20/09/19

Unit 02

MATRICES & SYSTEM OF LINEAR EQUATIONS

★ Row Elementary Transformation :- ① Interchange of any two rows.

$$R_i \longleftrightarrow R_j.$$

E.g.:
$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 2 \\ 0 & 3 & 2 \end{bmatrix} \begin{matrix} \dashrightarrow R_1 \\ \dashrightarrow R_2 \\ \dashrightarrow R_3 \end{matrix}$$

$$R_2 \longleftrightarrow R_3$$

$$A \underset{\text{(equivalent to)}}{\sim} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 2 \\ 0 & -1 & 2 \end{bmatrix} \begin{matrix} \dashrightarrow R_1 \\ \dashrightarrow R_2 \\ \dashrightarrow R_3 \end{matrix}$$

★② Multiplication of any row by a non-zero scalar 'K' :-

$$R_2 \longrightarrow 3R_2$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 2 \\ 0 & 3 & 2 \end{bmatrix} \qquad A \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & 6 \\ 0 & 3 & 2 \end{bmatrix}.$$

③ Addition to any row with constant multiple of any other row :-

$$R_3 \rightarrow R_3 + 3R_2$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 2 \\ 0 & 3 & 2 \end{bmatrix} \qquad A \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & +8 \end{bmatrix}$$

★ Equivalent matrices : Two matrice A & B are said to be equivalent if one matrix can be obtained from the other by finite no. of successive elementary row transformations. We write $\boxed{A \sim B}$.

★ Row Echelon Form: A non zero matrix A is said to be in row echelon form if the following conditions are satisfied:

(a) All the zero rows are below the non-zero rows.

(b) The entries below the leading entries in the same column are zero.

leading entry of 1st row. → non-zero (always for Echelon form).

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

→ leading entry of 2nd row.

zero

★ Rank of a matrix: The no. of non-zero rows in the Echelon form of matrix A. It is denoted by 'Rho', 'ρ'.

LP (1)

(i)

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

→ It should be non-zero. If not, then rearrange using elementary transformation.

$R_2 \to 3R_2 + R_1$

$R_3 \to 3R_3 - R_1$

$R_2 \to 3R_2 + R_1$

$R_3 \to 3R_3 - R_1$

$$A \sim \begin{bmatrix} 6 & -2 & 2 \\ 0 & 7 & -1 \\ 0 & -1 & 7 \end{bmatrix}$$

→ Next leading entry

→ to make ... 0, we use R_2 not R_1. ∵ using R_1 will change the previous 0 also.

$R_3 \to 7R_3 + R_2$

$$A \sim \begin{bmatrix} 6 & -2 & 2 \\ 0 & 7 & -1 \\ 0 & 0 & 48 \end{bmatrix}$$

→ Next leading entry.

∴ Keep R_2 as it is.

We have A which is req. echelon form of A.

∴ Rank = 03.

We have three non-zero rows, hence rank of matrix is 03.

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(ii) $\sim \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$

$$A \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

required Echelon form of A.
We have 1 non-zero rows.
$\therefore$ Rank = 1 = $\rho(A) = 3$.

(iii) $\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 10 \end{bmatrix}$

$R_3 \rightarrow R_3 - 2R_1$

$$A \sim \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 0 & -7 & 2 \end{bmatrix}$$

$R_3 \rightarrow 3R_3 + 7R_2$

writing important:

$$A \sim \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 0 & 0 & 20 \end{bmatrix}$$

which is required Echelon form of A.

Here, ~~two~~ three non-zero rows. $\therefore$ $\rho(A) = 3$.

(iv) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$

$$A \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & -5 & -7 \end{bmatrix}$$

$-7+25=$

$R_3 \to R_3 - 5R_2$

$$A \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & 0 & 18 \end{bmatrix}$$

→ required Echelon form.

Here, 03 non-zero rows.

$\therefore \rho(A) = 3.$

Ques :-

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 3 & 1 & 2 & 0 \\ 1 & -1 & 3 & 1 \\ 3 & 5 & 1 & 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & 3 & 1 \\ 3 & 1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 3 & 5 & 1 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_4 \to R_4 - 3R_1$

$1 \quad -1 \quad 3 \quad 1$

$0 \quad 4 \quad -4$

System of Linear Equation :

E.g.: $2x + 3y - z + 1 = 0$
$3x + z - 1 = 0$
$y + x + z = 0$

Rewriting the above eqn :-

$$\left.\begin{aligned} 2x + 3y - z &= -1 \\ 3x + 0y + z &= 1 \\ x + y + z &= 0 \end{aligned}\right\} \text{ — } (1)$$

$$AX = B$$

i.e.

$$\begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

here, $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

the matrix 'A' is called Coefficient matrix of eqn (1).

Augmented Matrix of eqn (1) can be written as :-

$$A|B = \left[\begin{array}{ccc:c} 2 & 3 & -1 & -1 \\ 3 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right]$$

* **Consistency & Inconsistency of the linear system of equation (LSE):**

LSE

Consistent
$\rho(A)=\rho(A|B)$

Inconsistent
$\rho(A)\neq\rho(A|B)$

Unique Sol^n^

$\rho(A)=\rho(A|B)=n$
'n' = no. of unknowns in LSE.

Infinite Solution
$\rho(A)=\rho(A|B)<n$

NOTE

1). If in the matrix form of LSE $B=0$ then the given linear system of eq^n^ (LSE) is called Homogeneous LSE.

Otherwise, LSE is called Non-Homogeneous LSE.

2). If the solution of unknowns (LSE) is zero (i.e. all unknowns are zero) then, we can say that the linear system of eq^n^ has Trivial sol^n^.
Otherwise, it has non-trivial sol^n^.

LP

Q. (i)
$$\left.\begin{aligned} 2x - 3y + 7z &= 5 \\ 3x + y - 3z &= 13 \\ 3x + 19y - 47z &= 32 \end{aligned}\right\} \text{— (1)}$$

The Augmented matrix of eqn. (1) is given below:

$x=5$
$y=14.5$
$z=5.5$

$$[A|B] = \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 3 & 19 & -47 & 32 \end{array}\right]$$

$R_2 \longrightarrow 2R_2 - 3R_1$
$R_3 \longrightarrow 2R_3 - 3R_1$

$\begin{array}{r} 26 \\ -15 \\ \hline 11 \end{array}$

$$= \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 0 & 47 & -115 & 49 \end{array}\right]$$

$47 \to 0$

$R_3 \to 11R_3 - 47R_2$

$$= \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 0 & 0 & 4 & 22 \end{array}\right] \text{— (2)}$$

→ Echelon form of Augmented matrix.

From eqn. (2)

$\rho(A) = 3 \qquad \rho[A|B] = 3 \qquad n = 3$

So $\because \rho(A) = \rho[A|B] = n$

$\therefore$ The given system is consisted & it has unique solution.

Eqn (2) can be written as :-

$$2x - 3y + 7z = 5 \quad \text{—} (3)$$
$$11y - 27z = 11 \quad \text{—} (4)$$
$$4z = 22 \quad \text{—} (5)$$

from eqn (5) $z = \frac{22}{4} = \frac{11}{2} = 5.5$

from (4) $y = \frac{1}{11}\left[11 + 27\left(11/_2\right)\right] =$

$$= 1 + \frac{27}{2}$$

$$= 1 + 13.5 = 14.5$$

$$x = \frac{1}{2}\left[3(14.5) - 7(5.5)\right] = 5.$$

Ques : $x = -y - z + 1$
$2x + 2y + 2z = 2$
$3x + 3y = -3z + 3$

$$\left.\begin{aligned} x + y + z &= 1 \\ 2x + 2y + 2z &= 2 \\ 3x + 3y + 3z &= 3 \end{aligned}\right\} \text{—} (1)$$

Augmented matrix of eqn (1) :-

$$[A|B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 0 & 0 & : & 0 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$\rho[A] = 1 \qquad \rho[A|B] = 1 < n$

$\therefore$ the given system is consisted & it has infinitely many solution.

let $z = c_1$

$y = c_2$

$\therefore$ we have. $x + y + z = 1$

$x = 1 - c_1 - c_2$.

The solⁿ of the eqⁿ ① is :-

- $x = 1 - c_1 - c_2$
- $y = c_2$
- $z = c_1$

Ques

$$\left.\begin{aligned} 2x + 2y + 2z &= 2 \\ 3x + 3y + 3z &= 2 \\ x + y + z &= 2 \end{aligned}\right\} - ①$$

Augmented matrix of eqⁿ ① :-

$$[A|B] = \begin{bmatrix} 2 & 2 & 2 & : & 2 \\ 3 & 3 & 3 & : & 2 \\ 1 & 1 & 1 & : & 2 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 3 & 3 & 3 & : & 2 \\ 2 & 2 & 2 & : & 2 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$2 - 3 \times 2$

$2 - 6 = -4$

$$\begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 0 & 0 & 0 & : & -4 \\ 0 & 0 & 0 & : & -2 \end{bmatrix}$$

$\rho[A] = 1 \qquad \rho[A|B] = 3$

$\because \rho[A] \neq \rho[A|B]$.

$\therefore$ The given system of eqn. is inconsistent.

LP ⑧.

$$\left.\begin{aligned} x+y+z &= 6 \\ x+2y+3z &= 10 \\ x+2y+\lambda z &= \mu \end{aligned}\right\} - ①$$

i) a unique soln.
ii) infinitely many solutions.
iii) no solution

Soln.: Converting into Echelon Form:
Augmented matrix of the matrix form of eqn. ① :-

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$.

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & \lambda-1 & : & \mu-6 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix}$$ → required Echelon form

Case 1 : It has no solution :-

$\Rightarrow \rho[A] \neq \rho[A|B]$

this happens if $\lambda - 3 = 0$ & $\lambda - 10 \neq 0$

$\lambda = 3$ & $\mu \neq 10$

Case 2 : For unique Solution :-

$\Rightarrow \rho[A] = \rho[A|B] = 3$ (no. of unknowns)

$\Rightarrow$ this happens if $\lambda - 3 \neq 0 \Rightarrow \boxed{\lambda \neq 3}$

$\mu - 10$ can be zero cannot be zero.

Case 3 :

For infinitely Solution :-

$\rho[A] = \rho[A|B] < 3$.

$\Rightarrow \lambda - 3 = 0$ & $\mu - 10 = 0$

$\Rightarrow \boxed{\lambda = 3}$ & $\boxed{\mu = 10}$

9.

$$-2x + y + z = a$$
$$x - 2y + z = b$$
$$x + y - 2z = c$$

First write linear system of eqn in matrix form then augmented matrix in exam:

$$[A|B] = \left[\begin{array}{ccc:c} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & c \end{array}\right]$$

$R_2 \to 2R_2 + R_1$

$R_3 \to 2R_3 + R_1$

$$[A|B] \sim \left[\begin{array}{ccc:c} -2 & 1 & 1 & a \\ 0 & -3 & 3 & a+2b \\ 0 & 3 & -3 & a+2c \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 0 & -3 & 3 & a+2b \\ 0 & 0 & 0 & 2a+2b+2c \end{array}\right] \quad \text{--- (2)}$$

which is req. Echelon form of $[A|B]$

It is consistent if $\rho[A|B] = \rho[A]$

$\Rightarrow 2a+2b+2c=0$

$\Rightarrow \boxed{a+b+c=0}$

Hence, proved.

Now, Put $a=1$ $b=1$ $c=-2$ in eqn (2)

$\therefore$ ~~$-2x+y+z=1$~~

$$\left[\begin{array}{ccc|c} -2 & 1 & 1 & 1 \\ 0 & -3 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

this can be written as L-S-E :-

$-2x+y+z=1$ --- (3)

$-3y+3z=3$ --- (4)

$z=c$

Eqn (4) becomes :- ~~$-3y+c=3$~~

~~$c=3+3y$~~

$\Rightarrow$ ~~$\boxed{\dfrac{c-3}{3}=y}$~~

$-y+z=1$

$-y+c=1$

$y=c-1$

$\therefore -2x+c-1+c=1$

$-2x+2c-2=0$

$-x+c-1=0$

$x=c-1$

Ques: $x+y+z=2$

$2x+y-z=K$

$3x+y=K^2$

~~Check if it is has consistent~~

Find the condⁿ, where the given LSE is consistent & inconsistent?

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & K \\ 3 & 1 & 0 & K^2 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$-1-2$

$K-$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & K-4 \\ 0 & -2 & -3 & K^2-6 \end{array}\right]$$

$K^2-6-2K+8$

K^2-2K+2

$R_3 \to R_3 - 2R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & K-4 \\ 0 & 0 & 3 & K^2-2K+2 \end{array}\right]$$

Consistent $\rho[A] = \rho[A|B]$

$\Rightarrow \underset{(3)}{\rho[A]} = \underset{(3)}{\rho[A|B]}$

$\therefore$ It is consistent for any value of 'K'.

So, it will never be inconsistent.

Gauss' Elimination Method:

In this method, the unknowns are eliminated successively & the system is reduced to an upper triangular system from which the unknowns are found by back substitution.

③ (i)
$$\left.\begin{aligned} 2x+y+z&=10 \\ 3x+2y+3z&=18 \\ x+4y+9z&=16 \end{aligned}\right\} -①$$

Augmented matrix of the matrix form of eqⁿ ①, we get :-

$$[A|B] = \left[\begin{array}{ccc:c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

upper Δ^r

$\downarrow 0$

$R_1 \leftrightarrow R_3$

$$\approx \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 3 & 2 & 3 & 18 \\ 2 & 1 & 1 & 10 \end{array}\right]$$

$R_2 \rightarrow R_2 - 3R_1$
$R_3 \rightarrow R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & -7 & -17 & -22 \end{array}\right]$$

$R_3 \rightarrow 10R_3 - 7R_2$

$$[A|B] \sim \begin{bmatrix} 1 & 4 & 9 & : & 16 \\ 0 & -10 & -24 & : & -30 \\ 0 & 0 & -2 & : & -10 \end{bmatrix} \quad -(2)$$

Eqn (2) can be written as follows:-

$$x + 4y + 9z = 16 \quad -(3)$$
$$-10y - 24z = -30 \quad -(4)$$
$$-2z = -10 \quad -(5)$$

from (5) $z = 5$

from (4) $-10y = -30 + 24 \times 5$

$y = -9$

from (3) $x = 16 + 36 - 45$

$x = 7$

Gauss' Jordan method : This is the extension/ modification of Gauss' Elimination method.

In this method, the given system is reducing into diagonal matrix form i.e. each eqn will have only one ~~algorit~~ unknown.

For each eqn & the unknowns can be obtained readily.

P.T.O

Same question by Gauss' Jordan method:

$$\left.\begin{aligned}2x+y+z&=10\\3x+2y+3z&=18\\x+4y+9z&=16\end{aligned}\right\}-①$$

Augmented matrix of the matrix form of eqn ①

$$[A|B] = \left[\begin{array}{ccc:c}2 & 1 & 1 & 10\\3 & 2 & 3 & 18\\1 & 4 & 9 & 16\end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$[A|B] \sim \left[\begin{array}{ccc:c}1 & 4 & 9 & 16\\3 & 2 & 3 & 18\\2 & 1 & 1 & 10\end{array}\right]$$

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - 2R_1$

$R_3 \to 10R_3 - 7R_2$

$$[A|B] \sim \left[\begin{array}{ccc:c}1 & 4 & 9 & 16\\0 & -10 & -24 & -30\\0 & 0 & -2 & -10\end{array}\right]$$

0, using R_2

$R_3 \to R_3/-2$ 0 0, using R_3

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$R_1 \to R_1 - 9R_3$
$R_2 \to R_2 + 24R_3$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 4 & 0 & -29 \\ 0 & -10 & 0 & 90 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$R_1 \to 10R_1 + 4R_2$

$$\left[\begin{array}{ccc|c} 10 & 0 & 0 & 70 \\ 0 & -10 & 0 & 90 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$R_1 \to R_1/10$
$R_2 \to R_2/10$

dia. should be 1, 1, 1

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & -1 & 0 & 9 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$R_2 \to -R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$\therefore x = 7$
$y = -9$
$z = 5$

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(4) (i)

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

$$[A|B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 2 & -3 & 4 & 13 \\ 3 & 4 & 5 & 40 \end{array}\right]$$

$$R_2 \to R_2 - 2R_1$$
$$R_3 \to R_3 - 3R_1$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 1 & 2 & 13 \end{array}\right]$$

$$R_3 \to 5R_3 + R_2$$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 0 & 12 & 60 \end{array}\right]$$

$$R_3 \to (1/12) R_3$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$R_1 \to R_1 - R_3$$
$$R_2 \to R_2 - 2R_3$$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 0 & 4 \\ 0 & -5 & 0 & -15 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$R_2 \rightarrow R_2 / -5$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 0 & : & 4 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$

$\therefore\ x = 1 \quad y = 3 \quad z = 5$

V.V.of
7M

★ Gauss Seidel Method :

Always, the LSE should be diagonally dominant.

$x + y + 10z = 12$
$x + 10y + z = 12$
$10x + y + z = 12$

Rewriting the above LSE :-

$10x + y + z = 12$
$x + 10y + z = 12$
$x + y + 10z = 12$

$\therefore\ x = \frac{1}{10}(12 - y - z)$

$y = \frac{1}{10}(12 - x - z)$

$z = \frac{1}{10}(12 - x - y)$

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Compute upto III Iterations.

Assume $x_0 = 0$, $y_0 = 0$, $z_0 = 0$:-

I st Iteration :-

$\therefore x^{(1)} = \frac{1}{10}(12-0-0) = \frac{12}{10} = 1.2$

$y^{(1)} = \frac{1}{10}(12-1.2-0) = 1.08$

$z^{(1)} = \frac{1}{10}(12-1.2-1.08) = \cancel{1.2}\ 0.972$

IInd iteration :-

$x^{(2)} = \frac{1}{10}(12-y^{(1)}-(z)^{1})$

$= \frac{1}{10}(12-1.08-0.972)$

$= 0.9948$

$y^{(2)} = \frac{1}{10}(12-x^{(2)}-z^{(1)})$

$= \frac{1}{10}(12-0.9948-0.972)$

$= 1.0033$

$z^{(2)} = \frac{1}{10}(12-x^{(2)}-y^{(2)})$

$= \frac{1}{10}(12-0.9948-1.0033)$

$= 1.0002$

IIIrd Iteration :

$(x)^3 = \frac{1}{10}(12-y^2-z^2)$

$= \frac{1}{10}(12-1.0033-1.0002)$

$= 0.9997$

$$(y)^{3} = \frac{1}{10}\left(12 - (x)^{3} - z^{2}\right)$$

$$= \frac{1}{10}\left(12 - 0.9996 - 1.0001\right)$$

$$= 1.0000$$

$$(z)^{3} = \frac{1}{10}\left(12 - x^{(3)} - y^{(3)}\right)$$

$$= 1.000$$

Use calculator to verify. Enter the given LSE.

∴ The solⁿ: for given LSE is

$$\left.\begin{aligned} x &= 0.9997 \\ y &= 1 \\ z &= 1 \end{aligned}\right\} \longrightarrow \text{1 Marks}$$

~~Eigen Value~~

NOTE:

While applying Gauss Seidel method, we always check whether the given system of LSE is diagonally dominant or not.

Eigen Value:

↳ can be applied on homogeneous LSE.

↳ square matrix.

Suppose we have :-

$$a_{11}x + a_{12}y + a_{13}z = 0$$
$$b_{21}x + a_{22}y + a_{23}z = 0$$
$$a_{31}x + a_{32}y + a_{33}z = 0$$

let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

let

$$a_{11}x + a_{12}y + a_{13}z = \lambda x$$
$$a_{21}x + a_{22}y + a_{23}z = \lambda y$$
$$a_{31}x + a_{32}y + a_{33}z = \lambda z$$

$\Rightarrow AX = \lambda X$

$\Rightarrow AX = \lambda IX$

$\Rightarrow (A - \lambda I)X = 0$

$X \neq 0$ (X cannot be zero because we are finding non-zero solutions).

$\therefore A - \lambda I = 0$

determinant of $A - \lambda I$ is called Characteristic Eq^n of 'A'.

NOTE: $A - \lambda I$ is given as

$$A - \lambda I = \begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{bmatrix}$$

* Working Procedure to find Eigen value :-

1). Given a square matrix 'A'. We form $A - \lambda I = 0$

2). Then, we find characteristic eqn: (in bracket) $|A - \lambda I| = 0$

3). We solve the above eqn & we get Eigen values (λ values).

4). Then we form the system of homogeneous equation $[A - \lambda I].X = 0$. & solve for x, y, z corresponding to every value of 'λ'.

Ques:

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{bmatrix}$$

$$|A - \lambda I| = (2-\lambda)\,(2-\lambda)^2 + 1(0 - 2 + \lambda)$$

$$= (2-\lambda)^3 - (2-\lambda)$$

$$= (2-\lambda)\left[(2-\lambda)^2 - 1\right]$$

$$|A-\lambda I| = (2-\lambda)\left((2-\lambda)^2-1\right)$$
$$= (2-\lambda)\left(4+\lambda^2-4\lambda-1\right) \rightarrow \text{characteristic eq}^n$$
$$= (2-\lambda)\left(3+\lambda^2-4\lambda\right)$$

$$|A-\lambda I| = 0$$
$$(2-\lambda)\left(\lambda^2-3\lambda-\lambda+3\right) = 0$$
$$(2-\lambda)\ \lambda(\lambda-3)-1(\lambda-3) = 0$$
$$(2-\lambda)(\lambda-1)(\lambda-3) = 0$$

$\therefore$ $\lambda = 2$ $\quad$ $\lambda = 1$ $\quad$ $\lambda = 3$

Case 1 : let $\lambda = 2$

$$[A-\lambda I]X = 0$$
$$[A-2I]X = 0$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\cancel{x/z \neq 0}$

$\therefore$ Echelon form of $[A-2I]$

$$\begin{array}{c} x \leftarrow \\ z \leftarrow \\ y \leftarrow \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cancel{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$\therefore x = 0$

$y = C$ $\quad$ where $C \neq 0$.

$z = 0$

$\therefore$ For $\lambda = 2$, we get eigen vector as follow $\begin{bmatrix} 0 \\ C \\ 0 \end{bmatrix}$.

Case 2 : let $\lambda = 1$

$$[A - I]X = 0$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

we have :- $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$R_3 \to R_3 - R_1$

$$A \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

the above matrix can be written as

$x + z = 0$

$y = 0$

let $z = c$ where $c \neq 0$

$\therefore x = -c$ where $c \neq 0$.

$\therefore$ for $\lambda = 1$, we get Eigen vector as follow: $\begin{bmatrix} -c \\ 0 \\ c \end{bmatrix}$ where $c \neq 0$.

Case 3 : let $\lambda = 3$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_1$

$$A \sim \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore -x + z = 0$

$y = 0.$

$z = c$ where $c \neq 0$

$\therefore x = +c$

$\therefore$ For $\lambda = 3$, Eigen vectors are :

$$\begin{bmatrix} c \\ 0 \\ c \end{bmatrix} \quad \text{where } c \neq 0.$$

<u>Ques</u> Find Eigen value & Eigen vector :-

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix}$$

Characteristic Eqn: $|A - \lambda I| = 0.$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (6-\lambda)\left[(3-\lambda)^2-1\right]+2\left[-2(3-\lambda)+2\right]+2\left(2-2(3-\lambda)\right)=0$$

$$\Rightarrow (6-\lambda)\left[9+\lambda^2-6\lambda-1\right]+2\left[-6+2\lambda+2\right]+2\left[2-6+2\lambda\right]=0$$

$$\Rightarrow (6-\lambda)(8+\lambda^2-6\lambda)-12+4\lambda+4+4-12+4\lambda=0$$

$$\Rightarrow 48+6\lambda^2-36\lambda-\cancel{8\lambda}-\lambda^3+6\lambda^2-16+\cancel{8\lambda}=0$$

$$\Rightarrow \cancel{48}\ -\lambda^3+12\lambda^2-36\lambda+32=0$$

$$\Rightarrow \lambda^3-12\lambda^2+36\lambda-32=0$$

$$\lambda = 8, 2, 2$$

Here, we get Eigen value as 2, 8.

<u>Case 1 :</u> $\lambda = 2$

we get $[A-2I]X=0$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} 4x-2y+2z&=0 \\ -2x+y-z&=0 \\ 2x-y+z&=0 \end{aligned}\right\}$$ Here all eqn's are same, ∴ can't apply cross multiplicⁿ method.

Here, we get only one eqⁿ :-

$$2x-y+z=0$$

let $y = c_1$

$z = c_2$

$x = \frac{1}{2}(c_1-c_2)$

∴ Required Eigeon vector for $\lambda=2$ is

$$\begin{bmatrix} \frac{1}{2}(c_1-c_2) \\ c_1 \\ c_2 \end{bmatrix}$$ where $c_1 \neq 0$ or $c_2 \neq 0$.

Case 2 : $\lambda = 8$

we get,
$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x - 2y + 2z = 0 \quad \text{--- (I)}$$
$$-2x - 5y - z = 0 \quad \text{--- (II)}$$
$$2x - y - 5z = 0$$

Consider first 2 eqn :-
By applying Cross multiplication method:-

$$\frac{x}{\begin{vmatrix} -2 & 2 \\ -5 & -1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -2 & 2 \\ -2 & -1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -2 & -2 \\ -2 & -5 \end{vmatrix}}$$

$$\frac{x}{2+10} = \frac{-y}{2+4} = \frac{z}{10-4}$$

$$\frac{x}{12} = \frac{y}{-6} = \frac{z}{6}$$

$$\frac{x}{2} = \frac{y}{-1} = \frac{z}{1} = K,$$

$\therefore$ Eigen vector will be $\begin{bmatrix} 2k \\ -k \\ k \end{bmatrix}$.

7 marks (V.V.I).
do atleast 6 Iterations

* POWER METHOD (To find Largest Eigen value & Corresponding Eigen Vector)

working Procedure to find largest eigen Value :-

Step 1) Find $AX^{(0)}$ where A is the given matrix & $x^{(0)}$ is Initial Eigen vector.
(If not given taken anything non zero value i.e. either (1,1,1) or (1,0,0) or (1,0,1)...)

Step 2: Take out largest numerical value from $AX^{(0)}$, then we get $\lambda^{(1)}X^{(1)}$ where $\lambda^{(1)}$ is the largest Eigen value in first Iteration & $X^{(1)}$ is their corresponding Eigen Vector.
consider modulus (not sign).

Step 3: Now find $AX^{(1)}$ & repeat the step 2 until we get our desired result.

Ques:
$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$
consider $x^{(0)} = (1,0,0)^T$

1st Iteration

$$AX^{(0)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 1 \\ -\frac{1}{2} \\ 0 \end{bmatrix}$$

$$= \lambda^{(1)} X^{(1)}$$

take biggest as common.

II^{nd} iteration :-

$$A\,X^{(1)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -\frac{1}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 2.5 = 5/2 \\ -2 \\ 0.5 = 1/2 \end{bmatrix}$$

$$= \frac{5}{2} \begin{bmatrix} 1 \\ -2/2.5 \\ \frac{0.5}{2.5} \end{bmatrix}$$

$$= \frac{5}{2} \begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix} = \lambda^{(2)}\, X^{(2)}.$$

III^{rd} Iteration :

$$A\,X^{(2)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 2.8 \\ -2.8 \\ 1.2 \end{bmatrix} = 2.8 \begin{bmatrix} 1 \\ -1 \\ 0.4286 \end{bmatrix}$$

$$= \lambda^{(3)}\, X^{(3)}$$

IV^{th} Iteration :

$$A\,X^{(3)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0.43 \end{bmatrix} = \begin{bmatrix} 3 \\ -3.43 \\ 1.86 \end{bmatrix}$$

$$= 3.43 \begin{bmatrix} 0.87 \\ -1 \\ 0.54 \end{bmatrix} = \lambda^{(4)}\, X^{(4)}$$

V^{th} Iteration

$$A\,X^{(4)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.87 \\ -1 \\ 0.54 \end{bmatrix} = \begin{bmatrix} 2.74 \\ -3.41 \\ 2.08 \end{bmatrix}$$

$$= 3.41 \begin{bmatrix} 0.8 \\ -1 \\ 0.61 \end{bmatrix} = \lambda^{(5)}\, X^{(5)}$$

use upto 04 decimal places always.

VIth Iteration :

$$A\,X^{(5)} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.8 \\ -1 \\ 0.61 \end{bmatrix} = \begin{bmatrix} 2.6 \\ -3.41 \\ 2.28 \end{bmatrix}$$

$$= 3.41 \begin{bmatrix} 0.76 \\ -1 \\ 0.65 \end{bmatrix} = \lambda^{(6)} X^{(6)}$$

∴ The largest Eigen value is 3.41 & their corresponding Eigen vector is

$$\begin{bmatrix} 0.76 \\ -1 \\ 0.65 \end{bmatrix} \text{ or } (0.76, -1, 0.65)^T.$$

- Gauss Siedel ✓ (Sure)
- Gauss Jordan/Elimination (Anyone).
- Rank | Eichelon form
- Power method / Eigen Value / Vector ✓ (Anyone)

10/10/19

VECTOR SPACE

Vector Addition (betn vector & vector)

i) $v_1 + v_2 \in V$ $\quad \forall\, u_1, u_2 \in V$

ii) $(v_1 + v_2) + v_3 = v_1 + (v_2 + v_3)$ $\quad \forall\, v_1, v_2, v_2 \in V$

iii) If $0 \in V$ such that $v_1 + 0 = v_1$

$0 + v_1 = v_1$

iv) $\forall\, v_1 \in V$ of $-v_1 \in V$ such that $v_1 + (-v_1) = 0$

$(-v_1) + v_1 = 0$

v) $v_1 + v_2 = v_2 + v_1$ $\quad \forall\, v_1, v_2 \in V$

let $v_1, v_2 \in V$ & $a, b, \in F$

↳ vector set. ↳ scalar field

Scalar multipeication (betn vector & scalar).

i) $a \cdot v_1 \in V$

ii) $(ab)v_1 = a(bv_1)$

iii) If $1 \in F$ such that $1 \cdot v_1 = v_1$

or $v_1 \cdot 1 = v_1$

iv) $(a+b)v_1 = av_1 + bv_1$

v) $(v_1 + v_2)a = av_1 + av_2$

If any set follows all these properties then the given set is vector space over a scalar ~~se~~ field 'F'.

E.g.

★ Matrix Space: $M_{2\times2}$ matrix space is a vector space over a scalar field F

let $\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}, \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$

Proof

★ Vector Addition:

$\begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}$

1). Closure property-

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix} \in M$$

2). Associative Property-

$$\left(\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}\right) + \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix} =$$

$$\begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix} + \left(\cancel{\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}} + \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} a_1+a_2+a_3 & b_1+b_2+b_3 \\ c_1+c_2+c_3 & d_1+d_2+d_3 \end{bmatrix} = \begin{bmatrix} a_1+(a_2+a_3) & b_1+(b_2+b_3) \\ c_1+(c_2+c_3) & d_1+(d_2+d_3) \end{bmatrix}$$

$$= \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \left(\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} + \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}\right).$$

3). Identity Property - If $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in M_{2\times2}$

Such that,

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a_1+0 & b_1+0 \\ c_1+0 & d_1+0 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}.$$

& $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} 0+a_1 & 0+b_1 \\ 0+c_1 & 0+d_1 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$

4). Inverse Property: $\forall \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \in M_{2\times2}$ If.

$\begin{bmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{bmatrix}$ in $M_{2\times2}$ such that.

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{bmatrix} = \begin{bmatrix} a_1+(-a_1) & b_1+(-b_1) \\ c_1+(-c_1) & d_1+(-d_1) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

& $\begin{bmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{bmatrix} + \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} -a_1+a_1 & -b_1+b_1 \\ -c_1+c_1 & -d_1+d_1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

5). Commutative Property:

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix} = \begin{bmatrix} a_2+a_1 & b_2+b_1 \\ c_2+c_1 & d_2+d_1 \end{bmatrix}$$

$$= \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} + \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}.$$

* Scalar Multiplication: let $p, q, r \in F$.

1). $p\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} pa_1 & pb_1 \\ pc_1 & pd_1 \end{bmatrix} \in M_{2\times2}$

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2). $p\left(\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}\right) = p\begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$

$$= \begin{bmatrix} p(a_1+a_2) & p(b_1+b_2) \\ p(c_1+c_2) & p(d_1+d_2) \end{bmatrix}$$

$$= \begin{bmatrix} pa_1+pa_2 & pb_1+pb_2 \\ pc_1+pc_2 & pd_1+pd_2 \end{bmatrix} = \begin{bmatrix} pa_1 & pb_1 \\ pc_1 & pd_1 \end{bmatrix} + \begin{bmatrix} pa_2 & pb_2 \\ pc_2 & pd_2 \end{bmatrix}$$

$$= p\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + p\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}.$$

3). $(p+q)\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} (p+q)a_1 & (p+q)b_1 \\ (p+q)c_1 & (p+q)d_1 \end{bmatrix}$

$$= \begin{bmatrix} pa_1+qa_1 & pb_1+qb_1 \\ pc_1+qc_1 & pd_1+qd_1 \end{bmatrix} = \begin{bmatrix} pa_1 & pb_1 \\ pc_1 & pd_1 \end{bmatrix} + \begin{bmatrix} qa_1 & qb_1 \\ qc_1 & qd_1 \end{bmatrix}$$

$$= p\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + q\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

4). $(pq)\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} pqa_1 & pqb_1 \\ pqc_1 & pqd_1 \end{bmatrix} = \begin{bmatrix} p(qa_1) & p(qb_1) \\ p(qc_1) & p(qd_1) \end{bmatrix}$

$$= p\begin{bmatrix} qa_1 & qb_1 \\ qc_1 & qd_1 \end{bmatrix}$$

5) If $1 \in F$ such that $1 \cdot \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} 1\cdot a_1 & 1\cdot b_1 \\ 1\cdot c_1 & 1\cdot d_1 \end{bmatrix}$

$$= \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

∴ The matrix space is a vector space over a scalar field F.

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V.V.I

* LINEAR Combination :-

let 'V' be a vector space over the field 'K'. A vector 'v' $\in V$ is a linear combination of vectors $u_1, u_2, \ldots u_m \in V$ if there exists scalars $a_1, a_2, \ldots a_m$ in the field 'K' such that $\boxed{v = a_1u_1 + a_2u_2 + \cdots + a_mu_m}$

NOTE :- 'v' is a linear combn of $u_1, u_2, \ldots u_m$ if there is a soln to the vector equation $v = x_1u_1 + x_2u_2 + \cdots + x_mu_m$ where $x_1, x_2, \ldots x_m$ are unknown scalars.

LP (3). Given $v = (1, -2, 5)$

$u_1 = (1, 1, 1)$

$u_2 = (1, 2, 3)$

$u_3 = (2, -1, 1)$

let x, y, z are unknown scalars in field 'K'.

we need to show $v = xu_1 + yu_2 + zu_3$.

$\therefore (1, -2, 5) = x(1,1,1) + y(1,2,3) + z(2,-1,1)$ — (1)

$$\begin{aligned} 1 &= x + y + 2z \\ -2 &= x + 2y - z \\ 5 &= x + 3y + z \end{aligned} \Rightarrow \left.\begin{aligned} x + y + 2z &= 1 \\ x + 2y - z &= -2 \\ x + 3y + z &= 5 \end{aligned}\right\} (2)$$

Augmented matrix of matrix form of eqn (2)

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & -1 & -2 \\ 1 & 3 & 1 & 5 \end{array}\right]$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 2 & : & 1 \\ 0 & 1 & -3 & : & -3 \\ 0 & 2 & -1 & : & 4 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$ $\qquad -\cancel{1}+6 = 5$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 2 & : & 1 \\ 0 & 1 & -3 & : & -3 \\ 0 & 0 & 5 & : & 10 \end{bmatrix} \quad \text{--- (3)} \qquad 4+6$$

$\because \rho(A) = 3 = \rho(A|B)$. → If not equal then no solⁿ, don't proceed further Just write no solⁿ.

$\therefore$ Eqⁿ (2) has solⁿ.

& $\because n = 3$.

$\therefore$ we get unique solⁿ.

$\therefore$ Eqⁿ (3) can be written as:-

$$x + y + 2z = 1 \quad \text{--- (4)}$$
$$y - 3z = -3 \quad \text{--- (5)}$$
$$5z = 10 \quad \text{--- (6)}$$

from (6) $z = 2$
from (5) $y = -3 + 6 = 3$
from (4) $x = 1 - 3 - 4 = 1 - 7 = -6$

$\therefore z = 2 \quad y = 3 \quad x = -6$.

$\therefore$ We can express 'v' as linear combⁿ of u_1, u_2, u_3 as :- $v = xu_1 + yu_2 + zu_3$

i.e. $\boxed{v = -6u_1 + 3u_2 + 2u_3}$

(4). Express M as a linear combination of the matrices A, B, C.

$$M = \begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$$

To check whether M can be expressed as

$M = xA + yB + zC$ —— (1)

$$\begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix} = x\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + y\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + z\begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$$

$$\begin{aligned} 4 &= x+y+z & & x+y+z=4 \\ 7 &= x+2y+z & \Rightarrow & x+2y+z=7 \\ 7 &= x+3y+4z & & x+3y+4z=7 \\ 9 &= x+4y+5z & & x+4y+5z=9 \end{aligned} \quad \Bigg\} \text{ —(2)}$$

Augmented matrix of eqn (2) :-

$$[A|B] = \begin{bmatrix} 1 & 1 & 1 & : & 4 \\ 1 & 2 & 1 & : & 7 \\ 1 & 3 & 4 & : & 7 \\ 1 & 4 & 5 & : & 9 \end{bmatrix}$$

0

$R_2 \to R_2 - R_1 \quad R_3 \to R_3 - R_1 \quad R_4 \to R_4 - R_1$

$$[A|B] \approx \begin{bmatrix} 1 & 1 & 1 & : & 4 \\ 0 & 1 & 0 & : & 3 \\ 0 & 2 & 3 & : & 3 \\ 0 & 3 & 4 & : & 5 \end{bmatrix}$$

0

$4 -. \quad 5-9 = -4 \quad 3-6 = -3$

$R_3 \to R_3 - 2R_2 \qquad R_4 \to R_4 - 3R_2$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 4 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 3 & : & -3 \\ 0 & 0 & \boxed{4} & : & -4 \end{bmatrix}$$

0

$R_4 \to 3R_4 - 4R_3 \qquad -12+12$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 4 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 3 & : & -3 \\ 0 & 0 & 0 & : & 0 \end{bmatrix} \quad \text{--- (3)}$$

$\because \rho(A) = \rho(A|B)$

$\therefore$ It has solⁿ.

$\therefore$ We can express M as linear combⁿ of A, B, C.

$\because \rho(A) = \rho(A|B) = 3 = n$

$\therefore$ We get unique solⁿ.

Then, from eqⁿ (3) :-

$x + y + z = 4 \quad \text{--- (4)}$

$y = 3 \quad \text{--- (5)}$

$3z = -3 \quad \text{--- (6)}$

from (6) :- $z = -1$

from (5) :- $y = 3$

from (4) :- $x = 4 - y - z = 4 - 3 + 1 = 2$

$\therefore$ from eqⁿ (1) :-

$$\boxed{M = 2A + 3B - C}$$

(2) Given $v = 3t^2 + 5t - 1$

$P_1 = t^2 + 2t + 1$

$P_2 = 2t^2 + 5t + 4$

$P_3 = t^2 + 3t + 6$

$$(3t^2+5t-1) = x(t^2+2t+1) + y(2t^2+5t+4) + z(t^2+3t+6)$$

We need to check v as linear ~~coeff~~ combn of P_1, P_2, P_3

$\Rightarrow$ We need to check whether v can be expressed as $v = xP_1 + yP_2 + zP_3$.

$$\therefore (3t^2+5t-1) = x(t^2+2t+1) + y(2t^2+5t+4) + z(t^2+3t+6)$$

Here, we need to compare coefficient of t^2 then coeff of t & then constant term.

$$\therefore \begin{aligned} 3 &= x+2y+z \\ 5 &= 2x+5y+3z \\ -1 &= x+4y+6z \end{aligned} \Rightarrow \left.\begin{aligned} x+2y+z &= 3 \\ 2x+5y+3z &= 5 \\ x+4y+6z &= -1 \end{aligned}\right\} (2)$$

Augmented matrix of (2)

$$[A|B] = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2 & 5 & 3 & 5 \\ 1 & 4 & 6 & -1 \end{array}\right]$$

0

$R_2 \to R_2 - 2R_1 \qquad R_3 \to R_3 - R_1$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 5 & -4 \end{array}\right]$$

0

$-4-2(-1)$
$-4+2$
-2

$R_3 \to R_3 - 2R_2$

$$[A|B] \sim \begin{bmatrix} 1 & 2 & 1 & : 3 \\ 0 & 1 & 1 & : -1 \\ 0 & 0 & 3 & : -2 \end{bmatrix} \quad -(3)$$

$\rho(A) = 3 = \rho(A|B) = n$

$\therefore$ we have unique solⁿ.

Now from (3) :-

$x + 2y + z = 3 \quad -(4)$

$y + z = -1 \quad -(5)$

$3z = -2 \quad -(6)$

from (6) $\quad z = -2/3$

from (5) $\quad y = -1 + \frac{2}{3} \Rightarrow \frac{2-3}{3} = -1/3$

from (4) $\quad x = 3 - 2y - z$

$= 3 + \frac{2}{3} + \frac{2}{3} = 3 + \frac{4}{3} = \frac{9+4}{3} = \frac{13}{3}$

$\therefore$ Eqⁿ (1) can be written as:-

$$(3t^2+5t-1) = \frac{13}{3}(t^2+2t+1) - \frac{1}{3}(2t^2+5t+4) - \frac{2}{3}(t^2+3t+6)$$

Ques $\quad v = 3t^2 + 5t - 1$

$P_1 = t^2 + 1$

$P_2 = t + 1$

$P_3 = t^2 + t$

* Spanning Set : or Linear Span :-

Let V be a vector space over a field K. Vectors $u_1, u_2, \ldots u_m$ in V are said to span V or to form a spanning set of V if every 'v' in V is a linear combination of the vectors $u_1, u_2, \ldots u_m$ i.e. if there exists scalars $a_1, a_2, \ldots a_m$ in K such that $v = a_1 u_1 + a_2 u_2 + \ldots + a_m u_m$.

9. LP

(a) $\{(1,0,0,1), (0,1,0,0), (1,1,1,1), (1,1,1,0)\}$

Let $u_1 = (1,0,0,1)$ $u_2 = (0,1,0,0)$ $u_3 = (1,1,1,1)$
$u_4 = (1,1,1,0)$

Let $v = (a,b,c,d)$

Let $v = xu_1 + yu_2 + zu_3 + tu_4$ where $x, y, z, t \in K$. (5)

$(a,b,c,d) = x(1,0,0,1) + y(0,1,0,0) + z(1,1,1,1) + t(1,1,1,0)$

$a = x + 0 \cdot y + z + t.$
$b = 0x + y + z + t.$
$c = 0x + 0y + z + t.$
$d = x + 0y + z + 0t.$

Augmented Matrix of above LSEq^{n} :-

$$[A|B] = \left[\begin{array}{cccc:c} 1 & 0 & 1 & 1 & a \\ 0 & 1 & 1 & 1 & b \\ 0 & 0 & 1 & 1 & c \\ 1 & 0 & 1 & 0 & d \end{array}\right]$$

$R_4 \rightarrow R_4 - R_1$

$$[A|B] \sim \left[\begin{array}{cccc:c} 1 & 0 & 1 & 1 & a \\ 0 & 1 & 1 & 1 & b \\ 0 & 0 & 1 & 1 & c \\ 0 & 0 & 0 & -1 & d-a \end{array}\right]$$

$\because \rho(A) = \rho(A|B) = 4.$

$\therefore$ we can express 'v' as linear combination ~~of~~ u_1, u_2, u_3, u_4.

$\therefore$

$x + z + t = a \quad - (1)$

$y + z + t = b \quad - (2)$

$z + t = c \quad - (3)$

$-t = d - a \quad - (4)$

from (4) : $t = a - d$.

from (3) $z = c - a + d$

from (2) $y = b - c + \cancel{a} - \cancel{d} - \cancel{a} + \cancel{d}$

$= b - c$

from (1) $x = a - c + \cancel{a} - \cancel{d} - \cancel{a} + \cancel{d}$

$= a - c$

from (5) :-

$v = (a-c)u_1 + (b-c)u_2 + (c-a+d)u_3 + (a-d)u_4$

$\therefore u_1, u_2, u_3, u_4$ spans R_4.

(b) $\{(1,2,1,0)\ (1,1,-1,0)\ (0,0,0,1)\}$

Let $u_1 = 1,2,1,0 \qquad u_2 = 1,1,-1,0 \qquad u_3 = 0,0,0,1$

~~$u_4 = 1,1,1,0$~~

Let $v = (a,b,c,d)$

$$v = xu_1 + yu_2 + zu_3 + \cancel{tu_4} \quad \text{---} ①$$ where $x, y, z, \cancel{t} \in K$.

$\therefore (a,b,c,d) = x(1,2,1,0) + y(1,1,-1,0) + z(0,0,0,1) \cancel{+ t(0,1,1,1)}$.

$$a = x + y + 0z \cancel{+ t}$$
$$b = 2x + y + 0z \cancel{+ t}$$
$$c = x - y + 0z$$
$$d = 0x + 0y + z.$$

$\therefore$ Augmented matrix of Above LSE :-

$$[A|B] = \left[\begin{array}{ccc:c} 1 & 1 & 0 & a \\ 2 & 1 & 0 & b \\ 1 & -1 & 0 & c \\ 0 & 0 & 1 & d \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$.

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 0 & a \\ 0 & -1 & 0 & b-2a \\ 0 & \boxed{-2} & 0 & c-a \\ 0 & 0 & 1 & d \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 0 & a \\ 0 & -1 & 0 & b-2a \\ 0 & 0 & 0 & (c-a)-2(b-2a) \\ 0 & 0 & 1 & d \end{array}\right]$$

~~$f(A) \neq f(A|B)$~~

$R_3 \leftrightarrow R_4$

$$\left[\begin{array}{ccc:c} 1 & 1 & 0 & a \\ 0 & -1 & 0 & b-2a \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & (c-a)-2(b-2a) \end{array}\right]$$

$\because \underbrace{f(A)}_{3} \neq \overbrace{f(A|B)}^{4}$

$\therefore$ The system does not have any solⁿ.

$\therefore$ 'v' cannot be expressed as linear combⁿ of u_1, u_2, u_3.

$\therefore$ u_1, u_2, u_3 cannot span R^4.

~~(10). (a) $\{t^2+1, t^2+1, t+1\}$.~~

~~$u_1 = t^2+1 \quad u_2 = t^2+1 \quad u_3 = t+1.$~~

~~Let $P_2 = at^2+bt+c$.~~

~~$P_2 = x(u_1) + yu_2 + zu_3$. — (1)~~

~~$\therefore (at^2+b) = x(t^2+1) + y(t^2+1) + z(t+1)$~~

~~$a = xt^2+yt^2+zt.$~~

~~$b = x+y+z.$~~

~~$\therefore$ Augmented matrix :-~~

~~$[A|B] = [$~~

P_2 : set of polynomials whose highest degree is 2.

(10). (a) $\{t^2+1,\ t^2+1,\ t+1\}$.

Let $u_1 = t^2+1$ $u_2 = t^2+1$ $u_3 = t+1$

Let $v = at^2+bt+c$.

let $x, y, z \in K$.

We need to check for span P_2 :

$\Rightarrow$ We need to check :-

$$v = xu_1 + yu_2 + zu_3.$$

$$(at^2+bt+c) = x(t^2+0t+1) + y(t^2+0t+1) + z(0t^2+t+1)$$

$$a = x + y + 0z$$
$$b = 0x + 0y + z$$
$$c = x + y + z$$

Augmented matrix of above LSE :-

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 0 & 0 & 1 & b \\ 1 & 1 & 1 & c \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 0 & 0 & 1 & b \\ 0 & 0 & 1 & c-a \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 0 & 0 & 1 & b \\ 0 & 0 & 0 & c-a-b \end{array}\right]$$

$\rho(A) \neq \rho(A|B)$.

$x + 5y - 3z = -4$
$-x + 4y + z = 3$

(11). $V_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$ $V_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}$ $V_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$ $Y = \begin{bmatrix} -4 \\ 3 \\ K \end{bmatrix}$

$$x + 5y - 3z = -4$$
$$-x - 4y + z = 3$$
$$-2x - 7y + 0z = K$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ -2 & -7 & 0 & K \end{array}\right]$$

$R_2 \to R_2 + R_1$
$R_3 \to R_3 + 2R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 3 & -6 & K-8 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & \underline{K-5} \end{array}\right]$$

$K - 5 = 0$
$\boxed{K = 5}$

This to span,
$\rho(A) = \rho(A|B) = 2$
$\therefore \underline{K - 5 = 0}$

* Linear dependency & Independency :-

Let V be a vector space over a scalar field 'K' and $u_1, u_2, \ldots u_m$ in V are linearly dependent if there exist scalar $a_1, a_2, \ldots a_m$ in K not all of them zero such that $a_1u_1 + a_2u_2 + \ldots + a_mu_m = 0$.

(Infinite solⁿ: dependent)

Otherwise, we say that the vectors $u_1, u_2, \ldots, u_m$ in V are linearly independent.

(unique solⁿ)

NOTE :

1). $u_1, u_2, \ldots, u_m$ in V are linear independent if we get zero solution for vector equation $x_1u_1 + x_2u_2 + \ldots + x_mu_m = 0$. (zero solⁿ = Trivial solⁿ). $\rho(A) = \rho(A|B) = n$.

no. of unknowns

2). $u_1, u_2, \ldots u_m$ in V are linear dependent if we get non-zero solⁿ (non-trivial solⁿ, infinitely many solⁿ).

(5) LP. (i) Let $u_1 = (1,2,3)$ $u_2 = (4,5,6)$ $u_3 = (2,1,0)$.

We need to find $x, y, z \in K$ such that.

$$xu_1 + yu_2 + zu_3 = 0.$$

$$x(1,2,3) + y(4,5,6) + z(2,1,0) = (0,0,0)$$

$$x + 4y + 2z = 0$$

$$2x + 5y + z = 0$$

$$3x + 6y + 0z = 0$$

if determinant of u_1, u_2, u_3 is $\neq 0$
then lin. independent.

Augmented Matrix of given LSE :-

$6-2(-3)$

$$[A|B] = \left[\begin{array}{ccc:c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 0 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1 \quad R_3 \to R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 2 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & -6 & -6 & 0 \end{array}\right]$$

$6+6$

$R_3 \to R_3 - 2R_2$

$6-2(-3)$

$6+6$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 4 & 2 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \rho(A) = 2$$

$\rho(A) = 2 = \rho(A|B) < n = 3$ $\quad$ n = no. of unknowns

$\Rightarrow$ We have infinitely many soln.

i.e. The given vectors (u_1, u_2, u_3) are linearly dependent.

(ii) $u_1 = (1, 2, -3) \quad u_2 = (1, -3, 6) \quad u_3 = (2, 1, 1)$

$$x(1,2,-3) + y(1,-3,6) + z(2,1,1) = (0,0,0)$$

$$x + y + 2z = 0$$
$$2x - 3y + z = 0$$
$$-3x + 6y + z = 0$$

$-3-2$

$1-4$

$6+3$

$$\therefore [A|B] = \left[\begin{array}{ccc:c} 1 & 1 & 2 & 0 \\ 2 & -3 & 1 & 0 \\ -3 & 6 & 1 & 0 \end{array}\right]$$

$1+6$

$R_2 \to R_2 - 2R_1 \quad R_3 \to R_3 + 3R_1$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 2 & 0 \\ 0 & -5 & -3 & 0 \\ 0 & 9 & 7 & 0 \end{array}\right]$$

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35 − 27
= 8

$R_3 \to 5R_3 + 9R_2$

$$\begin{bmatrix} 1 & 1 & 2 & : & 0 \\ 0 & -5 & -3 & : & 0 \\ 0 & 0 & 8 & : & 0 \end{bmatrix}$$

$\rho(A) = 3 = \rho(A|B) = n$

∴ The given vectors u_1, u_2, u_3 are linearly independent.

(6) $P_1(t) = t^2 + t + 2$ $P_2(t) = 2t^2 + t$ $P_3(t) = 3t^2 + 2t + 2$.

$x(t^2 + t + 2) + y(2t^2 + t + 0) + z(3t^2 + 2t + 2) = \underbrace{0t^2 + 0t + 0}_{= 0 \text{ (vector)}}$

$x + 2y + 3z = 0$
$x + y + 2z = 0$
$2x + 0y + 2z = 0$

Augmented matrix of above LSE :-

$$[A|B] = \begin{bmatrix} 1 & 2 & 3 & : & 0 \\ 1 & 1 & 2 & : & 0 \\ 2 & 0 & 2 & : & 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$ $R_3 \to R_3 - 2R_1$

$$[A|B] \sim \begin{bmatrix} 1 & 2 & 3 & : & 0 \\ 0 & -1 & -1 & : & 0 \\ 0 & -4 & -4 & : & 0 \end{bmatrix}$$ $-4+4$

$R_3 \to R_3 - 4R_2$

$$[A|B] \sim \begin{bmatrix} 1 & 2 & 3 & : & 0 \\ 0 & -1 & -1 & : & 0 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$\rho(A) = \rho(A|B) = 2 < 3 = n$
infinitely many sol^{n}s.
∴ linearly dependent.

no. of vectors = dim V_1

Ques Find whethere lin. dep. or Indep. ? in R^3

$u_1 = (1, 2, -1) \quad (1, -2, 1) = u_2$

$x(1, 2, -1) + y(1, -2, 1) = (0, 0, 0)$

$x + y = 0$

$2x - 2y = 0$

$-x + y = 0$

Augmented Matrix of LSE :-

$$[A|B] = \left[\begin{array}{cc:c} 1 & 1 & 0 \\ 2 & -2 & 0 \\ -1 & 1 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_1 \qquad R_3 \to R_3 + R_1$

$$[A|B] \sim \left[\begin{array}{cc:c} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 2 & 0 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$[A|B] \sim \left[\begin{array}{cc:c} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

~~This test can only be applied if no. of vectors = dimension of vector space~~

$\therefore \rho(A) = \rho(A|B) = 2 = n$

$\therefore$ ~~Linearly Independent~~ Linearly Independent.

- ~~Since, the dimension of R^3 is 3 & we have 2 vector. Therefore, the given vectors can't be linearly independent.~~
- ~~If given set of vectors are linearly independent the no. of vector set is always equal to dimension of vector space.~~

* ①

$$A = \begin{bmatrix} 1 & 6 & -2 \\ 3 & 1 & 29 \\ 5 & 1 & -17 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rho(A) = 3 = n$

$\therefore$ lin. Independent.

②

$$A = \begin{bmatrix} 6 & -1 & 9 \\ 5 & 3 & -4 \\ -4 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 < 3 = n$

$\therefore$ lin. dependent.

③

$$A = \begin{bmatrix} 1 & -6 & -26 & -16 \\ 2 & 2 & 4 & -34 \\ 6 & -2 & -20 & -14 \end{bmatrix}$$

no. of unknowns = no. of column = 4.

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\rho(A) < n$

$\therefore$ lin. dependent.

④

$$A = \begin{bmatrix} +3 & -18 & 2 & 8 \\ -3 & 18 & -6 & -24 \\ 2 & -12 & -6 & -24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -6 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) < n$.

$\therefore$ lin. dependent.

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(5)

$$A = \begin{bmatrix} -6 & 6 \\ 3 & 2 \\ -1 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$\rho(A) = n$ $\therefore$ Lin. Independent.

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18/10/19

★ BASIS:

- A set S, $u_1, u_2, \ldots u_m$ of vectors is a basis of a vector space 'V' if it has the following two properties:
  1. 'S' is linearly independent.
  2. 'S' span 'V'.

- If it is linearly dependent then do not check for span. ∵ 1st property is not satisfied.

- Definition of Dimension: If 'V' is spanned by a finite set then V is said to be finite dimensional and the dimension of 'V' is written as dim V. is the no. of vectors in basis of 'V'.

- The dimension of the zero vector space is defined to be zero.
- If 'V' is not spanned by finite set then 'V' is said to be $\boxed{\text{infinite dimensional}}$.

OR

- Suppose 'V' be a vector space over a scalar field 'K' and 'S' is a set of vectors which form basis for 'V' then the no. of vectors in set 'S' is called dimension of the vector space 'V'.

LP 12

P.T.O

LP (12) Given $X_1 = (1,5,2)$ $X_2 = (0,0,1)$ $X_3 = (1,1,0)$

$V \Rightarrow R^3.$

If don't have time in exam ← Since the dimension of R is 03 & we have 03 vectors. ∴ We only need to check for linear dependency or independency.

Property 1. let $x, y, z \in K$.

Here we need to check whether X_1, X_2, X_3 are lin. Independent or not.

$$xX_1 + yX_2 + zX_3 = 0$$

$$x(1,5,2) + y(0,0,1) + z(1,1,0) = (0,0,0)$$

$$x + 0y + z = 0$$

$$5x + 0y + z = 0$$

$$2x + y + 0z = 0$$

Augmented matrix of above LSE :-

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 5 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \end{array}\right]$$

$R_2 \to R_2 - 5R_1$ $R_3 \to R_3 - 2R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right]$$ $R_2 \leftrightarrow R_3$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right]$$

$\therefore$ $\rho(A) = \rho(A|B) = 3 = n$. n = no. of unknowns. → Homogeneous

$\therefore$ Soln: will be $x=0$, $y=0$, $z=0$

$\therefore$ X_1, X_2, X_3 are linearly independent.

Property 2

Now, we need to check whether X_1, X_2, X_3 spans R^3 or not.

$\therefore$ $x(1,5,2) + y(0,0,1) + z(1,1,0) = (a,b,c)$.

$$x + 0y + z = a$$
$$5x + 0y + z = b$$
$$2x + y + 0z = c$$

$$[A|B] = \left[\begin{array}{ccc:c} 1 & 0 & 1 & a \\ 5 & 0 & 1 & b \\ 2 & 1 & 0 & c \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 5R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & a \\ 0 & 0 & -4 & b-5a \\ 0 & 1 & -2 & c-2a \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 0 & 1 & a \\ 0 & 1 & -2 & c-2a \\ 0 & 0 & -4 & b-5a \end{array}\right]$$

$\because$ $\rho(A) = \rho(A|B) = n = 3$.

$\therefore$ we get unique soln.

$\therefore$ X_1, X_2, X_3 spans R^3.

$\{X_1, X_2, X_3\}$ form basis of R^3.

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LP (15)

(a) $\{(1,0,0,1), (0,1,0,0), (1,1,1,1), (1,1,1,0)\}$ on R^4.

$X_1 = 1,0,0,1$ $\quad X_2 = 0,1,0,0$ $\quad X_3 = 1,1,1,1$ $\quad X_4 = 1,1,1,0$

$$x(1,0,0,1) + y(0,1,0,0) + z(1,1,1,1) + t(1,1,1,0) = (0,0,0,0)$$

$$x + 0y + z + t = 0$$
$$0x + y + z + t = 0$$
$$0x + 0y + z + t = 0$$
$$x + 0y + z + 0t = 0$$

$$[A|B] = \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{array}\right]$$

$R_4 \to R_4 - R_1$

$$[A|B] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right]$$

$$\rho(A) = 4 = \rho(A|B) = n.$$

$\therefore x_1, x_2, x_3, x_4$ are Linearly Independent.

Now, checking for span :-

$$[A|B] = \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & a \\ 0 & 1 & 1 & 1 & b \\ 0 & 0 & 1 & 1 & c \\ 1 & 0 & 1 & 0 & d \end{array}\right] \quad R_4 \to R_4 - R_1$$

$$[A|B] \sim \left[\begin{array}{cccc:c} 1 & 0 & 1 & 1 & a \\ 0 & 1 & 1 & 1 & b \\ 0 & 0 & 1 & 1 & c \\ 0 & 0 & 0 & -1 & d-a \end{array}\right]$$

$\therefore$ It spans over $\mathbb{R}^4$

$\Rightarrow \{x_1, x_2, x_3, x_4\}$ forms basis on $\mathbb{R}^4$.

(14) LP let $u_1 = t^3 + 2t^2 + 3t$.

$u_2 = 2t^3 + 0t^2 + 0t + 1$.

$u_3 = 6t^3 + 8t^2 + 6t + 4$

$u_4 = t^3 + 2t^2 + t + 1$.

let x, y, z & $w \in F$(~~vector~~ scalar Field)

$x + 2y + 6z + w = 0$

$2x + 0y + 8z + 2w = 0$

$3x + 0y + 6z + w = 0$

$0x + y + 4z + w = 0$

$$[A|B] = \left[\begin{array}{cccc:c} 1 & 2 & 6 & 1 & 0 \\ 2 & 0 & 8 & 2 & 0 \\ 3 & 0 & 6 & 1 & 0 \\ 0 & 1 & 4 & 1 & 0 \end{array}\right] \quad R_2 \to 2R_1, \; R_3 \to R_3 - 3R_1$$

$$[A|B] \sim \left[\begin{array}{cccc:c} 1 & 2 & 6 & 1 & 0 \\ 0 & -4 & -4 & 0 & 0 \\ 0 & -6 & -12 & -2 & 0 \\ 0 & 1 & 4 & 1 & 0 \end{array}\right]$$

$R_2 \to R_2/-4$.

$$[A|B] \sim \left[\begin{array}{cccc:c} 1 & 2 & 6 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -6 & -12 & -2 & 0 \\ 0 & 1 & 4 & 1 & 0 \end{array}\right]$$

$R_3 \to R_3 + 6R_2$ $\quad -12+6$

$R_4 \to R_4 - R_2$ $\quad -2+$

$$[A|B] \sim \left[\begin{array}{cccc:c} 1 & 2 & 6 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -6 & -2 & 0 \\ 0 & 0 & \boxed{3} & 1 & 0 \end{array}\right]$$

$R_4 \to 2R_4 + R_3$ $\quad 2-2$

$$\left[\begin{array}{cccc:c} 1 & 2 & 6 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -6 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Here $n=4$ & $\rho(A)=3$ $\because \rho(A)<n$

$\therefore$ The given vectors are linearly dependent.

$\therefore$ They cannot form basis for the set of polynomials whose degree is 03. (P_3)

{$\because$ we don't get unique Sol^{n}}

(14) (b) let $u_1 = t^3+t^2+1$

$u_2 = t^3-1$ $\quad$ dimension for $P_3 = 4$.

$u_3 = t^3+t^2+t$

{ It won't ~~be~~ span ~~linearly dependent~~ $\because$ no. of vectors < dimension. But show in exam if asked for 6-M. }

Let $x, y, z \in F$.

we get following system of linear eqn's.

$x + y + z = 0$

$x + 0y + z = 0$

$0x + 0y + z = 0$

$x - y + 0z = 0$

$$[A|B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & -1 & 0 \end{array}\right] \quad R_4 \to R_4 - 2R_2$$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] \quad R_4 \to R_4 + R_3$$

$$[A|B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\because$ $\rho(A)$ = no. of unknowns,

$\therefore$ Set u_1, u_2, u_3, u_4 are linearly independent.

$\therefore$ 1st condn is true.

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II^{nd} condⁿ: let $v = at^3 + bt^2 + ct + d$.

we get following LSE :-

$$x + y + z = a$$
$$x + 0y + z = b$$
$$0x + 0y + z = c$$
$$x - y + 0z = d$$

$$[A|B] = \begin{bmatrix} 1 & 1 & 1 & : & a \\ 1 & 0 & 1 & : & b \\ 0 & 0 & 1 & : & c \\ 1 & -1 & 0 & : & d \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_4 \to R_4 - R_1 \end{matrix}$$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 1 & : & a \\ 0 & -1 & 0 & : & b-a \\ 0 & 0 & 1 & : & c \\ 0 & -2 & -1 & : & d-a \end{bmatrix} \quad R_4 \to R_4 - 2R_2$$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 1 & : & a \\ 0 & -1 & 0 & : & b-a \\ 0 & 0 & 1 & : & c \\ 0 & 0 & -1 & : & d-a-2(b-a) \end{bmatrix} \quad R_4 \to R_4 + R_3$$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 1 & : & a \\ 0 & -1 & 0 & : & b-a \\ 0 & 0 & 1 & : & c \\ 0 & 0 & 0 & : & d+a-2b+c \end{bmatrix}$$

$\because \rho(A) \neq \rho[A|B]$

$\therefore$ It has no solⁿ.

$\therefore$ The vectors u_1, u_2, u_3 can not span P_3.

$\therefore$ II^{nd} condⁿ is not true.

$\Rightarrow S = \{u_1, u_2, u_3\}$ can not form Basis.

★ **SUBSPACE**: Let 'V' be a vector space over a field 'K' and 'W' be a subset of 'V' then 'W' is a subspace of 'V' if 'W' is itself a vector space over the same field 'K' with respect to the operations of vector addition & scalar multiplication on 'V'.

• **Dimension of a Subspace**: No. of the non-zero rows in the echelon form of matrix A is called the dimension of a subspace.

(8) LP
$u_1 = (1,2,0)$
$u_2 = (-1,1,2)$
$u_3 = (3,0,-4)$

let $v = (a,b,c)$

If it is linear span then we can express as $v = xu_1 + yu_2 + zu_3$ where $x,y,z \in K$

$\therefore (a,b,c) = x(1,2,0) + y(-1,1,2) + z(3,0,-4)$.

$a = x - y + 3z$

$b = 2x + y + 0z$

$c = 0x + 2y - 4z$

$\therefore$ Augmented Matrix:

$$[A|B] = \left[\begin{array}{ccc:c} 1 & -1 & 3 & a \\ 2 & 1 & 0 & b \\ 0 & 2 & -4 & c \end{array}\right] \qquad R_2 \to R_2 - 2R_1$$

$$= \left[\begin{array}{ccc:c} 1 & -1 & 3 & a \\ 0 & 3 & -6 & b-2a \\ 0 & 2 & -4 & c \end{array}\right]$$

$-12+12$

$R_3 \to 3R_3 - 2R_2$

$$= \left[\begin{array}{ccc:l} 1 & -1 & 3 & a \\ 0 & 3 & -6 & b-2a \\ 0 & 0 & 0 & 3c-2(b-2a) \\ & & & =3c-2b+4a \end{array}\right]$$

Here $\rho(A) \neq \rho(A|B)$
$\Rightarrow u_1, u_2, u_3$ can not span R^3.

This vector can span if it has a soln.
For this to have a soln
$\rho(A) = \rho(A|B)$

$\therefore \boxed{3c-2b+4a=0}$.

If $3c-2b+4a=0$, then $\rho(A) = \rho[A|B]$
$\Rightarrow$ the set u_1, u_2, u_3 span W.

★ **COORDINATES:** Let 'V' be an n-dimensional vector space over 'K' with basis $S = \{u_1, u_2, \ldots$ then any vector 'v' in 'V' can be expressed uniquely as linear combination of the basis vector in 'S' say $v = a_1u_1 + a_2u_2 + \ldots + a_nu_n$. These n-scalars $a_1, a_2, \ldots a_n$ are called the coordinates of v relative to the basis S & they form a vector $[a_1, a_2, \ldots a_n]$ in K^n called coordinate vector of v relative to S.

We denote this vector by $[v]_S$ or $[v]$.

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Theorem : Suppose U & W are finite dimensional subspaces of vector space V. Then $U+W$ has finite dimension &:-

$$\boxed{\dim(U+W) = \dim(U) + \dim(W) - \dim(U \cap W)}$$

LP (16) $u_1 = (1,1,1,1)$ $u_2 = (1,2,3,2)$ $u_3 = (2,5,6,4)$.
$u_4 = (2,6,8,5)$.

let $x, y, z, w \in K$.

To check the given vector are Lin. Inde or dep. we need to find the Solⁿ for Homogeneous LSE.

$$xu_1 + yu_2 + zu_3 + wu_4 = 0$$

$$x(1,1,1,1) + y(1,2,3,2) + z(2,5,6,4) + w(2,6,8,5) = (0,0,0,0)$$

$$x + y + 2z + 2w = 0$$

$$x + 2y + 5z + 6w = 0$$

$$x + 3y + 6z + 8w = 0$$

$$x + 2y + 4z + 5w = 0$$

$$[A|B] = \left[\begin{array}{cccc:c} 1 & 1 & 2 & 2 & 0 \\ 1 & 2 & 5 & 6 & 0 \\ 1 & 3 & 6 & 8 & 0 \\ 1 & 2 & 9 & 5 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$[A|B] \sim \left[\begin{array}{cccc:c} 1 & 1 & 2 & 2 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 2 & 4 & 6 & 0 \\ 0 & 1 & 2 & 3 & 0 \end{array}\right] \quad \begin{array}{l} R_3 \to R_2 - 2R_2 \\ R_4 \to R_4 - R_2 \end{array}$$

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$$[A|B] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & -2 & -2 & 0 \\ 0 & 0 & -1 & -1 & 0 \end{array}\right] \quad R_4 \to 2R_4 - R_3$$

$$[A|B] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & -2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\because$ $\rho(A) <$ no. of unknowns scalars

$\therefore$ The set of u_1, u_2, u_3, u_4 are linearly dependent

$\Rightarrow$ $\{u_1, u_2, u_3, u_4\}$ can not form basis.

$\Rightarrow$ Now, we want to find dimension of subspace.

$\because$ no. of non-zero rows in Echelon form of $[A|B]$ is 03.

$\therefore$ Dim. of subspace $= 03$.

LP

(18). Coord. vector can only be found if it forms the basis.

Given $S = \{u_1, u_2\}$ $\quad v = (4, -3)$

where $u_1 = (1,1)$ $\quad u_2 = (2,3)$

let $x, y \in K$ (two vectors $\therefore$ two scalars).

$$v = xu_1 + yu_2$$

$$(4,-3) = xu_1 + yu_2$$

$$(4,-3) = x(1,1) + y(2,3)$$

$\therefore \quad 4 = x + 2y$
$\quad -3 = x + 3y$

$$[A|B] = \begin{bmatrix} 1 & 2 & : & 4 \\ 1 & 3 & : & -3 \end{bmatrix} \qquad R_2 \to R_2 - R_1$$

$$[A|B] \sim \begin{bmatrix} 1 & 2 & : & 4 \\ 0 & 1 & : & -7 \end{bmatrix}$$

$$x + 2y = 4 \quad \text{——} \ (1)$$
$$y = -7$$

from eqn: (1) $\quad x + 2(-7) = 4$
$\quad x = 18$

$\therefore$ Coordinates of R^2 is $x = 18$ & $y = -7$.
$\therefore$ Coordinate Vector $[v]_s = [18, -7]$.

Ques (17) $\dim U = 4 \quad \dim W = 5 \quad \dim V = 7$.

Extra self { $\dim(U+W) = 5$ or 6 or 7.
$\dim(U+W) = \dim(U) + \dim(W) - \dim(U \cap W)$ }

Soln:

Here, U & W are the subspace of a vectorspace 'V'. We know that : ~~dv~~
$\therefore \dim(U \cap W) = \dim(U) + \dim(W) - \dim(U+W)$.
& $\dim(U+W) = 5$ or 6 or 7.

Case 1 : let $\dim(U+W) = 5$:
then, $\dim(U \cap W) = 4 + 5 - 5 = 4$

Case 2 : let $\dim(U+W) = 6$
then, $\dim(U \cap W) = 4 + 5 - 6 = 3$

Case 3 : let $\dim(U+W) = 7$
then, $\dim(U \cap W) = 4 + 5 - 7 = 2$

$\therefore \dim(U \cap W) = 4$ or 3 or 2.

REVISION

* Def^n of vector space. [6-M]
  - Write all 11 cond^n properly.
* Explain vector space with example [7-M]

* Linear combination → sol^n (check)
  NO L.C no sol^n, for particular v

* Linear span → check for sol^n $v = (a, b, c)$ arbitrary.

} Non Homogeneous

* Linear Independence → Unique sol^n
  $\rho(A) = n$

* Linear Dependence → Infinitely many sol^n
  $\rho(A) < n$

} Homogeneous LSE

* Basis → Linear Indep. & Linear span.
  NO Basis, if one of the cond^n is not true.

* Coordinates → values of unknowns { will always get unique sol^n }
  coord. vector $[V_\phi]_S$ → [set of values of unknowns]

* Subspace → def^n subspace & find dim. of sub W which is subspace of V.

dim. of subspace : no. of non zero rows in Echelon form.
dim. of basis : no. of vectors in Basis.