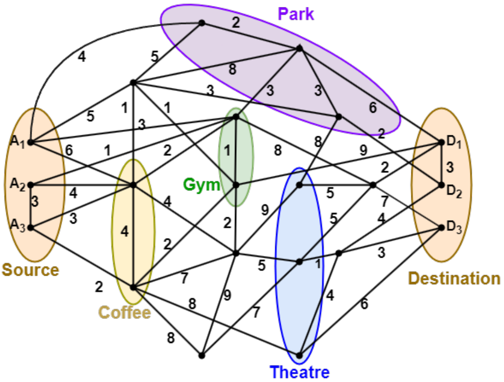
Fairness Driven Efficient Algorithms for Sequenced Group Trip Planning Query Problem

Napendra Solanki[1], Shweta Jain[1], Suman Banerjee[2], Yayathi Pavan Kumar S[1]

[1] Indian Institute of Technology Ropar
[2] Indian Institute of Technology Jammu

Sequenced Group Trip Planning Query Problem



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Goal: Find an optimal path minimizing the total distance while ensuring **fairness**.

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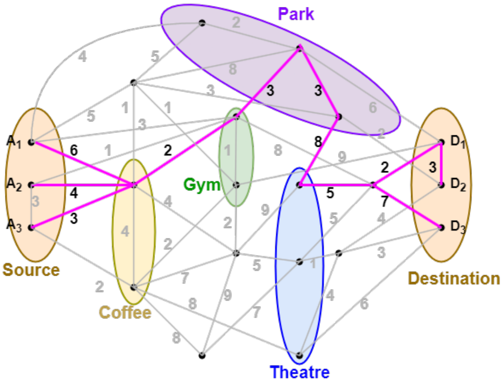

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Fair Sequenced-GTP

For a pair of agents i and j, and a given path p, define the **envy** by agent i from agent j as follows: $\mathcal{E}_{i \to j}(p) = \max\{D_i(p) - D_j(p), 0\}$.

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Definition (Envy of a Path)

Envy of a path p is the **maximum envy by any pair of agents**, i.e. $\max_{i,j} \mathcal{E}_{i \to j}(p)$.

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Definition ($\varepsilon-$Envy-Free Path ($\varepsilon-$EFP))

A path p is $\varepsilon-$envy-free path iff **the maximum envy is bounded by ε**, i.e., $D_i(p) \leq D_j(p) + \varepsilon\ \forall i,j \in B, \varepsilon > 0$.

Lemma

$\varepsilon-$EFP always exists for $\varepsilon \geq min_{p\in\mathcal{P}} \max_{i,j} \mathcal{E}_{i\to j}(p)$.

Related Work

- Hashem et al.[1] studied the problem and gave an efficient heuristic solution.
- Ahmadi et al.[2] gave a Progressive Group Neighbour Exploration Approach to compute the optimal solution.
- Later, Ahmadi et al.[3] proposed a dynamic programming-based solution to compute the optimal solution.

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- Recently, Singhal and Banerjee[4] solved fair sequenced GTP problem in $O(n^k)$ time.

We propose $O(n^4)$ time algorithm to fair sequenced GTP

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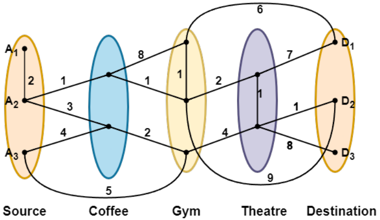
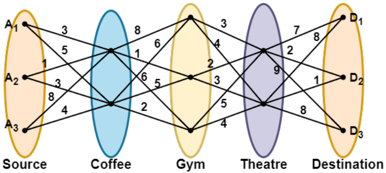
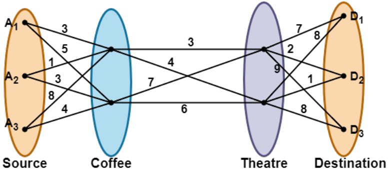


Run single source shortest path algorithm on the graph and create a (K+2)-partite graph



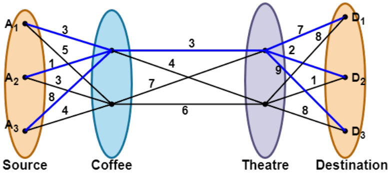
Run single source shortest path algorithm on the first category of the graph and create the 4-partite graph



	Path 1	Path 2	Path 3	Path 4
Agent 1	13	15	19	19
Agent 2	6	6	12	17
Agent 3	20	20	20	18

The path matrix

Optimal Path and Minimum Envy Path



	Path 1	Path 2	Path 3	Path 4
Agent 1	13	15	19	19
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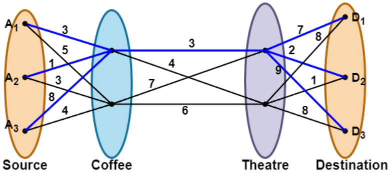
Agent 3	20	20	20	18
Sum	39	41	51	54

Optimal

The **optimal path** can be **very envious**. The envy in the optimal path is 14.

Optimal Path and Minimum Envy Path



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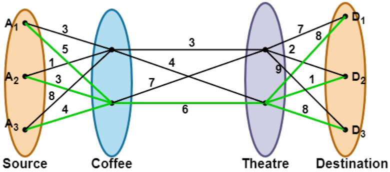

	Path 1	Path 2	Path 3	Path 4
Agent 1	13	15	19	19
Agent 2	6	6	12	10
Agent 3	20	20	20	18
Sum	39	41	51	54

Minimum envy

Path 4 is the minimimum envy path with envy 2.

Price of Fairness

Definition (Price of fairness)

Let the optimal path be denoted by p^* and the minimum envy path be denoted by p_f^*. Then, PoF is given by:

$$PoF = \frac{\sum_{i \in B} D_i(p_f^*)}{\sum_{i \in B} D_i(p^*)} \tag{1}$$

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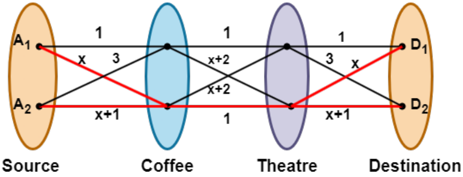

The price of fairness, in general, can be **unbounded**. PoF: $\frac{4x+3}{9}$.

Pareto-optimal Space

Definition (Pareto-optimal Path)

A path p' is called a Pareto-optimal path if there does not exist any other path p'' such that:

$$\forall i \in [b],\; D_i(p'') \leq D_i(p') \text{ and } \exists j \in [b] \text{ s.t. } D_j(p'') < D_j(p')$$

Here, $D_i(p)$ denotes the distance traveled by i^{th} agent in path p.

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	Path 1	Path 2	Path 3	Path 4
Agent 1	13	15	19	19
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Non Pareto-Optimal Paths

	Path 1	Path 4
Agent 1	13	19
Agent 2	6	17
Agent 3	20	18

Pareto-Optimal path
with minimum envy

The algorithm will take $O(n^4)$ time to compute minimum envy path.

The bounds on the Pareto-Optimal path

Theorem

The price of fairness among all Pareto-optimal solutions is bounded by 3 *when the number of agents is 2.*

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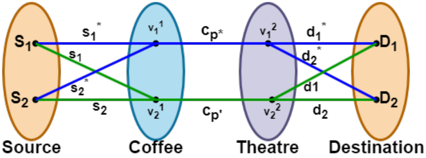
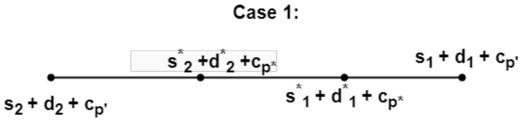


- Let p^* be the optimal path and $p^{'}$ be the pareto-optimal path with minimum envy.
- W.L.O.G. assume that $s_1^* + d_1^* \geq s_2^* + d_2^*$, otherwise we can always reorder agents.
- We have two cases.

The bounds on the Pareto-Optimal path



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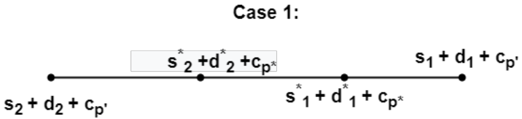
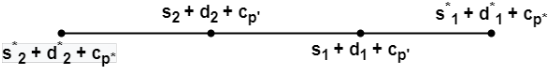


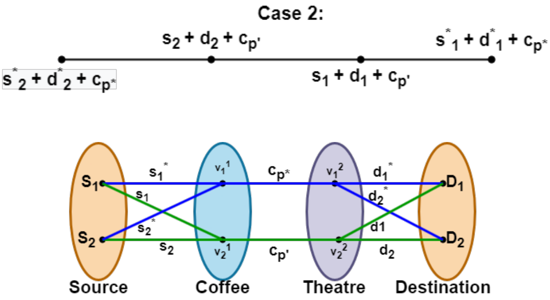
Contradiction to p' being the minimum envy path.

The bounds on the Pareto-Optimal path

Case 2:



The bounds on the Pareto-Optimal path



Bounds can be provided using triangular inequality,
$s_2 \leq s_2^* + s_1 + s_1^*$ and $d_2 \leq d_2^* + d_1 + d_1^*$.

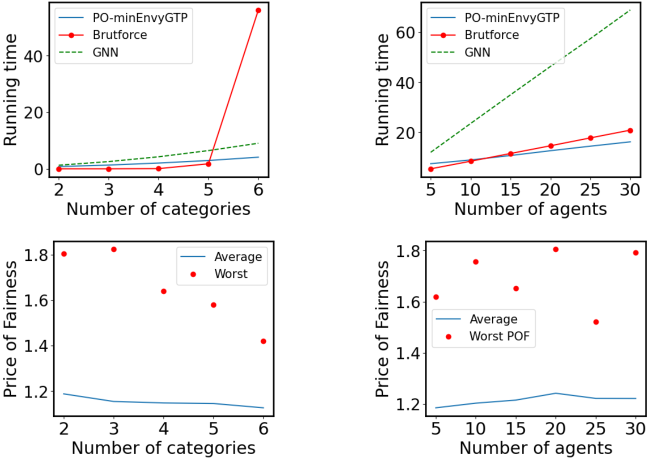
Other Results

Theorem

For b agents, the price of fairness among all Pareto-optimal solutions is bounded by $(2b - 1)$.

Theorem

There exists an example where the price of fairness among all Pareto-optimal solutions with b agents is approximately equals to $(2b - 1)$.



The experiments are performed on real-world[5] datasets.

[5] https://users.cs.utah.edu/ lifeifei/SpatialDataset.htm

Conclusion and Future Work

- We have introduced an algorithm that computes the minimum envy path in polynomial time.
- We have given tight bounds over the price of fairness for b agents.
- The unordered GTP is an open problem and can be seen as future work.
- Other fairness and efficiency notions can be studied in the GTP problem.

Napendra Solanki
napendra.21csz0001@iitrpr.ac.in
Indian Institute of Technology Ropar
India

Dr. Shweta Jain
shwetajain@iitrpr.ac.in
Indian Institute of Technology Ropar
India

Dr. Suman Banerjee
suman.banerjee@iitjammu.ac.in
Indian Institute of Technology Jammu
India

Yayathi Pavan Kumar S
yayathi.21csz0014@iitrpr.ac.in
Indian Institute of Technology Ropar
India