# GRAPH THEORY MA5234 EXERCISE SHEET 3

- If you would like feedback on your work, choose 2 problems from the sheet, write up your solutions, and submit an electronic copy via moodle.
- If you prefer to write solutions by hand, scan or photograph your work and submit this (but please make sure your writing is legible in the electronic copy).
- If you want to submit work, you should do this before the exercise classes on Thursday, 2 June. See moodle for the precise deadline.
- You are welcome to work in groups, but everyone should submit their own assignment. If you work with other students, please tell me who you worked with in a note at the top of your assignment.

Problem 1. Consider the graph G:
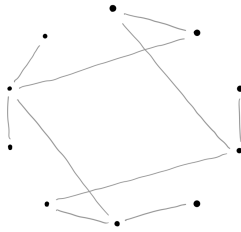
What is the maximum cardinality of a matching of G? Prove that your answer is correct.

Problem 2. This problem completes the proof of Theorem 1.8.1. Say G is a connected graph with $||G|| > 0$ and $d(v)$ even for all $v \in V(G)$. Show G has a closed walk W with at least one edge and such that W contains no edge of G more than once.

Problem 3. Say $G = (V, E)$ is a graph and $M \subseteq E$ is a matching. Say there is a matching $M' \subseteq E$ such that $|M'| > |M|$. Show that G contains an augmenting path for M.

Problem 4. Let $k \geq 1$. Say G is a k-connected graph. Let $v, u_1, \ldots, u_k$ be $k + 1$ distinct vertices of G. Show that there are paths P_i from v to u_i for $1 \leq i \leq k$ such that $P_i \cap P_j = \{v\}$ whenever $i \neq j$.

Problem 5. Consider the graph H:
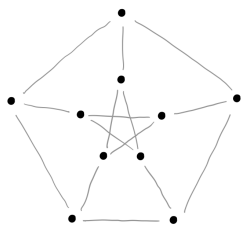
What is the connectivity $\kappa(H)$? What is the edge-connectivity $\lambda(H)$? Prove that your answers are correct.

Problem 6. Let $k \geq 2$. Let G be k-connected with $|G| \geq 2k$. Show that G contains a cycle of length at least $2k$.