The following data give the annual incomes (in thousands of dollars) and amounts (in thousands of dollars) of life insurance policies for 8 persons, where X denotes the annual income and Y denotes amount of life insurance policy for each person:

X	Y	X^2	XY
42	150	1764	6300
58	175	3364	10150
27	25	729	675
36	75	1296	2700
70	250	4900	17500
24	50	576	1200
53	250	2809	13250
37	100	1369	3700
$\sum X = 347$	$\sum Y = 1075$	$\sum X^2 = 16807$	$\sum XY = 55475$

Here, we use procedure for a fitting a least-squares line

Step 1- Find out $\bar{X}$ **&** $\bar{Y}$

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$
$$= \frac{347}{8}$$
$$= 43.38$$

$$\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$$
$$= \frac{1075}{8}$$
$$= 134.38$$

Comment

Step 4 of 10

Step 2 - Compute S_{XX} **and** S_{XY}

$$S_{XX} = \sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}$$
$$= 16807 - \frac{(347)^2}{8}$$
$$= 16807 - \frac{120409}{8}$$
$$= 16807 - 15051.13$$
$$= 1755.87$$

$$
\begin{aligned}
S_{XY} &= \sum_{i=1}^{n} X_i Y_i - \frac{\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right)}{n} \\
&= 55475 - \frac{(347)(1075)}{8} \\
&= 55475 - \frac{373025}{8} \\
&= 55475 - 46628.13 \\
&= 8846.87
\end{aligned}
$$

Step 3 Compute the $\hat{\beta}_0$ and $\hat{\beta}_1$ by substituting the computed quantities from step 1&2

$$
\begin{aligned}
\hat{\beta}_1 &= \frac{S_{XY}}{S_{XX}} \\
&= \frac{8846.87}{1755.87} \\
&= 5.04
\end{aligned}
$$

$$
\begin{aligned}
\hat{\beta}_0 &= \bar{Y} - \hat{\beta}_1 \bar{X} \\
&= 134.38 - (5.04)(43.38) \\
&= 134.38 - 218.64 \\
&= -84.26
\end{aligned}
$$

Note*- The Estimators of the E(Y) denoted by $\hat{Y}$ can be obtained by using the estimators β_1 and $\hat{\beta}_1$ of the parameters β_0 and β_1

Comment

Step 4 The fitted least squares line is

$$\begin{aligned}\hat{Y} &= \hat{\beta}_0 + \hat{\beta}_1 X \\ &= -84.26 + 5.04X \\ &= -84.26 + 5.04(59) \\ &= -84.26 + 297.36 \\ &= 213.10\end{aligned}$$

Step 5 compute the SSE (sum of squares for errors) and S (Standard deviation)

SSE means difference between observed Y_i **from its predicted value** $\hat{Y}_i$.

$$\begin{aligned}S_{yy} &= \sum_{i=1}^{n}(Y_i - \bar{Y})^2 \\ &= \sum_{i=1}^{n}(150-134.38)^2 + (175-134.38)^2 \ldots\ldots\ldots + (100-134.38)^2 \\ &= 52421.84\end{aligned}$$

$$\begin{aligned}SSE &= S_{yy} - \hat{\beta}_1 S_{xy} \\ &= 52421.84 - (5.04)(8846.87) \\ &= 52421.84 - 44588.22 \\ &= 7833.62\end{aligned}$$

$$\begin{aligned}S &= \sqrt{\frac{SSE}{n}} \\ &= \sqrt{\frac{7833.62}{8}} \\ &= 31.29\end{aligned}$$

Step 7 of 10

Step 6 A 90% confidence interval for Y is,

$$=\hat{Y} \pm t_{\alpha/2} S \sqrt{\left[1+\frac{1}{n}+\frac{(X-\bar{X})^2}{S_{XX}}\right]}$$

$$=213.10 \pm (1.860)(31.29)\sqrt{\left[1+\frac{1}{8}+\frac{(59-43.38)^2}{1755.87}\right]}$$

$$=213.10 \pm 58.20\sqrt{[1+0.125+0.14]}$$

$$=213.10 \pm 58.20(1.12)$$

$$=213.10 \pm 65.18$$

$$=(147.92, 278.28)$$

Comment

Step 8 of 10

Thus, we can conclude with at least 90% confidence that the true value of Y at the point X = 59 will be somewhere between 147.92 and 278.28.

Comment

Step 9 of 10

Note*- The value of t is obtained from t-table at 90% confidence interval.