

# Homework 1

ECON312 Time Series Analysis

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## Instructions

- The homework is due at due-time on **due-date**.
- Homeworks must be typeset in Latex and submitted (uploaded to the course page) in pdf format named `HW1_Name_Surname.pdf`.

## Assignment 1

Consider an  $AR(p)$  process given by

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t \quad \varepsilon_t \sim WN(0, \sigma^2)$$

with  $\sigma^2 > 0$ .

The impulse response function of an autoregressive process may be generated by setting  $\varepsilon_0 = \sigma$  and  $\varepsilon_t = 0$  for  $t > 0$ , and simulating the process for the required horizon  $H$ . The resulting series is the impulse response function.

For the following exercises, set  $\sigma = 0.4$  and  $H = 20$ .

1. Generate the impulse response function for an  $AR(1)$  process for the following values of  $\phi_1$ :  $\phi_1 = 0.2$ ,  $\phi_1 = 0.4$ ,  $\phi_1 = 0.6$ ,  $\phi_1 = 0.8$ ,  $\phi_1 = 0.9$  and plot them on the same graph. On a separate graph, plot the impulse response function for  $\phi_1 = -0.6$  and  $\phi_1 = -0.9$ .
2. Discuss the effect of  $\phi_1$  on the impulse response function.
3. Find example values of  $\phi_1$  and  $\phi_2$  for which the impulse response function of an  $AR(2)$  process is a) hump-shaped, b) oscillatory, c) explosive. Plot the impulse response functions for each case.
4. Discuss the effects of  $\phi_1$  and  $\phi_2$  on the impulse response function.

## Assignment 2

Consider an  $ARMA(1, 1)$  process given by

$$y_t = \phi y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1} \quad \varepsilon_t \sim WN(0, \sigma^2)$$

with  $|\phi| < 1$  and  $\sigma^2 > 0$ .

1. Find a representation of  $y_t$  in terms of  $\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \dots$
2. Find the mean and variance of  $y_t$
3. Find the first autocovariance of  $y_t$
4. Find the first autocorrelation of  $y_t$