Additional Notes Central Limit Theorem

1. The Central Limit Theorem (CLT) states that for random samples of size n (usually n>=30) taken from a single population with $mean\ \mu\ and\ standard\ deviation\ \sigma$, the sampling distribution of mean follows a normal distribution with $mean\ \mu\ and\ standard\ deviation\ \sigma/\sqrt{n}$
2. What this means is that the CLT can be used to answer questions about sample means the same way the normal distribution can be used to answer questions about individual values in the population.
3. For doing this a new formula must be used for the $z\ values$ as shown below

$$z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$$

4. Also note that when original variable is normally distributed then the distribution of sample means will be normally distributed for any sample size, n.
5. However, if the population is not normal then we need a sample size of 30 or greater to apply the CLT.

Example

It is reported that children below 5 years spend an average of 25 hours in the internet per week. Assume the variable is normally distributed and the standard deviation is 5 hours. If 16 children less than 5 years are selected at random, find the probability that the mean number of hours they spend in the internet is greater than 26 hours.

Solution

Since the variable is normally distributed, we can apply the CLT even though sample size=20

The z value is given by

$$z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} = \frac{26-25}{5/\sqrt{16}} = 0.8$$

$$P(\overline{X} > 28) = P(z > 0.8) = 1 - P(z < 0.8)$$

$$= 1 - 0.78814$$

$$= 0.21186$$

****