Exponential Distribution Simulation

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Overview

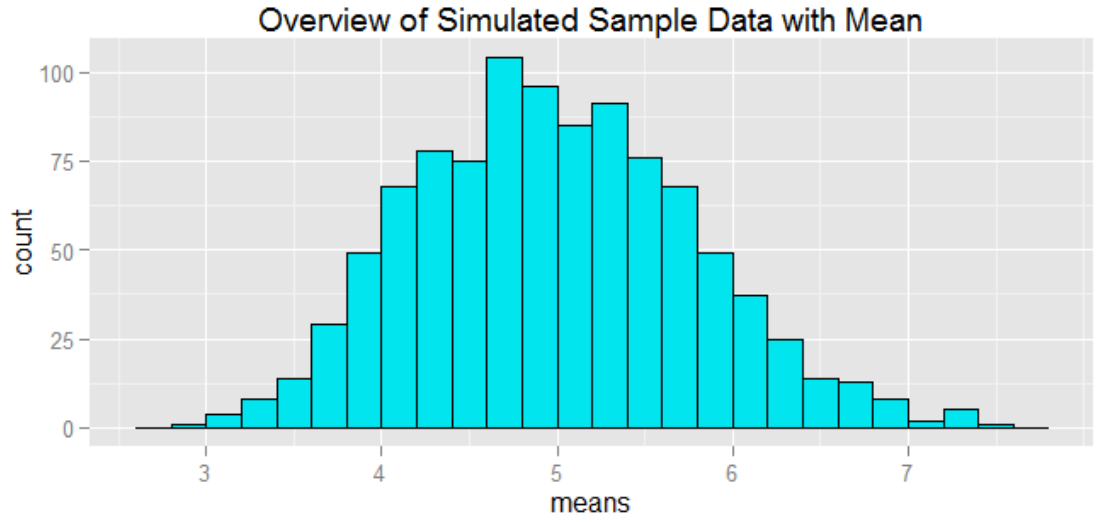
This report is part of Coursera **Statistical Inference** project to investigate the exponential distribution in R and compare it with the Central Limit Theorem through simulation .Scope of this investigation report is to

1. Show the sample mean and compare it to the theoretical mean of the distribution.
2. Show the sample variance and compare it to theoretical variance of the distribution.
3. Show that the distribution is approximately normal.

Simulations

The exponential distribution is simulated in R with `rexp(n, lambda)` where `lambda` λ is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. For this simulation, we set $\lambda = 0.2$. In this simulation, we investigate the distribution of averages of 40 numbers sampled from 1000 exponential distribution with $\lambda = 0.2$.

```
lambda <- 0.2 # lambda for rexp
n <- 40 # number of exponetials
num_sim <- 1000 # number of simulation
set.seed(3) # to reproduce same simulation

exp_dist <- matrix(data=rexp(n * num_sim, lambda), nrow=num_sim)
exp_dist_mean <- data.frame(means=apply(exp_dist, 1, mean))
```



Sample Mean versus Theoretical Mean

The expected mean μ of a exponential distribution of rate λ is $\mu = \frac{1}{\lambda}$

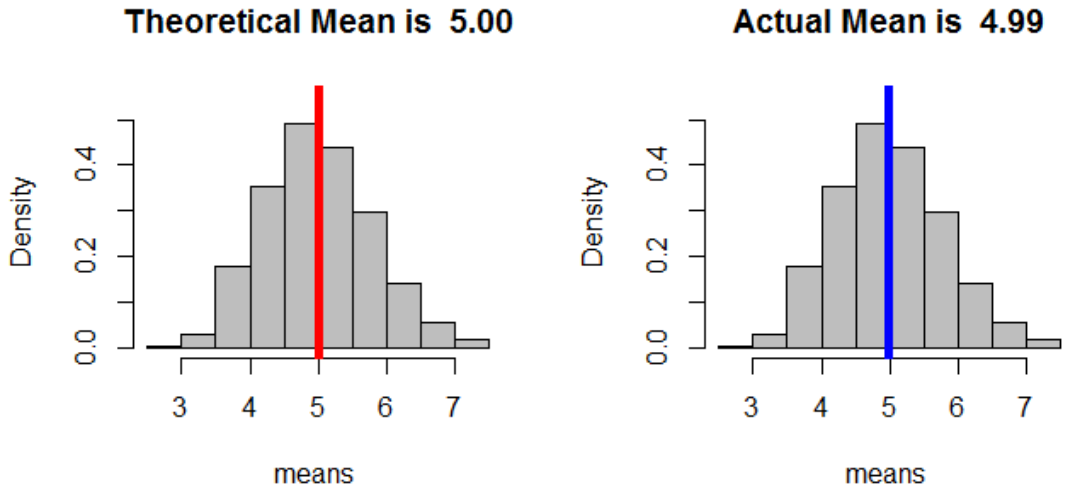
```
theory_mean <- 1/lambda
theory_mean
```

```
## [1] 5
```

The average sample mean of 1000 simulations of 40 randomly sampled exponential distributions.

```
sample_mean <- mean(exp_dist_mean$means) #Take mean Of Means
sample_mean
```

```
## [1] 4.98662
```



The above Histograms shows that value of sample mean **4.9866197** is very close to theoretical mean value **5**

Sample Variance versus Theoretical Variance

The theoritical variance Var of standard deviation σ is

$\text{Var} = \sigma^2$ where $\sigma = \frac{1/\lambda}{\sqrt{n}}$

```
std_dev <- 1/lambda/sqrt(n)
theory_Var <- std_dev^2
theory_Var
```

```
## [1] 0.625
```

The variance of the average sample mean of 1000 simulations of 40 randomly sampled exponential distribution is

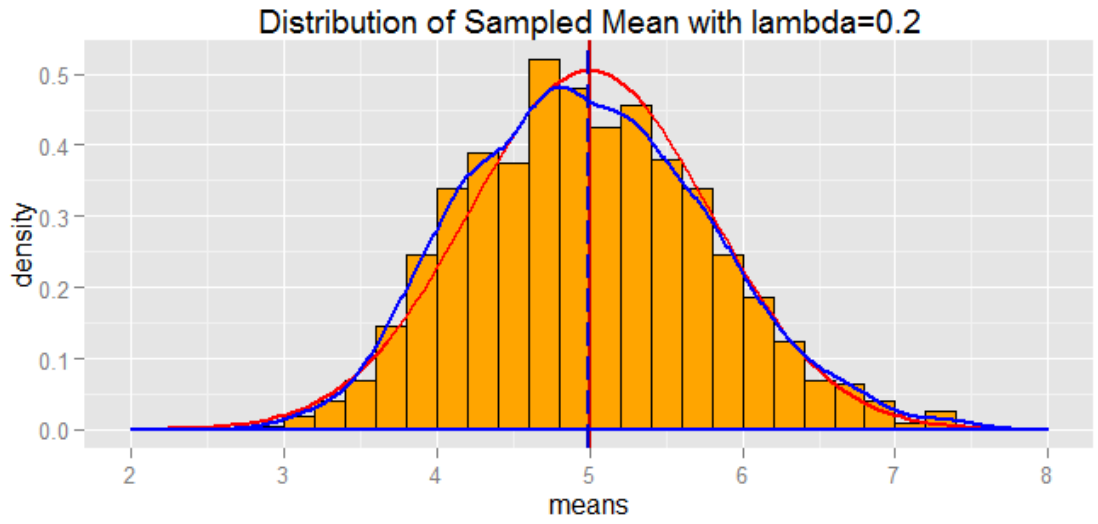
```
sample_Var <- var(exp_dist_mean$means)
sample_Var

## [1] 0.6257575
```

The value of sample variance **0.6257575** is almost equal to the theoretical variance **0.625** .Therefore, the variability in distribution of averages of 40 exponentials is close to the theoretical variance of the distribution.

Distribution

Finally Compare the sample distribution with a normal distribution of the theoritical values by plotting

- the distibution for sample values in Blue and theoritical values in Red .
- the sampled mean in dotted blue line and theoritical mean in red line.



It is clear from the graph, that sample distribution is approximately normal as the distribution of means of random sampled exponantial distributions overlaps closely with the normal distribution for $\lambda = 0.2$

Conclusion

For $\lambda = 0.2$,the sample distribution is approximately normal with Mean and Variance value match closely with theoritical mean $\mu = \frac{1}{\lambda}$ and variance $Var = \sigma^2$

Appendix

For complete report source code refer to GitHub