

## Homework 2

age	income	student	credit_rating	buys computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

$$\text{Info}(D) = I(9,5) = -\frac{9}{14} \log_2 \left( \frac{9}{14} \right) - \frac{5}{14} \log_2 \left( \frac{5}{14} \right) = 0.940$$

$$\begin{aligned} \text{Info}_{\text{age}}(D) &= \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2) \\ &= \frac{5}{14} \left[ -\frac{2}{5} \log_2 \left( \frac{2}{5} \right) - \frac{3}{5} \log_2 \left( \frac{3}{5} \right) \right] + \frac{4}{14} \left[ -\frac{4}{4} \log_2 \left( \frac{4}{4} \right) - \frac{0}{4} \log_2 \left( \frac{0}{4} \right) \right] + \\ &\quad \frac{5}{14} \left[ -\frac{3}{5} \log_2 \left( \frac{3}{5} \right) - \frac{2}{5} \log_2 \left( \frac{2}{5} \right) \right] = 0.694 \end{aligned}$$

$$\begin{aligned} \text{Info}_{\text{income}}(D) &= \frac{4}{14} I(2,2) + \frac{6}{14} I(4,2) + \frac{4}{14} I(3,1) \\ &= \frac{4}{14} \left[ -\frac{2}{4} \log_2 \left( \frac{2}{4} \right) - \frac{2}{4} \log_2 \left( \frac{2}{4} \right) \right] + \frac{6}{14} \left[ -\frac{4}{6} \log_2 \left( \frac{4}{6} \right) - \frac{2}{6} \log_2 \left( \frac{2}{6} \right) \right] + \\ &\quad \frac{4}{14} \left[ -\frac{3}{4} \log_2 \left( \frac{3}{4} \right) - \frac{1}{4} \log_2 \left( \frac{1}{4} \right) \right] = 0.911 \end{aligned}$$

$$\begin{aligned} \text{Info}_{\text{student}}(D) &= \frac{7}{14} I(6,1) + \frac{7}{14} I(3,4) \\ &= \frac{7}{14} \left[ -\frac{6}{7} \log_2 \left( \frac{6}{7} \right) - \frac{1}{7} \log_2 \left( \frac{1}{7} \right) \right] + \frac{7}{14} \left[ -\frac{3}{7} \log_2 \left( \frac{3}{7} \right) - \frac{4}{7} \log_2 \left( \frac{4}{7} \right) \right] = 0.788 \end{aligned}$$

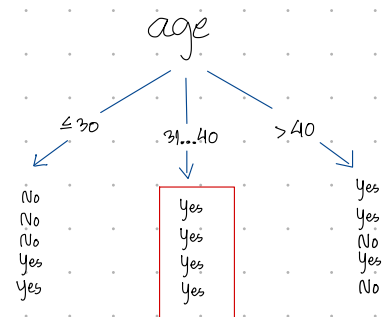
$$\begin{aligned} \text{Info}_{\text{credit\_rating}}(D) &= \frac{6}{14} I(3,3) + \frac{8}{14} I(6,2) \\ &= \frac{6}{14} \left[ -\frac{3}{6} \log_2 \left( \frac{3}{6} \right) - \frac{3}{6} \log_2 \left( \frac{3}{6} \right) \right] + \frac{8}{14} \left[ -\frac{6}{8} \log_2 \left( \frac{6}{8} \right) - \frac{2}{8} \log_2 \left( \frac{2}{8} \right) \right] = 0.892 \end{aligned}$$

$$\text{Gain}_{\text{age}} = 0.940 - 0.694 = 0.246 *$$

$$\text{Gain}_{\text{income}} = 0.940 - 0.911 = 0.029$$

$$\text{Gain}_{\text{student}} = 0.940 - 0.788 = 0.152$$

$$\text{Gain}_{\text{credit\_rating}} = 0.940 - 0.892 = 0.048$$



$$\text{Age} \leq 30$$

$$\text{Info}_{\text{age} \leq 30}(D) = I(2,3) = -\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right) = 0.971$$

$$\begin{aligned} \text{Info}_{\text{income}}(D) &= \overset{\text{high}}{\frac{2}{5} I(0,2)} + \overset{\text{medium}}{\frac{2}{5} I(1,1)} + \overset{\text{low}}{\frac{1}{5} I(1,0)} \\ &= \frac{2}{5} \left[ -\frac{0}{2} \log_2\left(\frac{0}{2}\right) - \frac{2}{2} \log_2\left(\frac{2}{2}\right) \right] + \frac{2}{5} \left[ -\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] + \\ &\quad \frac{1}{5} \left[ -\frac{1}{1} \log_2\left(\frac{1}{1}\right) - \frac{0}{1} \log_2\left(\frac{0}{1}\right) \right] = 0.4 \end{aligned}$$

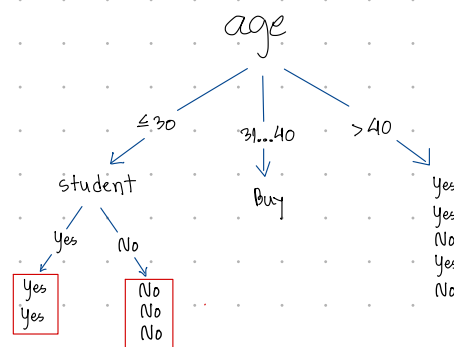
$$\begin{aligned} \text{Info}_{\text{student}}(D) &= \overset{\text{yes}}{\frac{2}{5} I(2,0)} + \overset{\text{no}}{\frac{3}{5} I(0,3)} \\ &= \frac{2}{5} \left[ -\frac{2}{2} \log_2\left(\frac{2}{2}\right) - \frac{0}{2} \log_2\left(\frac{0}{2}\right) \right] + \frac{3}{5} \left[ -\frac{0}{3} \log_2\left(\frac{0}{3}\right) - \frac{3}{3} \log_2\left(\frac{3}{3}\right) \right] = 0 \end{aligned}$$

$$\begin{aligned} \text{Info}_{\text{credit-rating}}(D) &= \overset{\text{excellent}}{\frac{2}{5} I(1,1)} + \overset{\text{fair}}{\frac{3}{5} I(1,2)} \\ &= \frac{2}{5} \left[ -\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] + \frac{3}{5} \left[ -\frac{1}{3} \log_2\left(\frac{1}{3}\right) - \frac{2}{3} \log_2\left(\frac{2}{3}\right) \right] = 0.951 \end{aligned}$$

$$\text{Gain}_{\text{income}} = 0.971 - 0.4 = 0.571$$

$$\text{Gain}_{\text{student}} = 0.971 - 0 = 0.971 *$$

$$\text{Gain}_{\text{credit-rating}} = 0.971 - 0.951 = 0.020$$



$$\text{Age} > 40$$

$$\text{Info}_{\text{age} > 40}(D) = I(3,2) = -\frac{3}{5} \log_2 \left( \frac{3}{5} \right) - \frac{2}{5} \log_2 \left( \frac{2}{5} \right) = 0.971$$

$$\begin{aligned} \text{Info}_{\text{income}}(D) &= \overset{\text{medium}}{\frac{3}{5} I(2,1)} + \overset{\text{low}}{\frac{2}{5} I(1,1)} \\ &= \frac{3}{5} \left[ -\frac{2}{3} \log_2 \left( \frac{2}{3} \right) - \frac{1}{3} \log_2 \left( \frac{1}{3} \right) \right] + \frac{2}{5} \left[ -\frac{1}{2} \log_2 \left( \frac{1}{2} \right) - \frac{1}{2} \log_2 \left( \frac{1}{2} \right) \right] = 0.951 \end{aligned}$$

$$\begin{aligned} \text{Info}_{\text{student}}(D) &= \overset{\text{yes}}{\frac{3}{5} I(2,1)} + \overset{\text{no}}{\frac{2}{5} I(1,1)} \\ &= \frac{3}{5} \left[ -\frac{2}{3} \log_2 \left( \frac{2}{3} \right) - \frac{1}{3} \log_2 \left( \frac{1}{3} \right) \right] + \frac{2}{5} \left[ -\frac{1}{2} \log_2 \left( \frac{1}{2} \right) - \frac{1}{2} \log_2 \left( \frac{1}{2} \right) \right] = 0.951 \end{aligned}$$

$$\begin{aligned} \text{Info}_{\text{credit-rating}}(D) &= \overset{\text{excellent}}{\frac{2}{5} I(0,2)} + \overset{\text{fair}}{\frac{3}{5} I(3,0)} \\ &= \frac{2}{5} \left[ -\frac{0}{2} \log_2 \left( \frac{0}{2} \right) - \frac{2}{2} \log_2 \left( \frac{2}{2} \right) \right] + \frac{3}{5} \left[ -\frac{3}{3} \log_2 \left( \frac{3}{3} \right) - \frac{0}{3} \log_2 \left( \frac{0}{3} \right) \right] = 0 \end{aligned}$$

$$\text{Gain}_{\text{income}} = 0.971 - 0.951 = 0.020$$

$$\text{Gain}_{\text{student}} = 0.971 - 0.951 = 0.020$$

$$\text{Gain}_{\text{credit-rating}} = 0.971 - 0 = 0.971^*$$

