Country Portfolios and Optimal Monetary Policy

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Abstract

In a world with rising financial globalization, exchange rate fluctuations play an increasingly important role in determining asset returns, thereby strengthening the exchange rate channel of monetary policy. Motivated by this premise, I study the optimal monetary policy regime in a small open economy in relation to its portfolio structure. I show that the optimal policy deviates from price stability by inducing optimal co-movement of domestic and foreign inflation, thus providing insurance against foreign shocks. Moreover, the optimal policy strategy and the exchange rate regime depend on the structure of country portfolios and openness and thus imply positive or negative co-movements between domestic and foreign variables. Nevertheless, in either case, the optimal policy framework promotes higher welfare than inflation targeting or an exchange rate peg.

Keywords: monetary policy, portfolio choice, exchange rates, inflation targeting.

JEL Classification Codes: E52, F31, F41

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Introduction

Financial globalization has led to significant growth in external assets held by both advanced and emerging economies, resulting in unprecedented levels of external wealth. Notably, between the years 1995 and 2015, most countries witnessed a two- to three-fold surge in their external wealth (Lane and Milesi-Ferretti, 2018). Nonetheless, this growth has concurrently increased the countries' vulnerability to external shocks such as exchange rate fluctuations, foreign inflationary pressures, and monetary policy shocks.

The integration of financial markets creates incentives for central banks to leverage exchange rates as a means to influence asset returns and country portfolios. Motivated by this premise, I study the optimal monetary policy regime in relation to the external wealth of an economy. Specifically, I develop a small open economy with overlapping generations and foreign asset holdings to study their policy implications. In the model, monetary non-neutrality arises because of nominal financial contracts and the fact that monetary policy can affect the returns of both domestic and foreign assets, and hence overall portfolio returns. Consequently, I show that the central bank can mitigate the country's exposure to foreign shocks by the provision of insurance in the form of optimal co-movement of domestic and foreign asset returns, thereby completing the markets.

Then, I evaluate the welfare effects of various monetary policy regimes and find that, contrary to the conventional wisdom, the optimal policy does not imply price stability. Instead, I show that the optimal policy strategy depends on the country's portfolio structure and its openness. Specifically, I find that domestic inflation with positive or negative co-movement (depending on the country's characteristics) with foreign inflation can improve household welfare. For a wide range of parameter values, the optimal policy implies a negative correlation between the domestic and foreign inflation rates, with positive inflation volatility. This policy framework achieves higher welfare compared with inflation targeting or an exchange rate peg.

The welfare-improving effects of domestic inflation arise because of its effects on the real returns of domestic and foreign assets, while the resulting exchange rate fluctuations affect asset returns via valuation effects.[1] Therefore, the central bank can make use of these two mechanisms for its policy objectives by altering the correlation between domestic and foreign asset returns. With such a policy, domestic assets take a complementary role, enabling households to hedge against foreign shocks. Specifically, I find that under the optimal policy strategy, the central bank induces negative co-movement between domestic

[1] See also Tille (2008) and Benigno (2009a) for discussions on the role of exchange rates as determinants of portfolio returns and Céspedes et al. (2004) for a model of balance sheet effects of exchange rate fluctuations.

and foreign inflation, which in turn increases the portfolio's expected return.[2]

Gains from the optimal co-movement between domestic and foreign inflation also extend beyond the insurance channel and improved portfolio returns. I find that the provision of insurance increases the demand for domestic assets and thus reduces the cost of borrowing for the government, thereby lowering the government debt level in the long run. Hence, while lower interest rates result in lower portfolio returns, they also taper government debt and consequently alleviate the tax burden on households.[3] Therefore, in this environment with non-Ricardian households, the central bank faces a trade-off between the effects of its policies on the returns of household portfolios and the taxes levied on them. I find that the wealth effect dominates the portfolio return effect and then characterize the optimal policy framework that achieves a balance between these two.

Furthermore, I extend the model to feature tradable and non-tradable goods with price rigidities and find that the optimal policy regime also depends on country openness, measured by the share of tradable goods in the consumption basket. This result is due fact that the share of imported goods determines the optimal co-movement between domestic and foreign inflation. I find that the optimal policy induces a *positive* pass-through of foreign inflation to imported goods inflation in economies with a low share of tradable goods. Conversely, as country openness increases, *negative* pass-through of foreign inflation to imported goods inflation becomes optimal. Nevertheless, in either case, the optimal policy framework promotes higher welfare than inflation targeting or an exchange rate peg.

The model developed here has several attractive features relative to the other frameworks in the literature. First, the modeling strategy allows to obtain exact analytical solutions, while preserving the non-linearities of the model. This is crucial since portfolio decisions always involve a trade-off between risk and return; hence, proper modeling of risk becomes an important feature for modeling portfolio choices. Other methods in the literature mainly rely on approximation methods that often do not allow for exact analytical solutions (e.g., Devereux and Sutherland (2009, 2010); Tille and van Wincoop (2010)).[4]

Different from these approaches, I solve the model for a finite number of states of exogenous disturbances, which provides several advantages over other methods. It enables an exact analytical solution of the model while maintaining the model's non-linearities. Consequently, it preserves the riskiness of the assets and avoids portfolio indeterminacy, which

[2] This effect is similar to the insurance channel studied by Fanelli (2023), but unlike the latter works as a portfolio effect, without interacting with price rigidities and capital controls.

[3] Although this mechanism works through the interaction of the monetary and fiscal policies, I show in Section 4 that the optimal policy strategy is independent of the fiscal policy conduct.

[4] Fanelli (2023) is among the few exceptions, as he develops an approximation method that allows for an analytical study of monetary and capital control policies in small open economies with external asset holdings.

provides valuable insights into the portfolio choices. Hence, the model allows us to get a well-defined analytical representation of country portfolios and capital flows, which is important for the analysis of the effects of monetary policy.

The paper contributes to the literature on price stability and optimal monetary policies in open economies by Benigno and Benigno (2003); Benigno (2009b); Corsetti et al. (2010); Devereux and Engel (2003); Svensson (2000); Coulibaly (2023); Egorov and Mukhin (2023) among others. Specifically, I illustrate two unconventional channels through which monetary policy may have real effects. I characterize the optimal policy framework working through those channels by providing insurance against foreign shocks, better portfolio diversification and lower government debt. The insurance channel of monetary policy is similar to that of Obstfeld and Rogoff (2002); Corsetti et al. (2010), which has been also recently studied by Fanelli (2023).

I contribute to this strand of the literature by proposing alternative mechanisms through which a central bank can neutralize the effects of adverse shocks on households. Gains from inflation volatility have been shown to exist also by Siu (2004), which considers state-contingent inflation as an absorber of fiscal shocks. Similarly, I show that gains from inflation volatility can be realized as a source of insurance against foreign shocks. This result aligns with the view of Benigno (2009b) that the benefits of deviating from a policy of price stability can outweigh the welfare costs of incomplete markets.

The results obtained here also speak to the literature on optimal monetary and exchange rate policies (see e.g. Céspedes et al. (2004); Chang and Velasco (2006); Benigno (2009a); Schmitt-Grohé and Uribe (2001) and more recently Fornaro (2015); Drenik et al. (2021); Itskhoki and Mukhin (2023)). Similar to Schmitt-Grohé and Uribe (2001), I show that an exchange rate peg is not optimal and that household welfare can be improved (at least locally) if the central bank leans towards inflation targeting and a flexible exchange rate. Finally, this paper contributes to the growing literature on monetary policy in overlapping generations models initiated by Galí (2014, 2021).

The rest of this paper is structured as follows. In Sections 1 and 2 I present the model and characterize the dynamic properties of the model economy. Sections 3 and 4 discuss the welfare effects of monetary policy and the optimal policy framework, respectively. Section 5 extends the model into a production economy with sticky prices. Finally, Section 6 concludes.

1 The model

Consider a small open economy that is populated by households, a government, and a central bank. There is a single internationally traded good consumed by households.

The foreign economy is assumed to be exogenous.

1.1 Households

There are overlapping generations of two-period lived households. Each period, a new generation of young households is born, of measure one, who are endowed with Q_t units of the consumption good, and who must pay T_t units of the consumption good as taxes to the government. Young households also have access to domestic and foreign government bonds D_t and F_t with gross nominal interest rates R_t and R_t^* respectively.[5]

Households get utility from consumption when they become old. The preferences of the representative household born in period t are given by:

$$U_t = E_t \left[\log \left(C_{t+1}\right)\right]$$

The budget constraint of the household is given by:

$$D_t + F_t = Q_t - T_t \tag{1}$$

for the young households, and

$$C_{t+1} = \frac{R_t}{\Pi_{t+1}} D_t + \Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}} F_t \tag{2}$$

for the old households, where $\Pi_t \equiv P_t / P_{t-1}$ is gross inflation and $\Psi_t \equiv \mathcal{E}_t / \mathcal{E}_{t-1}$ is the rate of exchange rate depreciation.

Note that the domestic and foreign bonds are assumed to be nominal. That is, the nominal return of domestic (foreign) bonds is fixed in domestic (foreign) currency, and it is thus affected by realized inflation (inflation and exchange rates). This will be the source of monetary non-neutralities, as we will see later.

1.2 Government

The domestic government collects taxes T_t from households, as well as issues bonds D_t to them to roll over the debt from the previous period. The government does not have any

[5] In the baseline model, for the sake of simplicity, I assume that the endowment Q_t and the foreign interest rate R_t^* are constant. Then, in Section 5, I consider an extension with stochastic endowments and foreign interest rates.

spending needs and satisfies the following budget constraint:

$$\frac{R_{t-1}}{\Pi_t} D_{t-1} = D_t + T_t \tag{3}$$

I assume that the government sets taxes according to the following rule

$$T_t = \overline{T} + \tau \left(D_{t-1} - \overline{D} \right) \tag{4}$$

where $\overline{T}$ is the steady-state level of taxes, $\overline{D}$ is the steady-state level of government debt, and $\tau > 0$ denotes the sensitivity of taxes to government debt.

The specification of the tax rule above is similar to that of Bianchi et al. (2023) and aims at stabilizing the government debt in the long run. As we will see later, the steady-state level of government debt $\overline{D}$ and taxes $\overline{T}$ are important determinants of the optimal monetary policy. Nevertheless, the results of this paper do not depend on the policy parameter τ.[6]

1.3 Central bank

Goods prices are assumed to be flexible and there are no trade costs. Moreover, since there is a single internationally tradable good, inflation is determined by the law of one price identity

$$\Pi_t = \Psi_t \Pi_t^*$$

where Π_t^* is the foreign inflation.

Thus, given the foreign inflation (which I will assume is given exogenously), there is a one-to-one mapping between the rate of exchange rate depreciation and domestic inflation.[7] With this relationship in place, the central bank can choose different monetary policy regimes by choosing between targeting domestic inflation or the exchange rate.[8] Therefore, I assume that monetary policy is implemented through a targeting rule, such that

$$\Pi_t = \Pi_t^{*\phi} \qquad\qquad \Psi_t = \Pi_t^{*\phi-1} \tag{5}$$

[6] In Section 4 I study the optimal policy framework under alternative fiscal policy parameters and show that the results are robust to the choice of τ.

[7] In Section 5 I also consider an extension with non-tradable goods where the pass-through from foreign inflation and exchange rates to domestic inflation is less than one-to-one.

[8] Central bank's ability to control inflation can be micro-founded and derived as the cashless limit of an economy where the households get utility from holding domestic money balances. Then, the central bank can control domestic inflation and exchange rate change, for example, through a money growth rate rule that targets domestic inflation or exchange rate.

Parameter ϕ above defines the monetary policy strategy as well as the exchange rate regime. The policy rule above nests inflation targeting and exchange rate pegging strategies, as well as intermediate policy regimes. In particular, $\phi = 0$ corresponds to *inflation targeting* with fixed domestic prices and a volatile exchange rate. In this case, the central bank can fully stabilize domestic prices by setting $\Psi_t = 1/\Pi_t^*$ in every period.

On the contrary, $\phi = 1$ implies an *exchange rate peg* with a fixed exchange rate and volatile domestic inflation. It is easy to see that the central bank can set $\Psi_t = 1$, such that domestic inflation would be identical to that of the foreign economy. On the other hand, $0 < \phi < 1$ corresponds to *hybrid targeting* with a relatively stable exchange rate and inflation, and partial propagation of foreign inflation shocks into the domestic economy.[9]

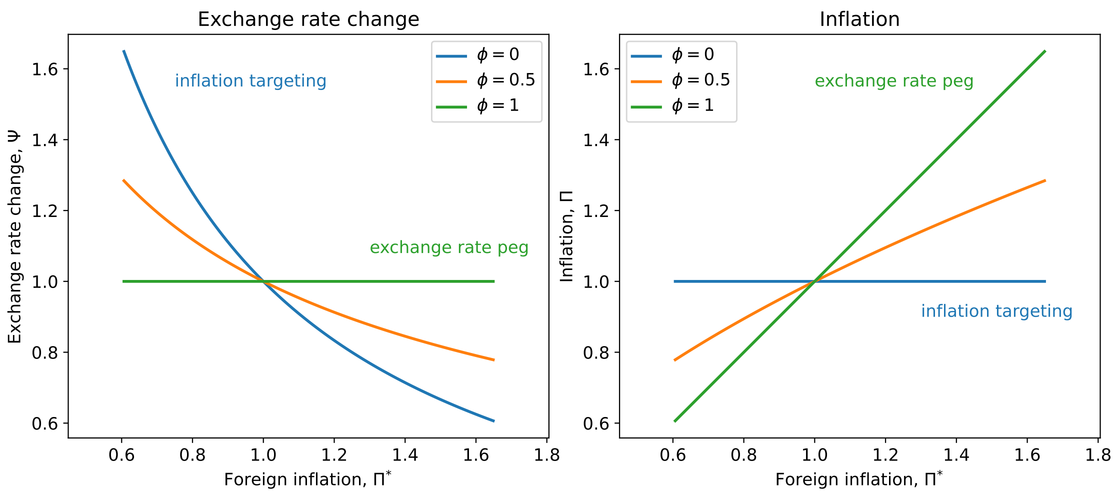

Figure 1: Effects of monetary policy strategy on exchange rate and inflation

Fig. 1 shows the relationship between domestic and foreign inflation rates as well as the exchange rate change for different values of ϕ. It can be seen that apart from defining the policy regime, the parameter ϕ also shows the degree of pass-through of foreign inflation to domestic inflation. For example, $\phi = 0$ implies that foreign inflation does not affect domestic inflation at all, whereas $\phi = 1$ implies that foreign inflation is replicated in the domestic economy. Generally, $\phi > 0$ implies a positive co-movement between foreign and domestic inflation, whereas $\phi < 0$ would imply a negative co-movement.

[9]See Ilzetzki et al. (2019) for a recent discussion on the history of exchange rate regimes (including hybrid arrangements) during the post-World War II period.

1.4 Foreign economy

The foreign economy is assumed to be exogenous and arbitrarily large relative to the domestic economy. I assume that foreign inflation shocks follow an i.i.d. process with high/low inflation states

$$\Pi_t^* = \begin{cases} \Pi^{*h} & \text{with probability} \quad p \\ \Pi^{*l} & \text{with probability} \quad 1-p \end{cases}$$

where $0 < p < 1$ and $\Pi^{*h} > \Pi^{*l} > 0$.[10]

The distribution of foreign inflation implies the following states for domestic inflation and exchange rate

$$\Pi_t,\ \Psi_t = \begin{cases} \Pi^h,\ \Psi^h & \text{if} \quad \Pi_t^* = \Pi^{*h} \\ \Pi^l,\ \Psi^l & \text{if} \quad \Pi_t^* = \Pi^{*l} \end{cases}$$

with $\Pi^l > 0$, $\Pi^h > 0$, $\Psi^l > 0$, $\Psi^h > 0$.

This mapping between the states of foreign inflation, domestic inflation and exchange rate is important for the analysis since it preserves the number of states of the model and allows us to obtain an exact solution of the model.

1.5 Equilibrium

Maximization of the utility function subject to the budget constraint yields the following Euler equation for the household portfolio choice:

$$E_t\left[\frac{1}{C_{t+1}}\frac{R_t}{\Pi_{t+1}}\right] = E_t\left[\frac{1}{C_{t+1}}\Psi_{t+1}\frac{R_t^*}{\Pi_{t+1}}\right] \tag{6}$$

which, together with the budget constraints in Eqs. (1) and (2), determines the household demand for domestic and foreign bonds for given interest rates R_t and R_t^*.

Combining Eq. (6) with the budget constraints in Eqs. (1) and (2) yields the following demand functions for domestic and foreign bonds

$$D_t = \frac{R_t^*\Psi_{t+1}^l\Psi_{t+1}^h - R_t\left(p\Psi_{t+1}^l + (1-p)\,\Psi_{t+1}^h\right)}{\left(R_t - R_t^*\Psi_{t+1}^l\right)\left(R_t - R_t^*\Psi_{t+1}^h\right)} R_t^* Z_t \tag{7}$$

[10]The assumption of only two states is made for analytical tractability. In Section 5, I consider an extension with multiple states of foreign inflation and show that the results are robust to the number of states.

for domestic bonds, and

$$F_t = \frac{R_t - R_t^* \left(p\Psi_{t+1}^h + (1-p)\,\Psi_{t+1}^l\right)}{\left(R_t - R_t^*\Psi_{t+1}^l\right)\left(R_t - R_t^*\Psi_{t+1}^h\right)} R_t Z_t \tag{8}$$

for foreign bonds, with

$$Z_t \equiv D_t + F_t = Q_t - T_t$$

where I keep the subscript on Ψ_{t+1} to emphasize the fact that it is realized in the period $t+1$.

Definition 1 (Equilibrium)
For sequences of Q_t, R_t^* and Π_t^*, competitive equilibrium is defined as the sequence of D_t, F_t, T_t, C_t, R_t, Π_t and Ψ_t satisfying Eqs. (1), (2), (3), (4), (5) and (6).

We can see from Eq. (3) and Eq. (4), that D_t is predetermined given the state variables D_{t-1}, R_{t-1} and Π_t^*. Therefore, with this inelastic supply of government debt, the equilibrium interest rate is determined from Eq. (7), such that it clears the government debt market. Foreign bonds are then determined from Eq. (8), as the residual amount of wealth that is left from meeting the inelastic supply of domestic bonds.

Lemma 1 (Existence and uniqueness of equilibrium)
Given a domestic debt level D_t, such that $0 < D_t < Z_t$, there exists a unique equilibrium interest rate R_t that satisfies the portfolio optimality conditions in Eqs. (7) and (8).

The equilibrium interest rate R_t is given by

$$R_t = \frac{1}{2}R_t^* \left[\left(\Psi_{t+1}^l + \Psi_{t+1}^h\right) - \frac{Z_t}{D_t}\left(p\Psi_{t+1}^l + (1-p)\,\Psi_{t+1}^h\right) + \Delta_t^{1/2}\right] \tag{9}$$

where

$$\Delta_t = \left(\Psi_{t+1}^l - \Psi_{t+1}^h\right)^2 - 2\frac{Z_t}{D_t}\left(\Psi_{t+1}^l - \Psi_{t+1}^h\right)\left(p\Psi_{t+1}^l - (1-p)\,\Psi_{t+1}^h\right) + \frac{Z_t^2}{D_t^2}\left(p\Psi_{t+1}^l + (1-p)\,\Psi_{t+1}^h\right)^2$$

Lemma 1 establishes the existence of a unique equilibrium interest rate R_t, which is the only variable that is not predetermined by the rest of the equilibrium conditions. Therefore, the lemma completes the uniqueness of the equilibrium.

To further characterize the equilibrium analytically, for the rest of this paper I assume that low and high foreign inflation states are equally likely, i.e. $p = 1/2$. This assumption is not essential for the results of this paper, but it simplifies the analysis and the exposition.

Moreover, the low and high states of foreign inflation are set to

$$\log \Pi^{*h} = \pi^*$$
$$\log \Pi^{*l} = -\pi^*$$

where $\pi^* \geq 0$ denotes the volatility of foreign inflation.[11]

On the other hand, using the definition of the exchange rate, we obtain the following implied states for the exchange rate change

$$\log \Psi^{l} = (\phi - 1)(-\pi^*) = \psi$$
$$\log \Psi^{h} = (\phi - 1)\pi^* = -\psi$$

where, given that $\phi \leq 1$ for monetary policy strategies considered in this paper, $\psi \geq 0$ denotes the volatility of exchange rate changes.

1.6 Determinants of the equilibrium interest rate

Under the assumed distribution of foreign inflation and exchange rate changes, the relative interest rate from Eq. (9) can be rewritten as

$$\frac{R_t}{R_t^*} = \frac{1}{2}\left[\left(e^{\psi} + e^{-\psi}\right)\left(1 - \frac{1}{2\omega_t}\right) + \Delta_t^{1/2}\right] \tag{10}$$

where

$$\Delta_t = \left(e^{\psi} - e^{-\psi}\right)^2\left(1 - \frac{1}{2\omega_t}\right)^2 + \frac{1}{\omega_t^2} \tag{11}$$

and $\omega_t \equiv D_t/Z_t$ is the share of domestic bonds in the portfolio of households.

In the remainder of this section, I study the determinants of the relative interest rate R_t/R_t^*.

Lemma 2 (First derivatives of R_t/R_t^*)
Given equilibrium relative interest rate R_t/R_t^* as a function of the share of domestic bonds ω_t and exchange rate volatility ψ, as in Eq. (10), the following statements hold:

- *R_t/R_t^* is weakly increasing in ω_t for all $0 < \omega_t < 1$ and $\psi \geq 0$.*

[11] Noting that $E_{t-1}[\log \Pi_t^*] = 0$, we can see that π^* equals to the standard deviation of foreign inflation

$$Var_{t-1}(\log \Pi_t^*) = \frac{1}{2}\log^2 \Pi^{*h} + \frac{1}{2}\log^2 \Pi^{*l} = \pi^{*2}$$

That is

$$\frac{\partial R_t/R_t^*}{\partial \omega_t} \geq 0 \quad \textit{for} \quad 0 < \omega_t < 1, \quad \psi \geq 0$$

with equality if and only if $\psi = 0$*.*

- R_t/R_t^* *is weakly decreasing in* ψ *for all* $\psi \geq 0$ *and* $0 < \omega_t \leq 1/2$*, and increasing for* $1/2 \leq \omega_t < 1$*.*

  That is, for all $\psi \geq 0$

$$\frac{\partial R_t/R_t^*}{\partial \psi} \leq 0 \quad \textit{if} \quad 0 < \omega_t \leq 1/2 \qquad \textit{and} \qquad \frac{\partial R_t/R_t^*}{\partial \psi} \geq 0 \quad \textit{if} \quad 1/2 \leq \omega_t < 1$$

  with equality if and only if $\omega_t = 1/2$*.*

Lemma 2 establishes the relationship between the share of domestic bonds ω_t, the volatility of the exchange rate change ψ the relative interest rate R_t/R_t^*. The first part of the lemma states that the relative interest rate is weakly increasing in the share of domestic bonds. This result is because household optimality conditions imply an increasing share of domestic bonds in the relative interest rate. Therefore, for a given level of foreign interest rate, a higher supply of domestic bonds will result in higher domestic interest rates. However, in the case of an exchange rate peg with $\psi = 0$, domestic and foreign become perfect substitutes, hence the relative interest rate is independent of the share of domestic bonds and is equal to 1.

On the other hand, the second part of the lemma states that the relationship between the relative interest rate and the volatility of the exchange rate change depends on the share of domestic bonds. In particular, the relative interest rate is weakly decreasing in the volatility of the exchange rate change for $0 < \omega_t \leq 1/2$ and increasing for $1/2 \leq \omega_t < 1$. To get a better understanding of the relationship between the relative interest rate and the volatility of the exchange rate change, Lemma 3 below provides the cross derivative of the relative interest rate with respect to the share of domestic bonds and the volatility of the exchange rate change.

Lemma 3 (Second derivatives of R_t/R_t^*)
Given equilibrium relative interest rate R_t/R_t^* *as a function of the share of domestic bonds* ω_t *and exchange rate volatility* ψ*, as in Eq. (10), the following statements hold:*

- *The sensitivity of* R_t/R_t^* *to* ω_t *is weakly increasing in* ψ *for all* $0 < \omega_t < 1$ *and* $0 \leq \psi \leq 1$*.*

$$\frac{\partial^2 R_t/R_t^*}{\partial \omega_t \partial \psi} \geq 0 \quad \textit{for} \quad 0 < \omega_t < 1, \quad 0 \leq \psi \leq 1$$

with equality if and only if $\psi = 0$.

Lemma 3 states that the sensitivity of the relative interest rate to the share of domestic bonds is weakly increasing in the volatility of the exchange rate change. This result is because because of the riskiness of foreign bonds, higher exchange rate volatility decreases the substitutability of domestic and foreign bonds as safe and risky assets. Consequently, households become more inclined to hold a well-diversified portfolio with equal shares of both assets. Hence, they demand higher domestic interest rates to increase the shares of domestic assets beyond the half. This result is illustrated in Fig. 2.

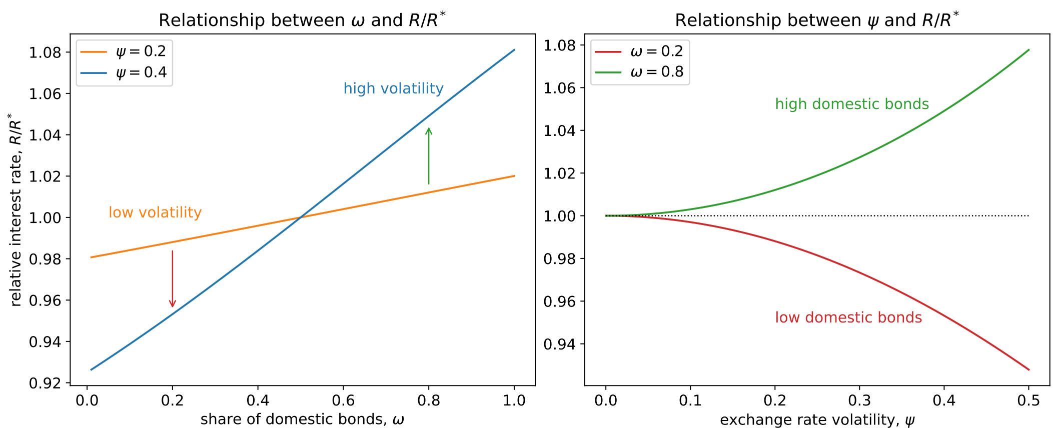

Figure 2: Relationship between ω, ψ and R/R^*

Fig. 2 plots the relationship between the share of domestic bonds ω_t, the volatility of the exchange rate change ψ and the relative interest rate R_t/R_t^*. It illustrates the results from Lemmas 2 and 3 for different levels of domestic bonds and exchange rate volatility.

2 Transition dynamics

The dynamics of the economy are governed by the government debt accumulation equation, which may take different forms depending on the monetary policy regime. In particular, the domestic real interest rate, which defines the rate of government debt accumulation depends on the monetary policy regime chosen by the central bank.

If the central bank implements an inflation-targeting strategy, then the domestic inflation rate is fixed at $\Pi_t = 1$, and foreign inflation shocks are completely absorbed by the exchange rate. As a result, the dynamics of the economy are not affected by foreign inflation shocks and the government debt evolves according to the following equation:

$$D_t = R_{t-1}D_{t-1} - T_t$$

On the other hand, for a given level of foreign inflation volatility π^*, the domestic real interest rate R_t is increasing in the share of domestic bonds ω_t, as a result of Lemma 2. Hence, for a given initial level of government debt D_0, the government debt converges to a steady state level $\mathcal{D}$, as shown in Fig. 3.

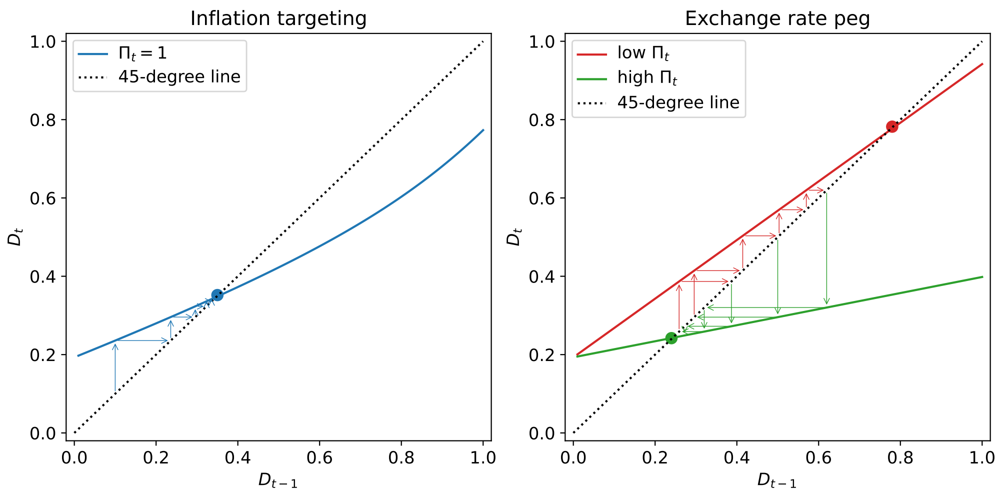

Figure 3: Transition dynamics under different monetary policy strategies

However, if the central bank implements an exchange rate targeting strategy with $\Psi_t = 1$, then foreign inflation shocks propagate into the economy and domestic inflation is equal to the foreign inflation $\Pi_t = \Pi_t^*$. Then, the real interest rate may take two different values depending on the state of the economy and the government debt will evolve according to the following equations:

- low inflation state $\Pi_t = \Pi^l$:

$$D_t = \frac{R_{t-1}}{\Pi^l}D_{t-1} - T_t$$

- high inflation state $\Pi_t = \Pi^h$:

$$D_t = \frac{R_{t-1}}{\Pi^h}D_{t-1} - T_t$$

As a result, the government debt accumulates faster in the low-inflation state and slower in the high-inflation state. On the other hand, under an exchange rate peg domestic interest rate is equal to the foreign interest rate $R_t = R_t^*$. Hence, unlike the case of inflation targeting, the government debt does not converge to a steady state level but instead oscillates between two levels $\mathcal{D}^l$ and $\mathcal{D}^h$, as shown in Fig. 3.

Fig. 4 plots the effects of monetary policy strategy on the lower and upper bounds of model variables. It is evident from the figure that the exchange rate targeting strategy with $\phi = 1$ implies a wide range of possible values for all variables, except for the domestic interest rate. Moreover, as the policy regime leans towards inflation targeting with $0 < \phi < 1$, the range of possible values for all variables shrinks into a single point at $\phi = 0$.

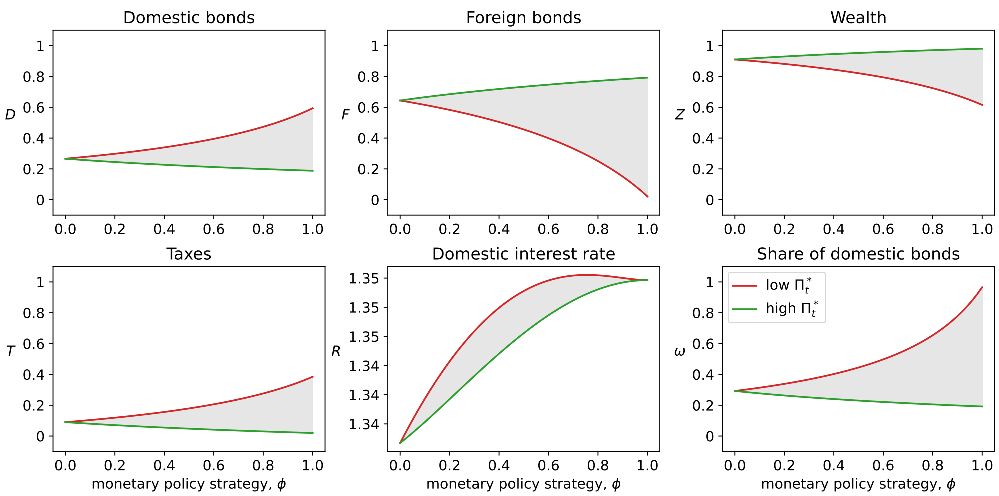

Figure 4: The effects of monetary policy strategy on the range of possible values of model variables

Inflation-targeting strategy with $\phi = 0$ leads to a single-steady state level for all domestic variables, except for consumption which fluctuates as a result of volatile returns on foreign assets. In the steady state, since domestic prices are fixed, there are no more disturbances that may drive government debt away from its steady-state level. Hence, the government debt, taxes, and the domestic interest rate are constant in the steady state.

3 Welfare

To analyze the welfare effects of the monetary policy regime, we need to define an aggregate welfare measure that will comprise the welfare of all generations. Given such an objective, the optimal monetary policy strategy will aim to maximize the aggregate household welfare measure $\mathcal{U}$, that is

$$\max_{\phi} \quad \mathcal{U}$$

subject to the equilibrium conditions of the model.

Recall that the welfare of the generation born at time t is

$$U_t = E_t \left[\log \left(C_{t+1} \right) \right]$$

which shows the expected utility from old-age consumption at time $t+1$.

An aggregate welfare measure comprising all generations may be defined as

$$\mathcal{U} \equiv E\left[U_t\right] = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T} U_t$$

The welfare measure above is defined as the unconditional expectation of the utility of the generations born at time t, or equivalently, the average utility of all generations. Note that, the existence of the aggregate welfare measure depends on the stationarity of the model, which is satisfied given the model assumptions.

Unlike in models with infinitely lived agents and time discounting, the welfare measure above assigns equal weights to all generations. The choice of the welfare measure is not unique and other measures can be defined. However, the choice of the welfare measure is motivated by the following reasons. Unlike in models with infinitely lived agents, the current model doesn't exhibit a deterministic steady state, or another natural state of the economy, in which the welfare could be evaluated. Therefore, in the case of a welfare measure with time discounting the optimal policy would depend on the initial conditions, namely the initial level of government debt. Nevertheless, the welfare measure above is consistent with the standard approach in the literature, where welfare is commonly evaluated assuming no time discounting.

The effects of the monetary policy regime on the aggregate welfare measure can be seen by studying the effects of the strategy on household consumption. For this purpose, substituting $\Psi_t = \Pi_t/\Pi_t^*$ from the low-of-one-price relation into the household budget constraint Eq. (2), we obtain

$$C_{t+1} = \frac{R_t}{\Pi_{t+1}} D_t + \frac{R_t^*}{\Pi_{t+1}^*} F_t$$

Hence, monetary policy may affect consumption through its effects on portfolio allocation between domestic and foreign bonds, as well as through its effects on the real returns of those assets. The latter expression, however, shows that monetary policy may only affect the real returns of domestic assets, whereas the real returns of foreign assets are determined by the foreign nominal interest rate R_t^* and the foreign inflation rate Π_{t+1}^*.

Therefore, monetary policy can affect consumption through the following channels: 1) real interest rate: through its effects on R_t and Π_{t+1}, 2) portfolio allocation: through its effects on D_t vs F_t, 3) interaction with fiscal policy: through its effects on $D_t \to T_t \to Z_t$.

To evaluate the effects of each transmission channel on household welfare across generations it is useful to apply a decomposition of the welfare measure into its components. For this purpose, household consumption can be expressed as

$$C_{t+1} = \frac{R_t}{\Pi_{t+1}} D_t + \Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}} F_t = X_{t+1} Z_t$$

where

$$X_{t+1} = \omega_t \frac{R_t}{\Pi_{t+1}} + (1 - \omega_t)\, \Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}} \tag{12}$$

is the real return on the household's portfolio.

Then, the welfare of generation t can be expressed as a sum of two components: 1) the log wealth of households, and 2) the expected log return on their portfolios. That is

$$U_t \equiv E_t \left[\log \left(C_{t+1}\right)\right] = \log Z_t + E_t \left[\log \left(X_{t+1}\right)\right]$$

Similarly, the aggregate welfare measure can be written as

$$\mathcal{U} \equiv E\left[U_t\right] = E\left[\log Z_t\right] + E\left[E_t \left[\log \left(X_{t+1}\right)\right]\right]$$

To study the effects of monetary policy on household welfare, I first analyze the effects of the policy parameter ϕ on the utility of each generation. Taking the total derivative of the utility of generation t with respect to ϕ, we obtain

$$\frac{dU_t}{d\phi} = \underbrace{\frac{d}{d\phi} \log Z_t}_{\text{Wealth effect}} + \underbrace{\frac{\partial}{\partial \omega_t} E_t \left[\log \left(X_{t+1}\right)\right] \frac{d\omega_t}{d\phi}}_{\text{Portfolio allocation effect}} + \underbrace{\frac{\partial}{\partial \phi} E_t \left[\log \left(X_{t+1}\right)\right]}_{\text{Portfolio return effect}} \tag{13}$$

The decomposition above breaks down the effects of monetary policy into three components: 1) the *indirect* effects through the wealth of the households, 2) *indirect* effects through the portfolio allocation, and 3) the *direct* effects through the real returns on portfolios. Note that the portfolio return X_{t+1} is a function of the monetary policy strategy parameter ϕ, as well as the share of domestic bonds ω_t, which itself also depends on ϕ. Therefore, the effects of monetary policy on the real returns of portfolios can be split into two different components.

3.1 Portfolio return effect

To understand the effects of monetary policy strategy on household utility, I study each of the three channels separately. For this purpose, first I study the effects of monetary policy through the real returns on portfolios. Proposition 1 below formulates the effects of portfolio structure and monetary policy regime on expected log returns.

Proposition 1 (Determinants of expected log returns)
From the perspective of a social planner, given the household optimality condition Eq. (6), the following statements hold:

- *The expected log return on the portfolio is weakly increasing in the share of domestic bonds*

$$\frac{\partial}{\partial \omega_t} E_t \left[\log \left(X_{t+1}\right)\right] \geq 0 \quad \textit{for all} \quad 0 < \omega_t < 1 \quad \textit{and} \quad \phi \leq 1$$

  with equality if and only if $\phi = 1$.

- *The expected log return on the portfolio is weakly decreasing in the monetary policy strategy parameter*

$$\frac{\partial}{\partial \phi} E_t \left[\log \left(X_{t+1}\right)\right] \leq 0 \quad \textit{for all} \quad 0 < \omega_t < 1 \quad \textit{and} \quad \phi \leq 1$$

  with equality if and only if $\phi = 1$.

Proposition 1 states that the expected log return is decreasing in the monetary policy strategy parameter ϕ. This result means that for a given wealth level and portfolio structure, the central bank may improve household welfare by implementing a policy corresponding to $\phi < 1$. It is easy to see that such a policy implies a domestic inflation rate that is different from the foreign inflation rate. Therefore, in terms of this channel, an exchange rate peg would be the least desirable strategy for the central bank being outperformed by inflation targeting as well as policies with $\phi < 0$.

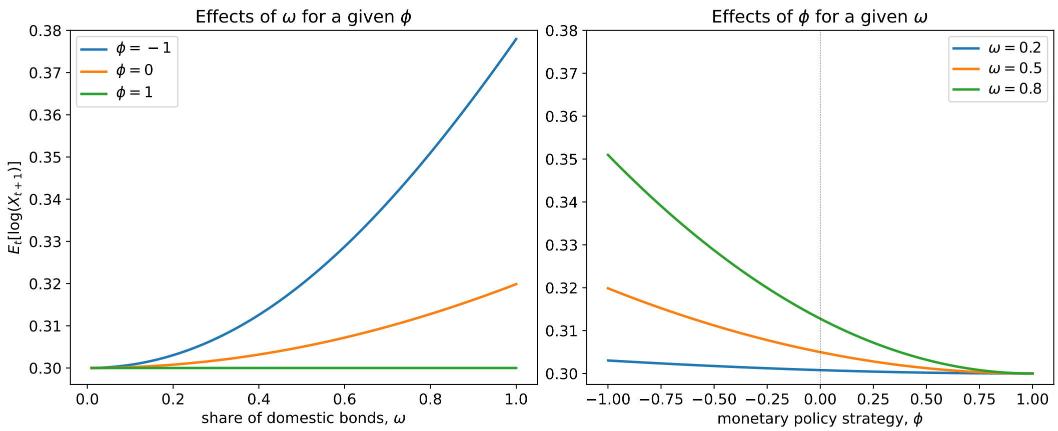

Figure 5: Partial effects of portfolio structure and monetary policy strategy on expected log returns

The results obtained by Proposition 1 are presented in Fig. 5, which shows the partial effects of portfolio structure and monetary policy regime on expected log returns and therefore on household utility. The left-side plot shows the effects of portfolio structure parameter ω_t for different monetary policy strategy parameters ϕ, while the right-side plot shows the effects of ϕ for different values of ω_t.

To understand the reason behind the benefits of lower ϕ note that different inflation rates reduce the correlation between the real returns between domestic and foreign assets. In particular, under inflation targeting with $\phi = 0$ and fixed domestic prices, the domestic government bond becomes a risk-free asset, providing households with some insurance against foreign inflation shocks. However, with policies corresponding to $\phi < 0$, domestic inflation moves in the opposite direction of foreign inflation, implying a negative correlation between the real returns of domestic and foreign assets. This further improves household welfare, as counter-cyclical domestic inflation offsets foreign inflation shocks.

3.2 Portfolio allocation effect

Apart from affecting the portfolio returns, monetary policy also can alter the portfolio allocation of households. This channel works through the ratio of government debt level D_t and the wealth level Z_t, which depend on the state of the economy, i.e. D_{t-1}, as well as the rate of government debt accumulation between periods t and $t-1$. To see how monetary policy affects the portfolio allocation, one can take the derivative of the share of domestic bonds ω_t with respect to the monetary policy strategy parameter ϕ, that is

$$\frac{d\omega_t}{d\phi} = \frac{dD_t/Z_t}{d\phi} = \frac{dD_t}{d\phi}\frac{1}{Z_t} - \frac{D_t}{Z_t^2}\frac{dZ_t}{d\phi}$$

The latter expression shows that the portfolio allocation effect depends on the ability of monetary policy to affect the domestic debt level D_t and household wealth Z_t, which in turn depend on the state of the economy, i.e. Π_t and D_{t-1}. Therefore, the portfolio allocation effect also depends on the state of the economy and implies a trade-off for the central bank between different generations.

To evaluate the aggregate portfolio allocation effect across generations, I take the expectation of it over all generations, which is the aggregate counterpart of the portfolio allocation in Eq. (13). The proposition below shows the sign of the aggregate portfolio allocation effect evaluated in inflation targeting and exchange rate pegging regimes.

Proposition 2 (Monetary policy and portfolio allocation)
The following statements hold for the mean portfolio allocation effect, i.e. average portfolio allocation effect over all generations.

- *Under exchange rate peg, i.e. at $\phi = 1$, mean portfolio allocation effect is non-positive. That is*

$$E\left[\frac{\partial}{\partial\omega_t}E_t\left[\log\left(X_{t+1}\right)\right]\frac{d\omega_t}{d\phi}\right]\bigg|_{\phi=1} \leq 0$$

  with equality if and only if $\pi^* = 0$.

- *Under inflation targeting, i.e. at $\phi = 0$*

  - *mean portfolio allocation effect is non-positive if $0 < \omega_t \leq 1/2$. That is*

$$E\left[\frac{\partial}{\partial\omega_t}E_t\left[\log\left(X_{t+1}\right)\right]\frac{d\omega_t}{d\phi}\right]\bigg|_{\phi=0} \leq 0 \qquad \textit{for} \qquad 0 < \omega_t \leq 1/2$$

  - *mean portfolio allocation effect is non-negative if $1/2 \leq \omega_t < 1$. That is*

$$E\left[\frac{\partial}{\partial\omega_t}E_t\left[\log\left(X_{t+1}\right)\right]\frac{d\omega_t}{d\phi}\right]\bigg|_{\phi=0} \geq 0 \qquad \textit{for} \qquad 1/2 \leq \omega_t < 1$$

  with equality if and only if $\omega_t = 1/2$.

Proposition 2 shows that the sign of portfolio allocation effect under inflation targeting depends on the share of domestic bonds in the portfolio of households. In particular, the portfolio allocation effect is non-positive if $0 < \omega_t \leq 1/2$, and non-negative if $1/2 \leq \omega_t < 1$. Nevertheless, the portfolio allocation effect is always non-positive under exchange rate pegging. This result implies that under inflation targeting, the central bank may improve household welfare by implementing a policy corresponding to $\phi < 0$ if $0 < \omega_t \leq 1/2$, while it may worsen household welfare if $1/2 \leq \omega_t < 1$.

3.3 Wealth effect

Finally, monetary policy also can alter the wealth of households through its effects on domestic debt levels and taxes. The interaction of monetary policy with fiscal policy affects the wealth transfer through household generations. Hence, the optimal policy regime should take into consideration its effects on the aggregate welfare through taxes and government debt. Note that taxes T_t depend on the state of the economy, i.e. D_{t-1}, hence effects of monetary policy on household wealth depend on its ability to affect the state of the economy D_{t-1}.

Therefore, to study the wealth effect of monetary policy, I take the derivative of the log wealth with respect to the monetary policy strategy parameter, that is

$$\frac{d}{d\phi}\log(Z_t) = \frac{1}{Z_t}\frac{dZ_t}{d\phi} = -\tau\frac{1}{Z_t}\frac{dD_{t-1}}{d\phi}$$

We have to note that this implies a trade-off for the central bank between different generations. Thus, to evaluate the aggregate wealth effect across generations, I take the expectation of it over all generations, which is the aggregate counterpart of the wealth effect in Eq. (13). The proposition below shows the sign of the aggregate wealth effect evaluated in inflation targeting and exchange rate pegging regimes.

Proposition 3 (Monetary policy and household wealth)
The following statements hold for the mean wealth effect, i.e. average wealth effect over all generations.

- *Under exchange rate peg, i.e. at $\phi = 1$, mean wealth effect is non-positive.*

  That is

$$E\left[\frac{d}{d\phi}\log(Z_t)\right]\bigg|_{\phi=1} \leq 0$$

  with equality if and only if $\pi^* = 0$.

- *Under inflation targeting, i.e. at* $\phi = 0$

  – *mean wealth effect is non-positive if* $0 < \omega_t \leq 1/2$.

    That is

$$E\left[\frac{d}{d\phi}\log\left(Z_t\right)\right]\Big|_{\phi=0} \leq 0 \qquad for \qquad 0 < \omega_t \leq 1/2$$

  – *mean wealth effect is non-negative if* $1/2 \leq \omega_t < 1$.

    That is

$$E\left[\frac{d}{d\phi}\log\left(Z_t\right)\right]\Big|_{\phi=0} \geq 0 \qquad for \qquad 1/2 \leq \omega_t < 1$$

  with equality if and only if $\omega_t = 1/2$.

Proposition 3 evaluates the aggregate wealth effect of monetary policy under inflation targeting and exchange rate peg. It shows that under an exchange rate peg, the central bank may improve household welfare by implementing a policy corresponding to $\phi < 1$. On the other hand, similar to the portfolio allocation effect, under inflation targeting, the central bank may improve household welfare by implementing a policy corresponding to $\phi < 0$ if $0 < \omega_t < 1/2$, while it may worsen household welfare if $1/2 < \omega_t < 1$.

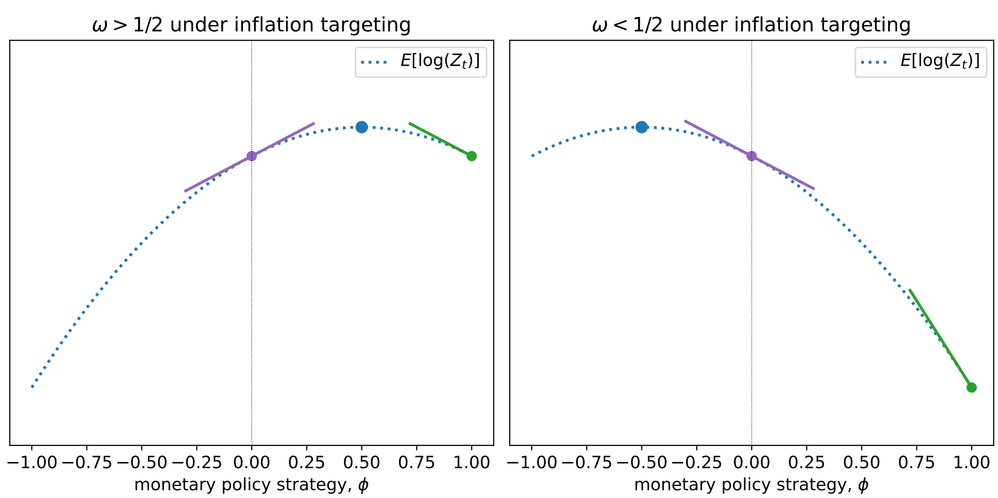

Figure 6: The effects of monetary policy strategy on expected log wealth

The results obtained in Proposition 3 are illustrated in Fig. 6, which shows the effects of monetary policy regime on expected log wealth evaluated under both strategies. Note that Proposition 3 implies that the optimal monetary policy strategy depends on the portfolio allocation of households under inflation targeting. In particular, if $0 < \omega_t < 1/2$, then the optimal monetary policy strategy corresponds to $\phi < 0$, while if $1/2 < \omega_t < 1$, then the optimal policy is given by $\phi > 0$.

4 Optimal policy

In the previous section, I studied the welfare effects of monetary policy through different channels. I showed that on the one hand, household welfare is decreasing in the monetary policy strategy parameter ϕ, due to the portfolio return effect. On the other hand, the portfolio allocation and wealth effects imply that the optimal monetary policy regime depends on the portfolio structure of households. In particular, if $0 < \omega_t < 1/2$, then the optimal monetary policy regime corresponds to $\phi < 0$, while if $1/2 < \omega_t < 1$, then the optimal policy is given by $\phi > 0$. Therefore, to study the net effects of monetary policy on household welfare, I study the optimal policy that maximizes the aggregate welfare measure $\mathcal{U}$.

4.1 Calibration

For the assessment of the welfare effects of the monetary policy regime, I simulate the model to evaluate the aggregate welfare measure $\mathcal{U}$. In the calibration, 1 period in the model is assumed to correspond to 10 years. Also, the foreign economy is calibrated to match the US economy.

I normalize the endowment size to $Q_t = 1$. Foreign inflation volatility π^* is set such that $\log \Pi^* = \pm\pi^*$ with $\pi^* = 0.2$ in order to match the standard deviation of US 10-year inflation, that is around 20% over the period 1960-2020.[12] Also, the foreign interest rate is set to $\log R^* = r^*$ with $r^* = 0.3$ in order to match the average US 10-year bond yield, approximately 6% (yearly) over the period 1960-2020.[13] I also calibrate $\overline{D}$ to target a steady-state share of domestic bonds $\omega = 0.3$ and set $\tau = 0.9$.

[12] Foreign inflation volatility π^* is calculated as the standard deviation of the log-difference of a price index

$$\pi^* = \widehat{\sigma}\left(\Delta \log\left(P_t^*\right)\right)$$

where P_t^* is the 10-year average of a price index in the US. Using the GDP deflator and the CPI as price indices, we obtain $\pi^* = 0.18$ and $\pi^* = 0.21$, respectively.

[13] Foreign interest rate r^* is calculated as

$$r^* = 10 \cdot \overline{r_t^*} - \overline{\Delta \log\left(P_t^*\right)}$$

where $\overline{r_t^*}$ is the average log yield on 10-year treasury bonds in the US and $\overline{\Delta \log\left(P_t^*\right)}$ is the average log-difference of the price index in the US. Using the GDP deflator and the CPI as price indices, we obtain $r^* = 0.3$ and $r^* = 0.26$, respectively.

Using the above calibration, the model is simulated for $1,000$ periods. The simulation results are available in the Online Appendix.

4.2 Optimal monetary policy

I begin by comparing the welfare effects of inflation targeting and exchange rate targeting policies as well as alternative policies that imply hybrid targeting regimes. Fig. 7 shows the aggregate welfare measure $\mathcal{U}$ evaluated under different monetary policy strategies ϕ, along with ranges of possible values bounded by the two levels that correspond to persistent low and high levels of inflation. It also plots the welfare components $\log Z_t$ and $E_t\left[\log\left(X_{t+1}\right)\right]$, that represent the natural log of the household wealth and the expected log real return on households portfolio, respectively.

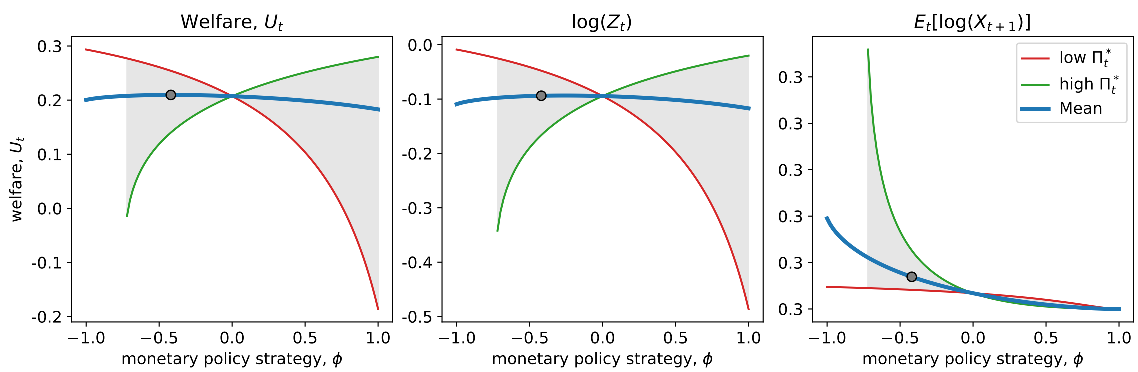

Figure 7: Household welfare and its components under the optimal policy

Exchange rate targeting policy with $\phi = 1$ provides the lowest welfare levels, while inflation with $\phi = 0$ provides the higher welfare. Also, the optimal policy implies $\phi < 0$, which corresponds to counter-cyclical domestic inflation relative to foreign inflation. Counter-cyclical inflation induced by the optimal policy results is a negative co-movement between the returns of the domestic and foreign bonds. Hence, the optimal policy improves the expected portfolio return, and therefore, household welfare.

This policy, as characterized by $\phi < 0$, provides the highest welfare levels, although, as shown in Fig. 7, implies a trade-off between the welfare components. On the one hand, monetary policy aims at maximizing the expected portfolio return, but on the other hand, affects the disposable endowment of households through its effects on government debt accumulation. One may view the first effect as a one-period effect of policy on household welfare working through the expected return channel discussed in Section 3.

The second effect works by altering the stochastic steady state of the economy and affecting household welfare through the wealth effect. In particular, higher expected portfolio returns improve household welfare but also affect the equilibrium interest rate R_t. Combined with the domestic inflation volatility of policies corresponding to $\phi < 0$, monetary policy also affects the long-run effects of government debt and hence taxes. Hence, the optimal policy tries to balance its effects on the wealth and expected return components.

Another important feature of the implied optimal policy is excessive exchange rate volatility. Recall that log-exchange-rate volatility is given by $\psi = (1-\phi)\pi^*$, hence exchange rate volatility increases as ϕ decreases. This result is consistent with the counter-cyclicality of domestic inflation, as a central bank can achieve domestic deflation in times of foreign inflation only through excessive exchange rate appreciation and vice versa.

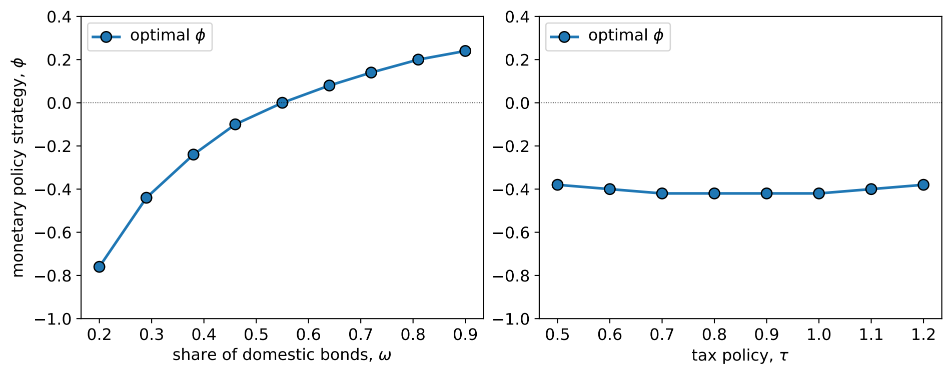

Figure 8: Optimal policy parameter ϕ for different values of ω and τ

Next, I study the effects of the portfolio structure ω and the tax rate τ on the optimal policy. Fig. 8 shows the optimal policy parameter ϕ for different values of steady-state values of ω and tax policy parameter τ. We can see from the figure that the optimal policy parameter ϕ is increasing in the share of domestic bonds ω, while it is almost constant in the tax rate τ. The fact that a high share of domestic bonds implies a higher policy parameter ϕ is due to the interaction between the portfolio allocation effect and the wealth effect. In particular, a high degree of government debt implies high taxes and therefore low wealth levels. This in turn implies that the portfolio allocation effect is dominated by the wealth effect. Hence, the large negative values of ϕ imply high inflation volatility and therefore high volatility of tax levels. Therefore the optimal policy implies a higher value of ϕ in order to reduce the volatility of tax levels.

4.3 Robustness checks

The analysis above was conducted assuming constant domestic endowment $Q_t = Q$ and foreign interest rate $R_t^* = R^*$, to focus on the effects of foreign inflation shocks. Thus, in this section, I study the robustness of the results to allow for time-varying endowments and foreign interest rates.

I assume that the domestic endowment and foreign interest rate follow AR(1) processes

$$\log Q_t = (1-\rho_q)\,\overline{Q} + \rho_q \log Q_{t-1} + \varepsilon_t^q \qquad \varepsilon_t^q \sim IID\left(0, \sigma_q^2\right)$$

and

$$\log R_t^* = (1-\rho_r)\,\overline{R}^* + \rho_r \log R_{t-1}^* + \varepsilon_t^r \qquad \varepsilon_t^r \sim IID\left(0, \sigma_r^2\right)$$

where $\overline{Q}$ and $\overline{R}^*$ are the steady-state values of the endowment and foreign interest rate, respectively.

We can see from model equations that Q_t and R_t^* are not forward-looking variables, thus they have little effect on the model dynamics. Since in each period the portfolio structure is predetermined by the government debt outstanding, Eq. (9) implies that endowment shocks affect the demand for domestic bonds hence the domestic interest rate R_t through their effects on the disposable endowment Z_t. On the other hand, Eq. (9) implies that the domestic interest rate R_t must respond one-to-one to the foreign interest rate R_t^* to clear the bond market.

Therefore, both endowment and interest rate shocks affect the domestic interest rate and borrowing costs for the government in Eq. (3). Hence, these shocks propagate to the domestic economy through the debt accumulation process. Nevertheless, this feature of the model does not affect the portfolio return effect, which is the main channel through which the central bank affects household welfare. To verify it, I simulate the model with time-varying endowments and foreign interest rates. The simulation results are available in the Online Appendix. The results show that the optimal policy regime is robust to allowing for time-varying endowments and foreign interest rates. In particular, the optimal policy strategy is still characterized by $\phi < 0$ for low shares of domestic bonds ω and $\phi > 0$ for high shares of domestic bonds ω.

5 Extensions

5.1 Production economy

In this section, I extend the model to a production economy with tradable and non-tradable goods with sticky prices. The extension allows us to study the optimality of monetary policy in a more general setting and to study the role of price rigidities in policy design.

5.1.1 Households

I assume that households supply labor to domestic firms at the young age and consume at the old age. The household utility function is

$$U_t = E_t\left[\log\left(C_{t+1}\right)\right] - \chi\frac{N_t^{1+\varphi}}{1+\varphi}$$

where φ is the inverse Frisch elasticity of labor supply and χ captures the disutility of labor.

The consumption aggregate is made of two consumption goods - tradable and non-tradable, given by

$$C_t = \left((1-\gamma)^{\frac{1}{\eta}}\left(C_{N,t}\right)^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}}\left(C_{T,t}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

with price index

$$P_t = \left((1-\gamma)\left(P_{N,t}\right)^{1-\eta} + \gamma\left(P_{T,t}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

where $P_{N,t}$ and $P_{T,t}$ are the prices of non-tradable and tradable goods, respectively and $\eta > 1$ is the elasticity of substitution between the two goods.

The budget constraint of the household is

$$D_t + F_t = \frac{P_{T,t}}{P_t}Q_t + W_t N_t + \Omega_t - T_t$$

where W_t denotes the real wage, Q_t is an endowment of the tradable good and Ω_t are the profits from domestic firms.

Maximization of household utility results in the following labor supply function

$$N_t^{\varphi} = \frac{1}{\chi}\frac{W_t}{Z_t}$$

where $Z_t \equiv D_t + F_t = \frac{P_{T,t}}{P_t}Q_t + W_t N_t + \Omega_t - T_t$.

5.1.2 Firms

Monopolistically competitive firms use labor as the only input to produce non-tradable goods according to a linear production function of the form

$$Y_t(j) = N_t(j)$$

Following Galí (2014), I introduce price stickiness for non-tradable goods assuming that the price $P_{N,t}(j)$ of each good is set in period $t-1$ to solve

$$\max_{P_{N,t}(j)} \quad E_{t-1}\left[\Lambda_{t-1,t}\left(\frac{P_{N,t}(j)}{P_t}Y_t(j) - W_t N_t(j)\right)\right]$$

subject to the demand schedule

$$Y_t(j) = \left(\frac{P_{N,t}(j)}{P_{N,t}}\right)^{-\epsilon} C_{N,t}$$

using $\Lambda_{t-1,t} \equiv (\lambda_{o,t}/\lambda_{y,t})$ as the discount factor.

The optimal price-setting rule is

$$E_{t-1}\left[\Lambda_{t-1,t}Y_t\left(\frac{S_{N,t}}{\Pi_t} - \mathcal{M}W_t\right)\right] = 0$$

where $S_{N,t} = \frac{P_{N,t}}{P_{t-1}}$ and $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$.

If firms are allowed to reset prices in each period and prices are flexible, then the optimal price-setting rule becomes

$$S_{N,t} = \mathcal{M}W_t\Pi_t$$

The price-setting rule above implies an inefficiently low level of employment and output due to monopolistic competition.[14] Thus, to restore the steady-state efficient level of output, I assume that the government implements a subsidy ν on labor income, which is financed by lump-sum taxes. The subsidy is set at $\nu = 1/\epsilon$, such that $1-\nu = 1/\mathcal{M}$ and the efficient level of output is restored in the steady state. As a result, the price-setting rule becomes

$$E_{t-1}\left[\Lambda_{t-1,t}Y_t\left(\frac{S_{N,t}}{\Pi_t} - W_t\right)\right] = 0$$

[14] See Galí (2015) for a discussion of the distortion of the efficient steady state due to monopolistic competition.

5.1.3 Central bank

Due to the free flow of tradable goods, tradable goods inflation in the domestic economy is given by the law of one price

$$\Pi_{T,t} = \Psi_t \Pi_t^*$$

The central bank chooses the monetary policy strategy parameter ϕ such that

$$\Pi_{T,t} = \Pi_t^{*\phi} \qquad\qquad \Psi_t = \Pi_t^{*\phi-1} \tag{14}$$

Note that the policy parameter ϕ captures the pass-through of foreign inflation on tradable goods inflation.

The rest of the setup is the same as in the baseline model.

5.1.4 Optimal policy

To evaluate household welfare in the model with production, I simulate it under different monetary policy strategies. In simulations, I set the inverse Frisch elasticity of labor supply to $\varphi = 1$, and the elasticity of substitution between the varieties of non-tradable goods to $\epsilon = 6$, which are standard in the literature. In the baseline calibration, I set the share of tradable goods to $\gamma = 0.5$, and the elasticity of substitution between tradable and non-tradable goods to $\eta = 0.5$. Later, I also assess the effects of different values of γ on the optimal policy regime.

I simulate the model for $T = 2,000$ periods and present the simulation results in Fig. 9.

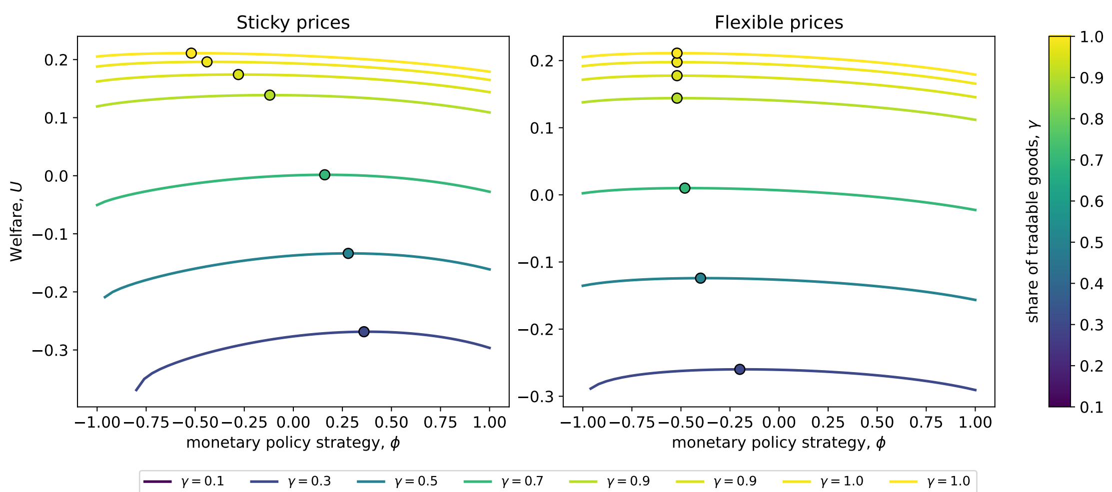

Figure 9: Effects of monetary policy strategy on consumer welfare in the production economy

[Fig. 9](#) plots the household welfare versus the monetary policy parameter ϕ in sticky-price and flexible-price economies. The figure shows that the welfare-maximizing policy parameter ϕ is decreasing in the share of tradable goods γ in both settings. This result shows that the optimal policy strategy is more inclined towards negative pass-through of foreign inflation on tradable goods inflation. Thus, similar to the baseline model, the optimal policy strategy implies a negative correlation between foreign inflation and imported goods inflation.

Next, I also study the effects of the country's portfolio structure on the optimal policy regime. In particular, in Fig. 10 I plot the optimal monetary policy strategy ϕ as a function of the share of domestic bonds ω for different degrees of openness γ.

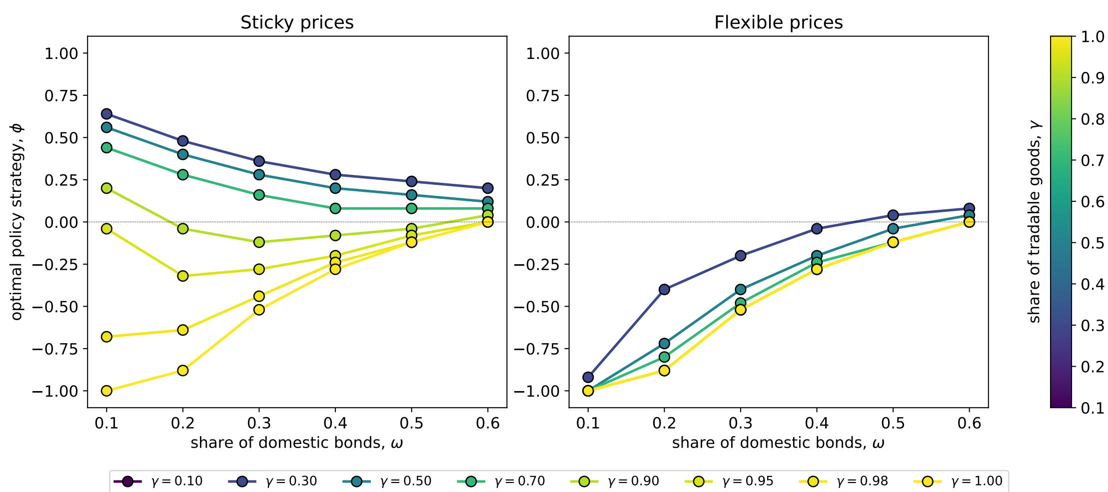

Figure 10: Optimal monetary policy strategy and country openness

The simulation results show that the optimal policy regime depends critically on the share of tradable goods γ in the consumption basket and also on the share of domestic bonds ω in the portfolios. In particular, Fig. 10 shows that in economies with a high share of tradable goods γ, i.e. a high degree of openness, the optimal policy regime implies a negative pass-through of foreign inflation on imported goods inflation, i.e. $\phi < 0$, for a wide range of ω values. Notably, the optimal policy implies a negative correlation between foreign inflation and imported goods inflation mostly under flexible prices. This result is consistent with the finding in the baseline model that the optimal policy strategy implies a negative correlation between foreign inflation and imported goods inflation.

However, in economies with a low degree of openness, i.e. with lower γ, the optimal policy strategy implies a positive pass-through of foreign inflation on imported goods inflation, i.e. $\phi > 0$. In contrast to the baseline model, the optimal policy strategy implies a positive correlation between foreign inflation and imported goods inflation. Nevertheless, the optimal policy regime still implies moderate inflation volatility thereby deviating from price stability.

Hence, except for specific calibrations of the model, the optimal policy regime implies a non-zero inflation volatility.

5.2 Multi-state economy

In previous sections, I developed the model with only two states of the world - low and high foreign inflation. This formulation allowed us to obtain an analytical solution for the model and provided valuable insights into the effects of monetary policy. In this section, I extend the number of states of the model economy to study the model economy in a more general setting.

Next, I assume that foreign inflation shocks follow an i.i.d. process with five possible states and corresponding probabilities

Π_t^*	Π^{*1}	Π^{*2}	Π^{*3}	Π^{*4}	Π^{*5}
Probability	$(1-p)^4$	$4p(1-p)^3$	$6p^2(1-p)^2$	$4p^3(1-p)$	p^4

where $0 < p < 1$ and $0 < \Pi^{*1} < \Pi^{*2} < \Pi^{*3} < \Pi^{*4} < \Pi^{*5}$.[15]

As in the baseline model, I assume $p = 1/2$ and set $\log \Pi^{*1} = -2\pi^*$, $\log \Pi^{*2} = -\pi^*$, $\log \Pi^{*3} = 0$, $\log \Pi^{*4} = \pi^*$, and $\log \Pi^{*5} = 2\pi^*$, such that π^* denotes the volatility of foreign inflation.[16] As a result, domestic inflation and exchange rate may take five possible values, which are determined by the policy parameter ϕ.

Using the multi-state model, I simulate the model for $T = 2,000$ periods and present the results in Fig. 11.

[15]The probabilities are set according to the binomial distribution

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where $n = 4$ and $k = 0, 1, 2, 3, 4$.

[16]Noting that $E_{t-1}[\log \Pi_t^*] = 0$, we can see that π^* equals to the standard deviation of foreign inflation

$$Var_{t-1}(\log \Pi_t^*) = \frac{1}{16}\log^2 \Pi^{*1} + \frac{1}{4}\log^2 \Pi^{*2} + \frac{3}{8}\log^2 \Pi^{*3} + \frac{1}{4}\log^2 \Pi^{*4} + \frac{1}{16}\log^2 \Pi^{*5} = \pi^{*2}$$

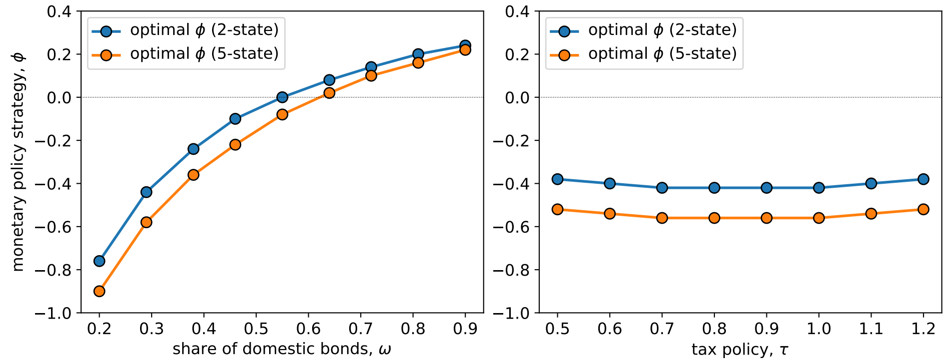

Figure 11: Optimal policy parameter ϕ in two-state and five-state economies

The simulation results show that the optimal policy regime slightly leans towards a negative pass-through of foreign inflation on domestic inflation relative to the baseline model. Nevertheless, the optimal policy strategy still implies a non-zero inflation volatility and it depends on the share of domestic bonds ω in the portfolios.

6 Conclusions

This paper develops an overlapping generations model with portfolio choice to analyze the role of monetary policy in an economy with external asset holdings. I study the effects of different monetary policy strategies on the domestic economy and household welfare and show that the optimal policy depends on the portfolio structure of households and country openness. As a result, this paper lays forward a new policy framework that, instead of inflation targeting, implies inducing a negative correlation between domestic and foreign inflation to complete asset markets.

The model features nominal debt contracts, hence monetary policy can affect real interest rates and therefore the real return on household portfolios, as well as the domestic debt accumulation rate. In this setting, the optimal policy regime implies a negative correlation between the returns of domestic and foreign assets, as a means of insurance against foreign inflation shocks. This result is achieved by inducing optimal co-movements between domestic inflation relative to foreign inflation. In the long run, monetary policy may also affect the domestic debt levels and the tax burden on households via its effects on the debt accumulation rate.

These channels of policy transmission generate a policy trade-off for the central bank. On the one hand, higher real interest rates increase the real return on household portfolios and therefore improve welfare. At the same time, higher real interest rates increase the domestic debt accumulation rate and lead to an equilibrium with higher government debt. This effect, however, reduces welfare, since higher government debt implies higher taxes and implies lower disposable income for households. Therefore, the optimal policy implies a trade-off between the effects on the real return on the household portfolio and the domestic debt accumulation rate.

The optimal monetary policy seeks balance between the effects of these two channels of policy transmission. For a wide range of parameter values, the optimal policy implies a negative correlation between the domestic and foreign inflation rates, with positive inflation volatility. This policy promotes higher welfare compared with exchange rate targeting as well as inflation targeting, contrary to the conventional wisdom.

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