The following are midterm and final examination test scores for 10 calculus students, where X denotes the midterm score and Y denotes the final score for each student:

X	Y	X^2	XY
68	74	4624	5032
87	89	7569	7743
75	80	5625	6000
91	93	8281	8463
82	88	6724	7216
77	79	5929	6083
86	97	7396	8342
82	95	6724	7790

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82	95	6724	7790
75	89	5625	6675
79	92	6241	7268
$\sum X = 802$	$\sum Y = 876$	$\sum X^2 = 64738$	$\sum XY = 70612$

Here, we use procedure for a fitting a least-squares line

Step 1- Find out $\bar{X}$ **&** $\bar{Y}$

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{802}{10} = 80.2$$

$$\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n} = \frac{876}{10} = 87.6$$

Step 2 - Compute S_{XX} and S_{XY}

$$
\begin{aligned}
S_{XX} &= \sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n} \\
&= 64738 - \frac{(802)^2}{10} \\
&= 64738 - \frac{643204}{10} \\
&= 64738 - 64320.4 \\
&= 417.6
\end{aligned}
$$

$$
\begin{aligned}
S_{XY} &= \sum_{i=1}^{n} X_i Y_i - \frac{\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right)}{n} \\
&= 70612 - \frac{(802)(876)}{10} \\
&= 70612 - \frac{702552}{10} \\
&= 70612 - 70255.2 \\
&= 356.8
\end{aligned}
$$

Step 3 Compute the $\hat{\beta}_0$ and $\hat{\beta}_1$ by substituting the computed quantities from step 1&2

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$$\begin{aligned}
\hat{\beta}_1 &= \frac{S_{XY}}{S_{XX}} \\
&= \frac{356.8}{417.6} \\
&= 0.85
\end{aligned}$$

$$\begin{aligned}
\hat{\beta}_0 &= \overline{Y} - \hat{\beta}_1\overline{X} \\
&= 87.6 - (0.85)(80.2) \\
&= 87.6 - 68.17 \\
&= 19.43
\end{aligned}$$

Note*- The Estimators of the E(Y) denoted by $\hat{Y}$ can be obtained by using the estimators β_1 and $\hat{\beta}_1$ of the parameters β_0 and β_1

Step 4 The fitted least squares line is

$$\begin{aligned}\hat{Y} &= \hat{\beta}_0 + \hat{\beta}_1 X \\ &= 19.43 + 0.85X \\ &= 19.43 + 0.85(92) \\ &= 19.43 + 78.2 \\ &= 97.63\end{aligned}$$

Step 5 compute the SSE (sum of squares for errors) and S (Standard deviation)

SSE means difference between observed Y_i from its predicted value $\hat{Y}_i$.

$$\begin{aligned}S_{yy} &= \sum_{i=1}^{n}(Y_i - \bar{Y})^2 \\ &= \sum_{i=1}^{n}(74-87.6)^2 + (89-87.6)^2 \ldots\ldots\ldots + (92-87.6)^2 \\ &= 512.4\end{aligned}$$

$$\begin{aligned}SSE &= S_{yy} - \hat{\beta}_1 S_{xy} \\ &= 512.4 - (0.85)(356.8) \\ &= 512.4 - 303.28 \\ &= 209.12\end{aligned}$$

$$S = \sqrt{\frac{SSE}{n}}$$

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$$= \sqrt{\frac{209.12}{10}}$$

$$= 4.57$$

Comment

Step 7 of 10

Step 6 A 95% confidence interval for Y is,

$$= \hat{Y} \pm t_{\alpha/2} S \sqrt{\left[1 + \frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{S_{XX}}\right]}$$

$$= 97.63 \pm (2.306)(4.57)\sqrt{\left[1 + \frac{1}{10} + \frac{(92 - 80.2)^2}{417.6}\right]}$$

$$= 97.63 \pm 10.54\sqrt{[1 + 0.1 + 0.33]}$$

$$= 97.63 \pm 10.54(1.20)$$

$$= 97.63 \pm 12.65$$

$$= (84.98, 110.28)$$

Comment