

Forecasting Methods :
 → Mean / Median
 → Naïve
 → Seasonal Naïve
 → Drift
 } Simple Methods

Forecasting / Smoothing Methods :

Real-world Data has:

↳ Noise

↳ one-off events

↳ human behaviour

↳ Business decisions don't care about NOISE

↓

↳ overall trend

↳ direction

↳ pattern

① Moving Average

② SES - Simple Exponential Smoothing

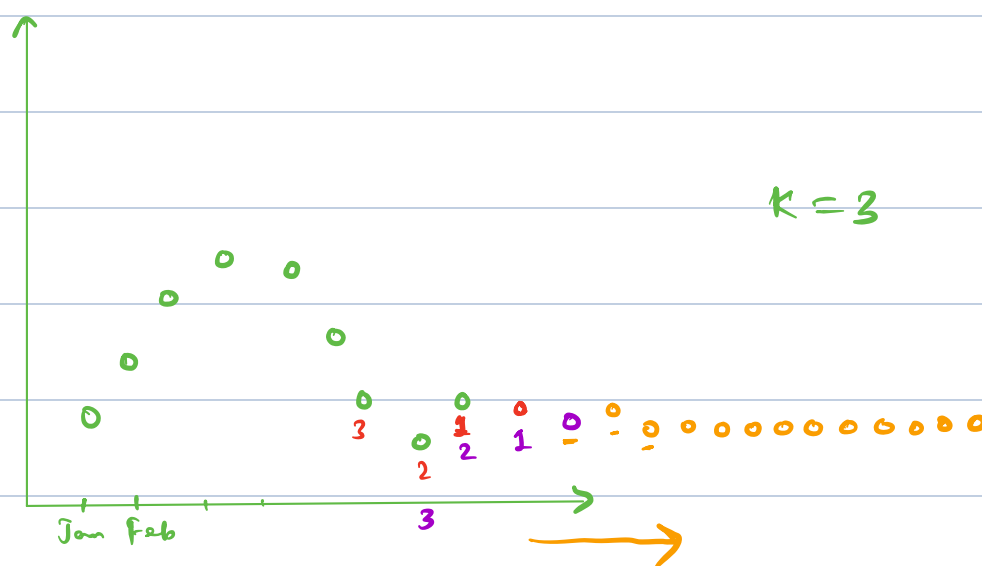
③ DES - Double " " (Holt's Method)

④ TES - Triple " " (Holt-Winter's Method)

Smoothing techniques are methods that:

- ① reduce short-term fluctuations
- ② highlight long-term patterns
- ③ prepare data for forecasting

① MOVING AVERAGE → rolling()



Sales

[k=3]

Original Data {
Jan 2001
Feb 2001
⋮
Jan 2019

Feb 2019 NaN

Mar 2019 NaN

Apr 2019 NaN

pred = [Nov 2018, Dec 2018, Jan 2019]

= [100, 200, 300]

↓

= [100, 200, 300, 200]

(1st)

Conclusion: Forecast is certainly better than simple methods like mean/median. However, it is not able to predict the variations and it gives a "flat line after few predictions".

MAPE : 9.4 %

② Simple Exponential Smoothing (SES) - [only level]

→ Instead of ignoring all past values → give lesser weights
recent values → give more weights

Let's consider the weight we assign to most recent value is ' α '

α → Smoothing parameter

$$0 \leq \alpha \leq 1$$

$$\text{New Smoothed Value} = (\alpha * \text{Current Value}) + (1 - \alpha) * \text{Previous Smoothed Value}$$

1st $\rightarrow S_t = \alpha Y_t + (1-\alpha) S_{t-1}$ \nearrow Previously smoothed value

\hookrightarrow Forecasted Smoothed value at t

$$S_{t-1} = \alpha Y_{t-1} + (1-\alpha) S_{t-2}$$

Substitute S_{t-1} in 1st equation.

$$S_t = \alpha \cdot Y_t + (1-\alpha) [\alpha Y_{t-1} + (1-\alpha) S_{t-2}]$$

$$S_t = \alpha \cdot Y_t + \alpha (1-\alpha) Y_{t-1} + \alpha (1-\alpha)^2 Y_{t-2} +$$

.....

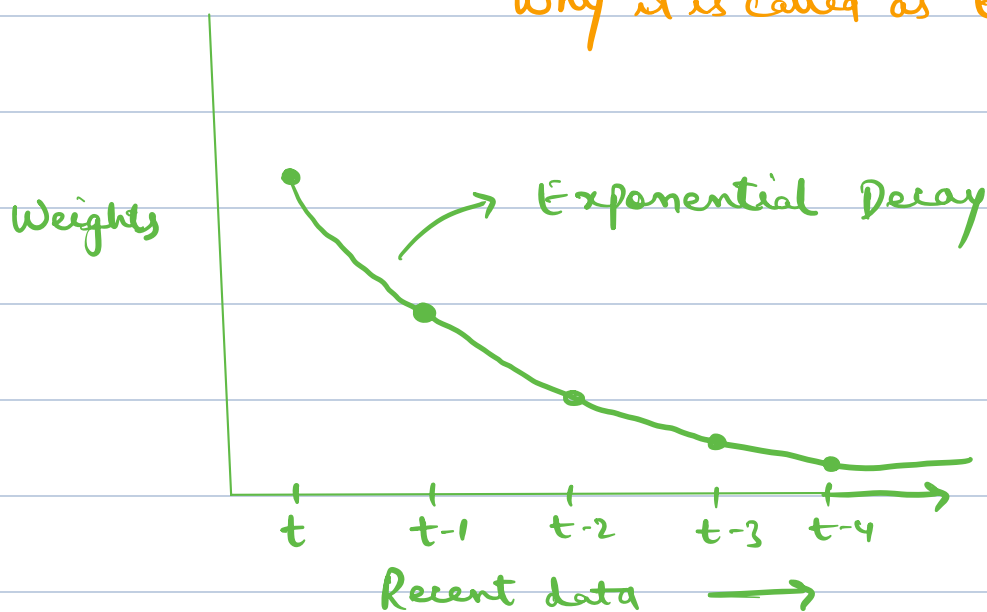
Weights \rightarrow 1st recent $\rightarrow \alpha$
2nd $\rightarrow \alpha(1-\alpha)$
3rd $\rightarrow \alpha(1-\alpha)^2$

$$\alpha = 0.8$$

$$2^{\text{nd}} \text{ weight} \rightarrow 0.16$$

$$3^{\text{rd}} \text{ weight} \rightarrow 0.032$$

Why it is called as Exponential Smoothing



- Weights decrease exponentially
- Never becomes exactly zero

$$\text{Sum of weights} \rightarrow \sum_{k=0}^{\infty} \alpha(1-\alpha)^k = 1 \quad [\text{Proper weighted average}]$$

Smoothing-level $\rightarrow \alpha$

\hookrightarrow Recommended starting value is equal to

$$\frac{1}{2 \times \text{Seasonality}} \quad [m=12]$$

SES does not consider any:

- ① Trend
- ② Seasonality

SES models only level, not change.

$$\text{MAPE} = 11.3\%$$

Conclusion: Looks even worse than MA forecast.

Gives a straight line.

Gives the right level ✓

③ DES \rightarrow Holt's Method (level + trend)

Shortcomings of SES:

- ① doesn't capture trend / seasonality
- ② gives one unique value (flat line)

2 components:

① Level

② Trend

① Level

$$L_t = \alpha * y_t + (1-\alpha) * (L_{t-1} + T_{t-1})$$

α = smoothing cons. (for level)

$$0 \leq \alpha \leq 1$$

② Trend

$$T_t = \beta (L_t - L_{t-1}) + (1-\beta) * T_{t-1}$$

β = smoothing cons. (for Trend)

$$0 \leq \beta \leq 1$$

③ Final Equation:

$$\hat{y}_{t+h} = L_t + h * T_t$$

h = horizon $\rightarrow 1, 2, 3, 4, \dots$

Y_t Today's sales = 120

Y_{t-1} Yesterday's sales = 110

T_{t-1} Yesterday's trend = 5

$$\hat{Y}_{t+1} = ?$$

$$\alpha = 0.5$$

$$\beta = 0.3$$

DES = SES + trend tracking

Seasonality \times MAPE $\rightarrow 8.9\%$

④ TES Method (Holt - Winter's method)

(level + trend + seasonality)

TES = DES + seasonality tracking

$$\hat{Y}_{t+h} = l_t + h \times T_t + S_{t+h-m}$$

$m \rightarrow$ frequency of seasonality

quarterly data $\rightarrow m = 4$

monthly data $\rightarrow m = 12$

MAPE $\rightarrow 5.1\%$ (odd)

$$\text{MAPE} \rightarrow 4.6\% \text{ (med)}$$

Conclusion: Perform best compared to all other methods.

$$\text{Level} \rightarrow \lambda_t = \alpha (\gamma_t - s_{t-m}) + (1-\alpha) (\lambda_{t-1} + T_{t-1})$$

$$\text{Trend} \rightarrow T_t = \beta (\lambda_t - \lambda_{t-1}) + (1-\beta) T_{t-1}$$

$$\text{Seasonality} \rightarrow s_t = \gamma (\gamma_t - \lambda_{t-1} - T_{t-1}) + (1-\gamma) s_{t-m}$$

γ = smoothing cons. (for seasonality)

→ Stationarity

→ ARIMA