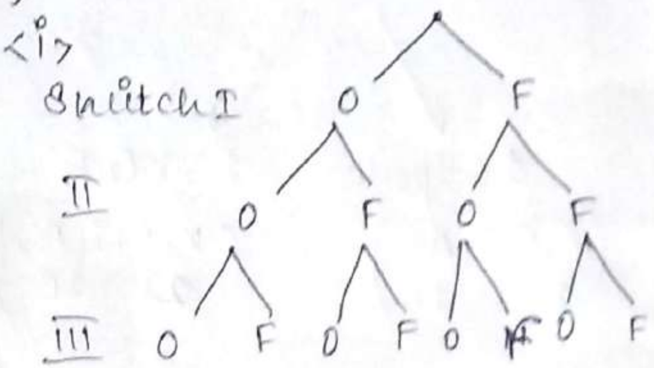
AS

(1)

Chapter 03 : PROBABILITY

6).

<i>



<ii> $S = \{(O,O,O), (O,O,F), (O,F,O), (O,F,F),$
$(F,O,O), (F,O,F), (F,F,O), (F,F,F)\}$

<iii> $A = \{(O,O,O), (O,O,F), (O,F,O), (O,F,F), (F,O,O),$
$(F,O,F), (F,F,O)\}$

$B = \{(O,O,O), (O,O,F), (O,F,O), (O,FF)\}$

$C = \{(F,F,F)\}$ $D = \{\phi\}$

<iv> • A & B are not mutually exclusive
(since, they can occur simultaneously)

• A & C are mutually exclusive, ∵ they cannot occur at the same time.

• A & D are mutually exclusive ∵ they can't occur simultaneously.

<v> Impossible event.

<vi> P(no switch is on) = 1/8 $\left\{ \begin{array}{l} \because n(S) = 8 \\ n(E) = 1 \\ E = \{(FFF)\} \end{array} \right.$

7).

52 Cards

26 Red | 26 Black

13 Diamond | 13 Heart | 13 Spade | 13 Club

$\begin{pmatrix} K\,Q\,J\,A \\ 2 \ldots 10 \end{pmatrix}$ $\begin{pmatrix} K\,Q\,J\,A \\ 2 \ldots 10 \end{pmatrix}$ $\begin{pmatrix} K,Q,J,A, \\ 2 \ldots 10 \end{pmatrix}$ $\begin{pmatrix} K,Q,J,A, \\ 2 \ldots 10 \end{pmatrix}$

$S = {}^{52}C_1$ ($\because$ 1 card is drawn)

$= 52$ ways

$P(\text{queen} \cup \text{Heart}) = P(\text{queen}) + P(\text{heart}) - P(\text{queen} \cap \text{heart})$

$= \left(\frac{4}{52}\right) + \left(\frac{13}{52}\right) - \left(\frac{1}{52}\right)$

$= 16/52$

8). $S = 6^2 = 36$

$= \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6),$

$(2,1) \ldots (2,6),$

$(3,1) \ldots (3,6),$

$(4,1) \ldots (4,6),$

$(5,1) \ldots (5,6),$

$(6,1) \ldots (6,6) \}$

E = 05 will occur at least once

$n(E) = 11$

$\therefore P(E) = 11/36$

9). 20 DVDs → 16 are non-defective ③
→ 04 are defective

∵ 02 DVDs are selected without replacement:

$S = {}^{20}C_2$ ways

E = selecting 02 defective DVDs

$= {}^{4}C_2$ ways

$\therefore P(E) = \dfrac{{}^{4}C_2}{{}^{20}C_2}$

10).

a).

1	2	3	4	5	6
26 ways (a....z)	26 ways (a--z)	26 ways (a--z)	26 ways (a--z)	26 ways (a...z)	10 ways (0--9)

$\therefore$ No. of total possible passwords $= 26 * 26 * 26 * 26 * 26 * 10$

$= (26)^5 * 10$

b).

1	2	3	4	5	6
					05 ways (0,2,4,6,8)

$\underbrace{1\ 2\ 3}_{3\ A's}$ $\underbrace{4\ 5}_{2\ B's}$

$\therefore$ Total no. of passwords possible with this condition is :- $\dfrac{5!}{3! * 2!} * 5 = 10 * 5$

$= 50$ ways

c). P(to guess this password correct) $= 1/50$

11). E = Exam is conducted

A = Delay in conduction of exam

$\bar{A}$ = No Delay in conduction of exam

Given, $P(E|\bar{A}) = 0.95$

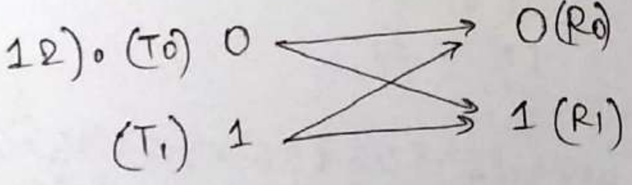
$P(E|A) = 0.60$

$P(A) = 0.20$

$\therefore P(\bar{A}) = 1 - P(\bar{A}) = 1 - 0.20 = 0.80$

$P(E) = ?$

$$P(E) = [P(A) * P(E|A)] + [P(\bar{A}) * (P(E|\bar{A}))]$$
$$= (0.20 * 0.60) + (0.80 * 0.95)$$
$$= 0.88$$

12). 

T_0 : '0' is transmitted

T_1 : '1' is transmitted

R_0 : '0' is received

R_1 : '1' is received

$P(R_0|T_0) = 0.94$

$P(R_1|T_1) = 0.91$

$P(T_0) = 0.45$

$\therefore P(T_1) = 1 - P(T_0) = 1 - 0.45 = \underline{0.55}$

Probability that 1 is received :-

a). Probability that (0 is transmitted & 1 is received)

$P(R_1)$ = Probability that (0 is transmitted & 1 is received) or (1 is transmitted & 1 is received)

$$= [P(T_0 \cap R_1) \cup P(T_1 \cap R_1)]$$
$$= P(T_0 \cap R_1) + P(T_1 \cap R_1)$$
$$= [P(T_0) * P(R_1|T_0)] + [P(T_1) * P(R_1|T_1)]$$

$= [P(T_0) * P(R_1|T_0)] + [P(T_1) * P(R_1|T_1)]$ — (1)

$P(T_0) = (0.45)$, $P(R_1|T_0)$ = (not known), $P(T_1) = (0.55)$, $P(R_1|T_1) = (0.91)$

$$\left\{ \begin{aligned} P(R_1|T_0) &= 1 - P(\overline{R_1}|T_0) \\ &= 1 - P(R_0|T_0) \\ &= 1 - 0.94 \\ &= 0.06 \end{aligned} \right.$$

Now, substituting all the values in eqn (i),
we get :-

$= (0.45 * 0.06) + (0.55 * 0.91)$

$= 0.5275 = P(R_1)$

b). Probability that 0 is received :-

$P(R_0)$ = 1 - probability that 1 is received

$= 1 - 0.5275$

$= 0.4725 = P(R_0)$

OR

$= [P(T_0 \cap R_0) \cup P(T_1 \cap R_0)]$

$= P(T_0 \cap R_0) + P(T_1 \cap R_0)$

$= [P(T_0) * P(R_0|T_0)] + [P(T_1) * P(R_0|T_1)]$ — (ii)

$P(T_0) = 0.45$, $P(R_0|T_0) = 0.94$, $P(T_1) = 0.55$, $P(R_0|T_1)$ = (not known)

$$\left\{ \begin{aligned} P(R_0|T_1) &= 1 - P(\overline{R_0}|T_1) \\ &= 1 - P(R_1|T_1) \\ &= 1 - 0.91 \\ &= 0.09 \end{aligned} \right.$$

Now, substituting all the values in eqn (ii), we get :-

$= (0.45 * 0.94) + (0.55 * 0.09)$

$= 0.4725$

c). Probability that 1 is transmitted, given that a 1 was received :-

$= P(T_1 | R_1)$

$= \dfrac{P(T_1) * P(R_1|T_1)}{P(R_1)}$

$= \dfrac{0.55 * 0.91}{0.5275} = 0.9488$

d). Probability that 0 was transmitted, given that a 0 was received :-

$= P(T_0 | R_0)$

$= \dfrac{P(T_0) * P(R_0|T_0)}{P(R_0)}$

$= \dfrac{0.45 * 0.94}{0.4725} = 0.8952$

e). Probability of an error :-

Error occurs when :-

- 0 is transmitted & 1 is Received

or

- 1 is transmitted & 0 is Received

$\therefore P(error) = P(T_0 \cap R_1) \cup P(T_1 \cap R_0)$

$= P(T_0 \cap R_1) + P(T_1 \cap R_0)$

$= [P(T_0) * P(R_1|T_0)] + [P(T_1) * P(R_0|T_1)]$

$=$

$P(R_1|T_0) = 1 - P(\bar{R_1}|T_0) = 1 - P(R_0|T_0) = 1 - 0.94 = 0.06$

$P(R_0|T_1) = 1 - P(\bar{R_0}|T_1) = 1 - P(R_1|T_1) = 1 - 0.91 = 0.09$

$= (0.45 * 0.06) + (0.55 * 0.09)$

$= 0.0765$

13). E_1 = articles are from Machine A.
E_2 = articles are from Machine B.
E_3 = articles are from Machine C.
E = article produced is satisfactory.

Now, Given :-

$P(E_1) = 20\% = 0.20$ | $P(E|E_1) = 0.95$
$P(E_2) = 30\% = 0.30$ | $P(E|E_2) = 0.85$
$P(E_3) = 50\% = 0.50$ | $P(E|E_3) = 0.90$

a). $P(E) = P(E_1) * P(E|E_1) + P(E_2) * P(E|E_2) + P(E_3) * P(E|E_3)$
$= (0.20 * 0.95) + (0.30 * 0.85) + (0.50 * 0.90)$
$= \boxed{0.895}$

~~b). $P(E_2|E) = P(E_2) * P(E|E_2) / P(E)$~~
~~$= 0.30 * 0.85 / 0.895$~~
~~$= \boxed{0.255} / 0.895$~~

b). $P(E_1|E) = \dfrac{P(E_1) * P(E|E_1)}{P(E)} = \dfrac{0.20 * 0.95}{0.895} = \boxed{0.212}$

14).

	X	Y	Z
let	$2t$	t	t
Defective :	0.2%	0.2%	0.4%

$2t + t + t = 1$
$4t = 1$
$t = 1/4$
$= 0.25$

E_1 = manufactured by X
E_2 = manufactured by Y
E_3 = manufactured by Z

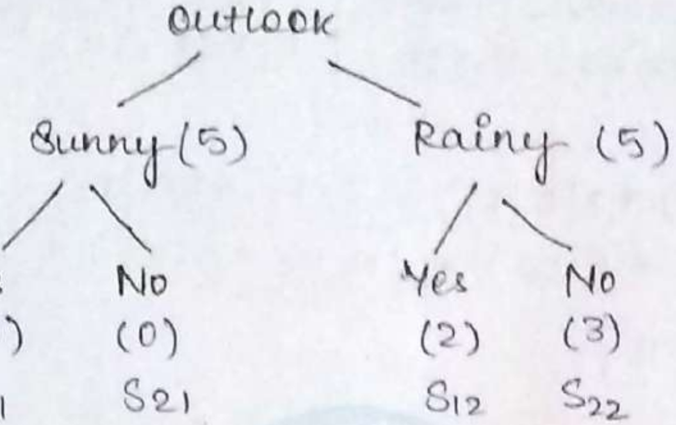
$P(E|E_1) = 0.2\% = 0.002$
$P(E|E_2) = 0.2\% = 0.002$
$P(E|E_3) = 0.4\% = 0.004$

Now, $P(E_2|E) = \dfrac{P(E_2) * P(E|E_2)}{P(E)}$
$= \dfrac{0.25 * 0.002}{(0.5 * 0.002) + (0.25 * 0.002) + (0.25 * 0.004)}$
$= 0.2$ //

15. $C_1 = Yes(7)$
$C_2 = NO(3)$

$\therefore$ $Info(D) = -\frac{7}{10}\log_2(7/10) - \frac{3}{10}\log_2(3/10)$
$= 0.8812$

* Now, for attribute Outlook :-



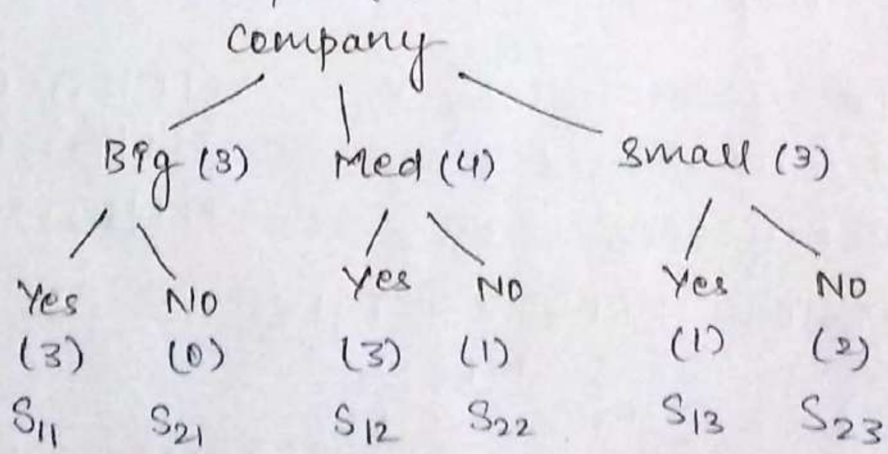
$I(S_{11}, S_{21}) = -5/5 \log_2(5/5) - 0/5 \log_2(0/5) = 0$

$I(S_{12}, S_{21}) = -2/5 \log_2(2/5) - 3/5 \log_2(3/5) = 0.9709$

$\therefore E(A) = \frac{5}{10} * 0 + \frac{5}{10} * 0.9709 = 0.4865$

$\therefore$ gain (outlook) = Info(D) − ~~gatter~~ E(A)
$= 0.8812 - 0.4865$
$= 0.3947$

* For attribute Company :-



$I(S_{11}, S_{12}) = -3/3 \log_2(3/3) - 0/3 \log_2(0/3) = 0$

$I(S_{12}, S_{22}) = -3/4 \log_2(3/4) - 1/4 \log_2(1/4) = 0.8112$

$I(S_{13}, S_{23}) = -1/3 \log_2(1/3) - 2/3 \log_2(2/3) = 0.9182$

$\therefore E(A) = \frac{3}{10} * 0 + \frac{4}{10} * 0.8112 + \frac{3}{10} * 0.9182$

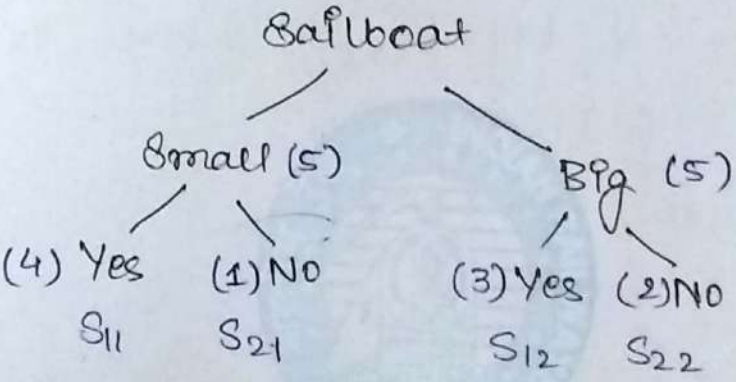
$= 0.5999 = 0.60$

$\therefore$ gain (company) = info(D) − E(A)

$= 0.8812 - 0.60$

$= 0.2812$

* For attribute Sailboat :-



$I(S_{11}, S_{21}) = -\frac{4}{5} \log_2(4/5) - 1/5 \log_2(1/5) = 0.7219$

$I(S_{22}, S_{22}) = -3/5 \log_2(3/5) - 2/5 \log_2(2/5) = 0.9709$

$\therefore E(A) = \frac{5}{10} * 0.7219 + \frac{5}{10} * 0.9709 = 0.8464$

$\therefore$ gain (sailboat) = info(D) − E(A)

$= 0.8812 - 0.8464$

$= 0.0348$

NOW, max [gain(outlook) , gain (company), gain (sailboat)]

= ~~max~~ 0.3947 = gain(outlook)

$\therefore$ outlook is classifying attribute.

Outlook

- Sunny
  - Yes
- Rainy

Company	Sailboat	Sail
NO	Small	NO
Med	Small	Yes
Big	Big	Yes
NO	Big	NO
Med	Big	NO

Now, $C_1 = Yes(2)$, $C_2 = NO(3)$ } → For Root Node

$\therefore$ $Info(D) = -2/5 \log_2(2/5) - 3/5 \log_2(3/5)$

$= 0.9709$

★ For attribute Company:

company

Small (2)		Med (2)		Big (1)	
Yes	NO	Yes	NO	Yes	NO
(0)	(2)	(1)	(1)	(1)	(0)
S_{11}	S_{21}	S_{12}	S_{22}	S_{13}	S_{23}

$I(S_{11}, S_{21}) = -0/2 \log_2(0/2) - 2/2 \log_2(2/2) = 0$

$I(S_{12}, S_{22}) = -1/2 \log_2(1/2) - 1/2 \log_2(1/2) = 1$

$I(S_{13}, S_{23}) = -1/1 \log_2(1/1) - 0/1 \log_2(0/1) = 0$

$\therefore E(A) = 2/5 * 0 + 2/5 * 1 + 1/5 * 0$

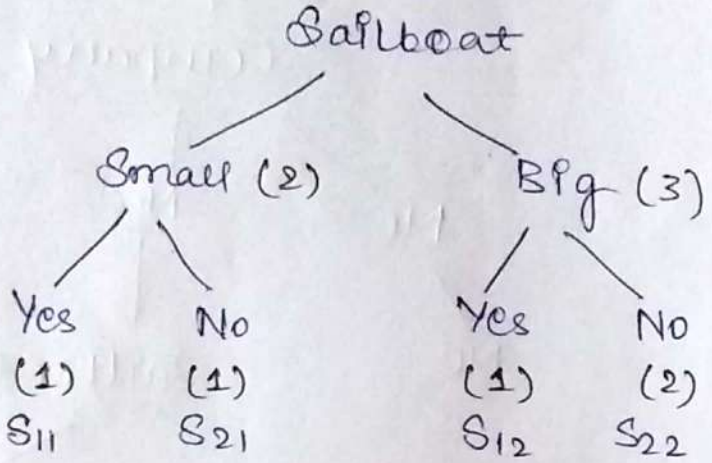
$= \underline{\underline{0.4}}$

$\therefore$ gain(company) = Info(D) − ~~ga~~ E(A)

$= 0.9709 - 0.4$

$= 0.5709$

★ For attribute, Sailboat :-



$I(S_{11}, S_{21}) = -\frac{1}{2}\log_2(\frac{1}{2}) - \frac{1}{2}\log_2(\frac{1}{2}) = 1$

$I(S_{12}, S_{22}) = -\frac{1}{3}\log_2(\frac{1}{3}) - \frac{2}{3}\log_2(\frac{2}{3}) = 0.9183$

$\therefore E(A) = \frac{2}{5} * 1 + \frac{3}{5} * 0.9183 = 0.9510$

$\therefore$ gain(sailboat) = info(D) − E(A)

$= 0.9709 - 0.9510 = \underline{0.02}$

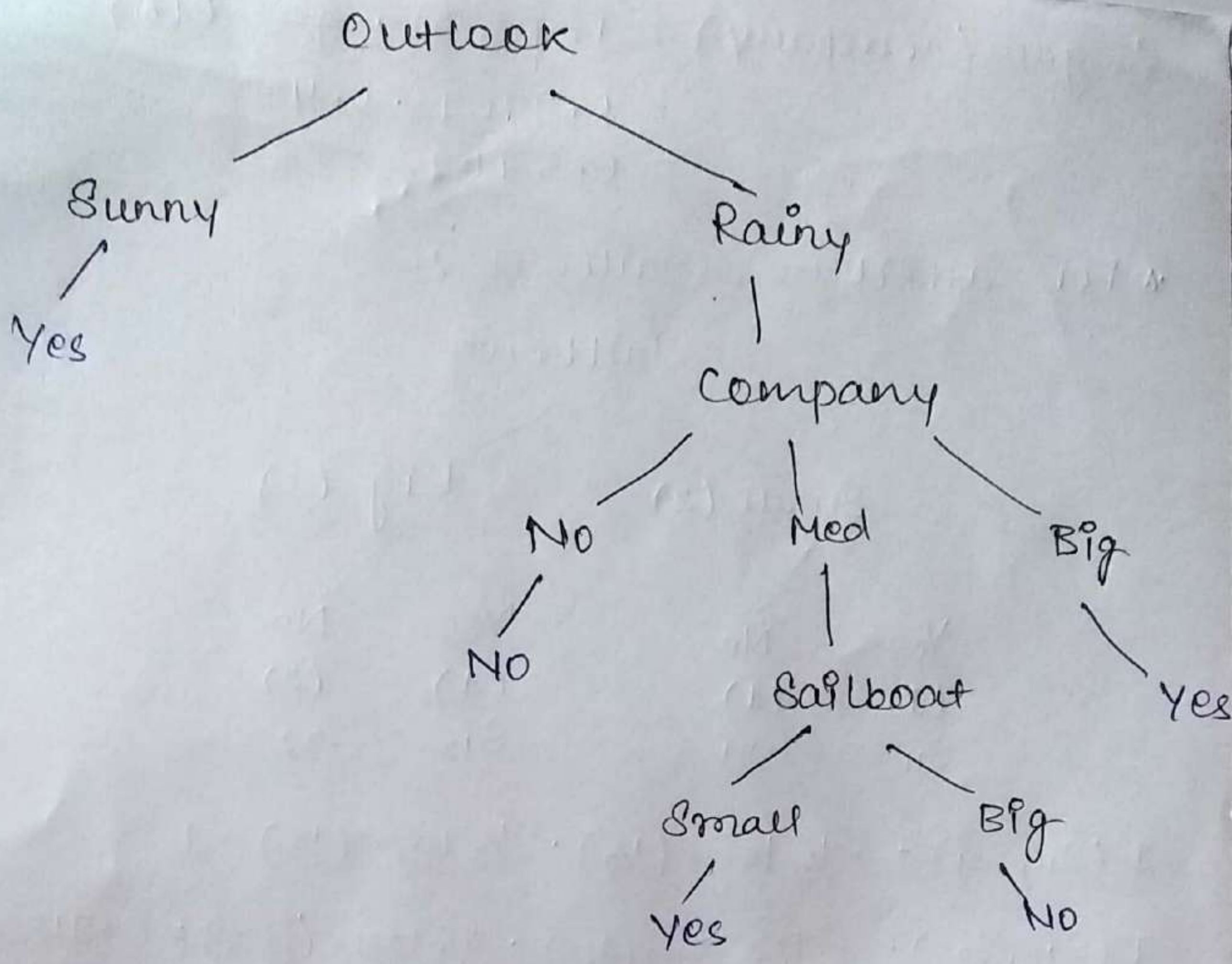
Now, max^{m} [gain(company), gain(sailboat)]

= gain(company) = 0.5709.

$\therefore$ company is the classifying attribute.

$\therefore$ Decision tree looks like :-

P.T.O



Now, for tuple :-

X = (outlook = "rainy", company = "big", sailboat = "small")

$\therefore$ $\boxed{X = \text{yes}}$

4). Different types of events :-

a). Equally Likely Event: The given events are said to be equally likely events, if none of them is expected to occur in preference to the other. i.e. all of them have equal preference.

b). Mutually Exclusive Event: A set of events is said to be mutually exclusive, if the happening of one excludes the happening of other i.e., both cannot occur simultaneously.

If A & B are mutually exclusive then $A \cap B = \phi$

c). Exhaustive event: The set of events is said to be exhaustive, if the performance of the experiment always results in the occurrence of atleast one of them. (13)

E.g If $E_1, E_2, \dots, E_n$ are exhaustive events, then $E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n = S$.

d). Basic terminologies related to Probability:-

- Experiment :- An operation which can produce some well-defined outcomes, is called an experiment.

- Outcomes :- A possible result of a random experi--ment is called its outcomes.

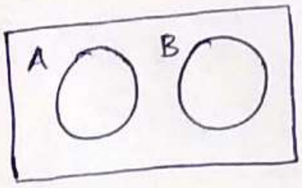
- Sample Space :- The set of all possible outcomes of a random experiment, is known as its sample space. It is denoted by S.

- Event :- A subset of the sample space associated with a random experiment is called an event.

- Trial :- When a random exp. is repeated under ideal conditions & it does not give the same result each time but may result in anyone of the several possible outcomes, then such exp. is called a trial & outcomes are called cases.

5). Mutual Exclusive & Independent Events :-

Mutual Exclusive events are the events that do not occur simultaneously i.e. they are disjoint in nature.



Here A & B are mutually exclusive events.

Two events A & B are said to be independent, if the occurrence or non-occurrence of one event does not affect the occurrence or non-occurrence of another event.

If A & B are independent events, then

$$P(E \cap F) = P(E) \cdot P(F)$$

Example: A coin is tossed.

Let A = Event of occurrence of Head
B = Event of occurrence of Tail

Here, A & B are mutually exclusive events, since, both A & B cannot occur simultaneously (at the same time)

$\therefore A \cap B = \phi$

Also, A & B are independent events, since occurrence of Head does not depend on the occurrence of Tail.

So, A & B are both mutually exclusive as well as independent events.