Number System

→ Number of factor of a number n expressed $a^p b^q c^r$--
is $(p+1)(q+1)(r+1)$ {including 1 and itself} where a, b, c are prime numbers

→ Number of even factors of $N = 2^p a^q b^r c^s = p(q+1)(r+1)(s+1)$

→ Number of odd factors = Total factors − Even factors / except 2 power

→ $HCF \times LCM = n_1 \times n_2$ (Only for two numbers, Can't say?)

→ GCD of two or more fractions = $\dfrac{\text{GCD of numerators}}{\text{LCM of denominators}}$

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→ If GCD = LCM of two number a, b then a = b.

→ for three numbers a, b, c if LCM of (a, b), (b, c) and (c, a) is L_{ab}, L_{bc}, L_{ca} and GCD of same numbers is G_{ab}, G_{bc}, G_{ca} then

$$\frac{LCM(a,b,c)}{GCD(a,b,c)} = \frac{L_{ab} \times L_{bc} \times L_{ca}}{a \times b \times c}$$

→ No. of ways to express a number as a product of two numbers

$$= \frac{1}{2}(p+1)(q+1)(r+1)$$

if number $= a^p b^q c^r$

→ Sum of all factors of $n = a^p b^q c^r$ is

$$n = \left\{ \frac{a^{p+1}-1}{a-1} \times \frac{b^{q+1}-1}{b-1} \times \frac{c^{r+1}-1}{c-1} \right\}$$

→ Numbers of ways to express a number aproduct of two co-prime factors.

$= 2^{K-1}$ (If K no. of prime factors are there)

→

Number	Number's in cycle	cyclicity.
1	1	1
2	2, 4, 8, 6	4
3	3, 9, 7, 1	4
4	4, 6	2
5	5	1
6	6	1
7	7, 9, 3, 1	4
8	8, 4, 2, 6	4
9	9, 1	2
10		

$4^{odd} = 4$ $4^{even} = 6$

$9^{odd} = 9$ $9^{even} = 1$

→

Numbers	Cyclicty
1, 5, 6	1
4, 9	2
2, 3, 7, 8	4.

→ Perfect number : If sum factors (except itself) equals that number Ex: 6, 28, 496, 8128 etc

→ Co-primes: Pairs with HCF is 1

→ Vulgar fractions:

* Both numerator and dinominator should be natural
* Denominator should not be power of 10

→ Perfect squares: 0, 1, 4, 9, 7 (Digital sum)

→ Perfect cubes: 0, 1, 8, 9 (Digitial sum)

→ Prime or not?

Given n find nearest square

ex: $n = 23$ square $= 25 = (5)^2$

consider root of it i.e. 5

→ find all primes less than 5 i.e 2, 3

if 2, or 3 divides it's not prime else prime.

→ Highest power in factorial → trailing zeroes is 5 in factorial.

↳ if prime then sum in prime factorization

eg: 5 in 100!

$$\begin{array}{r|l} 5 & 100 \\ \hline 5 & 20 \\ \hline & 4 \end{array} \quad 20 + 4 = 24$$

(sum < prime)

↳ non-prime: then express as product of primes and takes higher one

eg: 21 in 30! ⇒ $21 = 7 \times 3$ so take 7 ⇒ $\begin{array}{r|l} 7 & 30 \\ \hline & 4 \end{array}$ ⇒ 4

↳ non prime (same number)

eg: highest power of 8 in 98!

$8 = 2^3$ ⇒ $\begin{array}{r|l} 2 & 98 \\ \hline & \end{array}$ ⇒ $\dfrac{49+24+12+6+3+1}{3 \rightarrow \text{power}} = 31$

→ abcabc is presented as abc × 7 × 11 × 13

→ find pairs of numbers having LCM and HCF

$$a \times b = \frac{LCM}{HCF}$$

Express as pairs and consider only co-prime pairs.
And then forward as per question.
(If one of no's then multiply HCF to all no's in pairs)

→ LCM = Product of highest powers of all prime factors (union)

→ HCF = Product of lowest powers of all common prime factors (intersection)

→ Assume total strength as LCM

→ LCM of primes is product

→ HCF of primes is 1

→ Number of trailing zeroes in 100! and 10!

$$5 \underline{|100} \quad 5\underline{|20} \quad 4 \quad \} \; 24 \qquad 5\underline{|10} \quad 2 \rightarrow \underline{\underline{2}}$$

→ $(n)^x \% q == \frac{n_0}{q^0}$ eg: $\frac{(43)^4}{6} = \frac{43}{6} = 1$

→ $(19^4+2) \% 6 = \frac{19}{6} = 1+2 = 3$

→ $(19^4+7) \% 90 \Rightarrow \frac{(19)^2}{90} + \frac{7}{90} \Rightarrow \frac{361}{90} + 7 = 1+7 = 8$

Percentages

→ a% 100 = 100% a = a

→ Fractions ⟷ %

$\frac{1}{1}$ = 100%

$\frac{1}{2}$ = 50%

$\frac{1}{3}$ = 33.33%

$\frac{1}{4}$ = 25%

$\frac{1}{5}$ = 20%

$\frac{1}{6}$ = 16.66%

$\frac{1}{7}$ = 14.28%

$\frac{1}{8}$ = 12.5%

$\frac{1}{9}$ = 11.11%

$\frac{1}{10}$ = 10%

$\frac{1}{11}$ = 9.09%

$\frac{1}{12}$ = 8.33%

→ Extension

$\frac{2}{3} = 2\left(\frac{1}{3}\right)$ = 66.66%

$\frac{2}{9} = 2\left(\frac{1}{9}\right)$ = 22.22%

$\frac{7}{11} = 7\left(\frac{1}{11}\right)$ = 63.63%

$\frac{8}{11} = 8\left(\frac{1}{11}\right)$ = 72.72%

→ Express as 10% and 1%

eg: 72% of 2450

70% 2%

10% × 7 + 1% × 2

245 × 7 + 24.5 × 2

= 1764.

→ Note: Split the % if huge

15% 480

10% + 5%

48 ÷ 2 = 5%

48 + 24 = 72.

→ N = 1234

100% = 1234

10% = 123.4

1% = 12.34

0.1% = 1.234

* Percentage Increase

$\Rightarrow \frac{\text{Difference}}{\text{Small value / start point}} \times 100$

* Percentage Decrease

$\Rightarrow \frac{\text{Difference}}{\text{Large value / start point}} \times 100$

* Price * consumption/quantity = expenditure

-OR-

D = S×T -OR- A = l×b -OR- Total work = effort × time.

One increase other decrease to keep constant or so questions

Eg: If one increase by what other decrease to keep third value constant = $\frac{\text{difference}}{\text{new value}} \times 100$

→ Overall change happens consecutively

→ Overall % change = $a+b+a\%b$

(if decrease use negative)

→ $x\% \uparrow$ and $x\% \downarrow$ then $-\frac{x^2}{100}$

Ratio, Proportion and Variation

→ Compounded ratio = product of two or more ratios

→ Duplicate ratio $\left(\frac{a}{b}\right)^2$ → Sub-duplicate ratio $\left(\frac{a}{b}\right)^{1/2}$

→ Triplicate ratio $\left(\frac{a}{b}\right)^3$ → Sub-triplicate ratio $\left(\frac{a}{b}\right)^{1/3}$

→ $\frac{a}{b} = \frac{c+am}{d+bm}$ iff $\frac{c}{d} = \frac{a}{b}$

→ If a:b and b:c are given $a:b:c = (a\times b):(b\times b):(c\times b)$

→ Product of means = product of extremes

→ a, b, c are in proportion $a:b = b:c$ then they are in continued proportion

→ Note: fill with remaining number on left and rights for the large ratio's $(p:q:r:s)$

→ If a:b and b:c is given then a:c is asked

$$a:b = n_1:n_2$$
$$b:c = n_3:n_4.$$
$$a:c = (n_1 \times n_3):(n_2 \times n_4.)$$

→ Mean proportional of two number $= \sqrt{n_1 \times n_2}$

Rem: Express n_1 and n_2 as square.

→ Expenses: constantly and varying

$$x + n_1 \times y = ex_1$$
$$x + n_2 \times y = ex_2$$

find x and y

then for new n find it

→ Sum of n natural numbers = $\frac{n(n+1)}{2}$

→ Sum of n natural number square = $\frac{n(n+1)(2n+1)}{6}$

→ Sum of n natural number cube = $\left[\frac{n(n+1)}{2}\right]^2$

→ In ratio saving or notes if sum is given

eg. → saving in ratio 13:9:8 and sum is 3000

$13x+9x+8x=3000$

lly for note → if ₹10, ₹100, ₹500 is in ratio 5:3:11 and total is ₹25000

$10.5x+100.3x+500.11x=2500.$

→ If no sum 1rupee : 50ps × 2 equal.

→ If p is increased by x% by what % we have to decrease result to get original value = $\left(\frac{x}{x+100} \times 100\%\right)$

→ In fractions if p is increased by $\frac{n}{d}$ then decrease result by $\frac{n}{d+n}$.

→ If p is decreased increase resultant by $\left(\frac{x}{x-100} \times 100\%\right)$ for fractions

$\frac{n}{d-n}$

→ salary : a:b exp : c:d

$\frac{ax-sav1}{bx-sav2} = \frac{c}{d}$