

Number System

- Number of factor of a number n expressed $a^p b^q c^r$ is $(p+1)(q+1)(r+1)$ {including 1 and itself} where a, b, c are prime numbers
- Number of even factors of $N = 2^p a^q b^r c^s = p(q+1)(r+1)(s+1)$
- Number of odd factors = Total factors - Even factors / except 2^{power}
- $HCF \times LCM = n_1 \times n_2$ (Only for two numbers, Can't say?)
- GCD of two or more fractions = $\frac{\text{GCD of numerators}}{\text{LCM of denominators}}$
- LCM of two or more fractions = $\frac{\text{LCM of numerators}}{\text{GCD of denominators}}$
- If $GCD = LCM$ of two number a, b then $a = b$.
- for three numbers a, b, c if LCM of (a, b) , (b, c) and (c, a) is L_{ab}, L_{bc}, L_{ca} and GCD of same numbers is G_{ab}, G_{bc}, G_{ca} then

$$\frac{LCM(a, b, c)}{GCD(a, b, c)} = \frac{L_{ab} \times L_{bc} \times L_{ca}}{a \times b \times c}$$

- No. of ways to express a number as a product of two numbers
 $= \frac{1}{2} (p+1)(q+1)(r+1)$

$$\text{if number} = a^p b^q c^r$$

- Sum of all factors of $n = a^p b^q c^r$ is

$$n = \left\{ \frac{a^{p+1} - 1}{a - 1} \times \frac{b^{q+1} - 1}{b - 1} \times \frac{c^{r+1} - 1}{c - 1} \right\}$$

→ Number of ways to express a number as product of two co-prime factors.

$$= 2^{k-1} \quad (\text{If } k \text{ no. of prime factors are there})$$

→ Number	Number's in cycle	cyclicity.
1	1	1
2	2, 4, 8, 6	4
3	3, 9, 7, 1	4
4	4, 6	2
5	5	1
6	6	1
7	7, 9, 3, 1	4
8	8, 4, 2, 6	4
9	9, 1	2
10		

$$4^{\text{odd}} = 4 \quad 4^{\text{even}} = 6$$

$$q^{\text{odd}} = q \quad q^{\text{even}} = 1$$

→ Numbers Cyclicity

1, 5, 6 1

4, 9 2

2, 3, 7, 8 4.

→ Perfect number: If sum factors (except itself) equals that number Ex: 6, 28, 496, 8128 etc

→ Co-primes: Pairs with HCF is 1

→ Vulgar fractions:

- * Both numerator and denominator should be natural
- * Denominator should not be power of 10

→ Perfect squares: 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961, 1024, 1089, 1156, 1225, 1296, 1369, 1444, 1521, 1600, 1681, 1764, 1849, 1936, 2025, 2116, 2209, 2304, 2401, 2500, 2601, 2704, 2809, 2916, 3025, 3136, 3249, 3364, 3481, 3600, 3721, 3844, 3969, 4096, 4225, 4356, 4489, 4624, 4761, 4900, 5041, 5184, 5329, 5476, 5625, 5776, 5929, 6084, 6241, 6400, 6561, 6724, 6889, 7056, 7225, 7396, 7569, 7744, 7921, 8100, 8281, 8464, 8649, 8836, 9025, 9216, 9409, 9604, 9801, 10000

→ Perfect cubes: 0, 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728, 2197, 2744, 3375, 4096, 4913, 5832, 6859, 8000, 9268, 10648, 12167, 13924, 15939, 18216, 20768, 23607, 26750, 30207, 33988, 38103, 42562, 47375, 52548, 58091, 64014, 70327, 77040, 84163, 91706, 99679, 108092, 117055, 126578, 136671, 147354, 158637, 170530, 183043, 196186, 209969, 224402, 239495, 255258, 271691, 288804, 306607, 325110, 344323, 364256, 384919, 406312, 428445, 451318, 474941, 499314, 524447, 550340, 577003, 604436, 632649, 661642, 691425, 721998, 753361, 785514, 818457, 852190, 886713, 922026, 958129, 995032, 1032745, 1071268, 1110591, 1150724, 1191667, 1233420, 1275983, 1319356, 1363539, 1408532, 1454345, 1500978, 1548431, 1596704, 1645797, 1695710, 1746443, 1797996, 1850369, 1903562, 1957585, 2012438, 2068121, 2124634, 2181977, 2240150, 2299153, 2358986, 2419649, 2481142, 2543465, 2606618, 2670601, 2735414, 2799957, 2865230, 2931233, 2997966, 3065429, 3133622, 3202545, 3272198, 3342581, 3413694, 3485527, 3558080, 3631353, 3705346, 3779959, 3855192, 3930945, 4007318, 4084311, 4161924, 4240147, 4318980, 4398423, 4478476, 4559139, 4640412, 4722295, 4804788, 4887891, 4971504, 5055727, 5140560, 5225993, 5311926, 5398459, 5485582, 5573295, 5661598, 5750391, 5839674, 5929447, 6019710, 6110463, 6201706, 6293439, 6385652, 6478345, 6571518, 6665161, 6759274, 6853857, 6948900, 7044413, 7140396, 7236849, 7333772, 7431165, 7529028, 7627351, 7726134, 7825377, 7925080, 8025243, 8125856, 8226919, 8328432, 8429395, 8530808, 8632661, 8734954, 8837687, 8940850, 9044443, 9148456, 9252879, 9357712, 9462955, 9568608, 9674661, 9780924, 9887397, 9994070, 10000000

→ Prime or not?

Given n find nearest square

ex: $n=23$ square $=25=(5)^2$

consider root of it i.e. 5

→ find all primes less than 5 i.e. 2, 3

if 2, or 3 divides it's not prime else prime.

→ Highest power in factorial → trailing zeroes & 5 in factorial.

↳ if prime then sum in prime factorization

eg: 5 in 100!
$$\begin{array}{r} 5 \overline{) 100} \\ 5 \overline{) 20} \end{array} \left. \begin{array}{l} 20 \\ 4 \end{array} \right\} 20+4=24 \quad (\text{sum} < \text{prime})$$

↳ non-prime: then express as product of primes and takes higher one

eg: 21 in 30! $\Rightarrow 21=7 \times 3$ so take 7 $\Rightarrow 7 \overline{) 30} \Rightarrow 4$

↳ non prime (same number)

eg: highest power of 8 in 98!

$8=2^3 \Rightarrow 2 \overline{) 98} \Rightarrow \frac{49+24+12+6+3+1}{3} = 31$
(3) → power

→ $abcabc$ is presented as $abc \times 7 \times 4 \times 13$

→ find pairs of numbers having LCM and HCF

$$a \times b = \frac{\text{LCM}}{\text{HCF}}$$

Express as pairs and consider only co-prime pairs.

And then forward as per question.

(If one of no's then multiply HCF to all no's in pairs)

→ LCM = Product of highest powers of all prime factors (union)

→ HCF = Product of lowest powers of all common prime factors (intersection)

→ Assume total strength as LCM

→ LCM of primes & product

→ HCF of primes is 1

→ Number of trailing zeroes in $100!$ and $10!$

$$\begin{array}{r} 5 \overline{) 100} \\ \underline{20} \\ 4 \end{array} \Bigg\} 24$$

$$\begin{array}{r} 5 \overline{) 10} \\ \underline{2} \end{array} \rightarrow \underline{\underline{2}}$$

→ $(n)^x \% q = \frac{n}{q}$ eq: $\frac{(43)^4}{6} = \frac{43}{6} = 1$

→ $(19^4 + 2) \% 6 = \frac{19}{6} = 1 + 2 = 3$

→ $(19^4 + 7) \% 90 \Rightarrow \frac{(19)^2}{90} + \frac{7}{90} \Rightarrow \frac{361}{90} + 7 = 1 + 7 = 8$

Percentages

$$\rightarrow a\% \cdot 100 = 100\% \cdot a = a$$

\rightarrow Fractions $\longleftrightarrow \%$

$$\frac{1}{1} = 100\%$$

$$\frac{1}{5} = 20\%$$

$$\frac{1}{9} = 11.11\%$$

$$\frac{1}{2} = 50\%$$

$$\frac{1}{6} = 16.66\%$$

$$\frac{1}{10} = 10\%$$

$$\frac{1}{3} = 33.33\%$$

$$\frac{1}{7} = 14.28\%$$

$$\frac{1}{11} = 9.09\%$$

$$\frac{1}{4} = 25\%$$

$$\frac{1}{8} = 12.5\%$$

$$\frac{1}{12} = 8.33\%$$

\rightarrow Extension

$$\frac{2}{3} = 2 \left(\frac{1}{3} \right) = 66.66\%$$

$$\frac{2}{9} = 2 \left(\frac{1}{9} \right) = 22.22\%$$

$$\frac{7}{11} = 7 \left(\frac{1}{11} \right) = 63.63\%$$

$$\frac{8}{11} = 8 \left(\frac{1}{11} \right) = 72.72\%$$

\rightarrow Note: Split the % if huge

$$15\% \text{ of } 480$$

$$10\% + 5\%$$

$$480 \div 2 = 240$$

$$480 + 240 = 720$$

$$\rightarrow N = 1234$$

$$100\% = 1234$$

$$10\% = 123.4$$

$$1\% = 12.34$$

$$0.1\% = 1.234$$

\rightarrow Express as 10% and 1%

eg: 72% of 2450

$$\swarrow \quad \searrow$$

$$70\% \quad 2\%$$

$$10\% \times 7 + 1\% \times 2$$

$$245 \times 7 + 24.5 \times 2$$

$$= 1764$$

* Percentage Increase

$$\Rightarrow \frac{\text{Difference}}{\text{Small value / start point}} \times 100$$

* Percentage Decrease

$$\Rightarrow \frac{\text{Difference}}{\text{Large value / start point}} \times 100$$

* Price * consumption/quantity = expenditure
-OR-

$$D = S \times T \quad \text{-OR-} \quad A = l \times b \quad \text{-OR-} \quad \text{Total work} = \text{effort} \times \text{time}$$

One increase other decrease to keep constant or so questions

Eg: If one increase by what other decrease to keep third value constant = $\frac{\text{difference}}{\text{new value}} \times 100$

→ Small change happens consecutively.

→ Small % change = $a + b + a\%b$
(if decrease use negative)

→ $x\% \uparrow$ and $x\% \downarrow$ then $-\frac{x^2}{100}$

Ratio, Proportion and Variation

→ Compound ratio = product of two or more ratios

→ Duplicate ratio $\left(\frac{a}{b}\right)^2$ → Sub-duplicate ratio $\left(\frac{a}{b}\right)^{1/2}$

→ Triplicate ratio $\left(\frac{a}{b}\right)^3$ → Sub-triplicate ratio $\left(\frac{a}{b}\right)^{1/3}$

$$\rightarrow \frac{a}{b} = \frac{c+am}{d+bm} \quad \text{iff} \quad \frac{c}{d} = \frac{a}{b}$$

→ If $a:b$ and $b:c$ are given $a:b:c = (a \times b):(b \times b):(c \times b)$

→ Product of means = product of extremes

→ a, b, c are in proportion $a:b = b:c$ then they are in continued proportion

→ Note: fill with remaining numbers on left and right for the large ratio's $(p:q:r:s)$

→ If $a:b$ and $b:c$ is given then $a:c$ is asked

$$a:b = n_1:n_2$$

$$b:c = n_3:n_4$$

$$a:c = (n_1 \times n_3):(n_2 \times n_4)$$

→ Mean proportional of two numbers = $\sqrt{n_1 \times n_2}$

Rem: Express n_1 and n_2 as squares.

→ Expenses: constantly and varying

$$x + n_1 \times y = ex_1$$

$$x + n_2 \times y = ex_2$$

find x and y

then for new n find it

- Sum of n natural numbers = $\frac{n(n+1)}{2}$
- Sum of n natural numbers square = $\frac{n(n+1)(2n+1)}{6}$
- Sum of n natural numbers cube = $\left[\frac{n(n+1)}{2}\right]^2$

→ In ratio saving or notes if sum is given

eg. → saving in ratio 13:9:8 and sum is 3000

$$13x + 9x + 8x = 3000$$

if for note → if ₹10, ₹100, ₹500 is in ratio 5:3:11 and total is ₹2500

$$10 \cdot 5x + 100 \cdot 3x + 500 \cdot 11x = 2500$$

→ If no sum rupees & paise equal

→ If p is increased by $x\%$ by what % we have to decrease result to get original value = $\left(\frac{x}{x+100} \times 100\%\right)$

→ In fractions if p is increased by $\frac{n}{d}$ then decrease result by $\frac{n}{d+n}$

→ If p is decreased increase resultant by $\left(\frac{x}{x-100} \times 100\%\right)$ for fractions $\frac{n}{d-n}$

→ salary: $a:b$ exp: $c:d$

$$\frac{ax - \text{sav}_1}{bx - \text{sav}_2} = \frac{c}{d}$$