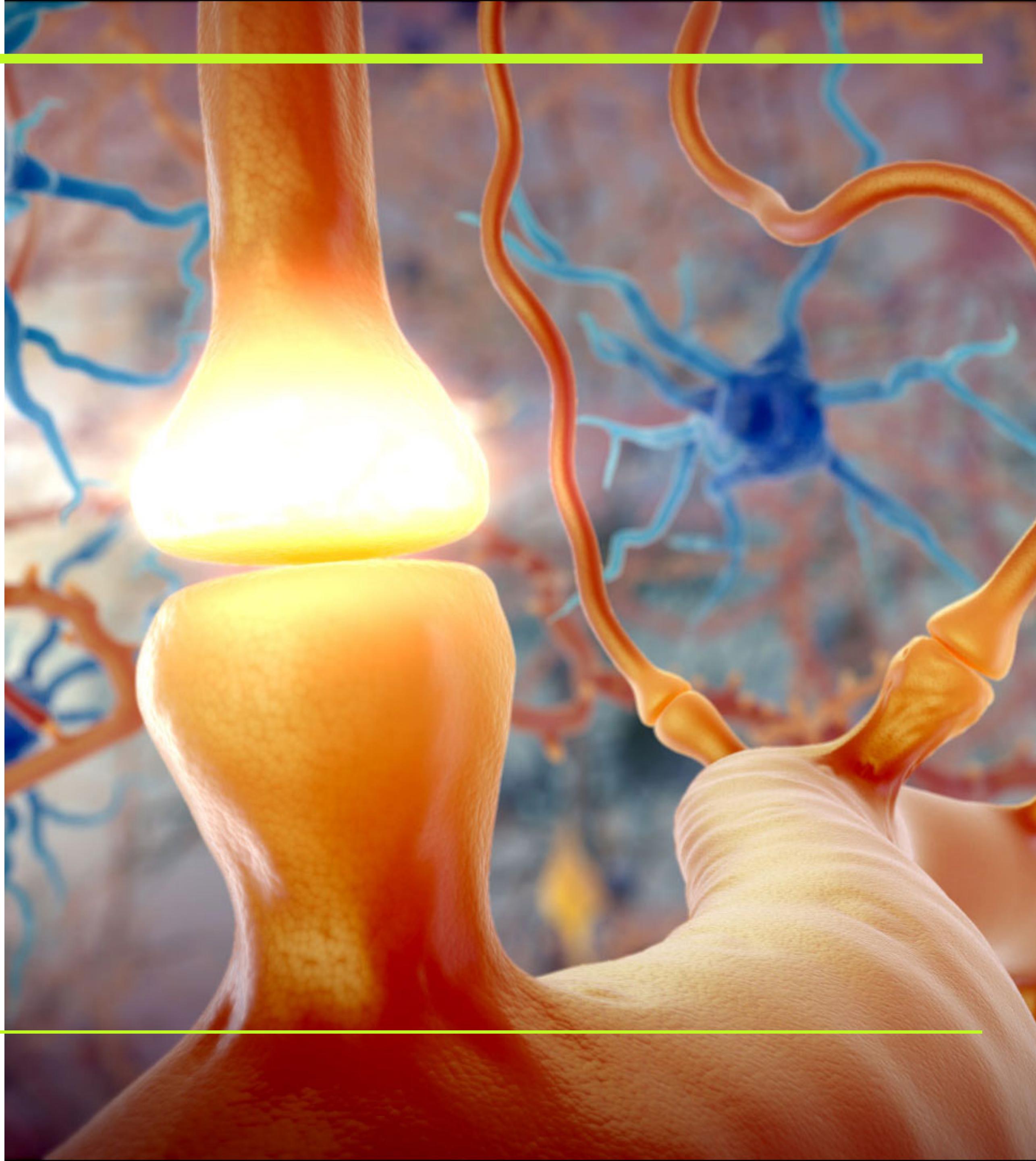


# 系统和计算神经科学导论

## 二、神经信号传导

# 1. 膜电位和动作电位

# 2. 数学模型



# 1. 膜电位和动作电位

## 2. 数学模型



# Cell membrane & Ion Channel

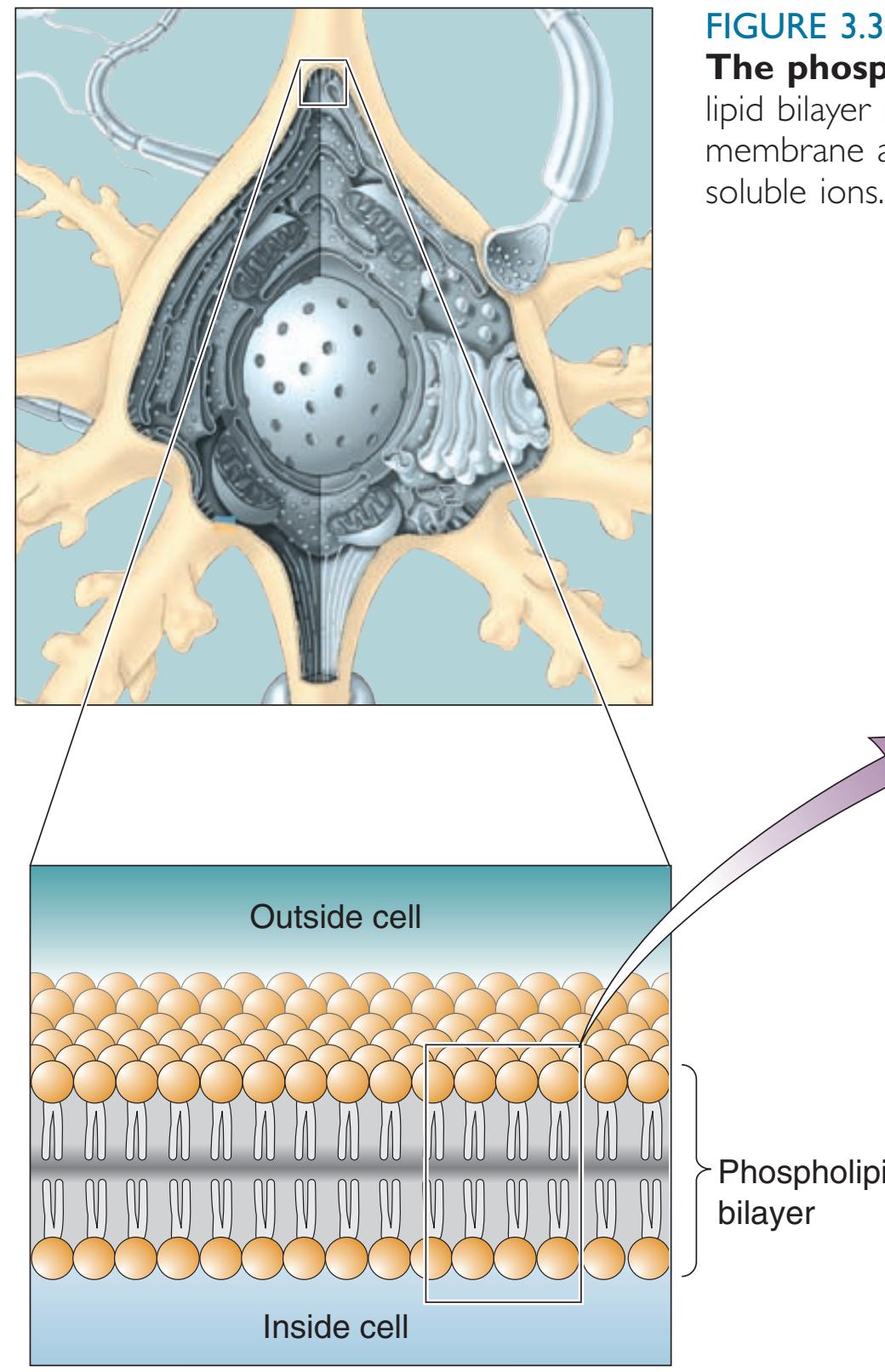


FIGURE 3.3

**The phospholipid bilayer.** The phospholipid bilayer is the core of the neuronal membrane and forms a barrier to water-soluble ions.

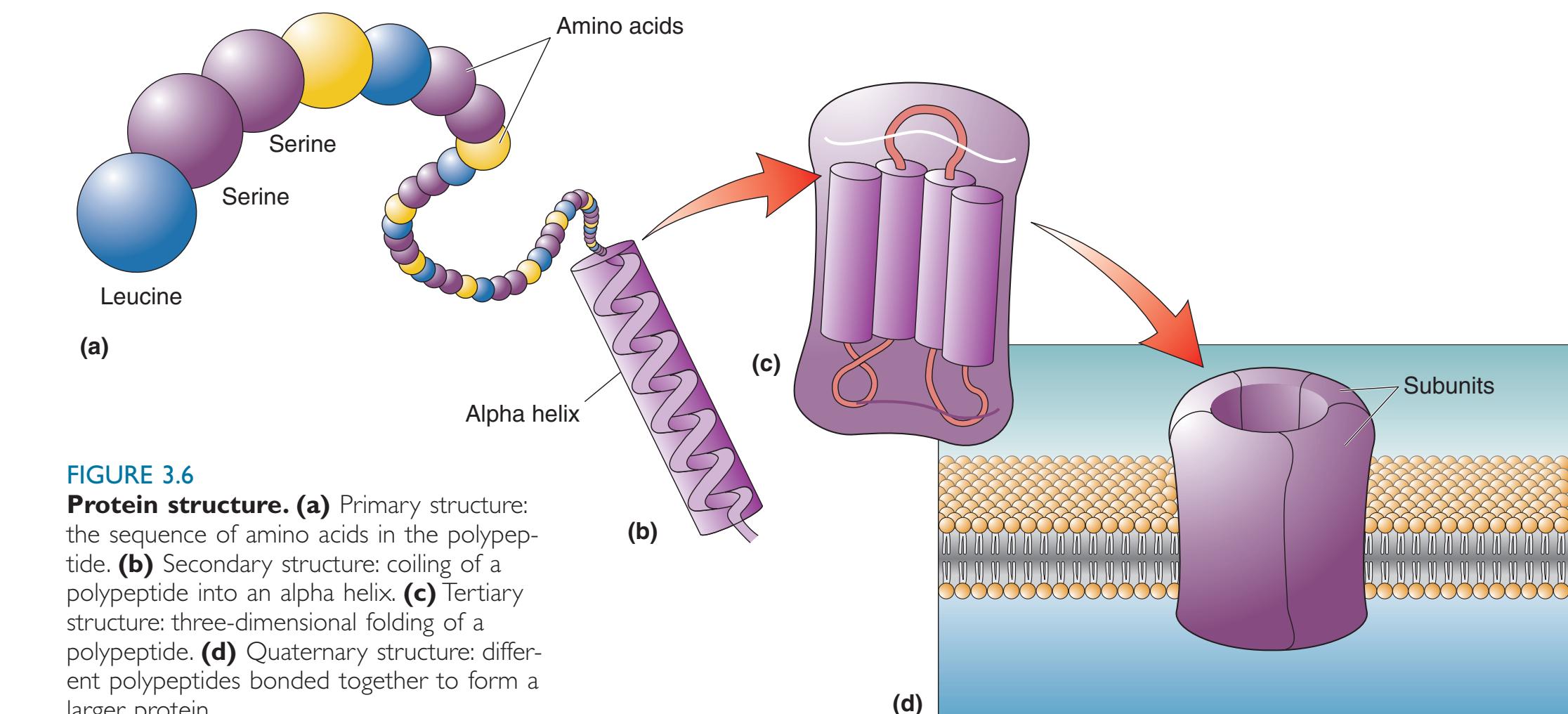
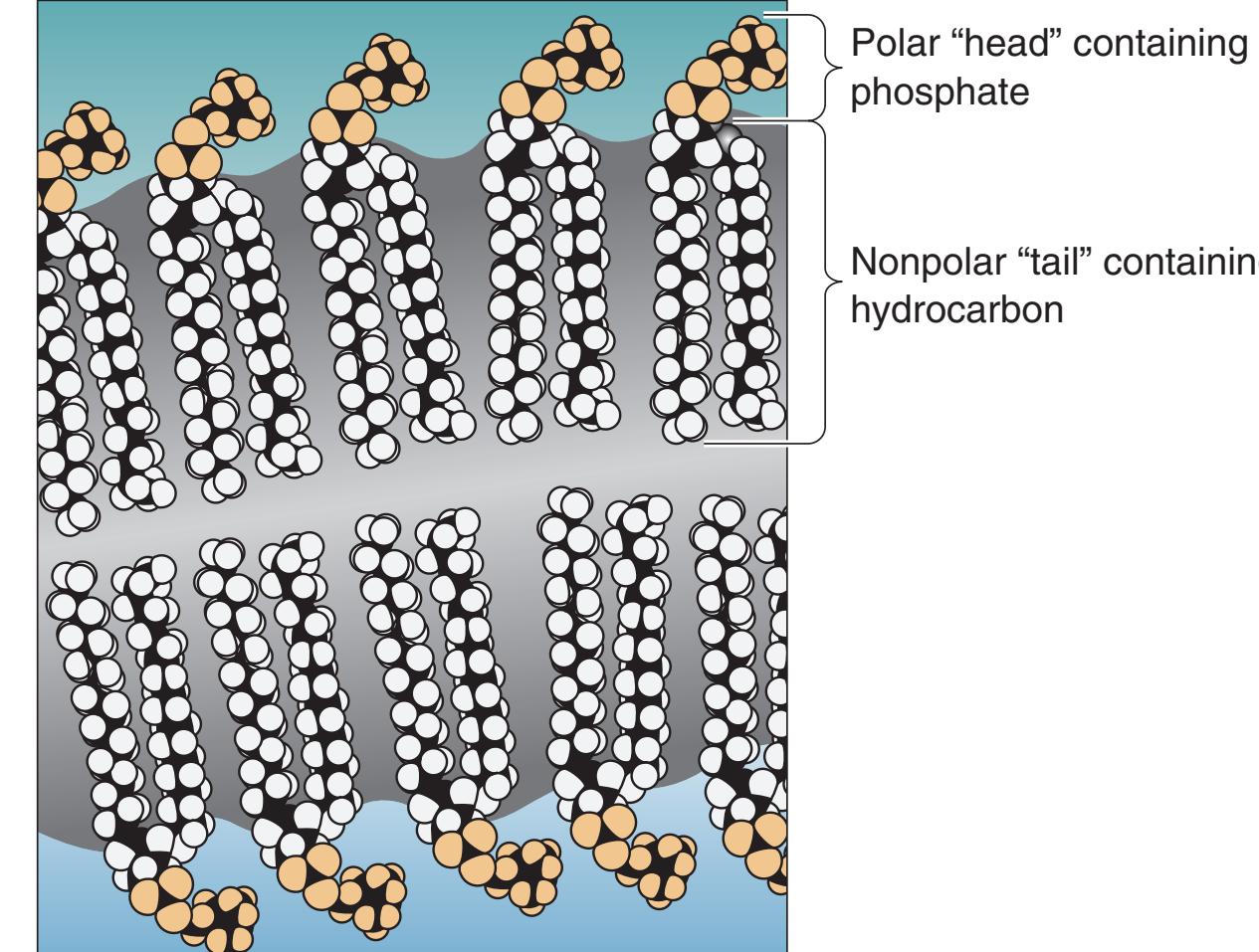
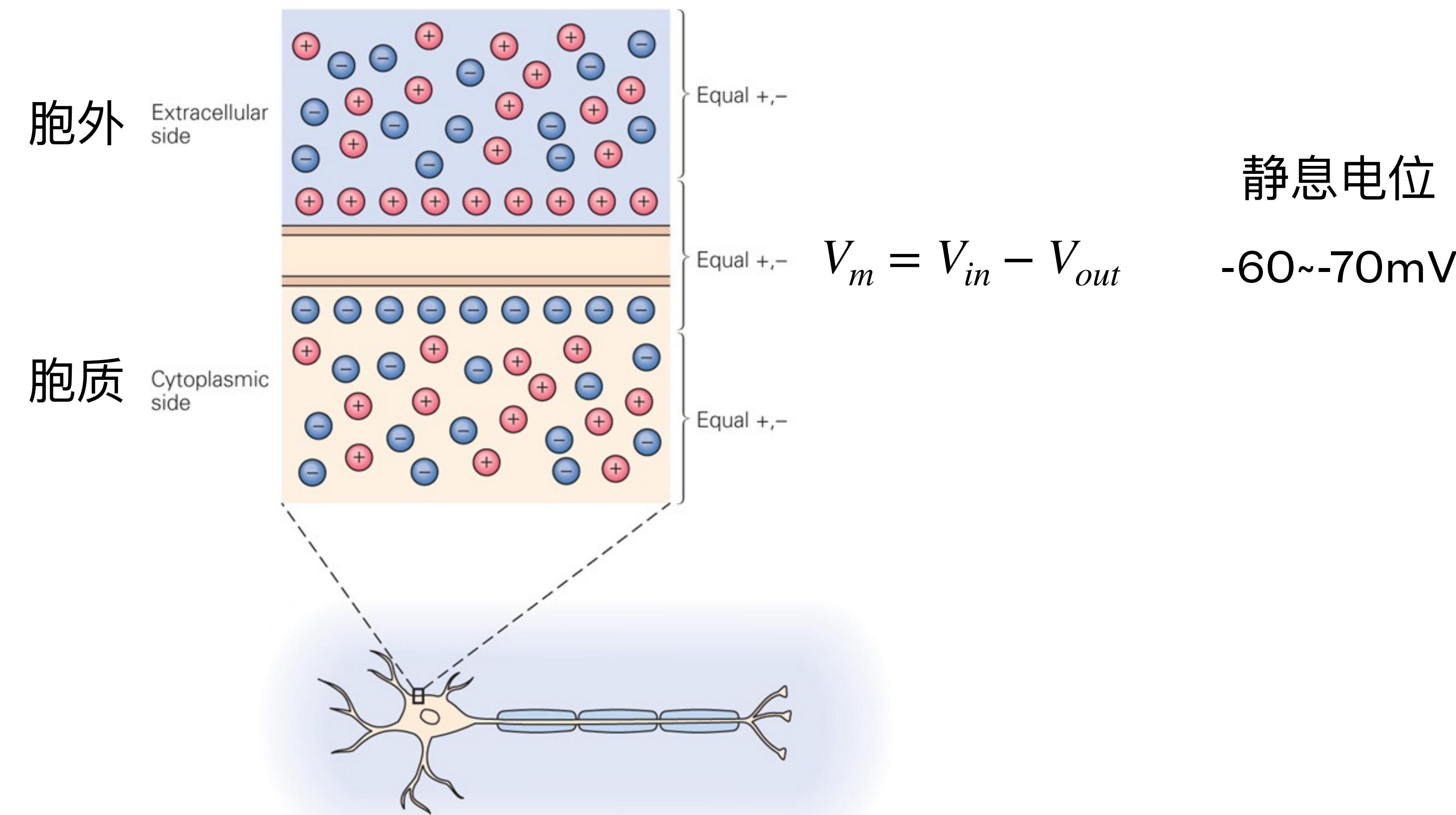


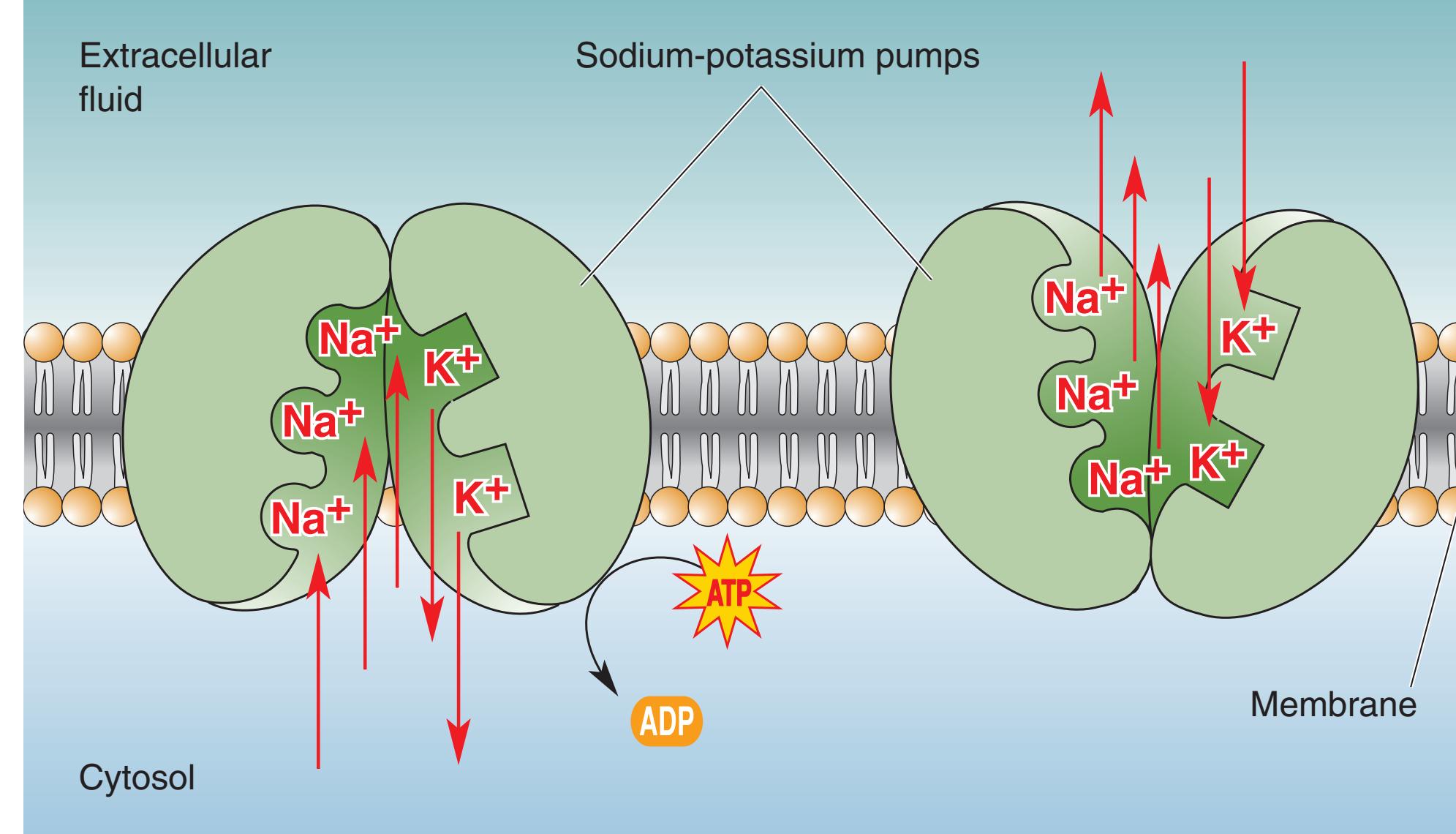
FIGURE 3.6

**Protein structure.** (a) Primary structure: the sequence of amino acids in the polypeptide. (b) Secondary structure: coiling of a polypeptide into an alpha helix. (c) Tertiary structure: three-dimensional folding of a polypeptide. (d) Quaternary structure: different polypeptides bonded together to form a larger protein.

# Membrane Potential



# Establishing the resting potential



气体常数 温度  
电荷数 法拉第常数

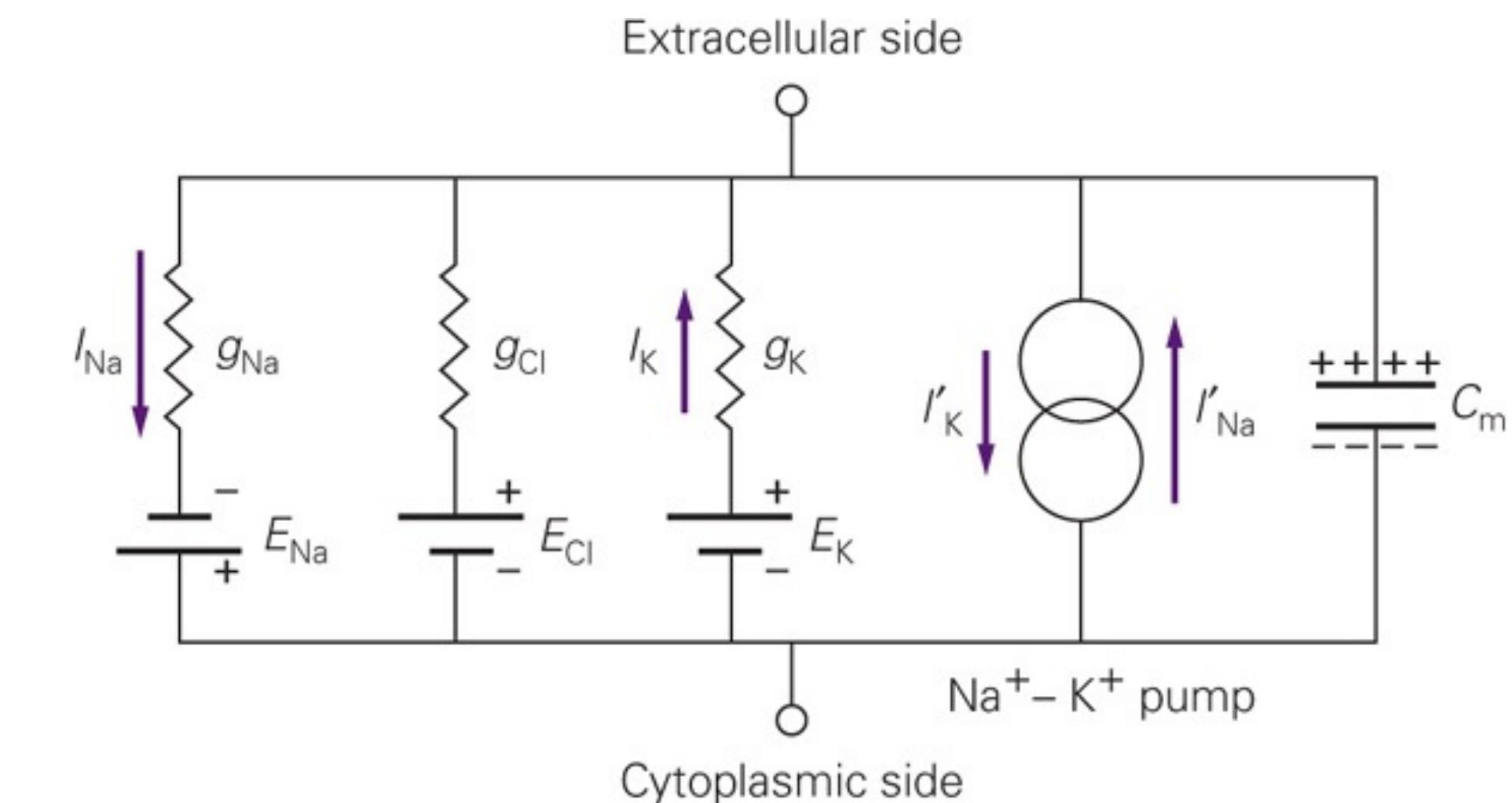
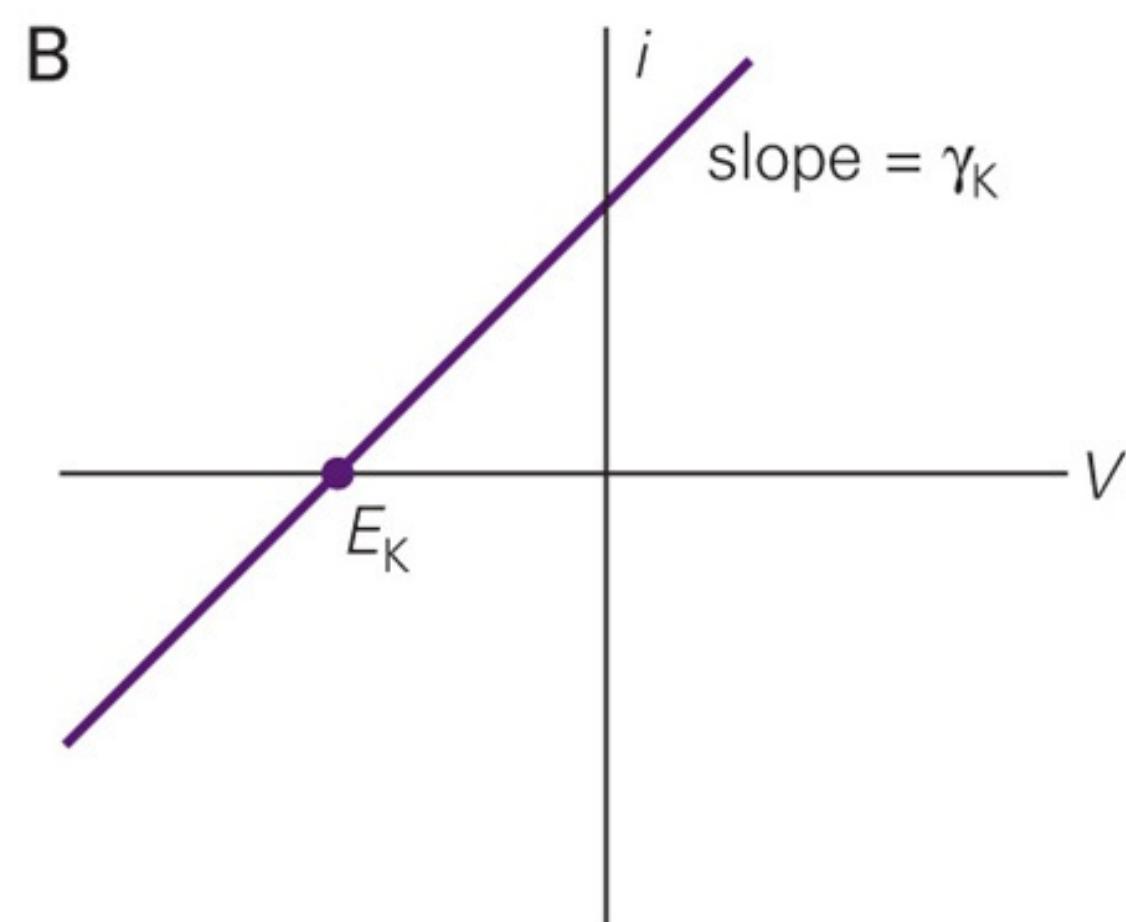
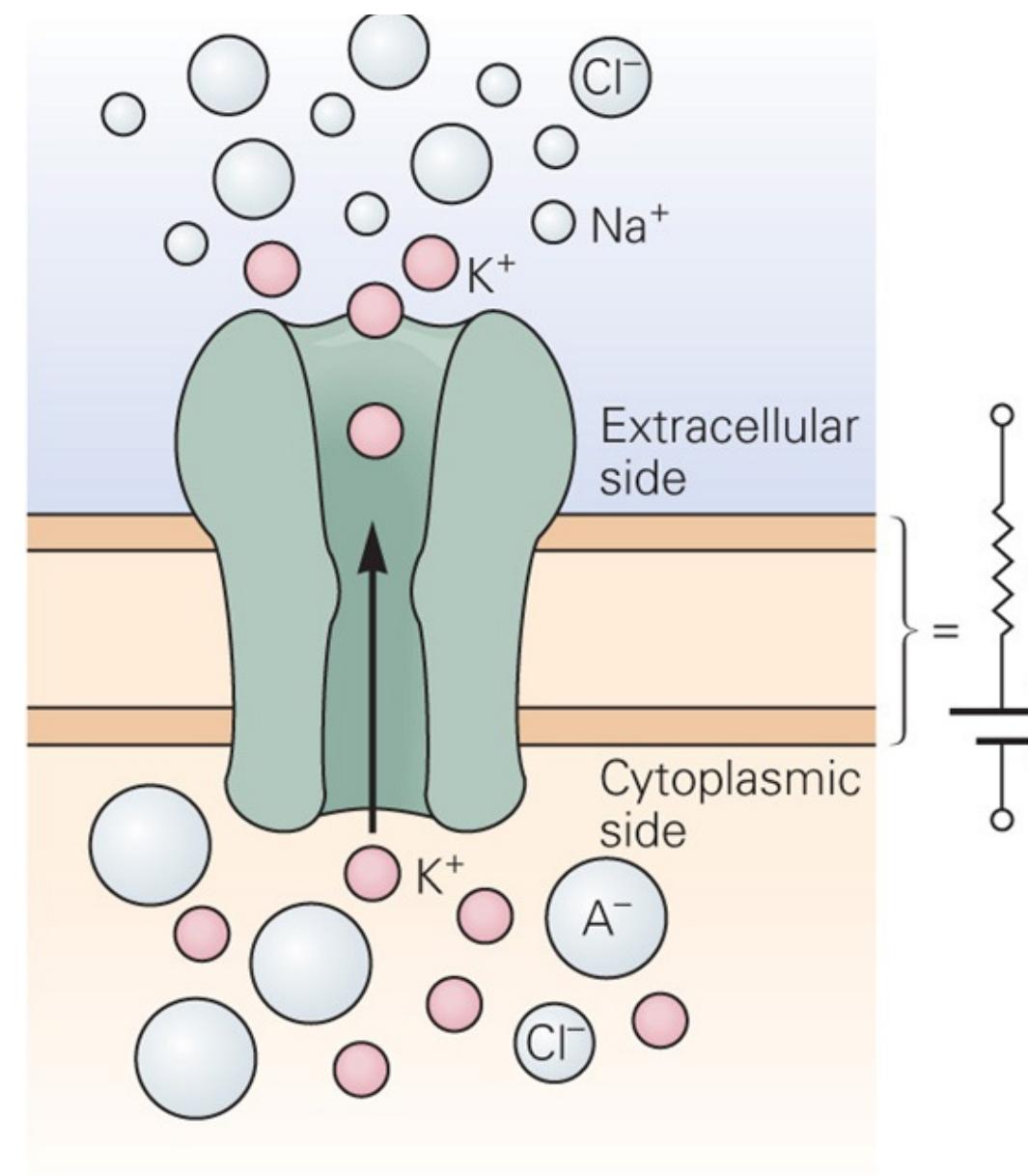
$$E_{ion} = \frac{RT}{zF} \ln \frac{[ion]_{out}}{[ion]_{in}}$$

Nernst Eq.

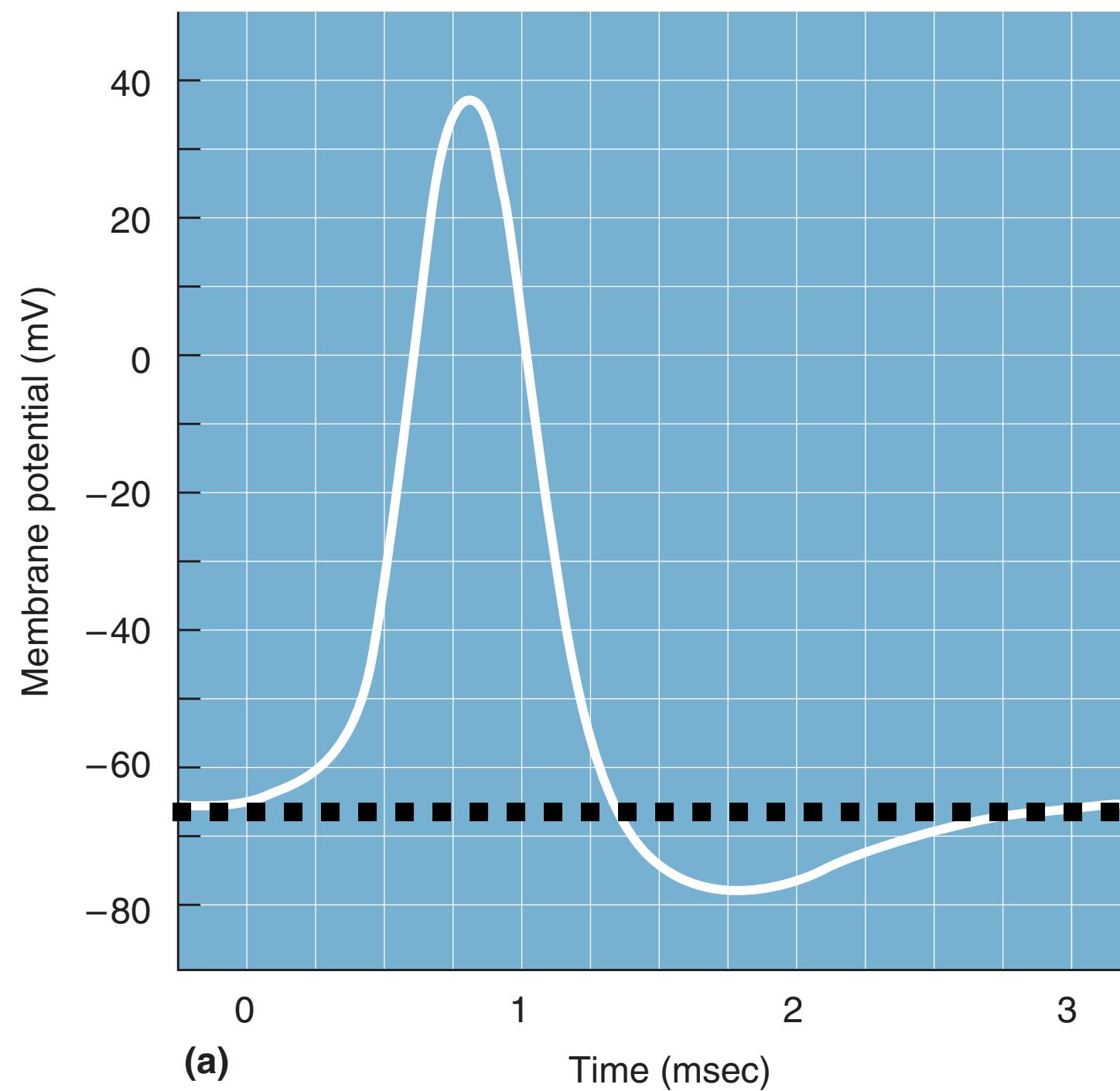
Ion	Concentration outside (in mM)	Concentration inside (in mM)	Ratio Out : In	$E_{ion}$ (at 37°C)
$\text{K}^+$	5	100	1 : 20	-80 mV
$\text{Na}^+$	150	15	10 : 1	62 mV
$\text{Ca}^{2+}$	2	0.0002	10,000 : 1	123 mV
$\text{Cl}^-$	150	13	11.5 : 1	-65 mV

Goldman Eq.  $V_m = \frac{RT}{F} \ln \frac{P[K^+]_o + P[Na^+]_o + P[Cl^-]_i}{P[K^+]_i + P[Na^+]_i + P[Cl^-]_o}$

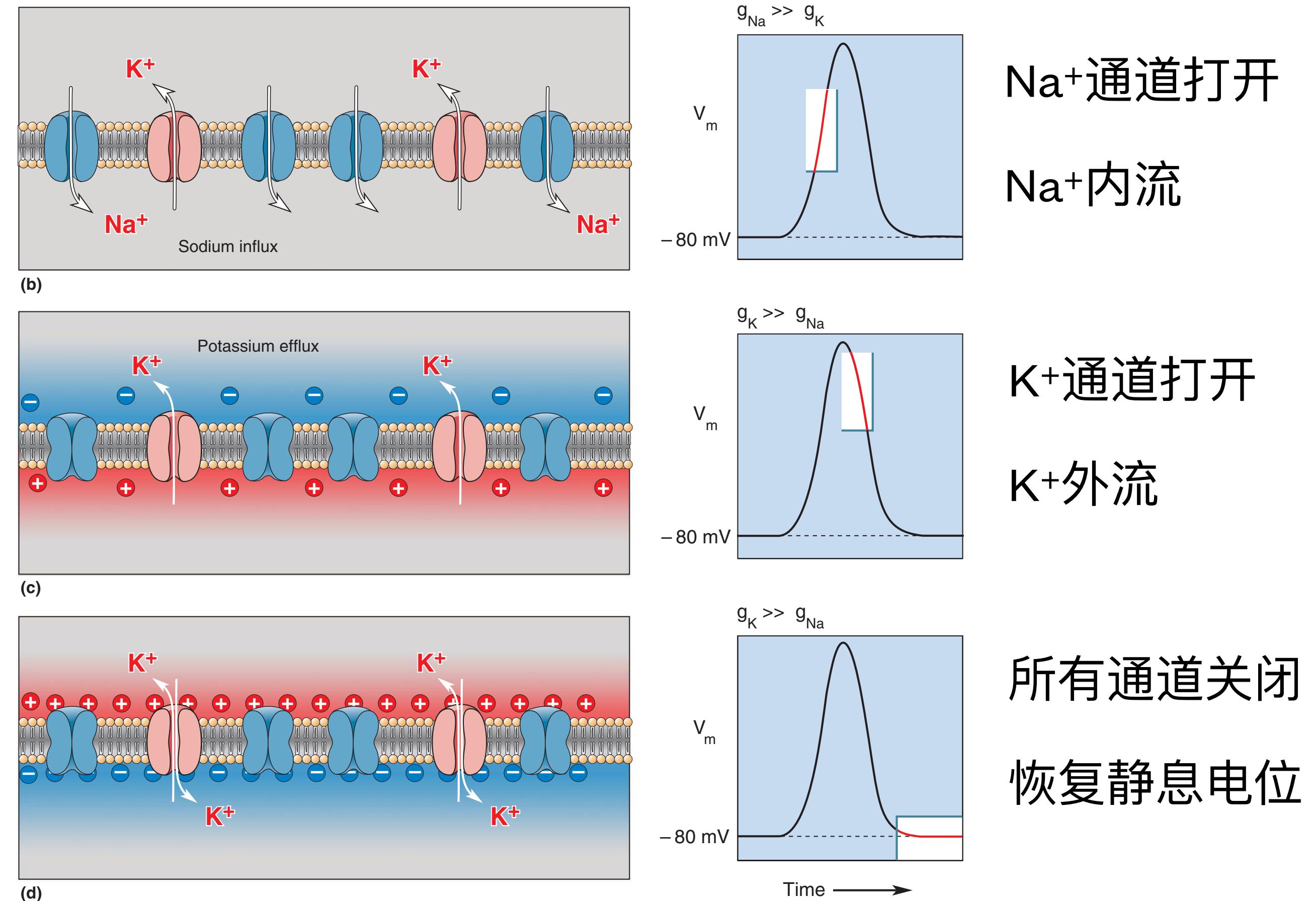
# Equivalent Circuitry



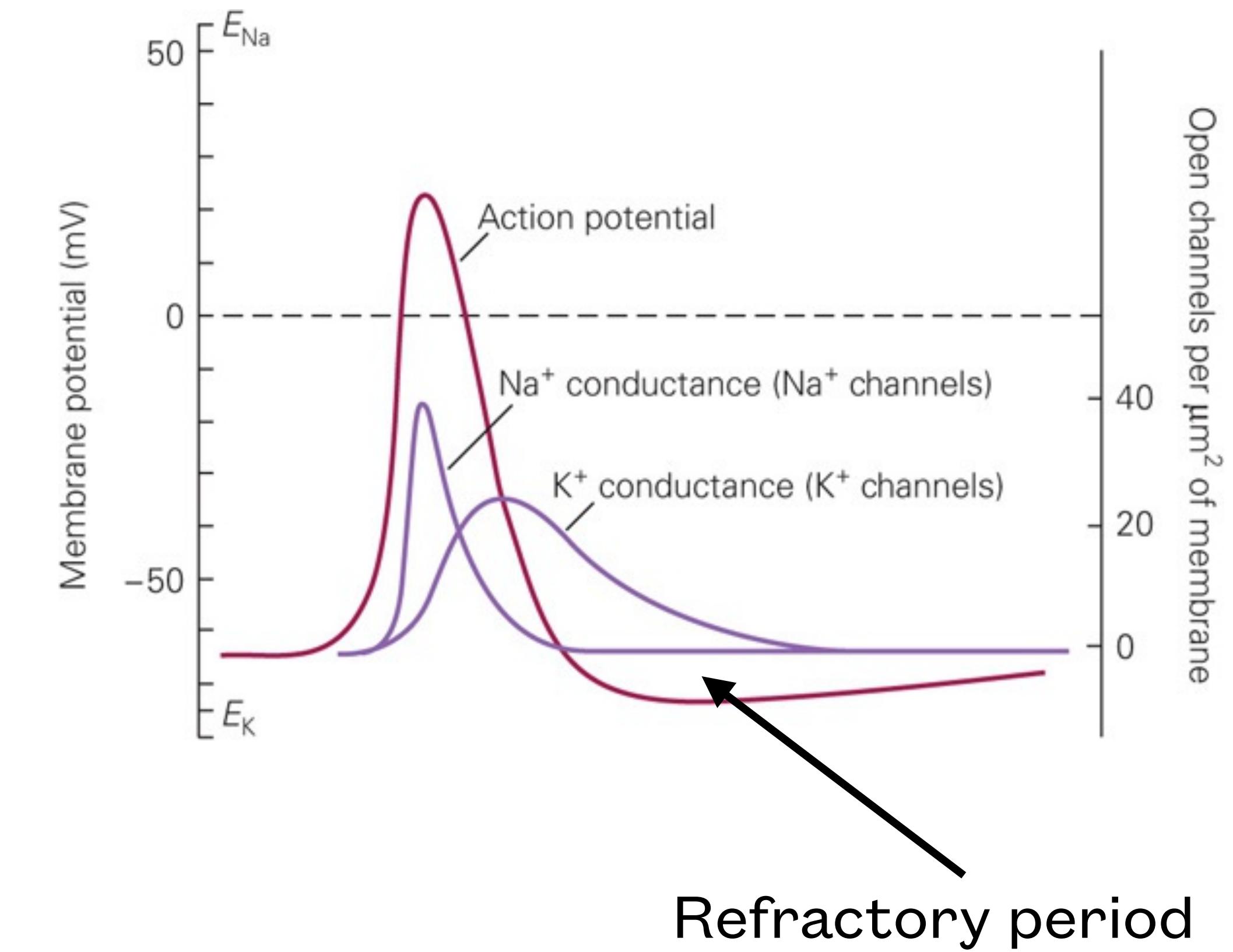
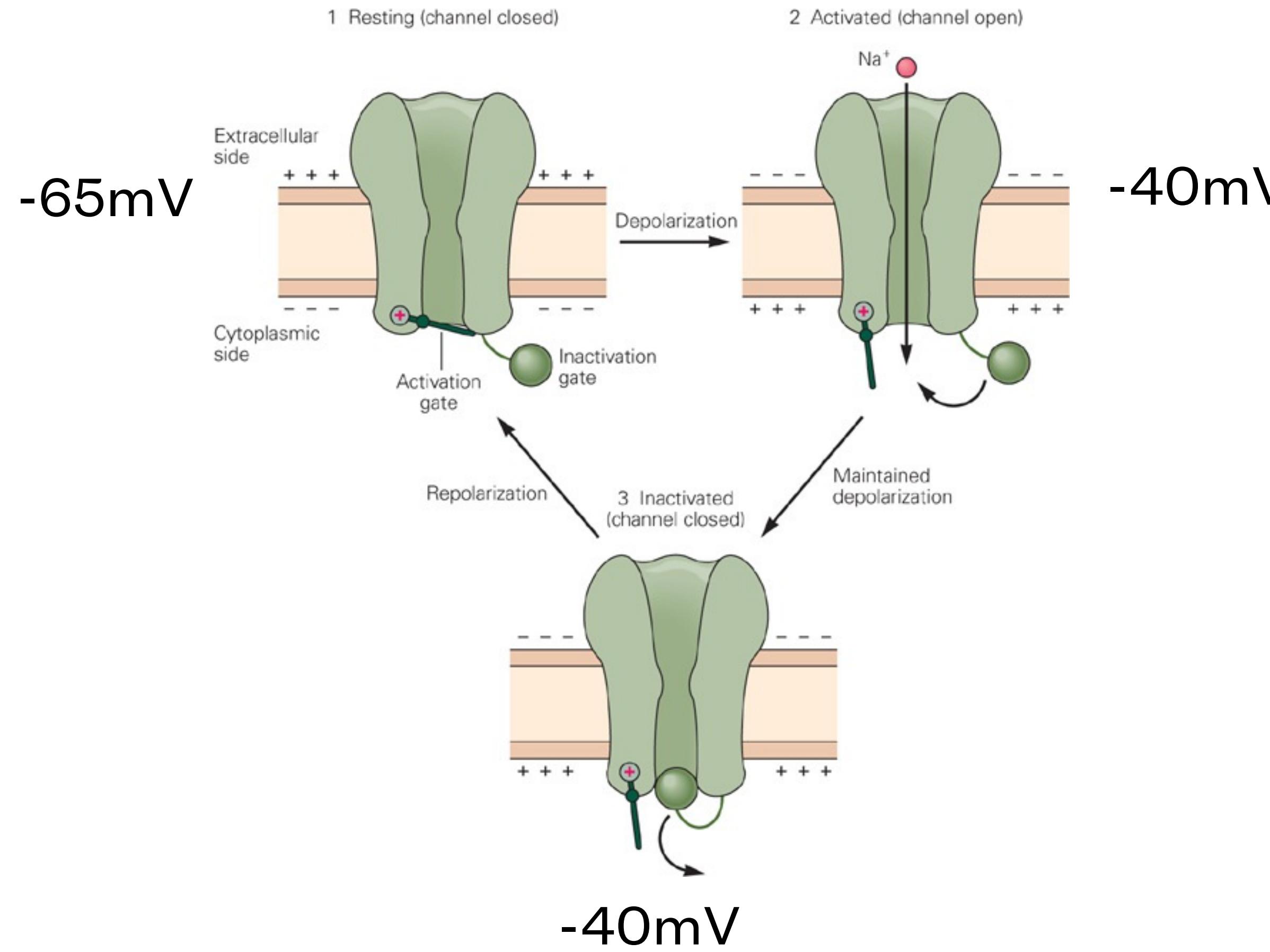
# Generation of Action Potentials



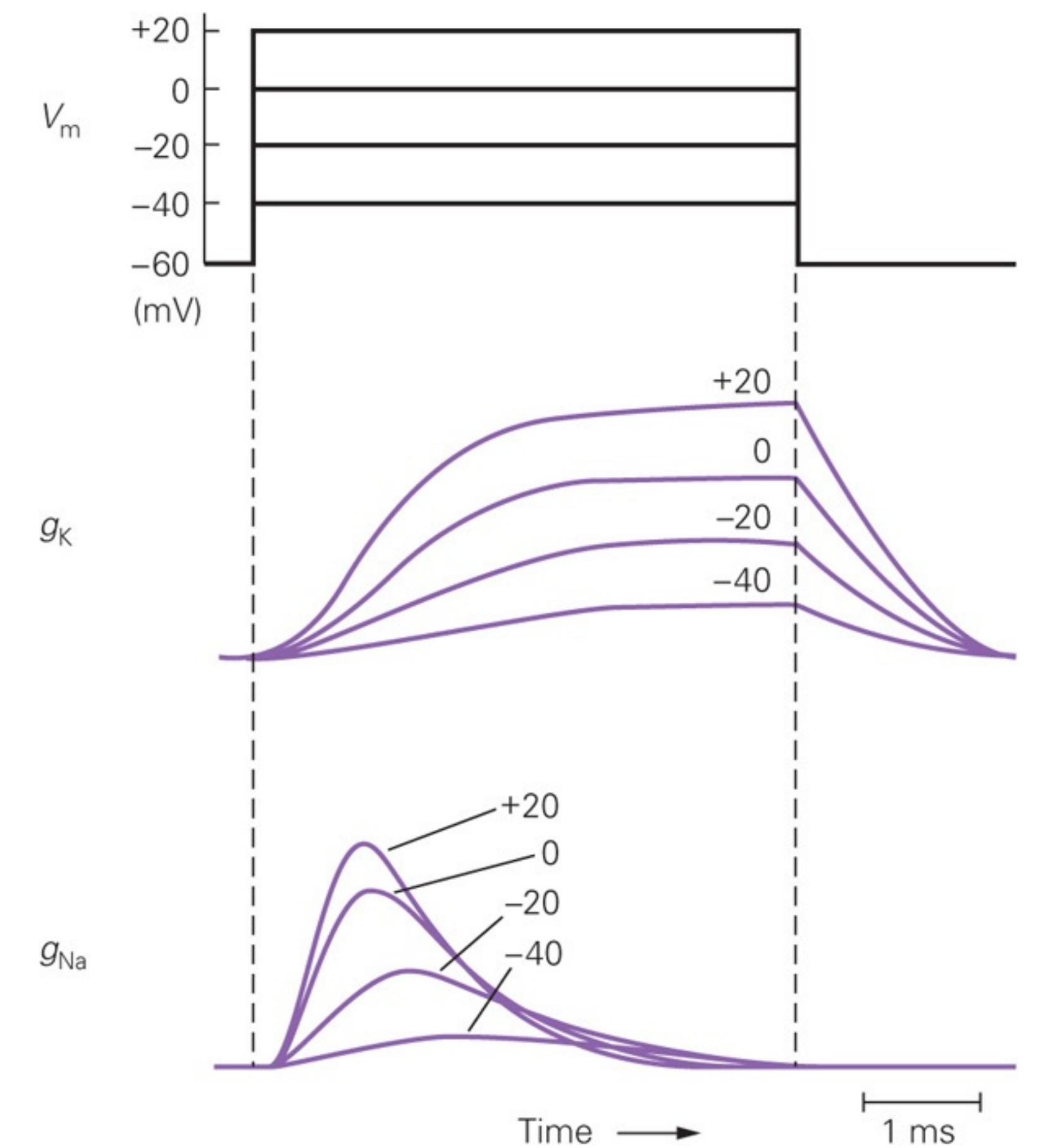
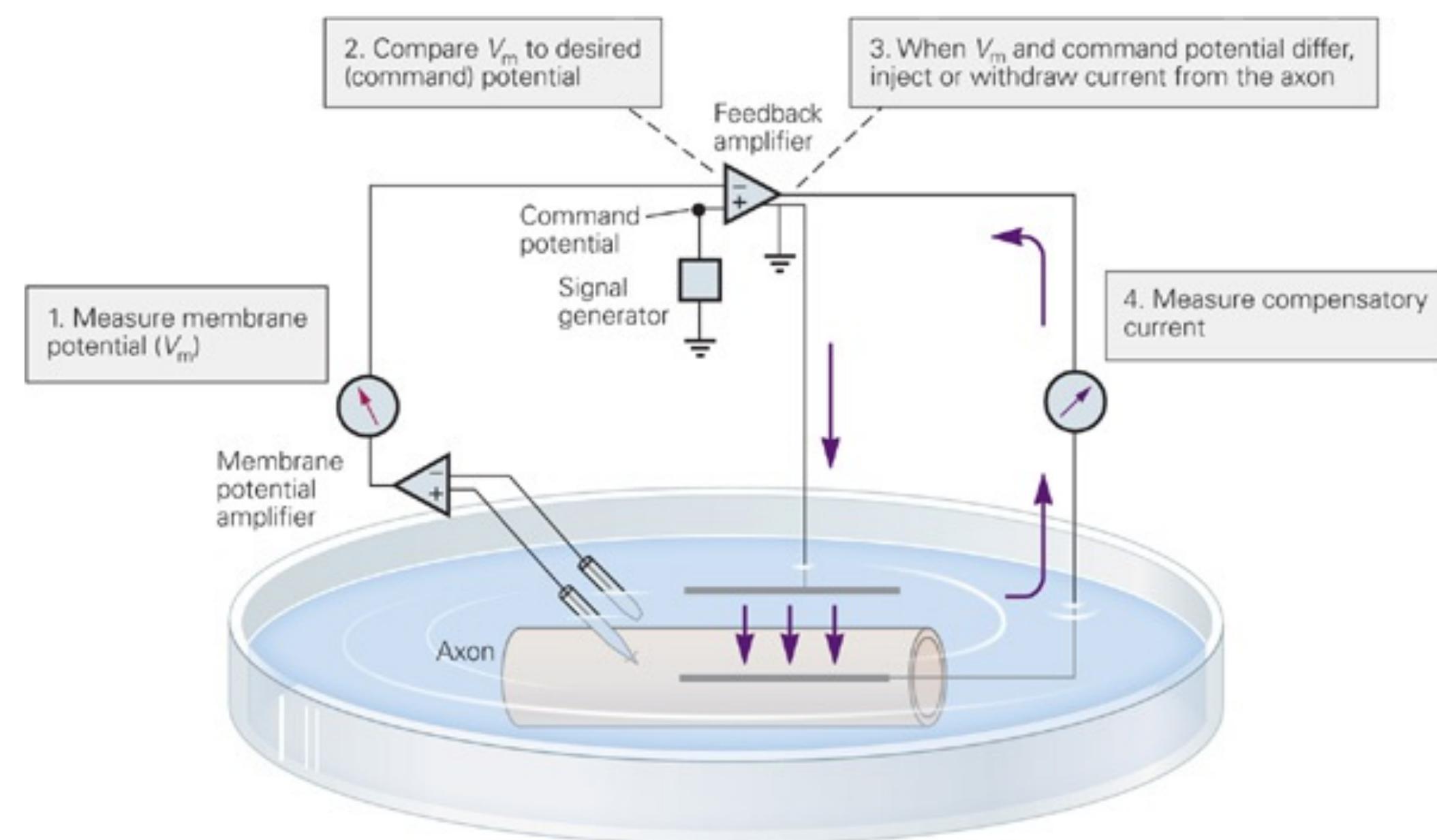
静息电位



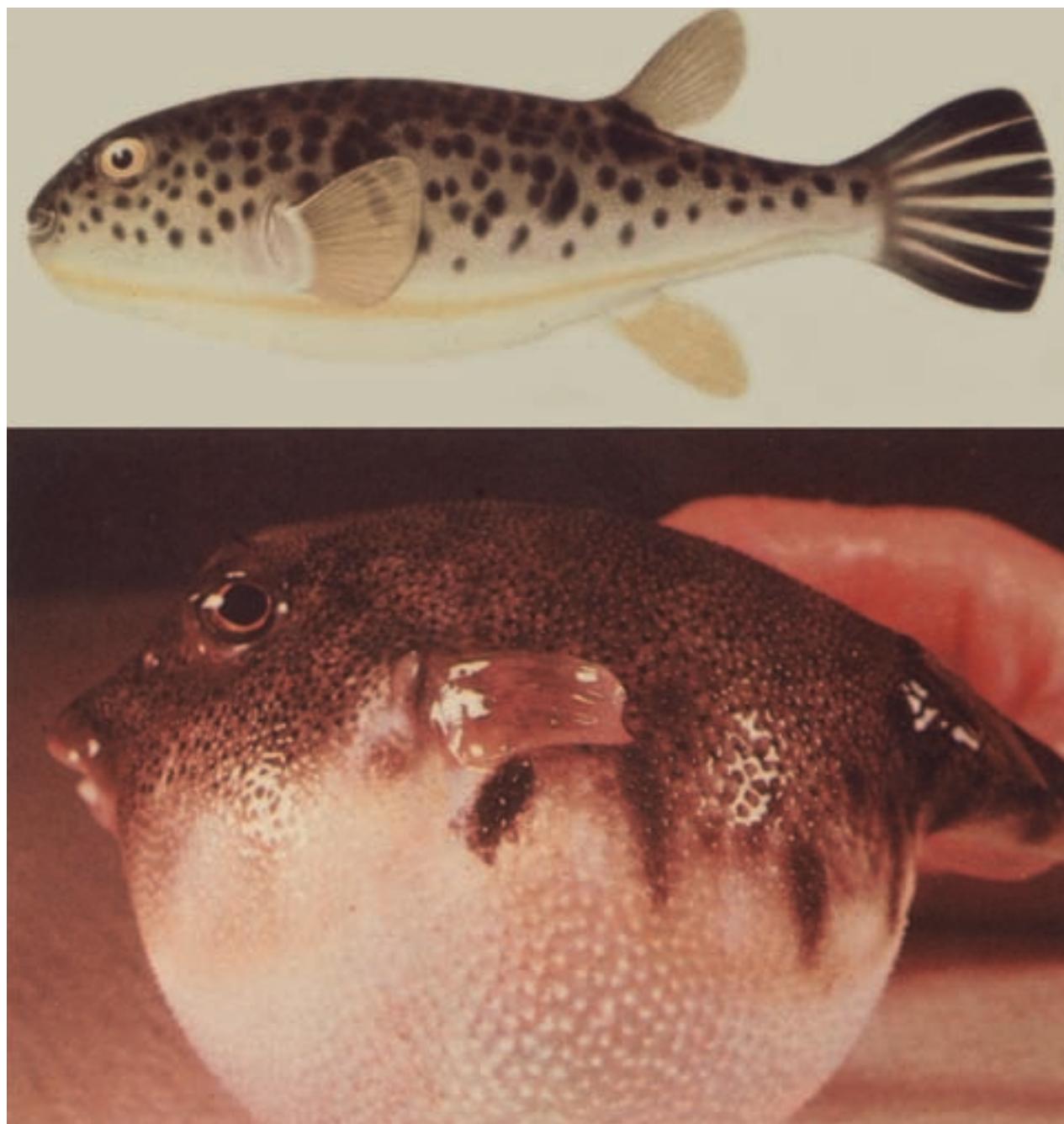
# Voltage-gate Na<sup>+</sup> Channel



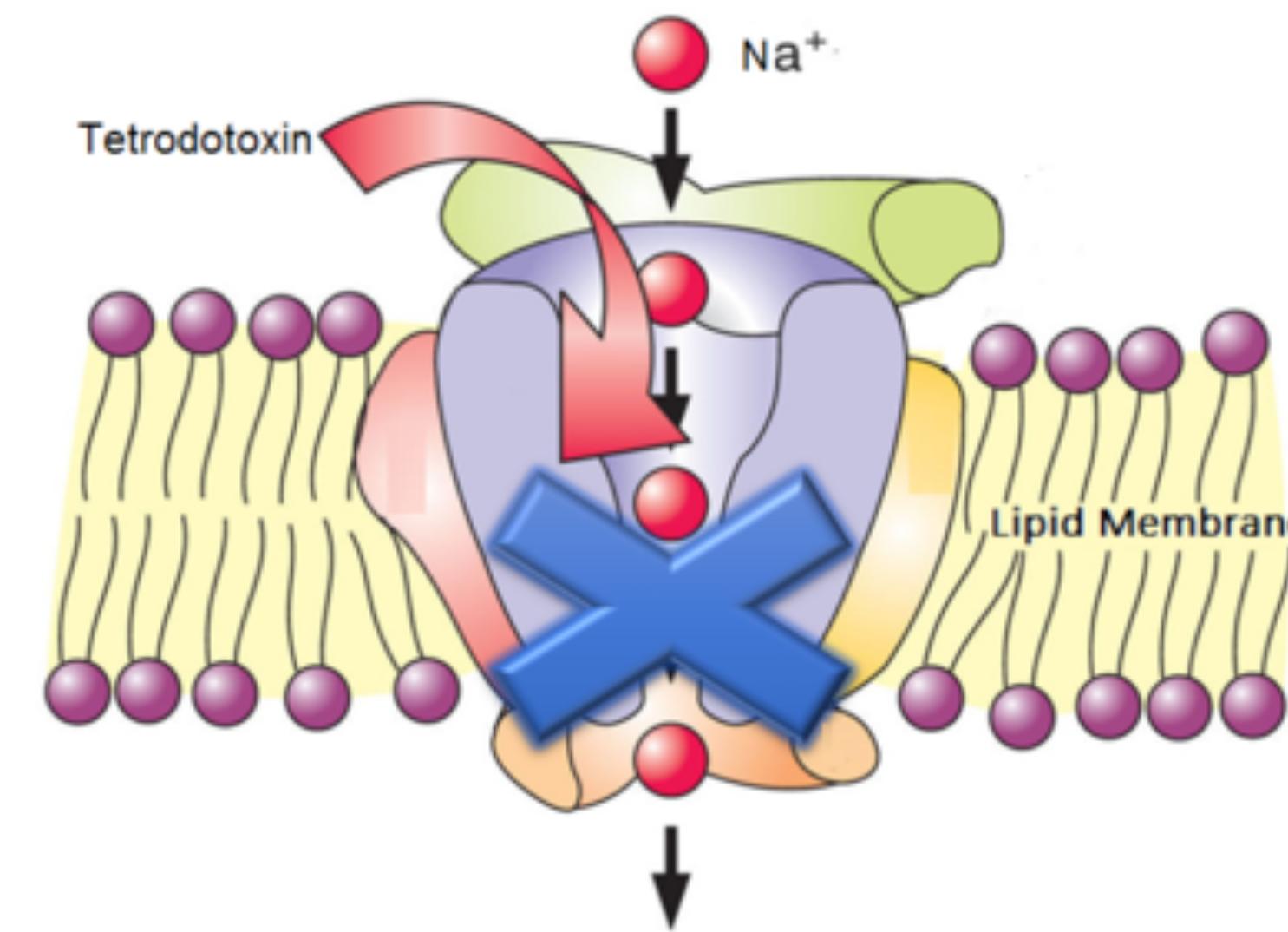
# Voltage Clamp



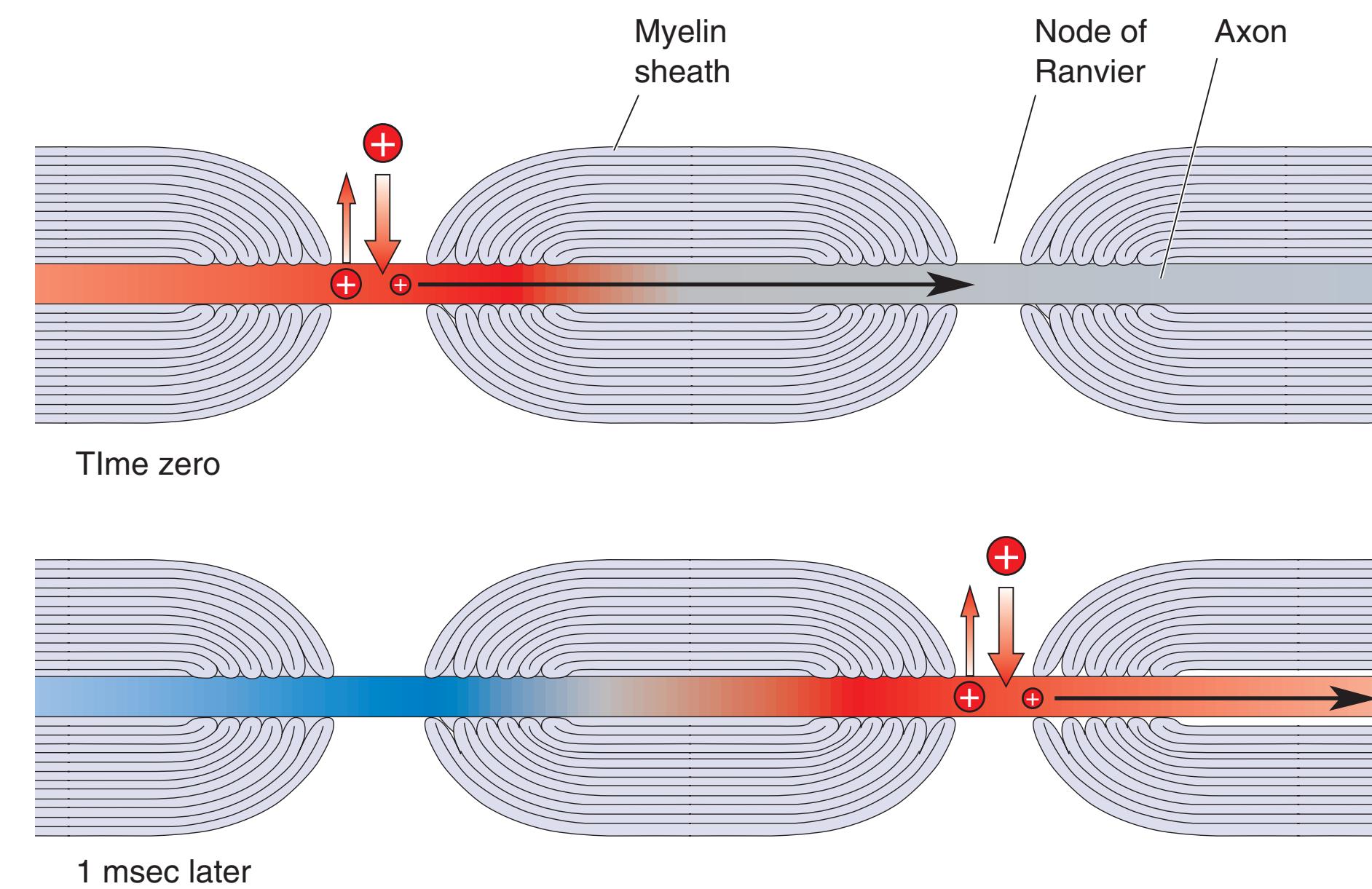
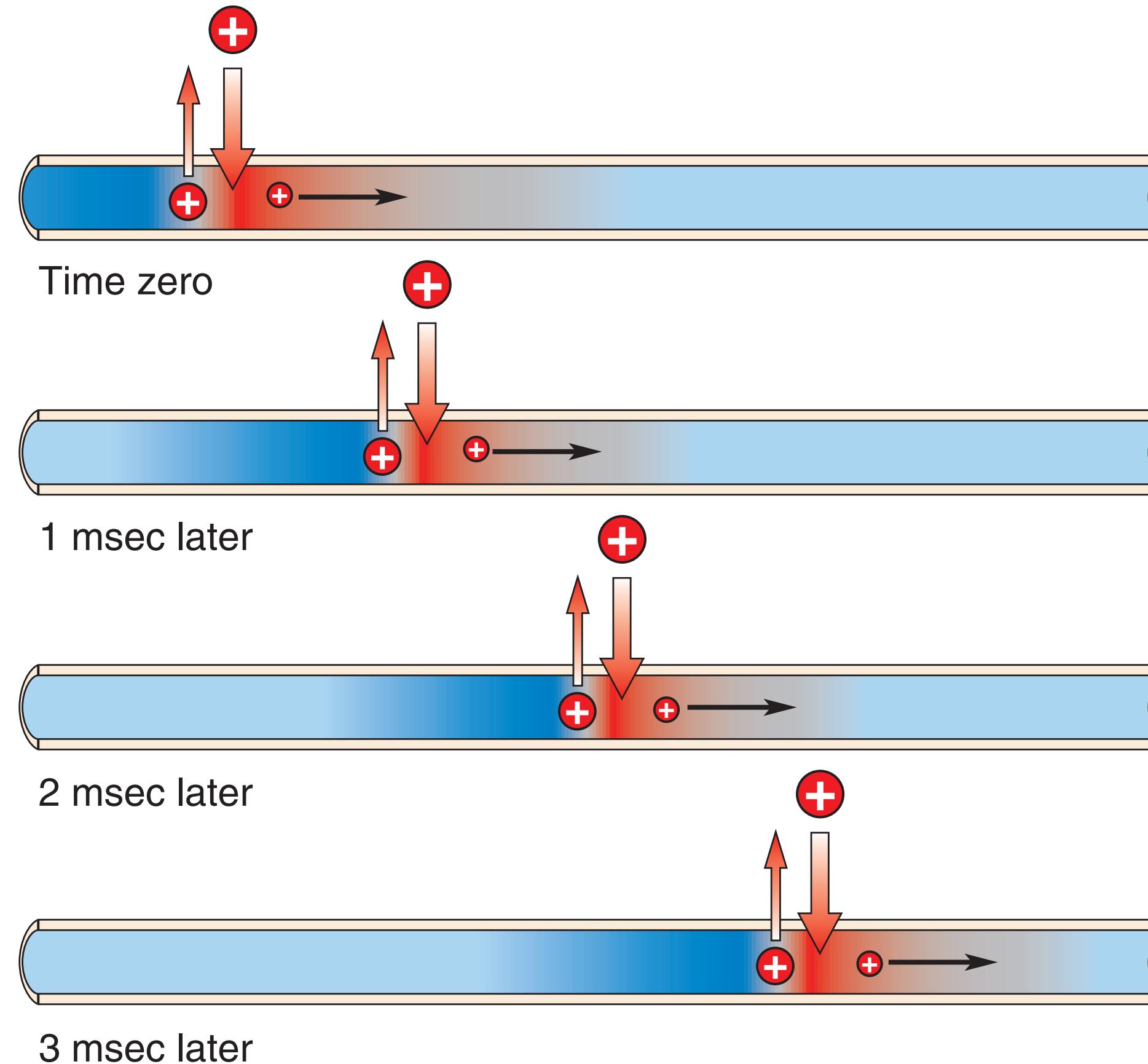
# TTX



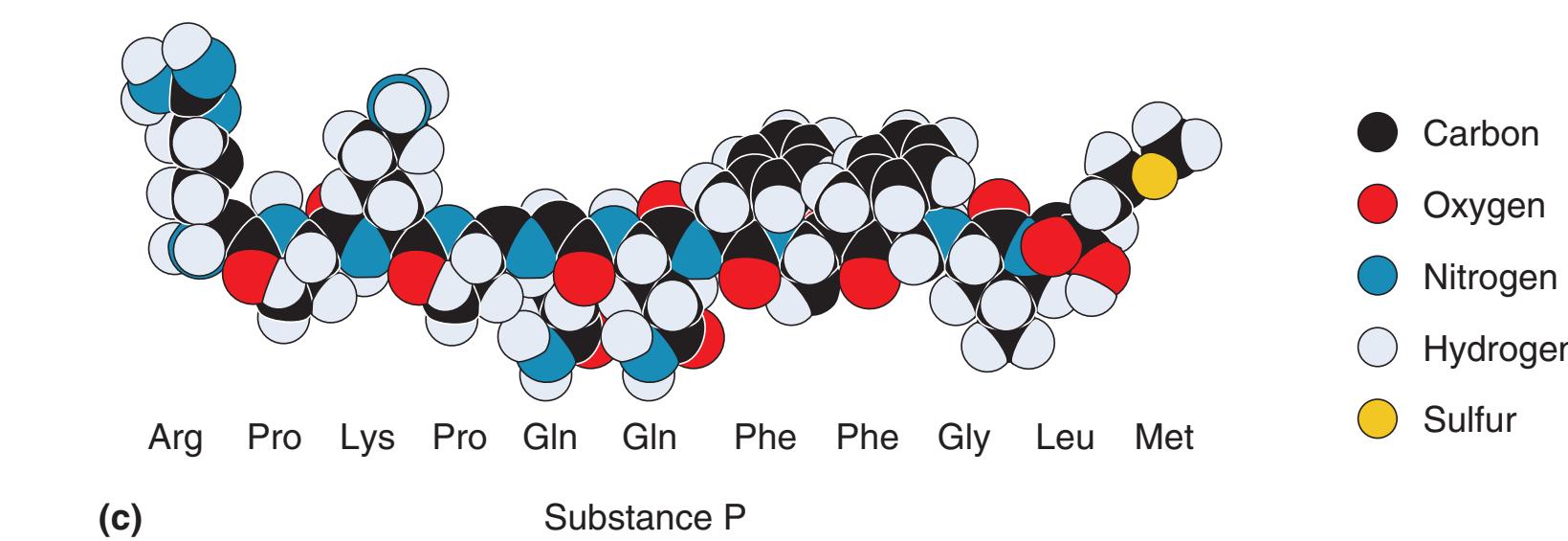
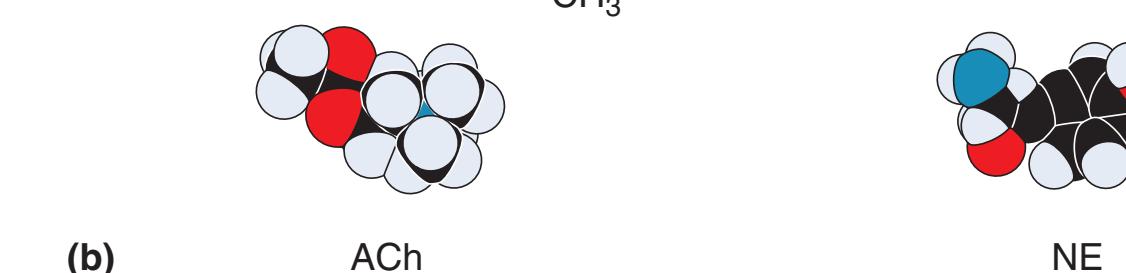
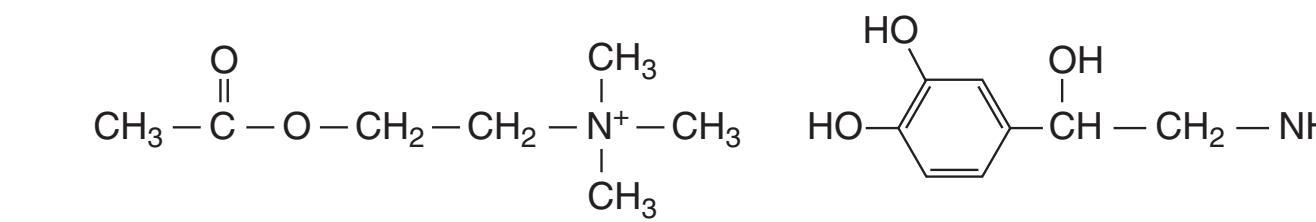
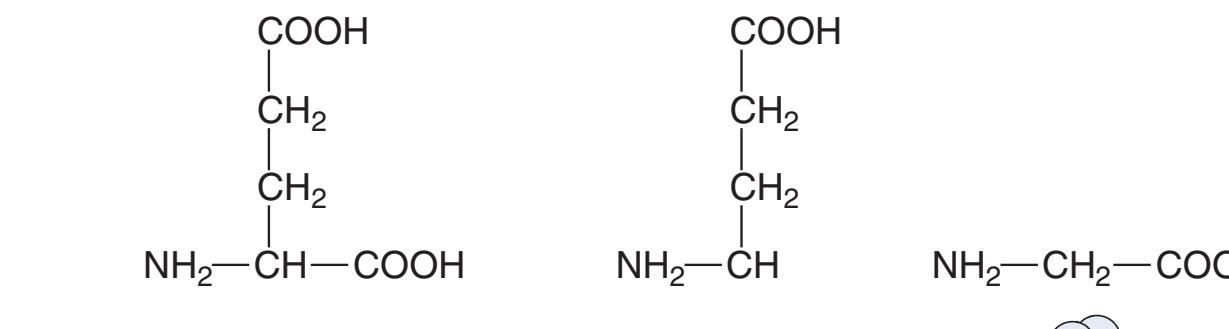
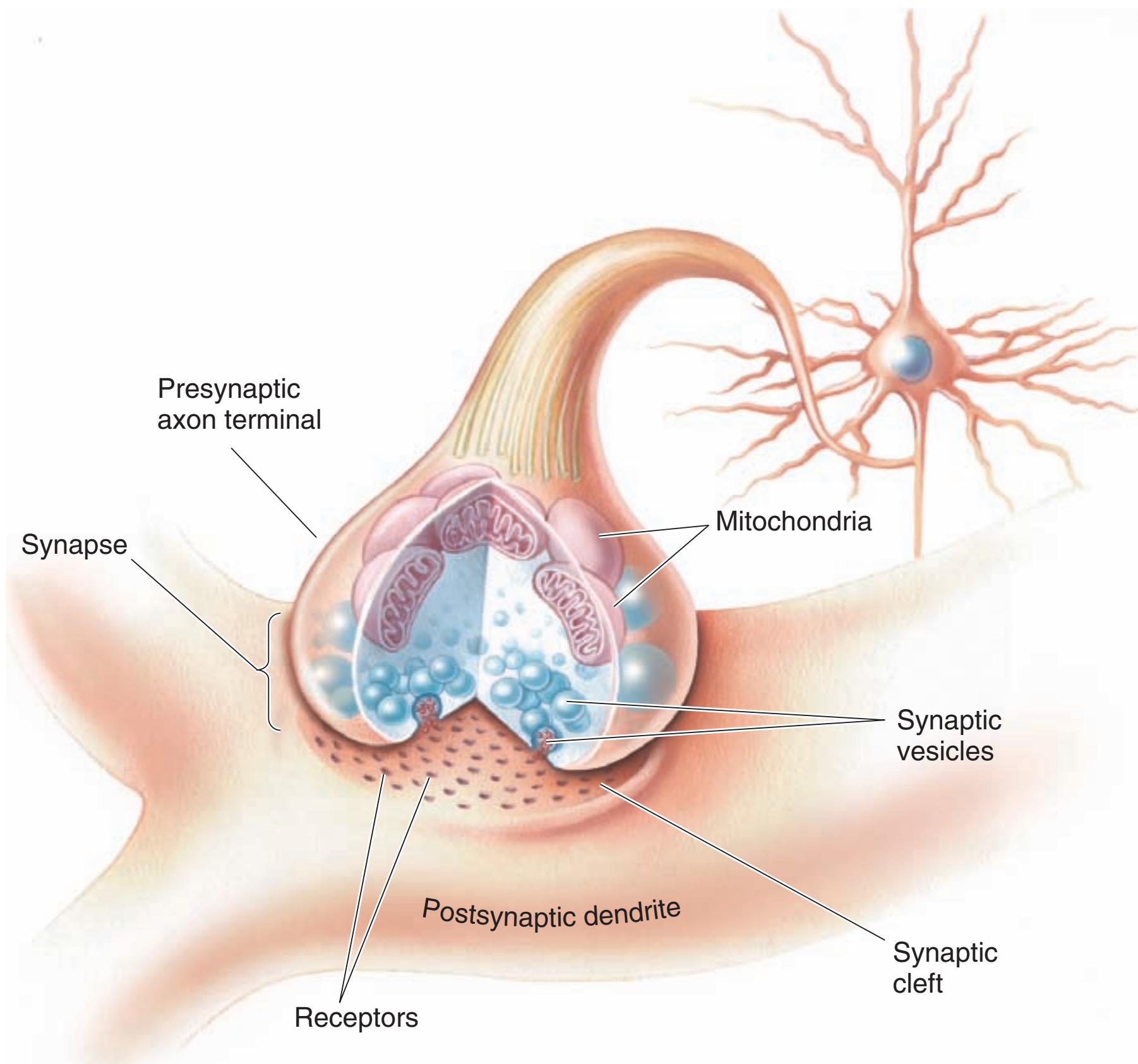
河豚



# Propagation of Action Potentials

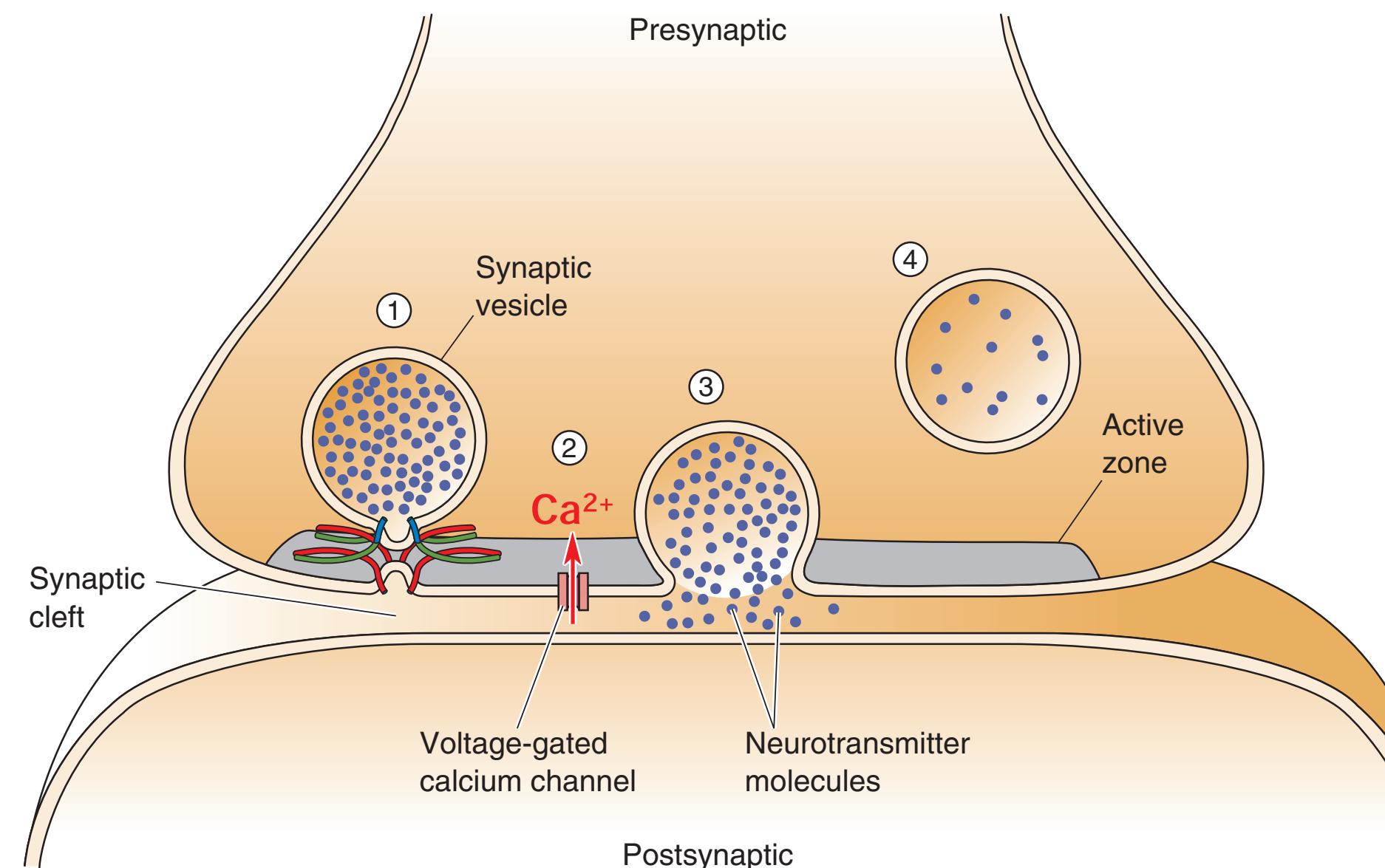


# Synapse



# Pre-synaptic reactions

Axon



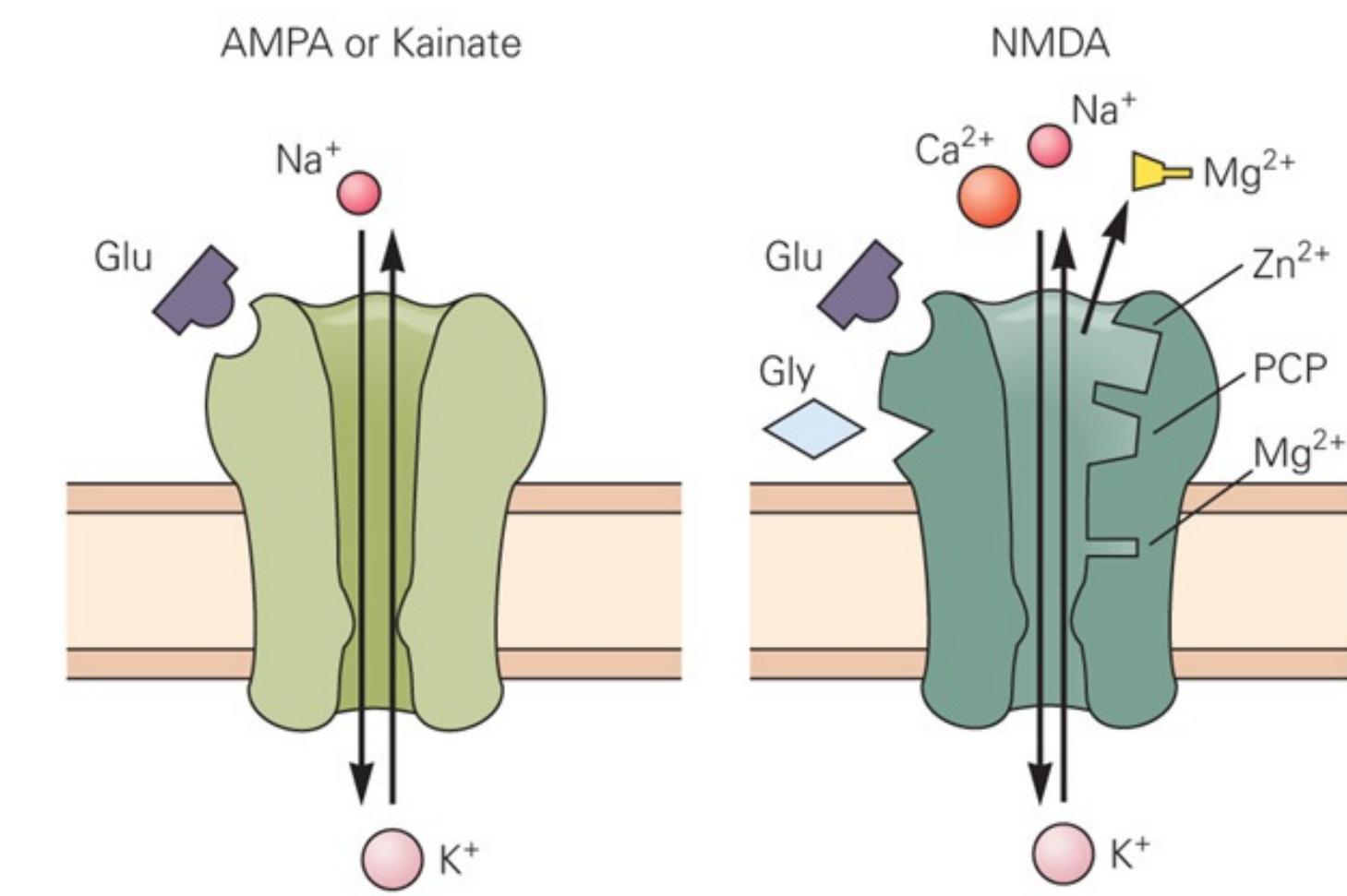
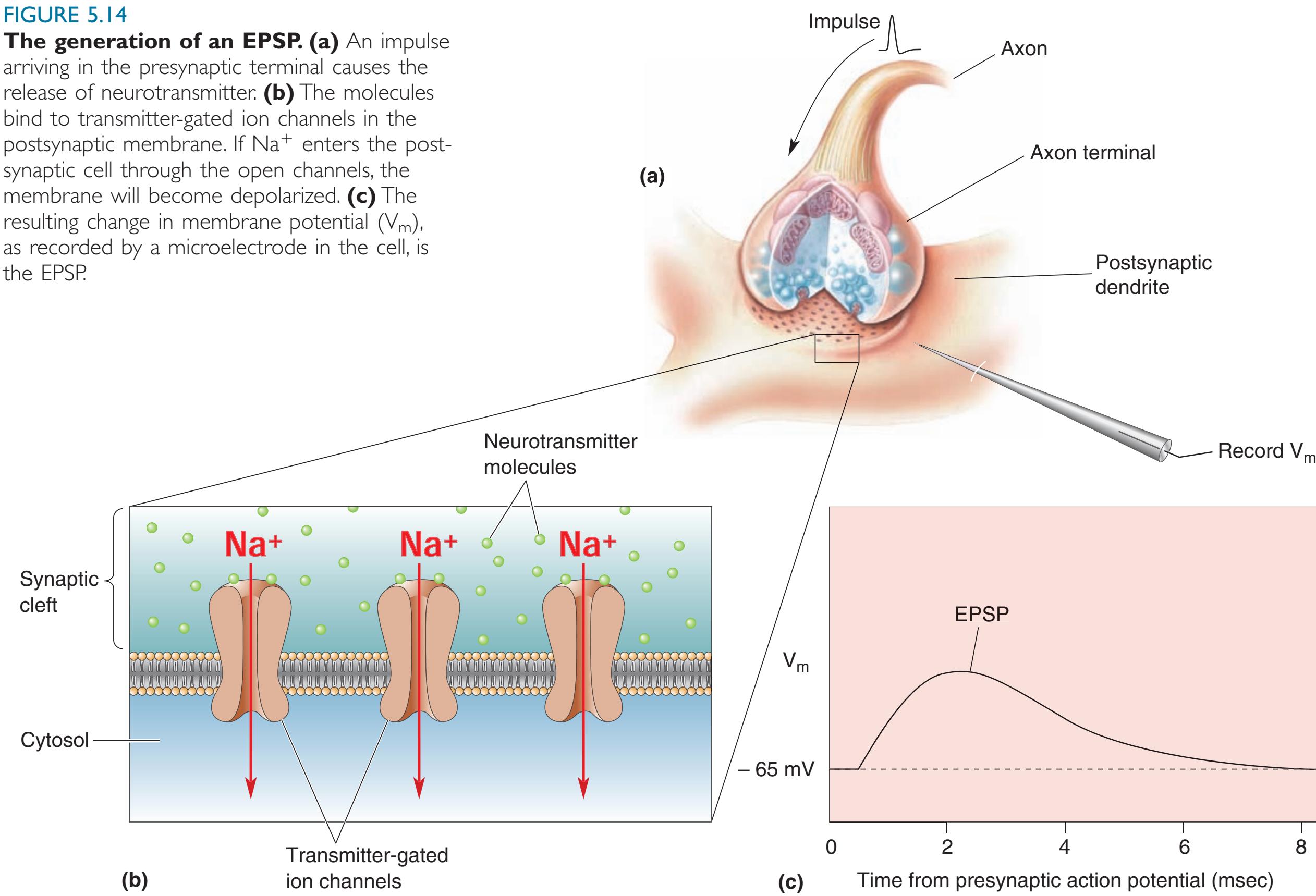
1. Action potential arrives
2. Voltage-gated  $\text{Ca}^+$  channel opens
3. Synaptic vesicle fuses with membrane
4. Neurotransmitter molecules are released

Dendrite / Soma

# Post-synaptic reactions (excitatory)

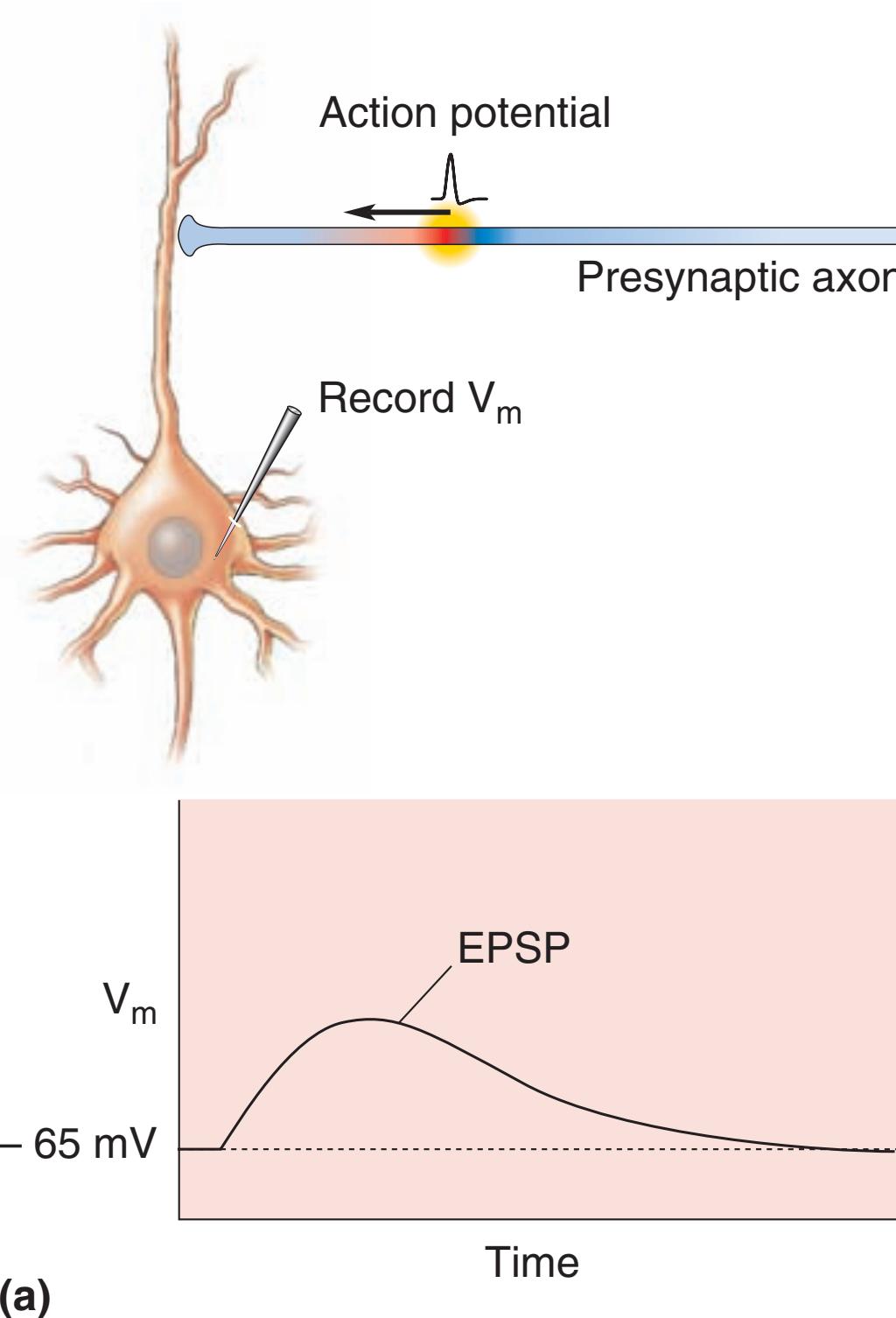
FIGURE 5.14

**The generation of an EPSP.** (a) An impulse arriving in the presynaptic terminal causes the release of neurotransmitter. (b) The molecules bind to transmitter-gated ion channels in the postsynaptic membrane. If  $\text{Na}^+$  enters the post-synaptic cell through the open channels, the membrane will become depolarized. (c) The resulting change in membrane potential ( $V_m$ ), as recorded by a microelectrode in the cell, is the EPSP.



Glutamate  
谷氨酸

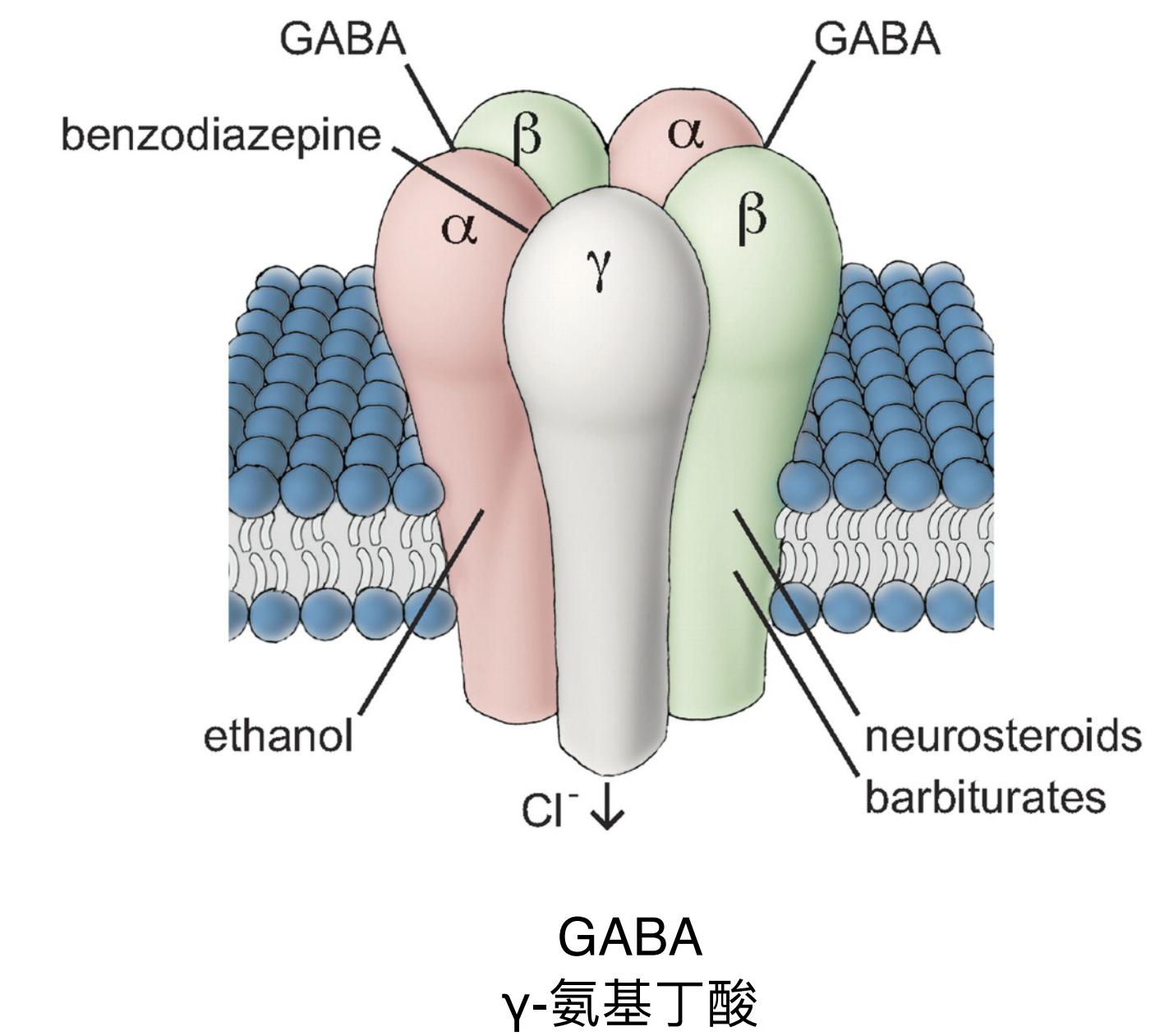
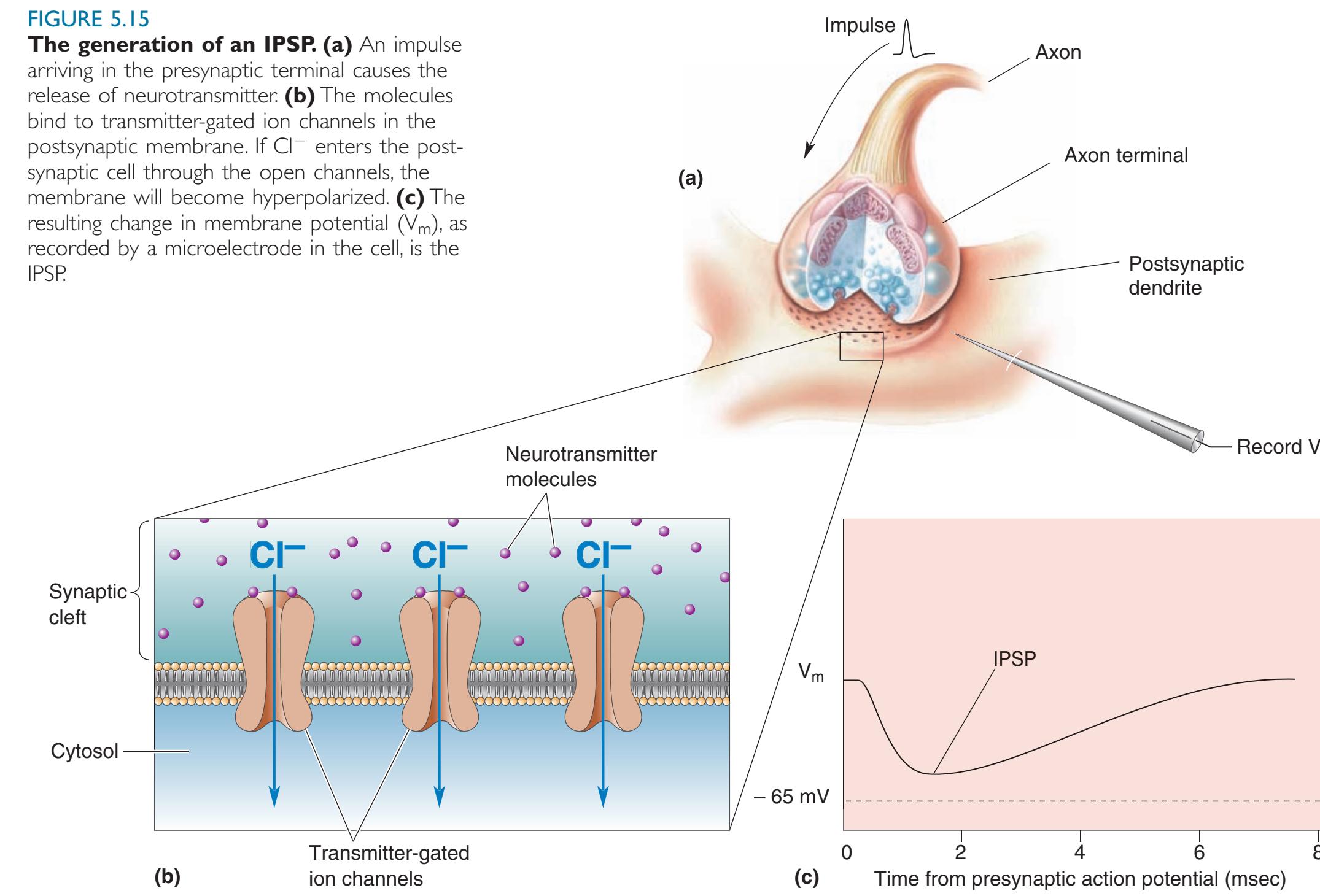
# Post-synaptic reactions (excitatory)



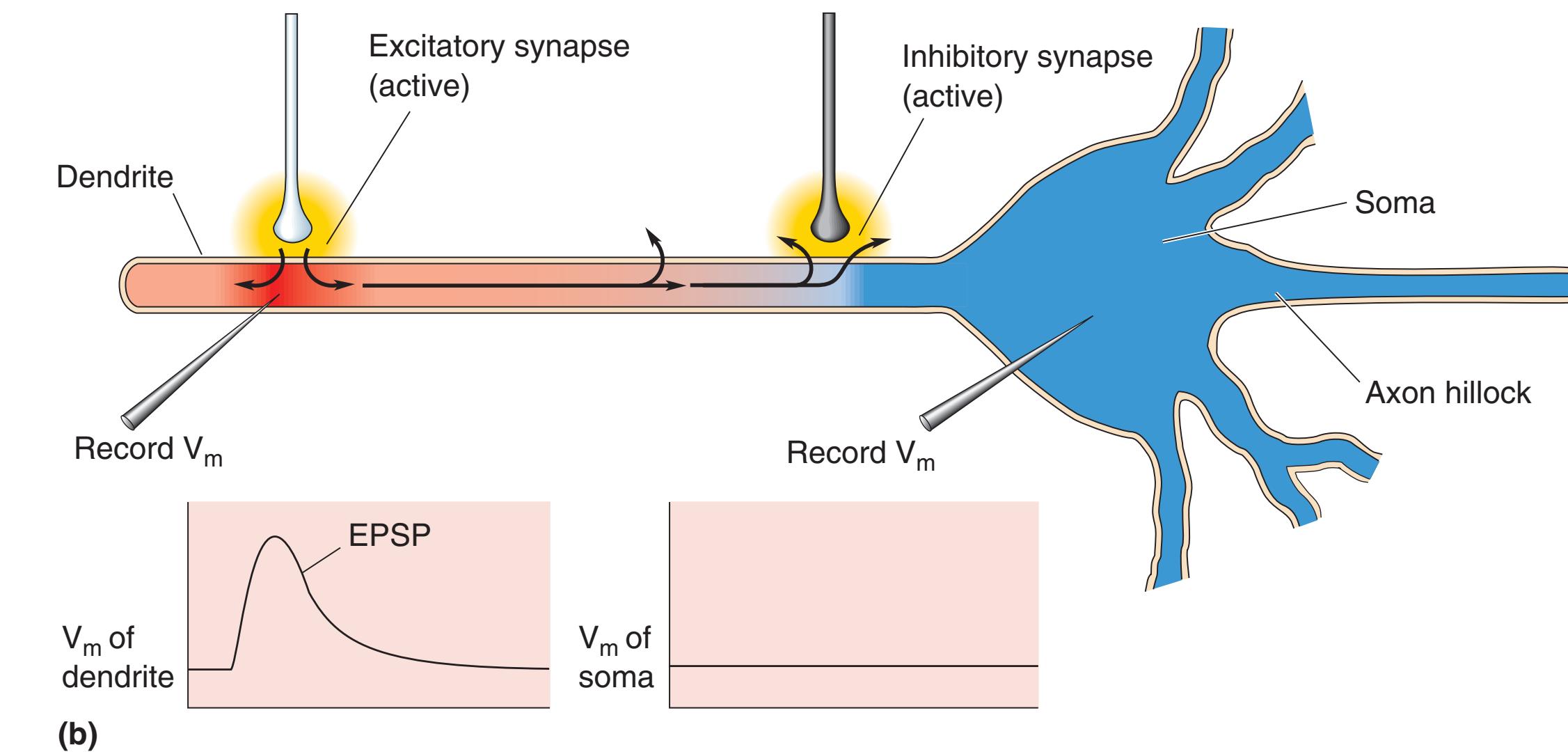
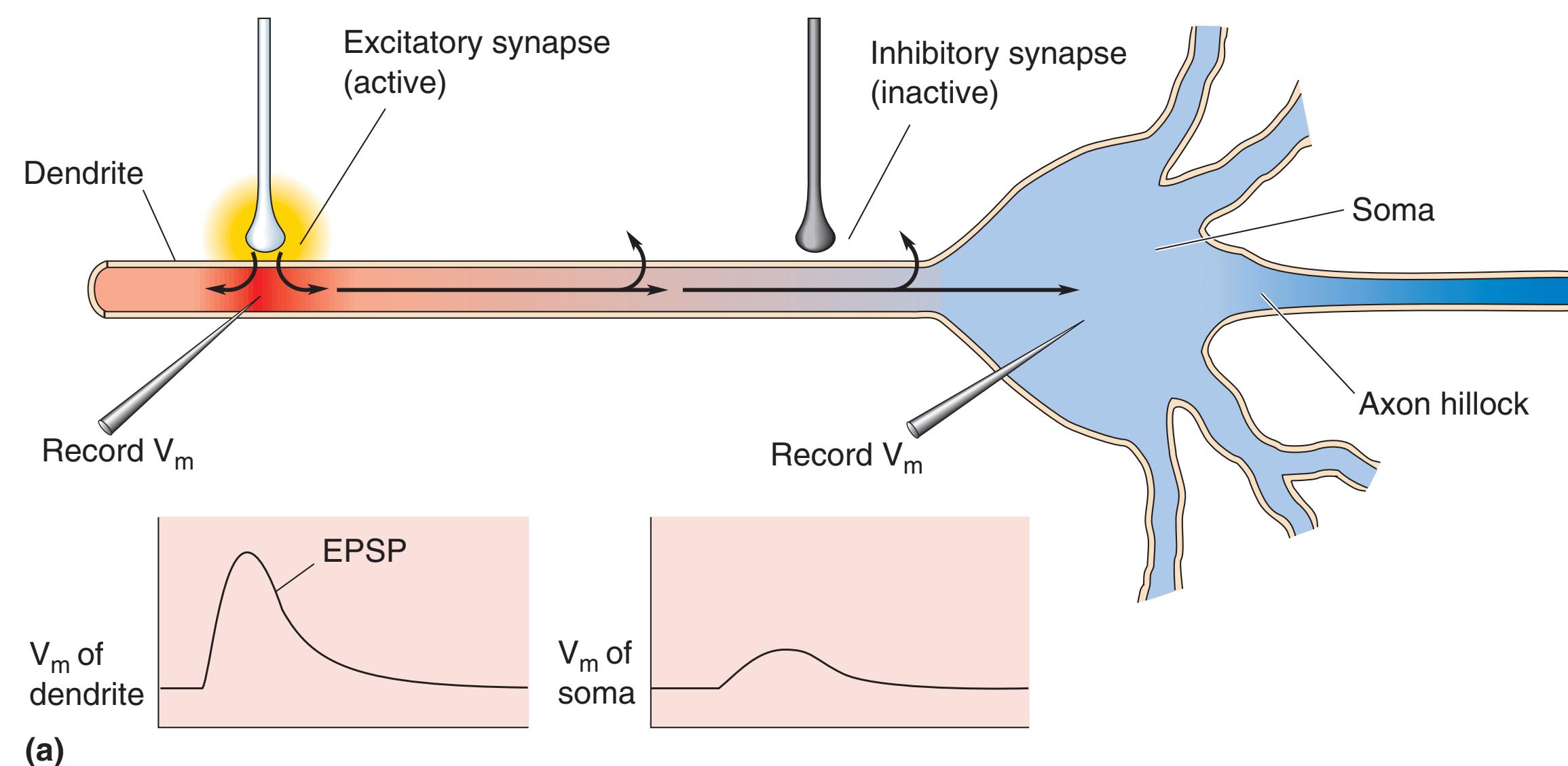
# Post-synaptic reactions (inhibitory)

**FIGURE 5.15**

**The generation of an IPSP.** (a) An impulse arriving in the presynaptic terminal causes the release of neurotransmitter. (b) The molecules bind to transmitter-gated ion channels in the postsynaptic membrane. If  $\text{Cl}^-$  enters the postsynaptic cell through the open channels, the membrane will become hyperpolarized. (c) The resulting change in membrane potential ( $V_m$ ), as recorded by a microelectrode in the cell, is the IPSP.



# Shunting Inhibition

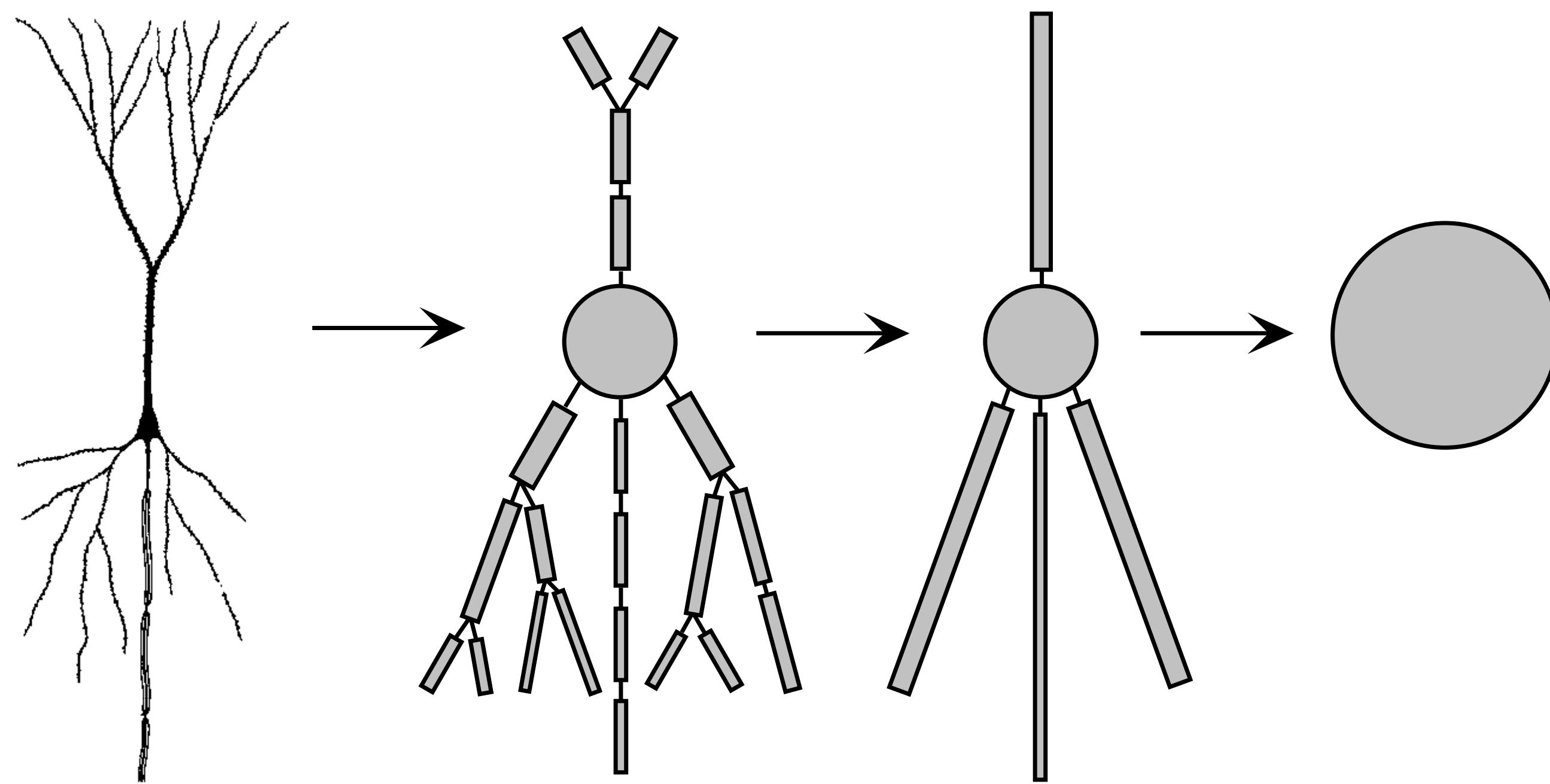


## 1. 膜电位和动作电位

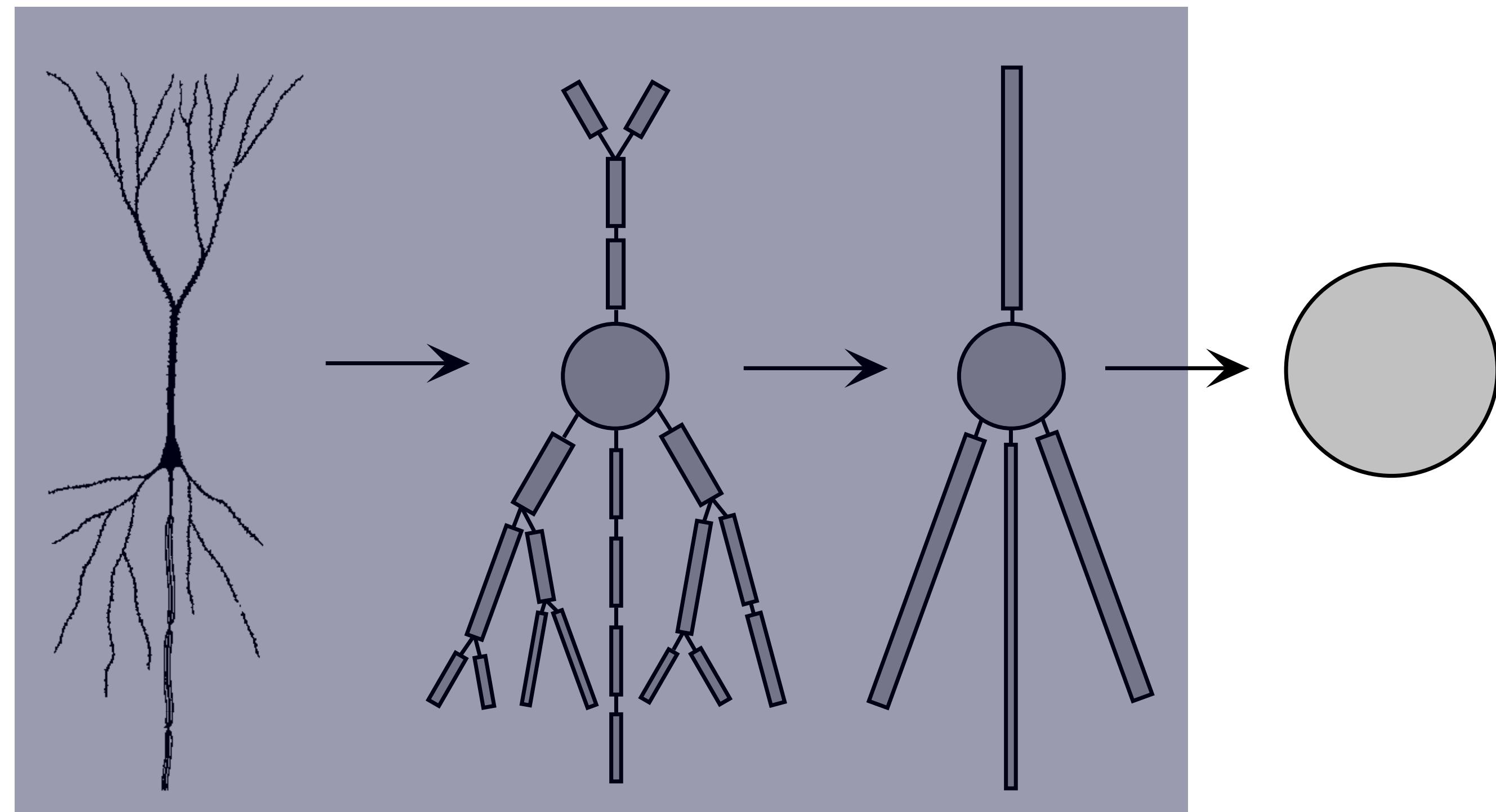
## 2. 数学模型



# Approximating a neuron



# Integrate-and-Fire Model



$$C_m \frac{dV}{dt} = -g_L(V - E_L) + I_e$$

$$V = V_{reset}$$

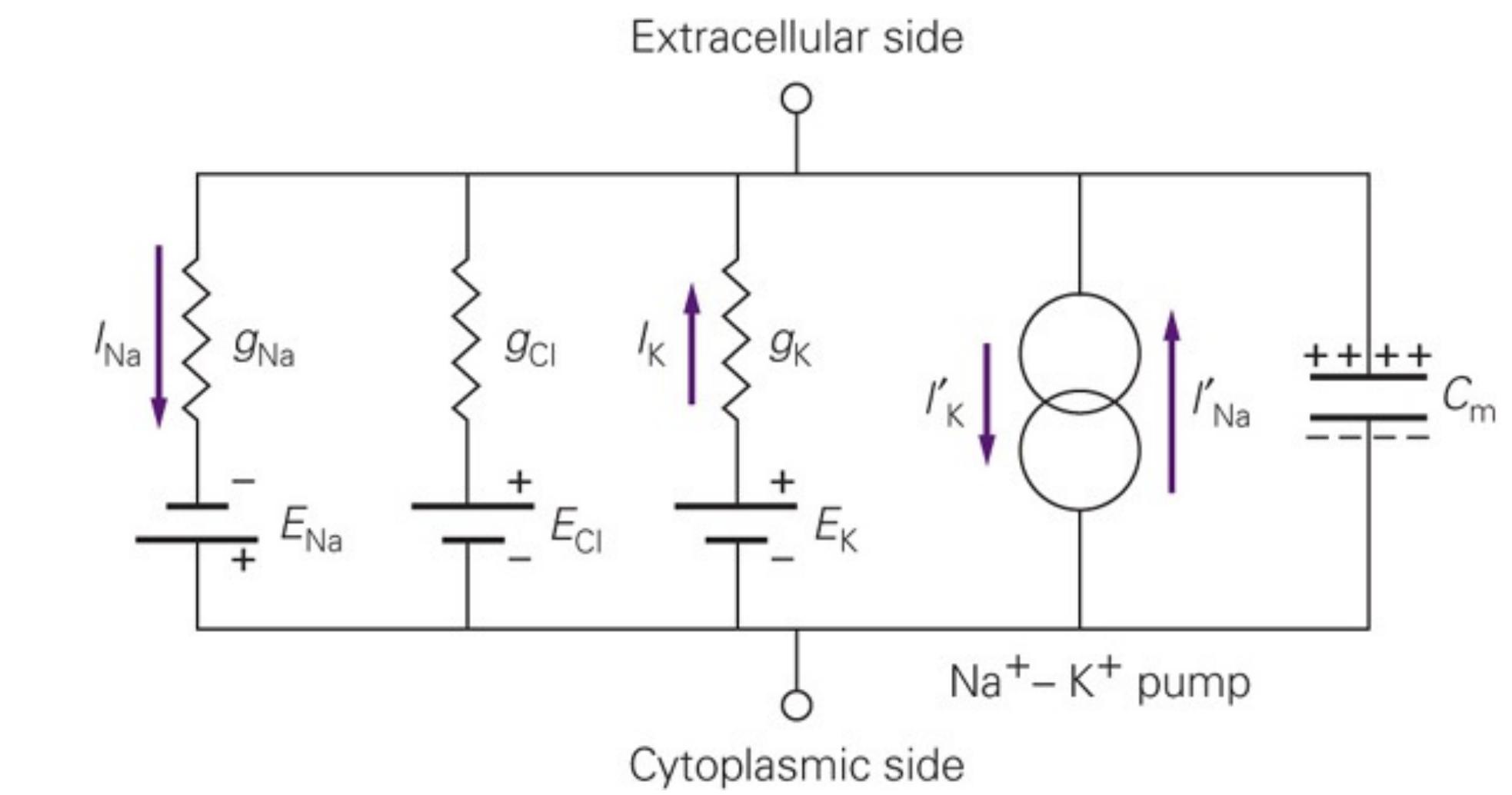
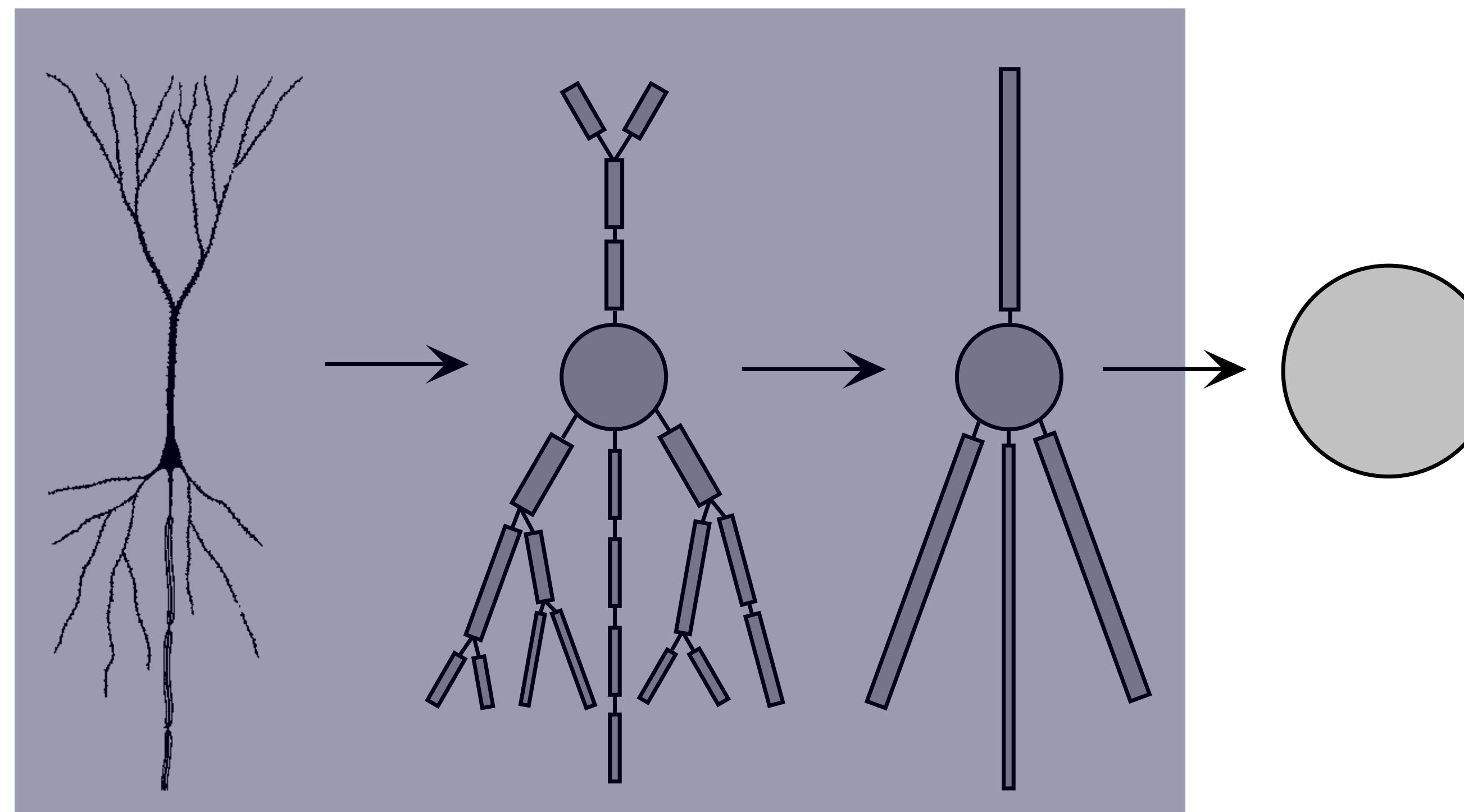
$$\frac{dV}{dt} = 0$$

$$V < V_{th}$$

$V \geq V_{th}$  Spike!

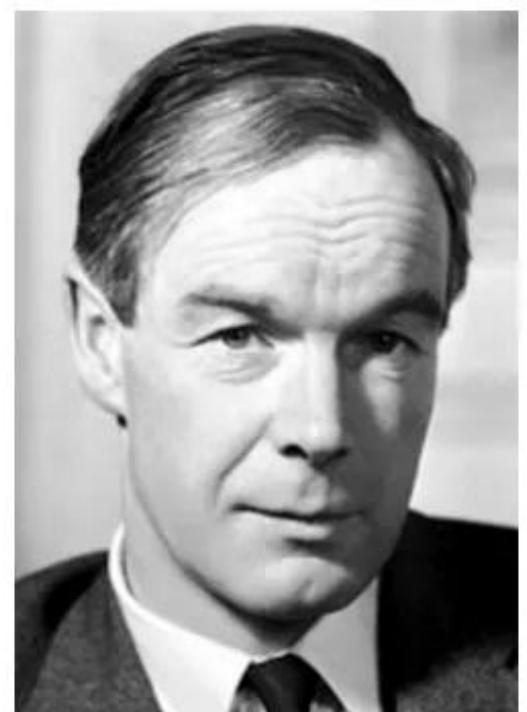
Refractory period

# Single Compartment Model



# Hodgkin-Huxley Model

$$C \frac{dV}{dt} = - g_L(V - E_L) - g_K n^4 (V - E_k) - g_{Na} m^3 h (V - E_{Na}) - I_E$$



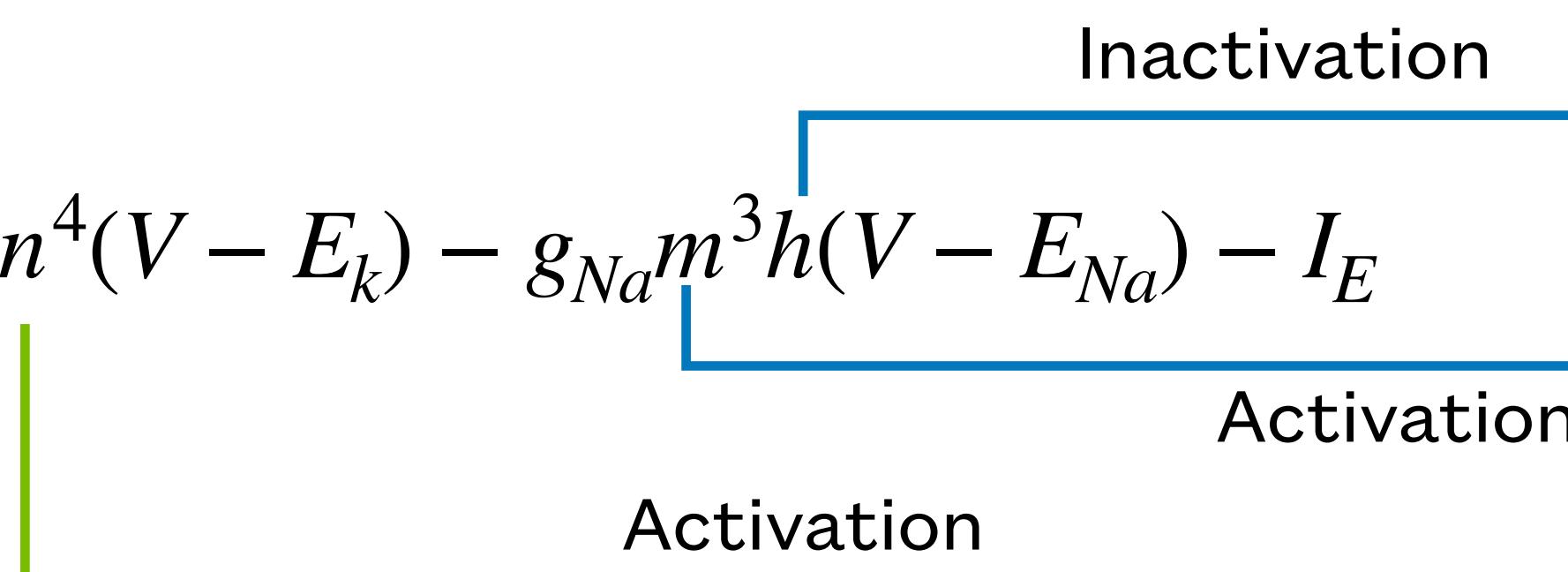
Alan Lloyd  
Hodgkin



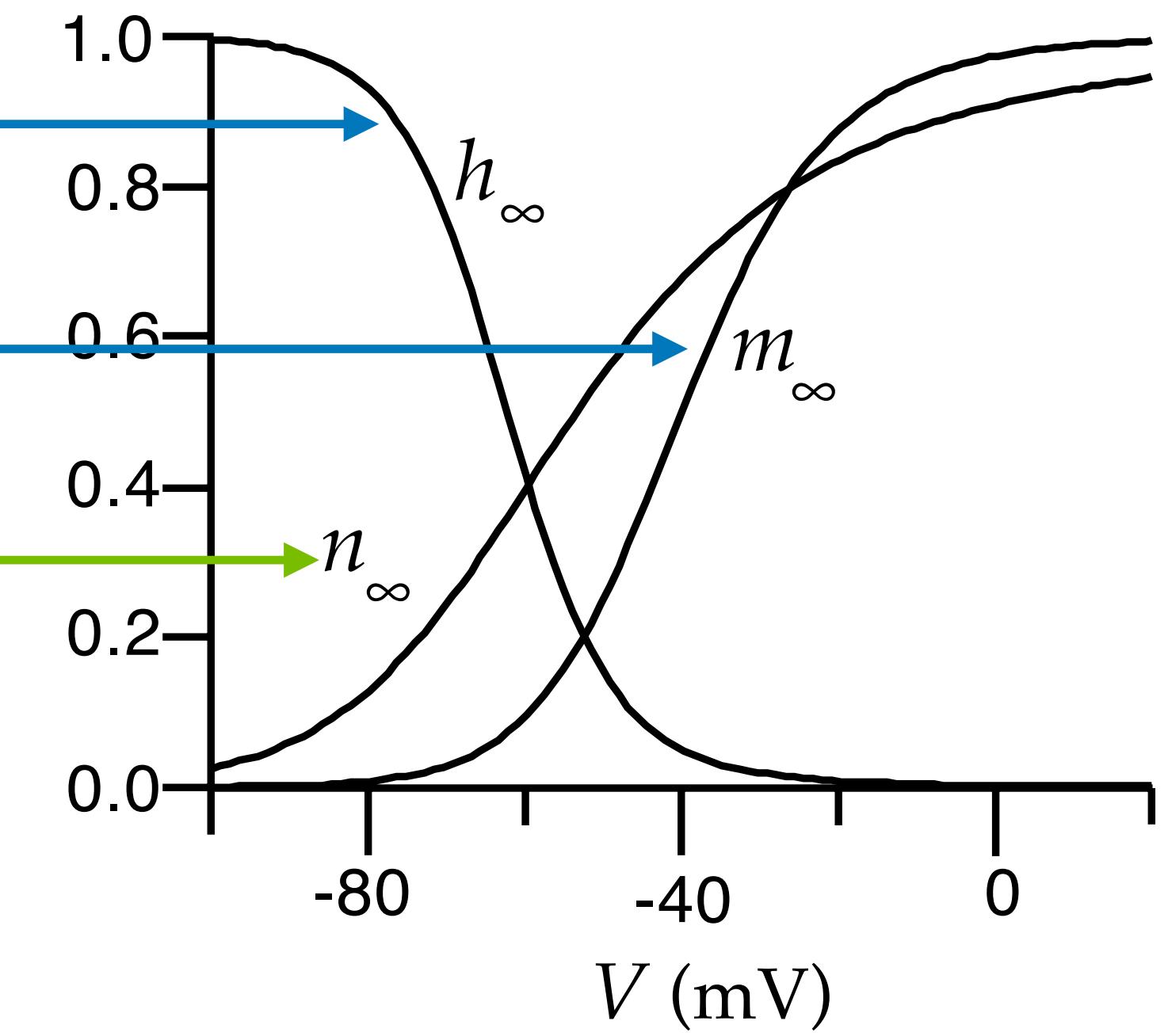
Andrew Fielding  
Huxley

Opening Rate  $\downarrow$   $\frac{dn}{dt} = \alpha_n(V)(1 - n) - \beta_n(V)n$

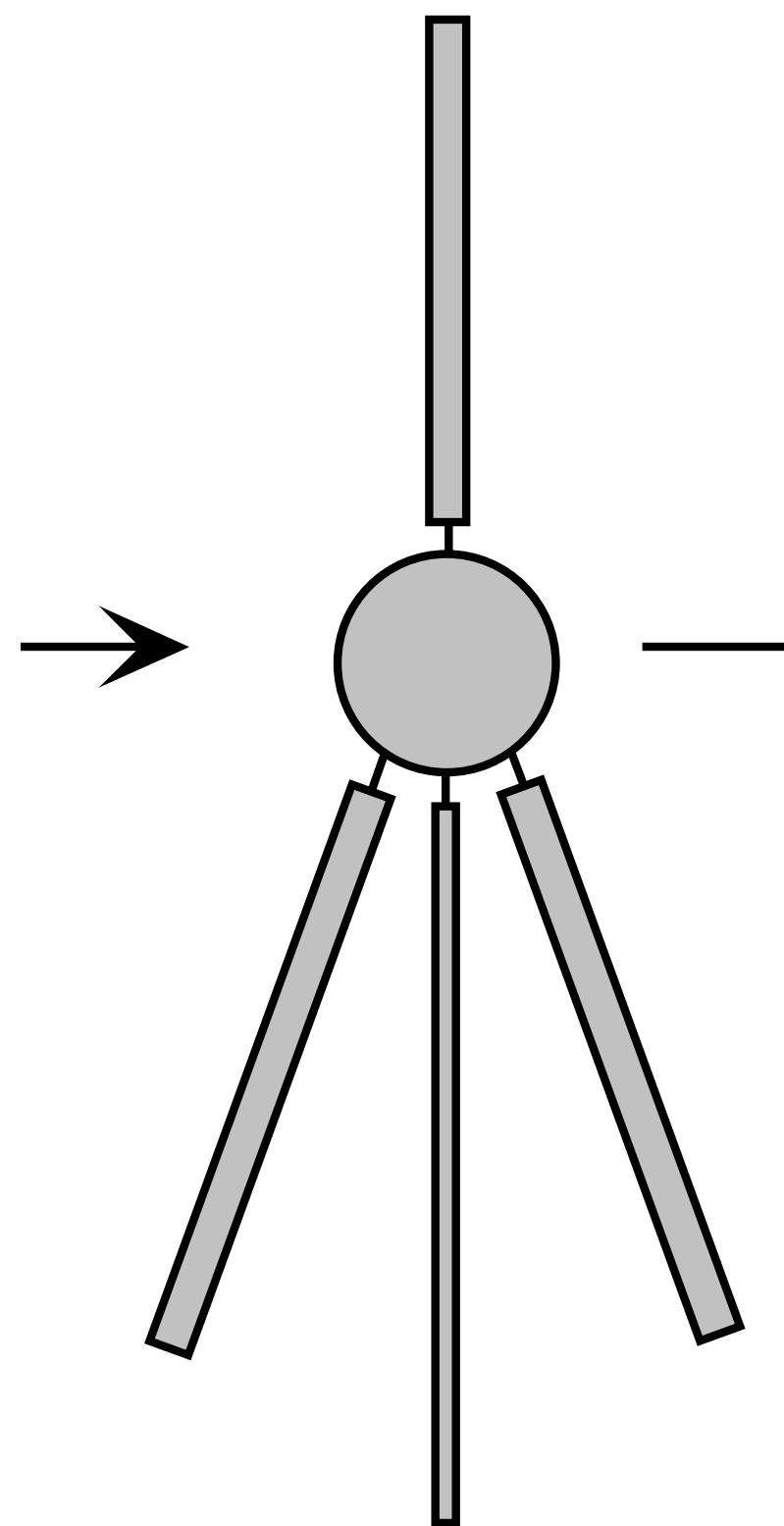
Closing Rate  $\downarrow$



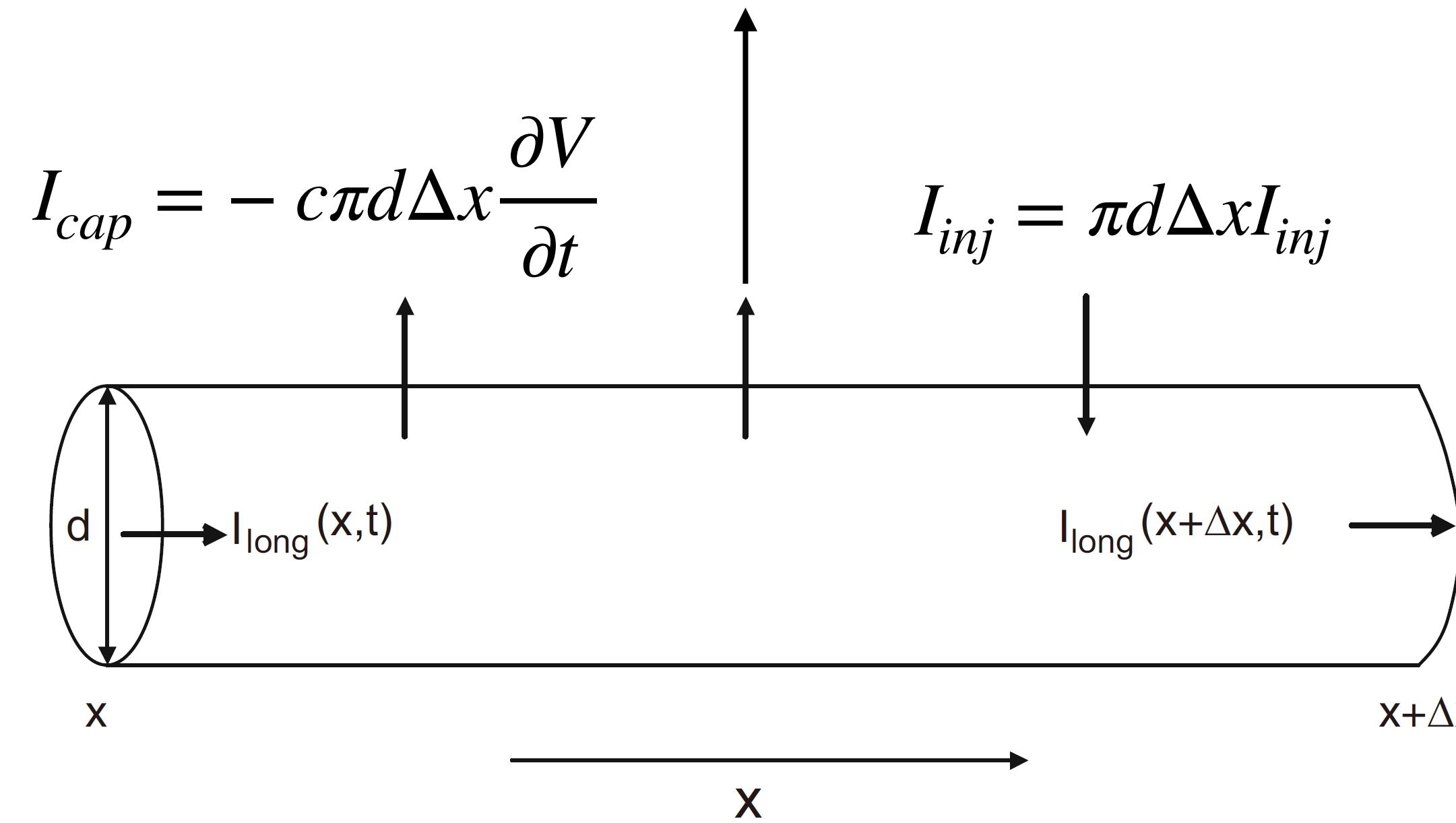
Gating Variables



# Cable Theory



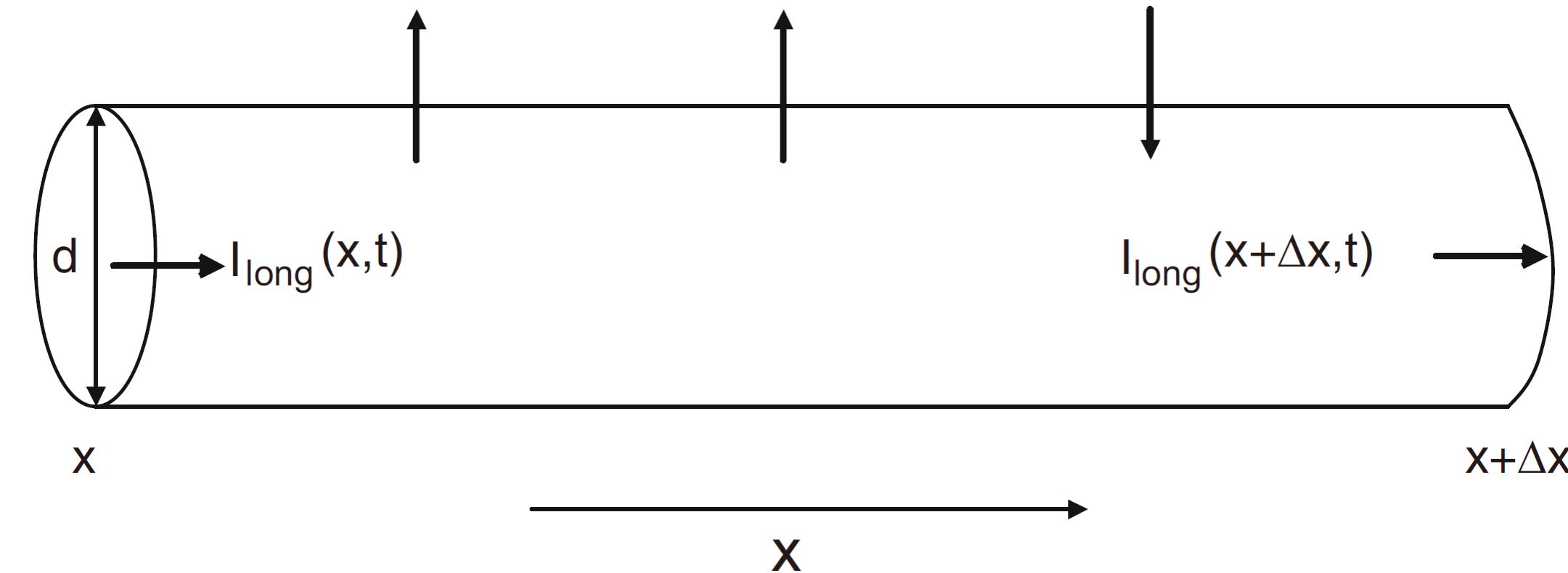
$$I_{leak} = -g_L \pi d \Delta x (E_L - V)$$



$$I_{long}(x) = -\frac{\pi d^2}{4r_a} \frac{\partial V}{\partial x}$$

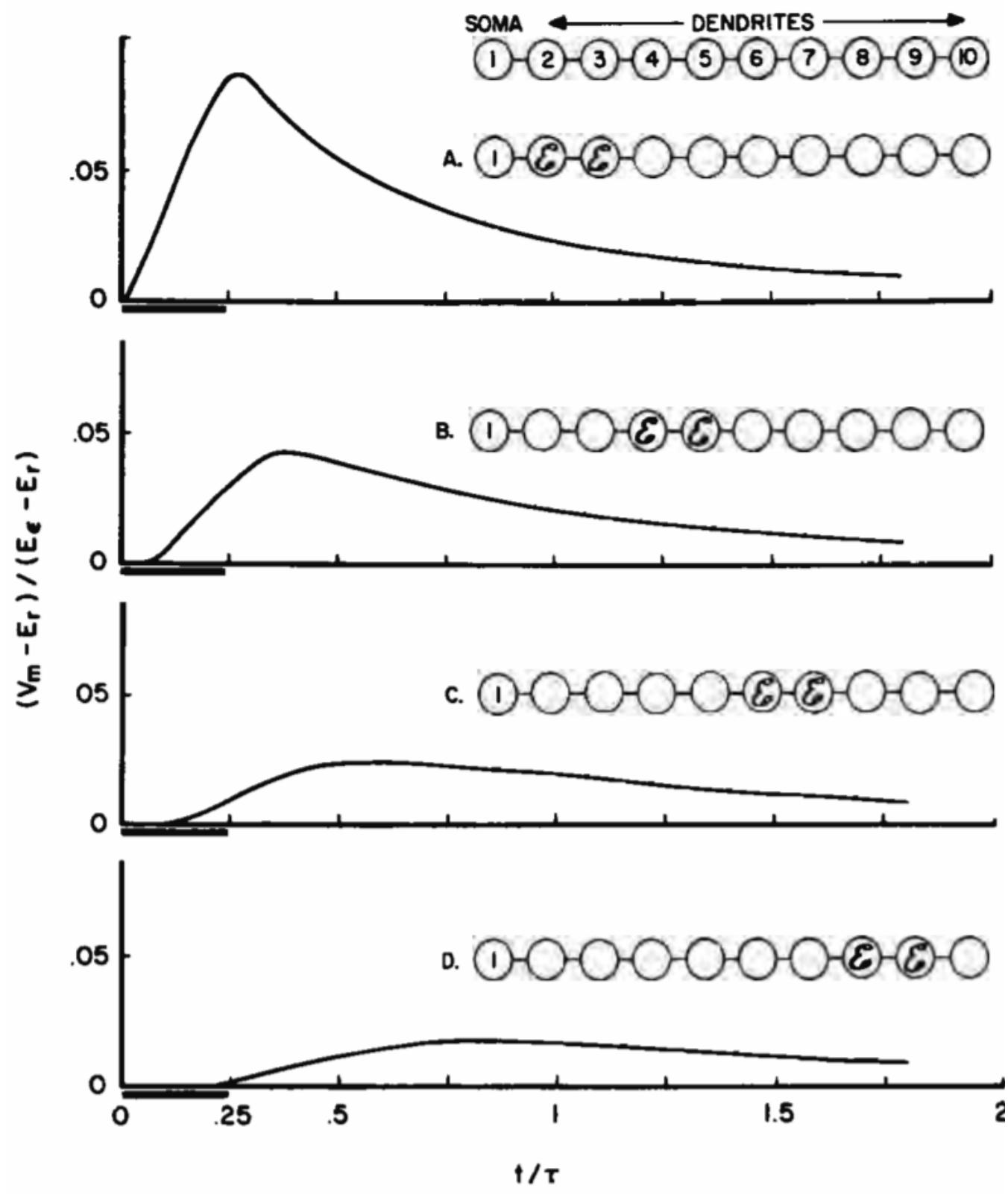
↑  
resistance

# Cable Theory



$$\frac{\pi d^2}{4r_a} \frac{\partial V}{\partial x} \Bigg|_{x+\Delta x} - \frac{\pi d^2}{4r_a} \frac{\partial V}{\partial x} \Bigg|_x = -c\pi d\Delta x \frac{\partial V}{\partial t} - g_L \pi d\Delta x (E_L - V) + \pi d\Delta x I_{\text{inj}}$$

$$c \frac{\partial V}{\partial t} = -g_L(V - E_L) + I_{\text{inj}} + \frac{d}{4r_a} \frac{\partial^2 V}{\partial x^2}$$



$$c \frac{\partial V}{\partial t} = -g_L(V - E_L) + I_{inj} + \frac{d}{4r_a} \frac{\partial^2 V}{\partial x^2}$$

↑  
Synaptic input

**Rall, 1964**

# Summary

- Membrane Potential
  - Ion pumps
- Action Potentials
  - Voltage-gated Ion channels
- Modeling
  - Single-compartment models: I&F, H&H
  - Cable theory

# Homework

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- Play with the simulation code that I sent out and learn how different patterns of input may affect the voltage response.

# Project Idea

- Combine a single-compartment model and the cable equation, and create a neuron that can carry out some interesting computations.
-