Solutions

Problems of Chap. 2

2.1 Mixture of pure states. It is easy to see that $\rho(\mathrm{x}) = \rho(0)\delta_{\mathrm{x}0} + \rho(1)\delta_{\mathrm{x}1}$ which suggests that the distribution of the mixing weights must coincide with the distribution ρ. Accordingly, in the case of continuous phase space, we can choose $w(\bar{x}) = \rho(\bar{x})$ to mix the pure states $\delta(x - \bar{x})$:

$$\rho(x) = \int w(\bar{x})\delta(x - \bar{x})\mathrm{d}\bar{x} \ .$$

2.2 Probabilistic or non-probabilistic mixing? Mixing n zeros and n ones is realized by their random permutation that amounts to the following $2n$-partite composite state:

$$\rho(\mathrm{x}_1, \ldots, \mathrm{x}_{2n}) = \frac{\delta_{\mathrm{x}_1 0} \ldots \delta_{\mathrm{x}_n 0}\delta_{\mathrm{x}_{n+1} 1} \ldots \delta_{\mathrm{x}_{2n} 1} + \text{permutations of } \mathrm{x}_1, \ldots \mathrm{x}_{2\mathrm{n}}}{\#\text{ of permutations}} \ .$$

In case of the probabilistic mixing, however, we make $2n$ random decisions as to choose a zero or a one and then we mix the $2n$ elements. The probabilistic mixing amounts to $\rho(\mathrm{x}_1) \ldots \rho(\mathrm{x}_{2n})$ which is obviously different from the above composite state.

2.3 Classical separability. If we choose $w(\bar{x}_A, \bar{x}_B) = \rho_{AB}(\bar{x}_A, \bar{x}_B)$ and replace summation over the weights by integration, it works:

$$\rho_{AB}(x_A, x_B) = \int w(\bar{x}_A, \bar{x}_B)\delta(x_A - \bar{x}_A)\delta(x_B - \bar{x}_B)\mathrm{d}\bar{x}_A\mathrm{d}\bar{x}_B \ .$$

Thus all ρ_{AB} are mixtures of the uncorrelated pure states $\delta(x_A - \bar{x}_A)\delta(x_B - \bar{x}_B)$.

2.4 Decorrelating a single state? The map $\rho_{AB} \to \rho_A\rho_B$ is nonlinear:

$$\rho_{AB}(x_A, x_B) \longrightarrow \int \rho_{AB}(x_A, x'_B)\mathrm{d}x'_B \int \rho_{AB}(x'_A, x_B)\mathrm{d}x'_A \ .$$

Hence the map is not a real operation.

2.5 Decorrelating an ensemble. The collective state $\rho_{AB}^{\times 2n}$ is granted to start with. We interchange the first n and the second n subsystems A. Then we trace over the second n composite systems AB. We get $(\rho_A \rho_B)^{\times n}$. This can be verified if, e.g., we calculate the expectation values of observables like $f(x_{A_1})g(x_{B_1})$, and $f_1(x_{A_1})f_2(x_{A_2})g_1(x_{B_1})g_2(x_{B_2})$, etc.

2.6 Classical indirect measurement. Imagine a detector of discrete state space $\{n\}$. Let the composite state of the system and the detector be $\rho(x,n) = \rho(x)\Pi_n(x)$ where $\rho(x)$ stands for the reduced state of the system. Observe that $\Pi_n(x)$ becomes the conditional state $\rho(n|x)$ of the detector provided the system is in the pure state x. Now we perform a projective measurement on the detector quantity n. Formally, the partition $\{P_m\}$ of the detector state space must be defined as $P_m(n) = \delta_{mn}$ for all m. According to the rules of projective measurement, the state change will be:

$$\rho(x,n) \to \rho_m(x,n) \equiv \frac{1}{p_m}\delta_{mn}\rho(x,n) \ ,$$

with probability $p_m = \int \rho(x,m)\mathrm{d}x$. For the reduced state $\rho(x) = \sum_m \rho(x,m)$, the above projective measurement induces the desired non-projective measurement of the effects $\{\Pi_n(x)\}$.

Problems of Chap. 3

3.1 Bohr quantization of the harmonic oscillator. The sum of the kinetic and potential energies yields the total energy $E = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2$ which is constant during the motion. Therefore the phase space point (q,p) moves on the ellipse of surface $2\pi E/\omega$. The surface plays a role in the Bohr-Sommerfeld q-condition because the contour-integral of $p\mathrm{d}q$ for one period is equal to the surface of the enclosed ellipse. Hence the canonical action takes the form E/ω and we get:

$$E/\omega = \hbar(n + \tfrac{1}{2}) \ .$$

3.2 The role of adiabatic invariants. The canonical actions I_k are adiabatic invariants of the classical motion. This means that they remain approximately constant against whatever large variations of the external parameters of the Hamilton-function provided the variations are slow with respect to the motion. So, the canonical action I of the oscillator will be invariant against the variation of ω in the Hamiltonian $\frac{1}{2}p^2 + \frac{1}{2}\omega^2(t)q^2$ provided $|\dot{\omega}| \ll \omega^2$. The q-condition remains satisfied with the same q-number n.

3.3 Classical-like or q-like motion. The Bohr-Sommerfeld q-condition restricts the continuum of classical motions to a discrete infinite sequence. For small q-numbers this restriction is relevant since the allowed phase space trajectories are well separated. For large q-numbers, typically, the allowed trajectories become quite dense in phase space and might fairly approximate any classical trajectory which does otherwise not satisfy the q-conditions.

Problems of Chap. 4

4.1 Decoherence-free projective measurement. Let us construct the spectral expansion $\hat{A} = \sum_\lambda A_\lambda \hat{P}_\lambda$ and the post-measurement state $\hat{\rho}' = \sum_\lambda \hat{P}_\lambda \hat{\rho} \hat{P}_\lambda$. If $[\hat{A}, \hat{\rho}] = 0$ then $\hat{\rho}' = \hat{\rho}$ since $[\hat{A}, \hat{\rho}] = 0$ is equivalent with $[\hat{P}_\lambda, \hat{\rho}] = 0$ for all λ. To prove the inverse statement, we consider the following identity:

$$[\hat{\rho}' - \hat{\rho}, \hat{P}_\lambda] = [\hat{P}_\lambda, \hat{\rho}] \; .$$

If $\hat{\rho}' = \hat{\rho}$ then the l.h.s. is zero for all λ which implies that the r.h.s. is zero for all λ which implies $[\hat{A}, \hat{\rho}] = 0$.

4.2 Mixing the eigenstates. Let us consider the spectral expansion of the matrix $\hat{\rho}$:

$$\hat{\rho} = \sum_\lambda \rho_\lambda \hat{P}_\lambda \; .$$

If $\hat{\rho}$ is non-degenerate then the $\hat{P}_\lambda$'s correspond to the pure eigenstates of $\hat{\rho}$ and their mixture yields the state $\hat{\rho}$ if the corresponding eigenvalues make the mixing weights: $w_\lambda = \rho_\lambda$. In the general case, the spectral expansion implies the mixture $\hat{\rho} = \sum_\lambda w_\lambda \hat{\rho}_\lambda$ with $w_\lambda = d_\lambda \rho_\lambda$ and $\hat{\rho}_\lambda = \hat{P}_\lambda / d_\lambda$ where d_λ is the dimension of $\hat{P}_\lambda$.

4.3 Separability of pure states. If $|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ then the composite density matrix $\hat{\rho}_{AB}$ is a single tensor product and it is trivially separable. The other way around, when the pure state satisfies the separability condition (4.47):

$$|\psi_{AB}\rangle \langle\psi_{AB}| = \sum_\lambda w_\lambda \hat{\rho}_{A\lambda} \otimes \hat{\rho}_{B\lambda} \; ,$$

then it follows that the matrices on both sides have rank 1. Accordingly, the r.h.s. must be equivalent to the tensor product of rank-one (i.e.: pure state) density matrices:

$$|\psi_{AB}\rangle \langle\psi_{AB}| = |\psi_A\rangle \langle\psi_A| \otimes |\psi_B\rangle \langle\psi_B| \; ,$$

which implies the form $|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$.

4.4 Unitary cloning? Let us suppose that we have duplicated two states $|\psi\rangle$ and $|\psi'\rangle$:

$$|\psi\rangle \otimes |\psi_0\rangle \longrightarrow |\psi\rangle \otimes |\psi\rangle \, ; \quad |\psi'\rangle \otimes |\psi_0\rangle \longrightarrow |\psi'\rangle \otimes |\psi'\rangle \; .$$

The inner product of the two initial composite states is $\langle\psi|\psi'\rangle$ while the inner product of the two final composite states is $\langle\psi|\psi'\rangle^2$. Therefore the above process of state duplication can not be unitary.

Problems of Chap. 5

5.1 Pure state fidelity from density matrices. Observe that $\langle \boldsymbol{m}|\boldsymbol{n}\rangle^2$ equals the trace of the product $|\boldsymbol{n}\rangle \langle\boldsymbol{n}|$ times $|\boldsymbol{m}\rangle \langle\boldsymbol{m}|$. Let us invoke the Pauli-representation of these two density matrices and evaluate the trace of their product:

$$|\langle \boldsymbol{m}|\boldsymbol{n}\rangle|^2 = \mathrm{tr}\left(\frac{\hat{I}+\boldsymbol{n}\hat{\boldsymbol{\sigma}}}{2}\,\frac{\hat{I}+\boldsymbol{m}\hat{\boldsymbol{\sigma}}}{2}\right) = \frac{1+\boldsymbol{n}\boldsymbol{m}}{2}\ ,$$

which yields $\cos^2(\vartheta/2)$.

5.2 Unitary rotation for $|\!\uparrow\rangle \longrightarrow |\!\downarrow\rangle$. Since $|\!\uparrow\rangle$ corresponds to the north pole and $|\!\downarrow\rangle$ corresponds to the south pole on the Bloch-sphere, we need a π-rotation around, e.g., the x-axis. The rotation vector is $\boldsymbol{\alpha} = (\pi, 0, 0)$ and the corresponding unitary transformation becomes:

$$\hat{U}(\boldsymbol{\alpha}) \equiv \exp\left(-\frac{i}{2}\boldsymbol{\alpha}\hat{\boldsymbol{\sigma}}\right) = -\mathrm{i}\hat{\sigma}_x\ .$$

We can check the result directly:

$$-\mathrm{i}\hat{\sigma}_x\,|\!\uparrow\rangle = -\mathrm{i}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = -\mathrm{i}\begin{bmatrix}0\\ 1\end{bmatrix} = -\mathrm{i}\,|\!\downarrow\rangle\ .$$

5.3 Density matrix eigenvalues and -states in terms of polarization. Consider the density matrix $\frac{1}{2}(\hat{I}+\boldsymbol{s}\hat{\boldsymbol{\sigma}})$ and find the spectral expansion of $\boldsymbol{s}\hat{\boldsymbol{\sigma}}$. We learned that if $\boldsymbol{s}$ is a unit vector then $\boldsymbol{s}\hat{\boldsymbol{\sigma}}\,|\!\Uparrow s\rangle = |\!\Uparrow s\rangle$ and $\boldsymbol{s}\hat{\boldsymbol{\sigma}}\,|\!\Downarrow s\rangle = -\,|\!\Downarrow s\rangle$. If $s \leq 1$, the two eigenstates remain the same and we keep the simple notations $|\!\Uparrow s\rangle$, $|\!\Downarrow s\rangle$ to denote qubits polarized along or, respectively, opposite to the direction $\boldsymbol{s}$. The eigenvalues will change trivially and we have $\boldsymbol{s}\hat{\boldsymbol{\sigma}}\,|\!\Uparrow s\rangle = s\,|\!\Uparrow s\rangle$ and $\boldsymbol{s}\hat{\boldsymbol{\sigma}}\,|\!\Downarrow s\rangle = -s\,|\!\Downarrow s\rangle$. Then we can summarize the eigenvalues and eigenstates of the density matrix in the following way:

$$\frac{\hat{I}+\boldsymbol{s}\hat{\boldsymbol{\sigma}}}{2}\,|\!\Uparrow s\rangle = \frac{1+s}{2}\,|\!\Uparrow s\rangle\ ,\qquad \frac{\hat{I}+\boldsymbol{s}\hat{\boldsymbol{\sigma}}}{2}\,|\!\Downarrow s\rangle = \frac{1-s}{2}\,|\!\Downarrow s\rangle\ .$$

5.4 Magnetic rotation for $|\!\uparrow\rangle \longrightarrow |\!\downarrow\rangle$. We must implement a π-rotation of the polarization vector and we can choose the rotation vector $(\pi, 0, 0)$ which means π-rotation around the x-axis. In magnetic field $\boldsymbol{\omega}$, the polarization vector $\boldsymbol{s}$ satisfies the classical equation of motion $\dot{\boldsymbol{s}} = \boldsymbol{\omega}\times\boldsymbol{s}$ meaning that $\boldsymbol{s}$ will rotate around the direction $\boldsymbol{\omega}$ of the field at angular velocity ω. Accordingly, we can choose the field to point along the x-axis: $\boldsymbol{\omega} = (\omega, 0, 0)$. The rotation angle π is achieved if we switch on the field for a period $t = \pi/\omega$.

5.5 Interrelated qubit physical quantities.

$$\hat{P}_n + \hat{P}_{-n} = \hat{I}$$

$$2\hat{P}_n - \hat{\sigma}_n = 2\hat{P}_{-n} + \hat{\sigma}_n = \hat{I}$$

5.6 Mixing non-orthogonal polarizations. Since the qubit density matrix is a linear function of the polarization vector, mixing the density matrices means averaging their polarization vectors with the mixing weights. Therefore our mixture has the following polarization vector:

$$\boldsymbol{s} = \frac{1}{3}\times(0,0,1) + \frac{2}{3}\times(1,0,0) = (1/3, 0, 2/3)\ .$$

Problems of Chap. 6

6.1 Universality of Hadamard and phase operations. The unitary rotations $\hat{U}$ of the qubit space are, apart from an irrelevant phase, equivalent to the spatial rotations of the corresponding Bloch sphere. This time the three Euler angles ψ, θ, ϕ are the natural parameters. We can write the unitary rotations, corresponding to the spatial ones, into this form:

$$\hat{U}(\psi,\theta,\phi) = \exp\left(-\frac{i}{2}\psi\hat{\sigma}_z\right)\exp\left(-\frac{i}{2}\theta\hat{\sigma}_x\right)\exp\left(-\frac{i}{2}\phi\hat{\sigma}_z\right) .$$

The middle factor, too, becomes rotation around the z-axis if we sandwich it between two Hadamard operations because $\hat{H}\hat{\sigma}_x\hat{H} = \hat{\sigma}_z$. We can thus express the r.h.s. in the desired form:

$$\hat{U}(\psi,\theta,\phi) = \hat{T}(\psi)\hat{H}\hat{T}(\theta)\hat{H}\hat{T}(\phi) .$$

6.2 Statistical error of qubit determination. Out of N, we allocate N_x, N_y, N_z qubits to estimate s_x, s_y, s_z, respectively. We learned that the estimated value of s_x takes this form:

$$\frac{N_{\uparrow x} - N_{\downarrow x}}{N_{\uparrow x} + N_{\downarrow x}} = \frac{2N_{\uparrow x}}{N_x} - 1 ,$$

because on a large statistics $N_x = N_{\uparrow x} + N_{\downarrow x}$ the ratio $N_{\uparrow x}/N_x$ converges to the q-theoretical prediction $p_{\uparrow x} = \langle\uparrow x|\,\hat{\rho}\,|\uparrow x\rangle \equiv \frac{1}{2}(1+s_x)$. The statistical error of the estimation takes the form $2\Delta N_{\uparrow x}/N_x$ and we are going to determine the mean fluctuation $\Delta N_{\uparrow x}$. The statistical distribution of the count $N_{\uparrow x}$ is binomial:

$$p(N_{\uparrow x}) = \binom{N_x}{N_{\uparrow x}} p_{\uparrow x}^{N_{\uparrow x}} p_{\downarrow x}^{N_{\downarrow x}} ,$$

hence the mean squared fluctuation of the count $N_{\uparrow x}$ takes the form $(\Delta N_{\uparrow x})^2 = N_x p_{\uparrow x} p_{\downarrow x} = N_x(1-s_x^2)/4$. This yields the ultimate form of the estimation error:

$$\Delta s_x = \sqrt{\frac{1-s_x^2}{N_x}} ,$$

and we could get similar results for Δs_y and Δs_z.

6.3 Fidelity of qubit determination. If the state $|\boldsymbol{n}\rangle$ sent by Alice and the polarization $\hat{\sigma}_m$ chosen by Bob were fixed then the structure of the expected fidelity of Bob's guess would be this:

$$|\langle\boldsymbol{n}|\boldsymbol{m}\rangle|^2 p_{\uparrow m} + |\langle\boldsymbol{n}|{-\boldsymbol{m}}\rangle|^2 p_{\downarrow m} .$$

Here we have understood that Bob's optimum guess must always be the post-measurement state $|\pm\boldsymbol{m}\rangle$ based on the measurement outcome $\hat{\sigma}_m = \pm 1$, respectively. Now we recall that $|\langle\boldsymbol{n}|\boldsymbol{m}\rangle|^2 = p_{\uparrow m} = \cos^2(\vartheta/2)$ where $\cos\vartheta = \boldsymbol{nm}$, and

$|\langle \boldsymbol{n}|{-\boldsymbol{m}}\rangle|^2 = p_{\downarrow m} = \sin^2(\vartheta/2)$. Hence the above fidelity takes the simple form $\cos^4(\vartheta/2) + \sin^4(\vartheta/2)$ which we rewrite into the equivalent form $\frac{1}{2} + \frac{1}{2}\cos^2(\vartheta)$. The average of $\cos^2(\vartheta) = (\boldsymbol{nm})^2$ over the random independent $\boldsymbol{n}$ and $\boldsymbol{m}$ yields $1/3$ therefore the expected fidelity of Bob's guess becomes $2/3$.

6.4 Post-measurement depolarization. Let $\hat{\boldsymbol{\sigma}}_n$ denote the polarization chosen by Bob. The non-selective measurement induces the change $\hat{\rho} \to \hat{P}_n\hat{\rho}\hat{P}_n + \hat{P}_{-n}\hat{\rho}\hat{P}_{-n}$ of the state. Inserting the Pauli-representation of $\hat{\rho}$ and the projectors $\hat{P}_{\pm n}$ yields:

$$\frac{\hat{I} + \boldsymbol{s}\hat{\boldsymbol{\sigma}}}{2} \to \frac{\hat{I} + \hat{\sigma}_n}{2}\,\frac{\hat{I} + \boldsymbol{s}\hat{\boldsymbol{\sigma}}}{2}\,\frac{\hat{I} + \hat{\sigma}_n}{2} + \frac{\hat{I} - \hat{\sigma}_n}{2}\,\frac{\hat{I} + \boldsymbol{s}\hat{\boldsymbol{\sigma}}}{2}\,\frac{\hat{I} - \hat{\sigma}_n}{2} =$$

$$= \frac{\hat{I} + \boldsymbol{s}\hat{\boldsymbol{\sigma}}}{4} + \frac{\hat{I} + \boldsymbol{s}\hat{\sigma}_n\hat{\boldsymbol{\sigma}}\hat{\sigma}_n}{4} .$$

Since Bob's choice is random regarding $\boldsymbol{n}$ we shall average $\boldsymbol{n}$ over the solid angle. Averaging the structure $\hat{\sigma}_n\hat{\boldsymbol{\sigma}}\hat{\sigma}_n$ yields $-\hat{\boldsymbol{\sigma}}/3$ hence the average influence of Bob's non-selective measurements can be summarized as:

$$\frac{\hat{I} + \boldsymbol{s}\hat{\boldsymbol{\sigma}}}{2} \to \frac{\hat{I} + \boldsymbol{s}\hat{\boldsymbol{\sigma}}/3}{2} .$$

6.5 Anti-linearity of polarization reflection. Let us calculate the influence of the anti-unitary transformation $\hat{T}$ on a pure state qubit:

$$\hat{T}\,|\boldsymbol{n}\rangle = \hat{T}\left(\cos\frac{\theta}{2}\,|{\uparrow}\rangle + \mathrm{e}^{\mathrm{i}\varphi}\sin\frac{\theta}{2}\,|{\downarrow}\rangle\right) = -\cos\frac{\theta}{2}\,|{\downarrow}\rangle + \mathrm{e}^{-\mathrm{i}\varphi}\sin\frac{\theta}{2}\,|{\uparrow}\rangle = \mathrm{e}^{-\mathrm{i}\varphi}\,|{-\boldsymbol{n}}\rangle \ .$$

6.6 General qubit effects. We can suppose that the weights w_n are non-vanishing. First, we have to impose the conditions $|\boldsymbol{a}_n| \leq 1$ since otherwise the matrices would be indefinite. Second, the request $\hat{\Pi}_n \geq 0$ implies the conditions $w_n > 0$. And third, the request $\sum_n \hat{\Pi}_n = \hat{I}$ implies the conditions $\sum_n w_n = 1$ and $\sum_n w_n\boldsymbol{a}_n = 0$.

Problems of Chap. 7

7.1 Schmidt orthogonalization theorem. Let r be the rank of $\hat{c}$ and let us consider the non-negative matrices $\hat{c}\hat{c}^\dagger$ and $\hat{c}^\dagger\hat{c}$ of rank r both. Their spectrum is non-negative and identical. Indeed, if $\hat{c}^\dagger\hat{c}\,|R\rangle = w\,|R\rangle$, i.e., w and $|R\rangle$ are an eigenvalue and a (normalized) eigenvector of $\hat{c}^\dagger\hat{c}$ then $|L\rangle = \hat{c}\,|R\rangle/\sqrt{w}$ will be a (normalized) eigenvector of $\hat{c}\hat{c}^\dagger$ with the same eigenvalue w. This can be seen by direct inspection. Now we determine the r positive eigenvalues w_λ for $\lambda = 1, 2, \ldots, r$ and the corresponding orthonormal eigenstates $|\lambda; R\rangle$ of $\hat{c}^\dagger\hat{c}$. Then, by $|\lambda; L\rangle = \hat{c}\,|\lambda; R\rangle/\sqrt{w_\lambda}$, we define the r orthonormal eigenstates of $\hat{c}\hat{c}^\dagger$ which belong to the common positive eigenvalues w_λ, for $\lambda = 1, 2, \ldots, r$. Now we can see that

$$\hat{c}\,|\lambda; R\rangle = \sqrt{w_\lambda}\,|\lambda; L\rangle \ ,$$

for all $\lambda = 1, 2, \ldots, r$. We have thus proven that there exists the following Schmidt decomposition of the matrix $\hat{\mathrm{c}}$:

$$\hat{\mathrm{c}} = \sum_{\lambda=1}^{r} \sqrt{w_\lambda}\, |\lambda; L\rangle \langle\lambda; R| \ .$$

7.2 Swap operation. For convenience, we introduce $\hat{\sigma}^{(\pm)} = (\hat{\sigma}_x \pm i\hat{\sigma}_y)/2$ instead of $\hat{\sigma}_x, \hat{\sigma}_y$. Now we express the Pauli matrices in the up-down basis:

$$\hat{\sigma}^{(+)} = |\uparrow\rangle\langle\downarrow| \ , \quad \hat{\sigma}^{(-)} = |\downarrow\rangle\langle\uparrow| \ , \quad \hat{\sigma}_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow| \ .$$

Substituting these expressions we obtain:

$$\frac{\hat{I}\otimes\hat{I} + \hat{\boldsymbol{\sigma}}\otimes\hat{\boldsymbol{\sigma}}}{2} = \frac{\hat{I}\otimes\hat{I} + 2\hat{\sigma}^{(+)}\otimes\hat{\sigma}^{(-)} + 2\hat{\sigma}^{(-)}\otimes\hat{\sigma}^{(+)} + \hat{\sigma}_z\otimes\hat{\sigma}_z}{2} =$$

$$= |\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\downarrow\downarrow\rangle\langle\downarrow\downarrow| + |\uparrow\downarrow\rangle\langle\downarrow\uparrow| + |\downarrow\uparrow\rangle\langle\uparrow\downarrow| \ ,$$

which is indeed the swap matrix $\hat{\mathrm{S}}$.

7.3 Singlet density matrix. The singlet state $\hat{\rho}(\mathrm{singlet})$ is invariant under rotations of the Bloch sphere. Therefore $\hat{\rho}(\mathrm{singlet})$ must be of the form:

$$\hat{\rho}(\mathrm{singlet}) = \frac{\hat{I}\otimes\hat{I} + \lambda\hat{\boldsymbol{\sigma}}\otimes\hat{\boldsymbol{\sigma}}}{4}$$

because there are no further rotational invariant mathematical structures. We could determine the parameter λ from the pure state condition $[\hat{\rho}(\mathrm{singlet})]^2 = \hat{\rho}(\mathrm{singlet})$, yielding $\lambda = -1$. However, we can spare these calculations if we recall the swap $\hat{\mathrm{S}}$. It is Hermitian, rotation invariant and idempotent: $\hat{\mathrm{S}}^2 = \hat{I}\otimes\hat{I}$. Hence we get the singlet state directly in the form:

$$\hat{\rho}(\mathrm{singlet}) = \frac{\hat{I}\otimes\hat{I} - \hat{\mathrm{S}}}{2} = \frac{\hat{I}\otimes\hat{I} - \hat{\boldsymbol{\sigma}}\otimes\hat{\boldsymbol{\sigma}}}{4} \ .$$

7.4 Local measurement of expectation values. Alice and Bob will determine $\langle\hat{A}\otimes\hat{B}\rangle$ and $\langle\hat{A}'\otimes\hat{B}'\rangle$ separately on two independent sub-ensembles and will finally add them since the expectation value is additive. Still we have to show that the expectation value of a tensor product, like $\langle\hat{A}\otimes\hat{B}\rangle$, can be determined in local measurements. We introduce the local spectral expansions $\hat{A} = \sum_\lambda A_\lambda \hat{P}_\lambda$ and $\hat{B} = \sum_\mu B_\mu \hat{Q}_\mu$. Alice and Bob perform local measurements of $\hat{A}$ and $\hat{B}$ in coincidence, yielding the measurement outcomes $A_1, B_1, A_2, B_2, \ldots, A_r, B_r, \ldots A_N, B_N$ where A_r is always an eigenvalue A_λ and the case is similar for the B_r's. Then Alice and Bob can calculate the q-expectation value asymptotically:

$$\langle\hat{A}\otimes\hat{B}\rangle = \lim_{N\to\infty}\frac{1}{N}\sum_r A_r B_r \ .$$

To see that this is indeed the right expression of $\langle \hat{A} \otimes \hat{B} \rangle$ we have to rethink the nonlocal measurement of $\hat{A} \otimes \hat{B}$ itself. Its spectral expansion is

$$\hat{A} \otimes \hat{B} = \sum_{(\lambda,\mu)} (A_\lambda B_\mu)(\hat{P}_\lambda \otimes \hat{Q}_\mu) \ ,$$

and the corresponding q-measurement will obviously yield the same statistics of the outcomes $A_r B_r$ like in case of the local-measurements.

7.5 Local measurement of certain nonlocal quantities. If we measure $\hat{\sigma}_z \otimes \hat{\sigma}_z$ on a singlet state we always get -1 and the singlet state remains the post-measurement state. In the attempted local measurement, the entanglement is always destroyed and we get either $|\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle$ for the post-measurement state. Obviously the degenerate spectrum of $\hat{\sigma}_z \otimes \hat{\sigma}_z$ plays a role in the nonlocality. If, in the general case, we suppose $\hat{A} \otimes \hat{B}$ has a non-degenerate spectrum then the post-measurement states will be the same pure states in both the nonlocal measurement of $\hat{A} \otimes \hat{B}$ and the simultaneous local measurements of $\hat{A}$ and $\hat{B}$.

7.6 Nonlocal hidden parameters. Let the further hidden parameter ν take values $1, 2, 3, 4$ marking whether Alice and Bob measures $\hat{A} \otimes \hat{B}$, $\hat{A}' \otimes \hat{B}$, $\hat{A} \otimes \hat{B}'$ or $\hat{A}' \otimes \hat{B}'$, respectively. Then, according to the hidden variable concept, the assignment of all four polarization values will uniquely depend on the composite hidden variable $r\nu$:

$$\hat{A} = A_{r\nu} = \pm 1 \ , \ \ \hat{A}' = A'_{r\nu} = \pm 1 \ , \quad \hat{B} = B_{r\nu} = \pm 1 \ , \ \ \hat{B}' = B'_{r\nu} = \pm 1 \ .$$

Contrary to the local assignment (7.36), the above assignments are called nonlocal since the hidden variable $r\nu$ is nonlocal: it depends on both Alice's and Bob's measurement setup. The statistical relationships, cf. (7.37), become modified:

$$\langle \hat{A} \otimes \hat{B} \rangle = \lim_{N_1 \to \infty} \frac{1}{N_1} \sum_{r \in \Omega_1} A_{r1} B_{r1}; \quad N_1 = |\Omega_1|,$$

$$\langle \hat{A}' \otimes \hat{B} \rangle = \lim_{N_2 \to \infty} \frac{1}{N_2} \sum_{r \in \Omega_2} A_{r2} B_{r2} \ ; \quad N_2 = |\Omega_2| \ ,$$

etc. for the other two cases $\nu = 3, 4$. The assignments are independent for the four different values of ν. There is no constraint combining the $A_{r\nu}$'s with different ν's. Hence it has become straightforward to reproduce the above said four q-theoretical predictions including of course correlations $\langle \hat{\mathbf{C}} \rangle$ that are higher than 2.

7.7 Does teleportation clone the qubit? The selective post-measurement state of the two qubits on Alice's side is one of the four maximally entangled Bell-states. Therefore the reduced state of the qubit that she had teleported is left in the totally mixed state independently of its original form as well as of the four outcomes of Alice's measurement. Note that the form (7.47) of the three-qubit pre-measurement state shows that Alice's measurement outcome is always random. The four outcomes have probability $1/4$ each.

Problems of Chap. 8

8.1 All q-operations are reductions of unitary dynamics. Given the trace-preserving q-operation $\mathcal{M}\hat{\rho} = \sum_n \hat{M}_n \hat{\rho} \hat{M}_n^\dagger$, we have to construct the unitary interaction matrix $\hat{U}$ acting on the composite state of the system and environment. Let us introduce the composite basis $|\lambda\rangle \otimes |n;E\rangle$ where $\lambda = 1,2,\dots,d$ and $n = 1,2,\dots,d_E$. Let us define the influence of $\hat{U}$ on a subset of the composite basis:

$$\hat{U}\left(|\lambda\rangle \otimes |1;E\rangle\right) = \sum_{n=1}^{d_E} \hat{M}_n |\lambda\rangle \otimes |n;E\rangle \ , \quad \lambda = 1,2,\dots,d \ .$$

This definition is possible because the above map generates orthonormal vectors:

$$\sum_{m=1}^{d_E} \langle\mu| \hat{M}_m^\dagger \otimes \langle m;E| \sum_{n=1}^{d_E} \hat{M}_n |\lambda\rangle \otimes |n;E\rangle = \sum_{n=1}^{d_E} \langle\mu| \hat{M}_n^\dagger \hat{M}_n |\lambda\rangle = \delta_{\lambda\mu} \ .$$

The further matrix elements of $\hat{U}$, i.e. those not defined by our first equation above, can be chosen in such a way that $\hat{U}$ is unitary on the whole composite state. Using this definition of $\hat{U}$ in the equation (8.3) of reduced dynamics we can directly inspect that the resulting operation is $\mathcal{M}\hat{\rho} = \sum_n \hat{M}_n \hat{\rho} \hat{M}_n^\dagger$, as expected.

8.2 Non-projective effect as averaged projection. Let us substitute the proposed form of the effects $\hat{\Pi}_n$ into the equation $p_n = \mathrm{tr}(\hat{\Pi}_n \hat{\rho})$ introduced for non-projective measurement in Sect. 4.4.2:

$$\mathrm{tr}\left(\hat{\Pi}_n \hat{\rho}\right) = \mathrm{tr}\left(\mathrm{tr}_E\, \hat{P}_n \hat{\rho}_E \hat{\rho}\right) = \mathrm{tr}\left(\hat{P}_n \hat{\rho}_E \hat{\rho}\right) \ .$$

In this formalism, i.e., without the $\otimes$'s, the matrices of different subsystems commute hence $\hat{\rho}_E \hat{\rho} = \hat{\rho}\hat{\rho}_E$. Thus we obtain the following result: $\mathrm{tr}(\hat{\Pi}_n \hat{\rho}) = \mathrm{tr}(\hat{P}_n \hat{\rho}\hat{\rho}_E)$. We recognize the coincidence of the r.h.s. with the r.h.s. of (8.18). Since this coincidence is valid for all possible $\hat{\rho}$, it verifies the proposed form of $\hat{\Pi}_n$.

8.3 Q-operation as supermatrix. We start from the Kraus representation $\mathcal{M}\hat{\rho} = \sum_n \hat{M}_n \hat{\rho} \hat{M}_n^\dagger$. We take the matrix elements of both sides and we also sandwich the $\hat{\rho}$ between the identities $\sum_{\lambda'} |\lambda'\rangle\langle\lambda'|$ and $\sum_{\mu'} |\mu'\rangle\langle\mu'|$ on the r.h.s.:

$$\langle\lambda| \mathcal{M}\hat{\rho} |\mu\rangle = \langle\lambda| \sum_n \hat{M}_n \sum_{\lambda'} |\lambda'\rangle\langle\lambda'| \hat{\rho} \sum_{\mu'} |\mu'\rangle\langle\mu'| \hat{M}_n^\dagger |\mu\rangle \ .$$

Comparing the r.h.s. with $\sum_{\lambda'\mu'} \mathcal{M}_{\lambda\mu\lambda'\mu'} \rho_{\lambda'\mu'}$, we read out the components of the supermatrix: $\mathcal{M}_{\lambda\mu\lambda'\mu'} = \sum_n \langle\lambda| \hat{M}_n |\lambda'\rangle\langle\mu'| \hat{M}_n^\dagger |\mu\rangle$.

8.4 Environmental decoherence, time-continuous depolarization. The equation takes the Lindblad form with $\hat{H} = 0$ and with three hermitian Lindblad matrices identified by the Cartesian components of $(\hat{\boldsymbol{\sigma}}/2\sqrt{\tau})$. For convenience, we shall use the Einstein convention to sum over repeated indices, e.g.: $\boldsymbol{s}\hat{\boldsymbol{\sigma}} = s_a \hat{\sigma}_a$. We write the r.h.s. of the master equation into the equivalent form $-(1/8\tau)[\hat\sigma_b,[\hat\sigma_b,\hat\rho]\,]$ and insert $\hat\rho=\frac{1}{2}(\hat I+s_a\hat\sigma_a)$ into it. The master equation reduces to:

$$\dot s_a\hat\sigma_a=-\frac{1}{8\tau}[\hat\sigma_b,[\hat\sigma_b,s_a\hat\sigma_a]\,]=-\frac{1}{\tau}s_a\hat\sigma_a\ ,$$

which means the simple equation $\dot{\boldsymbol s}=-\boldsymbol s/\tau$ for the polarization vector. Its solution is $\boldsymbol s(t)=\mathrm{e}^{-t/\tau}\boldsymbol s(0)$. Therefore the τ may be called depolarization time, or decoherence time as well.

8.5 Kraus representation of depolarization. The map should be of the form $\mathcal{M}\hat\rho=(1-3\lambda)\hat\rho+\lambda\hat{\boldsymbol\sigma}\hat\rho\hat{\boldsymbol\sigma}$ with $0\le\lambda\le1/3$ since there exist no other isotropic Kraus structures for a qubit. The depolarization channel decreases the polarization vector $\boldsymbol s$ by a factor $1-w$ and we have to find the parameter λ as function of w. Inserting $\hat\rho=\frac{1}{2}(\hat I+\boldsymbol s\hat{\boldsymbol\sigma})$ we get

$$\mathcal{M}\frac{\hat I+\boldsymbol s\hat{\boldsymbol\sigma}}{2}=\frac{\hat I+(1-4\lambda)\boldsymbol s\hat{\boldsymbol\sigma}}{2}\ ,$$

which means that $\lambda=w/4$. Four Kraus matrices make the depolarization channel: $\sqrt{1-3w/4}\hat I$ and the three components of $\sqrt{w/4}\hat{\boldsymbol\sigma}$.

Problems of Chap. 9

9.1 Positivity of relative entropy. We can write:

$$S(\rho'\|\rho)=\sum_{\mathrm x}\rho(\mathrm x)\log\frac{\rho(\mathrm x)}{\rho'(\mathrm x)}\ .$$

We invoke the inequality $\ln\lambda>1-\lambda^{-1}$ valid for $\lambda\neq1$ and apply it to $\lambda=\rho/\rho'$. This yields:

$$S(\rho'\|\rho)>\frac{1}{\ln 2}\sum_{\mathrm x}\rho(\mathrm x)\left[1-\frac{\rho'(\mathrm x)}{\rho(\mathrm x)}\right]=0\ ,$$

which always holds if $\rho'\neq\rho$.

9.2 Concavity of entropy. Suppose we have a long message $\mathrm x_1^{(1)}\mathrm x_2^{(1)}\dots\mathrm x_n^{(1)}$ where $\rho_1(\mathrm x)$ is the apriori distribution of one letter. Let S_1 stand for the single-letter entropy $S(\rho_1)$. The number of the typical messages is 2^{nS_1} so that their shortest code is nS_1 bits. Consider a second message from the same alphabet and suppose the single-letter distribution $\rho_2(\mathrm x)$ is different from $\rho_1(\mathrm x)$. Let us concatenate the two messages:

$$\mathrm x_1^{(1)}\mathrm x_2^{(1)}\mathrm x_3^{(1)}\dots\mathrm x_n^{(1)}\mathrm x_1^{(2)}\mathrm x_2^{(2)}\mathrm x_3^{(2)}\dots\mathrm x_m^{(2)}\ ,$$

where the two lengths n and m may be different. Obviously, the number of the typical ones among such composite messages is $2^{nS_1}\times2^{mS_2}$ and their shortest code is nS_1+mS_2 bits. Now imagine that we permute the $n+m$ letters randomly. On one hand, the composite messages become usual $(n+m)$-letter-long messages where the single letter distribution is always the same, i.e., the mixture $\rho = w_1\rho_1 + w_2\rho_2$ with weights $w_1 = n/(n+m)$ and $w_2 = m/(n+m)$. Therefore the number of the typical messages is $2^{(n+m)S(\rho)}$ and the shortest code is $(n+m)S(\rho)$ bits. On the other hand, we can inspect that the number of the typical messages $2^{(n+m)S(\rho)}$ is greater than $2^{nS_1} \times 2^{mS_2}$ because the number of inequivalent permutations has increased: the first n letters have become permutable with the last m letters. This means that $(n+m)S(\rho) > nS_1 + mS_2$ which is just the concavity of the entropy: $S(w_1\rho_1 + w_2\rho_2) > w_1S(\rho_1) + w_2S(\rho_2)$, for $\rho_1 \neq \rho_2$.

9.3 Subadditivity of entropy. We can make the choice $\rho'_{AB}(\mathrm{x},\mathrm{y}) = \rho_A(\mathrm{x})\rho_B(\mathrm{y})$. To calculate $S(\rho'_{AB}\|\rho_{AB}) = -S(\rho_{AB}) - \sum_{\mathrm{x},\mathrm{y}} \rho_{AB}(\mathrm{x},\mathrm{y}) \log \rho'_{AB}(\mathrm{x},\mathrm{y})$, we note that the second term is $-\sum_{\mathrm{x},\mathrm{y}} \rho_{AB}(\mathrm{x},\mathrm{y}) \log[\rho_A(\mathrm{x})\rho_B(\mathrm{y})] = S(\hat{\rho}_A) + S(\hat{\rho}_B)$. Hence the positivity of the relative entropy $S(\rho'_{AB}\|\rho_{AB}) \geq 0$ proves subadditivity: $S(\rho_{AB}) \leq S(\rho_A) + S(\rho_B)$.

9.4 Coarse graining increases entropy. Let us identify our system by the k-partite composite system of the k bits $\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_k$. Then the coarse grained system corresponds to the $(k-1)$-partite sub-system consisting of the first $k-1$ bits $\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_{k-1}$. The coarse grained state $\tilde{\rho}$ is just a reduced reduced state w.r.t. ρ. Hence we see that coarse graining increases the entropy because reduction does it.

Problems of Chap. 10

10.1 Subadditivity of q-entropy. Let us calculate

$$S(\hat{\rho}_A \otimes \hat{\rho}_B \| \hat{\rho}_{AB}) = -S(\hat{\rho}_{AB}) - \mathrm{tr}[\hat{\rho}_{AB} \log(\hat{\rho}_A \otimes \hat{\rho}_B)]$$

and note that the second term is $S(\hat{\rho}_A) + S(\hat{\rho}_B)$. Hence the Klein inequality $S(\hat{\rho}'\|\hat{\rho}) \geq 0$ proves the subadditivity: $S(\hat{\rho}_{AB}) \leq S(\hat{\rho}_A) + S(\hat{\rho}_B)$.

10.2 Concavity of q-entropy, Holevo entropy. We assume a certain environmental system E and a basis $\{|n;E\rangle\}$ for it. Let us construct a composite state:

$$\hat{\rho}_{big} = \sum_n w_n \hat{\rho}_n \otimes |n;E\rangle\langle n;E| \ .$$

Note that the reduced state of the system is invariably $\hat{\rho} = \sum_n w_n\hat{\rho}_n$ and the reduced state of the environment is $\hat{\rho}_E = \sum_n w_n |n;E\rangle\langle n;E|$. Subadditivity guarantees that $S(\hat{\rho}_{big}) \leq S(\hat{\rho}) + S(\hat{\rho}_E)$. Let us calculate and insert the entropies $S(\hat{\rho}_{big}) = S(w) + \sum_n w_n S(\hat{\rho}_n)$ and $S(\hat{\rho}_E) = S(w)$ which results in the desired inequality: $\sum_n w_n S(\hat{\rho}_n) \leq S(\hat{\rho})$.

10.3 Data compression of the non-orthogonal code. The density matrix of the corresponding 1-letter q-message reads:

$$\hat{\rho} = \frac{|\!\uparrow z\rangle\langle\uparrow z| + |\!\uparrow x\rangle\langle\uparrow x|}{2} = \frac{\hat{I} + \hat{\sigma}_n/\sqrt{2}}{2} \ ,$$

which is a partially polarized state along the skew direction $\boldsymbol{n} = (1,0,1)/\sqrt{2}$, cf. (6.29). The eigenvalues of this density matrix are the following:

$$p_+ = \frac{1+1/\sqrt{2}}{2} \ , \quad p_- = \frac{1-1/\sqrt{2}}{2} \ ,$$

hence its von Neumann entropy amounts to:

$$S(\hat{\rho}) = -p_+ \log p_+ - p_- \log p_- \approx 0.60 \ .$$

According to the q-data compression theorem, we can compress one qubit of the q-message into 0.6 qubit on average, and this is the best maximum faithful compression.

10.4 Distinguishing two non-orthogonal qubits: various aspects. In fact, we measure the polarization component orthogonal to the polarization of the single-letter density matrix. The measurement outcomes ± 1 on both q-states $|\Uparrow z\rangle, |\Uparrow x\rangle$ will appear with probabilities p_+ and p_- (cf. Prob. 10.3), in alternating order of course:

$$\begin{array}{ll} p(\mathrm{y}=+1|\mathrm{x}=0) = p_+, & p(\mathrm{y}=-1|\mathrm{x}=0) = p_- \\ p(\mathrm{y}=+1|\mathrm{x}=1) = p_-, & p(\mathrm{y}=-1|\mathrm{x}=1) = p_+ \ . \end{array}$$

Regarding the randomness of the input message the output message, too, becomes random: $H(Y) = 1$. Hence the information gain takes this form and value:

$$I_{gain} = H(Y) - H(Y|X) = 1 + p_+ \log p_+ + p_- \log p_- \approx 0.40 \ .$$

10.5 Simple optimum q-code. The q-data compression theory says that a pure state q-code is not compressible faithfully (i.e.: allowing the same accessible information) if and only if the single-letter average state has the maximum von Neumann entropy. In our case, we must assure the following:

$$\frac{|R\rangle\langle R| + |G\rangle\langle G| + |B\rangle\langle B|}{3} = \frac{\hat{I}}{2} \ ,$$

which is possible if we chose three points on a main circle of the Bloch-sphere, at equal distances from each other.

Problems of Chap. 11

11.1 Creating the totally symmetric state.

$$|S\rangle \equiv \frac{1}{2^{n/2}} \sum_{\mathrm{x}=1}^{2^n-1} |\mathrm{x}_n \mathrm{x}_{n-1} \dots \mathrm{x}_1\rangle = \sum_{\mathrm{x}_n=0}^{1} \sum_{\mathrm{x}_{n-1}=0}^{1} \cdots \sum_{\mathrm{x}_1=0}^{1} |\mathrm{x}_n\rangle \otimes |\mathrm{x}_{n-1}\rangle \otimes \dots \otimes |\mathrm{x}_1\rangle$$

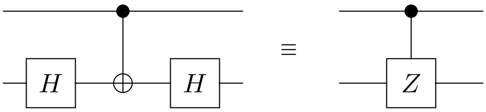
$$= \hat{H}|0\rangle \otimes \hat{H}|0\rangle \otimes \dots \otimes \hat{H}|0\rangle \equiv \hat{H}^{\otimes n}|0\rangle^{\otimes n} \ .$$

11.2 Constructing Z-gate from X-gate.

$$\hat{H}\hat{X}\hat{H} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \hat{Z}\,.$$

The inverse relationship follows from $\hat{H}^2 = \hat{I}$.

11.3 Constructing controlled Z-gate from cNOT-gate.

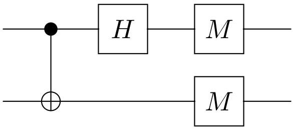


11.4 Q-circuit to produce Bell states. Let us calculate the successive actions of the H-gate and the cNOT-gate:

$$\begin{array}{llll}
|00\rangle & \longrightarrow |00\rangle + |10\rangle & \longrightarrow |00\rangle + |11\rangle & \longrightarrow \left|\Phi^{(+)}\right\rangle \\
|01\rangle & \longrightarrow |01\rangle + |11\rangle & \longrightarrow |01\rangle + |10\rangle & \longrightarrow \left|\Psi^{(+)}\right\rangle \\
|10\rangle & \longrightarrow |00\rangle - |10\rangle & \longrightarrow |00\rangle - |11\rangle & \longrightarrow \left|\Phi^{(-)}\right\rangle \\
|11\rangle & \longrightarrow |01\rangle - |11\rangle & \longrightarrow |01\rangle - |10\rangle & \longrightarrow \left|\Psi^{(-)}\right\rangle
\end{array}$$

The trivial factors $1/\sqrt{2}$ in front of the intermediate states have not been denoted.

11.5 Q-circuit to measure Bell states. The task is the inverse task of preparing the Bell states. Since both the H-gate and the cNOT-gate are the inverses of themselves, respectively, we can simply use them in the reversed order w.r.t. the circuit that prepared the Bell states (cf. Prob. 11.4):



The boxes $\boxed{\text{M}}$ stand for projective measurement of the computational basis.

References

1. M. A. Nielsen, I. L. Chuang: *Quantum Computation and Quantum Information*, (Cambridge University Press, Cambridge 2000)
2. J. Preskill: *Quantum Computation and Information*, (Caltech 1998); http://theory.caltech.edu/people/preskill/ph229/
3. D. Bouwmeester, A. Ekert, A. Zeilinger (editors): *The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computation*, (Springer, Berlin Heidelberg New York 2000)
4. L. D. Landau, E. M. Lifsic: *Course of Theoretical Physics: Mechanics*, (Pergamon, Oxford 1960)
5. M. C. Gutzwiller: *Chaos in Classical and Quantum Mechanics*, (Springer, Berlin Heidelberg New York 1991)
6. J. von Neumann: *Mathematical Foundations of Quantum Mechanics*, (Princeton University Press, Princeton 1955)
7. A. Peres: *Quantum Theory: Concepts and Methods*, (Kluwer, Dordrecht 1993)
8. K. Kraus: *States, Effects, and Operations: Fundamental Notions of Quantum Theory*, (Springer, Berlin Heidelberg New York, 1983)
9. P. Busch, P. J. Lahti, P. Mittelstadt: *The Quantum Theory of Measurement*, (Springer, Berlin Heidelberg New York 1991)
10. E. Joos, H. D. Zeh, C. Kiefer, D. Giulini, K. Kupsch, I. O. Stamatescu: *Decoherence and the Appearance of a Classical World in Quantum Theory*, 2nd edit. (Springer, Berlin Heidelberg New York 2003)
11. N. Gisin, G. Ribordy, W. Tittel, H. Zbinden: Rev. Mod. Phys. **74**, 145 (2002)
12. A. Ekert, P. Hayden, H. Inamori: *Basic Concepts in Quantum Computation*, (Les Houches lectures 2000); Los Alamos e-print arXiv: quant-ph/0011013
13. L. Diósi: Weak measurements in quantum mechanics. In: *Encyclopedia of Mathematical Physics*, vol 4, ed by J.P. Françoise, G.L. Naber, S.T. Tso (Elsevier, Oxford 2006) pp 276-282
14. R. F. Werner: Phys. Rev. A **40**, 4277 (1989)
15. W. K. Wootters, W. K. Zurek: Nature **299**, 802 (1982)
16. I. D. Ivanovic: Phys. Lett. A **123**, 257 (1987)
17. S. Wiesner: SIGACT News **15**, 77 (1983)
18. C. H. Bennett: Phys. Rev. Lett. **68**, 3121 (1992)
19. C. H. Bennett, G. Brassard: Quantum cryptography: Public key distribution and coin tossing, In: *Proceedings of IEEE International Conference on Computers, Systems and Signal Processing*, (IEEE Press, New York 1984)
20. A. Einstein, B. Podolsky, N. Rosen: Phys. Rev. **48**, 696 (1935)
21. W. F. Stinespring: Proc. Am. Math. Soc. **6**, 211 (1955)
22. A. Peres: Phy. Rev. Lett. **77**, 1413 (1996)

23. M. Horodecki, P. Horodecki, R. Horodecki: Phys. Lett. A **223**, 1 (1996)
24. J. S. Bell: Physics **1**, 195 (1964)
25. J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt: Phys. Rev. Lett. **49**, 1804 (1969)
26. S. Popescu: Phys. Rev. Lett. **74**, 2619 (1995)
27. C. H. Bennett, S. J. Wiesner: Phys. Rev. Lett. **69**, 2881 (1992)
28. C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. K. Wootters: Phys. Rev. Lett. **70**, 1895 (1993)
29. G. Lindblad: Commun. Math. Phys. **48**, 199 (1976)
30. V. Gorini, A. Kossakowski, E. C. G. Sudarshan: J. Math. Phys. **17**, 821 (1976)
31. C. E. Shannon: Bell System Tech.J. **27**, 379, 623 (1948)
32. B. Schumacher: Phys. Rev A **51**, 2738 (1995)
33. A. S. Holevo: Problems of Inf. Transm. **5**, 247 (1979)
34. R. P. Feynman: Int. J. Theor. Phys. **21**, 467 (1982)
35. D. Deutsch: Proc. Roy. Soc. **400**, 97 (1985)
36. D. Deutsch, R. Jozsa: Proc. Roy. Soc. London A **439**, 533 (1992)
37. L. K. Grover: A fast quantum mechanical algorithm for database search. In: *Proc. of the 28th annual STOC* (Assoc. for Comp. Machinery, New York 1996)
38. R. R. Tucci: *QC Paulinesia*, http://www.ar-tiste.com/PaulinesiaVer1.pdf 2004
39. P. W. Shor: Algorithms for quantum computation: discrete logarithm and factoring. In: *35th Annual Symposium on Foundations of Computer Science*, (IEEE Press, Los Alamitos 1994)

Index

Lecture Notes in Physics

For information about earlier volumes
please contact your bookseller or Springer
LNP Online archive: springerlink.com

Vol.667: W. D. Heiss (Ed.), Quantum Dots: a Doorway to Nanoscale Physics

Vol.668: H. Ocampo, S. Paycha, A. Vargas (Eds.), Geometric and Topological Methods for Quantum Field Theory

Vol.669: G. Amelino-Camelia, J. Kowalski-Glikman (Eds.), Planck Scale Effects in Astrophysics and Cosmology

Vol.670: A. Dinklage, G. Marx, T. Klinger, L. Schweikhard (Eds.), Plasma Physics

Vol.671: J.-R. Chazottes, B. Fernandez (Eds.), Dynamics of Coupled Map Lattices and of Related Spatially Extended Systems

Vol.672: R. Kh. Zeytounian, Topics in Hyposonic Flow Theory

Vol.673: C. Bona, C. Palenzula-Luque, Elements of Numerical Relativity

Vol.674: A. G. Hunt, Percolation Theory for Flow in Porous Media

Vol.675: M. Kröger, Models for Polymeric and Anisotropic Liquids

Vol.676: I. Galanakis, P. H. Dederichs (Eds.), Half-metallic Alloys

Vol.677: A. Loiseau, P. Launois, P. Petit, S. Roche, J.-P. Salvetat (Eds.), Understanding Carbon Nanotubes

Vol.678: M. Donath, W. Nolting (Eds.), Local-Moment Ferromagnets

Vol.679: A. Das, B. K. Chakrabarti (Eds.), Quantum Annealing and Related Optimization Methods

Vol.680: G. Cuniberti, G. Fagas, K. Richter (Eds.), Introducing Molecular Electronics

Vol.681: A. Llor, Statistical Hydrodynamic Models for Developed Mixing Instability Flows

Vol.682: J. Souchay (Ed.), Dynamics of Extended Celestial Bodies and Rings

Vol.683: R. Dvorak, F. Freistetter, J. Kurths (Eds.), Chaos and Stability in Planetary Systems

Vol.684: J. Dolinšek, M. Vilfan, S. Žumer (Eds.), Novel NMR and EPR Techniques

Vol.685: C. Klein, O. Richter, Ernst Equation and Riemann Surfaces

Vol.686: A. D. Yaghjian, Relativistic Dynamics of a Charged Sphere

Vol.687: J. W. LaBelle, R. A. Treumann (Eds.), Geospace Electromagnetic Waves and Radiation

Vol.688: M. C. Miguel, J. M. Rubi (Eds.), Jamming, Yielding, and Irreversible Deformation in Condensed Matter

Vol.689: W. Pötz, J. Fabian, U. Hohenester (Eds.), Quantum Coherence

Vol.690: J. Asch, A. Joye (Eds.), Mathematical Physics of Quantum Mechanics

Vol.691: S. S. Abdullaev, Construction of Mappings for Hamiltonian Systems and Their Applications

Vol.692: J. Frauendiener, D. J. W. Giulini, V. Perlick (Eds.), Analytical and Numerical Approaches to Mathematical Relativity

Vol.693: D. Alloin, R. Johnson, P. Lira (Eds.), Physics of Active Galactic Nuclei at all Scales

Vol.694: H. Schwoerer, J. Magill, B. Beleites (Eds.), Lasers and Nuclei

Vol.695: J. Dereziński, H. Siedentop (Eds.), Large Coulomb Systems

Vol.696: K.-S. Choi, J. E. Kim, Quarks and Leptons From Orbifolded Superstring

Vol.697: E. Beaurepaire, H. Bulou, F. Scheurer, J.-P. Kappler (Eds.), Magnetism: A Synchrotron Radiation Approach

Vol.698: S. Bellucci (Ed.), Supersymmetric Mechanics – Vol. 1

Vol.699: J.-P. Rozelot (Ed.), Solar and Heliospheric Origins of Space Weather Phenomena

Vol.700: J. Al-Khalili, E. Roeckl (Eds.), The Euroschool Lectures on Physics with Exotic Beams, Vol. II

Vol.701: S. Bellucci, S. Ferrara, A. Marrani, Supersymmetric Mechanics – Vol. 2

Vol.702: J. Ehlers, C. Lämmerzahl, Special Relativity

Vol.703: M. Ferrario, G. Ciccotti, K. Binder (Eds.), Computer Simulations in Condensed Matter Systems: From Materials to Chemical Biology Volume 1

Vol.704: M. Ferrario, G. Ciccotti, K. Binder (Eds.), Computer Simulations in Condensed Matter Systems: From Materials to Chemical Biology Volume 2

Vol.705: P. Bhattacharyya, B.K. Chakrabarti (Eds.), Modelling Critical and Catastrophic Phenomena in Geoscience

Vol.706: M.A.L. Marques, C.A. Ullrich, F. Nogueira, A. Rubio, K. Burke, E.K.U. Gross (Eds.), Time-Dependent Density Functional Theory

Vol.707: A.V. Shchepetilov, Calculus and Mechanics on Two-Point Homogenous Riemannian Spaces

Vol.708: F. Iachello, Lie Algebras and Applications

Vol.709: H.-J. Borchers and R.N. Sen, Mathematical Implications of Einstein-Weyl Causality

Vol.710: K. Hutter, A.A.F. van de Ven, A. Ursescu, Electromagnetic Field Matter Interactions in Thermoelastic Solids and Viscous Fluids

Vol.711: H. Linke, Controlled Nanoscale Motion in Biological and Artifical Systems

Vol.712: W. Pötz, J. Fabian, U. Hohenester (Eds.), Modern Aspects of Spin Physics

Vol.713: L. Diósi, A Short Course in Quantum Information Theory