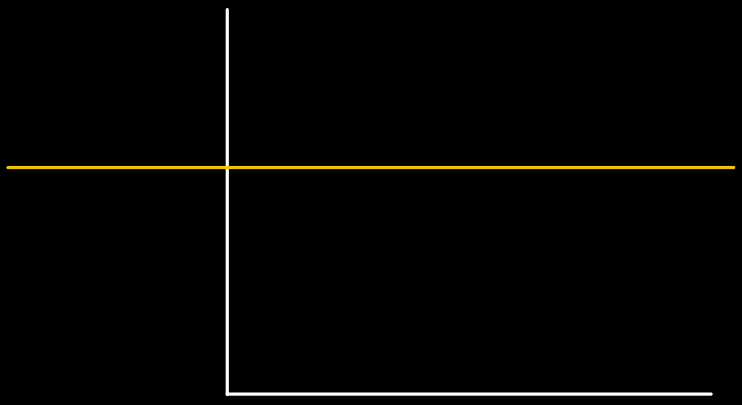
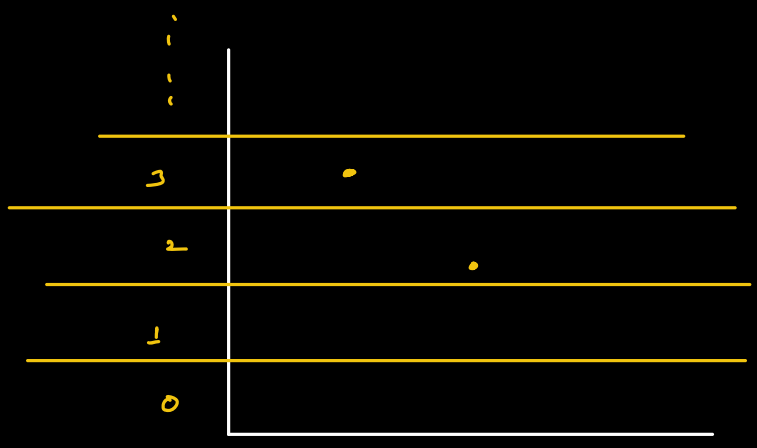
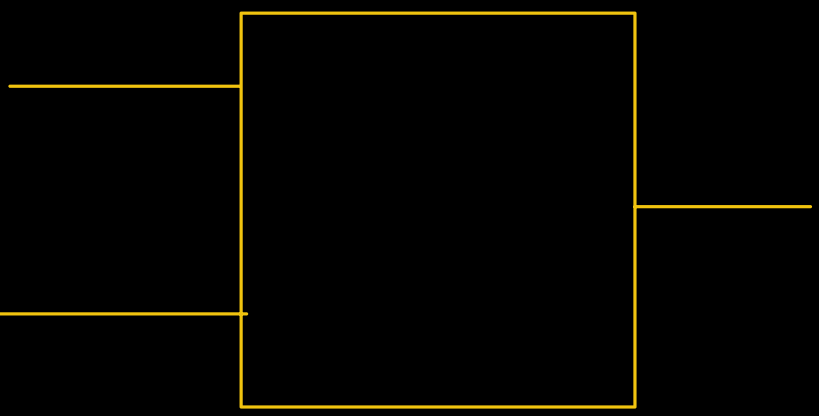
Bit Manipulation





O(1)

- a+b
- a-b
- a×b
- a÷b
- a%b
- a && b
- a || b



$$\begin{array}{r} {}^{1}\,{}^{1}\,{}^{1} \\ 9\,9\,9 \\ +\quad\quad 1 \\ \hline 1\,0\,0\,0 \end{array}$$

$(25)_{10}$

2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0	

$(11001)_2$

$(37)_{10}$

2	37	

$(100101)_2$

$(1010)_2 \longrightarrow 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 \Rightarrow (10)_{10}$

$(1010)_2$ (3 2 1 0)

```
    1  1  0  1  1
+   1  0  0  1  1
-------------------
```

```
6/10  13/10  10/10  16/10
  0    1      1      1
     3  4  5  9
+    2  8  4  7
-------------------
     6  13 10 16
     6  3  0  6
     ↑  ↑  ↑  ↑
  6%10 13%10 10%10 16%10
```

```
                3/2  2/2
  1  0  0  1  1
     1  1  0  1  1
+    1  0  0  1  1
-------------------
     2  1  1  3  2
  1  0  1  1  1  0
           ↑  ↑
        3%2  2%2
```

(&, |, ^, ~, >>, <<)

a	b	a & b	a | b	a ^ b
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

a = 4 b = 3

a & b ⟶ 0

```
  1 0 0
& 0 1 1
-------
  0 0 0
```

a | b ⟶ 7

```
  1 0 0
| 0 1 1
-------
  1 1 1
```

a ^ b ⟶ 7

```
  1 0 0
^ 0 1 1
-------
  1 1 1
```

10 | 1 ⟶ 11

```
  1 0 1 0
| 0 0 0 1
---------
  1 0 1 1
```

1 | 0 → 1
0 | 0 → 0

11 | 1 ⟶ 11

```
  1 0 1 1
| 0 0 0 1
---------
  1 0 1 1
```

A | 1 ⟶ A+1 (if A is even)
A | 1 ⟶ A (if A is odd)

A | 0 ⟶ A

```
  1011001 1
& 0000000 1
-----------
  0000000 1
```

$A \& 0 \longrightarrow 0$

```
  1011001 0
& 0000000 1
-----------
  0000000 0
```

$A \& 1$ → 0 (if A is even)

$A \& 1$ → 1 (if A is odd)

$0 \wedge 0 \rightarrow 0$

$1 \wedge 0 \rightarrow 1$

```
  1011001 1
^ 0000000 0
-----------
  1011001 1
```

$A \wedge 0 \longrightarrow A$

```
  1011001 1
^ 0000000 1
-----------
  1011001 0
```

$A \wedge 1$ → $A+1$ (if A is even)

$A \wedge 1$ → $A-1$ (if A is odd)

```
  1011001 0
^ 0000000 1
-----------
  1011001 1
```

```
  1011001 0
^ 1011001 0
-----------
  0000000 0
```

$$A \wedge A \longrightarrow 0$$

$$a \,\&\, b = b \,\&\, a$$

$$a \mid b = b \mid a$$

$$a \wedge b = b \wedge a$$

$$a \wedge b \wedge c = (a \wedge b) \wedge c = a \wedge (b \wedge c) = (a \wedge c) \wedge b$$

Q Given an array where all no. are present twice except one.
Find the single no.

2, 8, 3, 1, 2, 3, 5, 8, 1

$5 \wedge 5 \rightarrow 0$

$5 \wedge 5 \wedge 4 \rightarrow 4$

$5 \wedge 4 \wedge 5 \rightarrow 4$

$2 \wedge 5 \wedge 4 \wedge 5 \wedge 2 \rightarrow 4$

$2 \wedge 2 \wedge 5 \wedge 5 \wedge 4 \rightarrow 4$

```
ans = 0;
for (i=0; i<N; i++){
        ans = ans ^ A[i];
}
ret ans;
```

TC: O(N)
SC: O(1)

Q Given an array where all no. are present twice except two no.
Find both the single no.

2, 8, 3, 1, 2, 3, 5, 8, 1, 7

Left Shift (<<)

8 bit

x: 1 0 0 0 0 0 0 0 1 — x = x<<1

x: 2 0 0 0 0 0 0 1 0 — x = x<<1

x: 4 0 0 0 0 0 1 0 0 — x = x<<1

x: 8 0 0 0 0 1 0 0 0 — x = x<<1

x: 16 0 0 0 1 0 0 0 0 — x = x<<1

x: 32 0 0 1 0 0 0 0 0 — x = x<<1

x: 64 0 1 0 0 0 0 0 0 — x = x<<1

x: 128 1 0 0 0 0 0 0 0 — x = x<<1

x: ~~256~~ 0 0 0 0 0 0 0 0

0 0 0 0 1

1<<5 → 32 1 0 0 0 0 0 ⇒ 32

1<<10 → 1024

$$1 << n \longrightarrow 2^n$$

x: $(10)_{10}$

10: 1 0 1 0 — x = x<<1

20: 1 0 1 0 0 — x = x<<1

40: 1 0 1 0 0 0 — x = x<<1

80:

$$a << n \longrightarrow a \times 2^n$$

Right Shift

$x = 128$

128	1	0	0	0	0	0	0	0
64	0	1	0	0	0	0	0	0
32	0	0	1	0	0	0	0	0
16								
8								
4								
2								
1								
0								

$x = x >> 1$

$x = x >> 1$

$a >> n \longrightarrow a/2^n$

$1001 \rightarrow 9$

$100 \rightarrow 4$

$10 \rightarrow 2$

$50 >> 2$

$= 50/2^2$

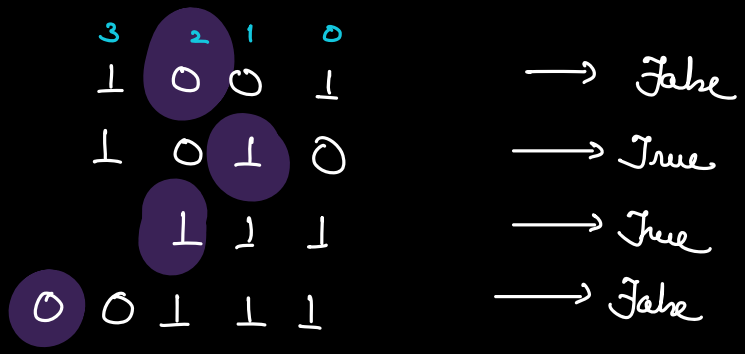
$= 12$

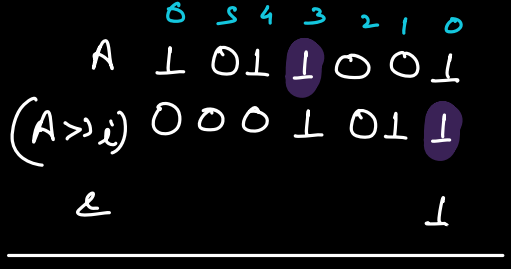
Set 1

unset 0

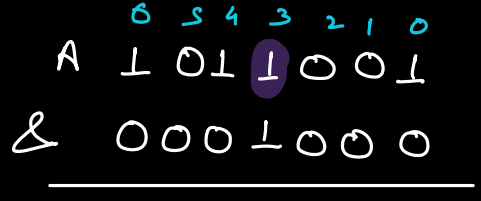
Q Write a fn to check if ith bit is set

A	i
9 ,	2
10 ,	1
7 ,	2
7 ,	4






```
boolean checkBit (int A, int i){
    ret ((A>>i) & 1) == 1;
}
```




```
boolean checkBit (int A, int i){
    ret ((1<<i) & A) > 0;
}
```

Q. Given a no. Set the ith bit

A	i
10	2
10	3

3 2 1 0

1 0 1 0 ⟶ 1 1 1 0 ⇒ 14

1 0 1 0 ⟶ 1 0 1 0 ⇒ 10

3 2 1 0

1 0 1 0

1 0 1 0 0

```
int setBit (int A, int i){
    ret A | (1<<i);
}
```