

Mean / median

Forecasting Methods : → Noise

→ Seasonal Naïve

→ Drift

} Simple  
Methods

Forecasting / Smoothing Methods :

Real-world Data has:

↳ Noise

↳ one-off events

↳ human behaviour

➢ Business decisions don't care about NOISE



↳ overall trend

↳ direction

↳ pattern

① Moving Average

② SES - Simple Exponential Smoothing

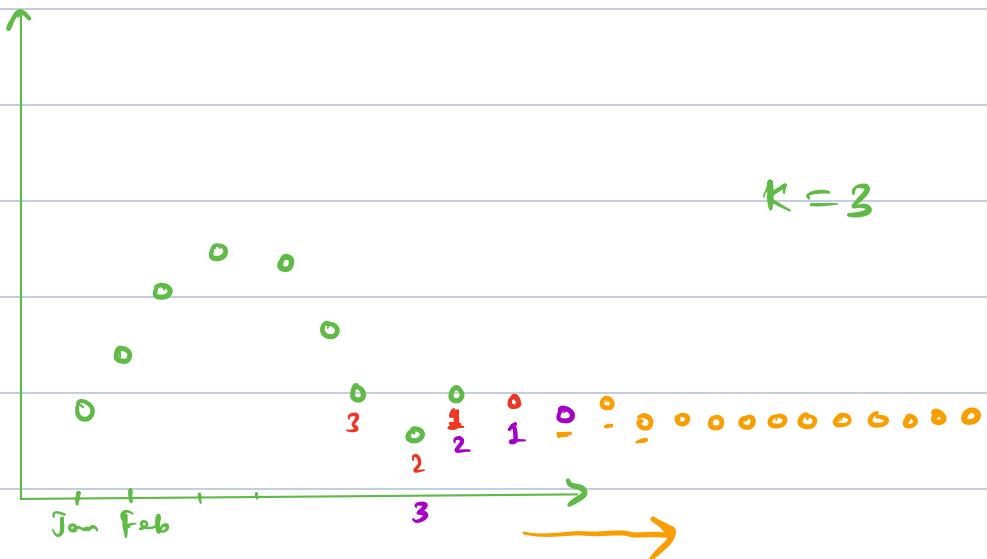
③ DES - Double " " (Holt's Method)

④ TES - Triple " " (Holt-Winter's Method)

Smoothing techniques are methods that:

- ① reduce short-term fluctuations
- ② highlight long-term patterns
- ③ prepare data for forecasting

## ① MOVING AVERAGE → rolling ( )



Sales

$[k=3]$

Original Data

Jan 2021
Feb 2021
:
Jan 2019
Feb 2019

$$\text{pred} = [\text{Nov 2018}, \text{Dec 2018}, \text{Jan 2019}]$$

$$= [100, 200, 300]$$



1st

$$= [100, 200, 300, 200]$$

Mar 2019	NaN
Apr 2019	NaN

Conclusion: Forecast is certainly better than simple methods like mean/median. However, it is not able to predict the variation and it gives a flat line after few predictions".

MAPE : 9.4 %.

## ② Simple Exponential Smoothing (SES) - [only level]

→ Instead of ignoring all past values → give lesser weights recent values → give more weights

Let's consider the weight we assign to most recent value is ' $\alpha$ '

$\alpha$  → Smoothing parameter

$$0 \leq \alpha \leq 1$$

$$\text{New Smoothed Value} = (\alpha * \text{Current Value}) + (1 - \alpha) * \text{Previous Smoothed Value}$$

$$1^{\text{st}} \rightarrow S_t = \alpha y_t + (1-\alpha) S_{t-1} \quad \begin{matrix} \nearrow \text{Previously smoothed} \\ \text{value} \end{matrix}$$

$\hookrightarrow$  Forecasted Smoothed value at t

$$S_{t-1} = \alpha y_{t-1} + (1-\alpha) S_{t-2}$$

Substitute  $S_{t-1}$  in 1<sup>st</sup> equation.

$$S_t = \alpha \cdot y_t + (1-\alpha) [\alpha y_{t-1} + (1-\alpha) S_{t-2}]$$

$$S_t = \alpha \cdot y_t + \alpha (1-\alpha) y_{t-1} + \alpha (1-\alpha)^2 y_{t-2} +$$

.....

Weights  $\rightarrow$  1<sup>st</sup> recent  $\rightarrow \alpha$

2<sup>nd</sup>  $\rightarrow \alpha(1-\alpha)$

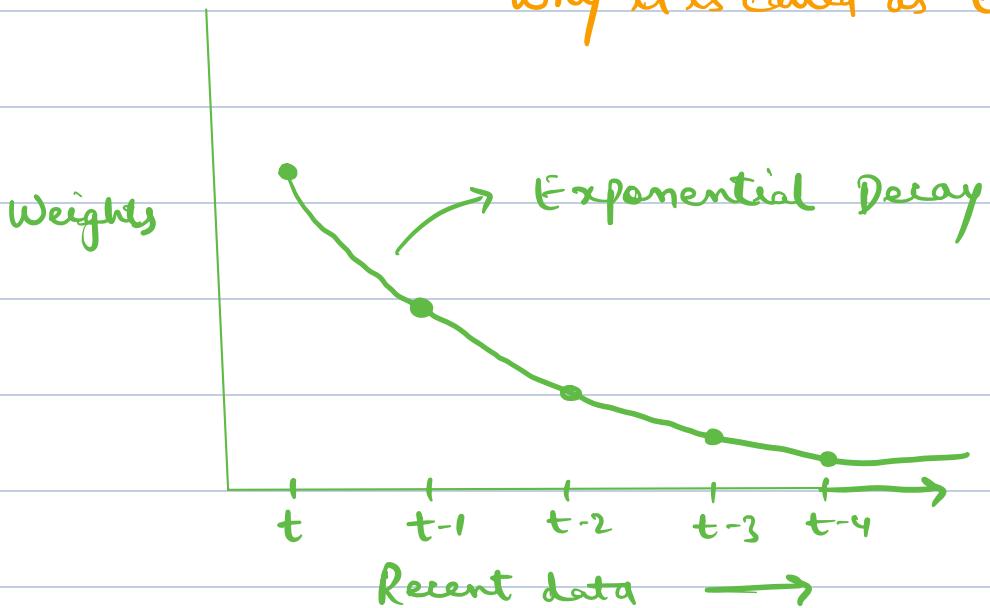
3<sup>rd</sup>  $\rightarrow \alpha(1-\alpha)^2$

$$\alpha = 0.8$$

$$2^{\text{nd}} \text{ weight} \rightarrow 0.16$$

$$3^{\text{rd}} \text{ weight} \rightarrow 0.032$$

Why it is called as Exponential Smoothing



- Weights decrease exponentially
- Never becomes exactly zero

$$\text{Sum of weights} \rightarrow \sum_{k=0}^{\infty} \alpha(1-\alpha)^k = 1 \quad [\text{Proper weighted average}]$$

Smoothing-level  $\rightarrow \alpha$

↳ Recommended starting value is equal to

$$\frac{1}{2 + \text{Seasonality}} \quad [m=12]$$

SES does not consider any:

- ① Trend
- ② Seasonality

SES models only level, not change.

MAPE = 11.3%.

Conclusion: Looks even worse than MA forecast.

Gives a straight line.

Gives the right level ✓

③ PES  $\rightarrow$  Holt's Method (level + trend)

Shortcomings of SES:

- ① doesn't capture trend / seasonality
- ② gives one unique value (flat line)

2 components:

① Level

② Trend

① Level

$$L_t = \alpha * y_t + (1-\alpha) * (L_{t-1} + T_{t-1})$$

$\alpha$  = smoothing cons. (for level)

$$0 \leq \alpha \leq 1$$

② Trend

$$T_t = \beta (L_t - L_{t-1}) + (1-\beta) * T_{t-1}$$

$\beta$  = smoothing cons. (for Trend)

$$0 \leq \beta \leq 1$$

③ Final Equation:

$$\hat{y}_{t+h} = L_t + h \times T_t$$

$h$  = horizon  $\rightarrow 1, 2, 3, 4, \dots$

$$Y_t \text{ Today's sales} = 120$$

$$Y_{t-1} \text{ Yesterday's sales} = 110$$

$$T_{t-1} \text{ Yesterday's trend} = 5$$

$$\hat{Y}_{t+h} = ?$$

$$\alpha = 0.5$$

$$\beta = 0.3$$

$$DES = SES + \text{trend tracking}$$

$$\text{Seasonality} \times \text{MAPE} \rightarrow 8.9\%$$

#### ④ TES Method (Holt - Winter's method)

(level + trend + seasonality)

$$TES = DES + \text{seasonality tracking}$$

$$\hat{Y}_{t+h} = l_t + h \times T_t + S_{t+h-m}$$

$m \rightarrow$  frequency of seasonality

quarterly data  $\rightarrow m=4$

monthly data  $\rightarrow m=12$

MAPE  $\rightarrow 5.1\%$  (add)

MAPE  $\rightarrow$  4.6 % (mult)

Conclusion: Perform best compared to all other methods.

$$\text{level} \rightarrow l_t = \alpha (\gamma_t - s_{t-m}) + (1-\alpha) (l_{t-1} + T_{t-1})$$

$$\text{Trend} \rightarrow T_t = \beta (l_t - l_{t-1}) + (1-\beta) T_{t-1}$$

$$\text{Seasonality} \rightarrow s_t = \gamma (\gamma_t - l_{t-1} - T_{t-1}) + (1-\gamma) s_{t-m}$$

$\gamma$  = smoothing cons. (for seasonality)

$\rightarrow$  Stationarity

$\rightarrow$  ARIMA