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Theoretical Prediction and Experimental Verification of Hitting Targets of Gravity-Powered Trebuchets

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Theoretical Prediction and Experimental Verification of Hitting Targets of Gravity-Powered Trebuchets

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Abstract: As one of the early weapons used by the ancient Chinese military, the gravity-powered trebuchet is listed as a mandatory program in science and technology games in several provinces. Due to its entertaining and informative features, this program has received continuous attention and discussion from scholars. Currently, most of the studies on this program are empirical, and few studies have been conducted quantitatively based on theoretical models. Therefore, this study established a theoretical model of the trebuchet projection process from the perspective of system energy conservation and gave an accurate analytical expression of hitting target points. In order to verify the reliability of the theoretical model, experimental tests were conducted. The results showed that the experimental results were relatively consistent with the theoretical predictions. These findings could provide a new research idea for facilitating gravity-powered trebuchets to hit targets accurately.

1. Introduction

As an indispensable weapon for long-range attack and defense in ancient battlefields at home and abroad, the trebuchet has had a prominent place in the history of weapons for more than 2,000 years, from its appearance to its retirement[1-5]. According to the literature, previous studies on trebuchets are mainly focused on two aspects. Scholars have mainly studied the types, structure, performance, and development history of trebuchets. In recent years, some researchers have made valuable research achievements by taking advantage of computer simulation software. However, few studies have been conducted quantitatively based on theoretical models from a physical view[6-8].

In this paper, a gravity-powered trebuchet was taken as an example to establish a theoretical model and analyze the kinetic law of the projection process from the perspective of system energy conservation and the motion law of the projectile from the perspective of kinematics. Through a series of theoretical derivations, the precise theoretical equations that are satisfied by the trebuchet in hitting the target point were obtained. In order to verify the reliability of the theoretical model, experimental tests were also conducted. The results showed that the experimental results were well consistent with the theoretical model.

2. Establishment of a Theoretical Model of Gravity-Powered Trebuchets

2.1. Construction of a gravity-powered trebuchet model

A gravity-powered trebuchet model was established (Figure 1), and it mainly consists of three parts: the base, mainframe, and working lever. The working lever, also called the throwing arm, works as a hand lever. The shorter arm of the throwing arm is the power arm, at the end of which a weight is placed to provide the throwing energy through the free fall of the counterweight, while the long arm of the throwing arm is the resistance arm, at the end of which a carrying device (usually in the shape of a flat spoon) is configured to carry the projectile. In order to achieve good throwing performance, the ratio of the length of the power arm to that of the resistance arm shall generally be within a certain range. Relevant practical experience shows that the throwing performance is optimal when the ratio is 1:5 to 1:2.5.

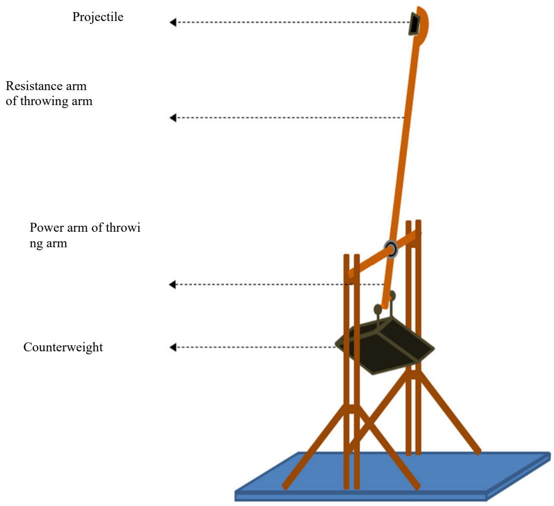

Figure 1. Gravity-powered Trebuchet

2.2. Brief analysis of the gravity-powered trebuchet model

In this section, the model was taken as the study object for theoretical analysis and in-depth study. To facilitate the following theoretical analysis, a schematic of the throwing process was made, as shown in Figure 2 below.

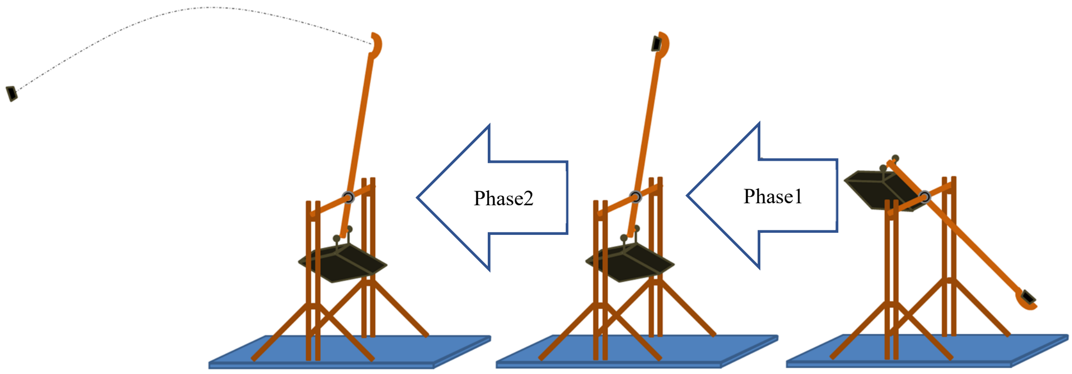

Figure 2. Schematic of the Throwing Process

The energy conversion in the throwing process involves potential energy, kinetic energy, rotational energy, frictional energy dissipation, vibration energy dissipation, etc.

3. Theoretical Analysis of Gravity-Powered Trebuchets

The theoretical analysis starts mainly from the throwing process of the gravity-powered trebuchet, which is divided into two phases. In Phase 1, the trebuchet changes from the energy storage state to the critical state, and the study object is the entire trebuchet, so this phase is called the energy phase. In Phase 2, the trebuchet changes from the critical state to the attack state, and the study object is the projectile, so this phase is called the motion phase.

3.1. Dynamics analysis of the gravity-powered trebuchet

The changes in energy of the gravity-powered trebuchet in the throwing process were obtained through analysis. The reduced energy includes the gravitational potential energy of the counterweight, the gravitational potential energy of the power arm of the throwing arm, and the vibration energy dissipation (the materials used for the construction of the trebuchet are mostly wood, bamboo, etc., which are of high elasticity.), the frictional energy dissipation of the mechanical device (it mainly refers to the energy dissipation of rolling bearing fiction), and the energy dissipation of the air friction (since air friction loss is very small, the calculation process is cumbersome, and many uncertainties exist, so this energy dissipation is ignored).

The increased energy includes the gravitational potential energy of the projectile, the gravitational potential energy of the resistance arm of the throwing arm, the kinetic energy of the projectile and the counterweight, and the rotation energy of the lever. According to the law of energy conservation and transformation, the energy transformation diagram was obtained, as shown in Figure 3.

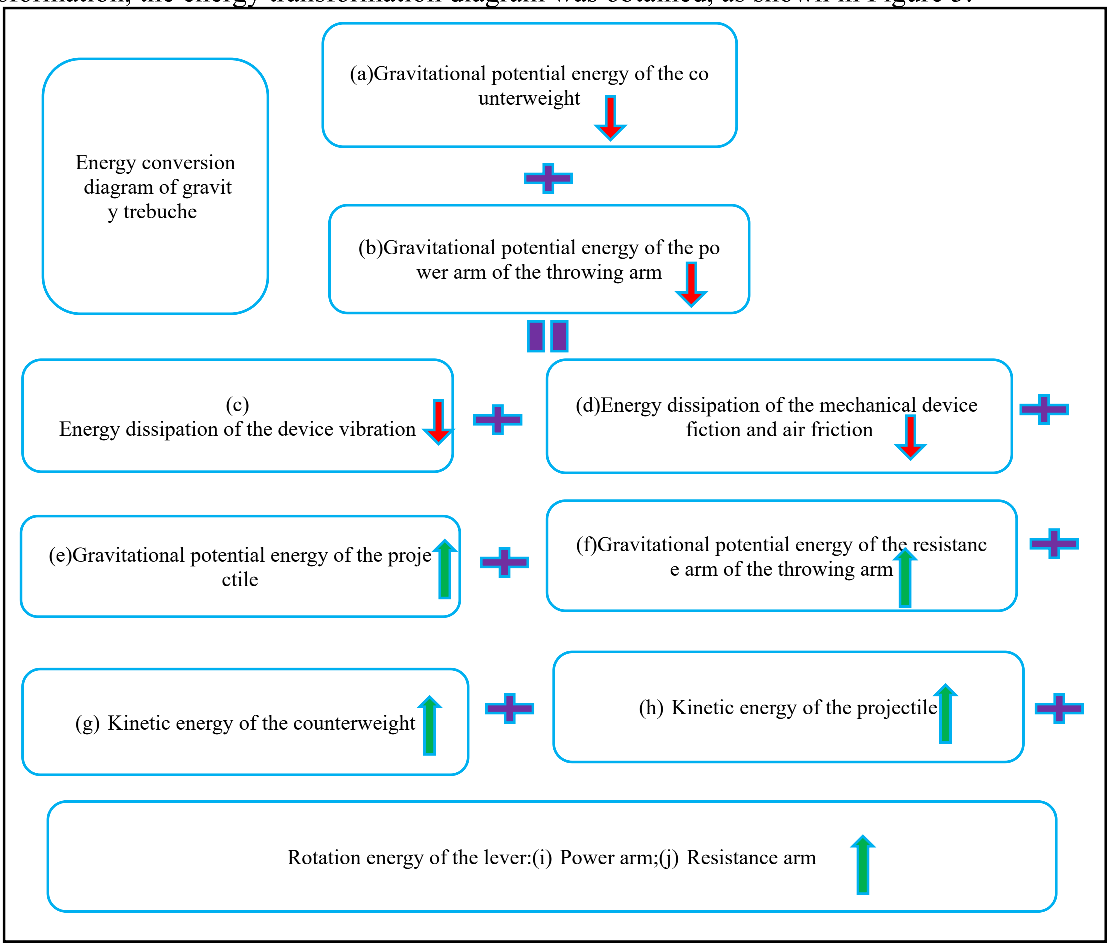

Figure 3. Energy Conversion During the Throwing Process of the Gravity-powered Trebuchet

Graphical analysis was conducted based on the schematic of the throwing process. Let the length of the power arm be L_1, the mass of the power arm be m_1, the length of the resistance arm be L_2 the mass of the resistance arm be m_2, the mass of the counterweight be M, and the mass of the projectile be m, the force diagram of the throwing arm was drawn, and a horizontal line was drawn at the pivot point. When the counterweight is above the horizontal line, the trebuchet is in the energy storage state (also called the pending state). When the counterweight is below the horizontal line, the trebuchet is in a state where the projectile just breaks free from the device (also called the critical state), and its velocity is v. In the two states, the angles between the resistance arm and the horizontal line are θ_1 and θ_2, respectively, and the angle between the velocity of the projectile just breaking free from the device and the horizontal line is θ, as shown in Figure 4.

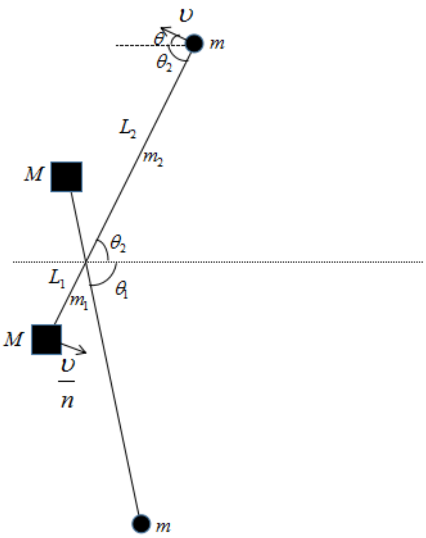

Figure 4. Kinetic Analysis. The dots are projectiles, and the squares are heavy objects.

The energy conversion equation was obtained from the above analysis, as shown in Equation (1) below.

$$MgL_1(\sin\theta_1+\sin\theta_2)(1-15\%)+\frac{m_1gL_1(\sin\theta_1+\sin\theta_2)}{2} = \frac{\mu(M+m+m_1+m_2)(\theta_1+\theta_2)D}{2}+mgL_2(\sin\theta_1+\sin\theta_2) + \frac{m_2gL_2(\sin\theta_1+\sin\theta_2)}{2}+\frac{Mv^2L_1^2}{2L_2^2}+\frac{mv^2}{2}+\frac{m_1L_1^2v^2}{6L_2^2}+\frac{m_2v^2}{6} \quad (1)$$

(Note: In the equation, 15% is the empirical value of vibration energy dissipation.)

In Equation (1), μ represents the friction coefficient, and D represents the nominal inner diameter of the bearing. Different devices have different vibration energy dissipation, and the empirical value was taken here. Generally, the value ranges from 15% to 20% and can be adjusted according to the actual situation, and the deviation is adjustable.

In actual situations, as the power arm of the throwing arm in the gravity-powered trebuchet drops, the resistance arm goes up, so the potential energy change of the arm is very small. Also, it was assumed that the material of the throwing arm is fine and light and has uniform density and the same cross-sectional area. Due to this, the potential energy change of the arm can be ignored. The calculated data showed that the energy dissipated by friction had little effect on the whole device and could be replaced by an empirical coefficient, which is generally 0.97–0.99. The above two are not definite values, and the empirical coefficient depends on the actual situation. Therefore, a simple expression of energy conversion can be obtained, as shown in Equation (2) below.

$$0.98MgL_1(\sin\theta_1+\sin\theta_2)(1-15\%) = mgL_2(\sin\theta_1+\sin\theta_2)+\frac{Mv^2L_1^2}{2L_2^2}+\frac{mv^2}{2}+\frac{m_1L_1^2v^2}{6L_2^2}+\frac{m_2v^2}{6} \quad (2)$$

By simplifying Equation (2), the expression for the velocity can be obtained, as shown in Equation (3) below.

$$v = \sqrt{\frac{6L_2^2(0.833MgL_1 - mgL_2)(\sin\theta_1 + \sin\theta_2)}{3ML_1^2 + 3mL_2^2 + m_1L_1^2 + m_2L_2^2}} \tag{3}$$

3.2 Kinematic analysis of gravity-powered trebuchets

In the first stage of the kinematic analysis of the trebuchet, the velocity of the projectile when it just breaks free from the trebuchet (called off-point velocity) can be calculated. Then, in the second stage, kinematic analysis can be performed for the motion made by the projectile after it just breaks free from the trebuchet from a geometric view. In the throwing process, the motion made by the projectile is oblique projectile motion (In the process, the projectile is only subject to the force of gravity and the influence of air resistance is so small that it can be ignored).

Suppose the height of the pivot point of the trebuchet is H. The projectile breaks free from the device at Point $A(x_0, y_0)$, and the coordinate of the target point is $B(x_p, y_p)$, the projection point of the pivot point on the ground was taken as the origin to establish the coordinate system. The schematic diagram of the motion of the projectile is shown in Figure 5.

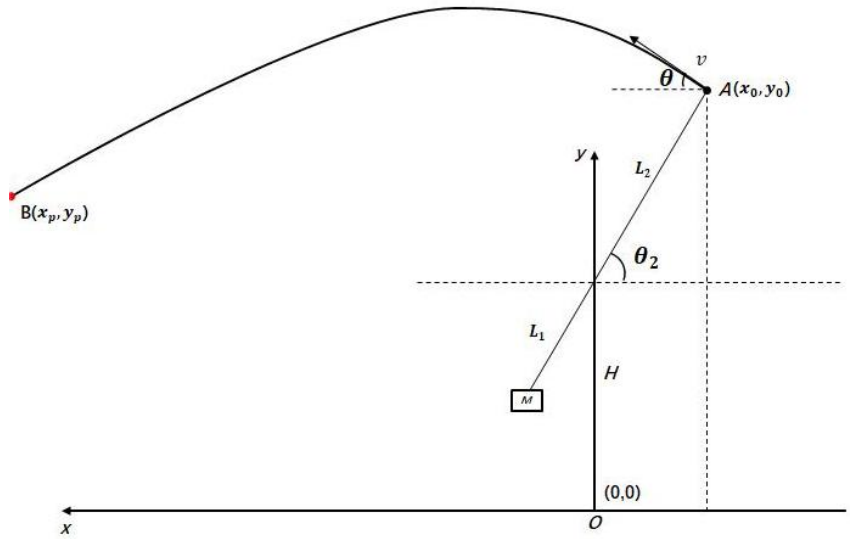

Figure 5. The Projectile Is out of Motion

According to Figure 5 and the geometric relations, we can obtain that

$$\theta = 90^\circ - \theta_2$$
$$x_0 = -L_2\cos\theta_2$$
$$y_0 = H + L_2\sin\theta_2$$

The velocity can be decomposed into a horizontal velocity and a vertical velocity as

$$x = x_0 + v\cos(90^\circ - \theta_2)\,t$$
$$y = y_0 + v\sin(90^\circ - \theta_2)\,t - \frac{gt^2}{2} \tag{4}$$

By combining the two equations in Equation (4) into an equation, we can eliminate t and obtain the trajectory equation of the projectile after it breaks free from the device as follows.

$$y = -\frac{g(x - x_0)^2}{2\sin^2\theta_2\, v^2} + \frac{x - x_0}{\tan\theta_2} + y_0 \tag{5}$$

By substituting the target point $B(x_p, y_p)$ into the trajectory equation and combining Equation (3) for the off-point velocity, the total equation can be obtained as follows.

$$y_p = -\frac{g\left(x_p + L_2 \cos\theta_2\right)^2 (3ML_1^2 + 3mL_2^2 + m_1L_1^2 + m_2L_2^2)}{12\,L_2^2 sin^2\theta_2\,(0.833MgL_1 - mgL_2)(\sin\theta_1 + \sin\theta_2)} + \frac{x_p + L_2\cos\theta_2}{\tan\theta_2} + \sin\theta_2 L_2 + H \tag{6}$$

4. Experimental Verification

4.1. Design of a test gravity-powered trebuchet

At the beginning of the design, the length L_1 and L_2 and the mass m_1 and m_2 of the power arm and resistance arm are measured first, and records are made. The mass M and m of the counterweight and the projectile can be measured first during the assembly process, and records are made. Figure 6 shows the trebuchet and relevant components used in this test plan.

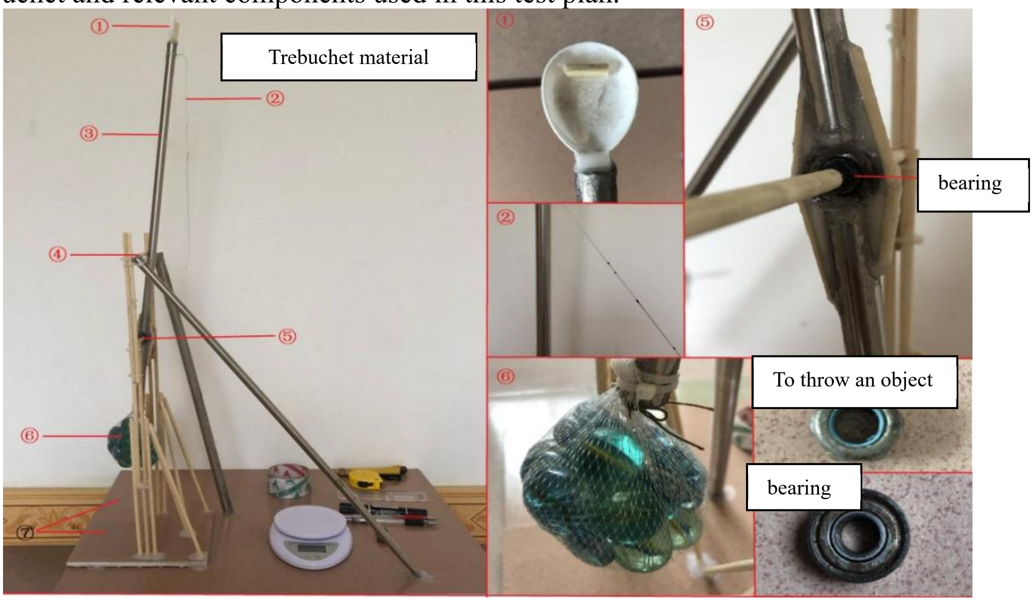

Figure 6. Test Gravity-powered Trebuchet

Materials used in the trebuchet and their roles are as follows.

1) Carrying device of the projectile: a flat plastic round spoon. In order to ensure that the projectile vertically breaks free from the throwing arm in the throwing process, a small wooden bar with a semi-circular cross-section is added at the top of the throwing arm. This ensures the vertical throwing of the projectile.

2) Pulling rope: a light thin rope. The rope is tied to the carrying device of the throwing arm and is mainly used to pull down the throwing arm and mark the throwing corner (see θ_1 marked in Figure 6).

3) Throwing arm: a stainless steel pipe with a diameter of 12 mm, a thickness of 1 mm, and a capacity of 304 L.

4) Crossbeam: disposable chopsticks erected on top of the frame, with a diameter of 5.0 mm and a length of 0.225 m.

5) Bearing: 608 zz deep groove bearings with an inner diameter of 9 mm and an outer diameter of 22 mm.

6) Counterweight: glass bouncing balls. The balls can provide power for the device. You can choose the material of the counterweight at will.

7) Base: a combination of 0.2 m×0.2 m tiles and 0.5 m×0.5 m tiles. The small tiles are used for the base of the frame of the trebuchet, and the large tiles mainly provide space for the frame to be fixed as well as to increase the gravity of the base, so as to improve the stability of the device.

Frame: a combination of disposable chopsticks of 5.0 mm diameter and 0.225 m length and a 304 L stainless steel pipe of 12 mm diameter and 1 mm thickness. Disposable chopsticks are mainly used to form the main frame, while the stainless steel pipe is mainly used to fix the device to reduce the device's vibration energy dissipation.

Projectile: M10 locknut.

Tools used: a utility knife, a needle-nosed plier, a scale, a protractor, a high-precision electronic scale, a hot-melt glue gun and glue stick, thin ropes, etc.

4.2. Design of a test plan

When the device is assembled and placed in the throwing area, the value of its pivot point height H, and the horizontal distance x_p of the selected pivot point from the preset target point are measured. According to the plan, the angle of θ_2 is predetermined first, and generally, an angle from 70° to 85° (depending on the situation) is set to calculate the coordinate point $A(x_0, y_0)$. Then, the angle of θ_1 is set, and the theoretical throwing height y_p is calculated by the total Equation (6). After that, throwing is performed, and each hit point is marked. Finally, the height coordinates of these hit points are measured and recorded as $y_i(i = 1,2, \dots, i)$, and the arithmetic mean of the hit points is calculated and compared with the theoretical height y_p for analysis, and the sample standard variance of the hit points is calculated to analyze the dispersion of the hit points in each test.

The above test process is repeated by adjusting the angle of θ_1, and the results of each test are analyzed. For the hit rate test, a dartboard is drawn with the target point $B(x_p, y_p)$ as the center of the dartboard, and the radius is as large as possible to ensure that each point can hit the dartboard, and then a target circle (recommended radius: 0.05 m) is drawn on the dartboard with the target point as the center of the circle, as shown in Figure 7.

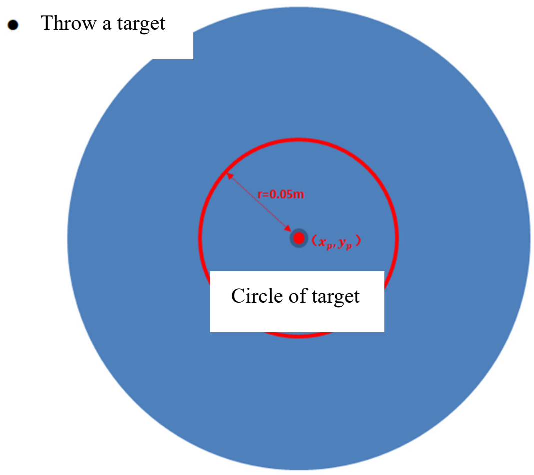

Figure 7. The Design of Target Throwing

The hit rate is equal to the number of times the target circle is hit divided by the total number of casting. Different conditions are formulated, and the hit rate is tested repeatedly. Finally, the results are analyzed.

4.3 Analysis of test results

In this study, several system tests and hit rate tests were conducted, and the on-site throwing mark images of four of these tests are shown in Figure 8.

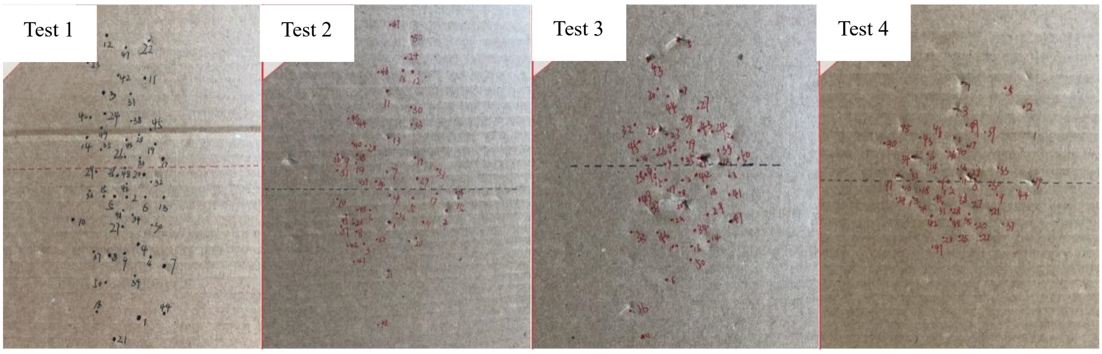

Figure 8. Hit the Dot Mark

Table 1. Experimental Equipment Parameters

Equipment parameters:	
Weight distribution (M):	0.435 kg
Mass of the projectile (m):	0.008 kg
Power arm length (L_1):	0.13 m
Length of resistance arm (L_2):	0.49 m
Power arm mass (m_1):	0.1174 kg
Mass of the resistance arm (m_2):	0.04426 kg
Shaft height (H):	0.36 m
Theta 2:	82°

The data from the four tests were processed as shown in Table 2, and the experimental equipment parameters are shown in Table 1.

Table 2. Test Data Processing

No.	Theta1 (°)	Throwing distance x_p(m)	Theoretical throwing height y_p(m)	The total Number of casting	Best estimated height(m)	Sample standard variance
1	70	2.5	0.147442	50	0.094	0.0325
2	60	2.5	0.105426	47	0.096	0.0301
3	60	2	0.422038	51	0.396	0.0209
4	30	2	0.246707	50	0.202	0.0149

According to the analysis of the results of several tests, theta1 set at 20° to 60° and theta2 set at 75° to 85° are the empirical conditions for the gravity-powered trebuchet to achieve the best throwing performance, which is further verified by several trial throws. This recommendation can be achieved simply by changing other conditions, such as the mass of the counterweight and the ratio of the length of the lever. In the data processing of the hit points, the arithmetic mean of each test was compared with the theoretical value, and the difference between the practical value and the theoretical value was analyzed to be around 0.05 m, which is called the mean error. Since many uncertainties existed in the vibration of the device and the operation of the thrower during the tests, the results fluctuated but not greatly.

In the hit rate tests, the radius of the target circle was set as 0.05 m. The analysis results of the eight tests are shown below. The test conditions are shown in Table 3.

Table 3. Hit Rate Test Condition

No.	Theta1(°)	Theta2(°)	Throwing distance x_p(m)	Theoretical throwing height y_p(m)
1	70	82	2.5	0.147442
2	60	82	2.5	0.105426
3	60	82	2	0.422038
4	40	82	2	0.324454
5	40	82	2	0.324454
6	10	82	2	-0.002613
7	20	82	2	0.141269
8	30	82	2	0.246707

After the data recording and statistical calculation, the average hit rate of these eight tests was obtained as 71%, and the statistics are shown in Table 4.

Table 4. Hit Rate Statistics

No.	Total number of flips	Number of hits in the target circle	Percentage points
1	50	27	54
2	50	38	76
3	50	41	82
4	50	36	72
5	50	42	84
6	50	30	60
7	50	29	58
8	50	40	80
Average hit rate:0.7075			

5. Conclusion

With a gravity-powered trebuchet as the object of study, this paper established a mechanical model to analyze the trebuchet in stages from the traditional dynamics and kinematics perspectives. With the energy conversion and energy dissipation in practical applications taken into account, the mechanical process of the trebuchet in detail was calculated, the total energy equation was listed, and then the off-point velocity expression was obtained by simplifying the conversion. Then, based on kinematics, the trajectory of the projectile after breaking free from the trebuchet was analyzed, and the equation of motion was listed. Finally, based on the obtained off-point velocity expression, the total equation for the trebuchet was obtained.

Based on the theoretical analysis, a self-designed gravity-powered trebuchet was introduced and used as the main study object of the test plan. Through the practical operation, the feasibility of the theoretical analysis was finally verified, and the performance indicators of the trebuchet were investigated. During the test process, problems were continuously discovered and solved, and then a complete test plan was concluded, which could provide valuable references for future trebuchet tests. Additionally, hit rate tests were conducted for the gravity-powered trebuchet. By combining different conditions of the tests, the mean hit rate was obtained as 71%.

Acknowledgments

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References

[1] Chen M. Y. and Jin M. B. Three new perspectives on historical archaeology: Wood and stone

composite weapons—Stone throwing ropes, trebuchets, and catapults [J]. Tribune of Social Sciences, 2014(3): 32–42.
[2] Ji C. Catapults in the Sui and Tang dynasties: form, performance, combat, and dissemination [J]. Tang Shi Lun Cong, 2016(1): 163–178.
[3] Zhang J. B. Research on catapults in ancient China [D]. Henan: Zhengzhou University, 2007. doi:10.7666/d.y1059827.
[4] Chevedden, P. E., Eigenbrod, L., Foley, V. & Soedel, W. 1995. The trebuchet. - Scientific American, July, 58–63.
[5] Hansen, P. V. 1992. Experimental reconstruction of a medieval trebuchet.
[6] Liu C. J. and Zheng J. Q. Dynamics simulation of trebuchet mechanism [J]. Mechanical Engineer, 2007(8):70–72. DOI: 10.3969/j.issn.1002-2333.2007.08.033.
[7] Xiao H. F. Research on a new high-efficiency trebuchet and its innovative design [D]. DOI:10.7666/d.y971180.
[8] Wang X. M. Physics of the operation and projection mechanism of ancient trebuchets[J]. Physics Teacher, 2021,42(12):59–62,66. doi:10.3969/j.issn.1002-042X.2021.12.019.