

DIGITALS

Boolean Expression :-

$$f(x,y,z) = x + \bar{y}z$$

It mainly has literals (variable such as x, y, z), a binary operator, '=' (and, or, +)

Truth table :-

Tabular representation of behavior of a function for different set of values

No. of rows = $2^n + 1$; No. of columns = $n+1$ (n - no. of literals)

- * Types of expression:
 - * Normal Expression
 - * Canonical Expression
- Any expression can have multiple equivalent functions
- 1's represent min-term (product) (All literals)
- 0's represent max-term (sum)
- * $f(x,y,z) = z + \bar{y}z$. Expressed as product terms
 ↳ SOP (sum of product)
- * $f(x,y,z) = (x+y^-) \cdot (x+z)$ Expressed as sum terms
 ↳ POS (product of sum)
- * Min-terms is a product term which represents one's of functional value either in normal or complementary form.

* Max-terms: Sum term represents 0 which has all literals but with its complementary form

$$f_1(x, y, z) = x + \bar{y}z \quad (\text{SOP}) \quad (\text{Disjunction})$$

$$f_2(x, y, z) = (x + \bar{y}) \cdot (x + z) \quad (\text{POS}) \quad (\text{Conjunction})$$

$$f_3(x, y, z) = \bar{x}\bar{y}z + x\bar{y}z + x\bar{y}\bar{z} + xy\bar{z} + xyz \quad (\text{SOP})$$

$$f_4(x, y, z) = (x + y + z) (x + \bar{y} + z) (x + \bar{y} + \bar{z}) \quad (\text{POS})$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Simpliest form of expression in terms of cost is minimal expression

Ex: $f(x, y, z)$

$f_1(x, y, z)$ is max-term canonical form (M)

$f_2(x, y, z)$ is min-term canonical form (m)

- OR -

$f_3(x, y, z)$ is Disjunctive canonical form
 $f_4(x, y, z)$ is Conjunctive canonical form.

Max-term canonical POS form of expression where every sum term is max-term

Min-term Canonical SOP form of expression where every product term is min-term

Equivalent expression or equivalent function:-
When the expressions give the same output for all set of values.

$$\begin{aligned} f_3(x, y, z) &= \bar{x}\bar{y}z + x\bar{y}z + x\bar{y}\bar{z} + xy\bar{z} + xyz \\ &= m_1 + m_2 + m_3 + m_4 + m_5 \\ &= \sum m (1, 4, 5, 6, 7) \end{aligned}$$

$$\begin{aligned} f_4(x, y, z) &= (x + y + z) (x + \bar{y} + z) (x + \bar{y} + \bar{z}) \\ &= M_0 \cdot M_1 \cdot M_2 \\ &= \prod M (0, 2, 3) \end{aligned}$$

→ Min or max term canonical expressions are not minimal expression. They are not efficient in terms of

Minimal sum:

$$\text{Ex: } \Sigma m(0, 3, 4, 6, 7)$$

$$y_2 \quad 00 \quad 01 \quad 11 \quad 10$$

$$y_1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0$$

$$1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0$$

$$a \quad b^c \quad 00 \quad 01 \quad 11 \quad 10$$

$$b = \overline{y_2} + y_2 \bar{x}$$

$$f = (\bar{y}_1 + \bar{z}) \cdot (x + \bar{y} + z)$$

$$c \quad 00 \quad 01 \quad 11 \quad 10$$

$$f = \bar{b} \bar{c} + bc + ab$$

$$f = (\bar{b} + \bar{c}) \cdot (a + b + c)$$

, minimal product

$$\text{Ex: } f(w, x, y, z) = \Sigma m(1, 3, 5, 8, 10, 11, 14)$$

$$y_2 \quad 00 \quad 01 \quad 11 \quad 10$$

$$y_1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0$$

$$w \quad 00 \quad 1 \quad 0 \quad 1 \quad 0$$

$$x \quad 01 \quad 1 \quad 0 \quad 1 \quad 0$$

$$z \quad 11 \quad 1 \quad 0 \quad 1 \quad 0$$

$$y \quad 10 \quad 0 \quad 1 \quad 0 \quad 1$$

Minimal product:

* Implicants are sum terms which are implied by

the function

All possible product terms are implicant

All possible sum terms are implicants

* An implicant is said to be prime implicant if it

does not subsume any other term in same expression

* Implicant is a product terms that implies the function

implies

$$f_1 \Rightarrow f_2$$

$$f_1 \Rightarrow f_1$$

both should satisfy.

$$0 \Rightarrow 0$$

(mo) (p.)

* Subsumes: $\bar{a}\bar{b}\bar{c}$, $\bar{b}\bar{c}$; $\bar{b}\bar{c}$ subsumes $\bar{a}\bar{b}\bar{c}$

$\therefore P_1$ subsumes no.

A term P_1 is said to subsume P_2 if all literals in P_1 are in P_2

are in P_2

* In any expression we have subsumes of one time then

it is not minimal

$$f = \bar{b}\bar{c} + b\bar{c}ab + a\bar{b}\bar{c}$$

This f is not minimal

A expression is said to be minimal if there are no subsumes of any product term

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* An implicate is said to be prime implicate if it doesn't have subsumes in same exp

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Minimizing Boolean
Simplifying in sum form
product not expression

$$f(w, x, y, z) = \sum m(0, 1, 3, 7, 8, 12)$$

wz	00	01	11	10	*
00	(1)	(1)	(1)	(0)	*
01	(0)	(0)	(1)	(0)	*
11	(1)	(0)	(0)	(0)	

subsuming
or is not
K-prime

$$f(w, x, y, z) = \bar{w}\bar{y}\bar{z} + w\bar{y}\bar{z} + \bar{w}\bar{x}\bar{z} + \bar{w}\bar{y}z$$

All prime implicants from K-map

subsuming
or is not
K-prime

$$f(w, x, y, z) = \bar{w}\bar{y}\bar{z} + w\bar{y}\bar{z} + \bar{w}\bar{x}\bar{z} + \bar{w}\bar{y}z$$

All prime implicants

Implicants: m0, m1, m3, m7, m8, m12.
Implicants: M2, M4, M5, M6, M9, M10, M11, M13, M14,

M15.

How many minimal sums are there:
 $f = \Sigma p_1 + \Sigma p_2 + \bar{w}\bar{x}\bar{y}$

Essential prime implicants:- $(w\bar{x}\bar{z}), (\bar{w}y\bar{z})$
Essential prime implicants:- $(\bar{y}+z), (\bar{w}+\bar{z})$

Terms essential for all minimal expression: essential terms
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essential prime implicant associated with exactly
at one cell

Prime implicants:-
 $w\bar{y}\bar{z}, \bar{x}\bar{y}\bar{z}, \bar{w}\bar{x}\bar{y}, \bar{w}\bar{x}\bar{z}, \bar{w}y\bar{z}$

Terms essential for all minimal expression: essential terms
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$$\text{Prime implicate: } (w+x+y)(w+\bar{x}+z)(\bar{x}+y+\bar{z})$$

$$f(w, x, y, z) = \sum m(0, 1, 3, 4) + d(7)$$

The boolean functions for which some input values
the output is not defined (it takes 0 or 1) then it is
incomplete boolean function.
The term is represented as - (Don't care)

$$f(w, x, y) = \sum m(0, 1, 3, 4) + d(7)$$

$$= Tm(2, 5, 6) + D(7)$$

$$\text{Prime implicate: } (w+x+y)(w+\bar{x}+z)(\bar{x}+y+\bar{z})$$

$$(y-\bar{x})(w-\bar{y})(\bar{w}-\bar{z})$$

$m_0 = \bar{a}\bar{b}$
 $m_1 = \bar{a}b$
 $m_2 = ab$
 $m_3 = ab$.

logic high / active high IC's: when enable
is higher logic 1
active low: when enable is low or zero.

state low: when enable is low or zero.

buffer

A	m_0	B	m_1	m_2	m_3
0	0	0	0	0	0
0	1	0	1	0	1
1	1	0	0	1	0
1	0	1	0	1	1

(MSB) a	a	m_0	m_1	m_2	m_3
0	0	0	0	0	0
0	1	1	0	0	0
1	0	1	0	1	0
1	1	0	1	0	1

(i) AND gate

$$\bar{f} = \bar{a}\bar{b} + \bar{a}\bar{c} + \bar{b}\bar{c}$$

$$a+b = \bar{a} \cdot \bar{b}$$

$$\bar{f} = \bar{a}\bar{b} \quad \bar{a}\bar{c} \quad \bar{b}\bar{c}$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$\bar{f} = (\bar{a}+\bar{b}) \quad (\bar{a}+\bar{c}) \quad (\bar{b}+\bar{c})$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$\bar{f} = \bar{M}_2 \cdot \bar{M}_3$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$\bar{f} = M_0 \cdot M_1$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$\bar{f} = \bar{m}_2 + \bar{m}_3$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$f = \bar{a}b + a\bar{c} + b\bar{c}$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$f = \bar{a}\bar{b} \cdot 1 + a \cdot \bar{c} + b \cdot \bar{c} \cdot 1$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$f = \bar{a}\bar{b} (1+\bar{c}) + a(\bar{b}+b) \bar{c} + (a+\bar{a}) b\bar{c}$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$f = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c} + abc + ab\bar{c} + ab\bar{c} + \bar{a}\bar{b}\bar{c}$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$f(a, b, c) = \bar{a}b + a\bar{c} + b\bar{c}$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$f(a, b, c) = \bar{a}b + a\bar{c} + b\bar{c}$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

$$f(a, b, c) = \bar{a}b + a\bar{c} + b\bar{c}$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}$$

group common

$$= m_3 + m_2 + m_6 + m_4 + m_0 = \Sigma m(3, 2, 6, 4, 0)$$

$$A + BC = (A + B) \cdot (A + C)$$

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$$\begin{aligned}
 f &= \bar{a}b + a\bar{c} + b\bar{c} \\
 f &= (\bar{a}b + a\bar{c} + b\bar{c}) \cdot (\bar{a}\bar{b} + a\bar{c} + \bar{c}) \\
 &= (\bar{a}b + a\bar{c} + b\bar{c}) \cdot (\bar{a}\bar{b} + \bar{c}(a+1)) \\
 &\quad \text{a} \cdot \bar{a} \\
 &= (\bar{a}b + a\bar{c} + b\bar{c}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= (\bar{a}\bar{c} + \bar{a} + \bar{b}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= (\bar{a}\bar{c} + \bar{a} + \bar{b}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= M_2 \cdot (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= M_2 \cdot (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= M_2 \cdot (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= M_2 \cdot (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= M_2 \cdot (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a}\bar{b} + \bar{c})
 \end{aligned}$$

a	b	c	O/P
0	0	0	-
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$\begin{aligned}
 f &= M_2 \cdot (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= M_2 \cdot (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= M_2 \cdot (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= M_2 \cdot (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a}\bar{b} + \bar{c}) \\
 &= M_2 \cdot (\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a}\bar{b} + \bar{c})
 \end{aligned}$$

If OR gate \rightarrow write in SOP form.
 AND gate \rightarrow write in POS form
 NOR gate \rightarrow take POS apply de-morgan's law
 NAND gate \rightarrow take SOP apply de-morgan's law

+ The odd parity generator using diodes and
 NOR gate only

OP		
nOR	bc	00 01 11 10
0	1	(0)
1	0	(0)
1	1	1

f = $(\bar{a} + b + \bar{c})$.

$$\begin{aligned}
 &(\bar{a} + \bar{b} + c) \cdot \\
 &(\bar{a} + b + \bar{c}) \cdot \\
 &(\bar{a} + b + c)
 \end{aligned}$$

$$f = M_1 \cdot M_2 \cdot M_4 \cdot M_7$$

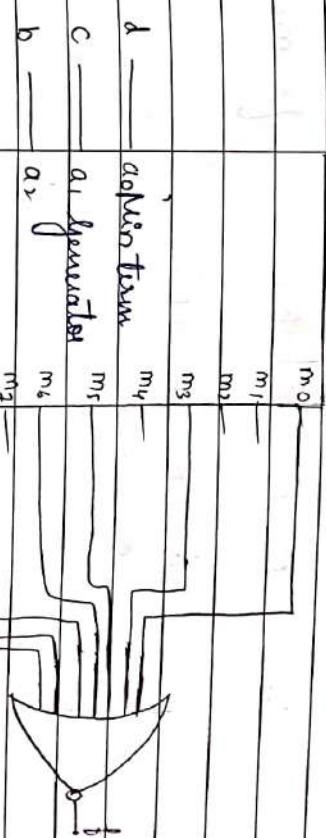
$$\begin{aligned}
 f &= \frac{\bar{f}}{f} = \frac{\bar{M}_1 \cdot \bar{M}_2 \cdot \bar{M}_4 \cdot \bar{M}_7}{M_1 \cdot M_2 \cdot M_4 \cdot M_7} = \frac{\bar{M}_1 + \bar{M}_2 + \bar{M}_4 + \bar{M}_7}{M_1 + M_2 + M_4 + M_7} \\
 &= m_1 + m_2 + m_4 + m_7
 \end{aligned}$$

3) 4-bit sum parity using NOR gate

a	b	c	d	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$$f = M_0 M_3 M_5 M_6 M_9 M_{10} M_{15}$$

$$f = m_0 + m_3 + m_5 + m_6 + m_9 + m_{10} + m_{15}$$



*

$$f = \bar{x}y + y\bar{z}$$

Using nor and decoder

$$\begin{aligned} f &= (a+b+c+d) \cdot (a+b+\bar{c}+\bar{d}) \\ &= (\bar{a}+\bar{b}+c+d) (\bar{a}+\bar{b}+\bar{c}+\bar{d}) \\ &= (\bar{a}\bar{b}\bar{c}\bar{d}) \\ &= \bar{m}_0 \bar{m}_1 \bar{m}_2 \bar{m}_3 \end{aligned}$$

$$= m_3 + m_2 + m_5 + m_1 = \overline{\text{IM}}(0, 1, 4, 6, 7, 1)$$

* Write the output

$$\begin{aligned} f &= \overline{m_3 + m_2 + m_5 + m_1} \quad \text{So} \quad f = \overline{m_0 m_4 \overline{m_6} m_7} \\ b &= \overline{m_3} \overline{m_2} \overline{m_5} \overline{m_1} \quad f = \overline{m_0} + \overline{m_4} + \overline{m_6} + \overline{m_7} \\ b &= M_3 \overline{M_2} \overline{M_5} \overline{M_1} \quad f = m_0 + m_4 + m_6 + m_7 \end{aligned}$$

$$\begin{aligned} b &= \overline{m_3} + \overline{m_2} + \overline{m_5} + \overline{M_1} \\ b &= m_3 + m_2 \end{aligned}$$

(y) y

$$\begin{array}{lll} A_1 & -xyz & (\bar{A}_1 \bar{A}_0 \epsilon_n) \\ A_0 & -xyz & (\bar{A}_1, \bar{A}_0, \epsilon_n) \\ \epsilon_n & xy\bar{z} & (A_1, \bar{A}_0, \epsilon_n) \\ & x\bar{y}\bar{z} & (A_1, A_0, \epsilon_n) \\ & \bar{x}y\bar{z} & (A_1, A_0, \epsilon_n) \\ & \bar{x}\bar{y}\bar{z} & (A_1, A_0, \epsilon_n) \end{array}$$

(x) z

$$\begin{array}{lll} 2 & -\bar{xy}2 & \\ A_0 & -\bar{x}y^2 & \\ \epsilon_n & xy^2 & \\ & x\bar{y}^2 & \\ & \bar{x}y^2 & \\ & \bar{x}\bar{y}^2 & \end{array}$$

m₁

$$\begin{aligned} m_0 & \\ m_1 & \\ m_2 & \\ m_3 & \\ m_4 & \\ m_5 & \\ m_6 & \\ m_7 & \end{aligned}$$

Change to k sequence

k - most significant

* Implement 4:0 to 7:0 using 2:0



Assume: 0 is not divisible
from OR

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* Design a 3-bit input system which checks if the no is divisible by 3 or 5. Using decoder and OR/NOR gate with least ilp's.

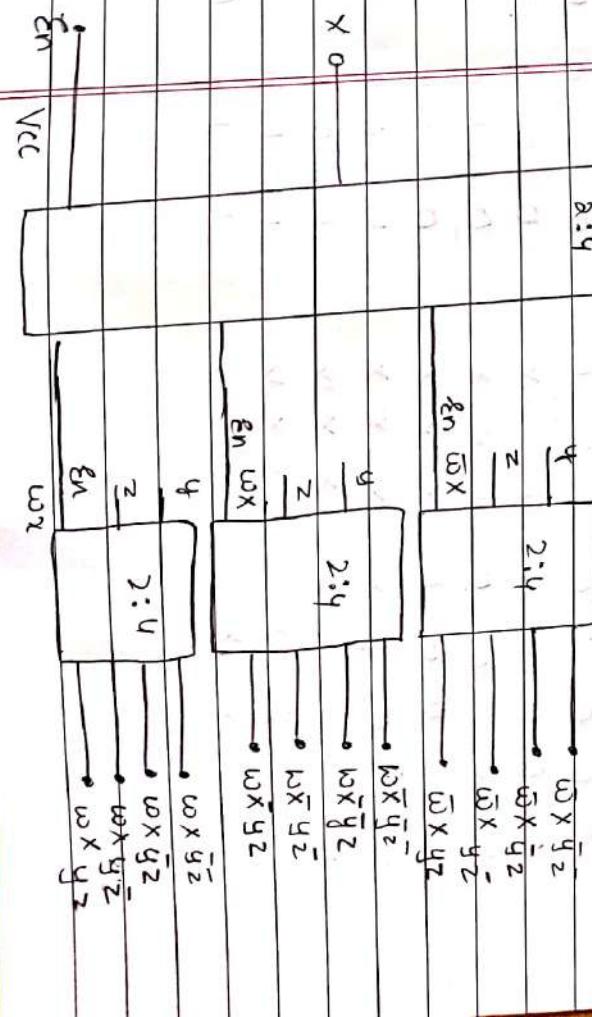
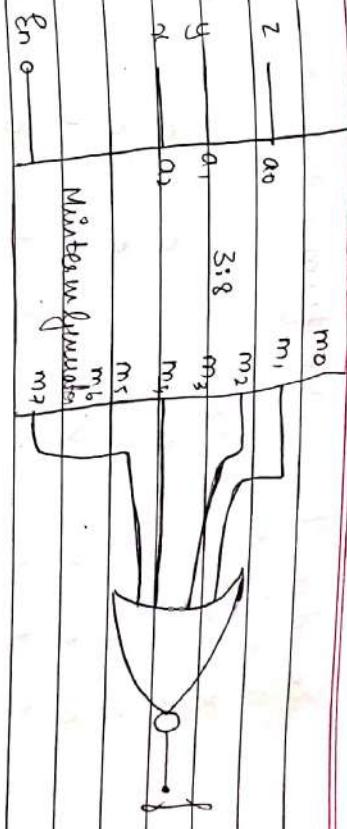
x	y	z	$f = m_0 + m_3 + m_6 + m_5$
0	0	0	$M_1 M_2 M_4 M_6$
0	0	1	$m_1 + m_3 + m_4 + m_6$
0	1	0	$m_0 + m_3 + m_6$
0	1	1	$m_0 + m_1 + m_3 + m_6$

because OR requires only 3 ilps and NOR requires 5 ilps.

$$f = \sum m(0, 3, 5, 6)$$

$$= \prod M(1, 2, 4, 7)$$

* Implemented 4:16 using 8:1 decoder.



-DR -

Encoder. It is $2^n:n$ form

Priority Encoder:

x_3	x_2	x_1	x_0	z_1	z_0	Valid
0	0	0	0	0	0	0
0	0	0	0	0	0	1
0	0	0	1	0	0	1
0	0	1	0	0	1	1
0	1	X	X	1	0	-
1	X	X	X	1	1	-

Encoder: 8:3

x_7	x_6	x_5	x_4	x_3	x_2	x_1	x_0	z_2	z_1	z_0	Valid
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	1	X	0	0	1	0	0
0	0	0	0	0	1	X	0	0	1	0	0
0	0	0	0	1	X	X	0	1	0	0	0
0	0	0	1	X	X	X	0	0	1	0	0
0	0	1	X	X	X	X	1	0	1	0	0
0	1	X	X	X	X	X	1	0	1	0	0
1	X	X	X	X	X	X	1	1	1	0	0

* Design a comparator that compares two numbers in the range 0 - 3. Using max. sum generator decoder and gate with least inputs.

A	B	$A \geq B$	$A < B$	$A = B$
0	0	0	0	0
0	0	0	0	1
0	1	1	0	0
1	0	1	1	0
1	1	1	1	1

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$$f_1 = M_0 M_1 M_2 M_3 M_5 M_6 M_7 M_8 M_{10} M_{11} M_{15}$$

(10)

$$f_2 = M_0 M_4 M_5 M_6 M_8 M_9 M_0 M_2 M_3 M_4 M_5$$

(10)

$$f_3 = M_1 M_2 M_3 M_4 M_6 M_7 M_8 M_9 M_0 M_2 M_3 M_4 M_5$$

(12)

$$f_1 = \sum m(4, 8, 9, 12, 13, 14) \quad (6)$$

$$= \sum m(4, 8, 9, 12, 13, 14) + M_4$$

$$f_2 = \sum m(1, 2, 3, 6, 7, 11) \quad (6)$$

$$f_3 = M_1 + M_2 + M_3 + M_6 + M_7 + M_{11}$$

$$f_3 = \sum m(0, S_{110}, 15) \quad (4) = M_0 + M_5 + M_{10} + M_{15}$$

$$A_1 \wedge A_2 \rightarrow M_0$$

$$A_0 \wedge A_2 \rightarrow M_1$$

$$S_1 \wedge A_1 \rightarrow M_2$$

$$B_0 \wedge A_0 \rightarrow M_3$$

$$M_4 \rightarrow M_4$$

$$M_5 \rightarrow M_5$$

$$M_6 \rightarrow M_6$$

$$M_7 \rightarrow M_7$$

$$M_8 \rightarrow M_8$$

$$M_9 \rightarrow M_9$$

$$M_{10} \rightarrow M_{10}$$

$$M_{11} \rightarrow M_{11}$$

$$M_{12} \rightarrow M_{12}$$

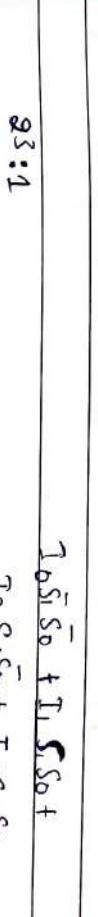
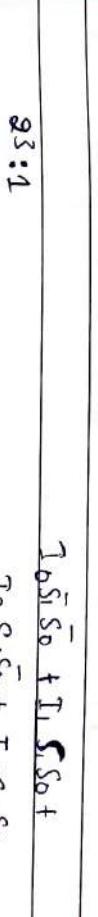
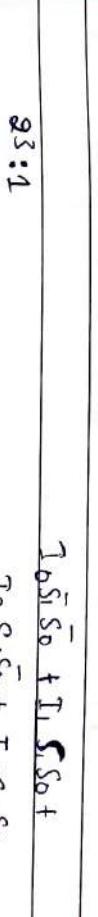
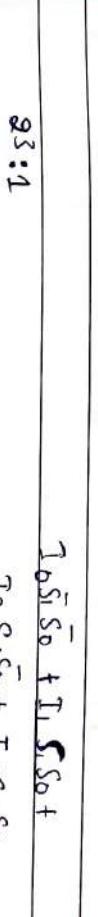
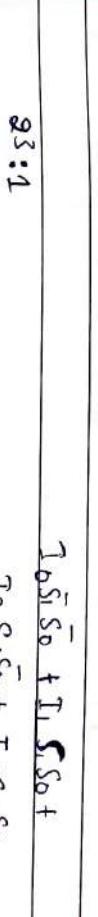
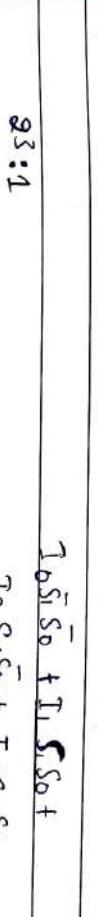
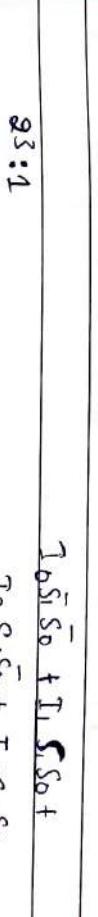
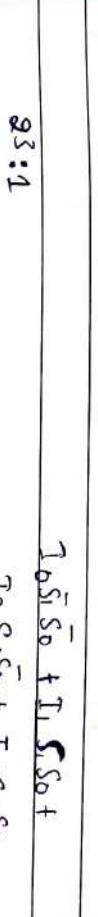
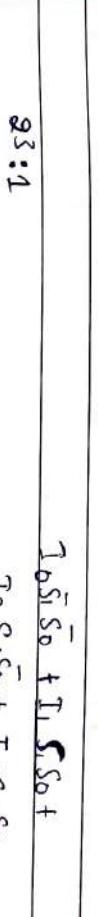
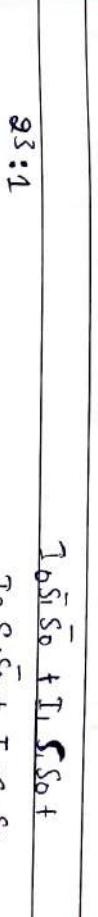
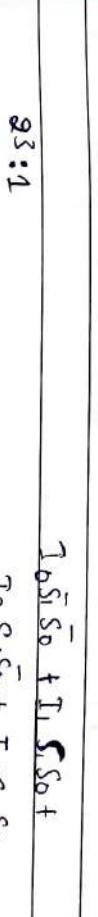
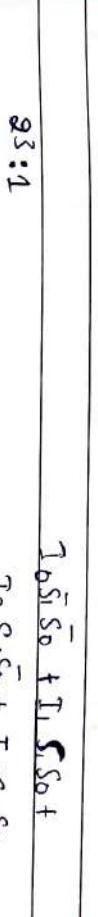
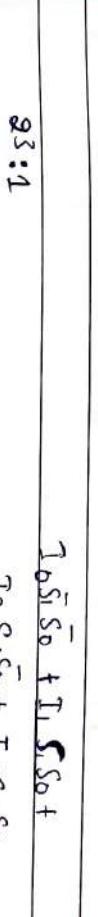
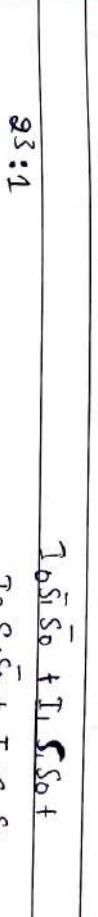
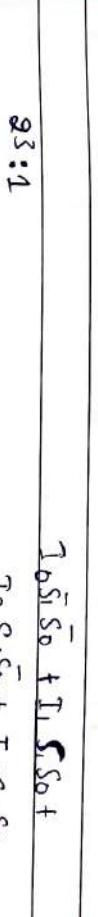
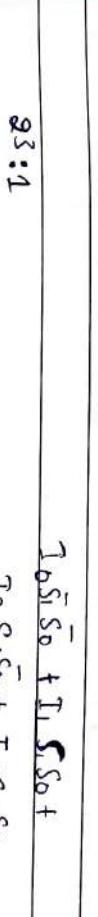
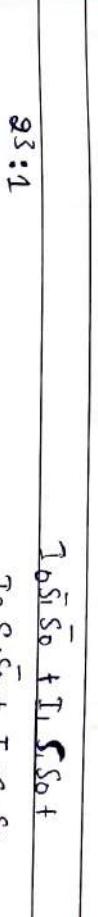
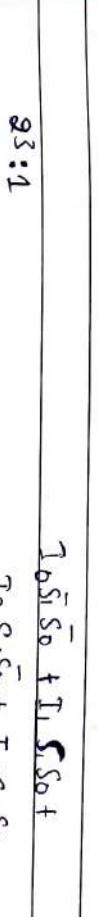
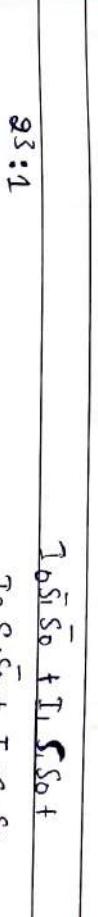
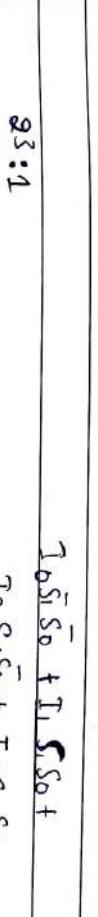
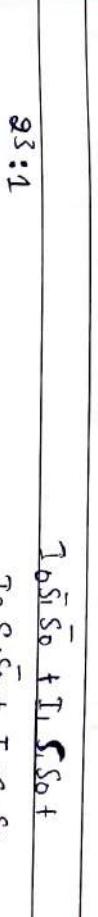
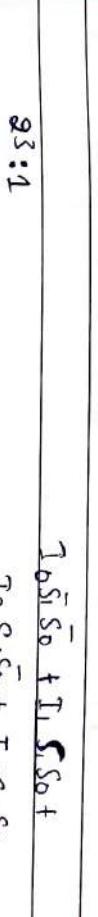
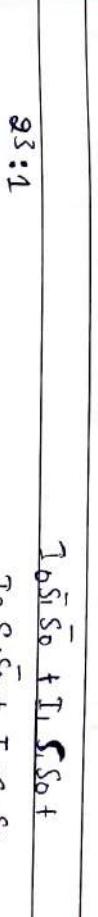
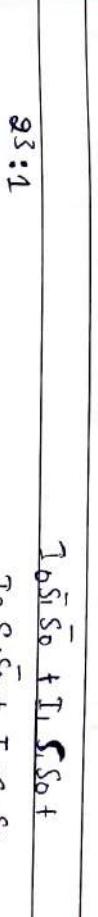
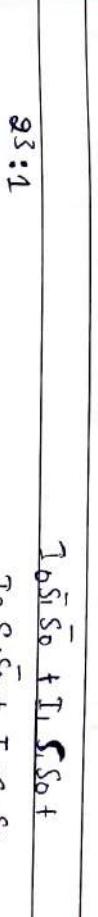
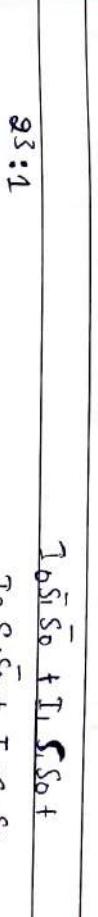
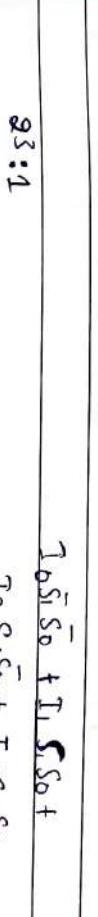
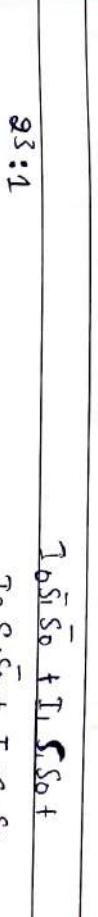
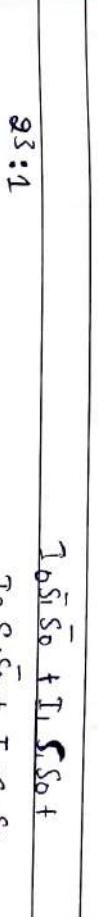
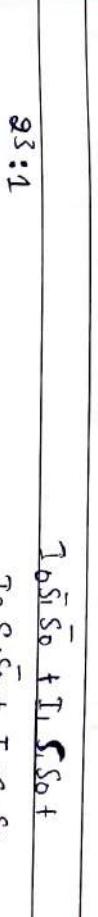
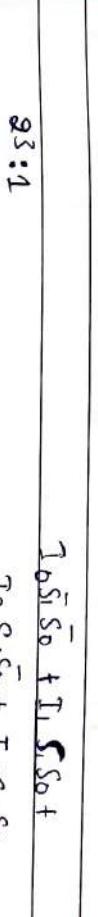
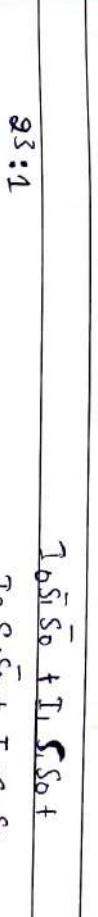
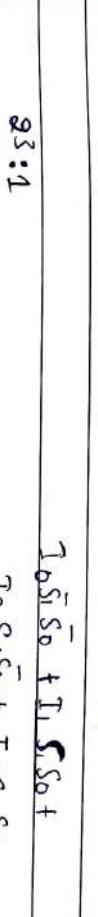
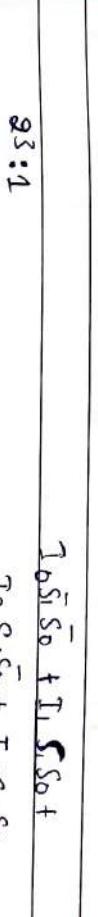
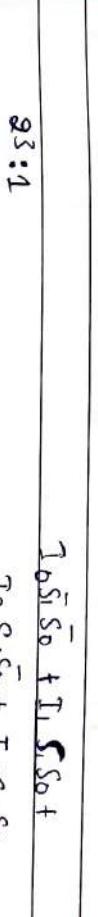
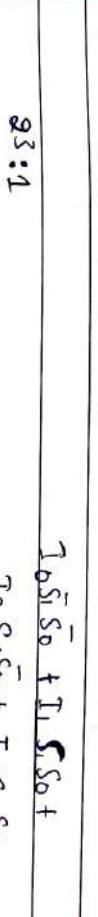
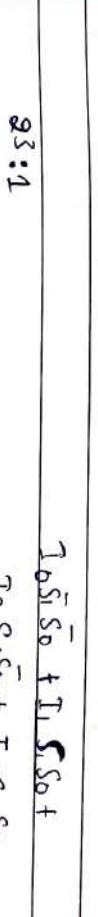
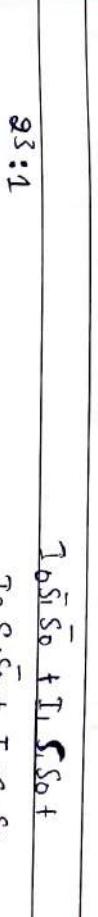
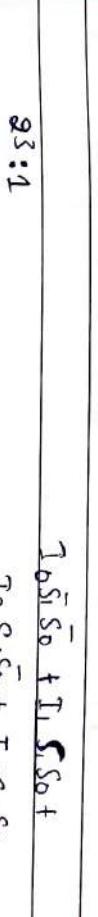
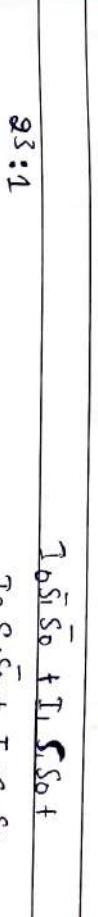
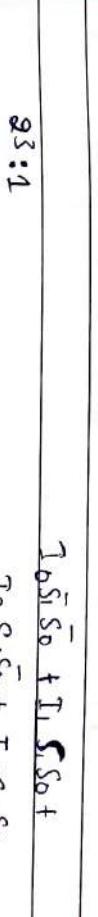
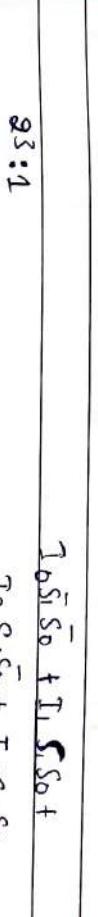
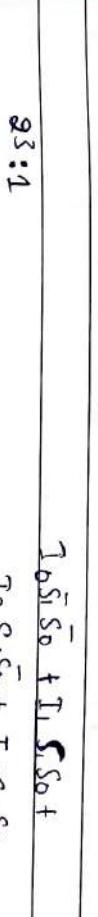
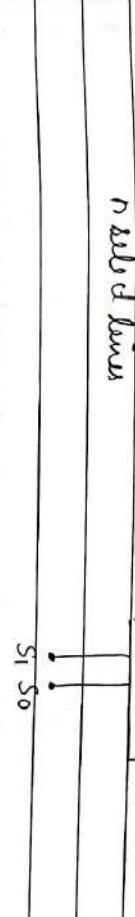
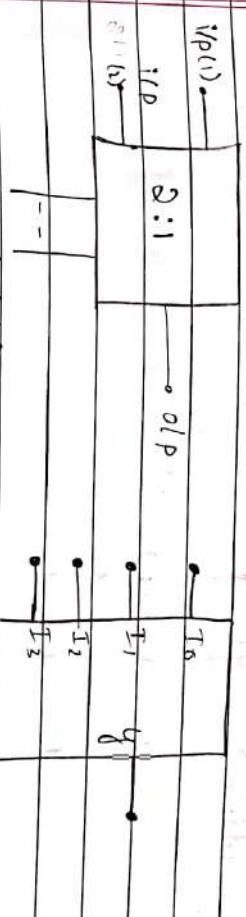
$$M_{13} \rightarrow M_{13}$$

$$M_{14} \rightarrow M_{14}$$

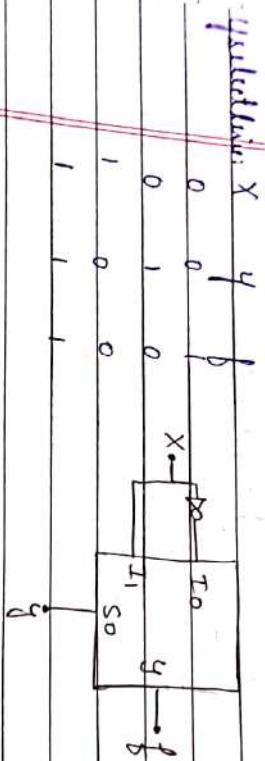
$$M_{15} \rightarrow M_{15}$$

Multiplexers:- It is MSI component (10-150 gates in IC)

If in 2^n:1 form

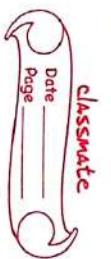
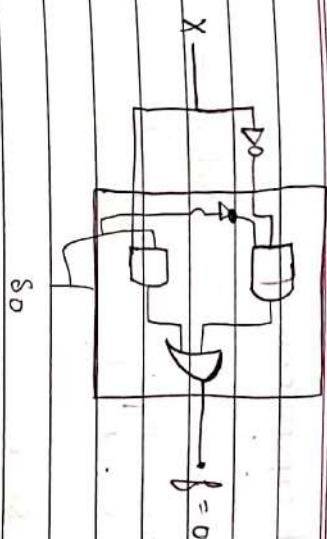


Ex: $f(x,y) = \Sigma m(0,1,3)$ using 2:1



Implement the following

$f(x_1, y_1, z) = \bar{x}_1 z + x_1 y_1 + \bar{y}_1 z$ Using 4:1 multiplexers & 2:1 as selective (without using truth table).



K-map:



Logic

vcc T₀

T₁

y₁

f

z

T₂

vcc

T₃

vcc

S₀

y₀

f

x₁

vcc

S₁

s₀

y₁

f

x₁

vcc

T₀

vcc

T₁

vcc

T₂

vcc

T₃

vcc

S₀

y₀

f

x₁

vcc

T₀

vcc

T₁

vcc

T₂

vcc

T₃

vcc

S₁

s₀

y₁

f

x₁

vcc

T₀

vcc

T₁

vcc

T₂

vcc

T₃

vcc

S₀

y₀

f

x₁

vcc

T₀

vcc

T₁

vcc

T₂

vcc

T₃

vcc

S₁

s₀

y₁

f

x₁

vcc

T₀

vcc

T₁

vcc

T₂

vcc

T₃

vcc

S₀

y₀

f

x₁

vcc

T₀

vcc

T₁

vcc

T₂

vcc

T₃

vcc

S₁

s₀

y₁

f

x₁

vcc

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vcc

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vcc

T₃

vcc

S₀

y₀

f

x₁

vcc

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vcc

S₁

s₀

y₁

f

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y₀

f

x₁

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vcc

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y₁

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vcc

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vcc

S₀

y₀

f

x₁

vcc

T₀

vcc

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vcc

T₂

vcc

T₃

vcc

S₁

s₀

y₁

f

x₁

vcc

T₀

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T₂

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y₁

f

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vcc

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T₃

vcc

S₀

y₀

f

x₁

vcc

T₀

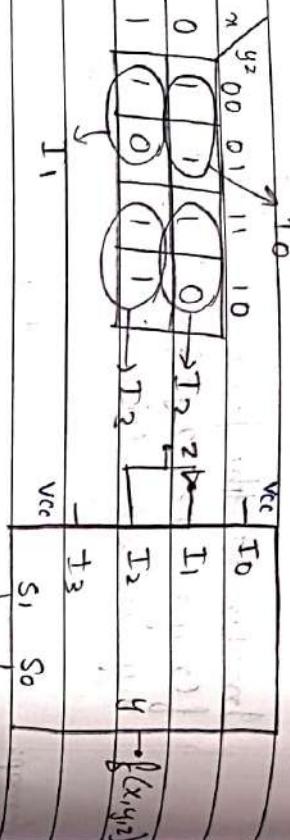
vcc

2. Mux input expressions.

$$\left. \begin{array}{l} (x,y) = (0,0) = 1 \\ (x,y) = (0,1) = z \\ (x,y) = (1,0) = \bar{z} \\ (x,y) = (1,1) = 1 \end{array} \right\} \text{dependent on } z$$

$$T_1^o = f(z)$$

② Taken, $S_1 = y$, $S_0 = x$



K-map:-

y^2	00	01	11	10	T_0
00	1	0	1	0	$\rightarrow T_2$
01	1	0	0	1	$\rightarrow T_3$
11	0	0	0	0	
10	0	1	1	0	$\rightarrow T_3$

y^2	0	1	T_1^o
0	0	0	0
1	1	0	1
0	1	1	1
1	0	1	0

Input mux expressions:

(S_1, S_0)

$$(y, x) = (0,0) \quad T_0 = \bar{z}$$

$$(y, x) = (0,1) \quad S_0 = x$$

$$(y, x) = (1,0) \quad T_1 = \bar{x}z$$

$$(y, x) = (1,1) \quad T_2 = z$$

OR

$$(x, y) = (0,0)$$

$$(x, y) = (1,0)$$

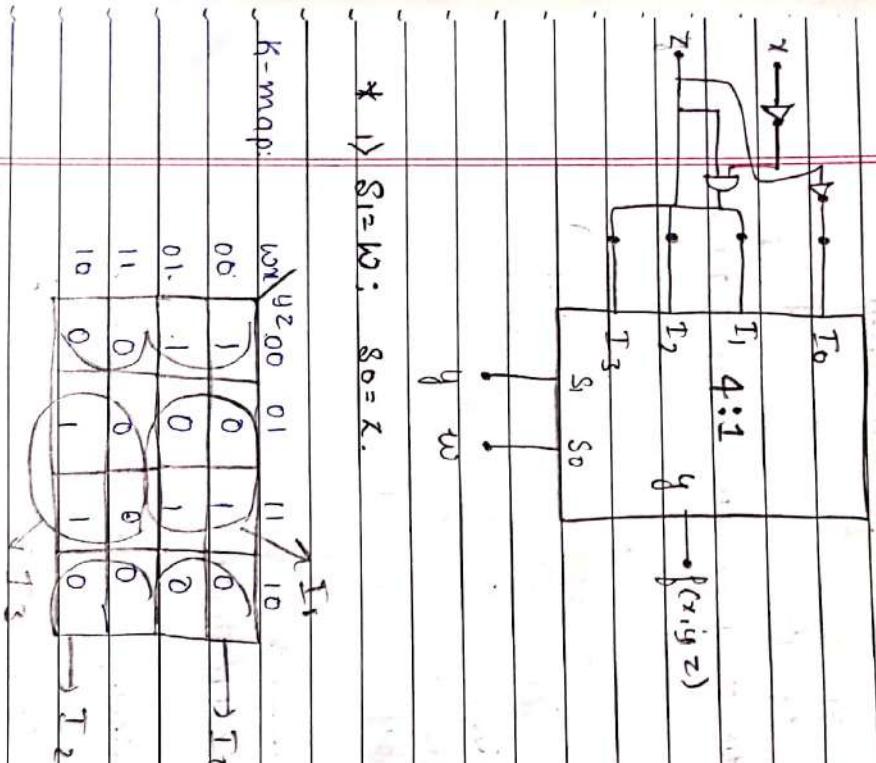
$$\text{Let say } f(x, y) = T_1^o$$

$$(x, y) = (0,1)$$

$$(x, y) = (1,1)$$

x^2	0	1									
0	1	0	0	0	1	0	0	1	0	0	1
1	0	1	1	0	0	1	1	0	1	0	0
0	0	0	0	1	0	0	1	0	0	1	0
1	1	0	1	1	0	1	0	1	1	0	0

Logic Diagram:



* 1) $S_1 = \bar{w}$; $S_0 = z$.

Input many expressions:

(S_1, S_0)

(w, z) $(0, 0)$ $T_0 = \bar{y}$

(w, z) $(0, 1)$ $T_1 = y$

(w, z) $(1, 0)$ $T_2 = \text{gnd}$

(w, z) $(1, 1)$ $T_3 = \bar{x}$

$$T_1 = f(x, y, z)$$

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K-map for individual terms:

		z\y	0	1	0	1
		w	0	0	0	1
		z	0	0	1	0
T ₀	0	1	0	0	0	1
T ₀	1	0	1	0	1	0

		z\y	0	1	0	1
		w	0	0	0	1
		z	0	0	1	0
T ₁	0	0	0	0	0	1
T ₁	1	0	1	0	1	0

T_0

T_1

T_2

T_3

$T_0 = \bar{y}$

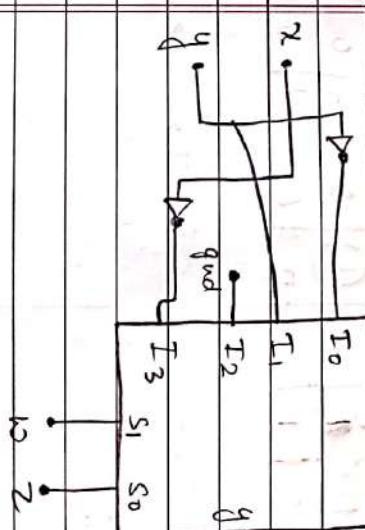
$T_1 = y$

$T_2 = \text{gnd}$

$T_3 = \bar{x}$

ground.

Logic Diagram:



*

$S_1 = x$ $S_0 = y$

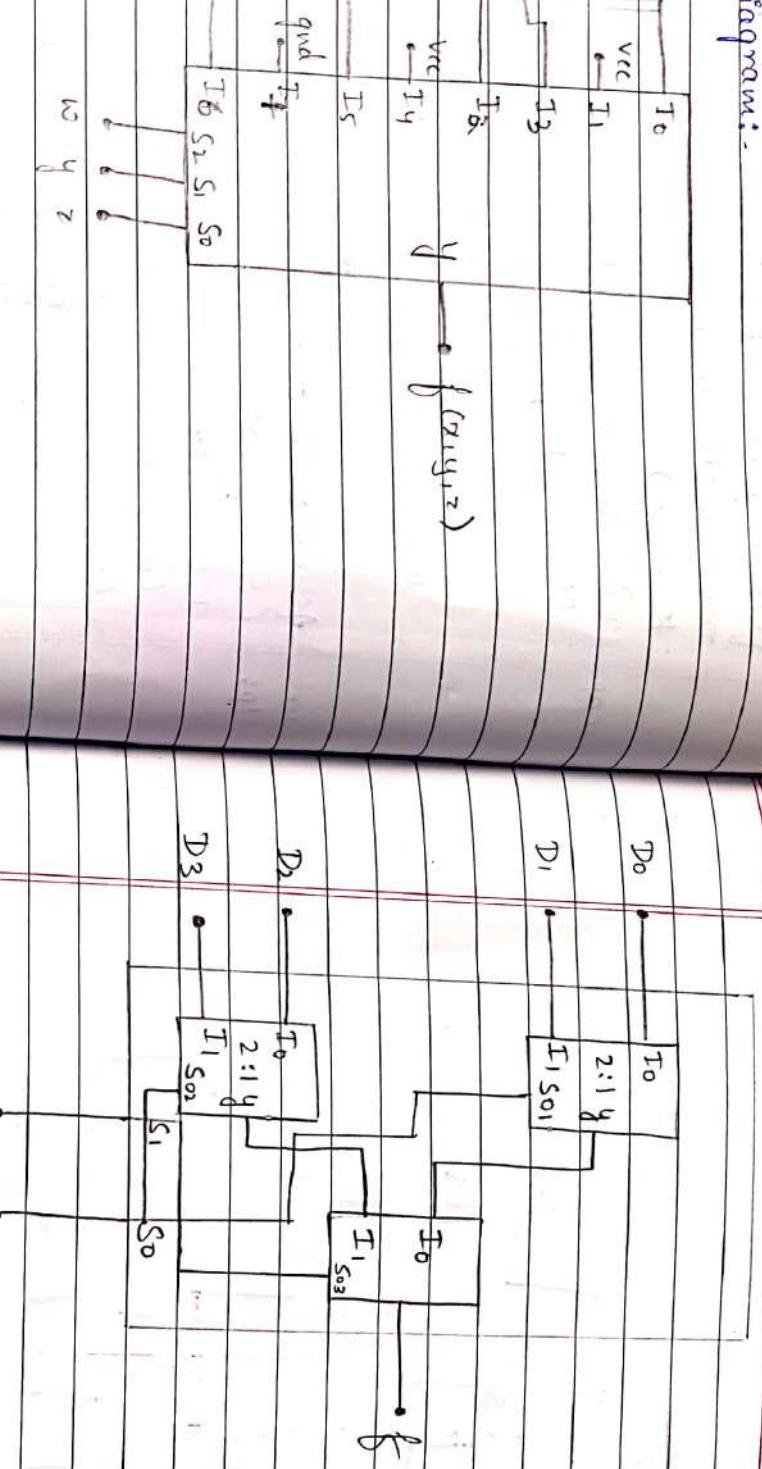
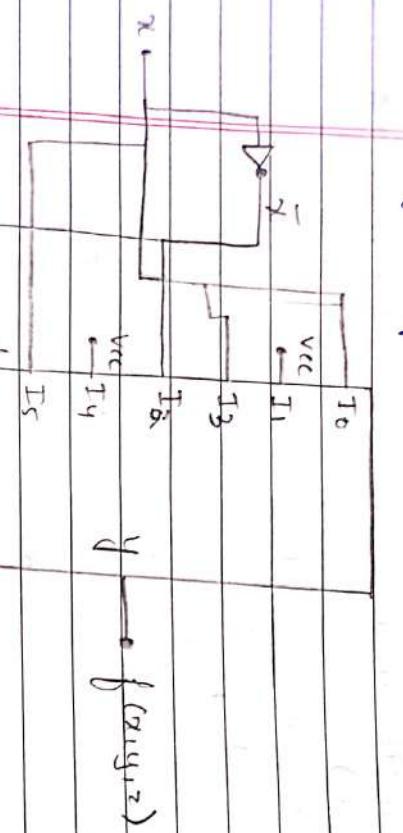
Assuming $S_1 = x$, $S_0 = y$

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Logic Diagrams:-

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* Implement 4:1 using 2:1

S_1, S_0 , S_0, S_0, S_{03}

0 0	D_0	0	0
0 1	D_1	1	0
1 0	D_2	0	1
1 1	D_3	1	1

* Implement 8:1 using 4:1

S_1, S_0 , S_0, S_0, S_{03}

Multiplexer tree

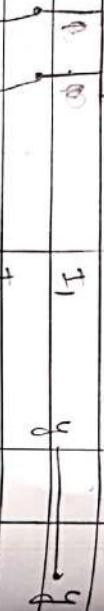
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K-map:

y_2	00	01	10	11
00	1	0	1	0
01	1	0	1	1
10	0	0	0	0

D_0	T_0	$4:1$	y	$0 \cup 1, 15, 2 \setminus 6$
D_1	T_1	y		
D_2	T_2			
D_3	T_3	$S_1 \ S_0$		



$T_3 \ S_1 \ S_0$

T_2

T_1

T_0

S_1

S_0

D_3

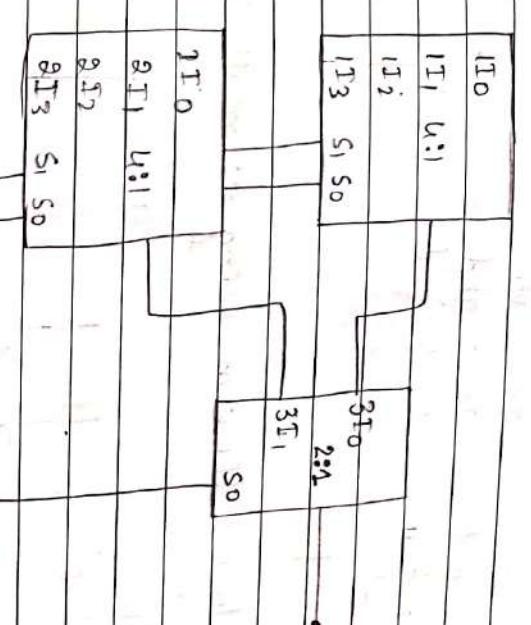
D_2

D_1

D_0

- * Implement the following function using a tree free with y as a select line to the second level mux and w_0 as select line to first level mux. Use 4:1 mux

$$f_{(x,y,z)} = \Sigma m(0,3,4,6,7).$$



$3T_0 \rightarrow T_0$

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00	11	10	11	10	00	01	11	10
0	1	0	0	1	0	1	1	0
1	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0

$\rightarrow 2T_1$

00	01	11	10	00	01	11	10	00	01	11	10
0	1	0	0	0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

$\rightarrow 2T_2$

$$f(\omega) = T_i$$

D ₀	D ₄	D ₆	D ₂	D ₀	D ₄	D ₆	D ₂	D ₀	D ₄	D ₆	D ₂
0	1	0	0	0	1	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

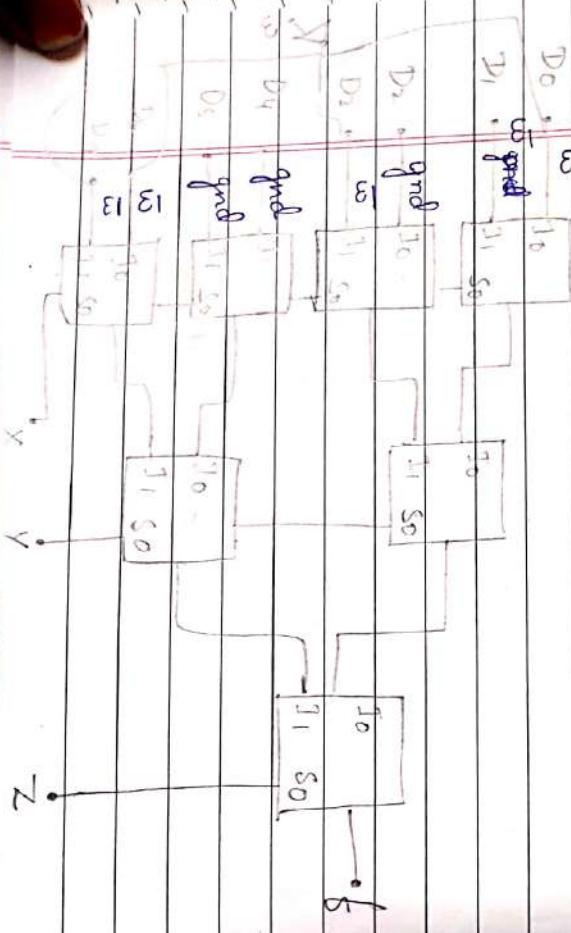
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$$\begin{aligned} T_0 &= \bar{z} \\ T_4 &= z \\ T_5 &= V_{CC} \\ T_2 &= \text{gnd} \quad T_6 = \text{gnd} \\ T_3 &= \text{gnd} \quad T_7 = \text{gnd}. \end{aligned}$$

$T_1 = f(C_2)$

Implement the same using 2:1 multiplexer.

$$\begin{aligned} D_0 &= T_0 = \bar{\omega} & \bar{\omega} &= T_1 \\ D_4 &= \text{gnd} & D_5 &= \text{gnd} \\ D_2 &= \text{gnd} & D_6 &= \bar{\omega} \\ D_3 &= \bar{\omega} & D_7 &= \omega \end{aligned}$$



SEQUENTIAL CIRCUITS

↑ NORMAL

Bistable element is capable of storing one bit

If $S=0$ then bit stored in this element is 0.

complemented the bit stored is called as state

if $S=1$ it is next state
if it is set state

Both of them can be one but it depends on manufacturing and diff in response times

If both occur same time then they don't become stable it is called metastable state.

At some point of time one gate will respond in less time than other gate to stable state. metastable is also called forbidden state.

The normal o/p is the value stored or the state of the flipflops. The complement o/p is always the complement of normal o/p. It is called bistable element :: it can take

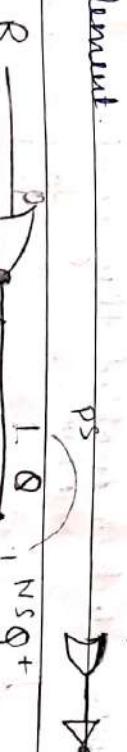
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two stable states (set or reset)

If we want to store the user intended state then we need external input

SR-latch

S and R can control the state of the bistable element. SR are external inputs to bistable element.



Present state
and next

state may

be same
but there

change of

previous input

Q+ and Q-bar+

is next state

NS state depends on change in IP & previous us inputs

S.C. are the circuits in which o/p's are not just function of IP's but also previous history of IP's

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Type of SC:

- i) Synchronous circuit (ASC)
ii) Asynchronous circuit (ASC)

ASC: Change in input will immediately change the o/p

SSC: Change in input will not immediately change out. They do it in discrete intervals of time.

Most of hardware are connected to single clock when clock is enabled they react. If it is connected to high level to make clock



\uparrow low level \uparrow

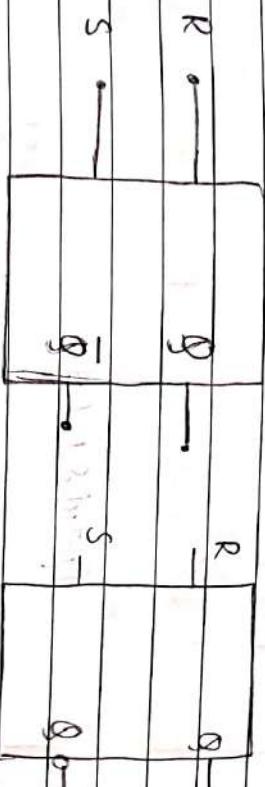
Block representation of SR-latch.

edge triggered : high or low
we can trigger it only when edge of that
prior to inputs
edge triggered.

latch SR latch is also asynchronous sc that
responds immediately on o/p change only
when enabled.

at that that o/p's corresponding
o/p is given

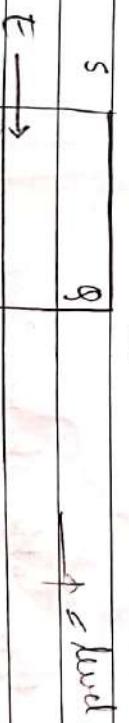
An input at the points other than triggered
points are omitted



R	S	Q^+	Q^-
0	0	0	0
0	1	1	0
1	0	0	1
1	1	0*	0*

metastable

R	0
	Q



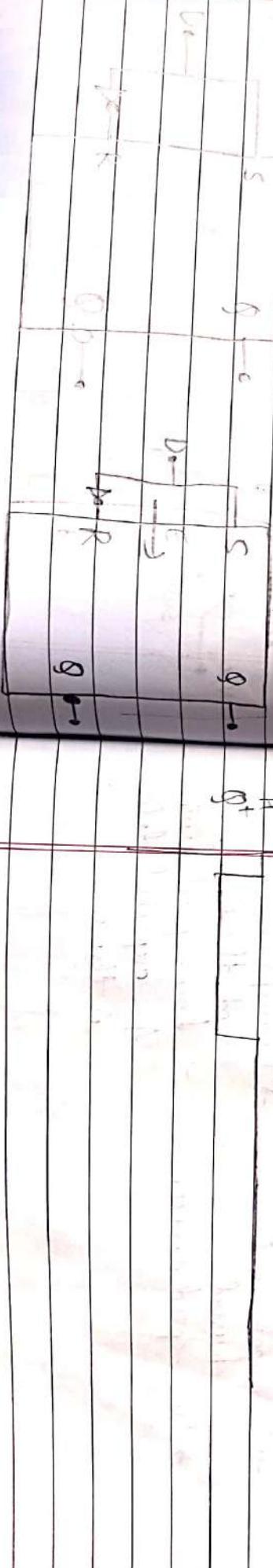
\uparrow high level \uparrow

bouncing effect

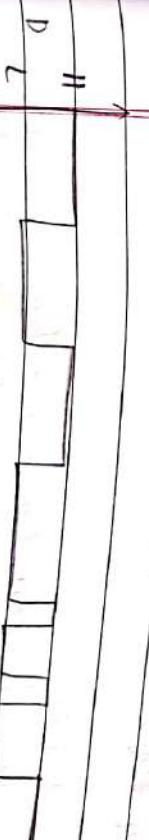
Q.P

when switch is high $S = 1$; $R = 0$ so it sets than during bouncing switch is low so $S = 0$; $R = 1$ so it retains against it bounces so $S = 1$; $R = 0$ so it is set and again retains the state to keep it in reducing the bouncing effect. Then some circuits are used to avoid it when during bouncing switch is low the methods pull it down the value to 0

D-latch:



D-latch



Initial low

Input D-latch.



O - 3
2 bits.



Flip-flops \leftarrow Pulse T / Level T (Master-Slave)
edge triggered

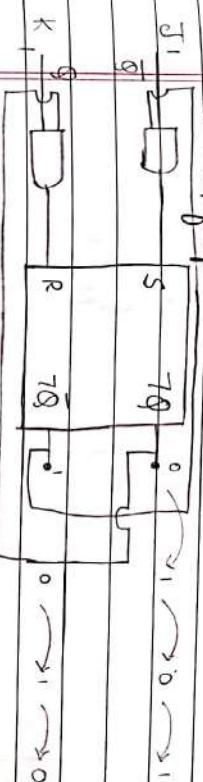


JK-Flipflop:-

In true pulse master takes the pulse in -ve pulse slave takes the pulse or state or it takes the state initially slave retains because there is no change in its up or no change in state.

The master retains because its disabled even while storing a state it takes the new ips and only after the complete cycle op is reflected the state of FF is effected after one complete cycle.

In middle one change but state of FF is not affected



JK-Flipflop

D-Flipflop (SR with D-latch):-

0	0	1	0	1	0	0
1	0	0	1	0	1	0
0	1	0	1	0	1	0
1	0	1	0	1	0	1

J

K

S

R

T

Q

Q-bar

flip-flop can store value even when external ip change

function table for SR flip-flop:

clk S R Q+ Q-bar+

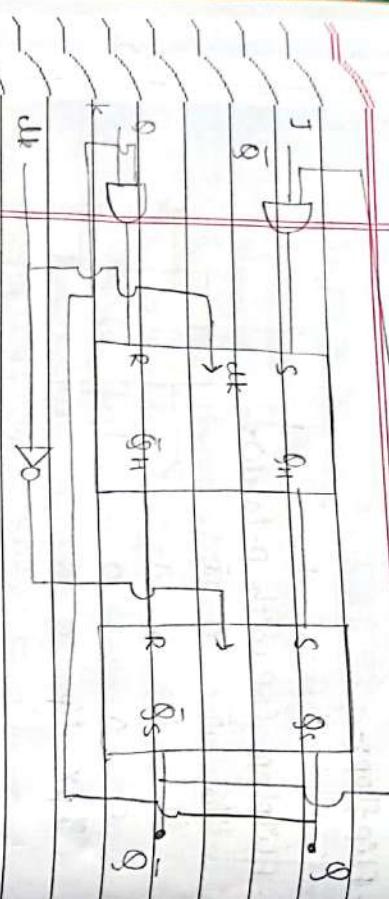
0	X	X	Q+	Q-bar+
1	0	0	Q+	Q-bar+
1	0	1	Q-	Q+
1	1	0	Q-	Q+
1	1	1	Q+	Q-

clk	J	K	Q+	Q-bar+
0	0	0	Q+	Q-bar+
0	0	1	Q-	Q+
1	0	0	Q-	Q+
1	1	0	Q+	Q-
1	1	1	Q+	Q-

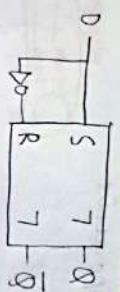
Limitation of LSR flip flop
which is overcome by JK flip flop.

forbidden

Jk flip flop:



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D -> S 7-Q
D -> R 7-Q-bar
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Sequential Circuits :-
Mo. Mo. M₁
Max. Max. M₀ S₀ M₀ M₁
Sx. Sx. S₁ M₁

Q



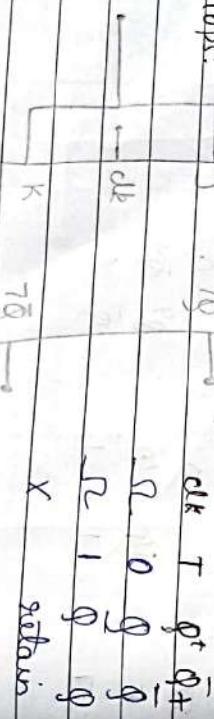
D
clk
Q

Jk flip flop :- T-flip flop

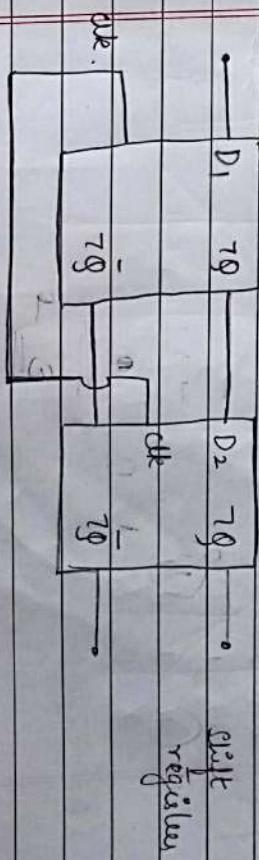
Serial in serial out
Serial in parallel out



T-Jk flip flop:



T -> J
T -> K
J -> Q
K -> Q-bar
clk
Q
Q-bar



clk
Q1
Q2
Q3
Q4
shift
register

Serial in serial out
Serial in parallel out

1 0 1 0 1 0 1

Edge triggered is more responsive
than pulse triggered

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Characteristic Equations:- (SR-flipflop)
Derivation of characteristic eqns for flip

flops

$$\cancel{Q^+} = f(i/p, Q)$$

S	R	\bar{Q}^+	Q^+
0	0	0	1
0	1	1	0
1	0	0	0
1	1	-	-

$\rightarrow P_1$

$$Q' = S + \bar{R}Q$$

$$-OR - R \cdot (S + Q)$$

Function-table:

T	K	Q^+	\bar{Q}^+	clk
0	0	0	1	0
0	1	0	1	0
1	0	1	0	0
1	1	1	0	0

i/p	i/p	Q^+	\bar{Q}^+
T	K	Q^+	\bar{Q}^+
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	1	1	0

Next-state table:

T	K	Q	Q^+	\bar{Q}^+
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	1	1	0	1

f is the function
is set of external
inputs and present

state. Using function
table we design
the next state table

T	K	Q	Q^+	\bar{Q}^+
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	1	1	0	1

$$Q^+ = \bar{K}Q + \bar{T}\bar{Q}$$

$$Q^+ = (T + Q) \cdot (\bar{T} + \bar{Q})$$

JK-flipflop:- (fun

T	K	Q^+	\bar{Q}^+
0	0	0	1
0	1	0	1
1	0	1	0
1	1	1	0

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* D-flipflop: Next state table

D	Q^+	\bar{Q}^+	D	Q	\bar{Q}
0	0	1	0	0	1
0	0	0	0	0	0
0	1	0	1	1	0
1	0	1	0	0	1
1	1	0	1	1	0

$$SOP = D = Q +$$

$$Q^+ = \bar{D}$$

* T flipflop: Next state table

T	Q	Q^+	\bar{Q}^+	T	Q	\bar{Q}
0	0	0	1	0	0	1
0	0	0	1	0	0	1
0	1	1	0	1	1	0
1	0	1	0	1	1	0

$$Q^+ = T\bar{Q} + \bar{T}Q$$

$$= T \oplus Q$$

T flipflop:- function table

$$Q^+ = \bar{T}Q + T\bar{Q}$$

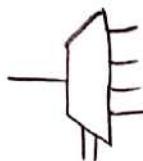
$$= (T + Q) \cdot (\bar{T} + \bar{Q})$$

SR-flipflops:-

clk	T	Q^+	\bar{Q}^+
0	0	0	1
1	0	1	0
x	1	1	1

clk	R	S	Q^+	\bar{Q}^+
0	0	0	0	1
1	1	0	1	0

clk	Set	Reset	Q^+	\bar{Q}^+
0	0	0	0	1
1	1	0	1	0



1:2ⁿ (demux)
n selected lines

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Decimal Adder:-

BCD - binary coded decimal and not the exact decimal (its all binary)

(0-9)

Answer

Decimal no

4

BCD

0100

Binary

0100

BCD

14

0001 0100

1110

256 entries

Every digit \rightarrow 4 bits

A = 9 B = 9

(0-9) (0-9)

0001 1000 (18)

✓ always 5 bits

D₃ D₀ D₃ D₀

4Bit BA

C S₃ S₂ S₁ S₀

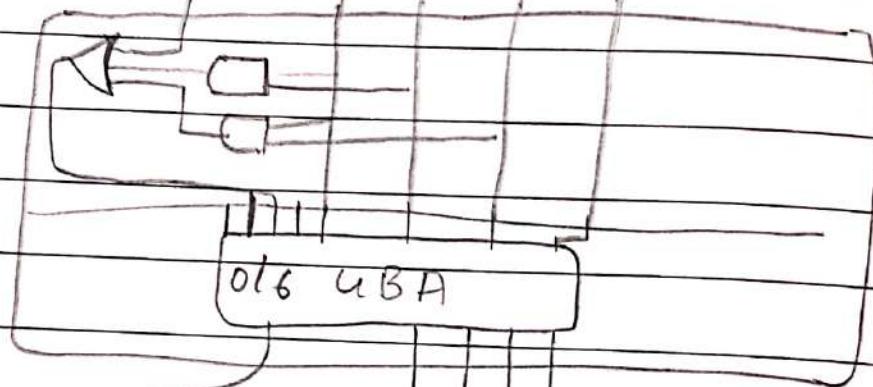
S₃ S₂ S₁ S₀

0	0	0	0	
0	0	0	0	
1	1	1	1	
0	0	1	1	

4-bit BA

Cu. | | 0 0

S3 S2 S1 SD



| 0 0 1 0

Z₃ Z₂ Z₁ Z₀