Homework 2 solutions

ECON312 Time Series Analysis
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Assignment 1

Consider the following AR(1) model

$$y_t = c + \phi y_{t-1} + e_t$$

where the error terms are themselves AR(1) processes

$$e_t = \rho e_{t-1} + u_t \qquad u_t \sim WN(0, \sigma^2).$$

with $|\phi| < 1$ and $|\rho| < 1$.

Solution

1. Substituting e_t from the second equation into the first, we get

$$y_t - c - \phi y_{t-1} = \rho \left(y_{t-1} - c - \phi y_{t-2} \right) + u_t$$

Then we have

$$y_t = c \left(1 - \rho \right) + \left(\phi + \rho \right) y_{t-1} - \rho \phi y_{t-2} + u_t$$

Subtracting y_{t-1} from both sides, we get

$$
\begin{aligned}
y_t - y_{t-1} &= c \left(1 - \rho \right) + \left(\phi + \rho - 1 \right) y_{t-1} - \rho \phi y_{t-2} + u_t \\
&= c \left(1 - \rho \right) + \left(\phi + \rho - \rho \phi - 1 \right) y_{t-1} + \rho \phi \left(y_{t-1} - y_{t-2} \right) + u_t
\end{aligned}
$$

Then denoting $\Delta y_t = y_t - y_{t-1}$

$$\Delta y_t = c \left(1 - \rho \right) + \left(\phi + \rho - \rho \phi - 1 \right) y_{t-1} + \rho \phi \Delta y_{t-1} + u_t$$

Finally

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_1 \Delta y_{t-1} + u_t$$

where $\alpha_0 = c\,(1 - \rho)$, $\gamma = \phi + \rho - \rho\phi - 1$, and $\alpha_1 = \rho\phi$.