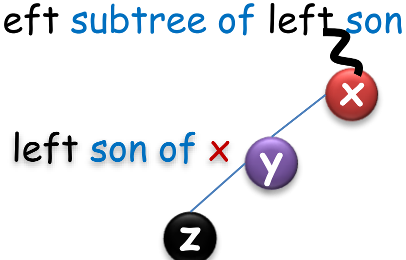
4 Insertions Re-balancing in AVL

Inserting AVL in a **long side** : ให้ x = node แรกที่ไม่ balance , ตาม path ที่ insert ให้ y เป็นลูกของ x และ z เป็นลูกของ y

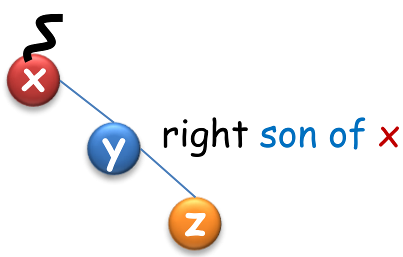


มี 4 กรณี เมื่อ insert ที่

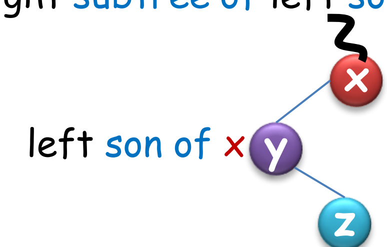
1. left subtree of left son of x



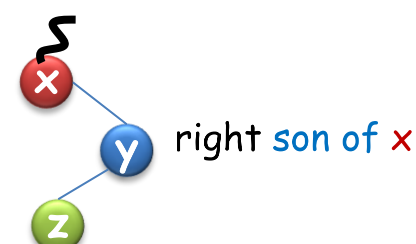
3. right subtree of right son of x



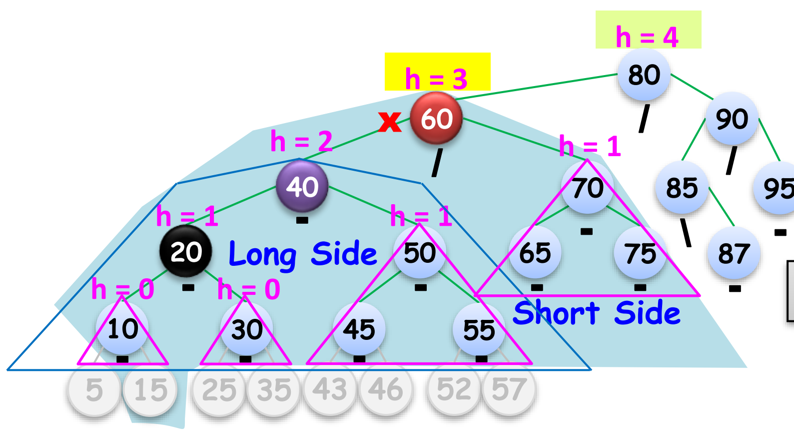
2. right subtree of left son of x

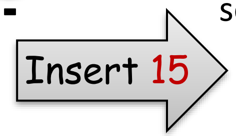


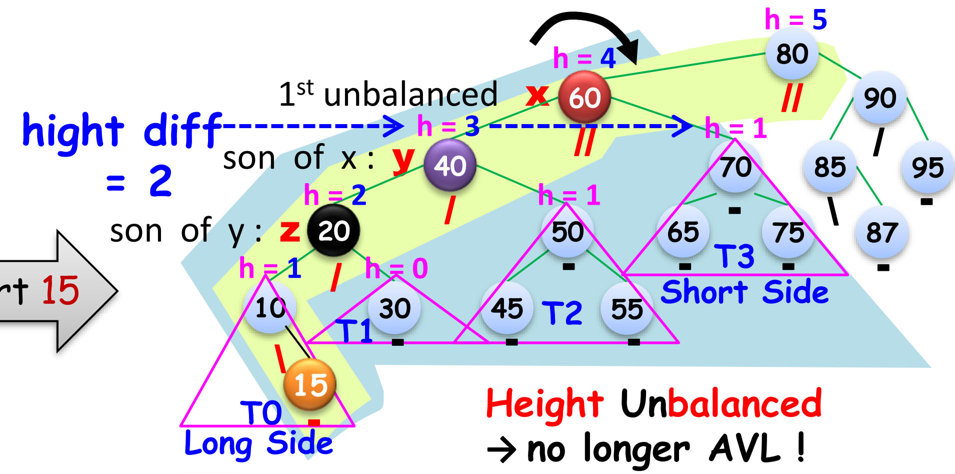
4. left subtree of right son of x

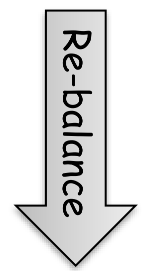


Rebalancing Left Subtree of Left Son of x (1st Unbalanced Node)

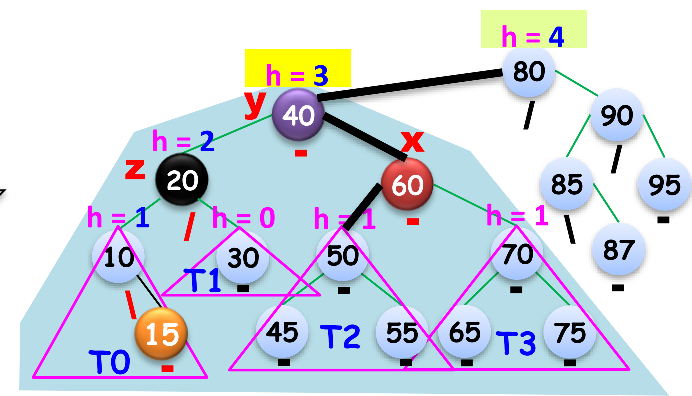








x = 1st unbalanced node. Along the path, let
y = son of x, z = son of y.



Height balanced again→ AVL

Left subtree of left son of x

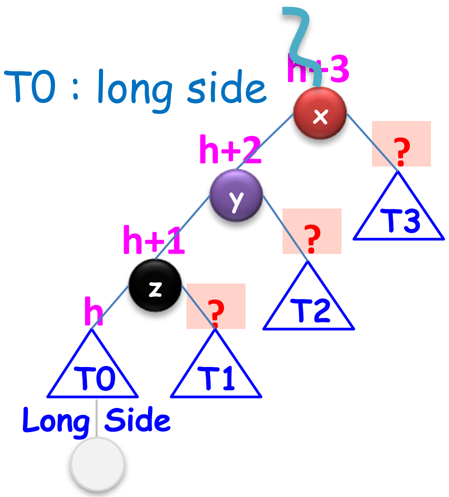
1. Right rotate at x
   - y ขึ้นไปแทนที่ x (เป็นลูกของพ่อของ x)
   - x ลงมาเป็น y's (right) son.
2. Re-arrange T0,T1,T2,T3 to preserve the properties of BST.
   - T0 ≤ z ≤ T1 ≤ y ≤ T2 ≤ x ≤ T3
3. Fixing some part of the shaded subtree's balance facters.
4. Height of the blue shaded subtree doesn't change -> also the whole AVL's.

Proof : Rebalancing Area' Height's unchange (Left Subtree of Left son of x)

Insert at a long side T0

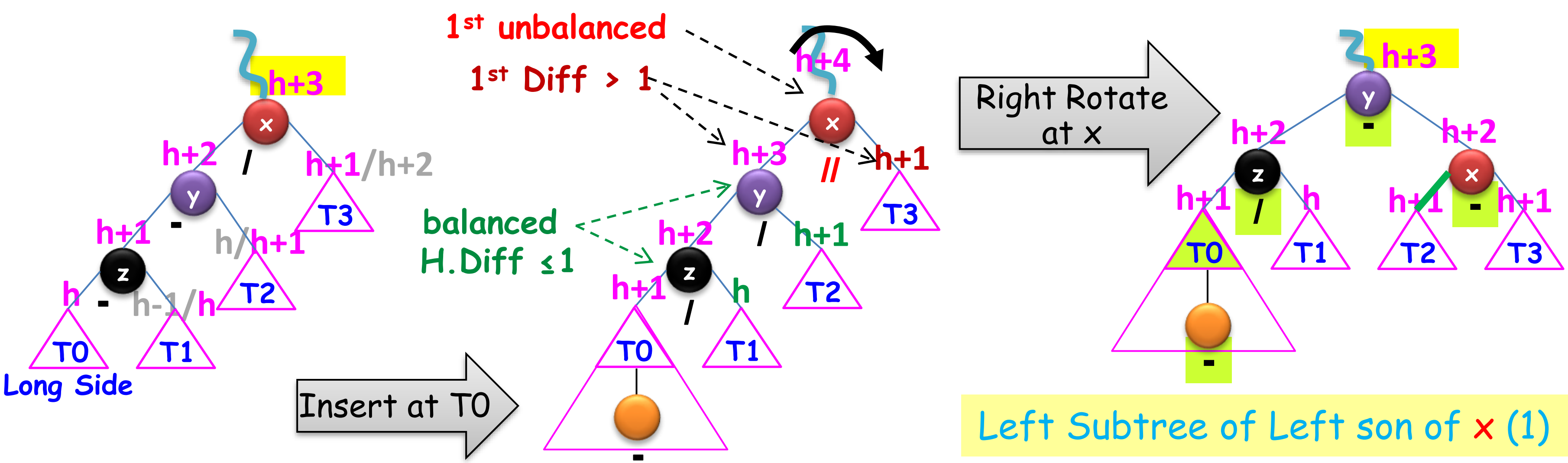
x = 1st unbalanced node, y = son of x, z = son of y. ก่อน insert:

- height(T1) = ? When height(T0) = h.
  = h-1 หรือ h ∵ heightDiff (T0,T1) ≤1 & ≠ h+1 ∵ T1 : short side, T0 : long side
  = h ∵ หลัง insert , z : balanced ∴ heightDiff ≤1
- height(T2) = h+1 ∵ หลัง insert , y : balanced
- height(T3) = h+1 ∵ หลัง insert , x : unbalanced



1. Rebalancing ไม่เปลี่ยน heights ของ subtree → ie. รวมทั้ง height ของ root.

2. ดังนั้นจึงแค่ แก้ balance facters ตาม inserted path



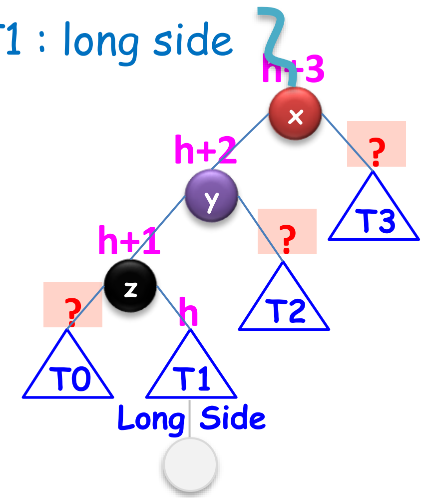
Left Subtree of Left son of x (1)

Proof : Rebalancing Area' Height's unchange (Left Subtree of Left son of x)2

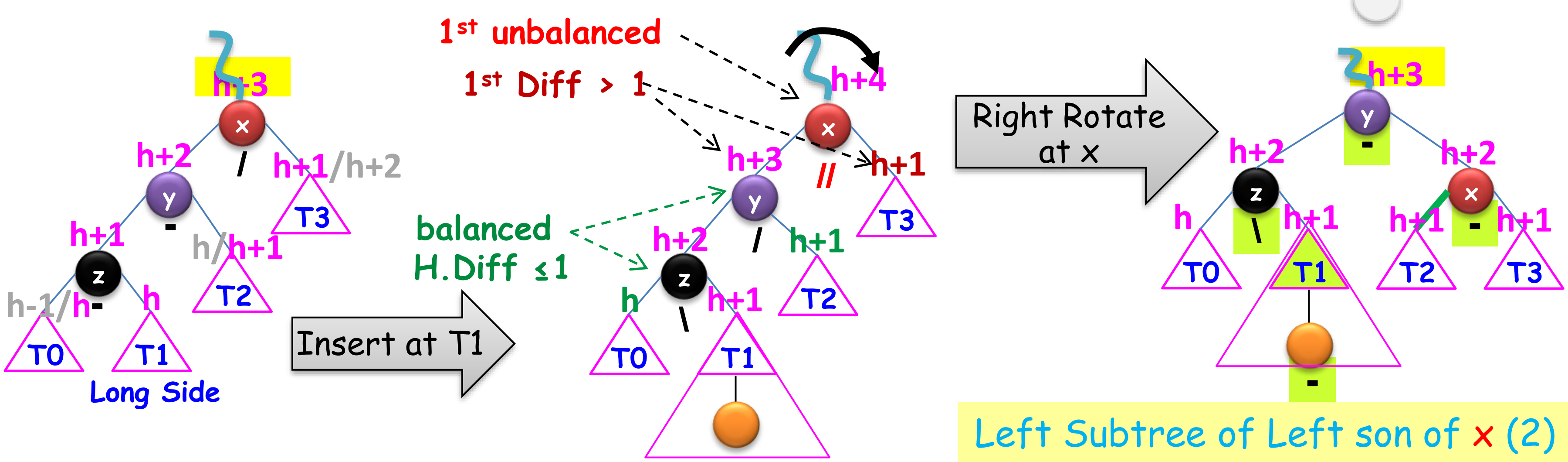
Insert at a long side **T1**

x = 1st unbalanced node, y = son of x, z = son of y. Before insertion :

- height(T0) = ? When height(T1) = h.
  = h-1 or h ∵ heightDiff (T0,T1) ≤1 & ≠ h+1 ∵ T0 : short side, T1 : long side
  = h ∵ After inserting , z : balanced ∴ heightDiff ≤1
- height(T2) = h+1 ∵ After inserting , y : balanced
- height(T3) = h+1 ∵ After inserting , x : unbalanced



1. Rebalancing does not change heights of the subtree → ie. Nor of root.
2. So only fix balance facters of inserted path to y and of x.

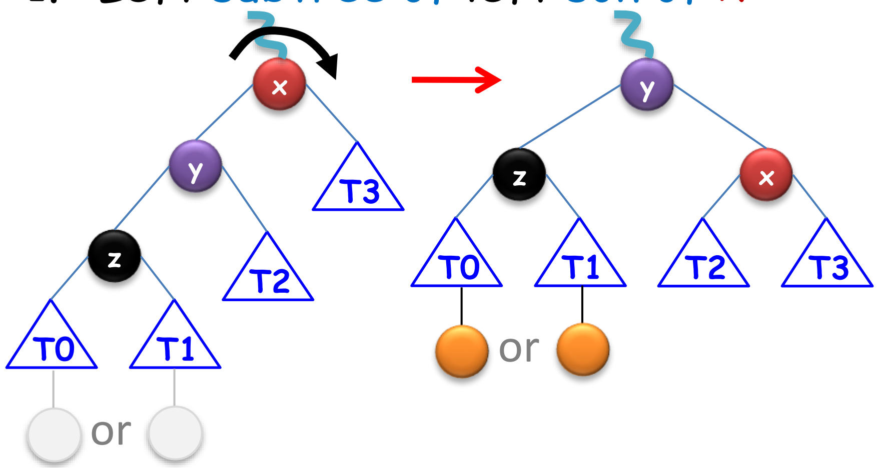


Left Subtree of Left son of x (2)

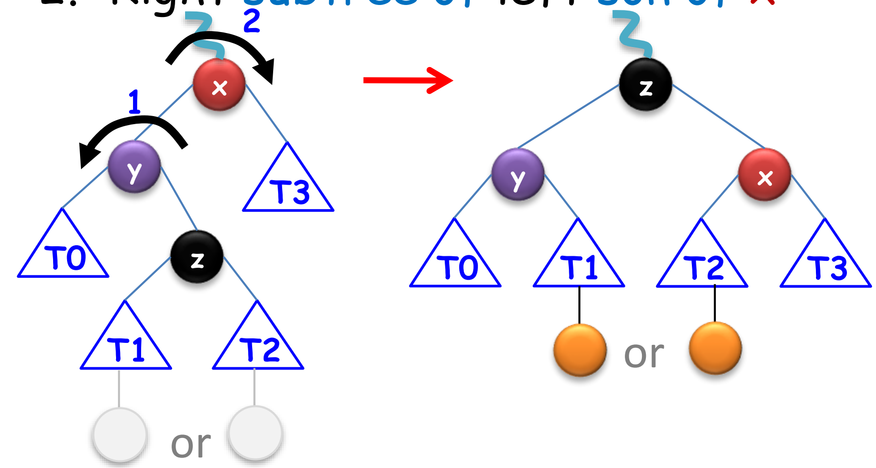
4 Kinds of Rebalancing in AVL

x = 1st unbalanced node, y = son of x, z = son of y

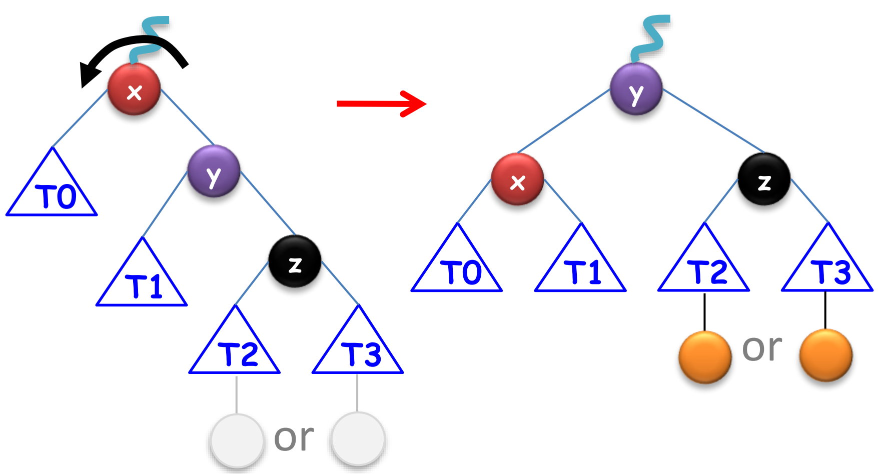
1. Left subtree of left son of x



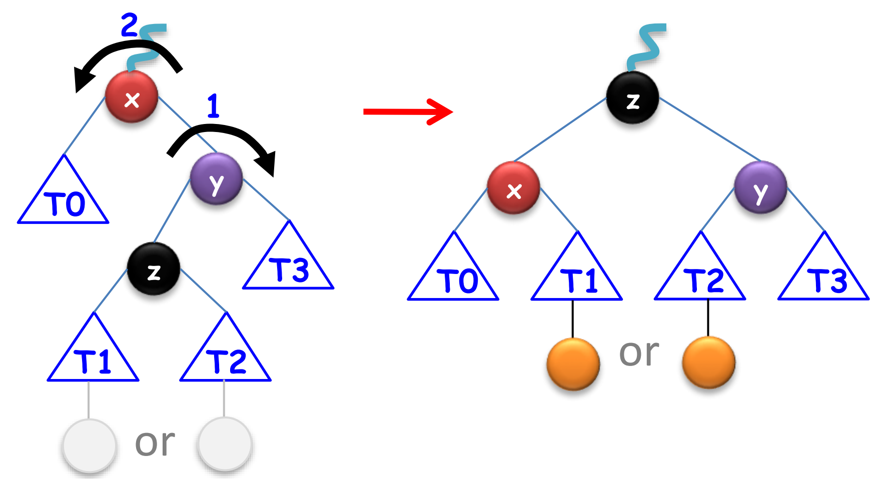
2. Right subtree of left son of x



3. Right subtree of right son of x



4. Left subtree of right son of x



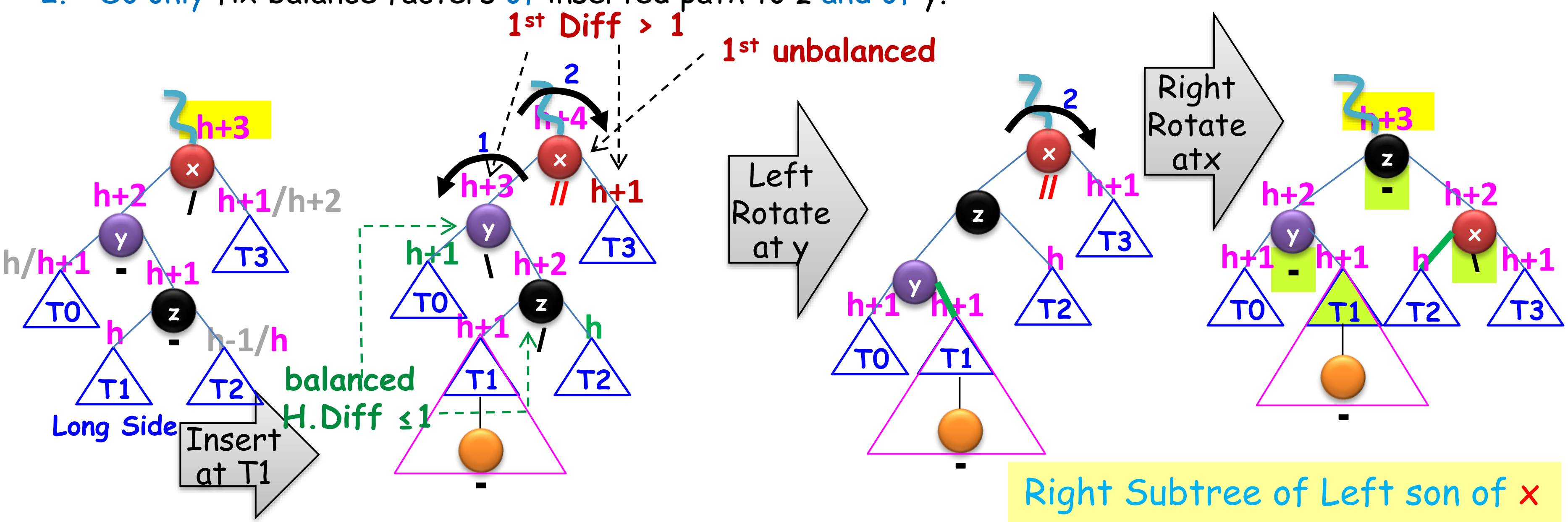
Proof : Rebalancing Area' Height's unchange (Right Subtree of Left son of x)

Insert at a long side **T1**

x = 1st unbalanced node, y = son of x, z = son of y. Before insertion :

- height(T2) = ? When height(T1) = h.
  = h-1 or h ∵ heightDiff (T2,T1) ≤1 & ≠ h+1 ∵ T2 : short side, T1 : long side
  = h ∵ After inserting , z : balanced ∴ heightDiff ≤1
- height(T0) = h+1 ∵ After inserting , y : balanced
- height(T3) = h+1 ∵ After inserting , x : <u>un</u>balanced

1. Rebalancing does not change heights of the subtree → ie. Nor of root.
2. So only fix balance facters of inserted path to z and of y.



Right Subtree of Left son of x

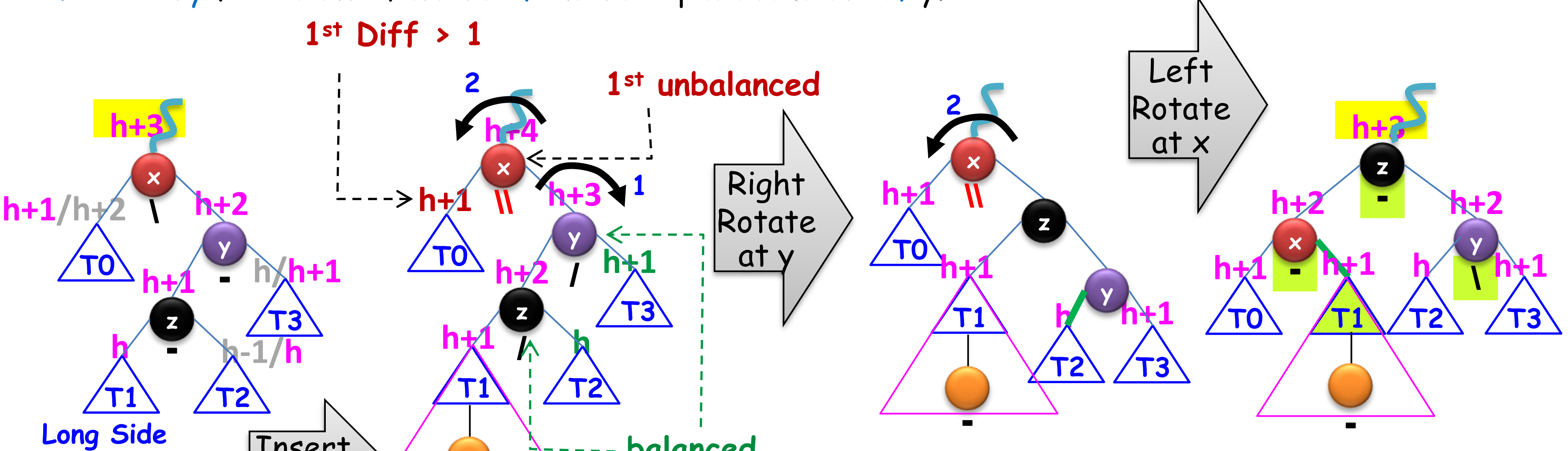
Proof : Rebalancing Area' Height's unchange (Left Subtree of Right Son of x)

Insert at a long side **T1**

x = 1st unbalanced node, y = son of x, z = son of y. Before insertion :

- height(T2) = ? When height(T1) = h.
  = h-1 or h ∵ heightDiff (T1,T2) ≤1 & ≠ h+1 ∵ T2 : short side, T1 : long side
  = h ∵ After inserting , z : balanced ∴ heightDiff ≤1
- height(T3) = h+1 ∵ After inserting , y : balanced
- height(T0) = h+1 ∵ After inserting , x : unbalanced

1. Rebalancing does not change heights of the subtree → ie. Nor of root.
2. So only fix balance facters of inserted path to z and of y.



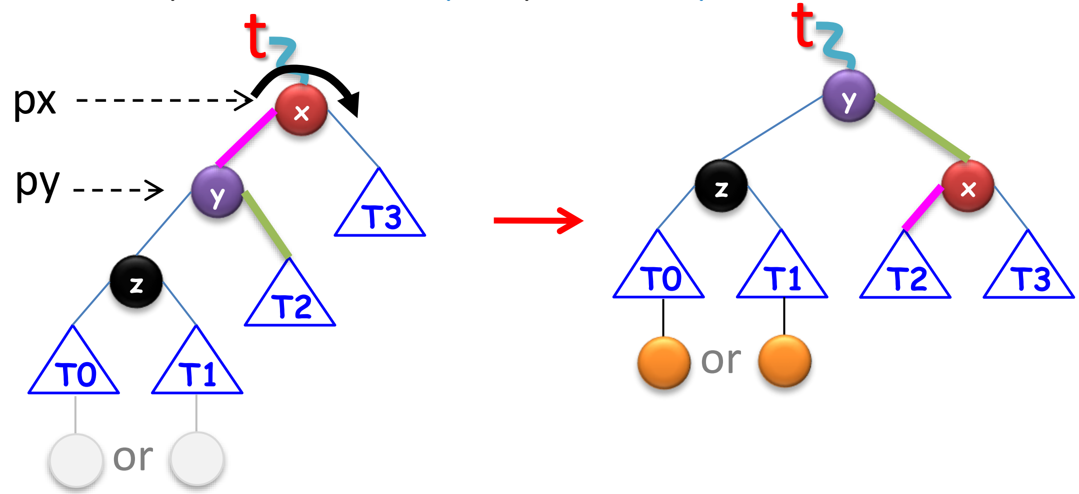
Left Subtree of Right son of x

Single Right Rotation & Single Left Rotation Algorithms

Python : no type node*

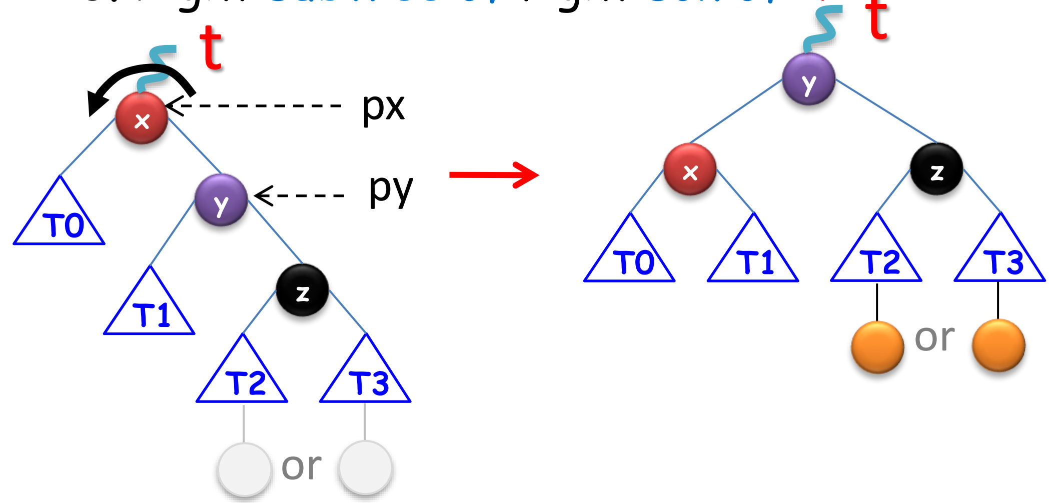
```
// Single Right Rotation
// LeftLeftRotate(t)
LeftLeftRotate(node *px)
    node* py = px.left  //px->left
    px.left = py.right
    py.right = px
    return py
```

1. Left subtree of left son of x




```
// Single Left Rotation
// RightRightRotate(t)
RightRightRotate(node *px)
    node* py = px.right
    px.right= py.left
    py.left = px
    return py
```

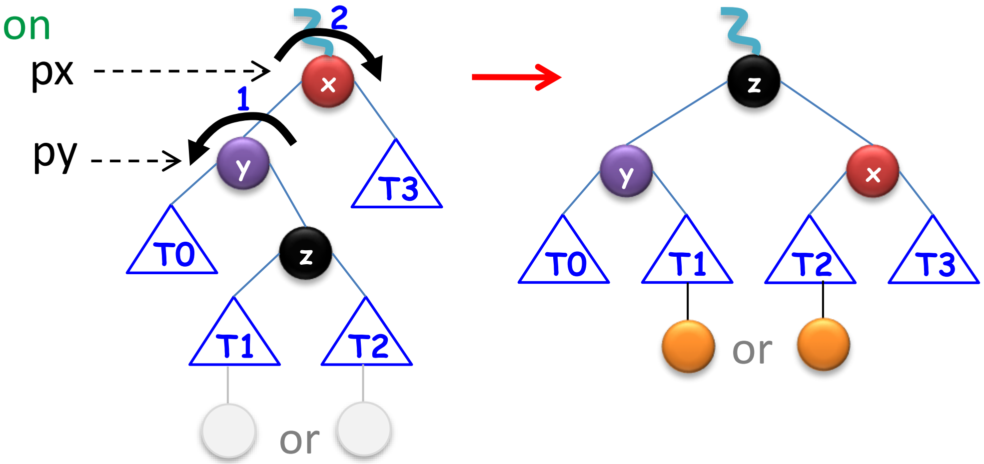
3. Right subtree of right son of x



Double Rotations Algorithms

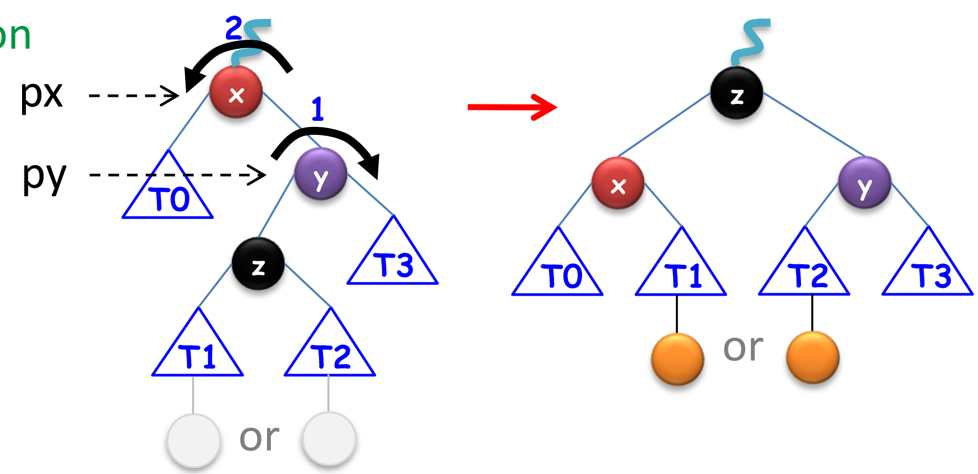
```
// Double Rotation :
// Case 2 : Left Rotation then Right Rotation
RightLeftRotate(node *px)
        node* py = px.left
        px.left = RightRightRotate(py)
        py = LeftLeftRotate(px)
        return py
```

2. Right subtree of left son of x



4. Left subtree of right son of x

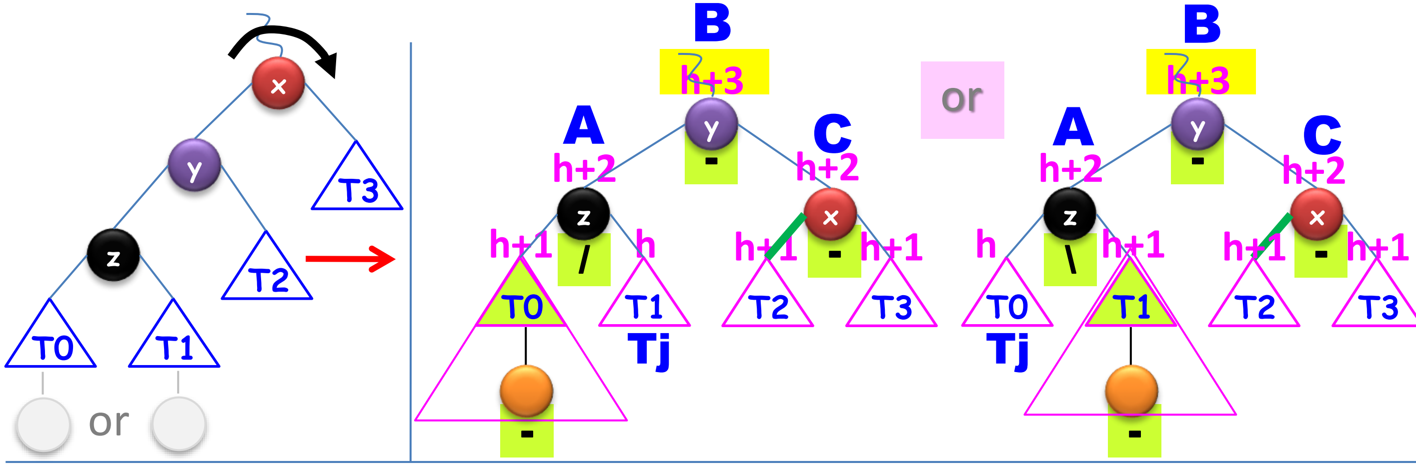
```
// Double Rotation :
// Case 4 : Right Rotation then Left Rotation
LeftRightRotate (node *px)
        node *py = px.right
        px.right = LeftLeftRotate(py)
        py = RightRightRotate(px)
        return py
```



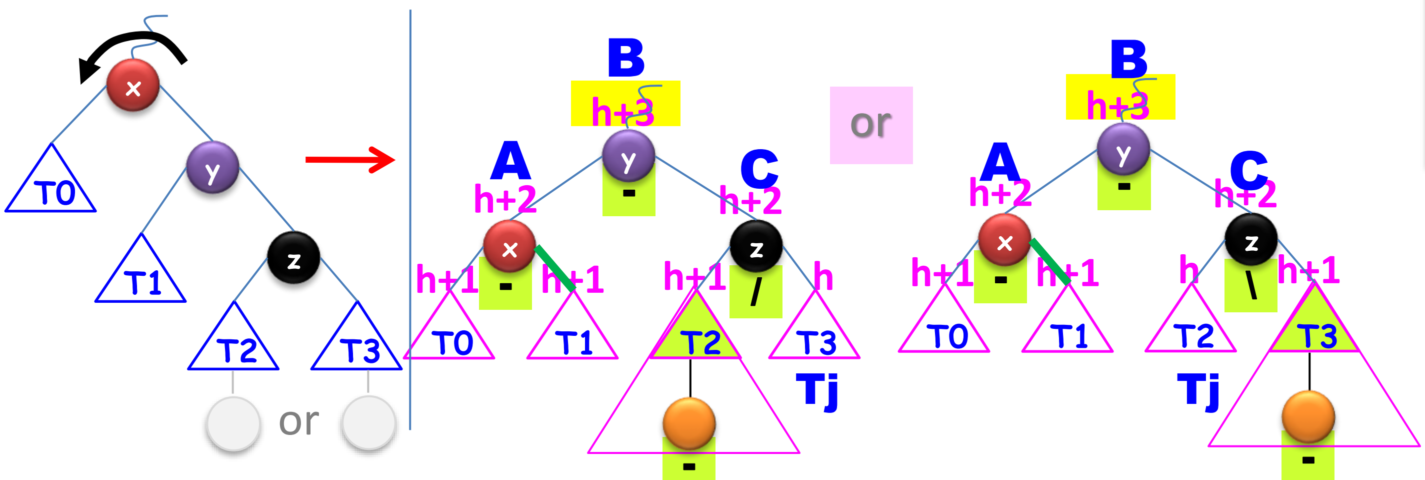
Single Rotation in AVL

x = 1st unbalanced node, y = son of x, z = son of y **A < B < C** **T0<T1<T2<T3**

1. Left subtree of left son of x



2. Right subtree of right son of x



สรุปรูปผลลัพธ์

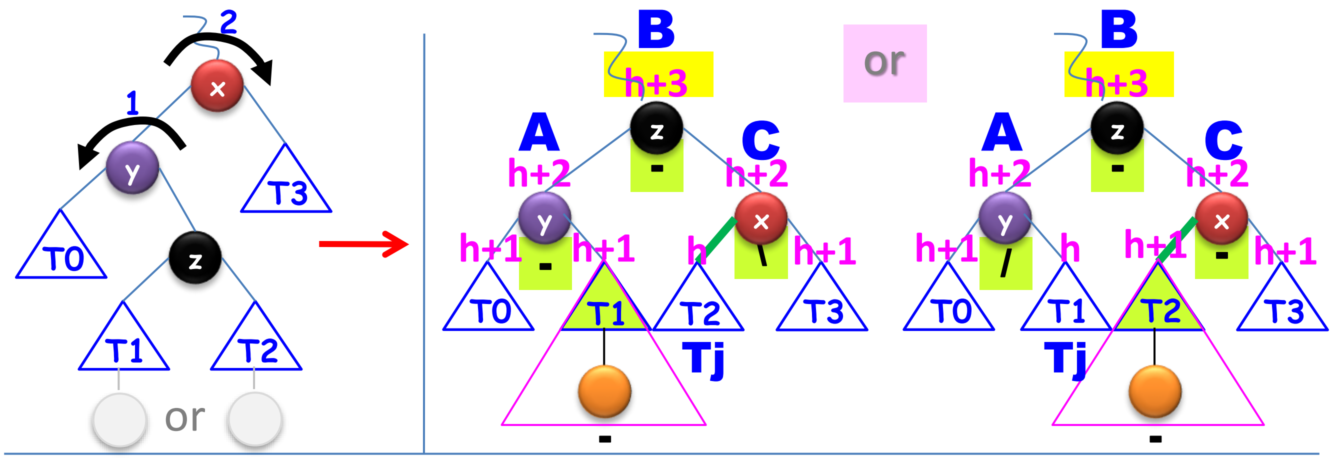
B ตัวค่ากลาง
เป็น root ใหม่ เสมอ

•**Height(Tj)** = h
ตัวเดียว **Tj** คือตัวที่คู่กับ **Ti** ที่ insert node ใหม่เข้าไป
•นอกนั้น height(T) = h+1 หมด

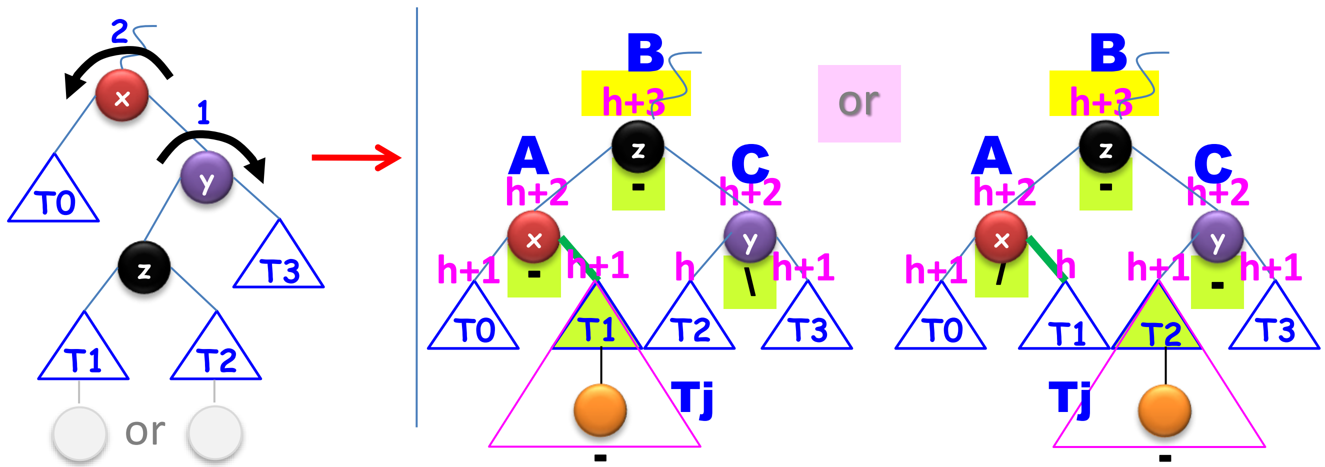
Double Rotations in AVL

x = 1st unbalanced node, y = son of x, z = son of y   **A < B < C**   **T0<T1<T2<T3**

1. Right subtree of left son of x



3. Left subtree of right son of x



สรุปรูปผลลัพธ์

B ตัวค่ากลาง
เป็น root ใหม่ เสมอ

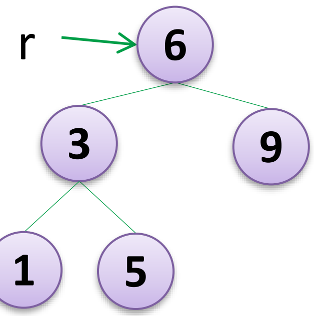
•**Height(Tj)** = h
ตัวเดียว **Tj** คือตัวที่คู่
กับ **Ti** ที่ insert
node ใหม่เข้าไป
•นอกนั้น height(T)
= h+1 หมด

3. AVL Tree
   Height Balanced Tree
4. **Which Representions ?**
5. n-ary Tree
6. Generic Tree
7. Multiway Search Tree
8. Top Down Tree
9. B-Tree

1. Tree Definitions
2. Binary Tree
   - Traversals
   - Binary Search Tree
   - Representations
   - Application : Expression Tree

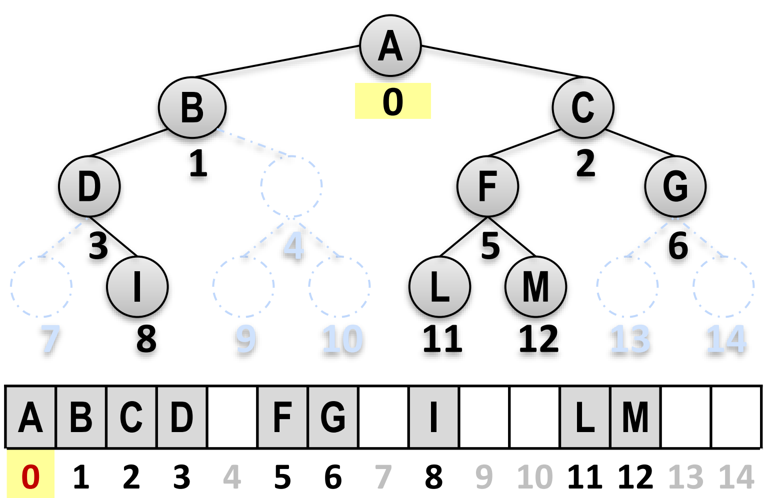
Binary Tree Representations

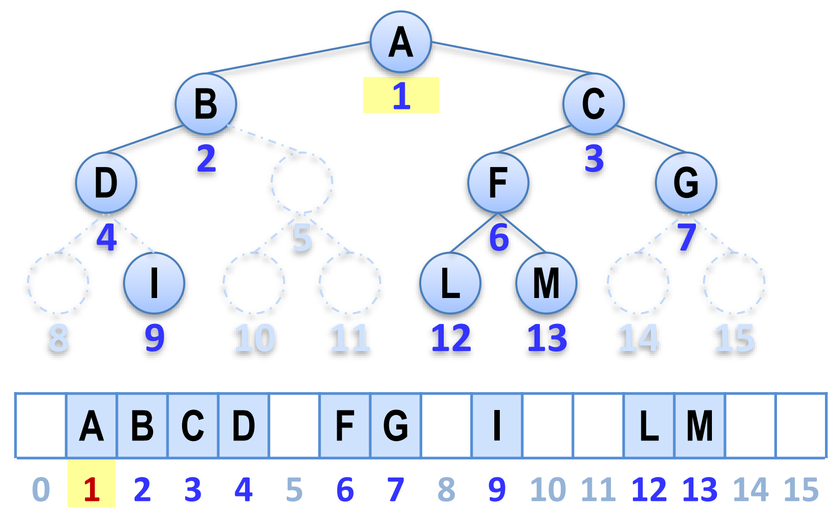
1. Dynamic



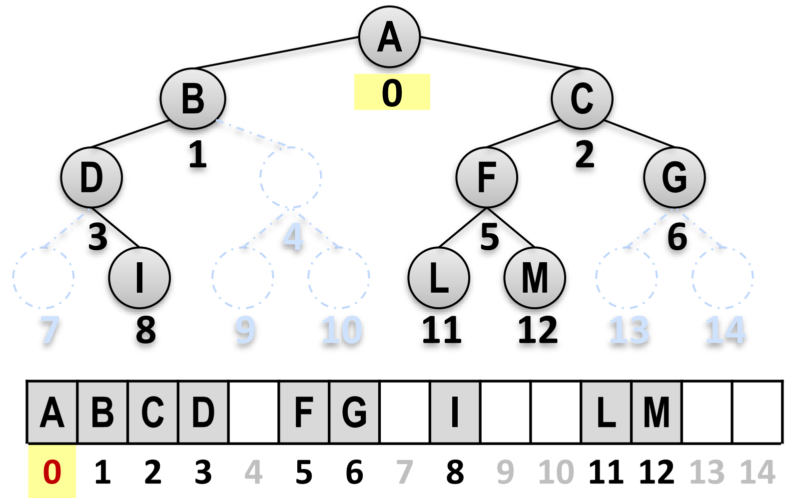

```
class node:

    def __init__(self,  data, left = None, right = None):
      self.data = data
      self.left = left if left is not None else None
      self.right = right if right is not None else None
```

2. Sequential (Implicit) Array



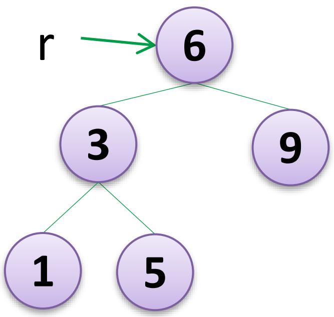


Sequential Array (Implicit Array)



- ง่าย
- ต้องกะขนาด array
- เพิ่มขึ้นอีก 1 level => array มีขนาด เพิ่มมากกว่าที่มีอยู่เดิม 1 (คิดเต็มที่ full binary tree)
- ถ้ากะขนาด array ถูก และ tree ใช้พื้นที่ส่วนใหญ่ของ array เช่น almost complete binary tree) จะ save space เพราะไม่ต้องมีฟิลด์ left, right, father

Dynamic



- จำนวน node ถูกจำกัดโดย memory

Tree 2

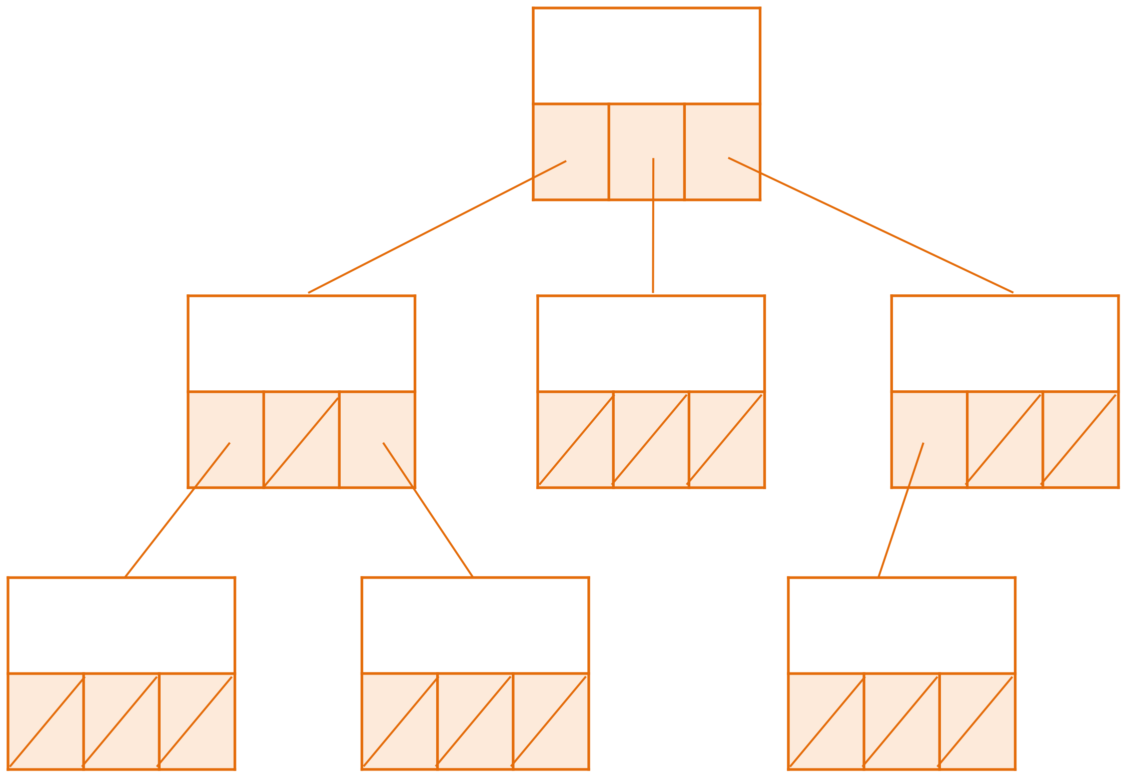
3. AVL Tree
   Height Balanced Tree
4. Which Representions ?
5. n-ary Tree
6. Generic Tree
7. Multiway Search Tree
8. Top Down Tree
9. B-Tree

1. Tree Definitions
2. Binary Tree
   - Traversals
   - Binary Search Tree
   - Representations
   - Application : Expression Tree

n-ary (multiway order n) Trees

bi-nary Tree แต่ละ node มีลูกได้อย่างมากที่สุด 2 ตัว

n-ary Tree แต่ละ node มีลูกได้อย่างมากที่สุด n ตัว



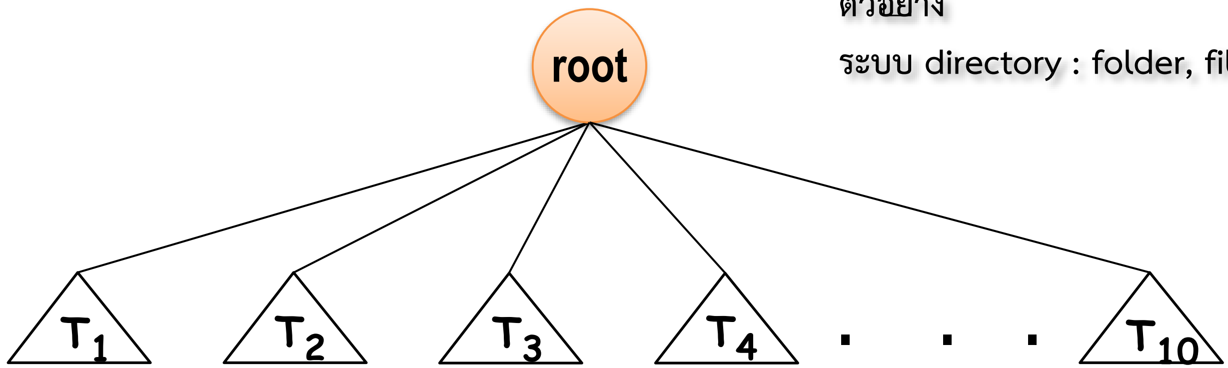
Ternary Tree n = 3

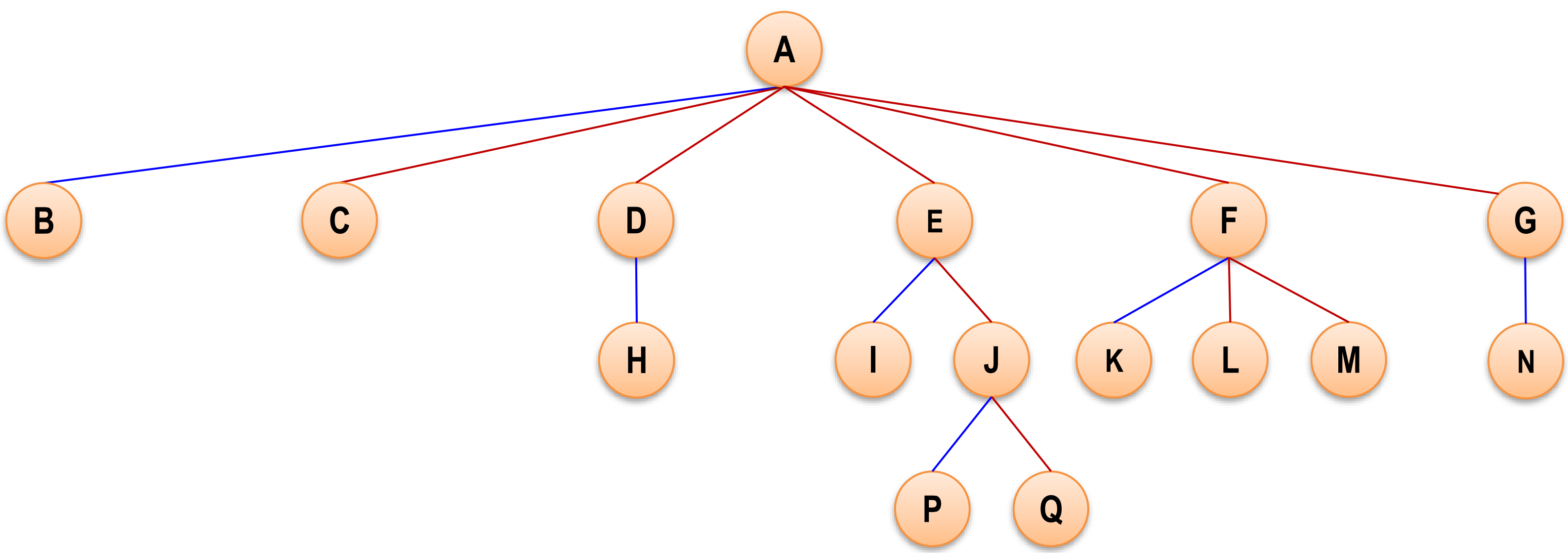
Generic Tree

Generic Tree : แต่ละ node มีลูกได้ไม่จำกัดจำนวน

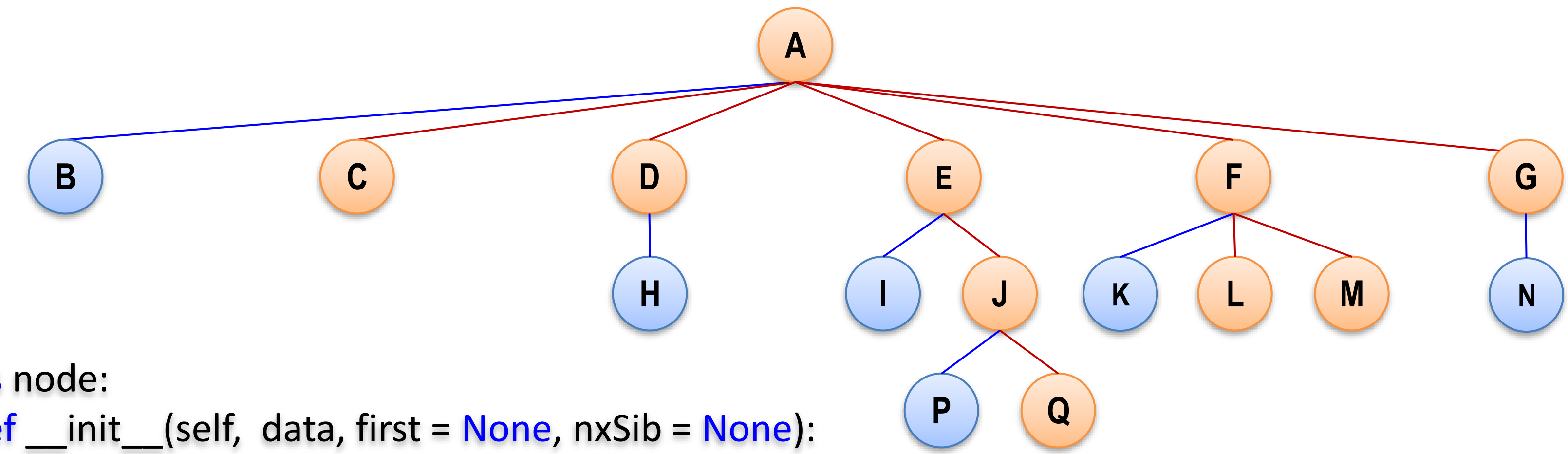
ตัวอย่าง

ระบบ directory : folder, file

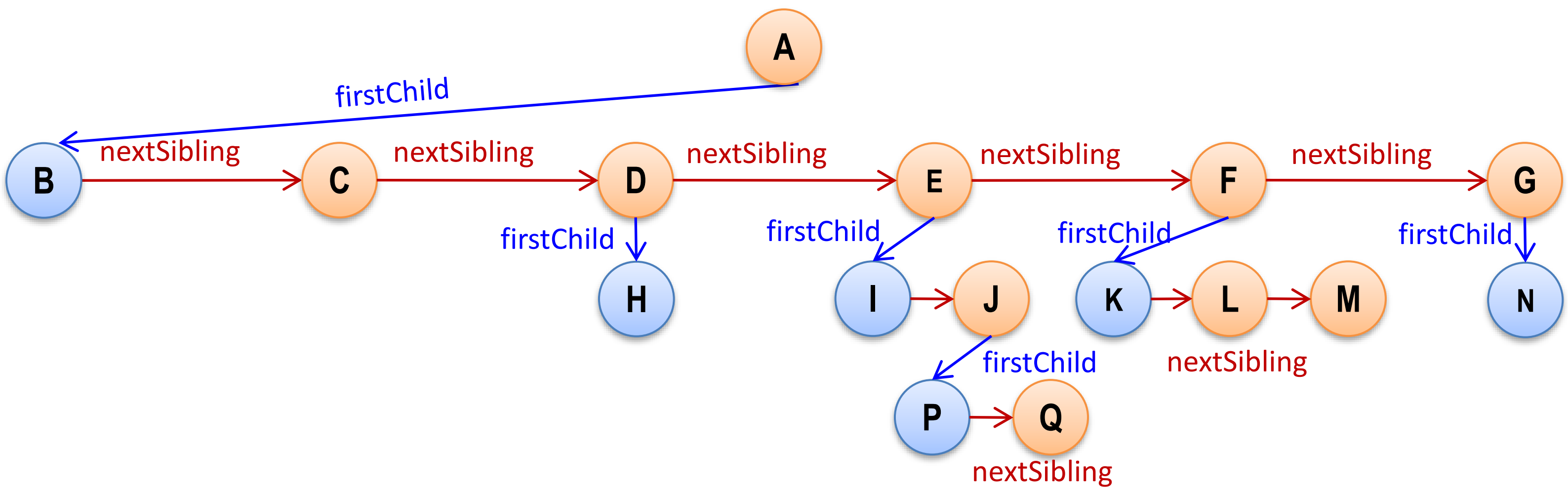




Implementation of Generic Tree




```
class node:
  def __init__(self,  data, first = None, nxSib = None):
    self.data = data
    self. firstChild = None if first is None else first
    self. nextSibling = None if nxSib is None else nxSib
```



Tree 2

3. AVL Tree
   Height Balanced Tree
4. Which Representions ?
5. n-ary Tree
6. Generic Tree
7. Multiway Search Tree
   - Top Down Tree
   - BalancedTree
     B-Tree

1. Tree Definitions
2. Binary Tree
   - Traversals
   - Binary Search Tree
   - Representations
   - Application : Expression Tree

Multiway Search Tree

Multiway Tree order n : มีลูกได้มากที่สุด n ตัว (0, 1,... or n subtrees) (#max. sons = n)

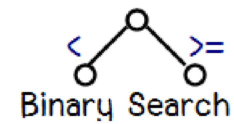
For order n :

- #max. sons = n
- #max. keys = n-1

Multiway Search Tree : for every key K

left decendents ≤ K

right decendents > K



leaf : node ที่ไม่มีลูกเลย

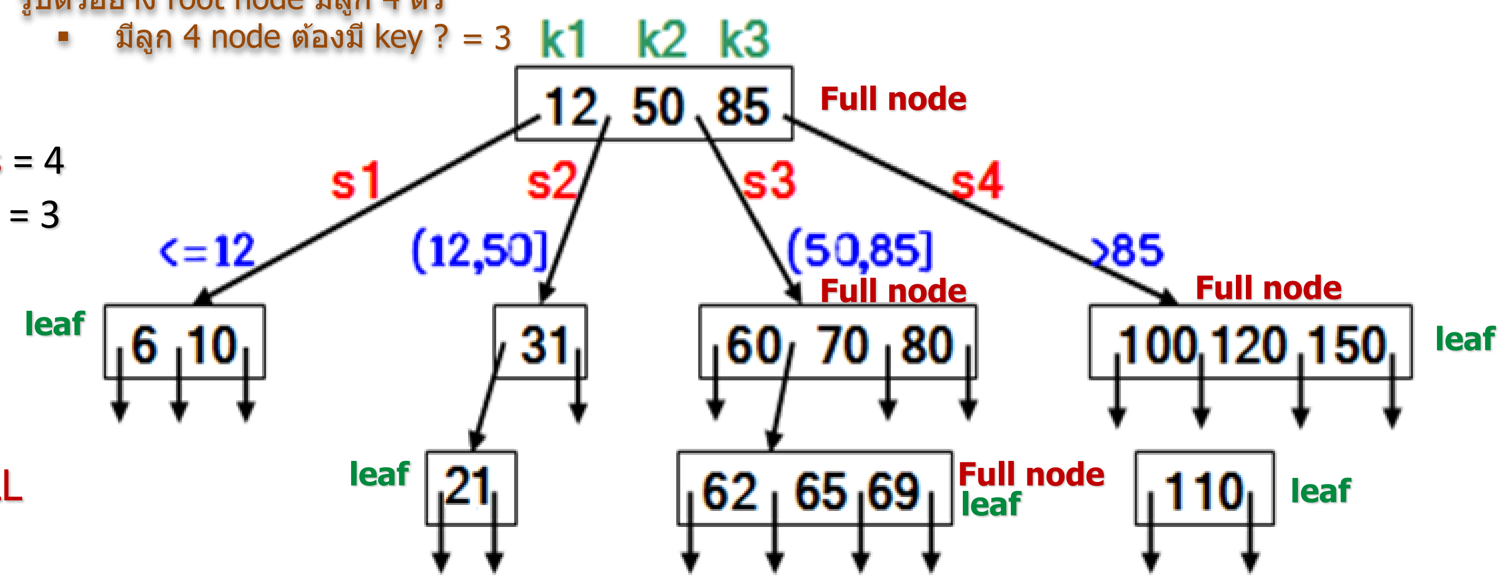
Full node : node ที่มีจำนวน key เต็มที่

รูปตัวอย่าง root node มีลูก 4 ตัว

- มีลูก 4 node ต้องมี key ? = 3

Order 4

- #max. sons = 4
- #max. keys = 3



↓ = NULL

Tree 2

3. AVL Tree
   Height Balanced Tree
4. Which Representions ?
5. n-ary Tree
6. Generic Tree
7. Multiway Search Tree
   - Top Down Tree
   - BalancedTree
     B-Tree

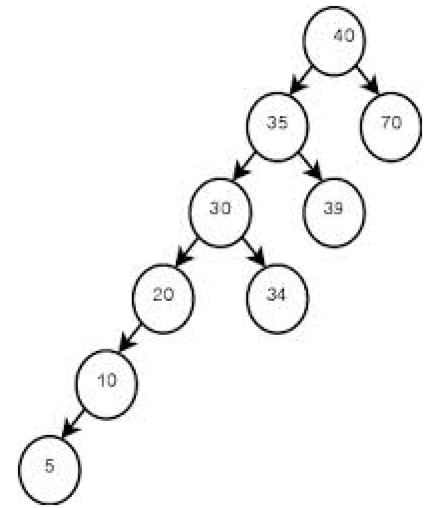
1. Tree Definitions
2. Binary Tree
   - Traversals
   - Binary Search Tree
   - Representations
   - Application : Expression Tree

Top Down Tree

Top Down Tree : Non full node must be leaf.
Node ที่ไม่เต็มจะเป็น leaf ได้เท่านั้น เป็น internal node ไม่ได้

- Fill up the existing nodes before creating a new node !
  เติม node ให้เต็มก่อนค่อยแตก node ใหม่

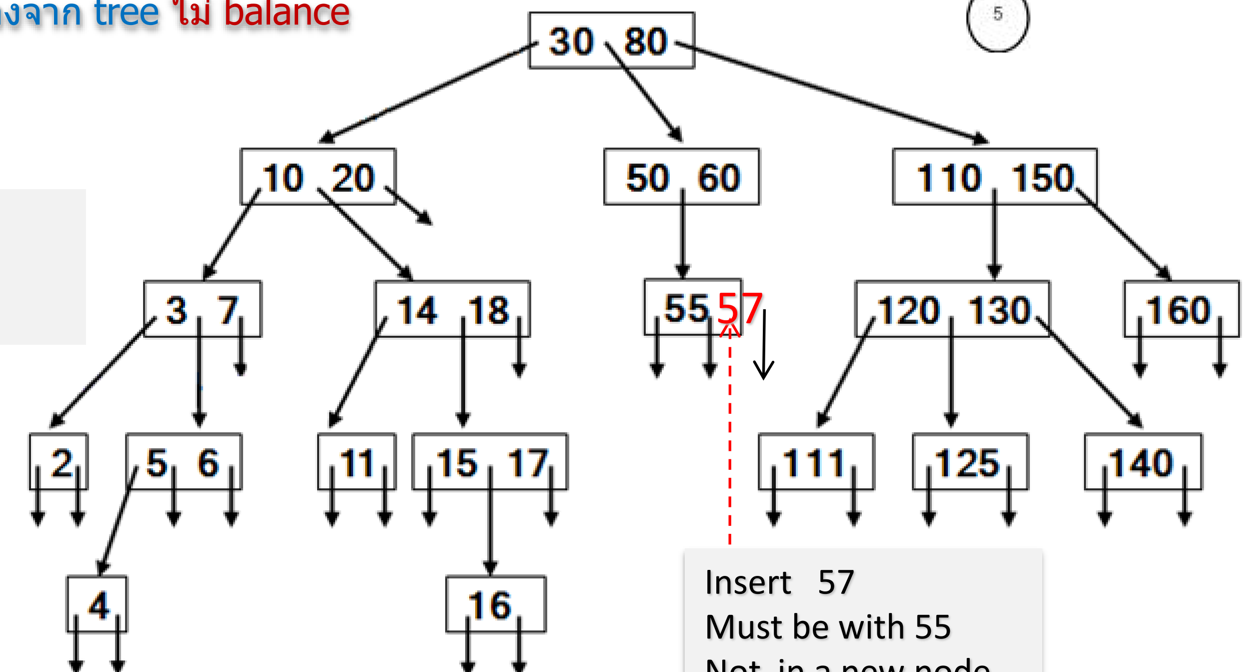
แนวคิด : มี nodes ให้น้อยที่สุด เพราะ อยากให้ height สั้น



แม้ว่าพยามทำให้ node น้อย โดยเติมให้เต็มก่อนสร้าง node ใหม่ ก็ตาม
height อาจยาวได้เนื่องจาก tree ไม่ balance

Order 3

- #max. sons = 3
- #max. keys = 2



Insert 57
Must be with 55
Not in a new node.

Tree 2

3. AVL Tree
   Height Balanced Tree
4. Which Representions ?
5. n-ary Tree
6. Generic Tree
7. Multiway Search Tree
   - Top Down Tree
   - BalancedTree
     B-Tree

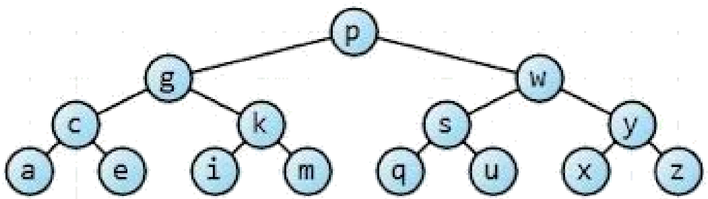
1. Tree Definitions
2. Binary Tree
   - Traversals
   - Binary Search Tree
   - Representations
   - Application : Expression Tree

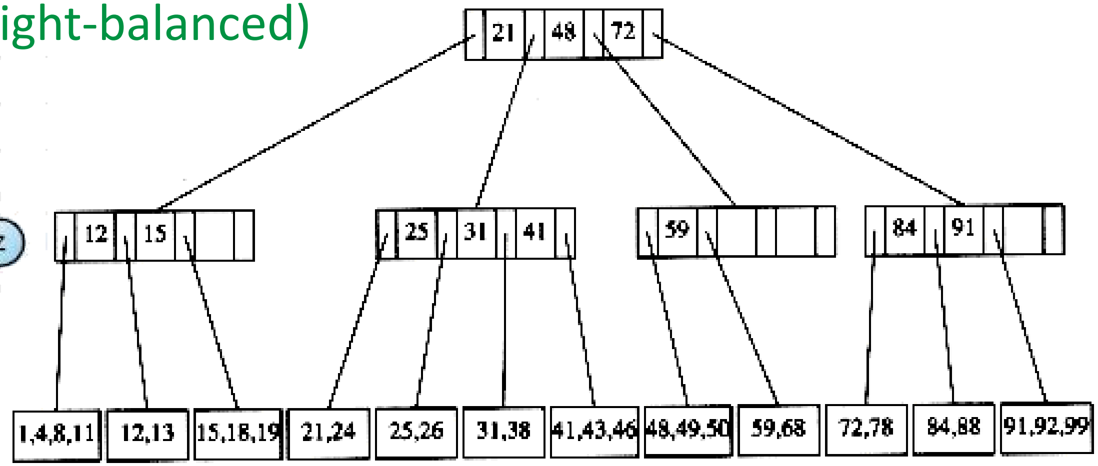
Balanced Tree

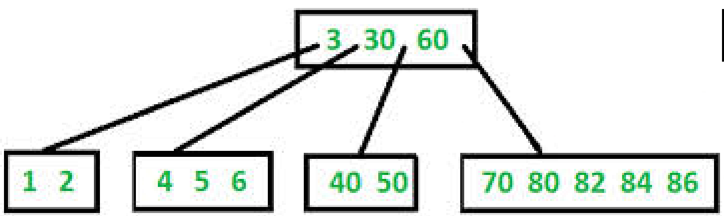
Balanced Tree : every leaf node is at the same level

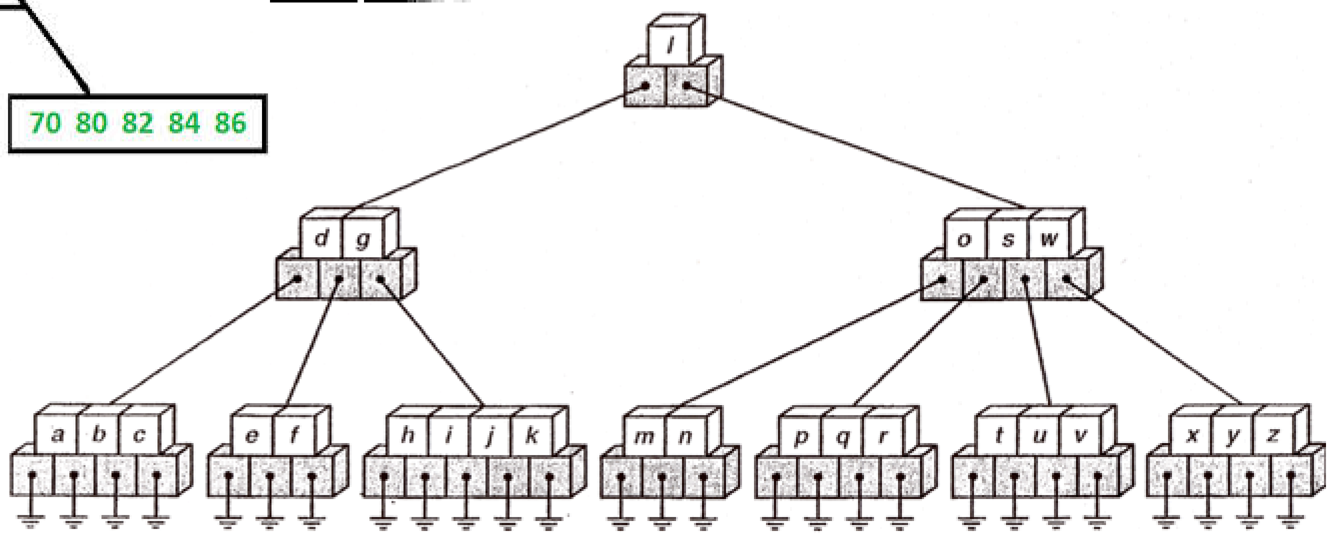
ทุก **leaf** อยู่ **level** เดียวกัน

(Perfect balance , perfectly height-balanced)









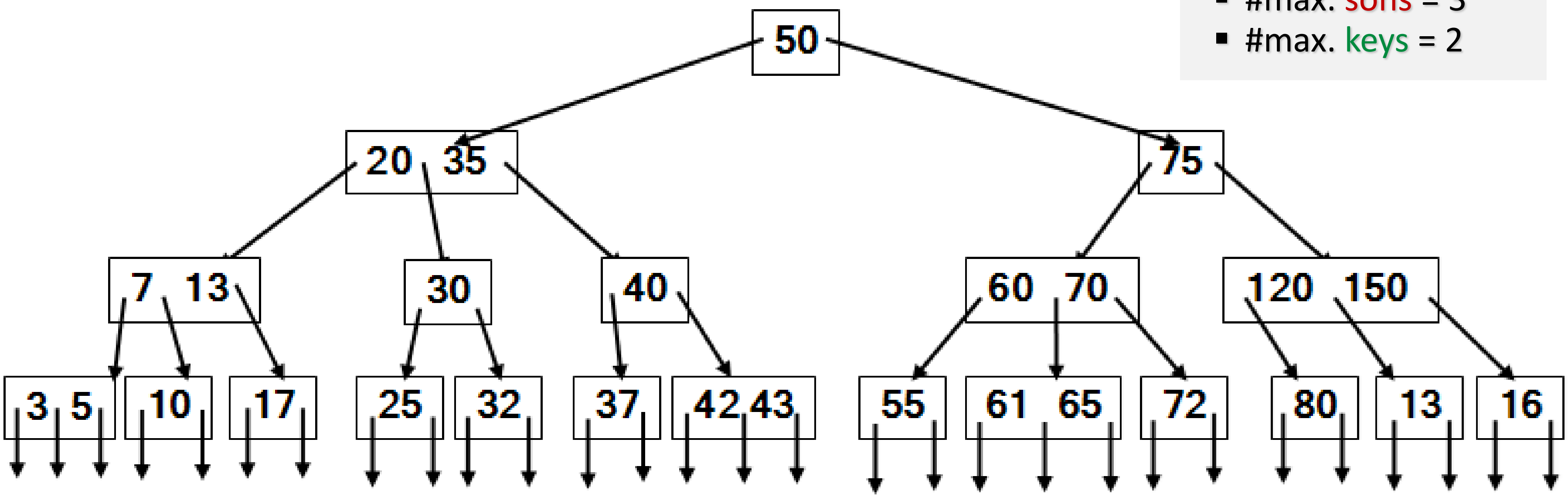
B-tree order n

1. Multi-way search tree order n : #max son = n, #max keys = n-1
2. Balanced tree : ทุก leaf อยู่ใน level เดียวกัน

   (Perfect balance , perfectly height-balanced)
3. => next page

Order 3

- #max. sons = 3
- #max. keys = 2



B-Trees

B-tree order n

1. Multi-way search tree order n

   #max son = n, #max keys = n-1

2. Balanced tree
3. ทุก node มีจำนวน non-empty subtree (มีลูก)อย่างน้อยที่สุด เท่ากับครึ่งหนึ่งของ n (มีน้อยกว่านี้ไม่ได้) คือ Half Full

   ยกเว้น root node ไม่ต้อง half full ก็ได้

   #min sons = $\lceil n/2 \rceil$ ยกเว้น root

-> ไม่มี node ที่เล็กมากๆ

-> ดังนั้น จำนวน node ทั้งหมดไม่มากนัก

-> height ไม่ยาว

EX.

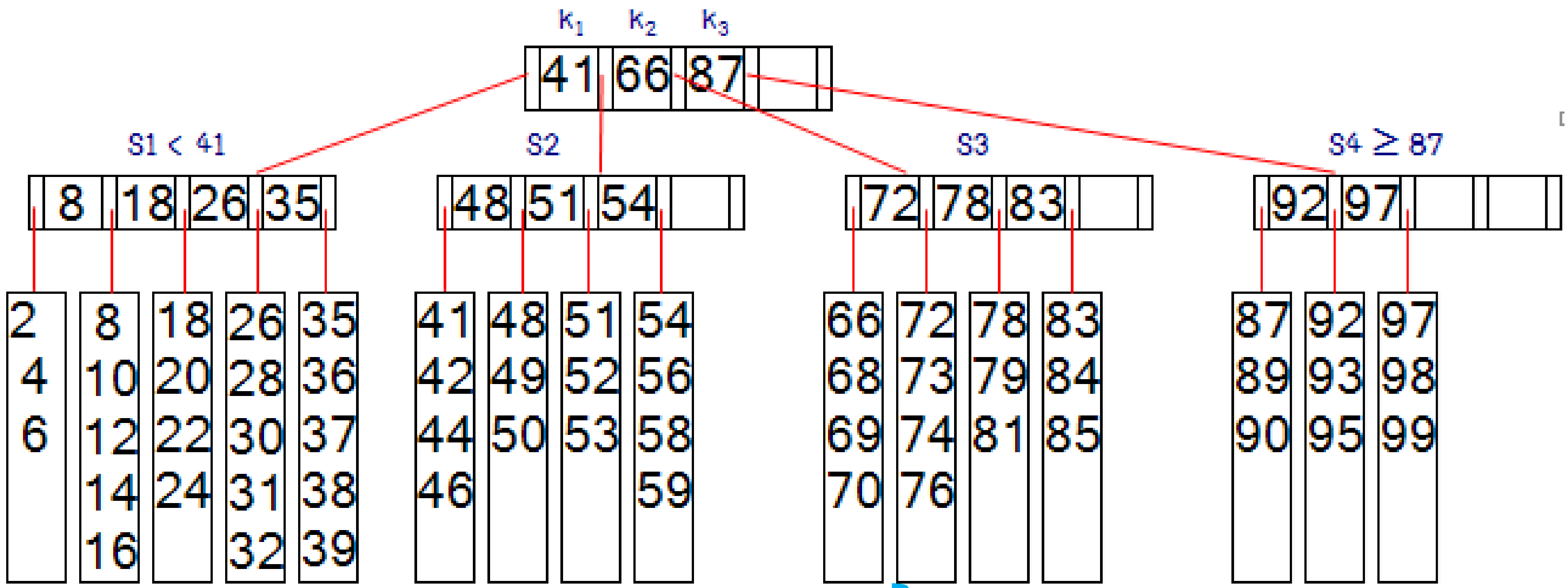
1. order 2 : #max sons =2 , #min sons = $\lceil 2/2 \rceil$ = 1

   ∴ Binary ที่เป็น (almost) complete binary tree จะเป็น B-tree

2 order 5 : #max sons = 5 , #min sons = $\lceil 5/2 \rceil$ = $\lceil 2.5 \rceil$ = 3

3 order 201 : #max sons = 201 , #min sons = $\lceil 201/2 \rceil$ = 101

B-Tree Variation



order 5 #max son = 5, #max keys = 4

- Half Full #min sons = $\lceil 5/2 \rceil = \lceil 2.5 \rceil = 3$ (ยกเว้น root node)

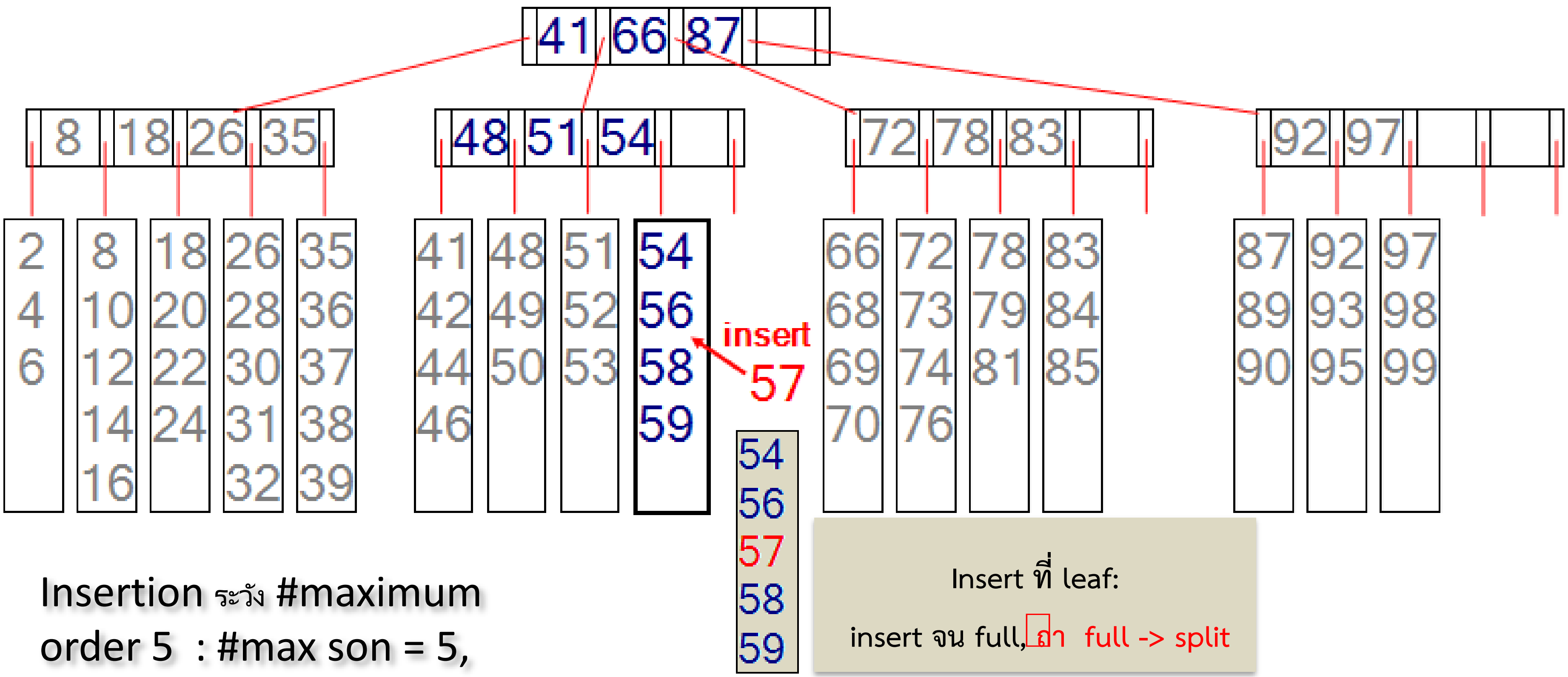
B-tree -> variations

- internal nodes : เก็บเฉพาะ key เพื่อประหยัดพื้นที่ (เก็บ data ได้มากกว่า เก็บทั้ง record), order กำหนดจำนวนลูก
- external nodes : เก็บ data ทั้ง record

  ค่า L กำหนดจำนวน record/node ของ leaf

ถ้า L = 5 : #max data = 5, #min data = $\lceil 5/2 \rceil = \lceil 2.5 \rceil = 3$

B-Tree (Variation) Insertion



Insert ที่ leaf:

insert จน full, ถ้า full -> split

Insertion ระวัง #maximum

order 5 : #max son = 5,

Internal : **#max keys = 4,**

#min sons = $\lceil 5/2 \rceil$ = 3

Leaf L = 5: **#max data = 5,**

จะ insert ต้องระวังไม่ให้เกินจำนวนที่กำหนด (max)

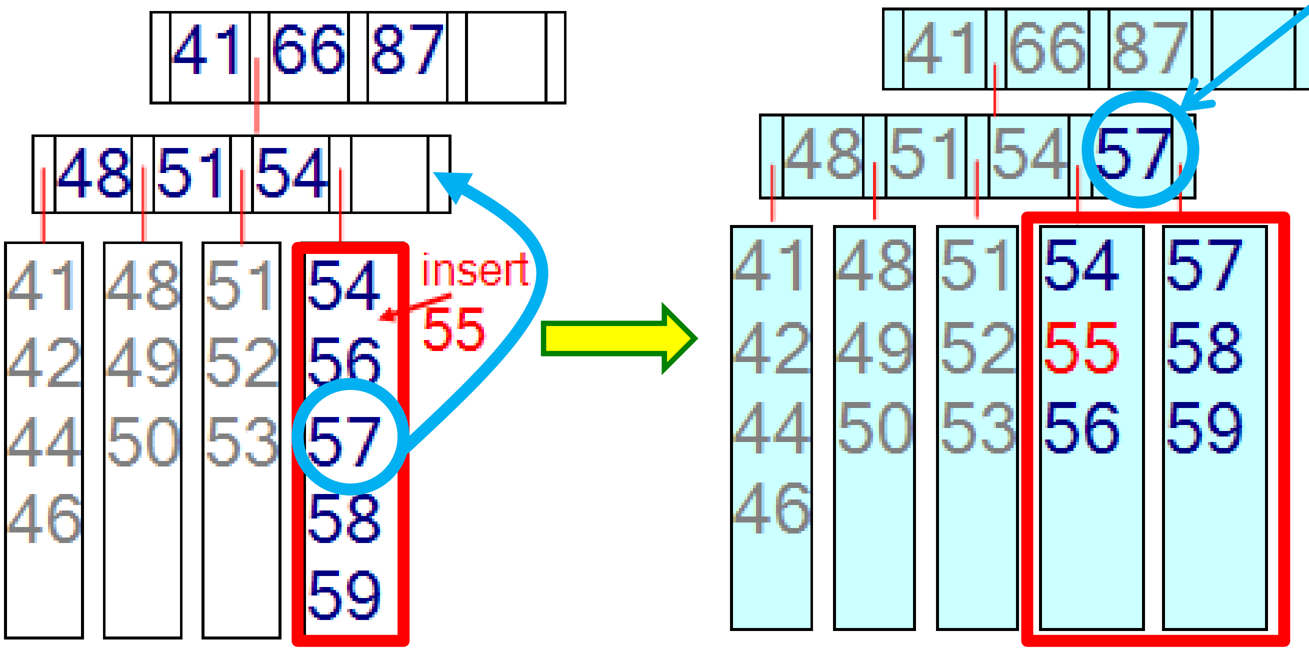
-> Insert ที่ leaf ดูค่า L

Leaf L = 5: #max data = 5

B-Tree (Variation) Insertion

order 5, #max key = 5-1=4
L=5 #leaf max data = 5

order 5 : internal node มี
#max son = 5
: **#max keys = 4,**
not full -> can insert



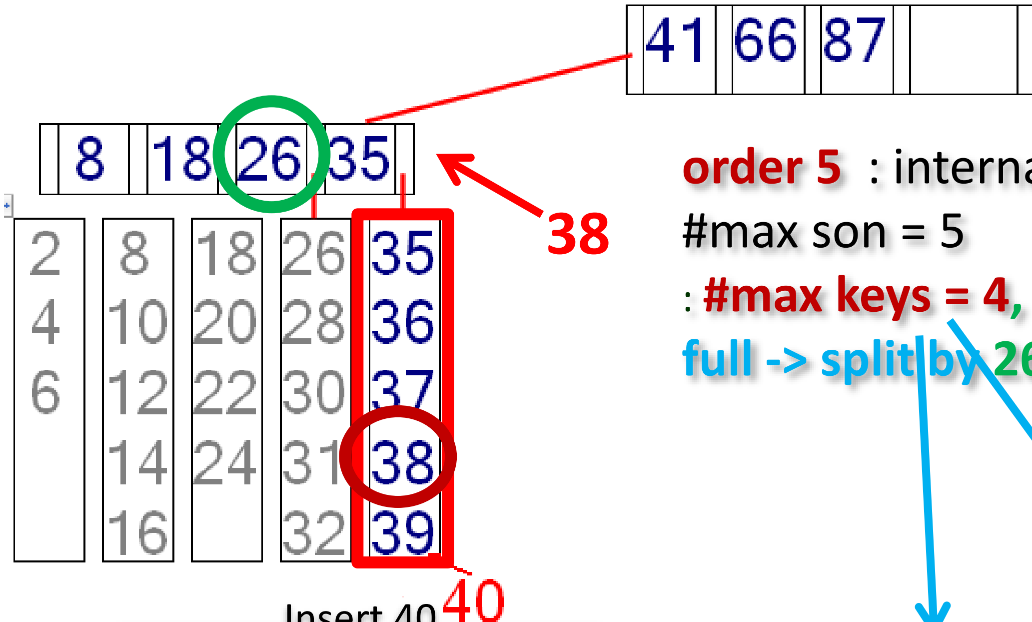
key 57 splits 2 nodes
key 57 ต้องถูก insert ใน father node

Insert 55
Insert ที่ leaf จน full :
L = 5: #max data = 5
Full

Full -> split half-half ที่ 57

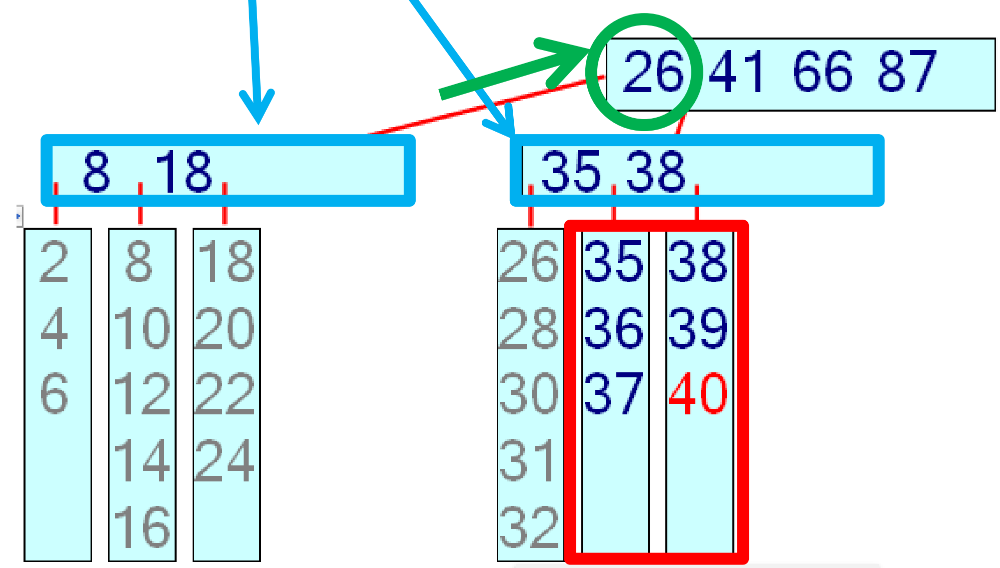
B-Tree (Variation) Insertion

Insert at leaf:
until full, if full -> split



order 5 : internal node :
#max son = 5
: **#max keys = 4,**
full -> split by 26

Insert 40
Insert ที่ leaf จน full :
L = 5: #max data = 5
Full

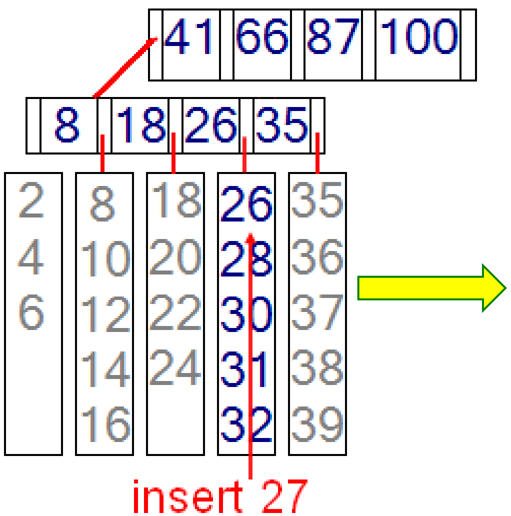


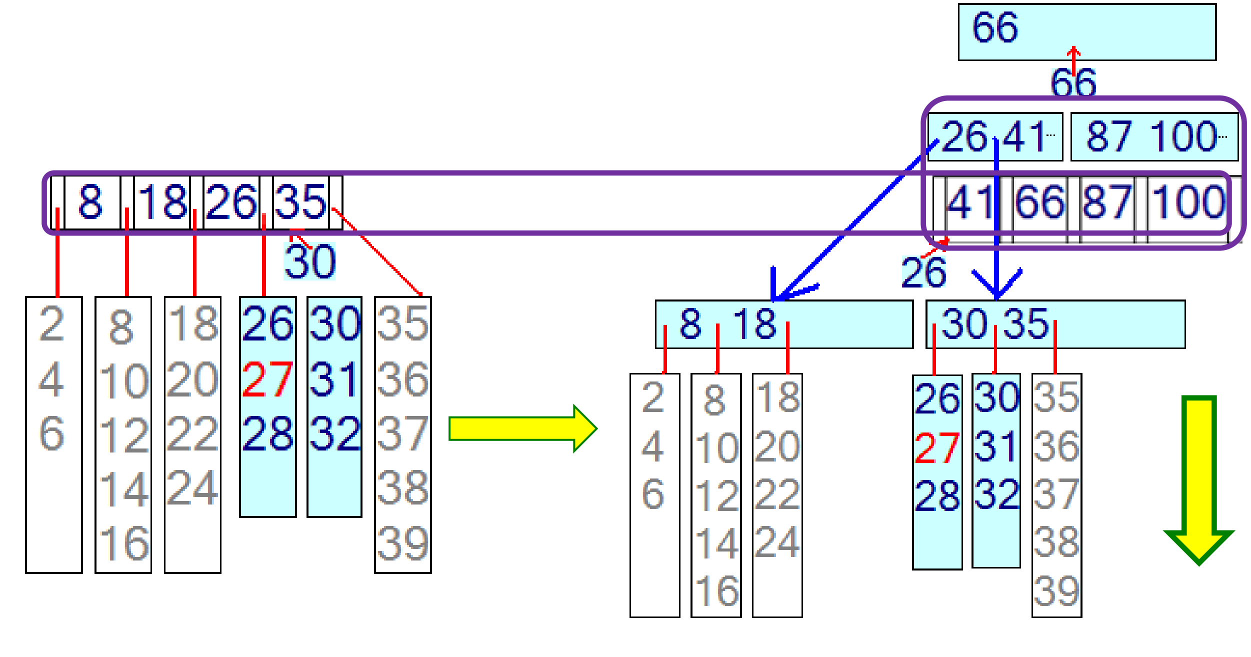
Full -> split by 38

B-Tree (Variation) Insertion

Insert at leaf:
until full, if full -> split

order 5, #max key = 5-1=4
L=5 #leaf max data = 5





Insertion ระวัง #maximum

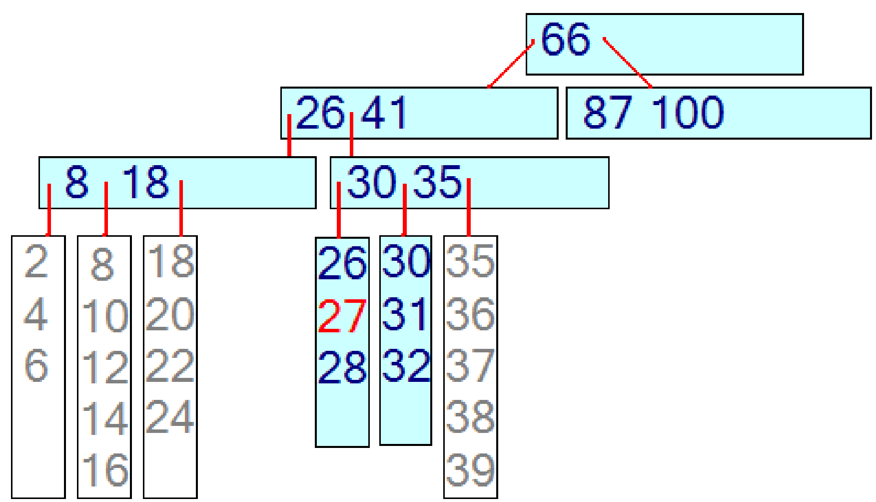
order 5 : #max son = 5,

Internal : **#max keys = 4,**

#min sons = $\lceil 5/2 \rceil = 3$

Leaf L = 5: **#max data = 5,**

#min data = $\lceil 5/2 \rceil = 3$



B-Tree (Variation) Deletion

Deletion consider
#minimum

Internal order 5 :

#min sons = $\lceil 5/2 \rceil$ = 3

#min keys = 2

Leaf L = 5:

#max data = 5,

#min data = $\lceil 5/2 \rceil$ = 3

Delete at leaf:

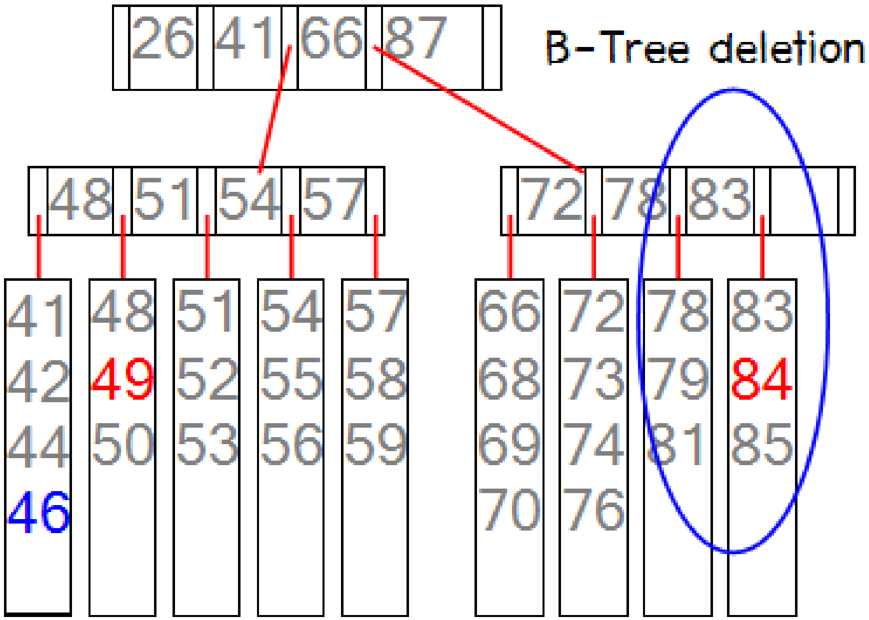
insert ระวังไม่ให้เกิน ต้องดู **max**
delete ระวังไม่ให้ขาด ต้องดู **min**
until low, if low -> borrow

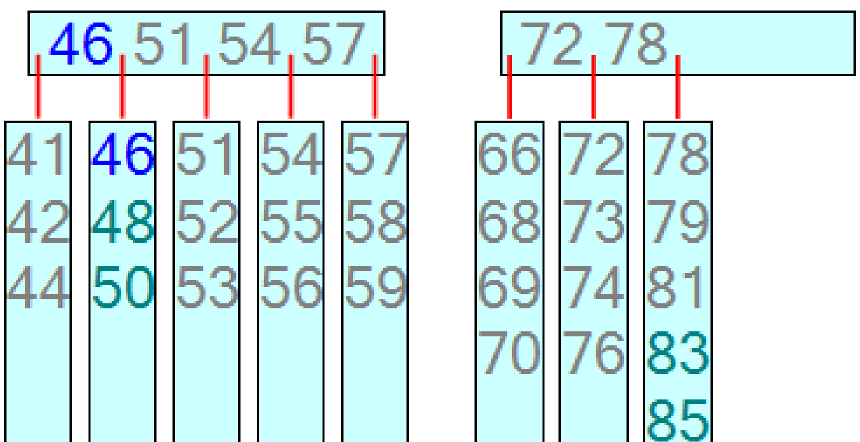
Borrowing
1. ยืมพี่น้อง(ผ่านพ่อ)
2. ยืมพ่อ ต้อง consolidate
   พ่อไม่มี พ่อยืม
   -พี่น้องพ่อ(ผ่านปู่)
   . . .

B-Tree (Variation) Deletion

Delete at leaf:
until low, if low -> borrow

B-Tree deletion : delete 49,84





Deletion consider #minimum

Internal order 5 : #min sons = $\lceil 5/2 \rceil$ = 3

#min keys = 2

Leaf L = 5: #max data = 5,

#min data = $\lceil 5/2 \rceil$ = 3

Borrowing

1. ยืมพี่น้อง(ผ่านพ่อ) เช่น del 49 เอา 46 ของพี่มา
2. ยืมพ่อ ต้อง consolidate เช่น del 84 พี่น้องไม่มีให้ยืม ยืมพ่อ โดยรวม consolidate กับพ่อ

ถ้าพ่อไม่มี พ่อยืม

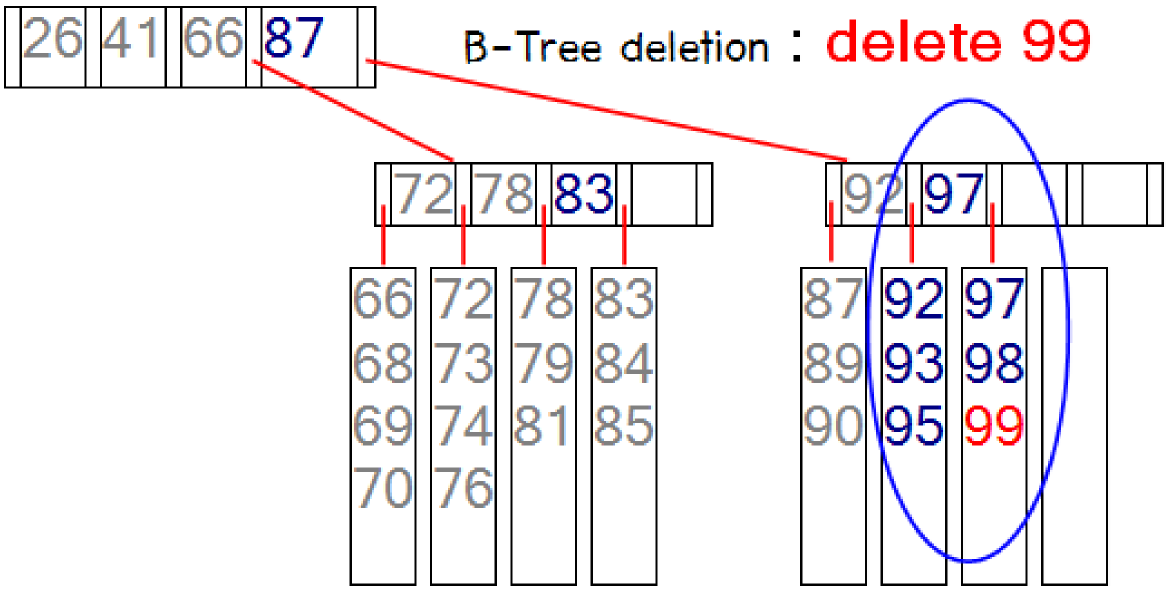
-พี่น้องพ่อ(ผ่านปู่)

. . .

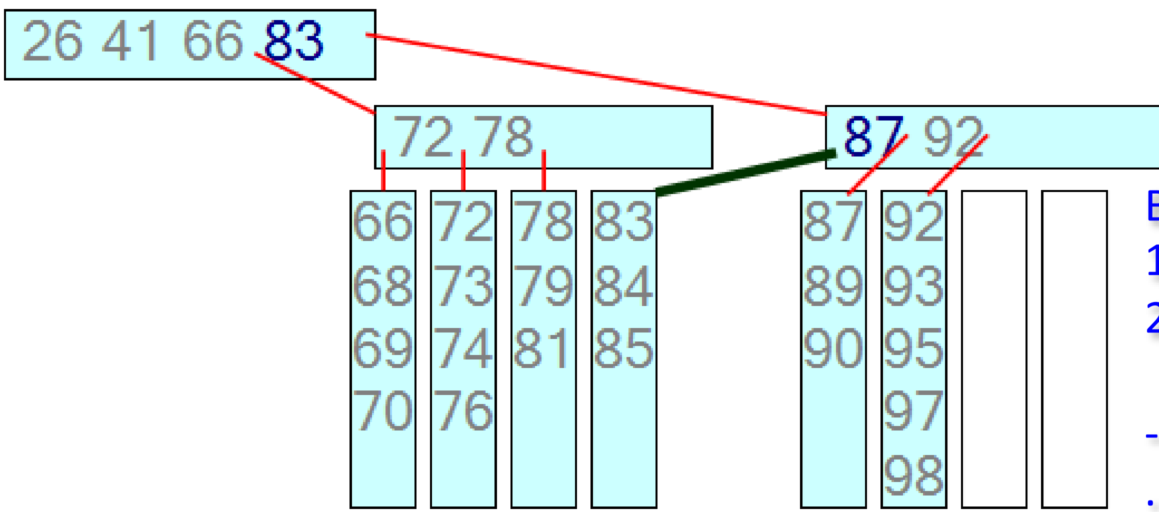
B-Tree (Variation) Deletion

Delete at leaf:
until low, if low -> borrow

B-Tree deletion : delete 99



Deletion consider
#minimum
Internal order 5 :
#min sons = $\lceil 5/2 \rceil$ = 3
#min keys = 2
Leaf L = 5:
#max data = 5,
#min data = $\lceil 5/2 \rceil$ = 3



Borrowing
1. ยืมพี่น้อง(ผ่านพ่อ)
2. ยืมพ่อ ต้อง consolidate
   พ่อไม่มี พ่อยืม
-พี่น้องพ่อ(ผ่านปู่)
. . .